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Final Scientific Report

by

Elmor L. Peterson

(October 1, 1976 through March 31, 1979)

GENERALIZED GEOMETRIC PROGRAMMING
WITH APPLICATIONS

During the grant period my research efforts have been concentrated in five major directions: (1) the effective formulation and study of two important optimization problems and two important equilibrium problems (as generalized geometric programming problems), (2) an investigation of the relations between suboptimization and parameter deletion, including the relations between ordinary duality, geometric duality, and Rockafellar duality, (3) an investigation of the relations between the fixed point problem and the geometric complementarity problem (a generalization of the ordinary complementarity problem), (4) an extension of the classical existence theorems for both the fixed point problem and the variational inequality problem, to deal with the geometric complementarity problem, (5) the preparation of a book that unifies and contrasts ordinary programming theory, geometric programming theory, parametric programming theory, ordinary complementarity theory, geometric complementarity theory, fixed point theory, and variational inequality theory; while showing how each of these theories supplies different insights into various important optimization problems and equilibrium problems.

The two important optimization problems and the two important equilibrium problems alluded to in (1) are: (i) generalized Fermat problems that

arise in the optimal location of facilities, (ii) discrete optimal control problems that have linear dynamics and arise in many different contexts, (iii) economic equilibrium problems in which spatial dependencies are accounted for, (iv) traffic equilibrium problems in which multimodal phenomena and intersection phenomena are accounted for. The first draft of a comprehensive treatment [11] of problem (i) is nearing completion; and although a treatment of problem (ii) has been accepted for publication as [2], it is still undergoing extensive revisions prior to its appearance. The treatment of problem (iii) originally described in [8] has been expanded from a "partial equilibrium model" to a "complete equilibrium model" and will be published as [9]. Moreover, an algorithmic treatment of the special affine case has been accepted for publication as [1]. Problem (iv) has recently been studied via the methods alluded to in (4), but a publication describing the results has not yet been drafted.

The relations between suboptimization and parameter deletion mentioned in (2) have been published as [5]. Moreover, the relations between ordinary duality and geometric duality have been accepted for publication as [6], while the relations between geometric duality and Rockafellar duality have been accepted for publication as [7].

The relations between the fixed point problem and the geometric complementarity problem referred to in (3) have been submitted for publication as [4]. In addition to their significance for geometric programming, they unify the Eaves and Kojima fixed point representations of the ordinary complementarity problem.

The extension of the classical existence theorems for both the fixed point problem and the variational inequality problem mentioned in (4) was

necessary because the existence theorems of Brouwer-Kakutani-Eilenberg-Montgomery, Leray-Schauder, and Hartman-Stampacchi-Saigal require boundedness of both the mapping's domain and its individual image sets--hypotheses that are too strong to deal effectively with the geometric complementarity problem because of its generally conical-like domain and the possible unboundedness of its image sets at certain domain boundary points. The results obtained so far extend such existence theorems by the introduction of hypotheses that essentially compactify the unbounded domain and eliminate some of its troublesome boundary points via coercivity conditions tailored to the specific nature of the geometric complementarity problem. Even in the special case of the ordinary complementarity problem (whose domain is the non-negative orthant), the results are stronger than the corresponding results previously obtained by More', Saigal and others. In the more general case, the results provide new existence theorems for the equilibrium problems (iii) and (iv) mentioned as part of (1). They also provide new existence theorems for certain non-monotone electrical network problems that might conceivably arise with the use of some non-monotone electronic devices. Although all of these results will soon appear in the form of a Ph.D. thesis [3] by Shu-Cherng Fang (a student who has received some financial support from the grant during the past year), they will eventually be published as several different papers by Mr. Fang and the author. The book [10] referred to in (5) will probably require at least two more years for completion. It will present a very comprehensive treatment of the listed topics; and close to one half of it now exists in the form of lecture notes. Some of the completed papers in the bibliography have been issued and

circulated in the form of AFOSR scientific reports or journal reprints.
The uncompleted papers will eventually be handled in a similar fashion.

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Bibliography

1. Asmuth, R., B.C. Eaves, and E.L. Peterson, "Studying Economic Equilibria on Affine Networks via Lemke's Algorithm", accepted for publication in Math. Oper. Res.
2. Dinkel, J.J., and E.L. Peterson, "A Duality Theory for Discrete Optimal Control via Geometric Programming", being revised for publication in Jour. Math. Anal. Appls.
3. Fang, S., "Fixed Points, Variational Inequalities, and Generalized Complementarity: Theory and Applications in Geometric Programming," Ph.D. thesis in Operations Research, Northwestern University, 1979.
4. Fang, S., and E.L. Peterson, "A Unification and Generalization of the Eaves and Kojima Fixed Point Representations of the Complementarity Problem", submitted to Math. Prog.
5. Peterson, E.L., "The Duality between Suboptimization and Parameter Deletion", Math. Oper. Res. 2, 4, 1977, 311-319.
6. _____, "Ordinary Duality vis-a-vis Geometric Duality", accepted for publication in Jour. Opt. Th. Appls.
7. _____, "Geometric Duality via Rockafellar Duality", accepted for publication in Jour. Opt. Th. Appls.
8. _____, "The Conical Duality and Complementarity of Price and Quantity for Multicommodity Spatial and Temporal Network Allocation Problems", Discussion Paper No. 207, Ctr. Math. Stud. Econ. & Mgt. Sci., Northwestern University, 1976.

9. _____, "Economic Equilibria between Spatially Separated Markets", to appear in Proc. Carnegie-Mellon Symp. on Appl. Math.
10. _____, The Mathematical Foundations of Linear and Non-linear Programming, lecture notes being expanded into a book.
11. Peterson, E.L, and R.E. Wendell, "Optimal Location via Geometric Programming", in preparation.