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Accelerated life testing of a product is commonly used to reduce test time and costs. Accelerated testing is achieved by subjecting the test units to conditions that are more severe conditions are then extrapolated to normal conditions to obtain an estimate of life distribution under normal condition. This problem is considered when the product is a 2-component system with lifetimes following the bivariate exponential distribution of Block and Basu (1974). Here our interest is in the joint analysis of data with two failure times.

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FINAL TECHNICAL REPORT  
TO THE AIR FORCE OFFICE OF  
SCIENTIFIC RESEARCH

(FOR THE PERIOD APRIL 1, 1987 - MAY 31, 1989)

For the research Program on  
STATISTICAL THEORY OF RELIABILITY  
(AIR FORCE GRANT NO. AFOSR-87-0139)

BY THE

Department of Statistics  
University of Missouri-Columbia

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Final Technical Report on Air Force Grant No. AFOSR-87-0139 on Statistical Theory Reliability.

The project started on April 1, 1987 and was completed on May 31, 1989. During this period the Principal Investigator, A.P. Basu, along with his collaborators (most of which were not supported by AFOSR) considered a number of problems. We describe them below briefly. Lists of technical reports and publications prepared under the project are given below along with their abstracts.

A. RESEARCH PAPERS AND OTHER PUBLICATIONS COMPLETED

1. On a Bivariate Accelerated Life Test. (by A.P. Basu and N. Ebrahimi). Published in Journal of Statistical Planning and Inference. Vol. 16 (1987) pages 297 - 304.

Abstract:

Accelerated life testing of a product is commonly used to reduce test time and costs. Accelerated testing is achieved by subjecting the test units to conditions that are more severe than the normal ones. The results obtained at the more severe conditions are then extrapolated to normal conditions to obtain an estimate of life distribution under normal condition. This problem is considered when the product is a 2-component system with lifetimes following the bivariate exponential distribution of Block and Basu (1974). Here our interest is in the joint analysis of data with two failure times.

2. On Estimating Change Point in a Failure Rate. (by A.P. Basu, J.K. Ghosh and S.N. Joshi). University of Missouri Technical Report No. 137 (June, 1987). Published in Proceedings of the Fourth Purdue Symposium on Decision Theory and Related Topics, Volume 2, Edited by S.S. Gupta and J.O. Berger p. 239 - 252.

Abstract:

Let  $F$  be a life ~~distribution~~ function (d.f.) with density  $f$  and failure rate  $r$ . It is assumed that  $f$  is the first part of a "bath-tub" model, that is,  $r(t)$  is non-increasing for  $t \leq 1$  and is constant for  $t > 1$ . In this paper the problem of estimating the change point or threshold has been considered. Two estimates for  $1$  have been proposed and their consistency have been proved.

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Nguyen, Rogers and Walker (1984) considered a specific parametric case where, with  $I(A)$  denoting the indicator function of  $A$ ,  $r(t) = a I(0 \leq t \leq 1) + b I(t > 1)$ , and proposed a consistent estimate. We have obtained the asymptotic distribution of their estimate using a new method which may have applications to other problems. We also propose a maximum likelihood estimate restricted to lie in a suitable compact set.

We report some simulations comparing the performance of these estimates.

3. A Test for Bivariate Exponentiality Against BIFRA Alternative. (by A.P. Basu and M. Habibullah). University of Missouri Technical Report No. 138 (July, 1987). Published in Calcutta Statistical Association Bulletin, Vol. 36 (1987) p. 79-84.

Abstract:

We present a statistical procedure to test whether a life distribution belongs to a class of bivariate exponential distributions with exponential minimum against the alternative that it is bivariate increasing failure rate average (BIFRA). The proposed test is applied to some actual data.

4. Reliability Growth Estimation with Missing Data (by A.P. Basu and Larry Crow). Proceedings of Reliability and Maintainability Symposium (1988). p. 248-253.

Abstract:

This paper considers a real life problem associated with data collection and analysis during a reliability growth program. The published reliability growth methods typically address the situation where the failure data are assumed to be complete. An attempt to use those methods for the situation discussed in this paper may result in an incorrect analysis or an analysis not utilizing all the available test data. This paper provides a practical methodology for obtaining statistically valid estimates of the growth rate and current MTBF which maximize the use of all available data.

5. Probabilistic Reliability (1987). Invited paper. Encyclopedia of Statistical Science, Vol. 8 p. 24-29.

Abstract:

Here a survey of reliability theory from probability point of view is considered. A number of view is considered. A number of useful concepts are considered and distributions with aging concepts are described.

6. Multivariate Harmonic New Better Than Used in Expectation Distributions. (by A.P. Basu and N. Ebrahimi). Journal of Statistical Planning and Inference. Vol. 20 p. 181-190.

Abstract:

The definition of multivariate harmonic new better than used in expectation (MHNBU) life distributions is introduced. The definition is multivariate generalization of the useful univariate aging property, harmonic new better than used in expectation (HNBUE), which is weaker than new better than used in expectation (NBUE). Various closure properties are proved. Unlike the previous extensions we also prove the very important result of closure under convolution. Some examples are given to illustrate the relationship between the proposed MHNBU and MHNBU properties by Basu, Ebrahimi, and Klefsjo (1983). We also study the dual property of multivariate harmonic new worse than used in expectation (MHNWUE).

7. Multivariate Exponential Distributions and their Applications in Reliability. Handbook of Statistics Vol. 7 (1988), p. 467-477.

Abstract:

In this paper we survey multivariate exponential distributions useful in reliability theory. Necessary inference problems are also considered.

8. Estimating the Intensity function of the Weibull process at the current time: Failure truncated case (with S. Rigdon). Journal of Statistical Computation and Simulation. Vol. 30, p. 17-38.

Abstract:

The Weibull process, a nonhomogeneous Poisson process with intensity function  $u(t) = (\beta/\theta)(t/\theta)^{\beta-1}$ , is often used to model the failure times of complex systems which are repaired after failure. Point estimation of the value of the intensity function at the current time is frequently an important problem and in this paper, two classes of estimators of this quantity are proposed when the data are failure truncated. Several members of these classes are suggested as estimators. Expressions for the bias and the mean squared error of these estimators are derived and are evaluated for several values of  $\beta$  and for sample sizes of 5, 10, 20 and 40. Some estimators have smaller mean squared error than the conditional MLE for a wide range of the parameters.

9. Accelerated Life Tests: Theory and Applications. (by Basu, A.P.). Reliability Key to Industrial Success ed. by Sundaresan and Kececilogly (1987) ASM Proceedings, p. 153-156.

Abstract:

The problem of accelerated life testing is described and a survey of some recent results are presented. The cases when several failure modes are present are described in detail with examples. Both dependent and independent causes of failure are considered.

10. A Two-Stage Test for Exponentiality Against IFRA Alternatives. (by Alam, M.S. and Basu, A.P.). Recent developments in Statistics and Their Applications (1989). ed by J. Klein and J. Leo, p. 69-86.

Abstract:

A two stage test is proposed for testing the hypothesis  $H_0$  : F is exponential versus  $H_1$  : F is IFRA and not exponential, on the basis of a random sample from F. To compare the performance of the two stage test with the corresponding one sample test, powers and expected sample sizes were computed by simulation for various alternatives. It is shown that, for the two stage test with approximately same power, expected sample size is considerably smaller. Critical values are tabulated to permit application of the test.

11. Bayesian Approach to Life Testing and Reliability Estimation Using Asymmetric Loss Function. (by Basu, A.P. and Ebrahimi, Nader)

Abstract:

In this paper the problem of estimating mean lifetime and reliability function has been considered using asymmetric loss functions. Exponential distribution has been considered as a model. A number of prior distributions have been considered and Bayesian estimates have been compared with corresponding estimates with squared error loss function.

12. Two Stage Testing Whether New is Better Than Used. (by Alam, M.S. and Basu, A.P.)

Abstract:

A two stage test is proposed for testing the hypothesis  $H_0$  : F is exponential versus  $H_1$  : F is NBU and not exponential, on the basis of a random sample from F. To compare the performance of the two stage test with the corresponding one sample test, powers and expected sample sizes are computed by simulation for various alternatives. It is shown that, for the two stage test with approximately same power, expected sample size is considerably smaller. Critical values are tabulated to permit application of the test. Finally, an illustrative example is given.

B. OTHER INVITED PUBLICATIONS AND CREATIVE ACTIVITIES  
COMPLETED UNDER AFOSR GRANT

Invited comment on the paper "Survey of Soviet work in Reliability" by Rukhin & Hsieh (1987) Statistical Science. Vol. 2, 502-503.

Book Review. An Introduction to Statistical Quality Control by D. Montgomery. In Journal of the American Statistical Association (1987). Vol. 82, p. 699.

Book Review Reliability and Life Testing, by S.K. Sinha in technometrics (1988) p. 131.

Invited paper presented at the International Conference on Recent developments of Statistical methods, at Seoul, Korea, August 1987.

Invited paper presented at the Symposium on dependence in Statistics and Probability at Somerset, Pennsylvania, 1987.

Presented paper at the 46th Session of the International Statistical Institute at Tokyo, Japan, 1987.

Invited paper presented at the Indo-U.S. conference in Bayesian statistics in Bangalore, India (1988).

C. CONCLUDING REMARKS

During this project a number of important problems have been considered. The present project raised a number of important problems that are currently being studied under grant no. AFOSR 89-0406.