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The Effects of Wind Tunnel Data Uncertainty on Aircraft Point Performance Predictions

J. D. Cloyd
Calspan Corporation/AEDC Operations

December 1989

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A study was conducted to determine the effects of uncertainties in wind tunnel test data on the point performance predictions for a typical fighter at medium altitude and subsonic velocity. The equations of motion of selected parameters were formulated in terms of the aerodynamic coefficients and thrust model. A computer code was written to calculate the performance parameters from the aerodynamic and thrust model inputs. An aerodynamic model closely approaching a typical fighter flying at 30,000 ft mean sea level was developed from wind tunnel, flight test, and flight manual data. The effects of the uncertainties in the aerodynamic coefficients on the performance parameters were determined by varying the aerodynamic coefficients input to the computer code.

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PREFACE

The work reported herein was conducted at Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Base, Tennessee, and sponsored by the Director of Technology (DOT), AEDC. The work was accomplished by Calspan Corporation/AEDC Operations, contract operator of the aerospace flight dynamics test facilities at the AEDC, under Project No. DC07PW. The Calspan project engineer was Mr. Daryl Sinclair. The report was submitted for publication on September 25, 1989.

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1.0 INTRODUCTION

Considerable attention has been focused on establishing the data accuracy requirements associated with wind tunnel testing. In Ref. 1, Steinle and Stanewsky suggest coefficient accuracy requirements of 0.01 for ΔC_L and 0.0001 for ΔC_D. The accuracy requirements for extrapolation of wind tunnel data to the flight condition were specifically addressed by AGARD Working Group 09 on boundary-layer simulation and control. For fighter aircraft the Group's conclusions were stated by Haines (Ref. 2) as a ΔC_L of 0.02 for buffet onset, a ΔC_D of 0.0002 for cruise and loiter, and a ΔC_D of 0.0005 for maneuver and high-speed dash. These goals serve as the impetus for continued improvements in testing equipment and methodology.

In contrast to the establishment of test data accuracy goals, very little has been published detailing the coefficient errors associated with testing in a particular wind tunnel. This is partly attributable to difficulty in establishing the relative magnitude of the errors associated with the multitude of sources (balance, humidity, wall interference, etc.), as the errors may vary considerably with configuration and flow field from test to test. Whoric and Hobbs (Ref. 3) found the total uncertainties for the aerodynamic coefficients in the Arnold Engineering Development Center (AEDC) Aerodynamic Wind Tunnel (4T) and Propulsion Wind Tunnel (16T) to be significantly outside the Ref. 1 and 2 requirements. Reference 3 establishes a ΔC_L range from 0.012 to 0.035 (3- to 16-percent nominal lift), and a ΔC_D range from 0.002 to 0.004 (3- to 14- percent nominal drag) for typical fighter and transport configurations.

The objective of this study was to determine the effects of uncertainties in wind tunnel test data on the point performance predictions for a typical fighter at medium altitude and subsonic velocity. A five-step process was used:

1. Specific point performance parameters were chosen which encompassed the spectrum of capabilities of a typical fighter at medium altitude,

2. The equations of the selected parameters were formulated in terms of the aerodynamic coefficients and thrust model,

3. A computer program was written which calculates the selected performance parameters from the input aerodynamic and thrust models,

4. An aerodynamic model which closely approximates a typical fighter at 30,000 ft mean sea level (MSL) was developed from a combination of wind tunnel, flight test, and flight manual data, and

5. The aerodynamic model was input into the computer program, varying the coefficients of lift and drag from −20 to +20 percent, with the percentage error of the selected performance parameters as a function of percentage error of the coefficients of lift and drag noted.

2.0 POINT PERFORMANCE PARAMETERS

2.1 DEFINITIONS AND ASSUMPTIONS

Point performance is defined as the aircraft performance at a given weight, altitude, and throttle setting. The equations for the parameters of interest are derived from the basic equations of motion in the horizontal and vertical planes. Throughout the determination of the point performance parameters the following assumptions are employed:

1. The aircraft is considered as a point mass,
2. The thrust vector incidence angle is zero,
3. Thrust acts down the chord line, and
4. Atmospheric properties are known functions of altitude.

The point performance equations and parameters have been extensively treated in the technical literature on aircraft design and performance. The real value of the point performance technique is that it reduces a problem with a large number of parameters to reasonably simple expressions. Although the assumptions made will lead to some degree of inaccuracy, the results are no less meaningful as they quantify the relationship between the primary parameters with reasonable accuracy.

Detailed derivations of the basic relations and point performance parameter equations are given in Ref. 4.

2.2 BASIC RELATIONS

The basic relations in the vertical plane are derived from the free-body diagram as shown in Fig. 1. Equating forces parallel and normal to the flight path yields the general equations in the vertical plane:

$$T \cos \alpha - D - \frac{W_o r_e^2 \sin \gamma}{(r_e + h)^2} = \frac{W_o}{g_o} \frac{dV}{dt} \tag{1}$$

$$T \sin \alpha + L - \frac{W_o r_e^2 \cos \gamma}{(r_e + h)^2} = \frac{W_o}{g_o} \left[V \frac{d\gamma}{dt} - \frac{V^2 \cos \gamma}{(r_e + h)} \right] \tag{2}$$

The basic relations in the horizontal plane are derived from the free-body diagram shown in Fig. 2. The turn is considered to be level, steady, and coordinated (zero side forces). The load factor (n) is defined by:

$$n = \frac{L}{W} = \frac{1}{\cos\phi} \tag{3}$$

The basic equation for the turn radius is:

$$r_t = \frac{V^2}{g\sqrt{n^2 - 1}} \tag{4}$$

The basic relation for the turn rate is:

$$\dot{\Psi} = \frac{g\sqrt{n^2 - 1}}{V} \tag{5}$$

2.3 ENERGY STATE APPROXIMATION

Assuming rotation to be zero, the aircraft's total energy at a specified altitude and velocity is:

$$E = Wh + \frac{WV^2}{2g} \tag{6}$$

Dividing through by weight and differentiating with respect to time gives the expression for the time rate of change of specific energy (referred to as specific excess power) with units of feet per second.

$$P_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} \tag{7}$$

The P_s value represents the aircraft's capability to accelerate and/or climb. In level flight ($dh/dt = 0$) Ps may be calculated by substituting for dV/dt in Eq. (1), multiplying through by V/W, and rearranging to yield:

$$P_s = \frac{V(T\cos\alpha - D)}{W} \tag{8}$$

2.4 POINT PERFORMANCE PARAMETERS

To determine the point performance parameters from the basic relations the following aircraft aerodynamic and thrust data at the desired altitude are assumed known:

1. The coefficients of lift and drag as a function of Mach number and angle of attack,

2. The net thrust at the desired throttle setting as a function of the Mach number,

3. The thrust specific fuel consumption as a function of the Mach number and the throttle setting, and

4. The basic aircraft data (sea-level weight, reference area, etc).

The point performance methods in this chapter are a compilation of the techniques detailed in Refs. 5-11. The techniques in these references assume that the angle of attack is zero. With the coefficient of lift as a function of Mach number and angle of attack known from experimental data, this assumption may be eliminated. An expression for the level nonaccelerating flight coefficient of lift is derived from Eq. (2):

$$C_{L_{\text{lvl flt}}} = \frac{2\left\{W\left[1 - \frac{V^2}{g(r_e + h)}\right] - T \sin \alpha\right\}}{Y_s p M^2 S} \tag{9}$$

A level flight coefficient of lift and angle of attack at each velocity is determined by the intersection of the known coefficient of lift versus angle-of-attack curve at that velocity with the curve generated from Eq. (9).

The maximum horizontal flight speed is obtained from the intersection of the thrust available and thrust required curves. The net thrust available curve is assumed to be known. The thrust required at each velocity is obtained from the level flight coefficient of lift, Eq. (9), and the known drag polar.

The level flight horizontal acceleration (dV/dt) is calculated by equating the right-hand sides of Eq. (7) and Eq. (8) with $dh/dt = 0$. Multiplying through by V/g yields:

$$\frac{dV}{dt} = \frac{g(T \cos \alpha - D)}{W} \tag{10}$$

The aircraft's rate of climb is given by $dh/dt = V \sin \gamma$. Holding the velocity constant in Eq. (7) yields:

$$P_s = \frac{dh}{dt} = V \sin \gamma \tag{11}$$

therefore:

$$\sin\gamma = \frac{P_s}{V} \tag{12}$$

For $0 < \gamma < \pi/2$ the value of P_s/V which maximizes $\sin\gamma$ also maximizes $\tan\gamma$. Therefore, the maximum climb angle and the corresponding velocity are determined by the angle between the abscissa and a line through the origin tangent to the P_s versus velocity curve.

The maximum climb rate at constant velocity $(dh/dt)_{max}$ is obtained at P_{smax} on the P_s versus velocity curve.

The equation for the level flight stall Mach is obtained by setting $C_L = C_{Lmax}$ in Eq. (9) and solving for Mach number:

$$M_{Stall} = \left[\frac{2(W - T\sin\alpha)}{\gamma_s p S C_{L_{max}} + \dfrac{2Wa^2}{g(r_e + h)}}\right]^{1/2} \tag{13}$$

The level flight stall Mach number is the Mach number at which the angle of attack required for level flight is equal to the angle of attack which produces C_{Lmax}. M_{stall} is calculated as a function of Mach number, and the intersection of that curve with a straight line through the origin with a slope of one yields the level flight stall Mach number.

The maximum instantaneous turn rate and minimum turn radius are achieved at the lowest flight velocity where the structurally limiting load factor can be obtained. The weight in Eq. (13) is replaced by the product of the limiting load factor and the weight. An iterative technique is used to find the lowest flight velocity for which Eq. (13) is satisfied. The turn radius is then calculated from Eq. (4) and the turn rate from Eq. (5).

Specific range (R_s) is the distance traveled divided by the weight of fuel consumed:

$$R_s = \frac{dR}{dW_f} = \frac{dR}{dt}\,\frac{dt}{dW_f} = \frac{V}{dW_f/dt} \tag{14}$$

The fuel flow rate (dW_f/dt) can be expressed as the product of the engine thrust (T) and the thrust specific fuel consumption (c). Substituting into Eq. (14):

$$R_s = V/Tc \tag{15}$$

The maximum range Mach number $(M_{R_{max}})$ is obtained by plotting specific range versus Mach number as shown in Fig. 3.

Specific endurance (E_s) is the flight time divided by the weight of fuel consumed. Employing the same methods used in obtaining Eq. (15):

$$E_s = 1/Tc \tag{16}$$

The maximum endurance Mach number is also obtained from Fig. 3. Noting that tan θ = R_s/V = $1/Tc$ = Es, the maximum endurance Mach number ($M_{E_{max}}$) is obtained by constructing a line through the origin tangent to the R_s versus Mach number curve as shown.

2.5 POINT PERFORMANCE COMPUTER PROGRAM

A Fortran computer program (PERCAL) was written to calculate the point performance parameters using the techniques of this section and the known aerodynamic coefficients and thrust data. The program employs a cubic spline routine for interpolating and uses the coefficients of the cubic equation on the appropriate interval to algebraically evaluate derivatives, maximum values, and curve intersections. The program can perturb the input lift and drag coefficients singularly or in combination to permit observation of their effects on the performance parameters.

3.0 AIRCRAFT MODEL

3.1 LIFT COEFFICIENTS

The low angle of attack (AOA) coefficient of lift data are given in Table 1. These data are the measured coefficients taken on a 0.05-scale model of a fighter aircraft in the AEDC Tunnel 4T. The high AOA data are given in Table 2. The Table 2 data are the measured coefficients, corrected for cavity and base pressure and weight tares, taken on a 0.047-scale model of the same fighter in the AEDC Tunnel 16T. Both sets of data were used to obtain C_L data over a full range of AOA (Table 3) for input into the PERCAL computer program.

The Table 3 data compare favorably with both the high and low AOA reference data, cover the entire subsonic range, provide lift coefficients at small Mach number intervals, and are considerably smoother than the reference data. The latter two conditions prevent the generation of erroneous local maxima or minima by the computer program spline fit routine.

3.2 DRAG COEFFICIENTS

Coefficient of drag data were obtained from flight tests of the selected fighter. The data were obtained on a clean configuration at 30,000 ft MSL using maximum thrust accelerations and maximum thrust accelerating and decelerating turns. The flight test low C_L drag polar is shown in Fig. 4 and the high C_L drag polar is shown in Fig. 5.

Drag coefficients for input into the PERCAL computer program were obtained from Figs. 4 and 5 using the Table 3 lift coefficients. The resulting C_D model is given in Table 4.

3.3 THRUST AVAILABLE AND FUEL CONSUMPTION

The total net thrust available was obtained from flight test data and is shown in Fig. 6. The data used were for the selected fighter with engines operating at military power at 30,000 ft MSL. Specific range as a function of Mach number and aircraft gross weight was obtained from the selected fighter flight manual and is shown in Fig. 7.

Thrust specific fuel consumption (TSFC) as a function of thrust level and Mach number was calculated to model the Fig. 7 specific ranges. From Eq. (1) the thrust required at small Mach number intervals for a range of aircraft gross weights was found. The corresponding specific ranges for these weights and Mach numbers were obtained from Fig. 7. The TSFCs were then calculated from Eq. (15). The results are given in Table 5.

The Table 1 C_L model, Table 2 C_D model, Fig. 6 thrust model, and Table 3 thrust specific fuel consumption model comprise the aircraft model input into the PERCAL computer program. This is the baseline (zero perturbation) model used to calculate the 0-percent error case for each of the point performance parameters.

4.0 EFFECT OF COEFFICIENT ERRORS

4.1 PERFORMANCE PARAMETER PERCENT ERROR CALCULATION

The PERCAL program was used to generate a value for each point performance parameter for 121 cases. Each case corresponded to a particular combination of errors in coefficients of lift and drag; each coefficient was perturbed by 20, 10, 5, 2, 1, or 0 percent. The percent error in the point performance parameters for each case was calculated using:

$$\% \text{ Error} = \left(\frac{\text{PVALVE}}{\text{NVALUE}} - 1.0 \right) 100 \tag{17}$$

where PVALUE = the parameter value for the specific case, and
NVALUE = 0-percent error value.

A positive percent error corresponds to a parameter value greater than the 0-percent error case.

A percent error was calculated for each of the nine selected point performance parameters for each of the 121 combinations of errors in the coefficients of lift and drag. The resulting percent errors for the 121 cases were input into the AEDC Tekplot computer program for graphic analysis. The output plots (Figs. 8 - 22) represent the percent error on the ordinate versus the percent error in the coefficient of lift or drag on the abscissa. The individual curves are for a constant percent error of the coefficient not used on the abscissa. The dashed lines indicate the maximum C_L and C_D errors as established by Ref. 3.

4.2 PERFORMANCE PARAMETER ERROR ANALYSIS

The relationship between the percent error in a given point performance parameter and the percent error in C_L or C_D was found to be approximately linear through the -20- to $+20$-percent coefficient error range. This was as expected since the point performance parameters are based on first-order terms only. In the performance parameters that are functions of both C_L and C_D, the effect of errors in C_D have at least twice the effect of errors in C_L, except for maximum specific endurance where the effect of errors for both coefficients was approximately equal. Errors which increase C_L or decrease C_D enhance all performance parameters, while decreasing C_L or increasing C_D was detrimental to all performance parameters.

Of the performance parameters calculated, the maximum horizontal flight speed (Figs. 8 and 9) was found to be the least sensitive to errors in either C_L or C_D. This is attributable to the analysis being restricted to the military power setting, thereby giving the model insufficient thrust to overcome transonic drag.

The horizontal acceleration, maximum climb angle, and climb rate (Figs. 10 - 15) are all basically functions of specific excess power. The effects of errors in C_D for these parameters were two to four times the effect of errors in C_L. Each 1-percent error in C_D resulted in approximately a 1-percent error in the performance parameter.

Level flight stall speed and best instantaneous turn radius and rate (Figs. 16, 17, and 18) are functions of C_L only. Each 1-percent error in C_L yielded a 0.5- to 1.0-percent error in those parameters.

Maximum specific range and endurance (Figs. 19 - 22) are significantly affected by errors in both C_L and C_D. For specific range the effect of errors in C_D is twice the effect of errors in C_L while for specific endurance the effect of errors in C_L and C_D is approximately equal. Each 1-percent error in C_D produces approximately a 1-percent error in specific range and endurance.

Table 6 summarizes the results of the investigation. The approximate effects (in percent) on each of the selected performance parameters for each 1-percent error in the coefficients of drag and lift are given.

5.0 APPLICATIONS

5.1 ENHANCEMENT OF TEST RESULTS

The relative importance of the selected performance parameters is dependent on the aircraft/mission combination. An interceptor on a lane or point defense mission would require a good maximum climb rate (assuming a scramble launch) to intercept altitude, and good specific endurance to maintain time on station once it is established in an orbit. In a strictly air-to-air mode a premium is placed on acceleration to gain energy for maneuvering, and a maximum turn rate to position the nose for weapons deployment. A close air support fighter would emphasize specific endurance for time on station, and good turn rate and radius to position itself for weapons delivery and for defense against surface-to-air threats. Maximum specific range is desirable for fighters on deep interdiction strikes and for their air-to-air escorts.

Despite the relative simplicity and ease of calculation, the point performance analysis was found to yield quite accurate results. This method can be used to enhance the results obtained in wind tunnel testing through the following:

1. An analysis of this type should be performed prior to wind tunnel testing of a proposed aircraft. The design phase lift curve, drag polar, and thrust estimates should be sufficiently accurate to establish where testing inaccuracies would lead to misconceptions about the proposed design's performance. Knowing the areas where testing should be concentrated would improve test efficiency and reduce costs.

2. A computer program similar to PERCAL should be available either on-line during test data acquisition or easily accessible off line to rapidly evaluate the data being taken. If the data yield gross performance parameter deviations from those expected, further investigation into the data accuracy would be warranted.

3. An extension of the point performance techniques to path performance should be pursued. Path performance would have the advantage of identifying performance parameters and associated accuracies critical to the particular aircraft/mission combination. For example, the performance parameters and accuracies required for a given aircraft might be substantially different for a high-altitude intercept mission as opposed to a low-level surface attack mission. Path performance techniques used in conjunction with testing as suggested in parts 1 and 2 of this section would be valuable tools in aircraft design and design analysis.

5.2 CURVE FITTING OF EXPERIMENTAL DATA

When doing computer performance calculations from input data points, researchers must pay particular attention to the cubic spline curve fits. If experimental data are used without smoothing, the inherent scatter, though quite small, may generate false local maxima and minima. To obtain acceptable spline fits may require the concentration of data in regions where a cubic equation is not likely to yield a good fit. For example, in the lift curve the cubic spline must be closely monitored over the linear portion and the transition region from linear to nonlinear. It was found by trial and error that the spline routine used in the PERCAL program required coefficient data at intervals of 0.05 Mach or less to produce consistently reliable results. The interval at which coefficient data are required may well be a function of the particular type of spline fit used. A different spline routine (Akima, for example) may allow the data intervals to be larger. Regardless of the routine selected, the user must be aware of the characteristics of the type of spline employed.

6.0 SUMMARY

Previous studies have investigated the effects of wind tunnel measurement uncertainty on the aerodynamic force coefficients determined from the wind tunnel measurements. The present investigation considered the propagation of the error in the aerodynamic force coefficients on the prediction of aircraft point performance parameters from wind tunnel data.

The equations of motion were formulated in terms of the force and moment coefficients. A computer program was written to calculate selected performance parameters from the force and moment coefficients and the aircraft thrust models.

The effects on the aircraft performance parameters attributable to coefficients were determined for a typical fighter aircraft at 30,000 ft mean sea level. The aerodynamic model for the fighter aircraft was developed from flight test and flight manual data.

The coefficients of lift, drag, and thrust parameters input into the computer program produced a 0-percent error performance model that very closely approximated a medium-weight fighter at 30,000 ft MSL. The calculated best instantaneous turn performance is slightly better than that of a production aircraft. This is most likely because of a higher C_{Lmax} as a result of the C_L data being taken at a horizontal stabilator setting of zero rather than deflected to produce the necessary angle of attack. This error is quite small, approximately 5 percent or less, and should have no significant effect on the error levels presented. The accuracy of the other performance parameters is within the differences between individual aircraft.

The effects on aircraft point performance parameters attributable to wind tunnel measurement errors in the force coefficients were determined as follows:

1 Each 1-percent error in C_D caused a corresponding 1-percent error in horizontal acceleration, minimum climb angle, and climb rate.

2 Each 1-percent error in C_L caused a 0.5- to 1-percent error in level flight stall speed and best instantaneous turn radius and rate.

3 Maximum specific range and endurance are significantly affected by errors in both C_L and C_D.

4 Maximum horizontal flight speed was the least sensitive performance parameter to errors in C_L and C_D.

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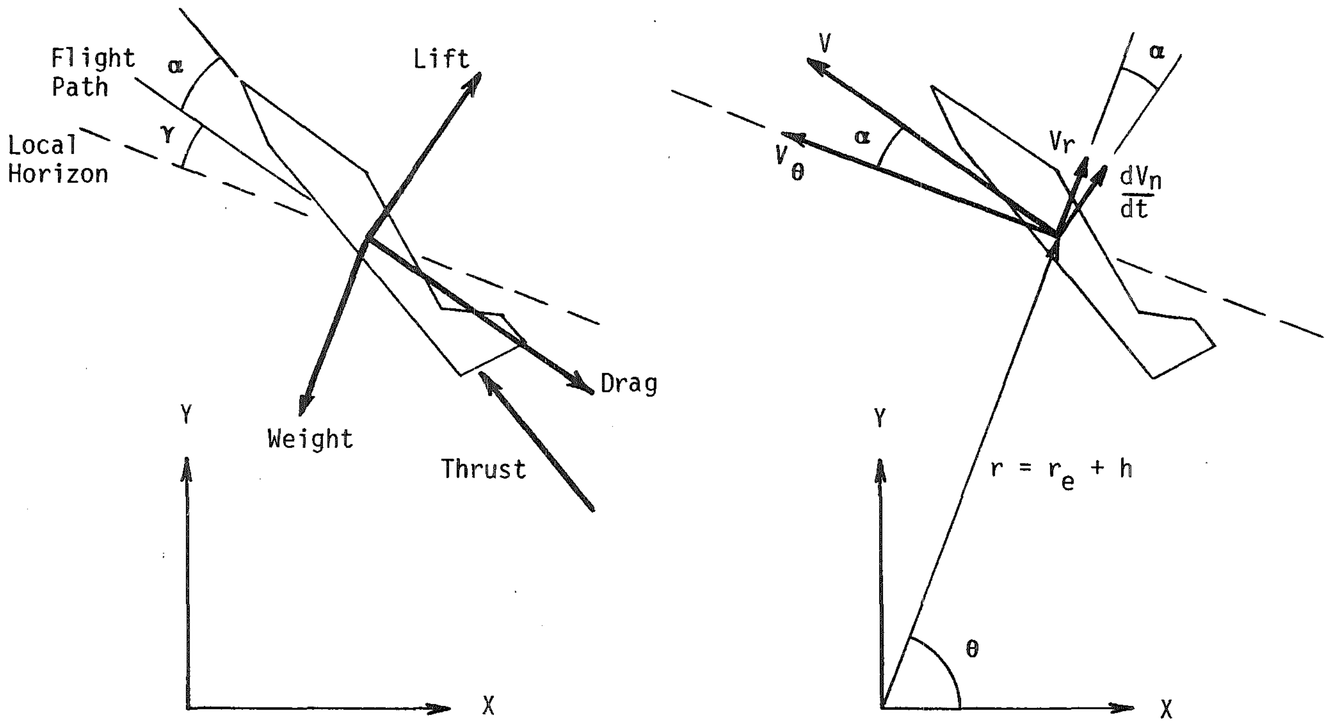

a. Forces　　　　b. Velocities

Figure 1. Basic relations in the vertical plane.

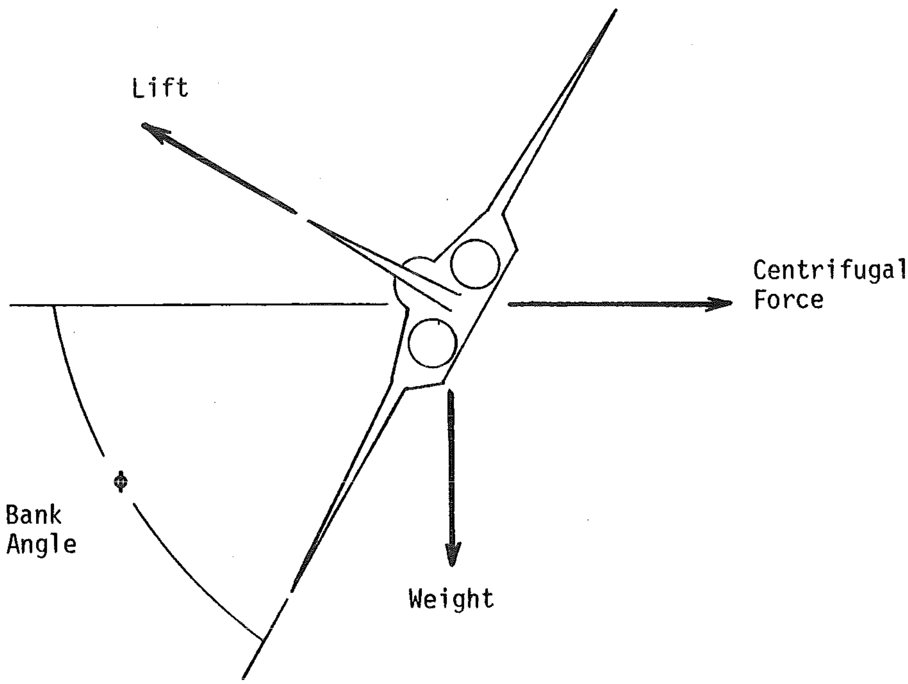

Figure 2. Basic relations in the horizontal plane.

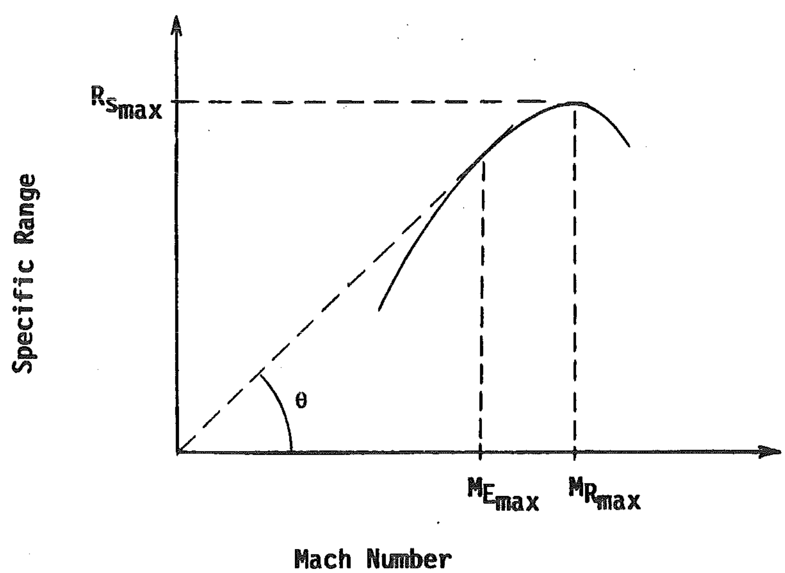

Figure 3. Specific range versus Mach number.

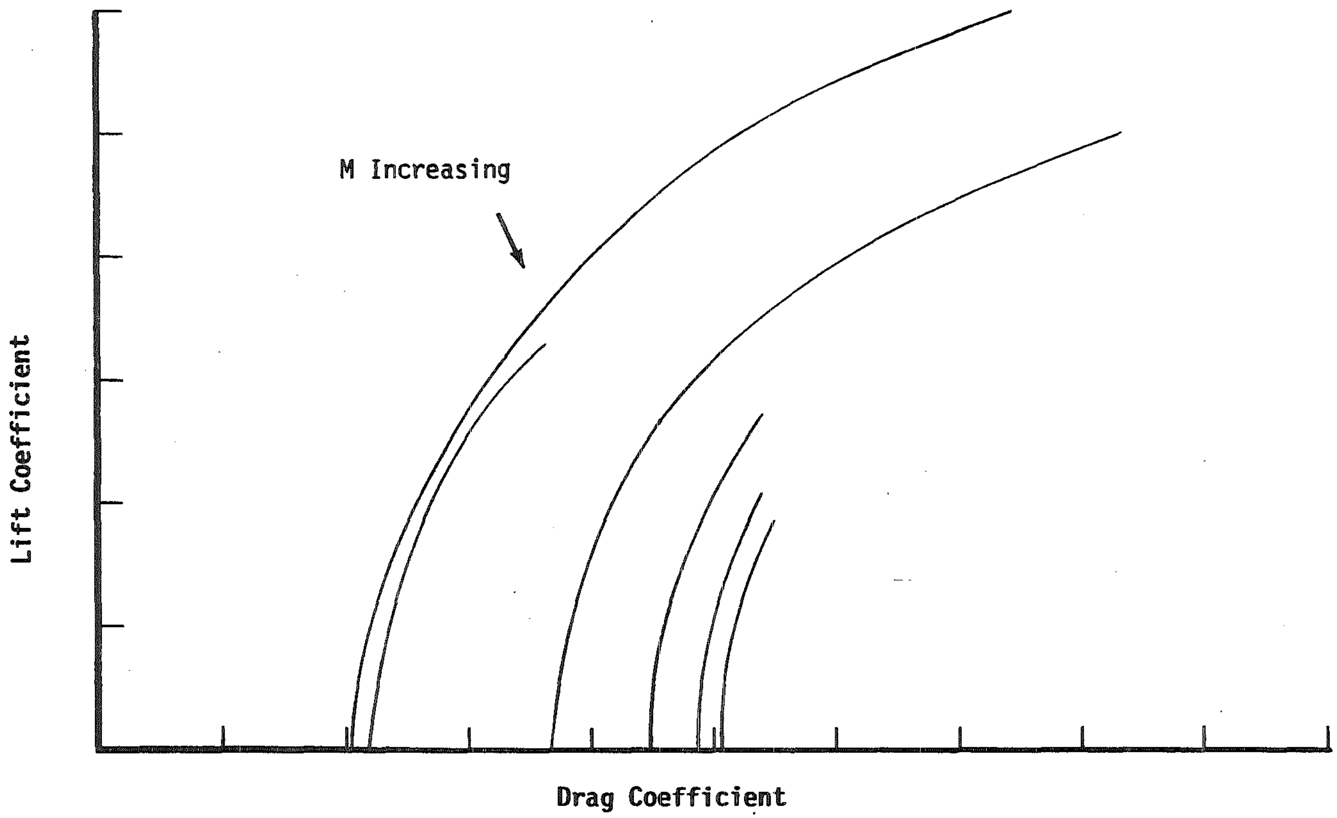

Figure 4. Flight test low C_L drag polar.

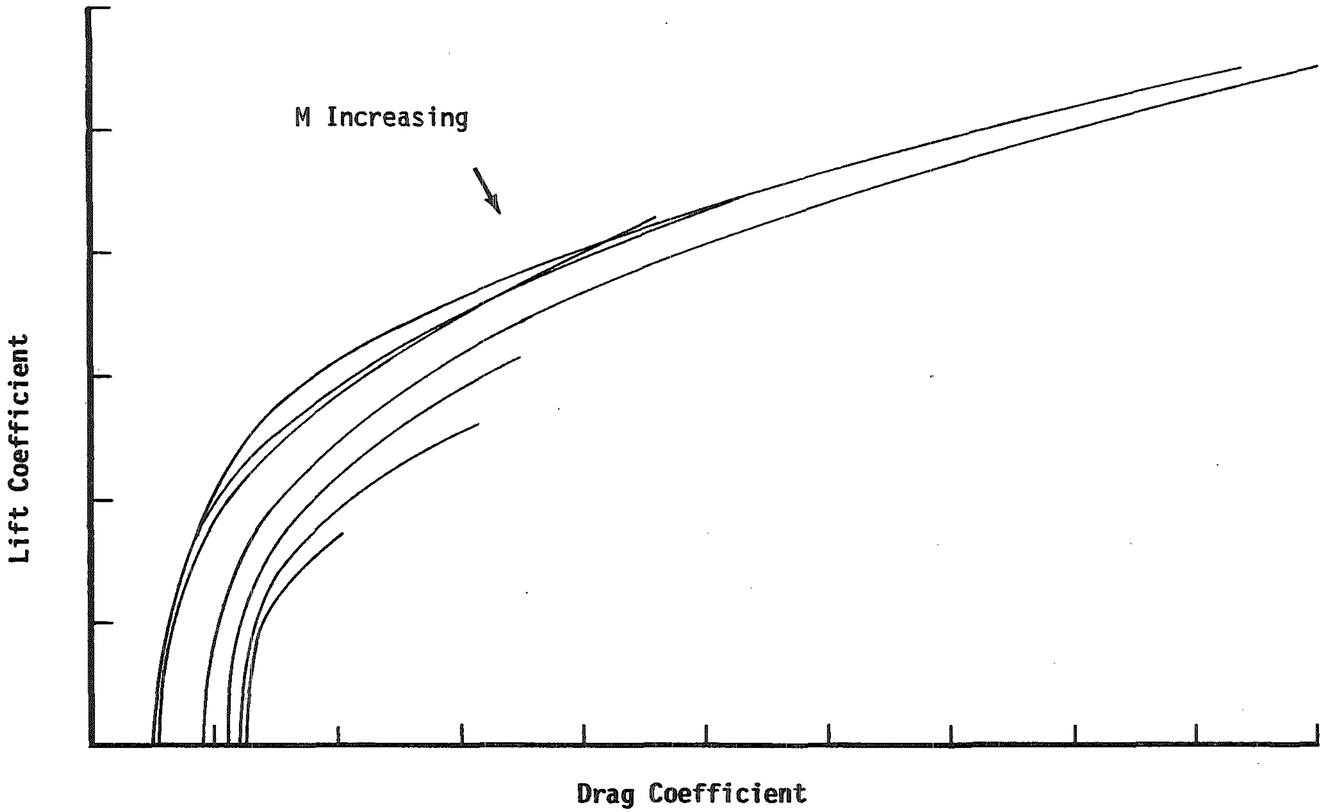

Figure 5. Flight test high C_L drag polar.

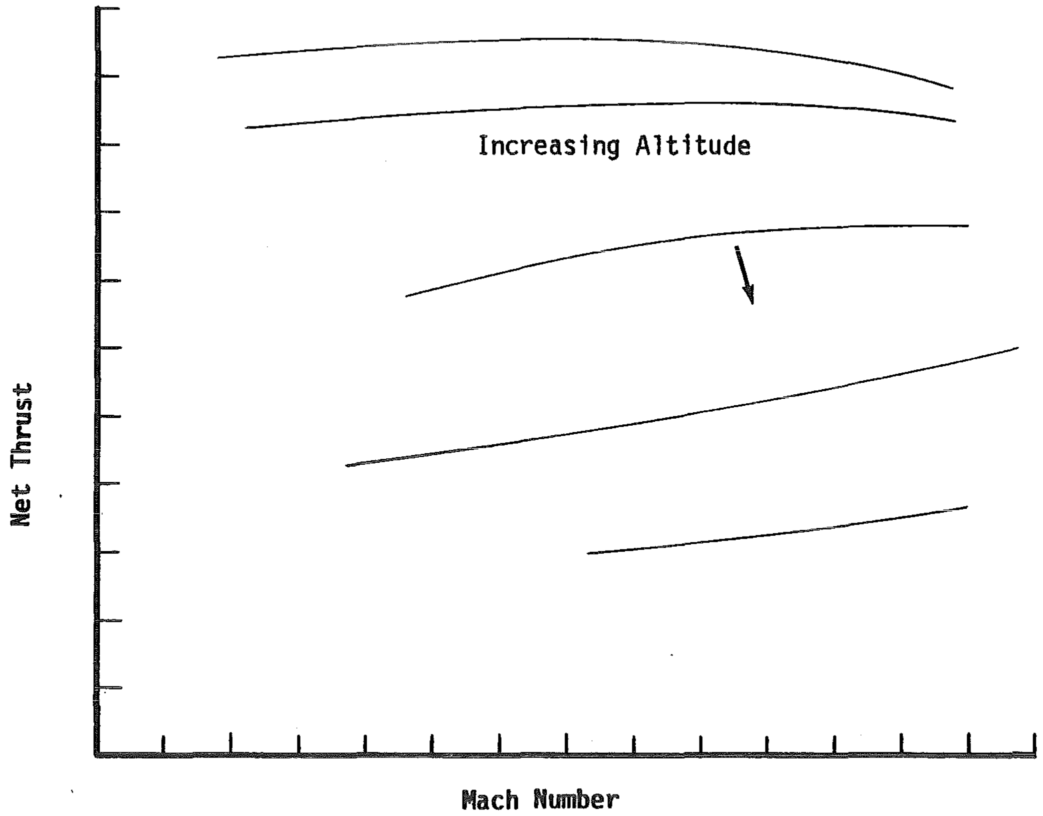

Figure 6. Net thrust available.

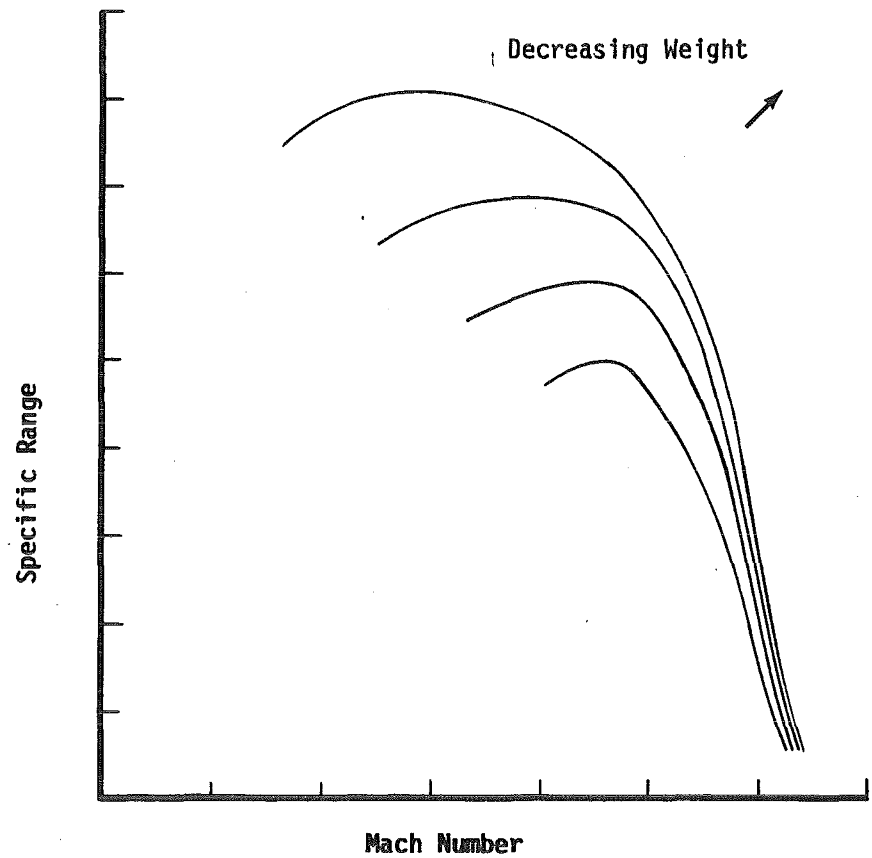

Figure 7. Specific range.

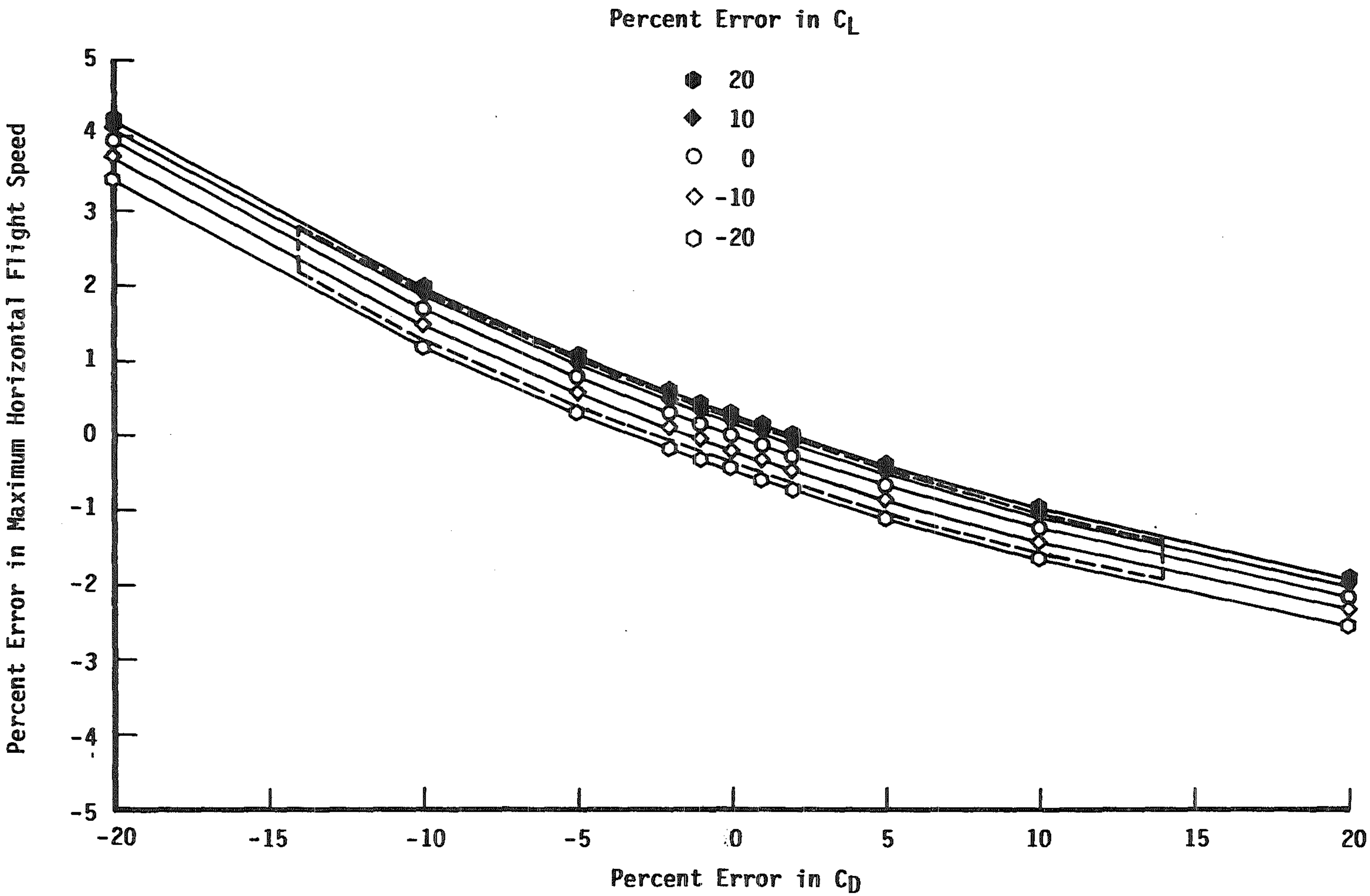

Figure 8. Effect of C_D error on maximum horizontal flight speed.

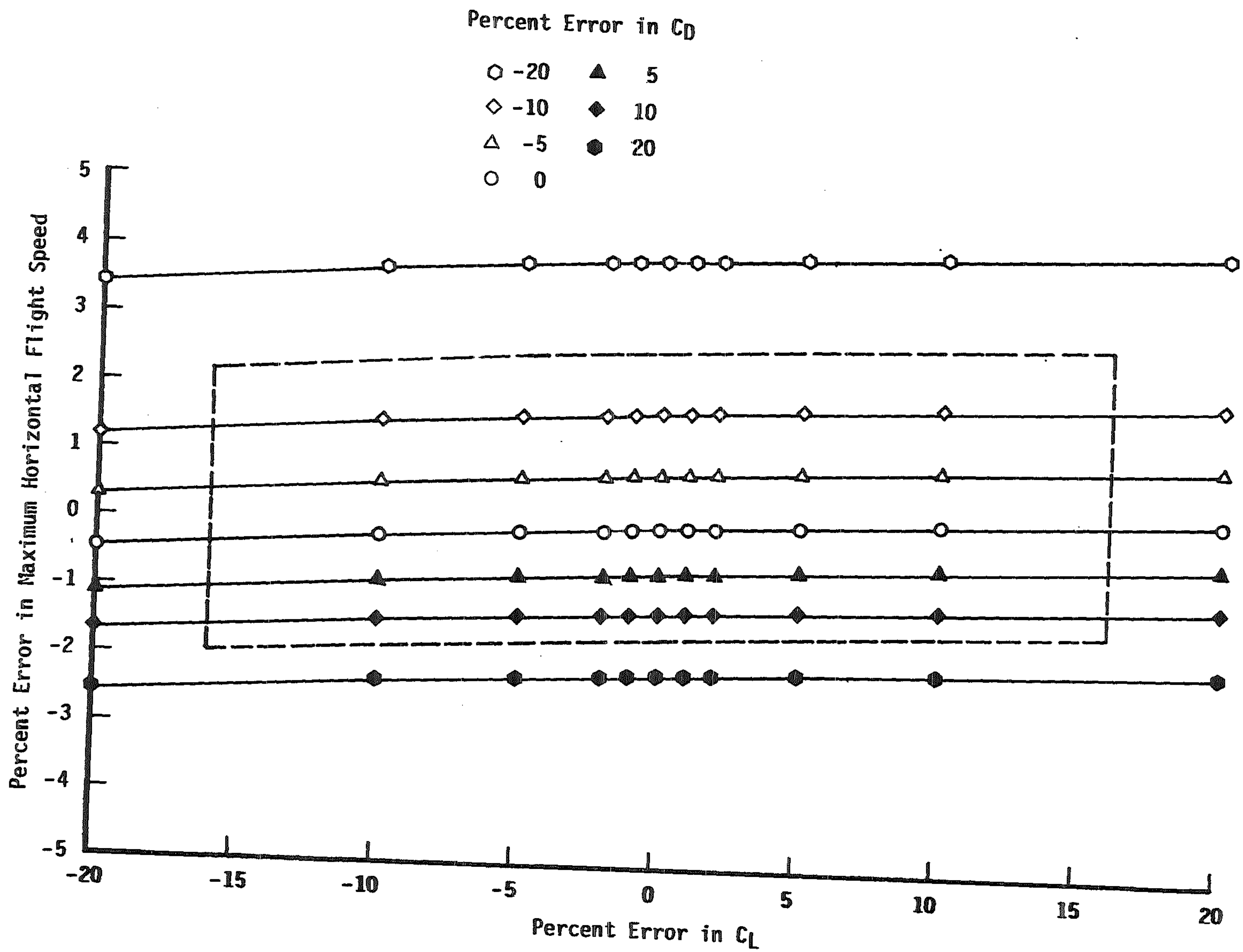

Figure 9. Effect of C_L error on maximum horizontal flight speed.

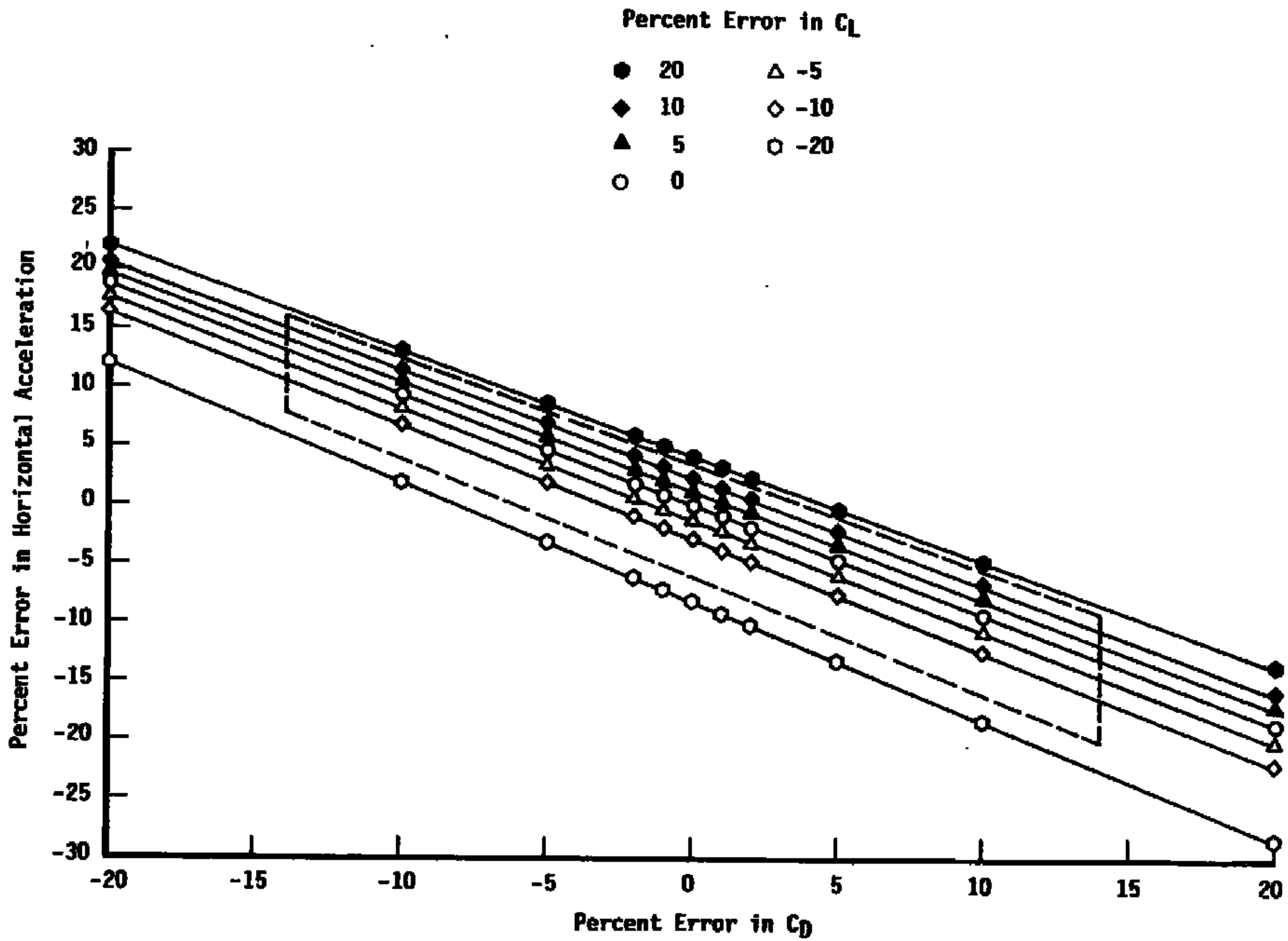

Figure 10. Effect of C_D error on horizontal acceleration.

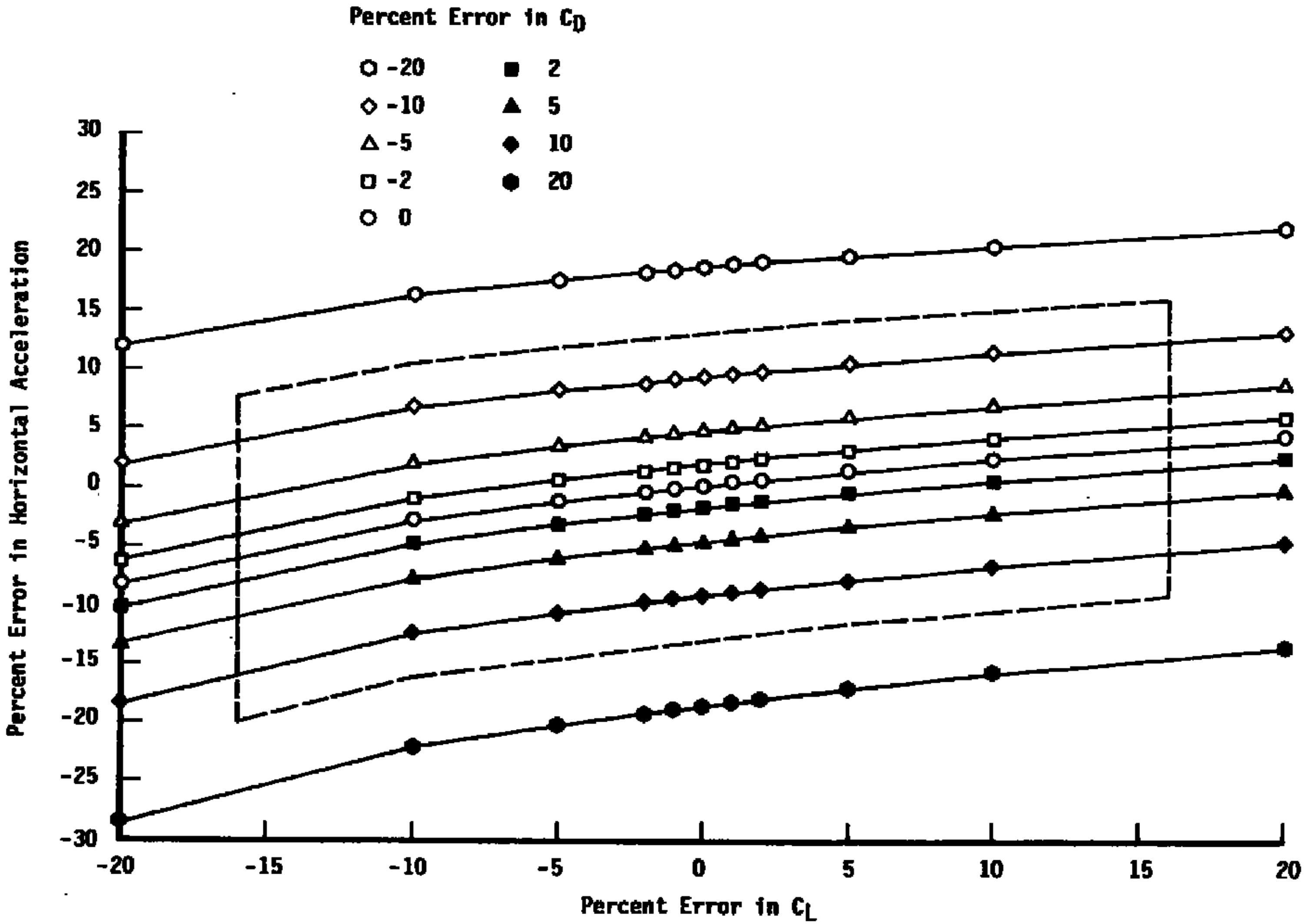

Figure 11. Effect of C_L error on horizontal acceleration.

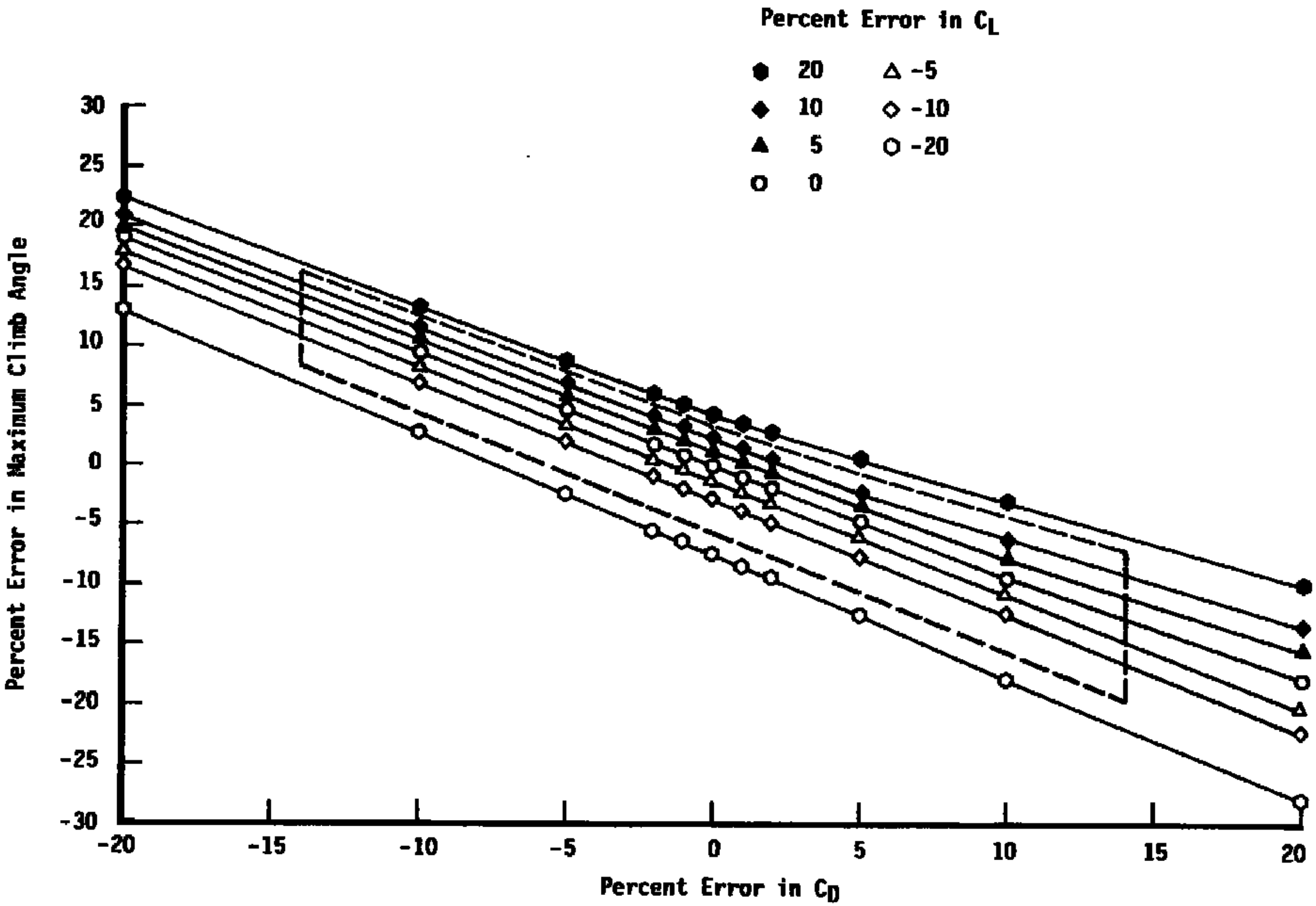

Figure 12. Effect of C_D error on maximum climb angle.

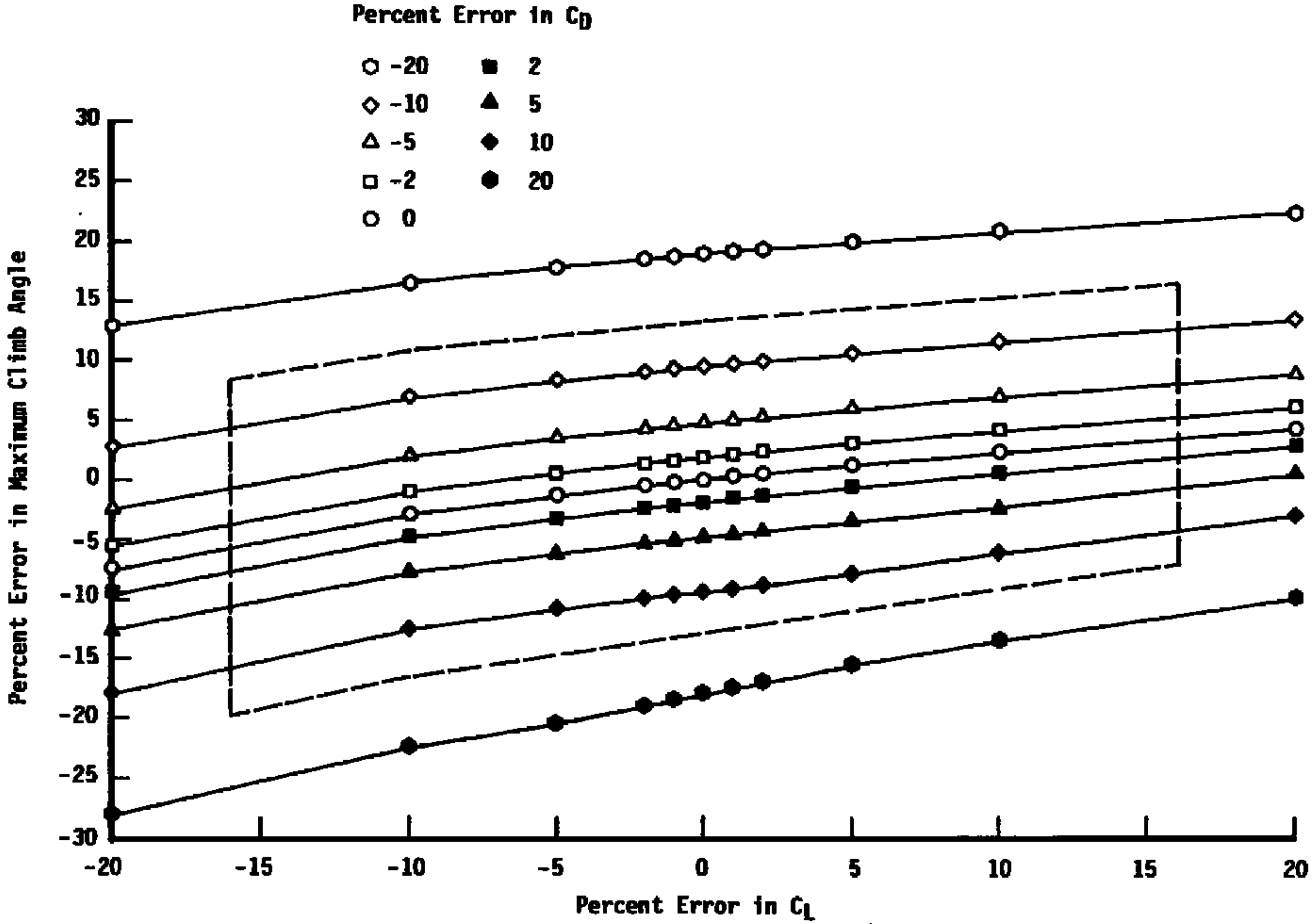

Figure 13. Effect of C_L error on maximum climb angle.

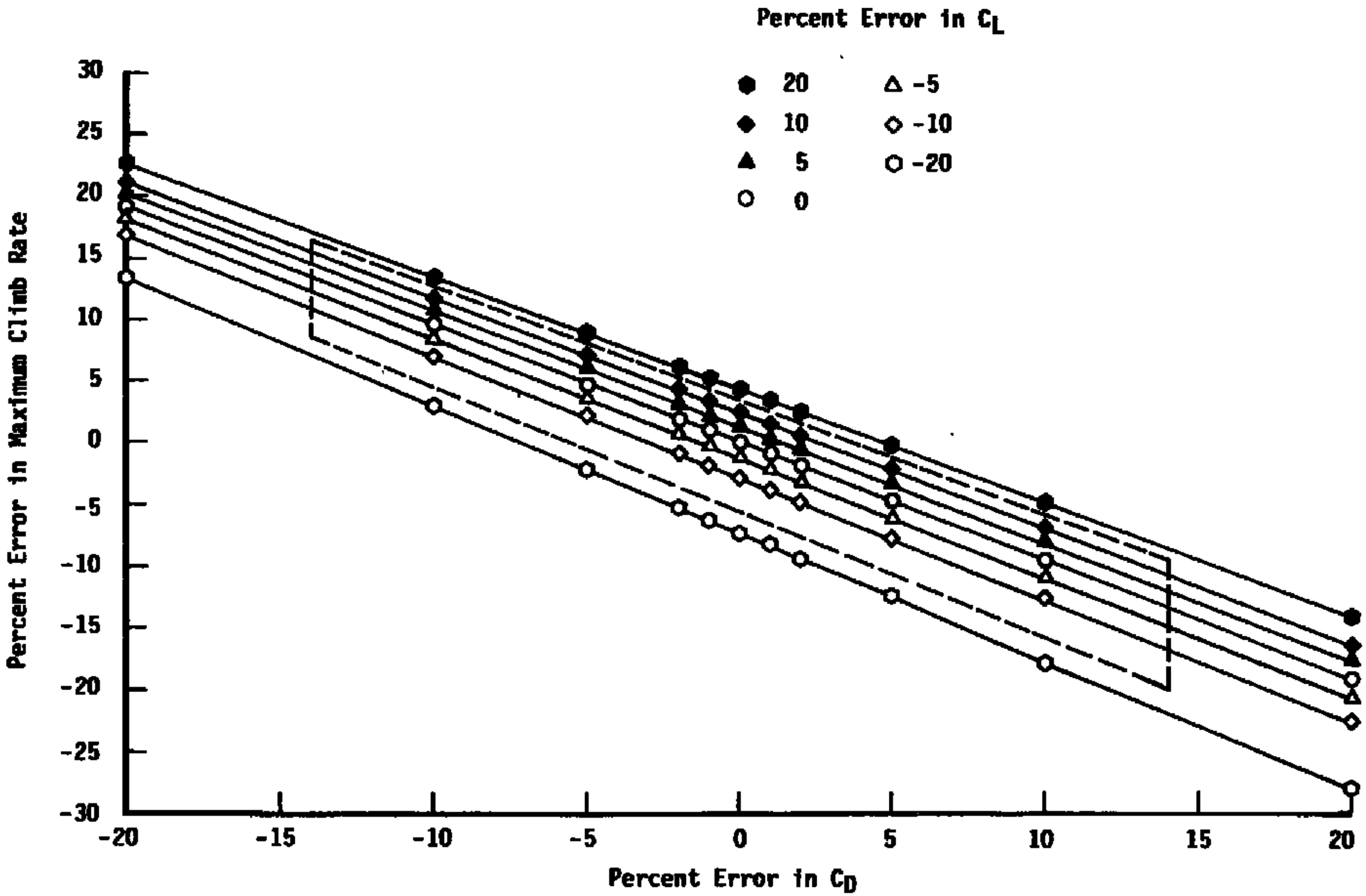

Figure 14. Effect of C_D error on maximum climb rate.

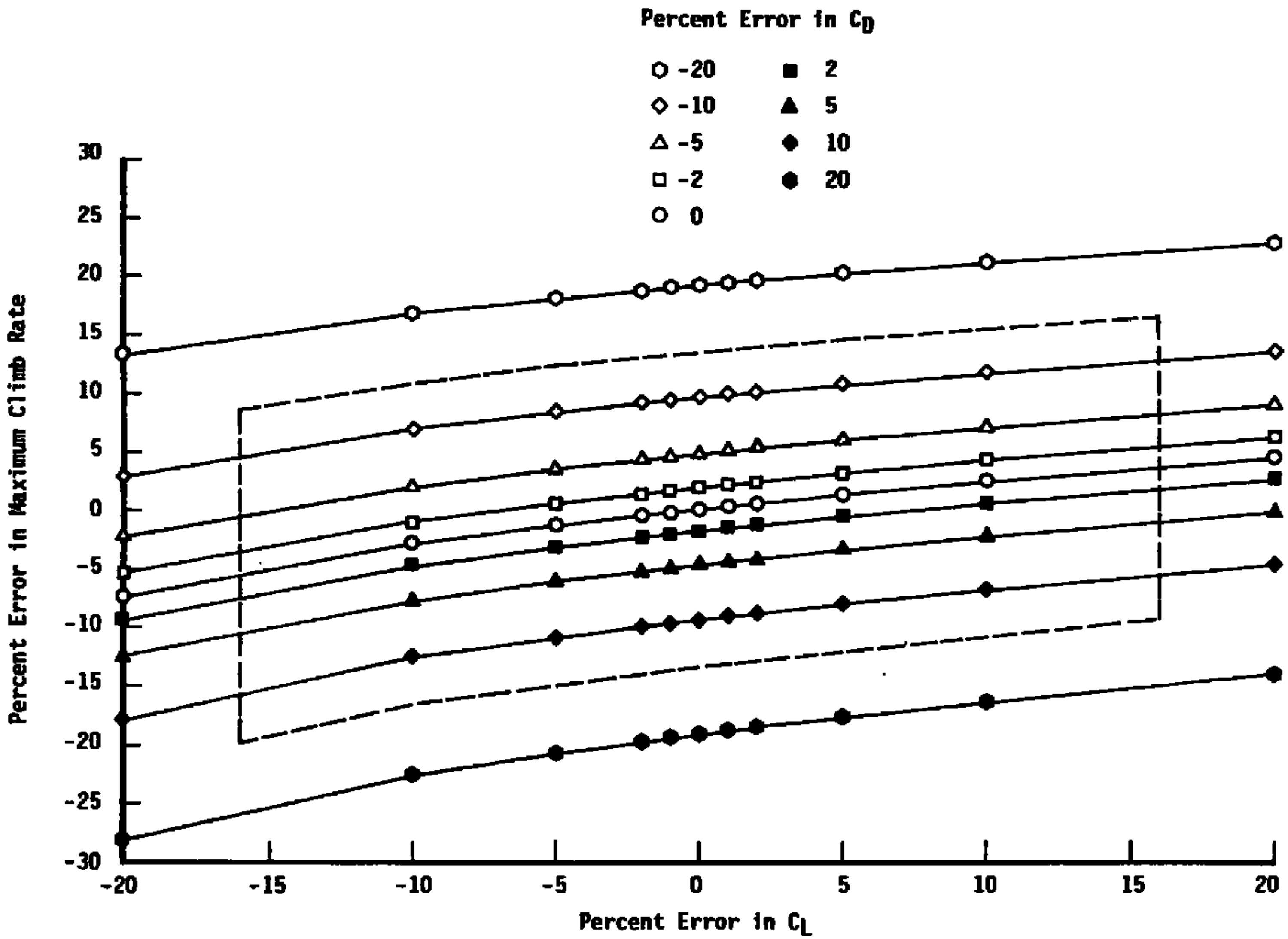

Figure 15. Effect of C_L error on maximum climb rate.

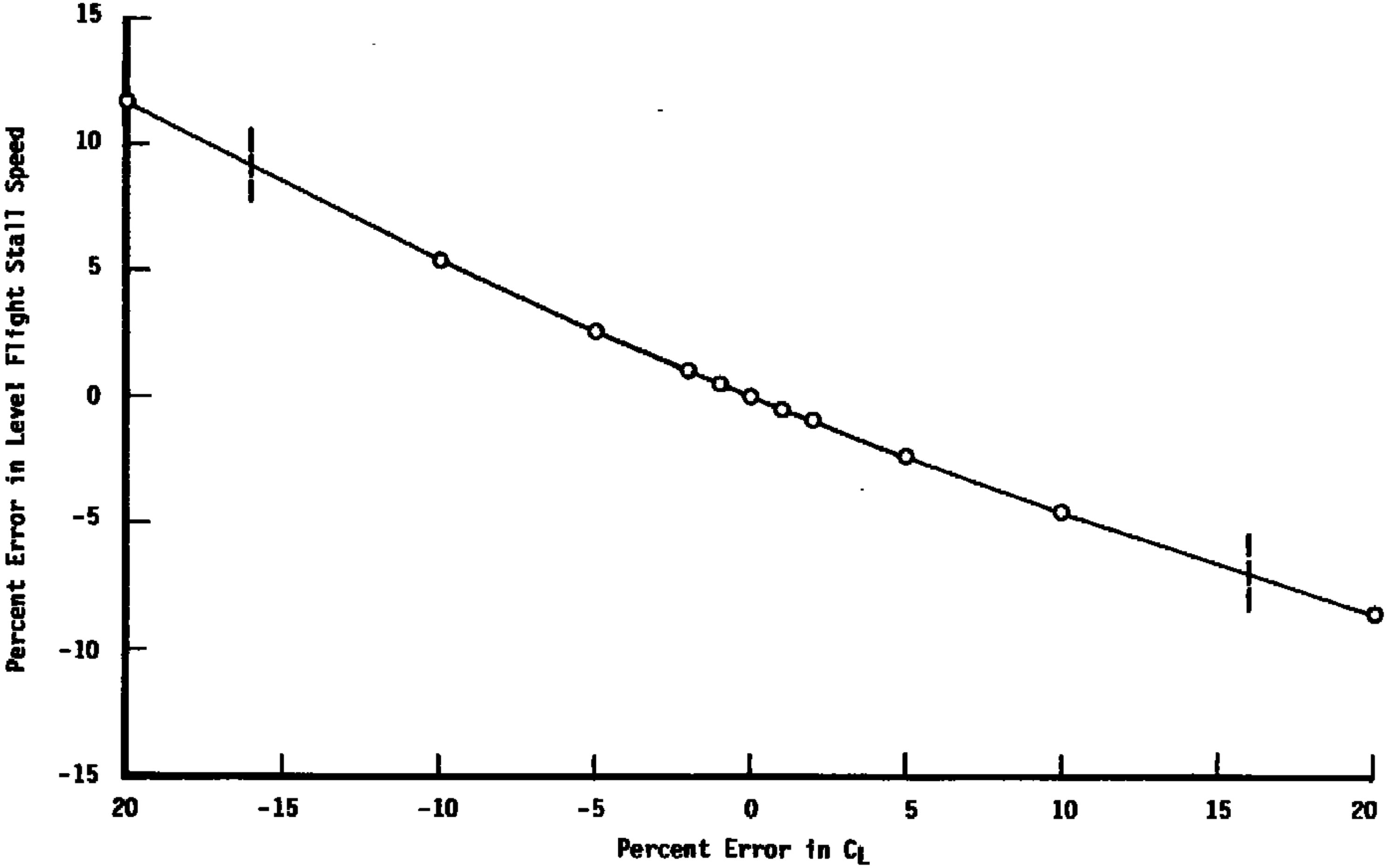

Figure 16. Effect of C_L error on level flight stall speed.

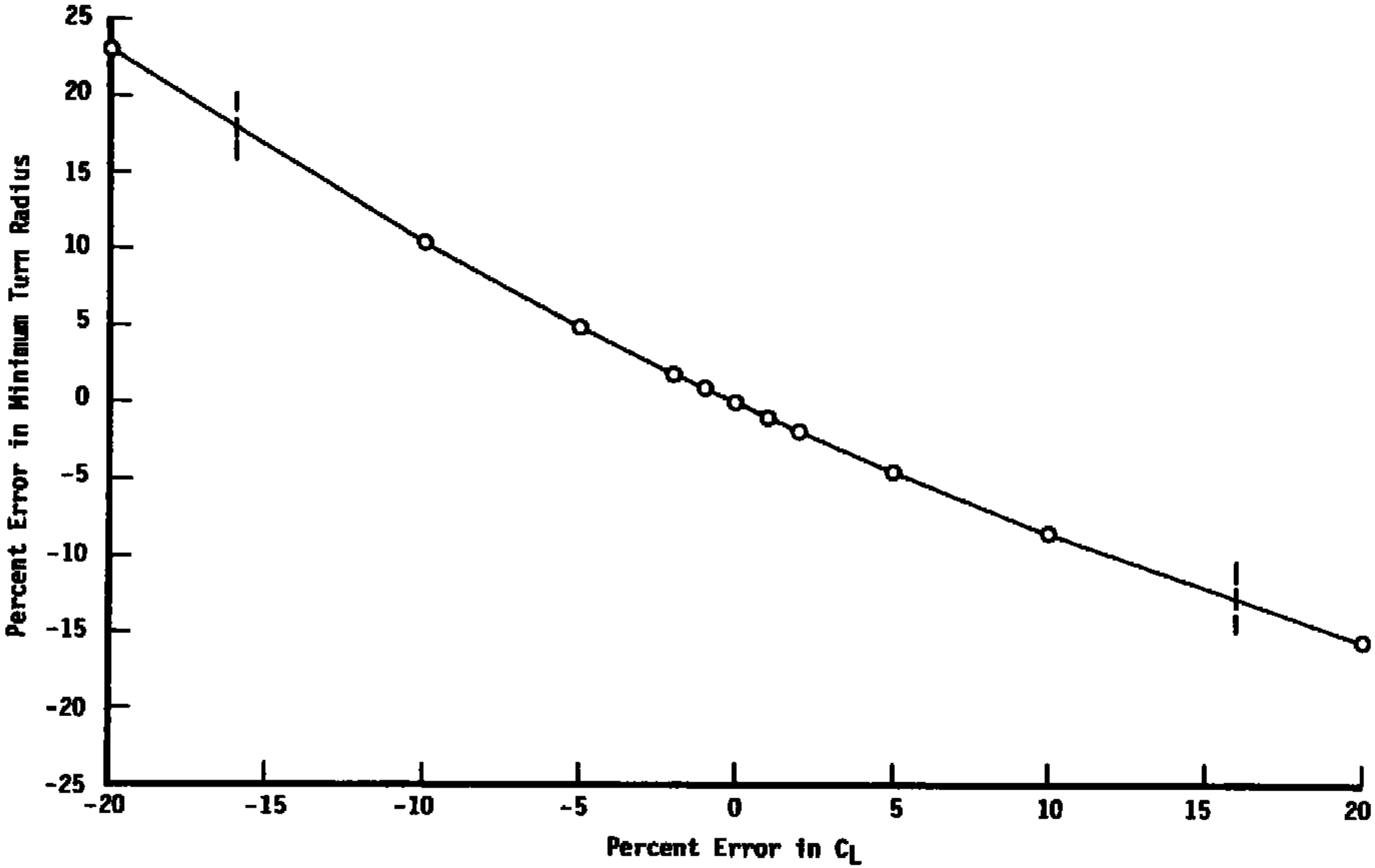

Figure 17. Effect of C_L error on minimum turn radius.

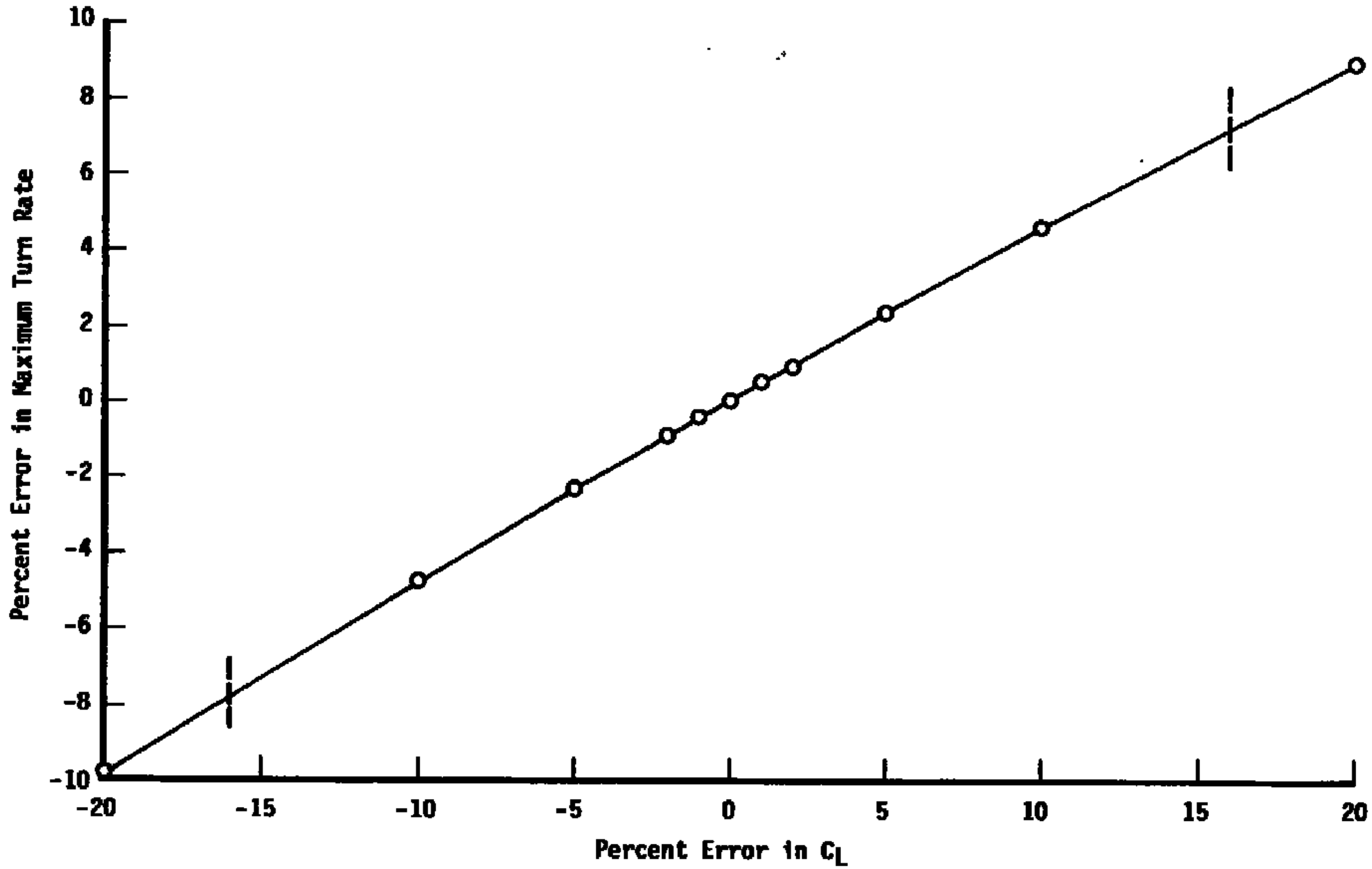

Figure 18. Effect of C_L error on maximum turn rate.

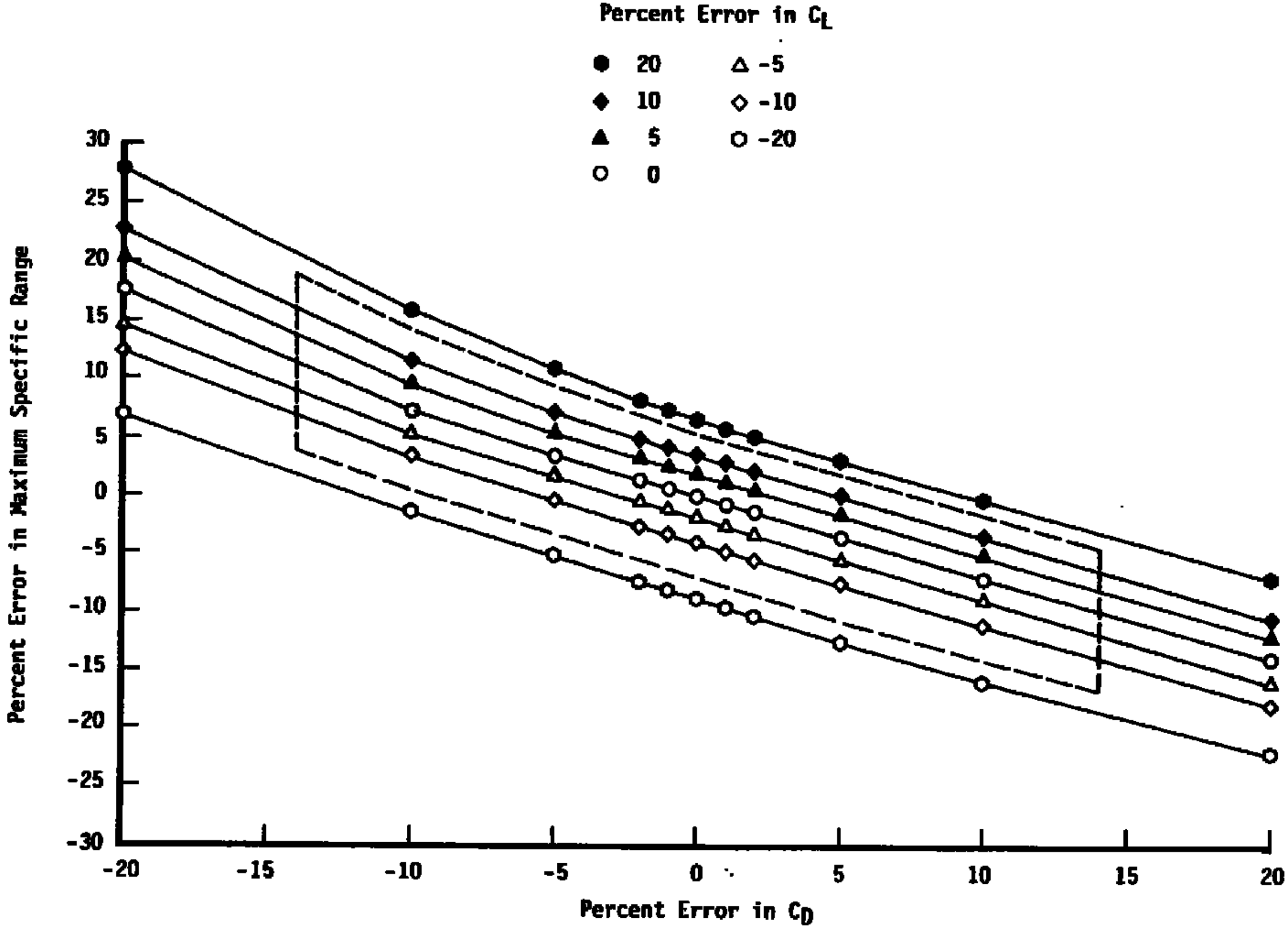

Figure 19. Effect of C_D error on specific range.

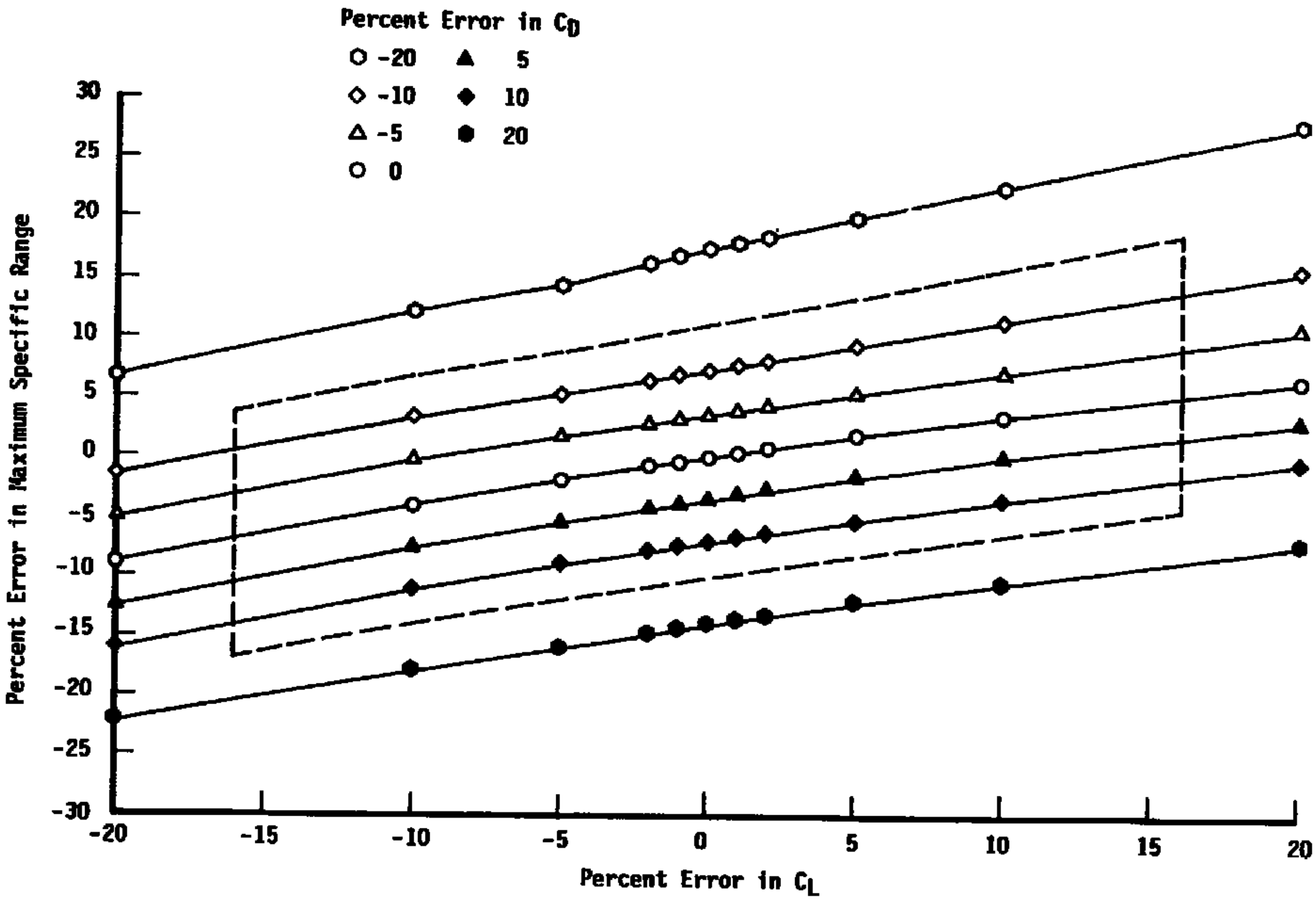

Figure 20. Effect of C_L error on specific range.

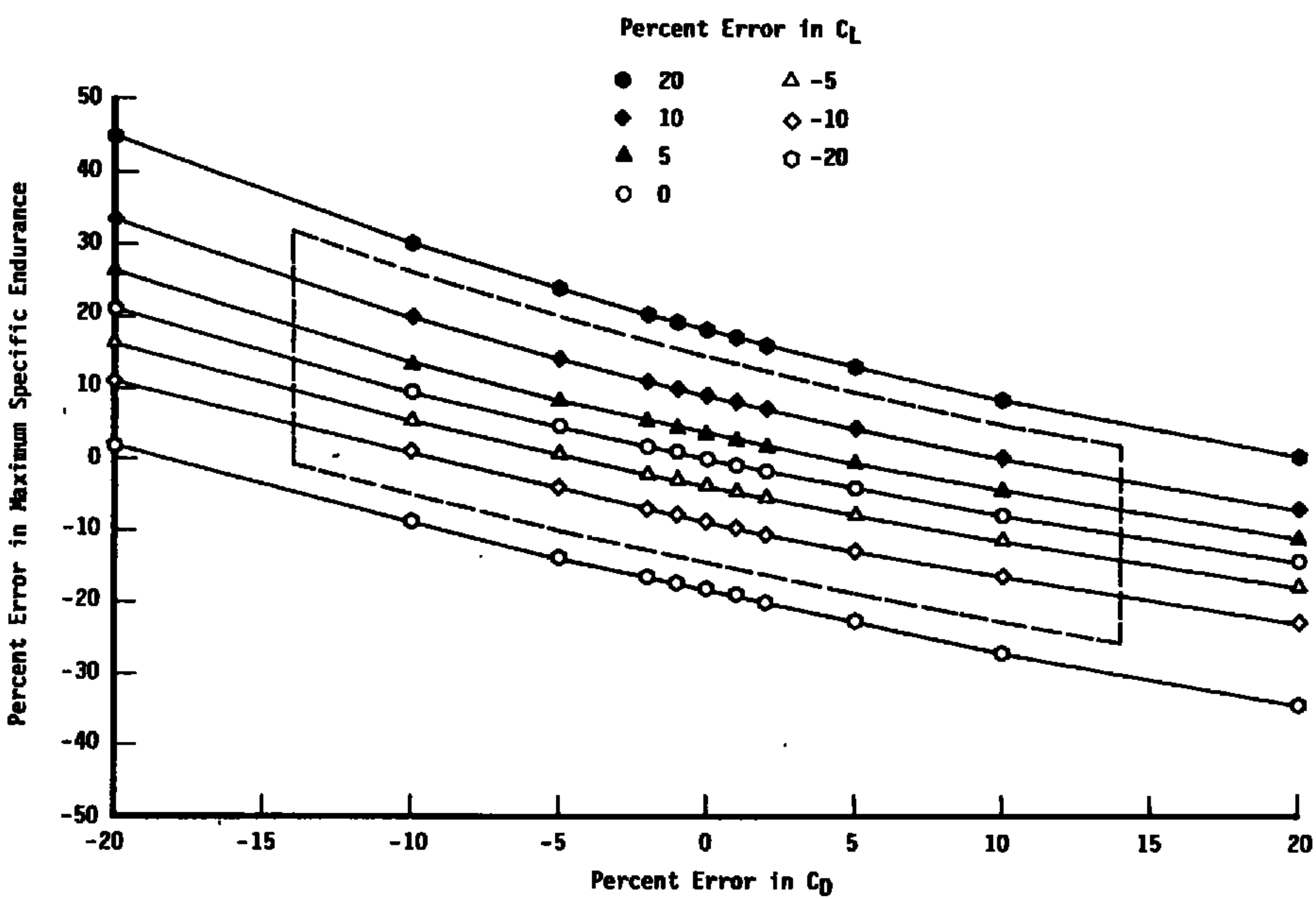

Figure 21. Effect of C_D error on specific endurance.

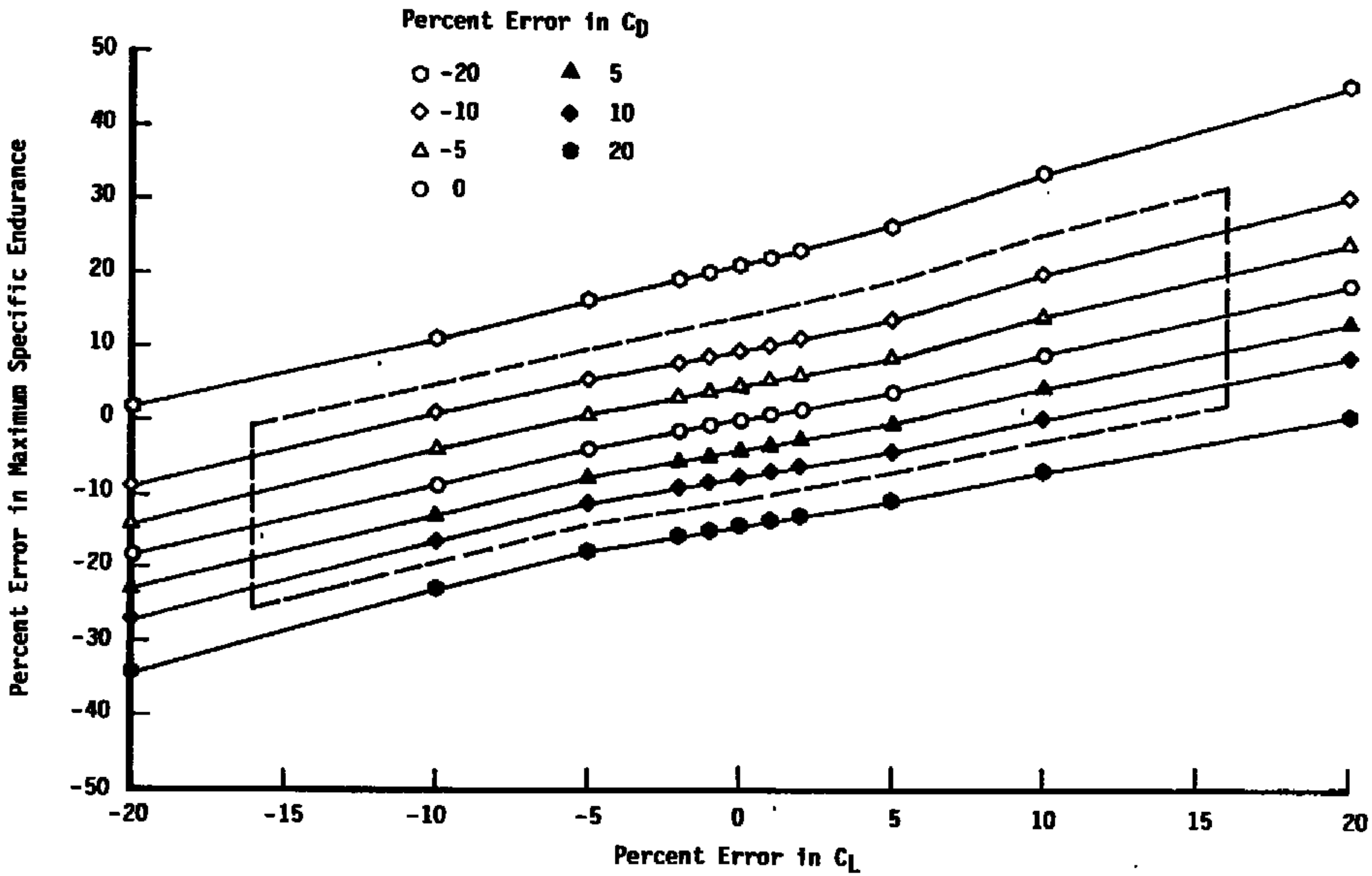

Figure 22. Effect of C_L error on specific endurance.

Table 1. PERCAL Computer Program Lift Coefficient Data

AOA	Mach Number Low →							
Low ↓	−0.111	−0.111	−0.111	−0.111	−0.111	−0.111	−0.111	−0.111
	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.026
	0.155	0.155	0.156	0.157	0.158	0.159	0.161	0.163
	0.288	0.288	0.290	0.291	0.293	0.295	0.297	0.300
	0.421	0.422	0.424	0.426	0.428	0.430	0.433	0.437
	0.554	0.555	0.557	0.559	0.562	0.565	0.569	0.574
	0.687	0.688	0.691	0.694	0.697	0.700	0.705	0.711
	0.806	0.808	0.812	0.814	0.817	0.822	0.827	0.835
	0.915	0.916	0.920	0.924	0.930	0.935	0.942	0.951
	1.018	1.020	1.024	1.027	1.032	1.038	1.046	1.056
	1.113	1.114	1.119	1.123	1.129	1.136	1.145	1.155
	1.201	1.202	1.206	1.210	1.215	1.222	1.231	1.242
	1.277	1.278	1.282	1.286	1.292	1.299	1.309	1.321
	1.343	1.344	1.349	1.354	1.360	1.368	1.378	1.389
	1.401	1.402	1.408	1.412	1.419	1.427	1.437	1.450
	1.450	1.451	1.456	1.462	1.468	1.476	1.486	1.498
	1.487	1.488	1.494	1.499	1.506	1.514	1.523	1.535
	1.514	1.515	1.521	1.526	1.533	1.541	1.551	1.564
	1.532	1.533	1.538	1.542	1.549	1.557	1.569	1.583
	1.532	1.534	1.543	1.549	1.556	1.564	1.574	1.584
	1.524	1.526	1.533	1.538	1.544	1.551	1.559	1.569
	1.492	1.494	1.502	1.508	1.515	1.523	1.532	1.543
High	1.452	1.454	1.459	1.463	1.468	1.475	1.484	1.494

Table 1. Concluded

AOA	Mach Number → High						
Low	−0.109	−0.107	−0.103	−0.099	−0.092	−0.092	−0.108
	0.029	0.032	0.038	0.044	0.054	0.062	0.061
	0.167	0.172	0.179	0.188	0.201	0.215	0.231
	0.305	0.311	0.320	0.331	0.347	0.369	0.400
	0.443	0.450	0.461	0.474	0.493	0.523	0.569
	0.581	0.590	0.602	0.618	0.640	0.676	0.739
	0.719	0.729	0.743	0.761	0.786	0.830	0.908
	0.844	0.856	0.872	0.890	0.915	0.960	1.049
	0.961	0.975	0.991	1.010	1.037	1.084	1.173
	1.068	1.082	1.100	1.121	1.150	1.198	1.285
	1.167	1.182	1.199	1.221	1.252	1.303	1.377
	1.255	1.271	1.292	1.316	1.349	1.396	1.457
	1.336	1.354	1.376	1.400	1.427	1.467	1.525
	1.404	1.423	1.446	1.471	1.500	1.535	1.576
	1.465	1.483	1.504	1.528	1.556	1.584	1.615
	1.513	1.531	1.552	1.575	1.598	1.623	1.650
	1.550	1.569	1.588	1.609	1.628	1.647	1.666
	1.579	1.595	1.612	1.629	1.646	1.662	1.679
	1.597	1.612	1.626	1.640	1.655	1.669	1.684
	1.595	1.607	1.620	1.634	1.648	1.663	1.679
	1.580	1.592	1.606	1.620	1.636	1.653	1.670
↓	1.555	1.562	1.585	1.601	1.619	1.637	1.656
High	1.506	1.520	1.537	1.558	1.581	1.607	1.636

Table 2. PERCAL Computer Program Drag Coefficient Data

AOA	Mach Number				
	Low ⟶				
Low	0.0222	0.0222	0.0222	0.0222	0.0222
	0.0208	0.0208	0.0208	0.0284	0.0208
	0.0236	0.0236	0.0237	0.0237	0.0237
	0.0308	0.0308	0.0309	0.0310	0.0311
	0.0423	0.0424	0.0426	0.0429	0.0431
	0.0624	0.0627	0.0632	0.0637	0.0645
	0.1044	0.1048	0.1060	0.1072	0.1083
	0.1635	0.1645	0.1665	0.1675	0.1690
	0.2289	0.2295	0.2323	0.2351	0.2392
	0.3068	0.3085	0.3119	0.3145	0.3187
	0.3876	0.3884	0.3927	0.3961	0.4012
	0.4624	0.4632	0.4666	0.4700	0.4743
	0.5270	0.5278	0.5312	0.5346	0.5397
	0.5831	0.5839	0.5882	0.5924	0.5975
	0.6324	0.6332	0.6383	0.6417	0.6477
	0.6740	0.6749	0.6791	0.6842	0.6893
	0.7055	0.7063	0.7114	0.7157	0.7216
	0.7284	0.7293	0.7344	0.7386	0.7446
	0.7437	0.7446	0.7488	0.7522	0.7582
	0.7437	0.7454	0.7531	0.7582	0.7641
	0.7369	0.7386	0.7446	0.7488	0.7539
↓	0.7097	0.7114	0.7182	0.7233	0.7293
High	0.6757	0.6774	0.6817	0.6851	0.6893

Table 2. Continued

AOA	Mach Number →				
Low	0.0222	0.0222	0.0222	0.0222	0.0222
	0.0208	0.0208	0.0208	0.0208	0.0209
	0.0238	0.0239	0.0239	0.0241	0.0243
	0.0312	0.0314	0.0317	0.0320	0.0325
	0.0433	0.0436	0.0440	0.0446	0.0453
	0.0653	0.0663	0.0676	0.0694	0.0717
	0.1095	0.1117	0.1143	0.1179	0.1223
	0.1715	0.1740	0.1780	0.1825	0.1895
	0.2427	0.2475	0.2538	0.2615	0.2723
	0.3238	0.3306	0.3391	0.3493	0.3612
	0.4071	0.4148	0.4233	0.4335	0.4462
	0.4802	0.4879	0.4972	0.5083	0.5219
	0.5457	0.5542	0.5644	0.5771	0.5924
	0.6043	0.6128	0.6222	0.6349	0.6511
	0.6545	0.6630	0.6740	0.6868	0.7021
	0.6961	0.7046	0.7148	0.7276	0.7429
	0.7284	0.7361	0.7463	0.7590	0.7752
	0.7514	0.7599	0.7709	0.7837	0.7973
	0.7650	0.7752	0.7871	0.7990	0.8117
	0.7709	0.7794	0.7879	0.7973	0.8075
	0.7599	0.7667	0.7752	0.7845	0.7947
↓	0.7361	0.7437	0.7531	0.7633	0.7692
High	0.6953	0.7029	0.7114	0.7216	0.7335

Table 2. Concluded

AOA	Mach Number			→ High
Low ↓	0.0218	0.0218	0.0220	0.0232	0.0383
	0.0209	0.0210	0.0213	0.0226	0.0374
	0.0246	0.0250	0.0256	0.0279	0.0435
	0.0332	0.0341	0.0356	0.0410	0.0610
	0.0465	0.0482	0.0577	0.0654	0.1005
	0.0749	0.0800	0.0995	0.1135	0.1625
	0.1284	0.1379	0.1590	0.1805	0.2570
	0.2000	0.2119	0.2280	0.2600	0.3585
	0.2846	0.3000	0.3230	0.3629	0.4570
	0.3764	0.3944	0.4190	0.4598	0.5461
	0.4607	0.4794	0.5057	0.5491	0.6192
	0.5397	0.5601	0.5882	0.6281	0.6828
	0.6111	0.6315	0.6545	0.6885	0.7369
	0.6706	0.6919	0.7165	0.7463	0.7774
	0.7199	0.7403	0.7641	0.7879	0.8084
	0.7607	0.7803	0.7998	0.8211	0.8363
	0.7913	0.8092	0.8253	0.8415	0.8490
	0.8117	0.8262	0.8406	0.8542	0.8593
	0.8236	0.8355	0.8483	0.8602	0.8633
	0.8185	0.8304	0.8423	0.8551	0.8593
	0.8066	0.8185	0.8321	0.8466	0.8522
	0.7888	0.8024	0.8177	0.8330	0.8410
High	0.7480	0.7658	0.7854	0.8075	0.8251

Table 3. PERCAL Computer Program Thrust Specific Fuel Consumption Data

Mach	Aircraft Weight: Light → Heavy							
	Treq.	TSFC	Treq.	TSFC	Treq.	TSFC	Treq.	TSFC
Low	3023.0	0.958	4775.0	0.928	8136.0	0.862	12984.0	0.890
	2992.0	0.966	4443.0	0.925	7243.0	0.863	11518.0	0.866
	3007.0	0.960	4261.0	0.925	6519.0	0.863	10206.0	0.903
	3037.0	0.950	4146.0	0.923	5988.0	0.877	9110.0	0.918
	3070.0	0.956	4055.0	0.930	5635.0	0.888	8223.0	0.948
	3115.0	0.961	4002.0	0.933	5394.0	0.897	7557.0	0.966
	3186.0	0.961	4002.0	0.929	5193.0	0.911	7069.0	0.978
	3276.0	0.956	4039.0	0.920	5040.0	0.923	6712.0	0.977
	3372.0	0.954	4083.0	0.916	4968.0	0.928	6426.0	0.979
	3469.0	0.954	4125.0	0.922	4978.0	0.921	6186.0	0.978
	3568.0	0.957	4181.0	0.927	5024.0	0.909	6026.0	0.972
	3674.0	0.958	4266.0	0.927	5061.0	0.902	5973.0	0.956
↓	3789.0	0.959	4374.0	0.923	5095.0	0.900	5993.0	0.935

Table 3. Concluded

Mach	Aircraft Weight: Light → Heavy							
	Treq.	TSFC	Treq.	TSFC	Treq.	TSFC	Treq.	TSFC
	3913.0	0.961	4487.0	0.919	5152.0	0.904	6028.0	0.919
↓	4447.0	0.961	4596.0	0.919	5238.0	0.902	6052.0	0.904
	4189.0	0.961	4703.0	0.922	5338.0	0.899	6083.0	0.899
	4340.0	0.959	4811.0	0.929	5437.0	0.899	6139.0	0.894
	4511.0	0.957	4936.0	0.930	5543.0	0.902	6225.0	0.893
	4711.0	0.951	5097.0	0.930	5671.0	0.904	6342.0	0.889
	4917.0	0.964	5278.0	0.929	5807.0	0.910	6466.0	0.903
	5059.0	0.973	5411.0	0.949	5895.0	0.936	6548.0	0.937
	5132.0	1.020	5491.0	0.993	5943.0	0.991	6594.0	0.993
	5451.0	0.991	5823.0	1.012	6255.0	1.017	6896.0	1.027
	6454.0	0.963	6829.0	0.952	7251.0	0.963	7854.0	0.989
	8212.0	0.868	8577.0	0.868	8993.0	0.879	9526.0	0.925
High	10342.0	0.908	10690.0	0.848	11101.0	0.863	11547.0	0.911

Table 4. Summary of the Effects of Coefficient Errors

Parameter	Effect (%) for each 1% error in:	
	C_D	C_L
Maximum Level Flight Speed	0.18	0.025
0.9 Mach Horizontal Acceleration	1.0	0.25 - 0.5
Maximum Climb Angle	0.9	0.25 - 0.35
Maximum Climb Rate	1.0	0.2 - 0.3
Level Flight Stall Speed	---	0.5
Minimum Turn Radius	---	1.0
Maximum Turn Rate	---	0.5
Maximum Specific Range	0.8	0.4
Maximum Specific Endurance	1.0	1.0

NOMENCLATURE

a	Local sonic velocity, ft/sec
a_c	Radial acceleration (horizontal turn), ft/sec^2
AOA	Angle of attack, deg
a_r	Acceleration perpendicular to local horizon, ft/sec^2
a_θ	Acceleration parallel to local horizon, ft/sec^2
c	Thrust specific fuel consumption, lbm/lbf-sec
C_D	Coefficient of drag
ΔC_D	Increment in coefficient of drag
C_L	Coefficient of lift
ΔC_L	Increment in coefficient of lift
CPU	Central processing unit
D	Total drag, lbf
E	Total energy, ft lbf
E_{a_θ}?	Error in horizontal acceleration, percent
E_{C_D}	Error in coefficient of drag, percent
E_s	Specific range, ft/lbm fuel
E_T	Error in thrust, percent
F_c	Centrifugal force (horizontal turn), lbf
g	Local gravitational acceleration, ft/sec^2

g_o	Sea-level gravitational acceleration, 32.174 ft/sec^2
h	Height above sea level, ft
L	Total lift, lbf
m	Mass, lbm
M	Mach number
MSL	Mean sea level
n	Load factor, g's
p	Local atmospheric pressure, lbf/ft^2
PERCAL	Point performance computer program
P_s	Specific excess power, ft/sec
q	dynamic pressure (= 0.5 pM^2), lbf/ft^2
R	Distance traveled, ft
r_e	Radius of the earth, 20890584.0 ft
R_s	Specific range, ft/lbm fuel
r_t	Turn radius, ft
S	Reference area, ft^2
T	Net thrust, lbf
Treq	Thrust required, lbf
TSFC	Thrust specific fuel consumption, lbm/lbf-sec
V	Velocity along flight path, ft/sec

V_n	Velocity normal to flight path, ft/sec
V_R	Velocity perpendicular to local horizon, ft/sec
V_θ	Velocity parallel to local horizon, ft/sec
W	Weight corrected for altitude above sea level, lbf
W_o	Sea-level weight, lbf
W_f	Fuel consumed, lbm
α	Angle between flight path and fuselage reference line, deg
γ	Angle between flight path and local horizon, deg
γ_s	Ratio of specific heats, 1.4 for air
ϕ	Bank angle, deg
$\dot{\psi}$	Turn rate, deg/sec