

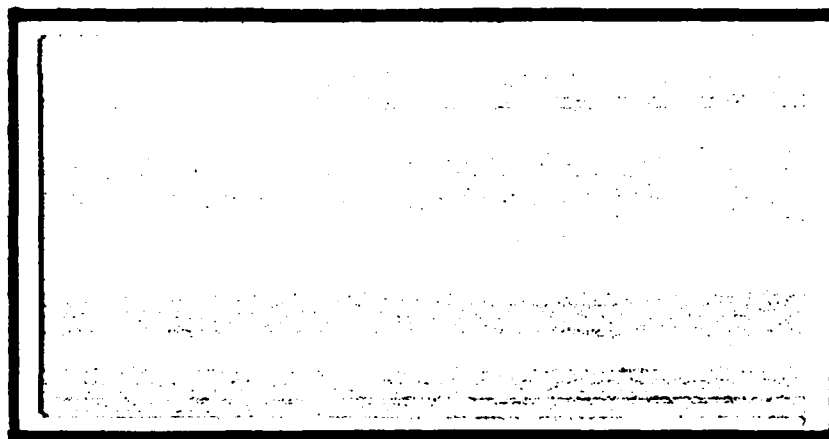
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A FINITE ELEMENT ANALYSIS OF POROSITY
EFFECTS ON MATERIALS

THESIS

Paul R. Woodmansee
Captain, USAF

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A FINITE ELEMENT ANALYSIS OF POROSITY
EFFECTS ON MATERIALS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Aeronautical Engineering

Paul R. Woodmansee, B.S.
Captain, USAF

December 1989

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Preface

The purpose of this thesis was to develop a finite element model, and from there develop an empirical equation, that could be used to predict a porous materials strength based on the average pore size and volumetric porosity. Such a model would allow a material designer to evaluate how strong a porous material would be, before producing a sample and doing extensive testing.

I would like to thank Captain H. Gans who's assistance, especially in learning the ins and outs of the computer code, was indispensable. I am indebted to Dr. A. Palazotto for his suggestions and encouragement to pursue this topic. I would also like to thank Dr R. Ruh of the Air Force Materials Laboratory for suggesting pertinent references when I was stuck and needed help.

Paul R. Woodmansee



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Table of Contents

	Page
Preface	ii
List of Figures	v
List of Tables	vii
List of Symbols	viii
Abstract	x
I. Introduction	1
II. Objective	3
III. Background	5
Elastic Modulus versus Porosity Equations . . .	5
Linear Equation	5
Empirical Exponential Equation	6
Semi-Empirical Equation	7
Empirical Power Equation	11
Stress and Strains about Pores	12
IV. Model Creation	15
Choosing Elements	15
The Model	19
Superelements	21
Primary Cell Configurations	27
Porous Cell Distribution	31
Evaluating Test Results	34
Global Local Analysis	36
V. Results	38
Stresses	38
Cobalt Oxide	45
Silicon Carbide	52
Aluminum Nitride	52
Derived Equation	57
Linear Derived Equation	67
Plasticity	78
VI. Conclusion	80

Appendix A: Sample Computer Model Input Data File . . .	82
Appendix B: Computer Model Runs, Input Data & Results .	98
Bibliography	135
Vita	137

List of Figures

Figure		Page
1	Heterogeneous Material Divided into Cells	8
2	Individual Cell	8
3	Kitagawa's Finite Element Models	13
4	Resulting Pore Deformations and Strains From Kitagawa's's Models	13
5	Test Model	16
6	Test Model, Isometric view	17
7	Test Model Displacement	18
8	20 Noded Single Pore Cell, Displaced	20
9	The Final Computer Model Used	22
10	Cobalt Oxide, x200 Magnification	23
11	Steps Followed in Superelement Analysis	24-25
12	A Single Pore Cell	28
13	A Double Pore Cell	29
14	A Cell with Three to Eight Pores	30
15	Single Pore Cell Displacement	39
16	Single Pore Cell Stress Contour	40
17	Double Pore Cell Exterior Stress Contour	41
18	Double Pore Cell Interior Stress Contour	42
19	Three Pore Cell Exterior Stress Contour	43
20	Three Pore Cell Interior Stress Contour	44
21	Four Pore Cell Exterior Stress Contour	46
22	Four Pore Cell Interior Stress Contour	47
23	Five Pore Cell Exterior Stress Contour	48

List of Figures (Continued)

Figure		Page
24	Five Pore Cell Interior Stress Contour	49
25	Cobalt Oxide Elastic Modulus vs Porosity	51
26	Silicon Carbide Elastic Modulus vs Porosity	53
27	Aluminum Nitride Elastic Modulus vs Porosity	55
28	5 Micro Meter Pore, AL N Elastic Modulus vs Porosity	58
29	8 Micro Meter Pore, AL N Elastic Modulus vs Porosity	59
30	10 Micro Meter Pore, AL N Elastic Modulus vs Porosity	60
31	15 Micro Meter Pore, AL N Elastic Modulus vs Porosity	61
32	Relationship Between Parameter A and Pore Size .	64
33	Relationship Between Parameter D and Pore Size .	66
34	Cobalt Oxide Elastic Modulus vs Porosity	68
35	Silicon Carbide Elastic Modulus vs Porosity	69
36	Relationship Between Parameter a and Pore Size .	72
37	Cobalt Oxide Elastic Modulus vs Porosity	74
38	Silicon Carbide Elastic Modulus vs Porosity	75
39	5 Micro Meter Pore, AL N Elastic Modulus vs Porosity	76
40	10 Micro Meter Pore, AL N Elastic Modulus vs Porosity	77
41	Representative Flexural Stress-Strain Behavior of Kyocera Sintered SiC	79

List of Tables

Table		Page
1	Microstructure Data for Aluminum Nitride	54
2	Equation 19 Parameters for Aluminum Nitride . . .	63
3	Slopes for Various Pores Sizes	70
4	Equation 24 Verification for Aluminum Nitride . . .	73

List of Symbols

A	constant, a parameter in the semi-empirical equation
A	area over which force is applied
a	slope, constant, a parameter in the linear equation
b	constant, a parameter in the empirical exponential equation
D	a parameter in the derived empirical nonlinear equation
E	Elastic modulus (Young's modulus)
E ₀	Elastic modulus without any porosity
E ₁	Elastic modulus of a dispersed substance
E _i	Elastic modulus for data point i
e	the Napier number, 2.71828...
F	applied force
L	original length before deformation
L	pore size (length, diameter, etc.) in micro meters
m	constant, a parameter in the empirical power equation
n	number of data points taken
P	volume fraction porosity
P(x)	probability of there being x pores in the cell
P _i	porosity for data point i
P ₁	a constant
P ₂	a constant
P ₃	a constant

List of Symbols (Continued)

V_1	volume of average cell with one pore
V_p	pore volume
$V_{\bar{r}}$	volume of actual cell used
x	variable, parameter of probability
x	variable, number of inclusions in the cell
$\bar{x}$	average number of pores per cell
z	1,2,3,...
ΔZ	deformation
ϵ	linear normal strain
σ	linear normal stress

Abstract

Porosity in materials greatly reduces the strength and load carrying characteristics of the material. Porosity is unavoidable in some materials (particularly ceramics) and is sometimes desirable for other reasons, such as radar reflection properties, diffusing a fluid through a material, and adjusting the heat transfer properties. In the past, the effects of porosity on material properties, particularly a materials modulus of elasticity (Young's modulus), have been determined, by fitting test data to one of three theoretical equation forms (linear, empirical exponential, and Hassin's semi-empirical equation, whichever fit the data best). These analytical equations all require that the effects of porosity on material properties be determined experimentally for several cases, and then other cases can be extrapolated. This limits accurate prediction before a material can be fully analyzed and produced.

In this thesis a finite element model using MSC/NASTRAN is developed that can numerically determine the material's modulus of elasticity using the limited information from one material sample. The model is three dimensional, and simulates pores by placing small elements that are non-load bearing into the structure. These voids are randomly, and unevenly, distributed (using a Poisson distribution) to better simulate the response of a real porous material to a load. How much this porous model deforms can be used directly to calculate the porous modulus of elasticity. The model is shown to be repeatedly accurate.

The same finite element model is used to demonstrate the effects

on material behavior of changing the pore size and distribution. From this information a trend was noticed and an empirical equation developed that predicted a materials elastic modulus based on porosity and pore size.

I. Introduction

Porosity exists in almost all materials to some extent. Therefore the effects of porosity should be of concern to any material developed for a design. Porosity is of particular interest to a designer who plans to use ceramic materials, because it is unavoidable in ceramics. Sometimes porosity is desirable for other than structural reasons such as heat transfer properties, easier production (bringing lower cost), radar reflection, *et cetera*.

When dealing with porous materials, there are two structurally oriented concerns: (1) the onset of a pore growing into a crack, and (2) the macroscopic effects of the the pores on material strength. This thesis is concerned primarily with the macroscopic effects of porosity. The growth of pores into cracks will be discussed only briefly for completeness. The modulus of elasticity, also called Young's modulus, is the major parameter of interest when looking for the macroscopic effects of porosity. Therefore, this thesis will deal primarily with the effects of porosity on Young's modulus.

Porosity is defined as the fraction of volume occupied by non load carrying voids in the material. Past work on the effect of porosity on Young's modulus (which will be talked about in detail in Chapter Three) dealt with fitting a curve to actual data for a specific material. Thus for every material one had to make dozens of samples, each with different porosities, and then try and fit a curve to the data points to show how Young's modulus varied with porosity. While this method has proven to

be reasonably accurate, it is also time consuming, requiring extensive lab work. Also, there was no way for a material engineer to predict the results of changing an untested material's porosity without actually producing the material and testing it. Furthermore, previous work did not adequately predict the effects of changing the pore sizes, or distribution.

This thesis is an attempt to create a finite element model that simulates a porous material. With this finite element model the porosity, pore size, and distribution can be varied to see the effects on the overall material. The finite element model uses a common "off the shelf" computer code, MSC/NASTRAN. The computer model used in this thesis only used linear elastic deformations, but it can be extended into the nonlinear regime. Even though plastic deformation and microcracking are significant effects on the microstructure, it will be apparent that a reasonably accurate model of the macroscopic effects of porosity can be made using only linear elastic calculations.

II. Objective

The ultimate objective of this thesis was to devise a method to predict the effect of porosity on a material's modulus of elasticity. This method was applied to another material to predict the effect of porosity. Finally, the analytical prediction was checked against a fully tested material.

Several predictors (i.e. equations showing how Young's Modulus varied with porosity) existed already, but each relied on testing the material at various porosities, and fitting them to a curve. None of the previously used equations, (18:458-459 and 7:327-329) (Chapter Three) took in to account pore size or distribution.

The first step in coming up with a predictor was to create a finite element model that simulated the microstructure of a porous material. Previous finite element models of porous materials, (11:909-913 and 6:2-15), used two dimensional circular voids evenly spaced in a material. Instead, a three dimensional model, with cubic voids, in a random and uneven distribution was chosen for this analysis. Also, unlike previous works, linear elastic deformation was chosen for this analysis. Cracking and plastic deformation were not considered. Fortunately the model selected, after some refinement, did a reasonably good job of predicting the properties of the studied materials.

The model was then used to look at various aspects of porous material behavior. Stress concentrations, the effects of changing the

pore distribution, the effects of varying the pore size, and an equation to estimate these effects on the modulus of elasticity were all examined.

The equations derived from this thesis allow an engineer to get an estimate of the effects that porosity will have on a material. However while the results are reasonable, there is still the possibility of error in the estimates. Some lab testing should be done to confirm the theoretical results.

III. Background

This section discusses some of the previously used equations for Young's modulus as a function of porosity, and how they were developed. Volumetric porosity is determined by comparing the theoretical material density with the actual total density. Also discussed in this chapter is the stresses and strains acting on a pore, and the effects of void growth and coalescence.

Elastic Modulus versus Porosity Equations

Linear Equation. The linear relationship between porosity and modulus of elasticity, Young's modulus, is the oldest and still the most commonly used expression. The following equation is most common way to express this linear relation (16:78-79 and 18:458-459);

$$E = E_0(1 - aP) \quad (1)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

a = slope, an empirical constant

P = volume fraction porosity

The values of a (slope) and E_0 (zero porosity modulus) are determined experimentally by taking several material specimens, each of

different porosity, and finding the elastic modulus. The accuracy of this linear equation is often quite good. For example: in Petrak (16:78-79) eight tests of porous cobalt oxide (CoO) were done. After doing a least squares curve fit using Equation (1) he found values for a and E_0 that fit the data with only a 2.56% error. It is worth noting that Petrak (16:78-79) went on to curve fit the data to the empirical exponential, and semi-empirical equations (discussed below) and came up with even smaller errors, but returned to the linear equation because of its simplicity and accuracy.

Empirical Exponential Equation. While investigating the microstructure features on the mechanical properties of hot-pressed ceramic oxides, Spriggs and Vasilos (21:187) found that the effects of porosity on the elastic modulus for alumina and magnesia fit very closely to an exponential expression. The exponential expression was of a form previously suggested by Knudsen (12:376-387) and by Duckworth (4:68). The expression is:

$$E = E_0 e^{-bp} \quad (2)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

e = the Napier number, 2.71828...

b = an empirical constant

P = volume fraction porosity

Since it was first introduced, the empirical exponential expression has been used extensively by people trying to correlate material strength with porosity. For some materials it is an excellent fit, for others the error is more significant and one of the other equations must be used to try and curve fit the solution.

Semi-empirical Equation. Unlike the linear and empirical exponential equations, which were primarily guesses of a curve that seem to fit the data, this method has some basis in a derivation. It was originally deduced by Hasselman (10:452-453) from work done by Hashin (9:143-150).

Hashin, (9:143-150), wanted a theoretical expression for the change in a material's overall elastic modulus if it were really a two material substance. Hashin assumed that on the microstructure level, a primary material made up most of the volume, and a secondary material occupied small islands in this sea of prime material. Next Hashin broke the material into smaller volumes called cells, where each cell had exactly one "island" of secondary material inside of it (Figure 1). Finally Hashin assumed that each cell was spherical, and the secondary material inside of it was also spherical, and body centered (Figure 2). This last assumption was needed to allow any work to be done. However, this assumption is untrue, since it becomes obvious that a group of spheres will not fit together to make up a solid material.

Once these assumptions are made the problem has radial symmetry. Hashin then derived an expression for the modulus of elasticity for

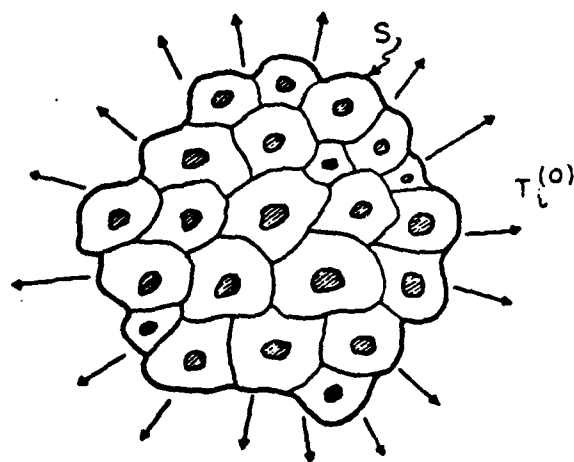


Figure 1. Heterogeneous Material Divided into Cells

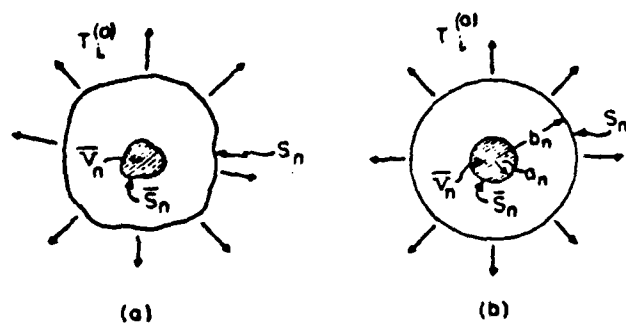


Figure 2. Individual Cell

a continuous phase containing a dispersed phase (the "islands") of spherical particles, where each phase had a different, but known, modulus of elasticity. The equation can be written in the form;

$$\frac{E}{E_0} = 1 + \frac{A \left(1 - \frac{E_1}{E_0} \right) P}{1 - (A + 1) \left[\frac{E_1}{E_0} + \left(1 - \frac{E_1}{E_0} \right) P \right]} \quad (3)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus of continuous substance

E_1 = Elastic modulus of dispersed substance

A = a constant (actual value dependent on many factors)

P = volume fraction of the dispersed substance to total volume

Hasselman, (10:453), took Equation 3 above and applied it to porosity. If it is assumed that the dispersed substance is a pore with no strength, compared to the primary material, then $E_1 = 0$ and result is a porous material. The equation then becomes:

$$E = E_0 \left[1 + \frac{AP}{1 - (A + 1)P} \right] \quad (4)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

A = a constant

P = volume porosity

Since then common usage of this equation has been changed, because A in Equation 4 was always negative. Letting $A = -A$ gives the more common equation, (16:78-79 and 7:327-329):

$$E = E_o \left[1 - \frac{AP}{1 + (A-1)P} \right] \quad (5)$$

where

E = Elastic modulus (Young's modulus)

E_o = Elastic modulus without any porosity

A = a constant

P = volume porosity

Equation (5) then is the semi-empirical equation used to estimate the effects of porosity on elastic modulus. The constants E_o and A are determined from the experimental data. This can be done most easily by using the following matrix equation derived by Hasselman (10:453) to best fit data to this curve:

$$\begin{pmatrix} E_o \\ A \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n \frac{1}{E_i^2} & \sum_{i=1}^n \frac{P_i}{E_i(1-P_i)} \\ \sum_{i=1}^n \frac{-P_i}{E_i(1-P_i)} & \sum_{i=1}^n \frac{-P_i^2}{(1-P_i)^2} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n \frac{1}{E_i} \\ \sum_{i=1}^n \frac{P_i}{1-P_i} \end{pmatrix} \quad (6)$$

where

n = number of data points taken

E_i = Elastic modulus for data point i

P_i = volume porosity for data point i

E_0 = Elastic modulus without any porosity (unknown being solved)

A = a constant (unknown being solved)

While the semi-empirical Equation 5 is difficult to deal with, it does seem to be more accurate than the linear and empirical exponential relationships. However, once again no provision is made to see the effects of changing pore size and/or distribution.

Empirical Power Equation. For completeness, there is one last equation that is used to correlate porosity with elastic modulus. In a Soviet paper by Bubnov (2:89-92) the following empirical equation derived by M. Yu. Bal'shin (1:166-172) was recommended:

$$E = E_0(1 - P)^m \quad (7)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

m = a constant

P = volume porosity

Again the value of the empirical constants m , and E_0 are derived from testing. In Bubnov (2:89-92) this equation was found to closely correlate to the tested effects of porosity on the elastic characteristics of Silicon Nitride (Si_3N_4). The maximum difference between the predicted elastic modulus and the tested value of elastic modulus was less than 3%.

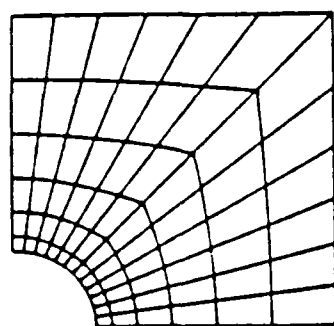
Stresses and Strains about Pores

Several researchers (mentioned below) have done finite element analysis of pores. They were primarily concerned with modeling one pore and the stresses and strains about it, or sometimes two pores and there interaction.

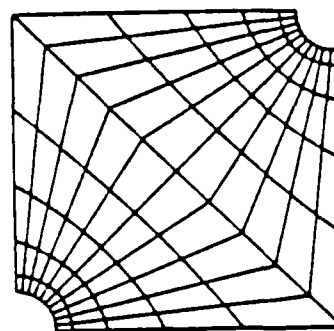
The first such study was done by McClintock (15:363-371) in which he looked at an isolated void in an infinite region of non hardening rigid plastic material. He assumed an infinite cylindrical pore so that he could do a two dimensional study. McClintock found that void growth rate for this case depends exponentially on the mean in plane normal stress. Later Rice and Tracy (17:201) treated the case of a spherical void in an infinite region of non hardening rigid plastic material. They found an exponential dependence of void growth rate on the mean normal stress.

Gurson (6:2-15) wanted to account for void interaction in an approximate way, so the voids were placed in a finite cell. For spherical voids this cell was a cube, and for infinite cylindrical voids (2 dimensional analysis) this cell was a square. Gurson then modeled the material as rigid and perfectly plastic, and approximated the local velocity field as a linear function of the macroscopic strain rate. Gurson obtained some extensive results for the yield functions of a porous solid.

More recently, using modern computational methods, Kitagawa and Honke (11:909-913) did a finite strain numerical analysis to investigate anisotropy and softening due to void growth. They used a two dimensional circular cell model much like Gurson's though unlike the Gurson model, it was not isotropic.



(a) Model A

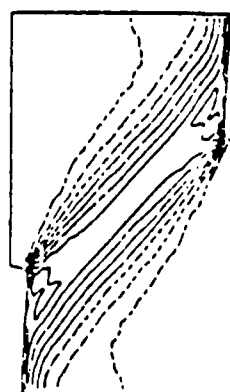


(b) Model B

Figure 3. Kitagawa's Finite Element Models



Model A



Model B

Figure 4. Resulting Pore Deformations and Strains

Figure 3 shows the finite element model used, and Figure 4 shows the strain distributions for this model under two different loading conditions. The results from model A show that the area surrounding the pore does plastically deform, and model B shows large stress concentrations and strain deformation between two pores.

When the strain deformation between two pores cause the stress to reach its critical yield point, the material will elongate and the two pores will coalesce into one large oval shaped pore. If the pore is long enough, then it can be considered a microcrack and crack growth theory applies. Similarly, if an isolated pore is deformed enough, it can be the beginning of a microcrack.

IV. Model Creation

Choosing Elements

Before constructing a finite element model of a porous material, it was necessary to determine what solid elements would give the most accurate results for this situation. Therefore, a conversion study was carried out similar to a small version of the final model. Forces were applied for which expected displacements could be calculated. If the actual displacements matched the theoretical, then the elements were assumed to be correct.

Figures 5 and 6 are two views of the test model used. The model consists of six cubic elements (MSC/NASTRAN CHEXA elements) arranged in a cube, called a cell, with a smaller cube inside of it. These seven elements make up a superelement which is copied seven times to make up a larger cubic model. The images of the superelement do not show up on Figure 5 and 6 which show the entire model.

The points originally on the $Z=0$ plane are constrained to the $Z=0$ plane, but can have X and Y displacements, i.e. on rollers. An upward z force is applied in some cases to the corners of the model, and in other cases as a pressure to the entire upper model surface. The whole model then displaces (Figure 7).

The 20 node CHEXA element was tried in the model first. MSC/NASTRAN forced the use of 3 point Gaussian integration.

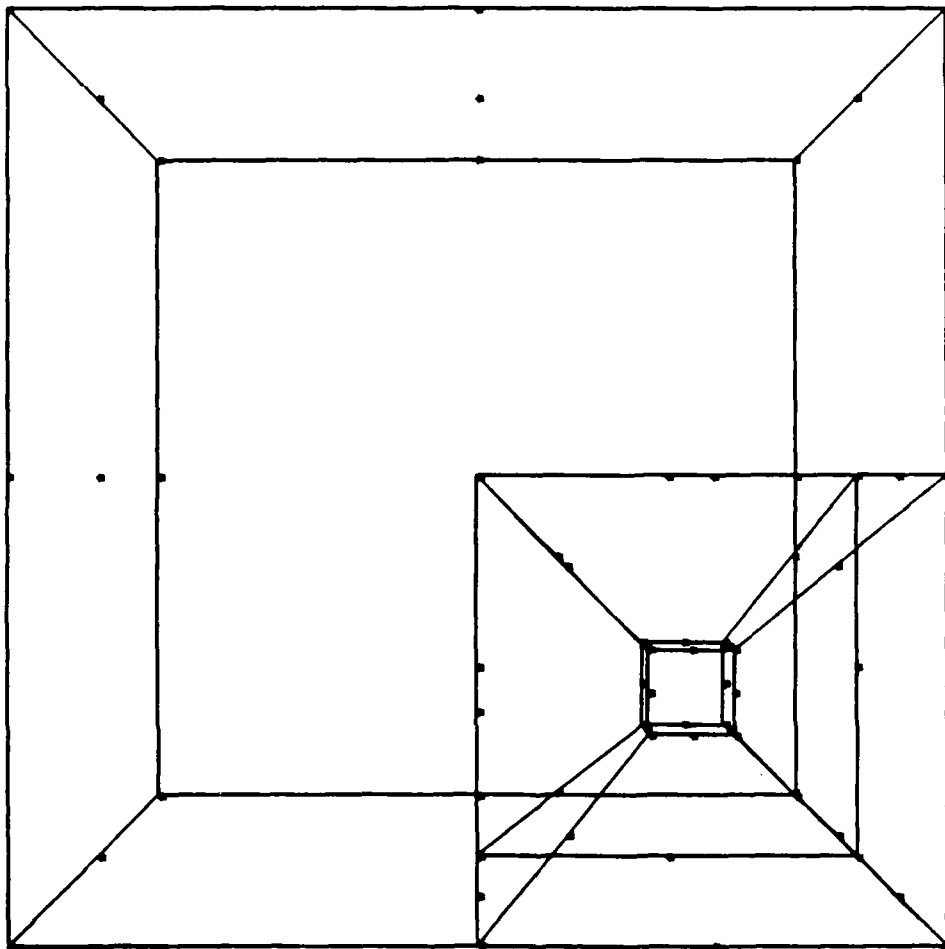


Figure 5. Test Model (Ref 20)

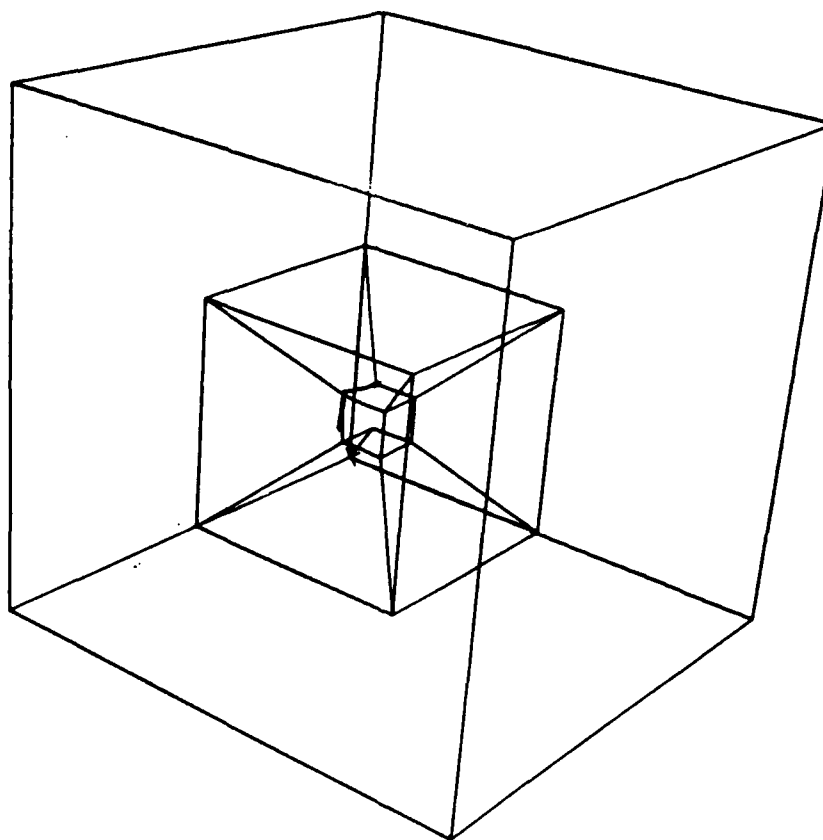


Figure 6. Test Model, Isometric View

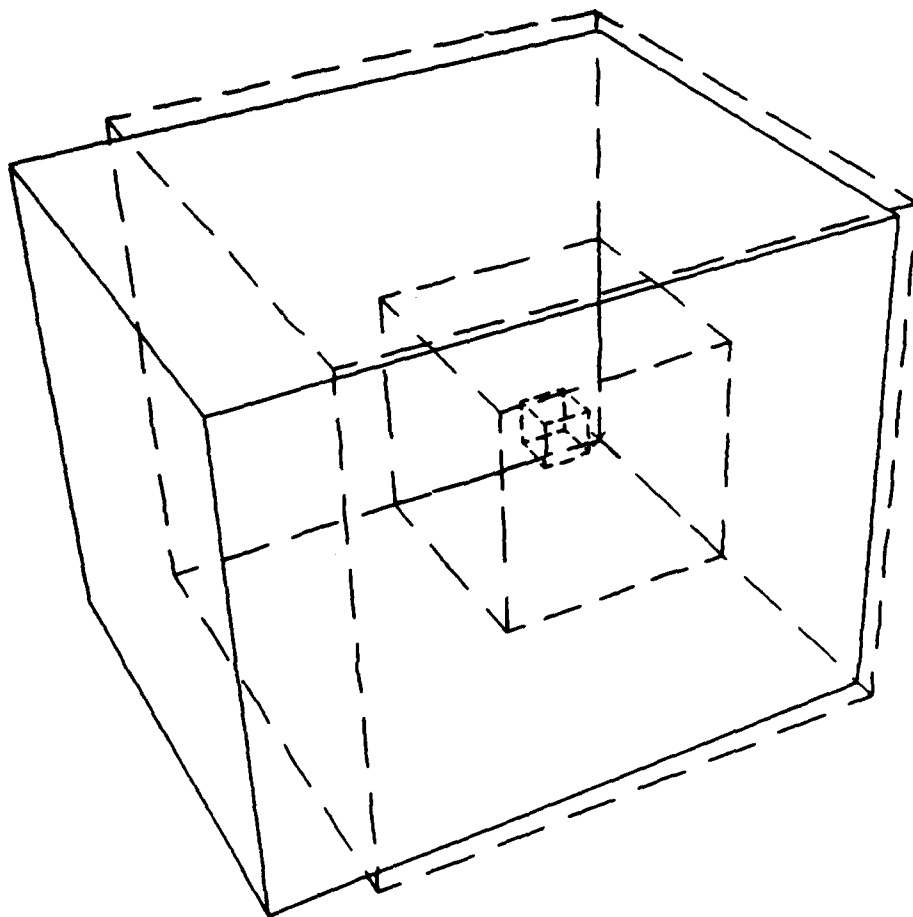


Figure 7. Test Model Displacement (Ref 20)

Since the center cube of the cell had the same material as the rest of the model (i.e. model a solid cube) one can easily calculate how much linear displacement in the Z direction is expected. The 20 node elements did not displace enough; they were too stiff. Next, the central cube was made into a pore by giving it an elastic modulus (E) much smaller than the material's. This caused the corners of a cell to displace too much, and the sides not to displace at all (see Figure 8). In short, a 20 node CHEXA element was not adequate.

Next, eight node CHEXA elements were tried. For the solid cube case the computer results matched the theoretical results exactly for both the corner forces and surface forces case. Also, when the central cube of each cell was made into a pore, by eliminating it and creating a void, the model deformed a bit more than the solid block case, as expected. The eight node model gave the identical results whether one, two, or three point Gaussian integration was used (specified with the PSOLID card). Therefore the default one point integration was used in the final model.

The Model

The model used in the main part of the analysis is similar in many ways to the test model used above. The overall model is a block, and the size varied with each test case (Appendix B). This block was divided into 80 cubes called cells. The block was four cells by four cells by five cells. More cells were put in the Z direction, as that was the direction force would be applied. Like the test case, the model was fixed to the Z=0 plane, and a distributed force applied to the top plane of the model.

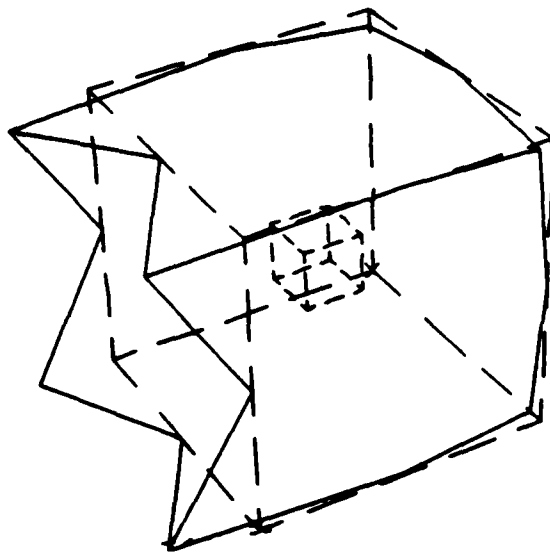


Figure 8. 20 Noded Single Pore Cell, Displaced (Ref 20)

Each cell was then made by an image superelement of one of the primary superelements. The primary superelements were a collection of elements simulating zero, one, two, or more pores inside of a cell. Figure 9 shows the overall model with only the primary superelements shown. The image superelements are placed in the cubes created by each of the residual data points shown in Figure 9.

Unlike previous models which assumed all pores to be round, this model assumed all pores to be cubes. This is done primarily to simplify the model. However, Figure 10 shows a picture of a porous material (cobalt oxide) and it is evident that the pores (black areas) are irregular angular shapes situated between grain boundaries. Therefore an argument can be made that pores are more accurately (or at least as accurately) simulated by cubes rather than by spheres.

Superelements. To simplify processing time, this model makes extensive use of superelements. A superelement is a collection of elements that make up some substructure of the whole model. In this case the substructure is the cell, so the whole model is partitioned into a number of cells.

Superelements are processed in the following way by MSC/NASTRAN. Use Figure 11, based on material from the MSC/NASTRAN Handbook for Superelement Analysis (5:1.1-2,1.1-3), to follow the steps taken to process a superelement model.

- 1) First the overall model is partitioned into the

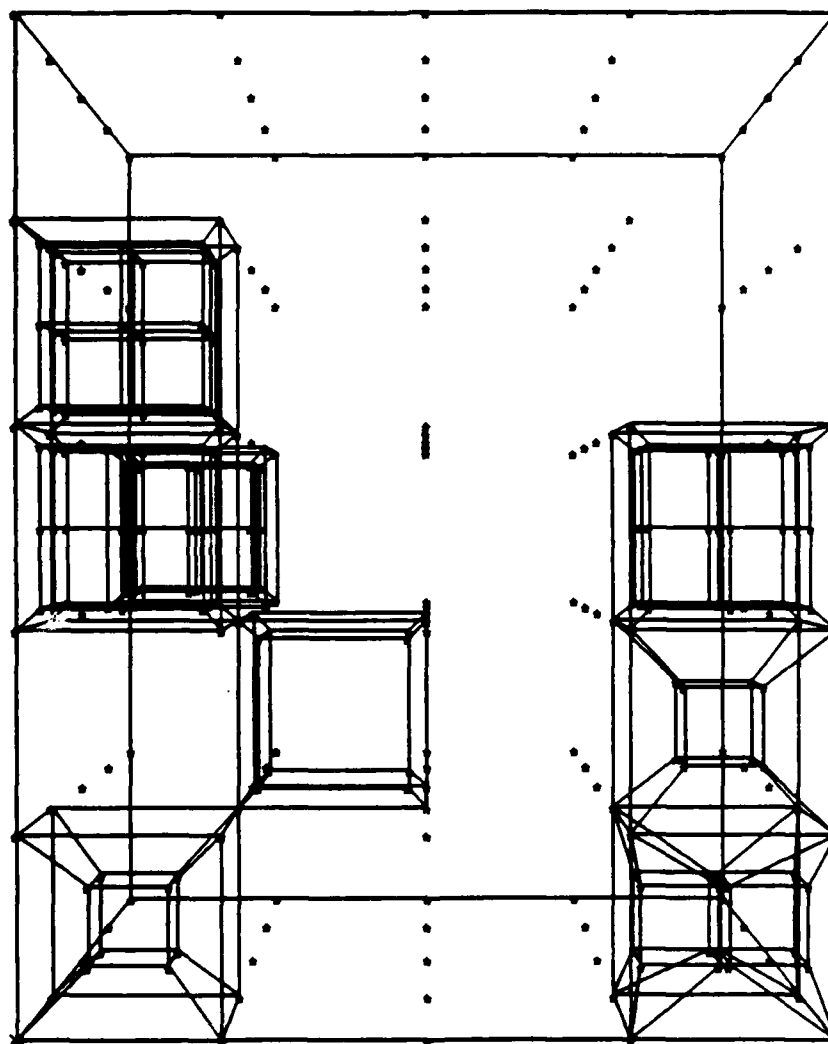


Figure 9. The Final Computer Model Used (Ref 20)

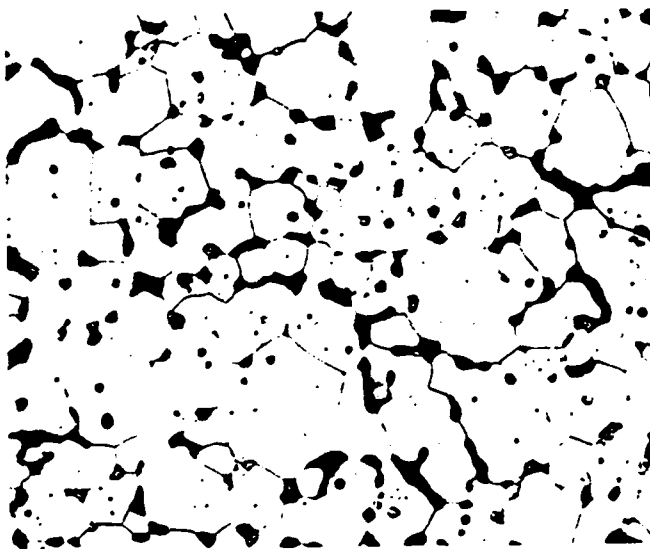
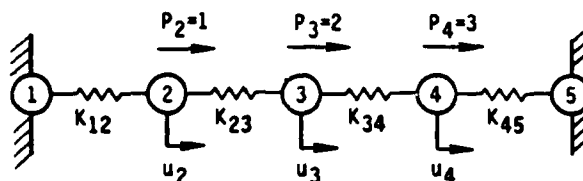


Figure 10. Cobalt Oxide, x400 Magnification

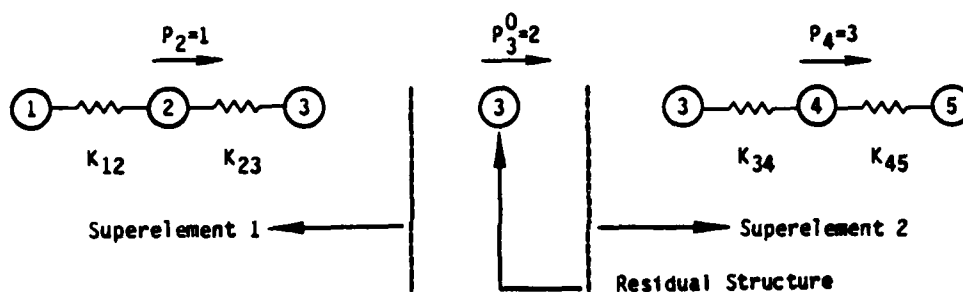
A. Conventional Analysis



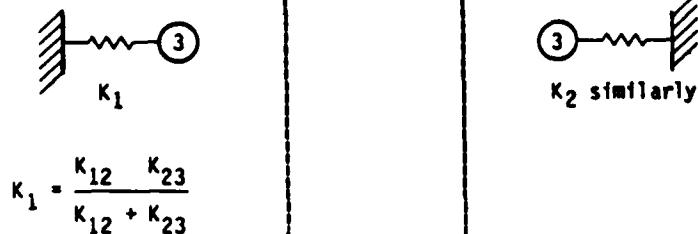
Given the loads P_i , find the resulting displacements u_i . Assume all springs have unit stiffness.

Superelement Analysis

B. Generation



C. Stiffness Reduction

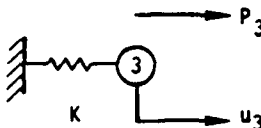


D. Load Reduction



Figure 11. Steps Followed in Superelement Analysis (Ref 5)

E. Assembly and Solution



$$P_3 = P_3^0 - P_3^1 - P_3^2 = 4$$

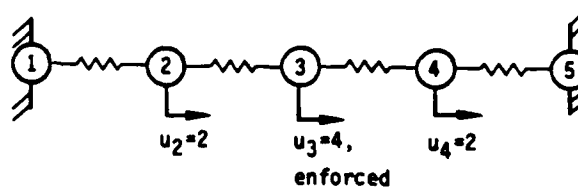
$$K = K_1 + K_2 = 1.0$$

$$u_3 = \frac{P_3}{K} = 4$$

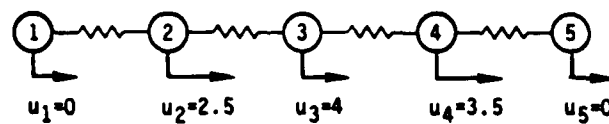
F. Data Recovery



Fixed Boundary Component, Plus



Enforced Boundary Motion Component, Equals



Total Solution

Figure 11. Steps Followed in Superelement Analysis (cont. Ref 5)

various superelements and residual data points, step B in Figure 11. The residual data points are points that don't belong to any superelement, but are used to connect superelements together.

2) All the points and elements in a superelement are then reduced to one equivalent stiffness matrix, step C in Figure 11. In this reduction operation each superelement is treated as a complete, disjointed structure, and solved independently of the rest of the structure using standard finite element matrix reduction techniques.

3) Any load on the superelements is then reduced to either a boundary load, or an equivalent load on a connecting residual data point. Figure 11 step D shows the boundary loads P_1 and P_5 being solved for.

4) Then the equivalent stiffness matrix for all superelements that connect to each residual data point are summed to get an equivalent residual structure, step E in Figure 11. Any load applied to a residual data point is applied. The displacement on all the residual data points, and thus the entire residual structure, can be easily solved using conventional finite element matrix manipulations.

5) The last step is full data recovery, step F in Figure 11. Here each partial solution due to reducing the loads, step D, is added to the partial solution due to the enforced motion of the residual data points, part E. This produces a solution for the motion of each data point that is identical to the solution produced by conventional analysis, shown in part A of Figure 11.

In this paper's model, all of the loads and boundary restraints are applied to residual data points, so steps D and F of Figure 11 are

superfluous. Once the equivalent stiffness matrixes for each superelement are found (step C) they can immediately be summed up for each residual data point and residual data point displacements found (step E).

Image superelements are superelements that are repeated several times in the model. This reduces calculations because once the primary superelement has been solved, each of the image superelements can use the exact same equivalent stiffness matrix. In the model used in this thesis there are 80 cells but only a few of them (9 or less) are primary superelements, the rest are simply copies of one of the primaries. This greatly reduces computer run time.

Primary Cell Configurations. Each of the primary cells had a number of pores inside of it. The solid cell and the one pore cell are identical (Figure 12), except the one pore cell had a different material in central cube that has a modulus of elasticity one million times less, making it act like a non load bearing pore.

The two pore cell is depicted in Figure 13. The two pores in this cell are so close together that the slab between them will possibly be in plastic deformation if the load is high. However, this effect was ignored in this model because the data on the tested material's plastic deformation was unavailable. The final results, shown later, are accurate enough that no further correction was needed.

For the case where three to eight pores are packed into a cell, it becomes likely that the pores are touching. Figure 14 shows a cell with eight pore sized cubes inside of it. For each of the three to eight pore cases, a number of the eight internal cubes equal to the number

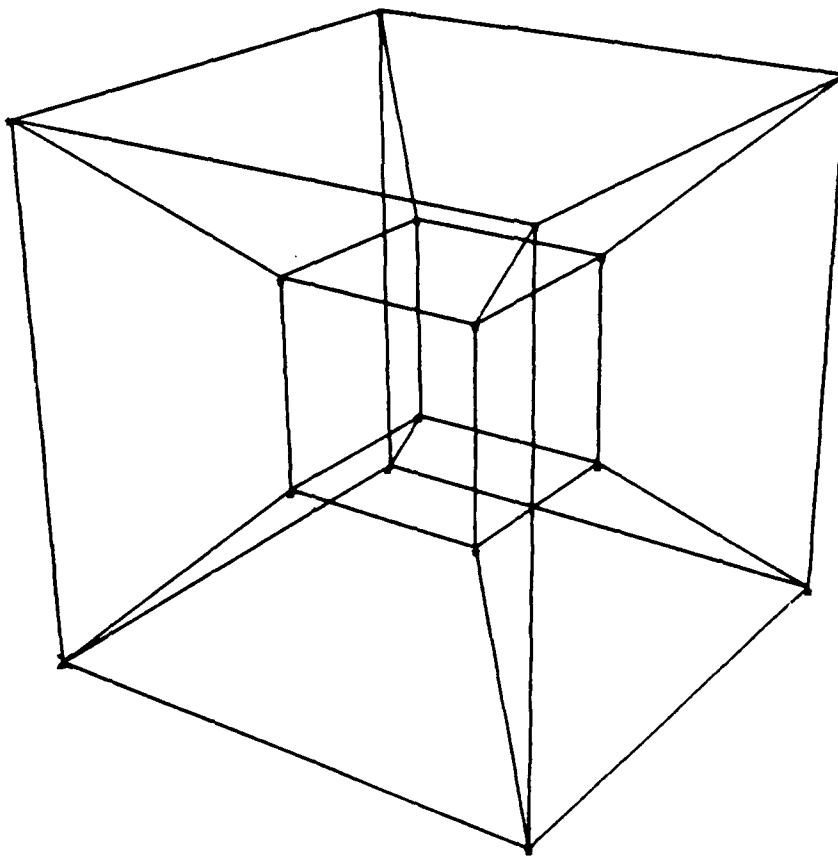


Figure 12. A Single Pore Cell (Ref 20)

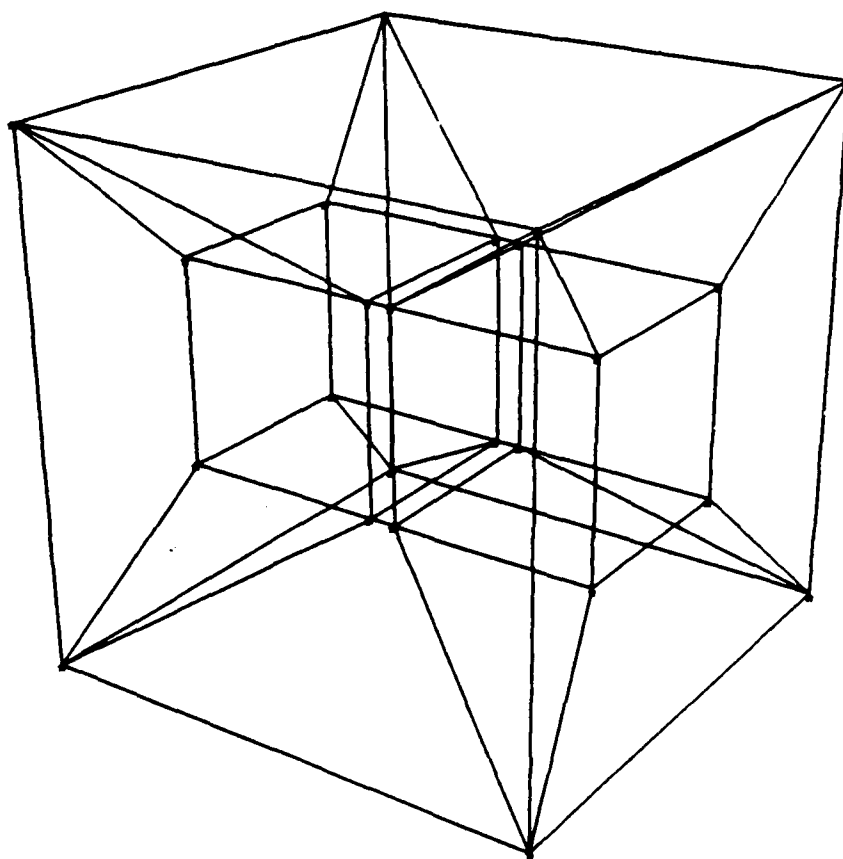


Figure 13. A Double Pore Cell (Ref 20)

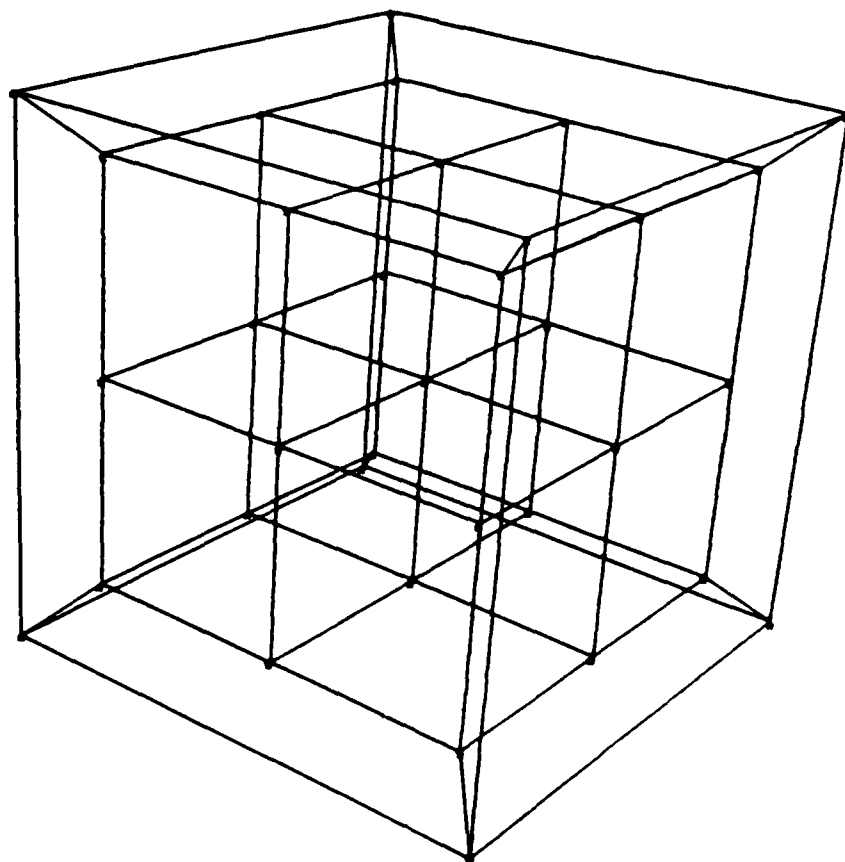


Figure 14. A Cell with Three to Eight Pores Inside (Ref 20)

of pores were assigned a material with almost no stiffness to simulate a pore. In the case of the eight pore cell, all eight central cubes have no stiffness, making the eight pore cell a cell with one large internal pore.

Porous Cell Distribution. Now that cells with different number of pores in them can be simulated, how should the model be populated? For some of the models an even distribution of one pore per cell was tried, but for most cases a Poisson distribution (explained below) was used.

In Haight's book Applied Probability, "A Poisson distribution is formed by normalizing the terms in the exponential sum to unity (8:8)." To do this one takes the exponential summation formula:

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \dots \quad (8)$$

where

x = parameter of probability

e = the Napier number, 2.71828...

Then divides it into discrete probabilities which must, by definition, sum up to 1. The discrete probabilities are therefore written:

$$\left(e^{-x}, x e^{-x}, \frac{1}{2} x^2 e^{-x}, \frac{1}{3!} x^3 e^{-x}, \dots \right) \quad (9)$$

where the symbology is the same as Equation 8 above.

Usually all terms in a probability distribution can be represented by

a single formula with a dummy variable introduced for compact notation.

This is true for Poisson distribution which can be written:

$$\frac{x^z e^{-x}}{z!} \quad (10)$$

where

$z = 1, 2, 3, \dots$

x = parameter of probability

e = the Napier number, 2.71828...

According to a paper by R.D. Thomson and J.W. Hancock which discussed the effects of non-homogeneous void distributions, "The probability that any cell selected at random will contain x inclusions is given by the Poisson distribution (22:108-109)." In the case of a cell with x inclusions in it, Thomson and Hancock (22:108-109) used the following Poisson distribution equation:

$$P(x) = \frac{\bar{x}^{(x-1)}}{e^{\bar{x}}(x-1)!} \quad (11)$$

where

$P(x)$ = probability of there being x pores in the cell

x = number of inclusions in the cell

e = the Napier number, 2.71828...

$\bar{x}$ = average number of pores per cell

The average number of inclusions per cell is obtained from:

$$\bar{x} = \frac{V_{\bar{x}}}{V_1} \quad (12)$$

where

$\bar{x}$ = average number of pores per cell

$V_{\bar{x}}$ = volume of actual cell used

V_1 = volume of average cell with one pore

And the volume of the average cell with one pore is:

$$V_1 = \frac{V_p}{P} \quad (13)$$

where

V_1 = volume of average cell with one pore

V_p = pore volume

P = volume porosity

Therefore, using known information; pore size, cell size, and volume porosity; Equations 11, 12, and 13 can be used to calculate the probability of a cell having x number of pores. Once the probability distribution is known, it is a simple matter to allocate a certain number of cells in the 80 cell model to match the probability distribution. Appendix B gives the actual distributions used for each case tested.

Evaluating Test Results

MSC/NASTRAN solves all finite element models in terms of displacements. Therefore, a way was needed to turn the displacement back into a value for the modulus of elasticity. Using the linear stress-strain relationship:

$$\sigma = E \epsilon \quad (14)$$

or

$$E = \frac{\sigma}{\epsilon} \quad (15)$$

where

σ = linear normal stress

ϵ = linear normal strain

E = modulus of elasticity

The definitions of normal linear stress and strain are given by Equations 16 and 17 below:

$$\sigma = \frac{F}{A} \quad (16)$$

$$\epsilon = \frac{\Delta Z}{L} \quad (17)$$

where

σ = linear normal stress

F = applied force

A = area over which force is applied

ϵ = linear normal strain

ΔZ = deformation

L = original length before deformation

Putting the equations for stress and strain (Equations 16 and 17) into Equation 15, and applying the result to this model, you can get the following relationship.

$$E = \frac{F \Delta Z}{A L} \quad (18)$$

Where

E = calculated modulus of elasticity

F = total applied upward force to the top of the model

A = model top end area

ΔZ = displacement of the top of the model

L = model height

Equation 18 was used throughout all of the testing to calculate what the total model's modulus of elasticity was for that test. For the distributed pore cases, since the cells were distributed randomly, the final displacements on the top end of the model are uneven. The actual

top end displacements are given in Appendix B. To calculate of modulus of elasticity, the average displacements were used in the distributed pore cases.

Global-Local Analysis

In a complex structure there are areas where a fine mesh of finite elements are needed to give detailed stress and strain information, such as bolt holes, corners, etc. Such a fine mesh is unnecessary for the entire structure, which can use a course mesh. Therefore, fine mesh area's are created in local areas of interest, and the results of the course models are applied to the fine mesh as boundary conditions to obtain local effects. This is called global-local analysis (14:1).

Normally, the global-local method treats the entire structure using the Rayleigh-Ritz method but adds finite elements in regions of interest. The Rayleigh-Ritz functions need not satisfy as many boundary conditions then, because there are more elements to smooth things out in the region of interest. Global-local analysis is useful in this context when the structure is regular enough for classical Raleigh Ritz treatment, and only some parts need more detail, thus reducing the number of degrees of freedom for a given level of accuracy.

Global-local analysis is used in this thesis in a slightly different way. The computer model of this thesis is a small portion of a large piece of material, and the global local principle is used to deduce that the larger material will behave the same way as the small sample. For example; most of the tests use 80, 20 micrometer cells, making the entire

model have dimensions of only 0.8 by 0.8 by 1.0 millimeters. This is much smaller than even the smallest real test specimen, but the global local principle says that the entire structure, which is made up the same way, will deform proportionately the same as the small computer model. This saves a great amount of computer time because now the entire large structure need not be analyzed in the fine detailed way that the small piece was analyzed.

V. Results

Stresses

Before looking at the effects of porosity on the elastic modulus, it is worthwhile to examine the effects that pores have on the stress immediately around them. To do this, several sample cells have been taken out of the model and stress contours plotted using SCRD-IDEAS (20).

Figure 15 shows the displacement of a cell with a single pore (like the one show in Figure 12). Notice that the displacement is uneven due to the effects of neighboring cells. Figure 16 shows the exterior stress contour on that same cell. The effects of a small central pore are minimal.

Figure 17 shows the external stress contour of a two pore cell, similar to the one in Figure 14. Notice that the stress contour lines are much closer together indicating a higher stress gradient across the cube. Figure 18 shows the stress contour internal to the cell along the surface of the two pores. The stress builds up and is largest along the thin wedge between the two pores.

Figure 19 shows the external stress contour of a three pore cell. In sharp contrast to this is Figure 20 which shows the internal stress contours of that same cell. The three pores are the cubes with the few stress concentration lines on them.

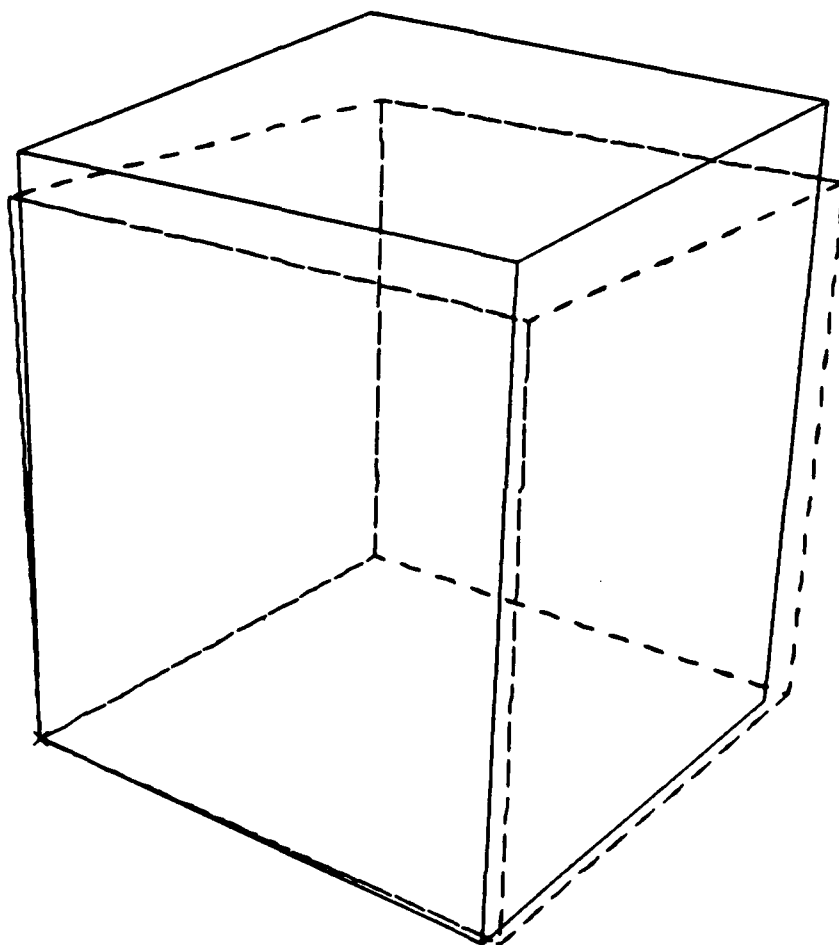


Figure 15. Single Pore Cell Displacement (Ref 20)

Single Porous Cube (Cell 1, PORE5-1)

Max. Stress = 1.66×10^{-2} GPa

Key

- 1 = 1.10×10^{-2} GPa
- 2 = 1.19×10^{-2} GPa
- 3 = 1.28×10^{-2} GPa
- 4 = 1.38×10^{-2} GPa
- 5 = 1.47×10^{-2} GPa
- 6 = 1.56×10^{-2} GPa

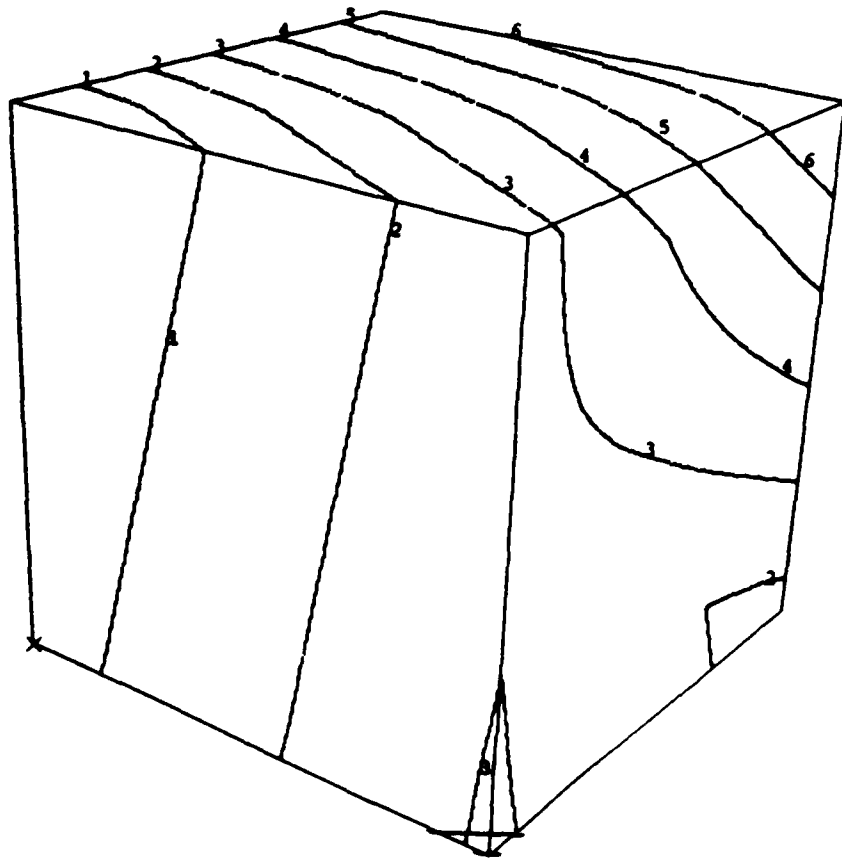


Figure 16. Single Pore Cell Stress Contour (Ref 20)

Double Pore Cube (Cell 4, PORE3-1)

Max. Stress = 1.15×10^{-2} GPa

Key

1 = 0.98×10^{-2} GPa

2 = 1.01×10^{-2} GPa

3 = 1.04×10^{-2} GPa

4 = 1.07×10^{-2} GPa

5 = 1.09×10^{-2} GPa

6 = 1.12×10^{-2} GPa

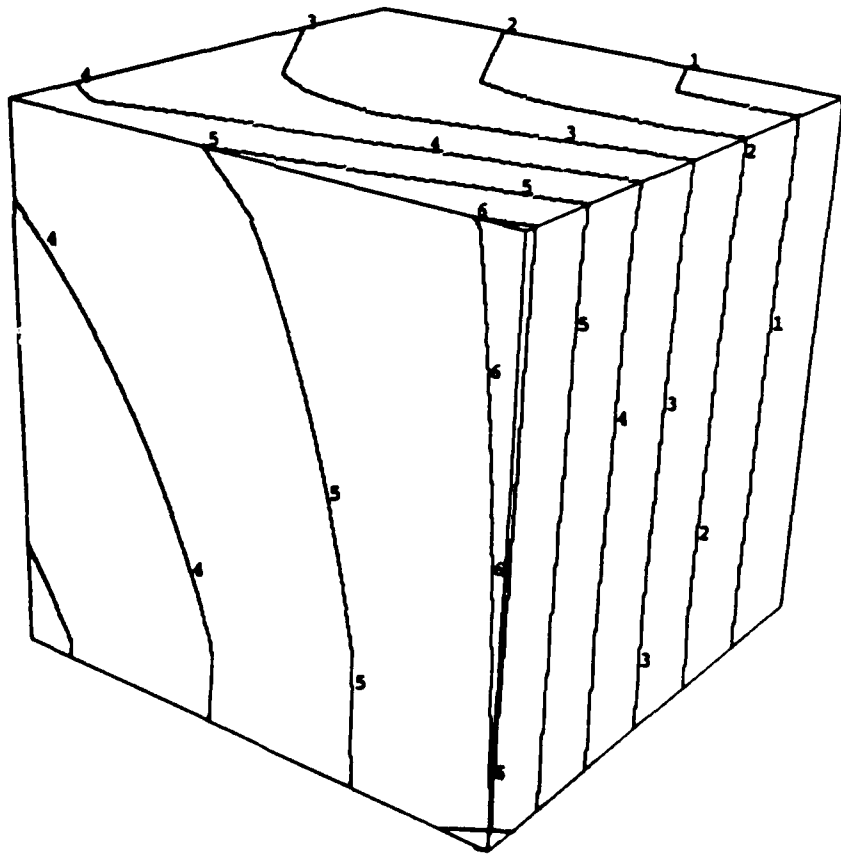


Figure 17. Double Pore Cell Exterior Stress Contour (Ref 20)

Double Pore Cube (Cell 4, PORE3-1)

Max. Stress = 1.60×10^{-2} GPa

Key

1 = 0.23×10^{-2} GPa

2 = 0.46×10^{-2} GPa

3 = 0.67×10^{-2} GPa

4 = 0.91×10^{-2} GPa

5 = 1.14×10^{-2} GPa

6 = 1.37×10^{-2} GPa

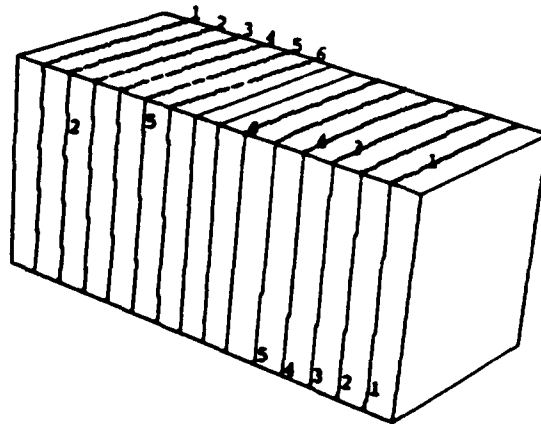


Figure 18. Double Pore Cell Interior Stress Contour (Ref 20)

Three Pore Cube (Cell 33, PORE3-1)

Max. Stress = 2.91×10^{-2} GPa

Key

1 = 0.36×10^{-2} GPa

2 = 0.79×10^{-2} GPa

3 = 1.21×10^{-2} GPa

4 = 1.63×10^{-2} GPa

5 = 2.06×10^{-2} GPa

6 = 2.48×10^{-2} GPa

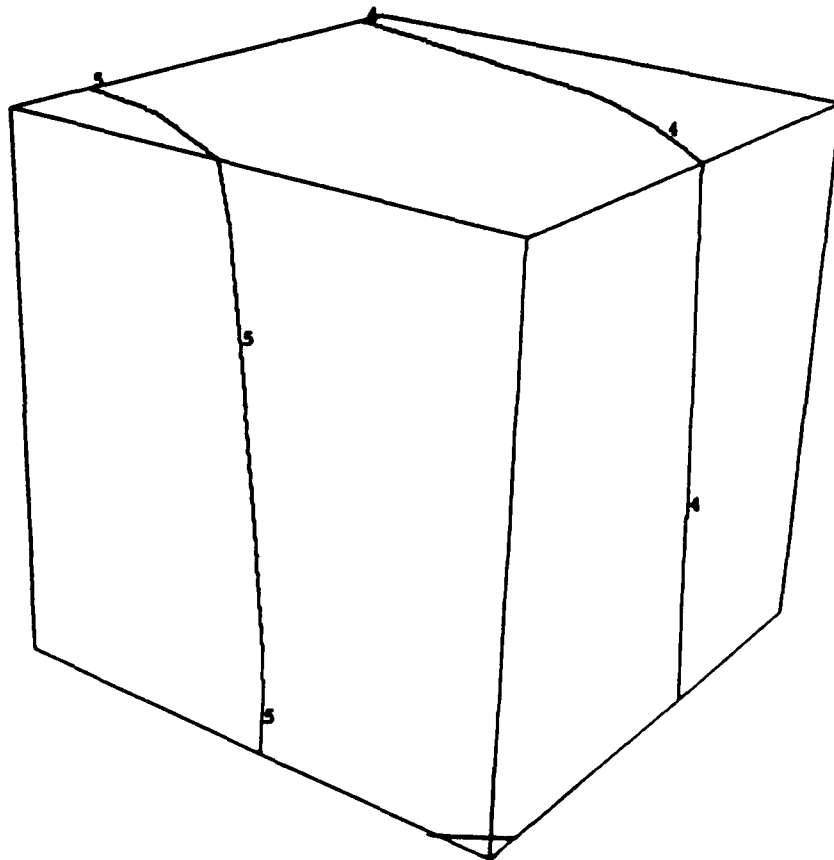


Figure 19. Three Pore Cell Exterior Stress Contour (Ref 20)

Three Pore Cube (Cell 33, PORE3-1)

Max. Stress = 3.96×10^{-2} GPa

Key

1 = 0.51×10^{-2} GPa

2 = 1.09×10^{-2} GPa

3 = 1.66×10^{-2} GPa

4 = 2.23×10^{-2} GPa

5 = 2.81×10^{-2} GPa

6 = 3.38×10^{-2} GPa

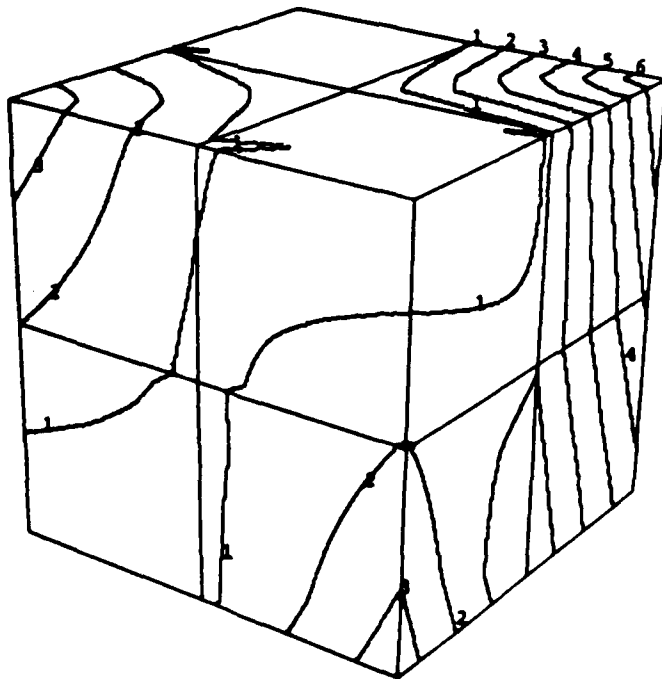


Figure 20. Three Pore Cell Interior Stress Contour (Ref 20)

Figure 21 shows the external stress contour of a four pore cell. Figure 22, the internal stress contours of the same four pore cell, shows much sharper stress gradients than Figure 21. Thus, the stress gradients close to the pores (Figure 22) are quite high, while the stress gradients less than one quarter of a pore radius away (Figure 21) are much less. The pores in Figure 22 are the four cubes with few stress lines on them, while the solid parts have a tight grid of stresses. Figure 23 and 24 show the external and internal stress contours, respectively, of a five pore cell.

The trends in each of these cases is clear enough with these examples. As the number of pores rise, both the stress and the stress gradient on the material near the pore rise dramatically, but the stress even one pore diameter away changes very slowly, if at all.

Cobalt Oxide

If the porous material computer model was going to be useful, the predicted results had to match laboratory tests for an actual material. The material initially chosen was cobalt oxide (CoO) because of the wealth of information known about it from Petrak (16:78-79). Figure 10, shown previously, is a sample of cobalt oxide at x200 magnification. From this picture one can measure the average pore size to be about 8 micrometers (i.e. 8×10^{-6} meters). This average ignores places where two pores have combined into one, since the model takes into account multiple pores that touch each other.

First, evenly distributing the pores, one per cell, was tried. For this case the total porosity was changed by changing the pore size.

Four Pore Cube (Cell 49, PORE5-1)

Max. Stress = 5.95×10^{-2} GPa

Key

1 = 0.33×10^{-2} GPa

2 = 1.27×10^{-2} GPa

3 = 2.20×10^{-2} GPa

4 = 3.14×10^{-2} GPa

5 = 4.08×10^{-2} GPa

6 = 5.01×10^{-2} GPa

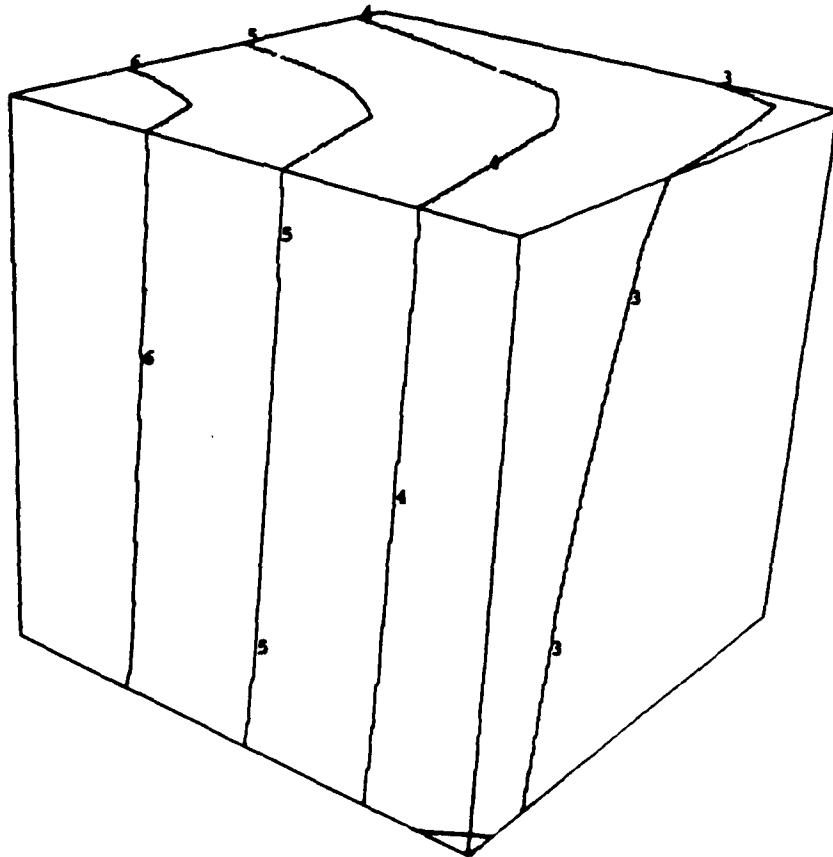


Figure 21. Four Pore Cell Exterior Stress Contour (Ref 20)

Four Pore Cube (Cell 49, PORE5-1)

Max. Stress = 5.28×10^{-2} GPa

Key

1 = 0.23×10^{-2} GPa

2 = 1.07×10^{-2} GPa

3 = 1.91×10^{-2} GPa

4 = 2.76×10^{-2} GPa

5 = 3.60×10^{-2} GPa

6 = 4.44×10^{-2} GPa

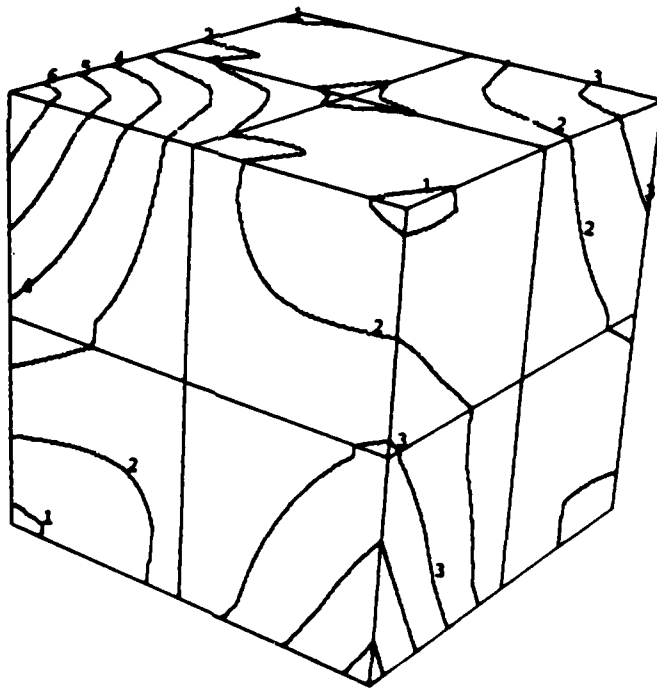


Figure 22. Four Pore Cell Interior Stress Contour (Ref 20)

Five Pore Cube (Cell 45, PORE5-1)

Max. Stress = 7.49×10^{-2} GPa

Key

1 = 0.96×10^{-2} GPa

2 = 2.05×10^{-2} GPa

3 = 3.14×10^{-2} GPa

4 = 4.22×10^{-2} GPa

5 = 5.31×10^{-2} GPa

6 = 6.40×10^{-2} GPa

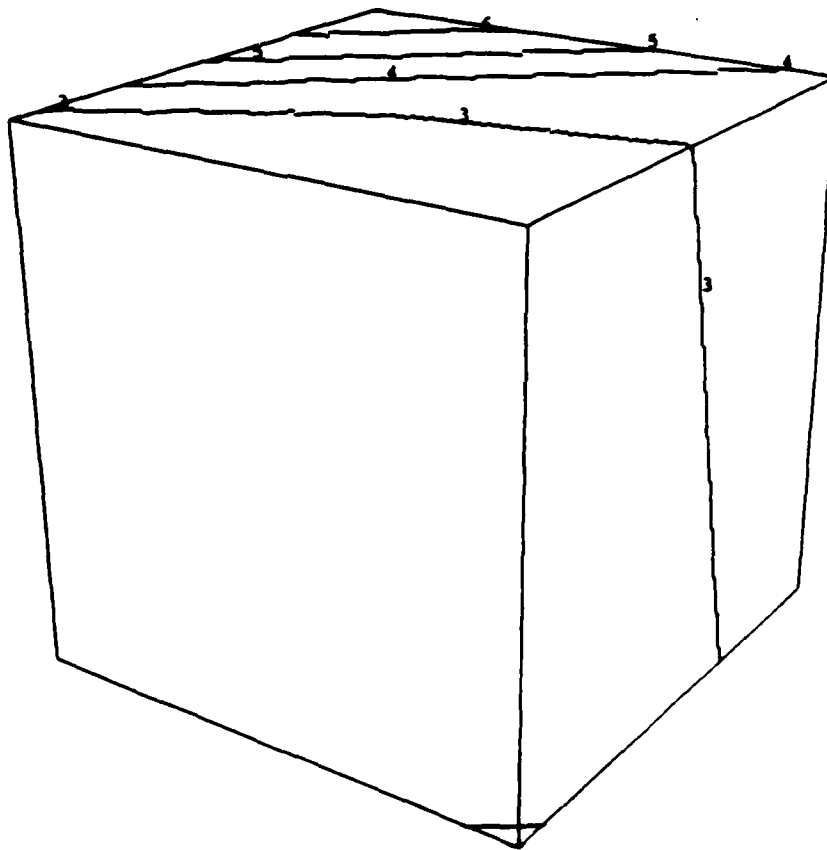


Figure 23. Five Pore Cell Exterior Stress Contour (Ref 20)

Five Pore Cube (Cell 45, PORE5-1)

Max. Stress = 5.19×10^{-2} GPa

Key

1 = 0.63×10^{-2} GPa

2 = 1.39×10^{-2} GPa

3 = 2.15×10^{-2} GPa

4 = 2.91×10^{-2} GPa

5 = 3.67×10^{-2} GPa

6 = 4.43×10^{-2} GPa

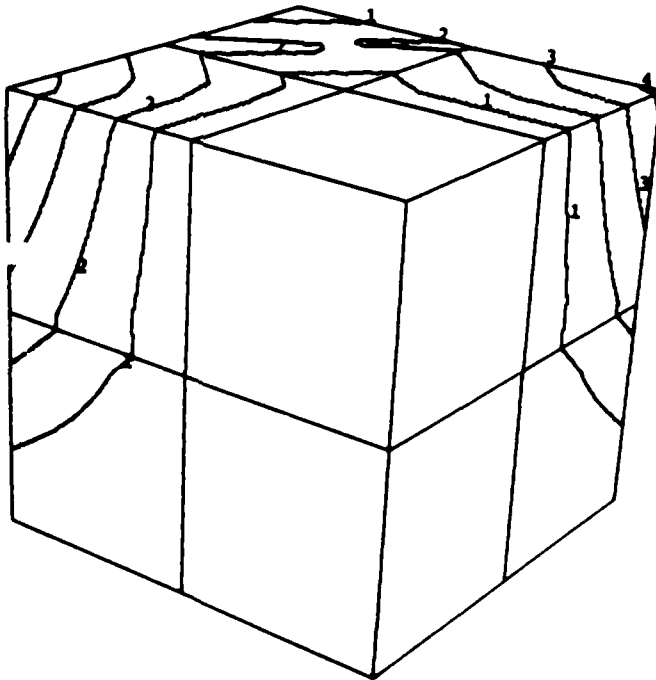


Figure 24. Five Pore Cell Interior Stress Contour (Ref 20)

The results are shown on Figure 25 (the symmetric distribution), and the raw data is given under test PORE1- in Appendix B. Comparing the results of this case to the actual test data, also shown on Figure 25, one can see that evenly distributing the pores causes the modulus of elasticity to be much higher, and drop much slower as porosity increases, than the real test values.

Next a Poisson distribution (explained previously) was tried. This put more pores in some cells and fewer in others, while keeping the pore size of 8 micrometers constant. Figure 25 also shows how this turned out, and in Appendix B the data is given under the PORE2- tests. Figure 25 shows that the slope using distributed pores is much steeper than the case for evenly distributed pores (i.e. modulus of elasticity for this case drops much faster as porosity increases). In fact, the slope is close to the actual test results once they are linearly curve fitted, but for some reason all the computer model values are high. This is unusual since the value of zero porosity Young's modulus (E_0) used for the solid elements is the same as the linear approximations intercept point for zero porosity ($E_0 = 29.3 \times 10^6$ psia). Thus, one would expect the interception with zero porosity to be about the same, and all the values to be close to the same since the slope is close.

To correct this problem, the value for the modulus of elasticity of the solid elements was dropped to 26.1×10^6 psia. This produced the results of test PORE2- ;2 also shown by the shortest dotted line on Figure 25. The results are close to the actual data.

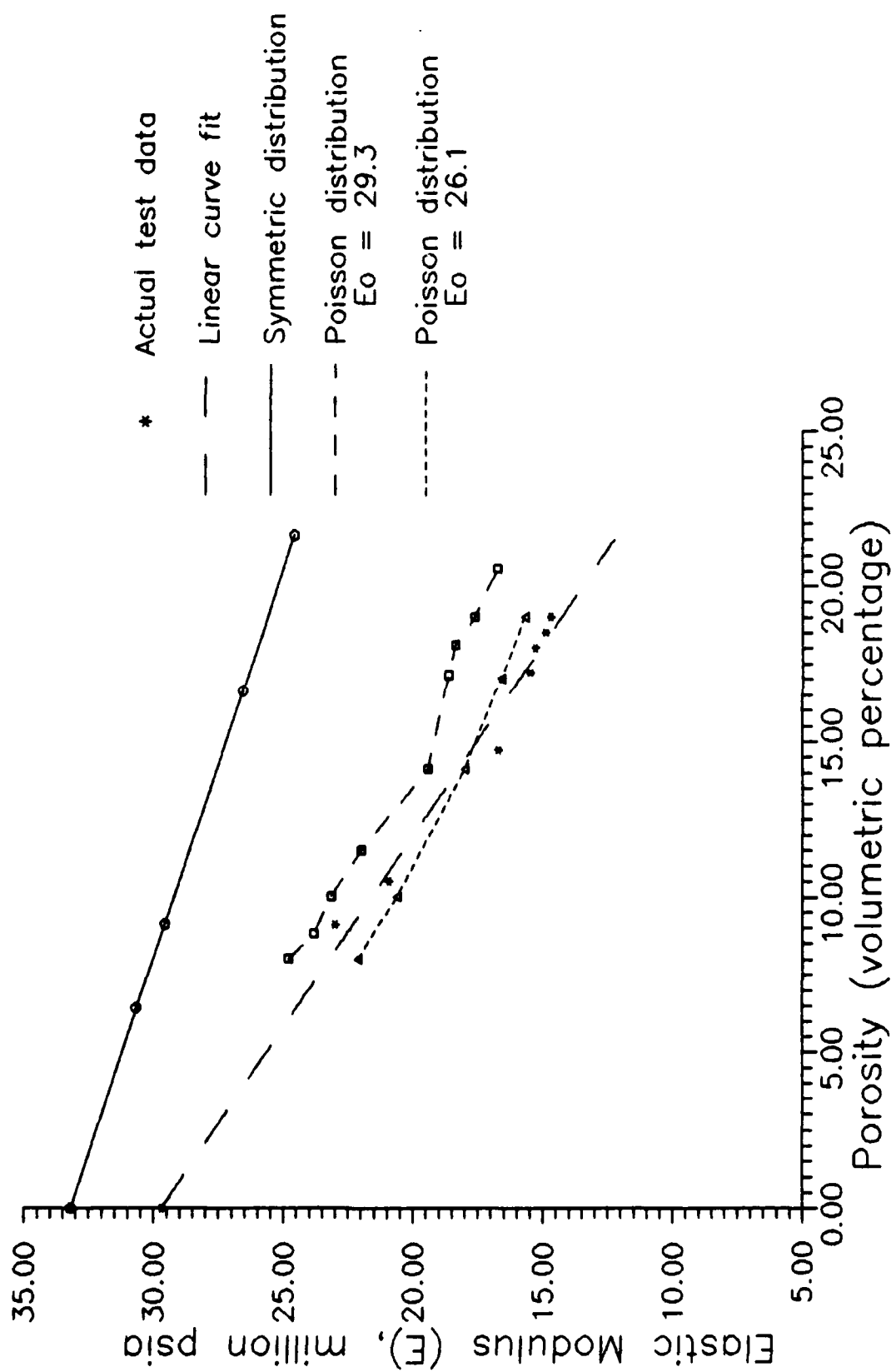


Figure 25 Colt Oxide Elastic Modulus vs Porosity

The results are not perfect, in fact even the linear least squares fit line is closer to the real test data. However, these results were computer simulated using only the value of zero porosity modulus of elasticity (E_0) as input.

Silicon Carbide

Next a demonstration that the model worked for other materials was needed. The material used next was silicon carbide (SiC). Ruh (19:1369) states that silicon carbide has a reasonably consistent average grain sizes of 2.5 micrometers (2.5×10^{-6} meters), and gives a value of zero porosity elastic modulus ($E_0 = 440$ Giga Pascals) as the linear equations zero porosity intercept point. Figure 26, and PORE3- cases in Appendix B, shows the results of using this data in the computer model. This first run was slightly high just like the first cobalt oxide case, but lowering the value of zero porosity elastic modulus (E_0) to 425 Giga Pascals gives results that almost exactly matches the actual test data.

This demonstrates that the model was accurate for more than one material. It also shows that the value of zero porosity elastic modulus used in the computer model should be about 5% lower than the linear equation would predict it to be.

Aluminum Nitride

It has always been difficult to get a close correlation between porosity and elastic modulus for aluminum nitride because aluminum nitride pores vary greatly in size and shape depending on the manufacturing techniques. Table 1, from Ruh (19:1370), shows how much pore size can vary. Notice that larger pore sizes seem to cause

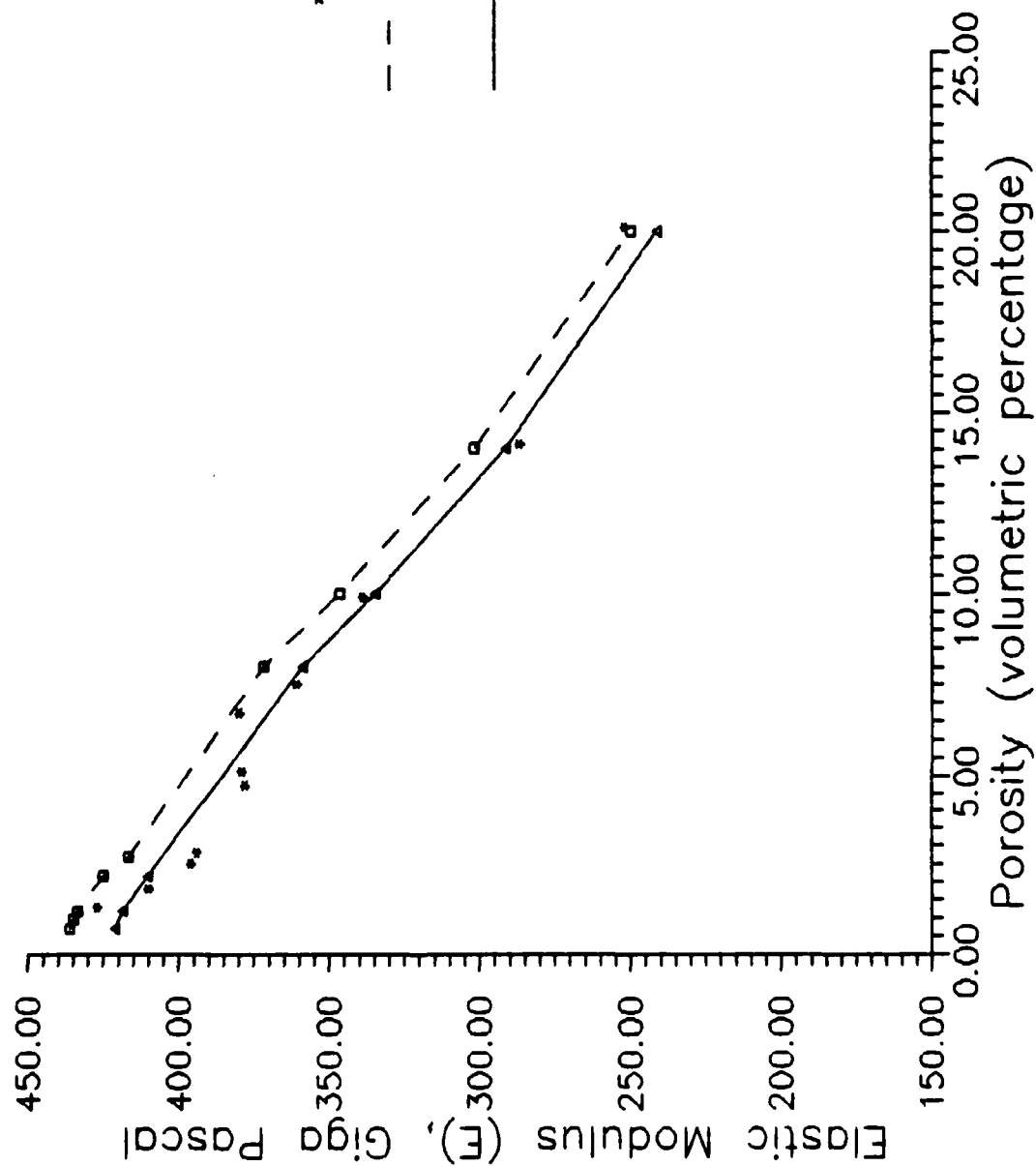


Figure 26 Silicon Carbide Elastic Modulus vs Porosity

a much larger elastic modulus even if the porosity is about the same.
For example: sample 6 has a larger porosity than sample 5 and a larger (not smaller as would normally be expected) elastic modulus due to much larger pore sizes.

Sample no.	Origin *	Porosity (%)	Pore size micro m	Elastic Modulus million psia
1	STK/-400/1700/5	0.5	2-10 (5)	46.6
2	Fabricated by GE	1.7	1-3	44.5
3	AT/ar/1700/30	4.1	1-20 (10)	40.8
4	AT/ar/1700/40	4.7	1-20 (10)	42.3
5	STK/ar/1700/30	5.4	2-8	38.6
6	AT/ar/2100/60	5.6	5-50 (10)	42.1
7	AT/ar/1750/15	6.0	1-5	37.9
8	STK/ar/1700/20	10.5	2-6	34.9
9	AT/ar/1850/30	12.9	2-10 (5)	34.0
10	AT/+400/1600/15	13.2	5-120	36.5
11	AT/+400/1700/15	14.0	5-120	36.4
12	STK/ar/1600/30	16.1	1-10 (5)	34.3
13	AT/ar/1950/30	18.4	2-20 (10)	29.7
14	STK/ar/1600/30	21.2	2-20 (10)	27.8
15	AT/+400/1700/20	23.9	3-250	23.4

* = Starting powder/ Particle size/ Hot-pressing temp (°C)/ Time (min)
STK = Stack, AT = Atommergic, ar = as received, GE = General Electric
Pore sizes in parenthesis used as actual test data in figure 27

Table 1: Microstructure data for Aluminum Nitride (AlN)

Aluminum nitride was modeled next to find out how pore size affected the macroscopic material behavior, and if the computer model simulated this change properly.

First cubic pores with each side 8 micrometers long were tested, Appendix B test results PORE4- show the results, using a zero porosity elastic modulus (E_0) of 46.7 million psia. The results fit quite close to the actual test data given in Table 1 (Figure 27).

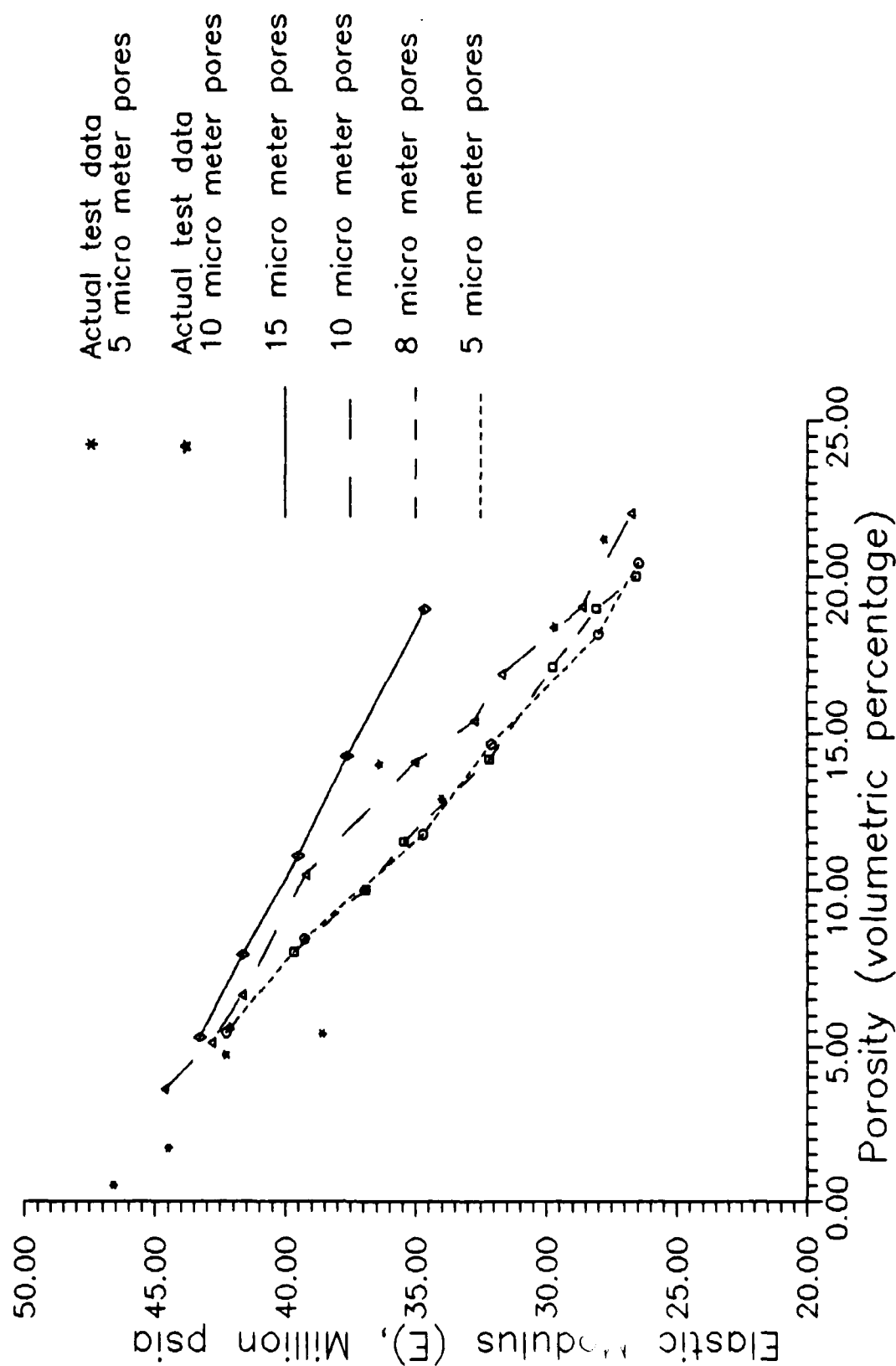


Figure 27 Aluminum Nitride Elastic Modulus vs Porosity

Next 5 micrometer pores were tested (Appendix B test results PORE6-). The results were much too high elastic modulus for porosities above two or three percent. The reason for this was that the cell size was too big for the smaller pore size. To get large porosities, i.e. above 5 percent, large conglomerations of up to ten pores in one cell were needed, and few, if any, cells with only a few pores in them were used. Essentially this means that the model was really modeling larger pores, one per cell, not small scattered pores.

This was solved by making the cell smaller, 12 micrometers instead of 20 micrometers. This made distributions of zero, one, and two pore cells much more prevalent, and eliminated eight, nine and 10 pore cells. The results, shown in Figure 27 and in Appendix B under PORE7- , were reasonably close to the actual data point for 5 micrometer pores, shown on Figure 27 by an asterisk. Figure 27 also shows that the 5 micrometer pore results are almost identical to the 8 micrometer pore results.

If the 10 micrometer pore results were not significantly different from the 5 and 8 micrometer pores, then the model wasn't properly accounting for pore size. The 10 micrometer pore tests (PORE5- in Appendix B) shown in Figure 27 were in fact higher than the 5 and 8 micrometer pore results. They were also quite close to the actual test results, shown by stars on Figure 27. The 10 micrometer tests showed that the model did, therefore, show the effects of increasing pore size, i.e. as pore size grows the elastic modulus decreases slower as porosity increases.

Lastly, 15 micrometer pores were tried (PORE8- in Appendix B) to see

if the trend continued. Figure 27 shows that the trend not only continues, but increases. In other words; as the pore size increases the negative slope of the elastic modulus versus porosity line decreases at a faster and faster rate. This would explain why there was very little difference between the 5 and 8 micrometer pore results.

Derived Equation

From the aluminum nitride model, a trend can be seen relating the pore size with the rate of change of elastic modulus to porosity. Therefore, the next task is to try and find an equation that describes this trend. This was done by taking the computer generated data and trying to fit it to the previously used equations; linear (Equation 1), empirical exponential (Equation 2), semi-empirical (Equation 5), and empirical power (Equation 7). I found the best fit to be with Equation 5, the semi-empirical equation, using Equation 6 to find the empirical constants. Figures 28, 29, 30, and 31 show the close correlation between the computer data and the empirical equation for pore sizes of 5, 8, 10, and 15 micrometers respectively. In all cases except the 10 micrometer pores, the error is less than 1 percent. Now all that was needed was a relationship between the pore sizes and the semi-empirical equation constants to get a good predictor.

To find this relationship the semi-empirical equation was rewritten a bit. Previously the semi-empirical equation was given as:

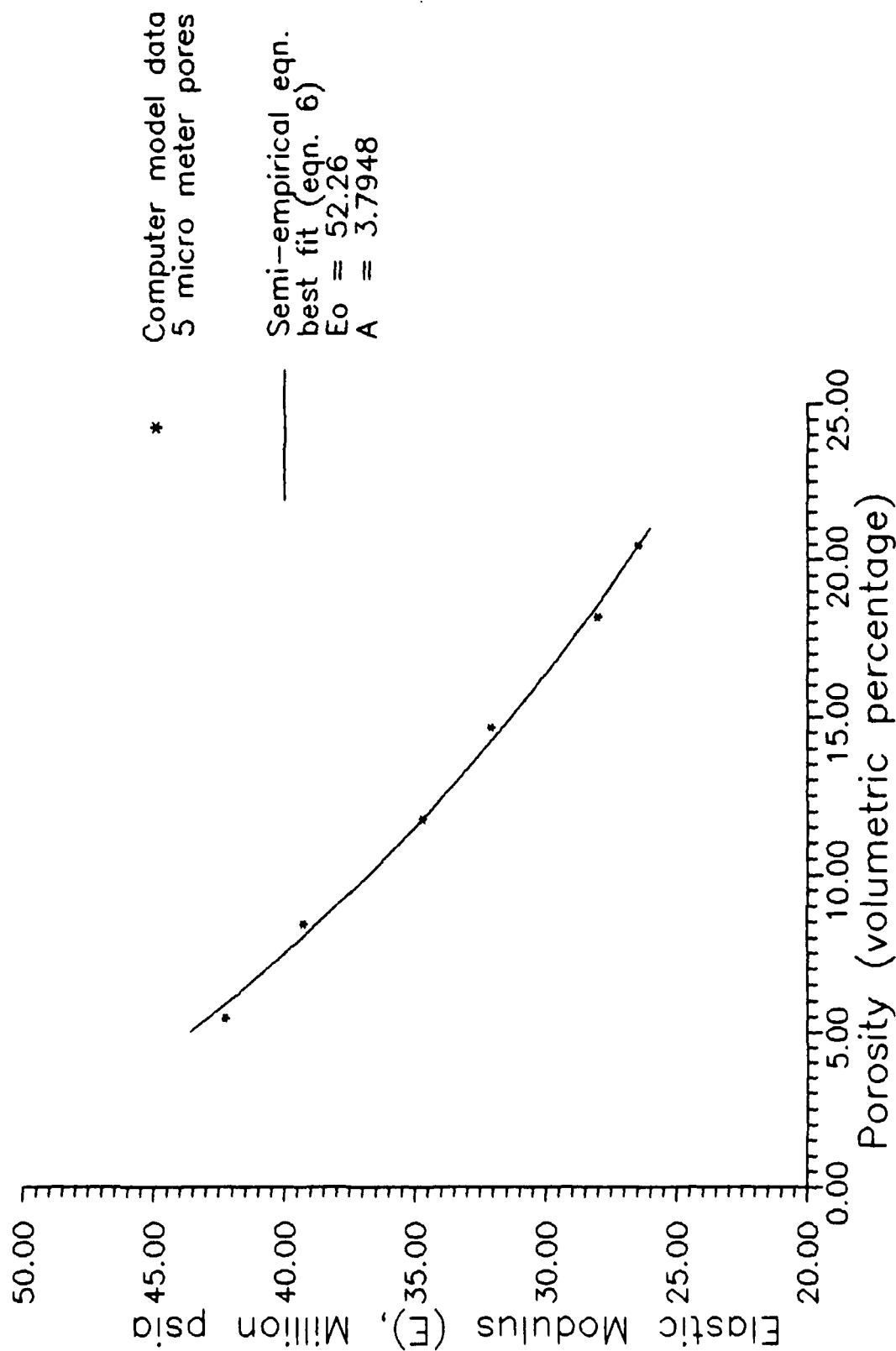


Figure 28 5 micro meter pore, Al N, Elastic Modulus vs Porosity

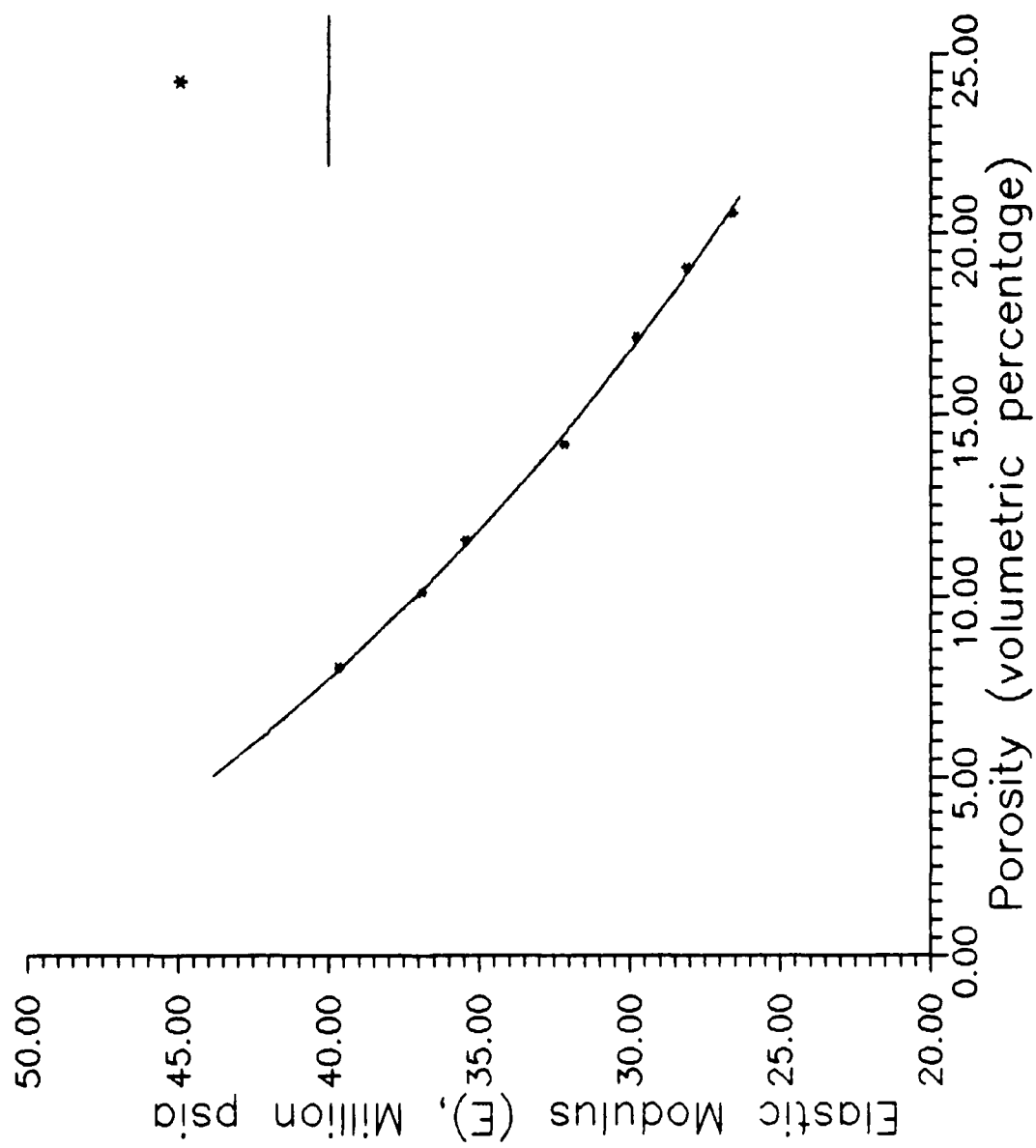


Figure 29 8 micro meter pore, Al N, Elastic Modulus vs Porosity

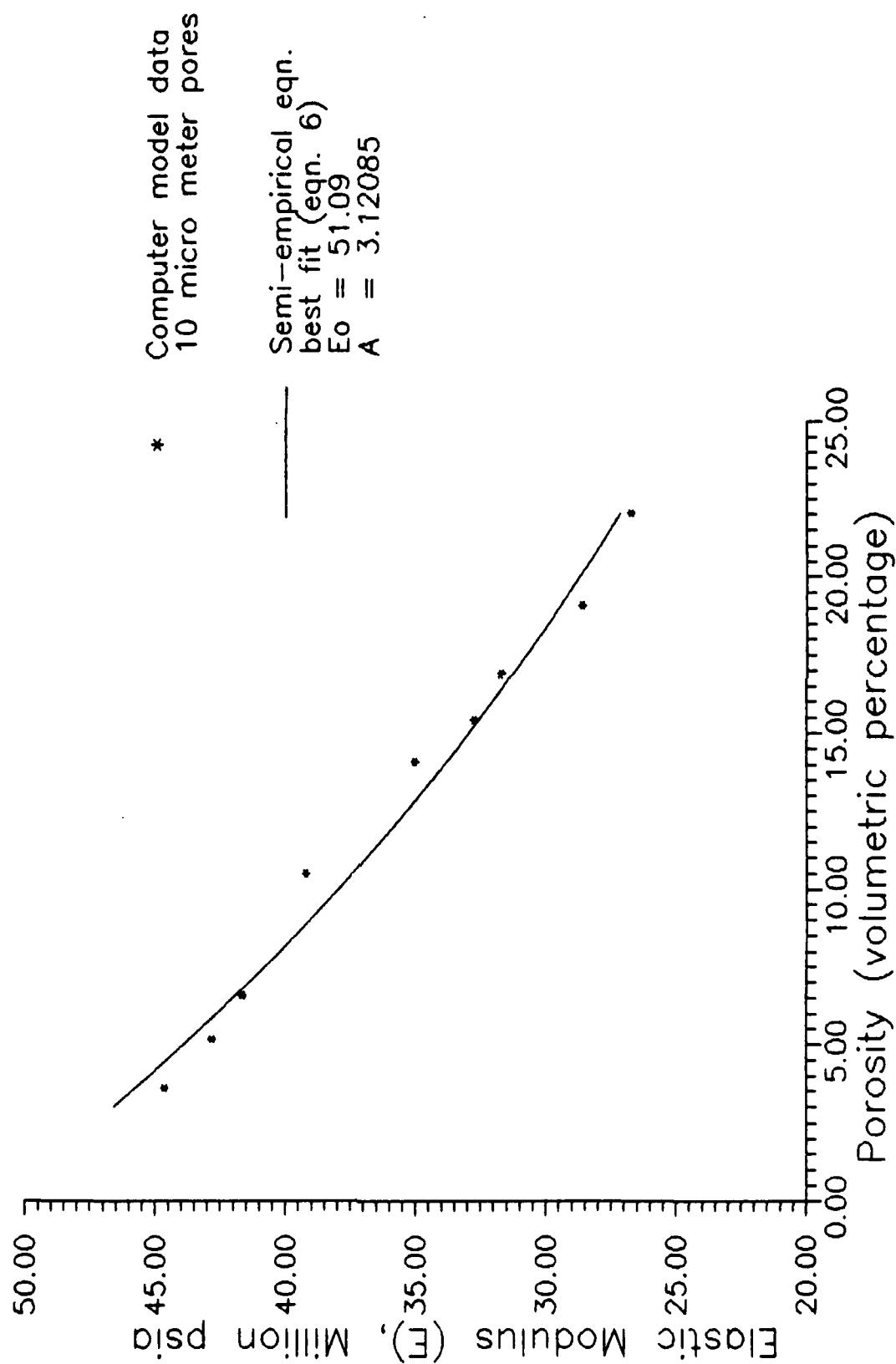


Figure 30 10 micro meter pore, Al N, Elastic Modulus vs Porosity

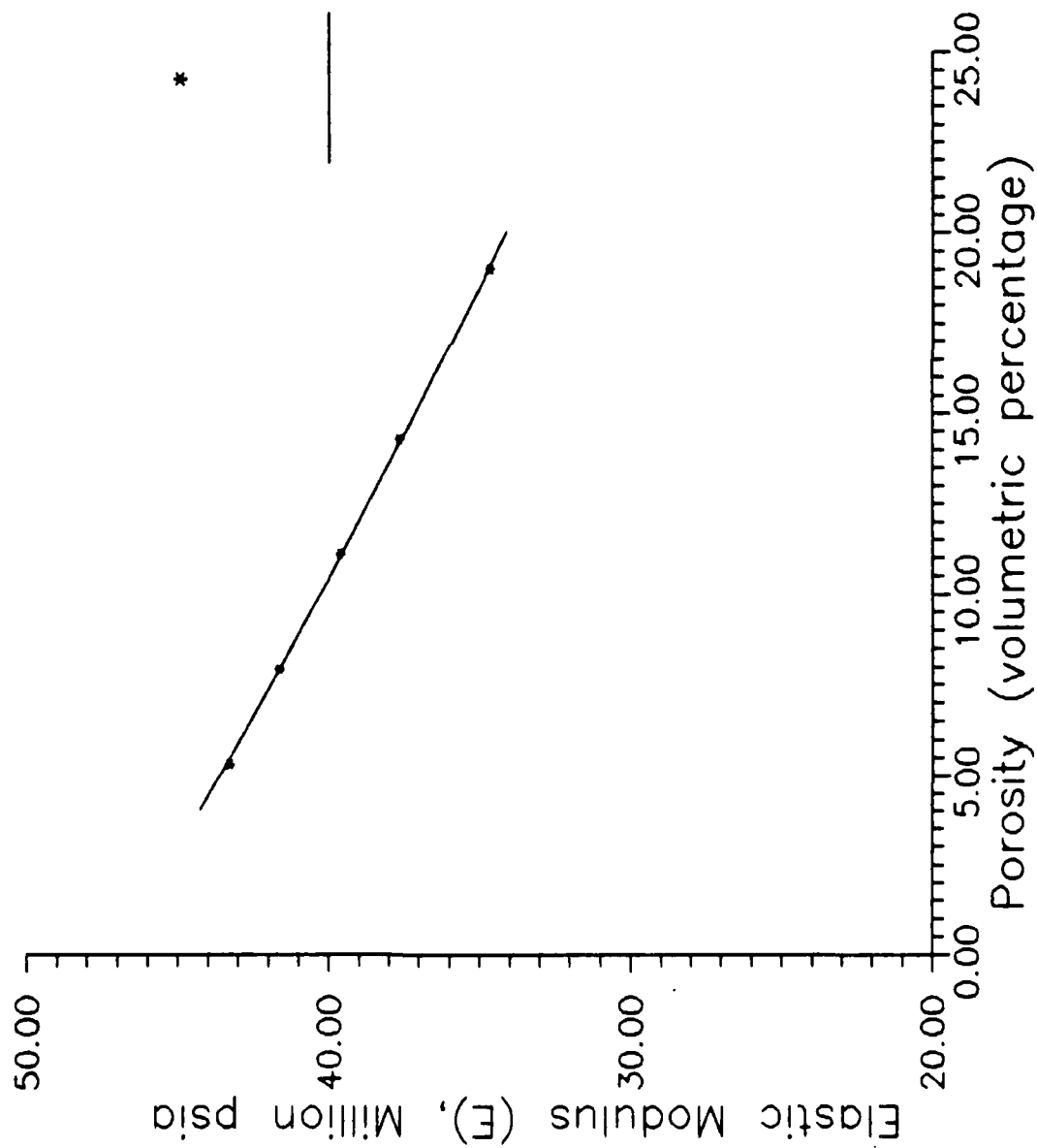


Figure 31 15 micro meter pore, Al N, Elastic Modulus vs Porosity

$$E = E_o \left[1 - \frac{AP}{1 + (A - 1)P} \right] \quad (5)$$

where

E = Elastic modulus (Young's modulus)

E_o = Elastic modulus without any porosity

A = a constant

P = volume porosity

This form is slightly misleading as E_o in Equation 5 is usually not the actual zero porosity elastic modulus. So another parameter, D , was added to correct this, making the equation:

$$E = DE_o \left[1 - \frac{AP}{1 + (A - 1)P} \right] \quad (19)$$

where

E = Elastic modulus (Young's modulus)

E_o = Elastic modulus without any porosity

A = a constant parameter

D = a constant parameter

P = volume porosity

In Equation 19 the value of E_o is the actual zero porosity elastic modulus, and the parameter D modifies it according to the pore size.

So now a relationship between pore size and the parameters A and D was needed. Table 2 shows the parameter values (calculated using Equation 6) and pore sizes for aluminum nitride.

Pore size Micro meters	Calculated E _o	Parameter D *	Parameter A
5	52.26	1.1191	3.7948
8	52.43	1.1227	3.7251
10	51.09	1.0931	3.12085
15	47.05	1.0075	1.51536

* = Assuming actual E_o is 46.7 million psia

Table 2 Equation 19 parameters for aluminum nitride

Using a Shareware program called CURVEFIT by Thomas Cox (3), an equation was found that almost perfectly matched the relationship between parameter A and the pore size. The program called it Hoerl's Equation;

$$A = (P1)(P2^L)(L^{P3}) \quad (20)$$

where

A = parameter to use in Equation 19

L = pore size (length, diameter, etc.) in micrometers

P1 = a constant, in this case 0.8624

P2 = a constant, in this case 0.7461

P3 = a constant, in this case 1.831

Figure 32 shows how nicely the Parameter A is described by Equation 20.

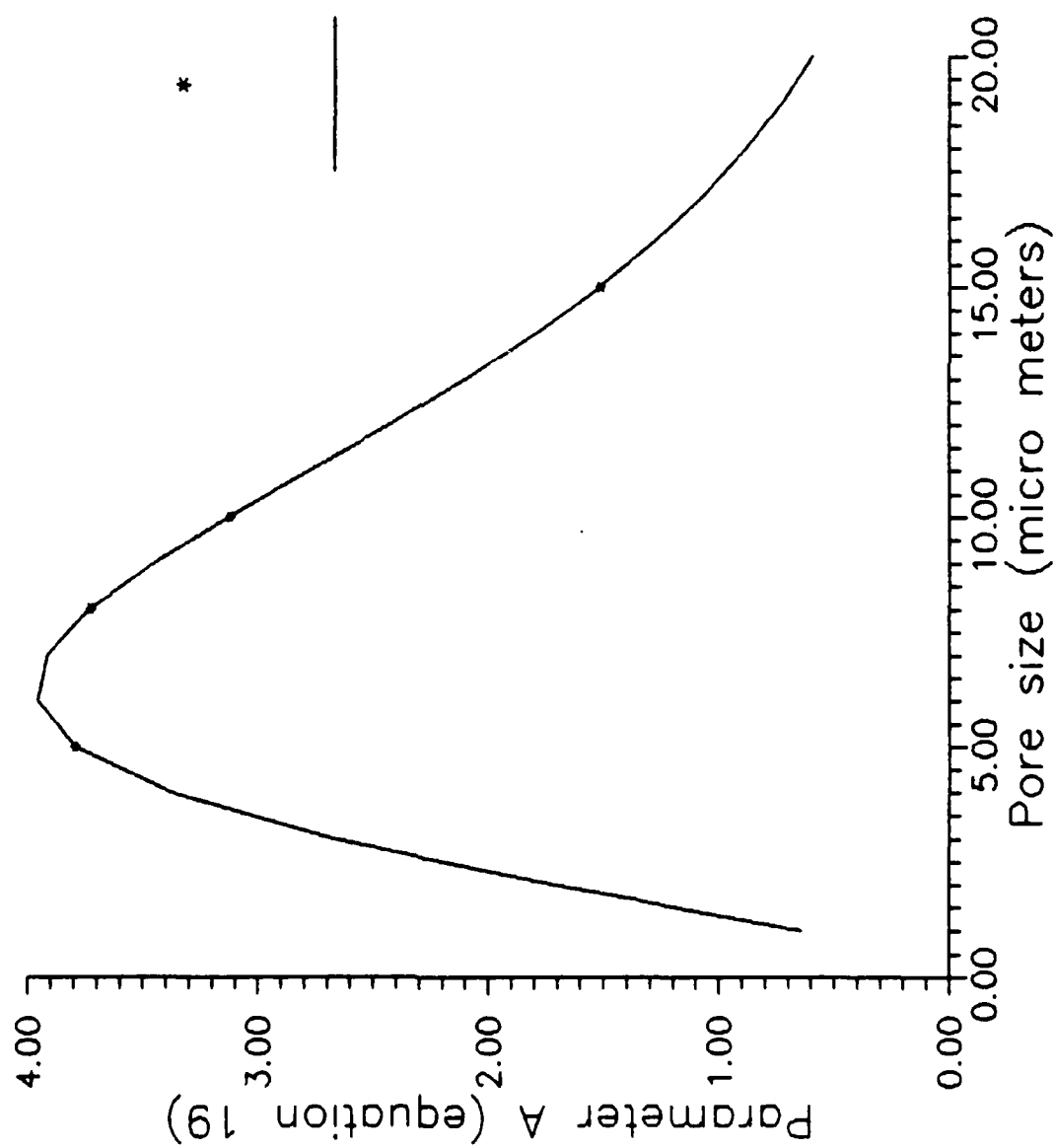


Figure 32 Relationship between parameter A and pore size

Using the same program (3) a relationship was also found between the parameter D and the pore size. A Cauchy distribution, Equation 21, best described the relationship between the two values.

$$D = \frac{1}{P_1(L + P_2)^2 + P_3} \quad (21)$$

where

D = parameter to use in Equation 19

L = pore size (length, diameter, etc.) in micrometers

P1 = a constant

P2 = a constant

P3 = a constant

The values of P1, P2, and P3 for Equation 21 that best fit the data in table 2 are shown by the solid line in Figure 33. However, If the condition that D goes to one when pore size goes to zero is imposed, then the constants take on the values given by the dotted line in Figure 33. Neither curve exactly fits the data, so making D go to one as pore size goes to zero is best.

Combining the results from Equations 20 and 21 into Equation 19 gives:

$$E = E_0 \left(\frac{1}{P_1(L + P_2)^2 + P_3} \right) \left[1 - \frac{AP}{1 + (A-1)P} \right] \quad (22)$$

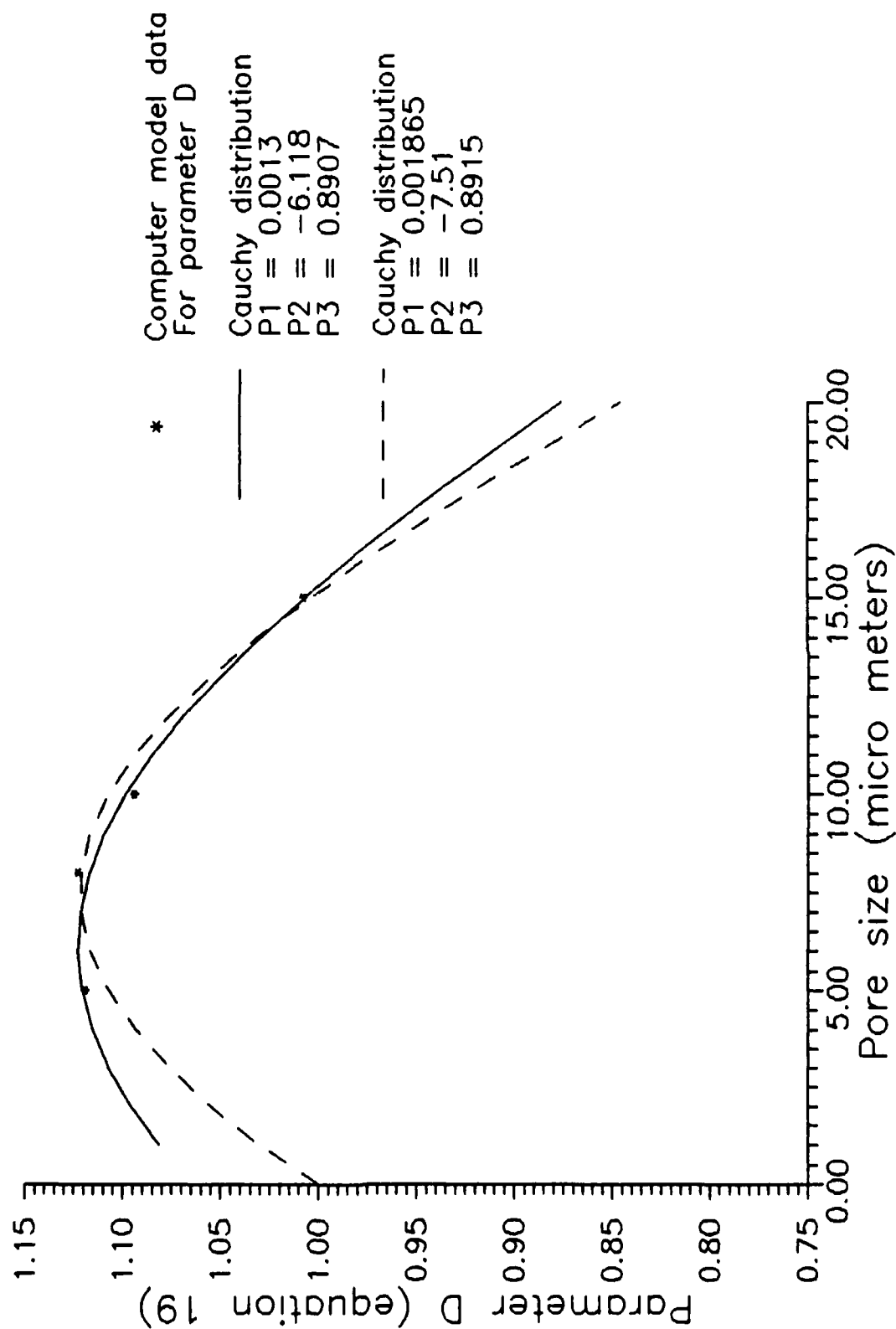


Figure 33 Relationship between parameter D and pore size

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

A = a constant parameter from Equation 20

L = pore size (length, diameter, etc.) in micrometers

P_1 = 0.001865

P_2 = -7.51

P_3 = 0.8915

P = volume porosity

Equation 22 is good for aluminum nitride with pore sizes between 5 and 15 micrometers since that's the data that was used to generate it. However, to be useful it must apply to other materials and pore sizes. Applying Equation 22 to cobalt oxide's 8 micrometer pores (and letting $E_0 = 26.1$) gives the results shown in Figure 34, these results match the data but not very closely. Figure 35 shows the results of Equation 22 when applied to silicon carbide (2 micrometer pores). These results are quite bad and to get them even that close one must use an E_0 of 400 Giga Pascals, not the actual 440 Giga Pascals.

Linear Derived Equation

Both Figure 34 and 35 show that the derived approximate equation using the semi-empirical curve fit didn't work to well. Better results might be achieved by using a linear approximation, because it is more generally applicable even if it isn't as accurate for any one case.

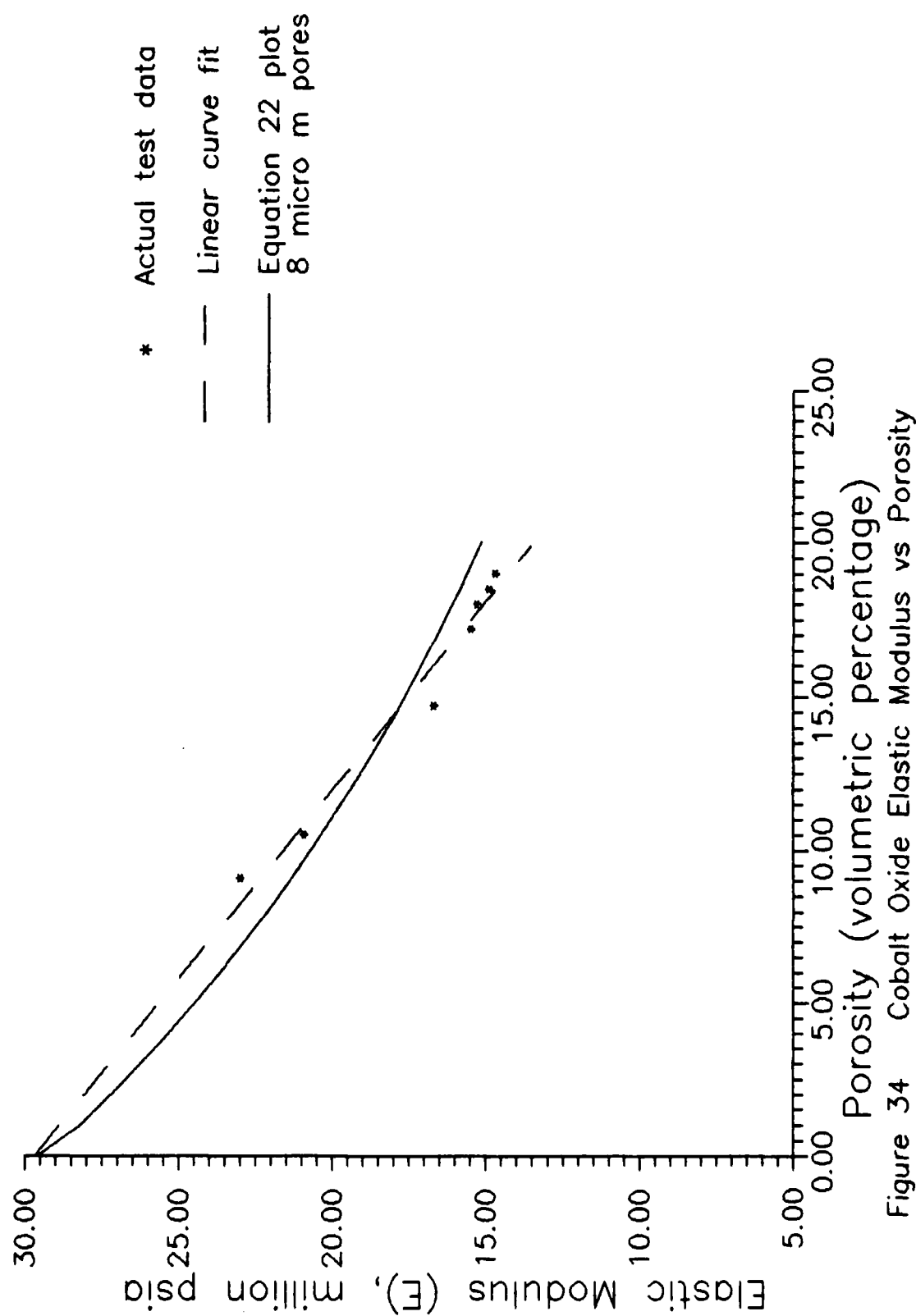


Figure 34 Cobalt Oxide Elastic Modulus vs Porosity

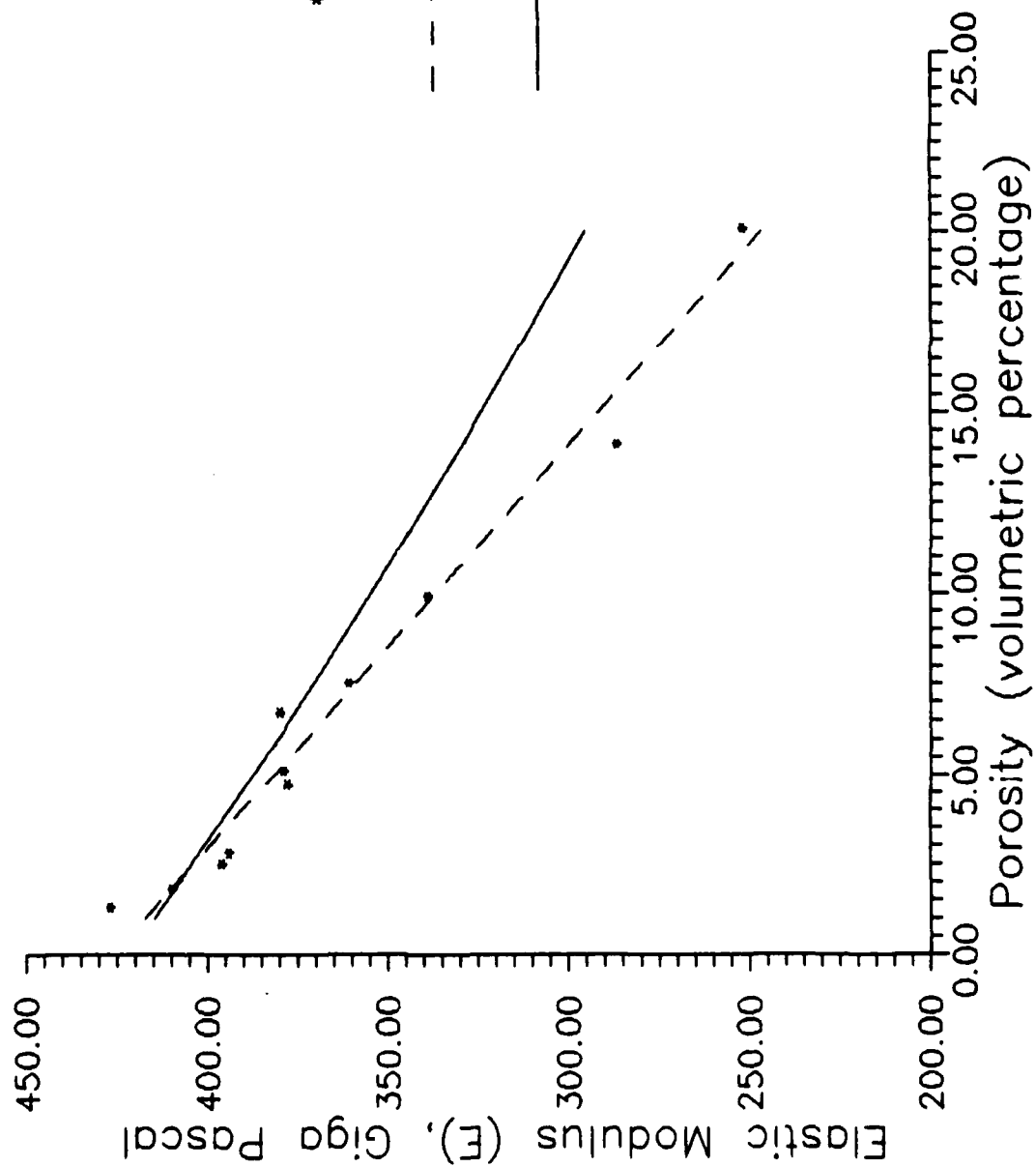


Figure 35 Silicon Carbide Elastic Modulus vs Porosity

Using the linear Equation 1, repeated below, means that only the parameter for the slope (a) needs to be modified for the various pore sizes.

$$E = E_o(1 - aP) \quad (1)$$

where

E = Elastic modulus (Young's modulus)

E_o = Elastic modulus without any porosity

a = slope, an empirical constant

P = volume fraction porosity

Table 3 shows all the data correlating slopes to pore size, for the cases studied in this paper.

Pore Size <u>Micro m.</u>	<u>Slope</u>	<u>Material</u>
2	2.103	Si C, actual test data
5	2.157	Al N, actual test data
5	2.247	Al N, computer data
8	2.720	Co O, actual test data
8	2.201	Al N, computer data
10	2.048	Al N, computer data
10	1.908	Al N, actual test data
15	1.351	Al N, computer data

Table 3 Slopes for various pore sizes

CURVEFIT (3) was used again to find an equation that describes how slope, a, changes with pore size. The result is again the Cauchy distribution equation, this time with very different parameters:

$$a = \frac{1}{P_1 (L + P_2)^2 + P_3} \quad (23)$$

where

a = parameter, slope, for Equation 1

L = pore size (length, diameter, etc.) in micrometers

P_1 = a constant, in this case 0.003866

P_2 = a constant, in this case -5.988

P_3 = a constant, in this case 0.4275

Figure 36 shows how Equation 23 is only an approximation to the actual data given in Table 3. Putting the result from Equation 23 into Equation 1 gives:

$$E = E_0 \left(1 - \frac{1}{P_1 (L + P_2)^2 + P_3} P \right) \quad (24)$$

where

E = Elastic modulus (Young's modulus)

E_0 = Elastic modulus without any porosity

L = pore size (length, diameter, etc.) in micrometers

P_1 = a constant, in this case 0.003866

P_2 = a constant, in this case -5.988

P_3 = a constant, in this case 0.4275

P = volume fraction porosity

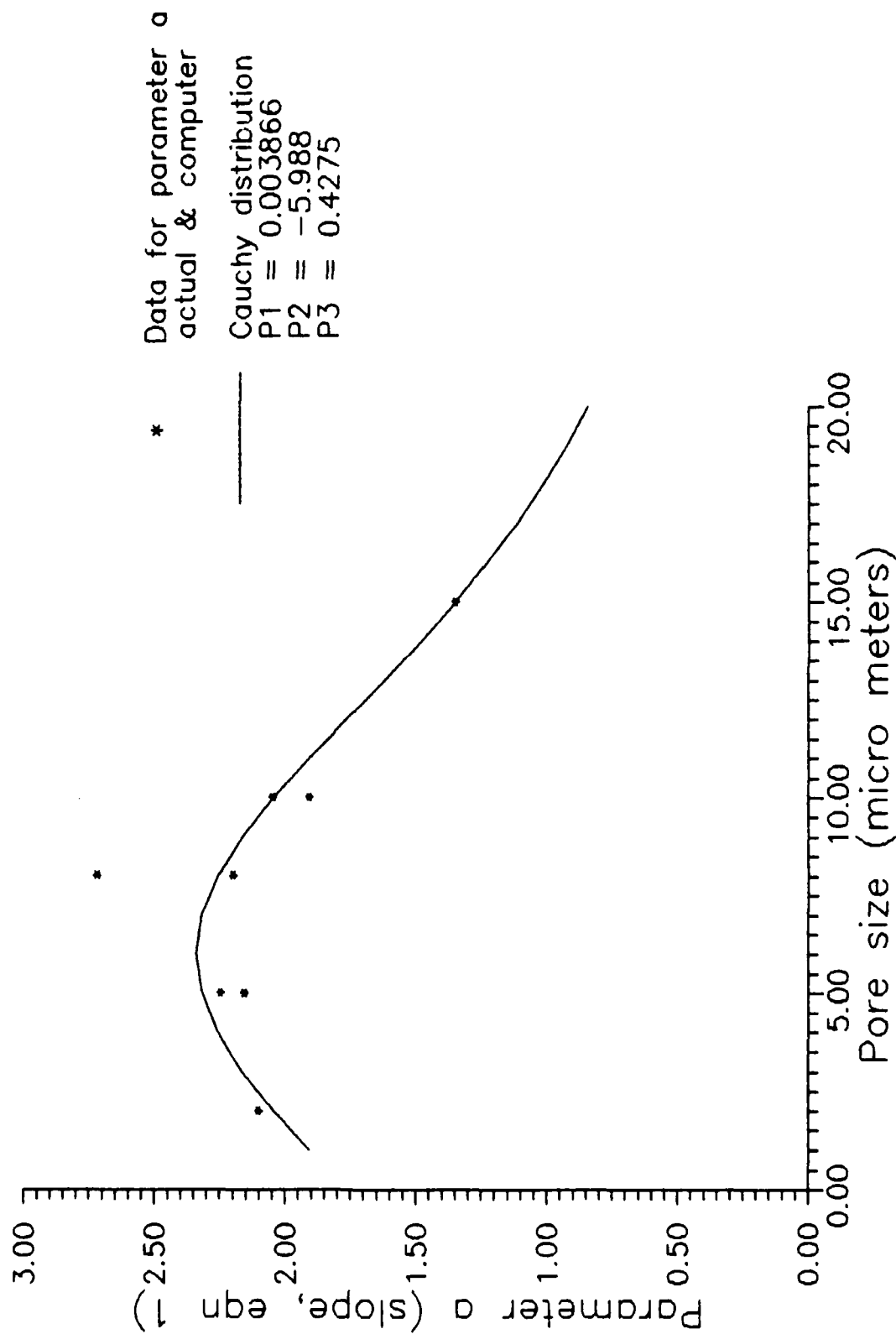


Figure 36 Relationship between parameter a and pore size

Applying Equation 24 to the cases that have been looked at in this paper (i.e. cobalt oxide, silicon carbide, and aluminum nitride) give very accurate results, shown in Figures 37, 38, 39, and 40. While Equation 24, which estimates elastic modulus based on pore size and porosity is not perfect, it is close enough to give a material designer a good starting estimate of how a new material would behave.

To verify Equation 24, it should be used for some test case. Information on the pore size of materials is scarce, so aluminum nitride will be used again, but this time for different pore sizes not previously examined. Referring back to Table 1, data point 2 has 1.7% porosity, 2 micrometer pores (average), and the same zero porosity elastic modulus for aluminum nitride is about 46.7 million psia. Putting this data into Equation 24 gives an elastic modulus of 45.07 million psia, where the actual tested number is 44.5 million psia. This is only a 1.28 percent error, extremely good for an empirical first approximation. Several other data points from Table 1 were tested, and Table 4 below shows the results. In all cases the error was small, but usually not ignorable.

Sample (Table 1)	Porosity (%)	Pore size micro m	E (tested) million psia	E (calculated) million psia	Error (%)
2	1.7	2.0	44.5	45.07	1.28
5	5.4	4.0	38.6	41.00	6.22
7	6.0	2.5	37.9	40.79	7.62
8	10.5	3.0	34.9	36.08	3.38
10	13.2	12.0	36.5	35.80	1.92
11	14.0	12.0	36.4	35.1	3.57

Table 4 Equation 24 Verification for Aluminum Nitride (AlN)

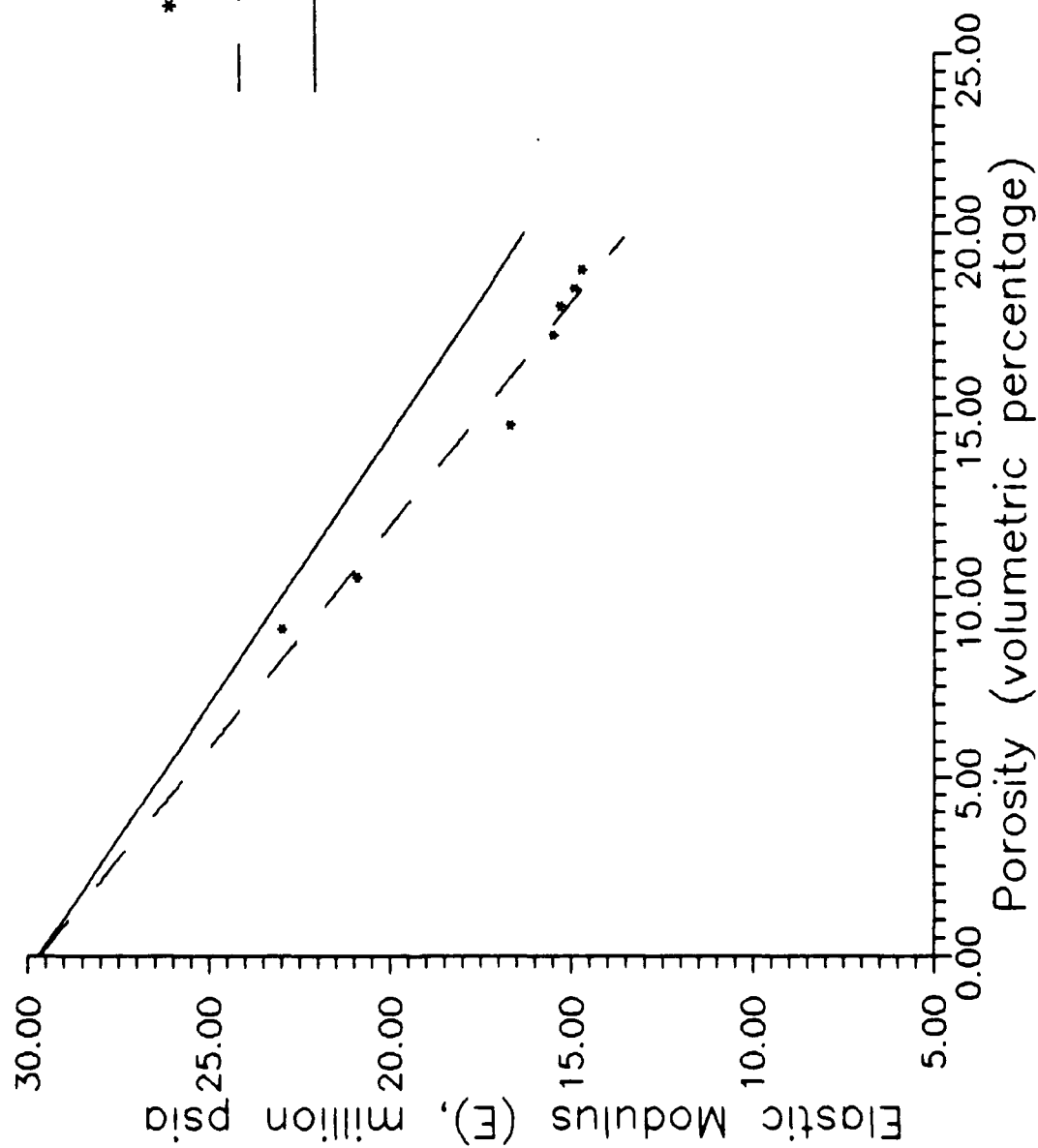


Figure 37 Cobalt Oxide Elastic Modulus vs Porosity

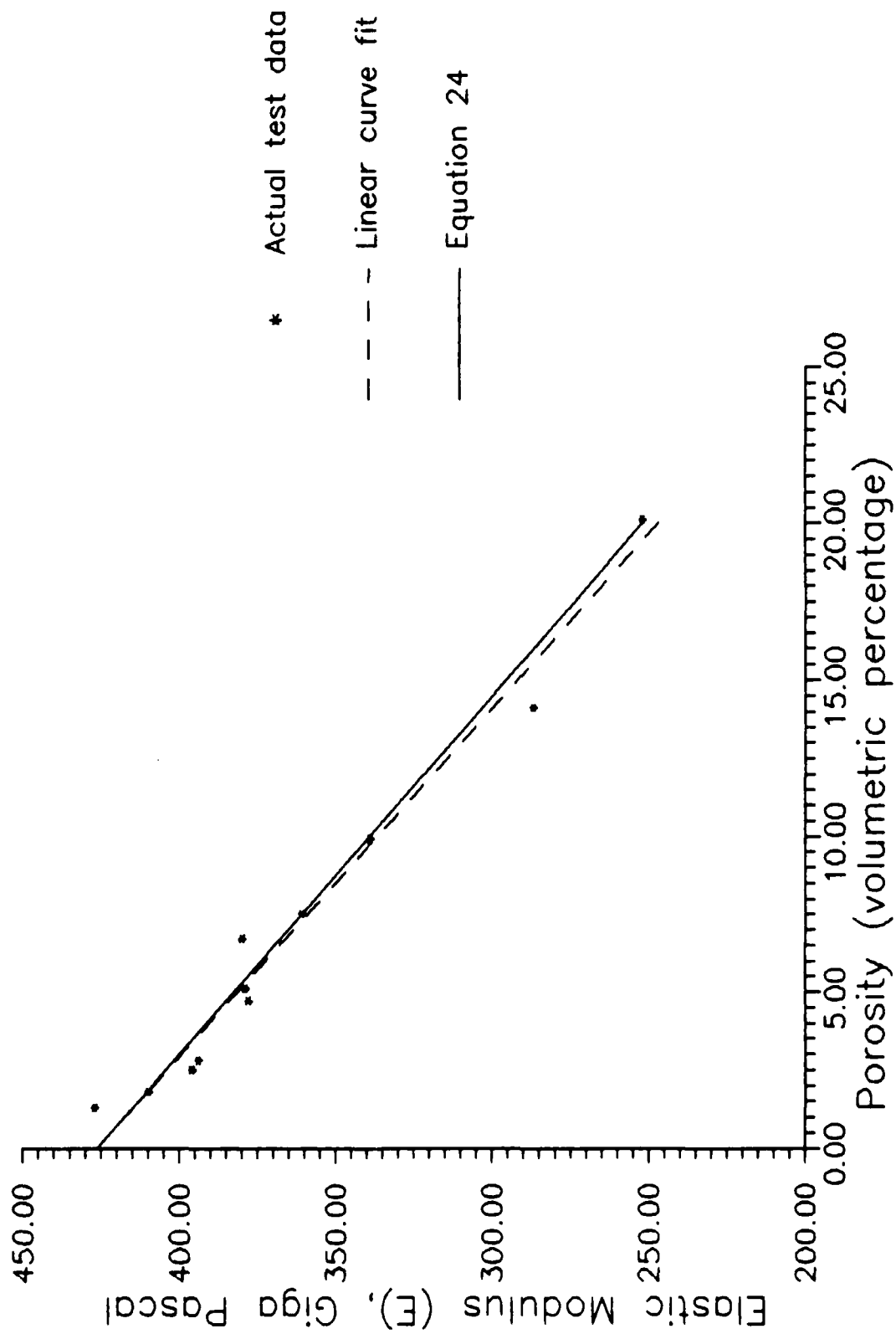


Figure 38 Silicon Carbide Elastic Modulus vs Porosity

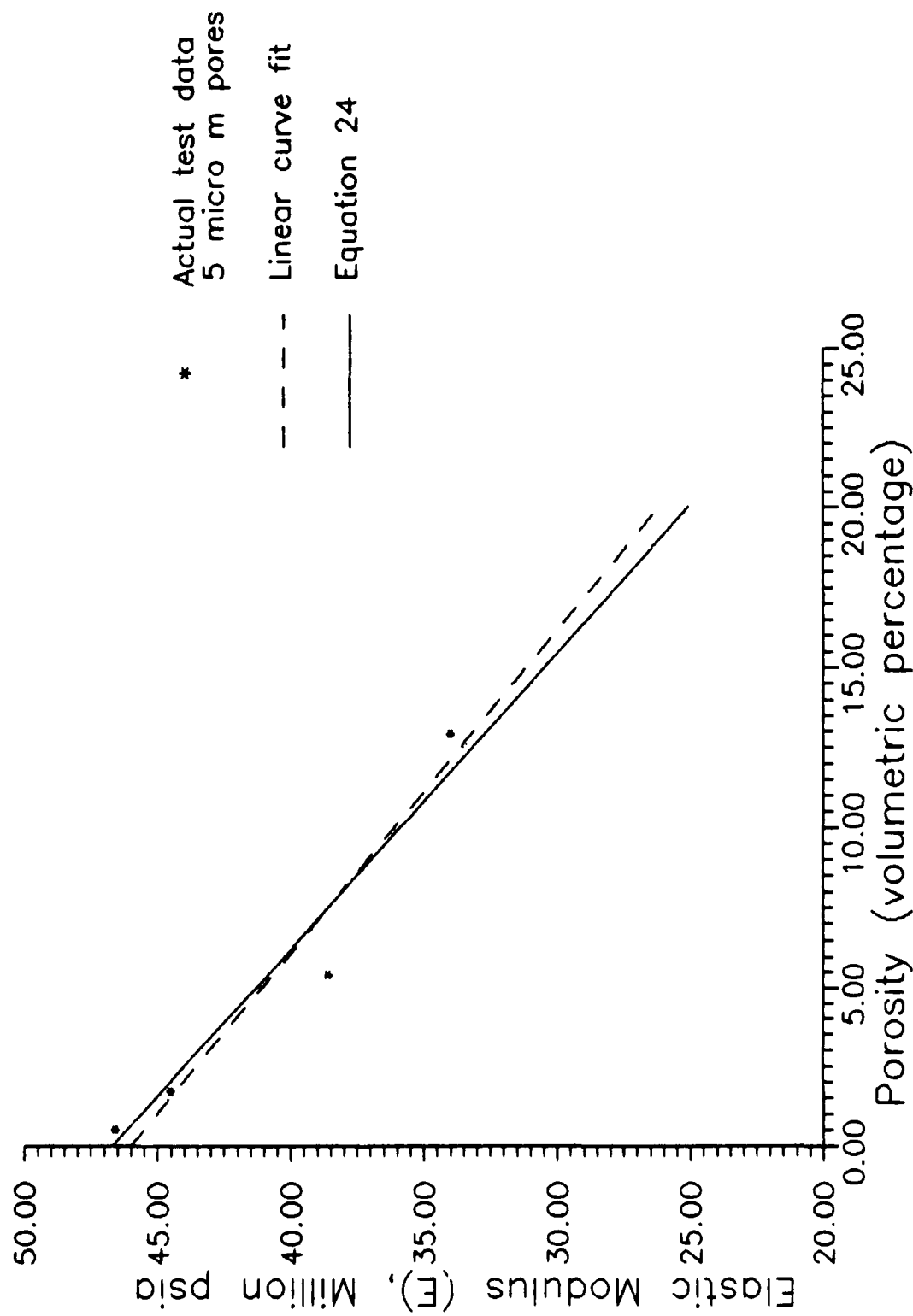


Figure 39 Aluminum Nitride Elastic Modulus vs Porosity

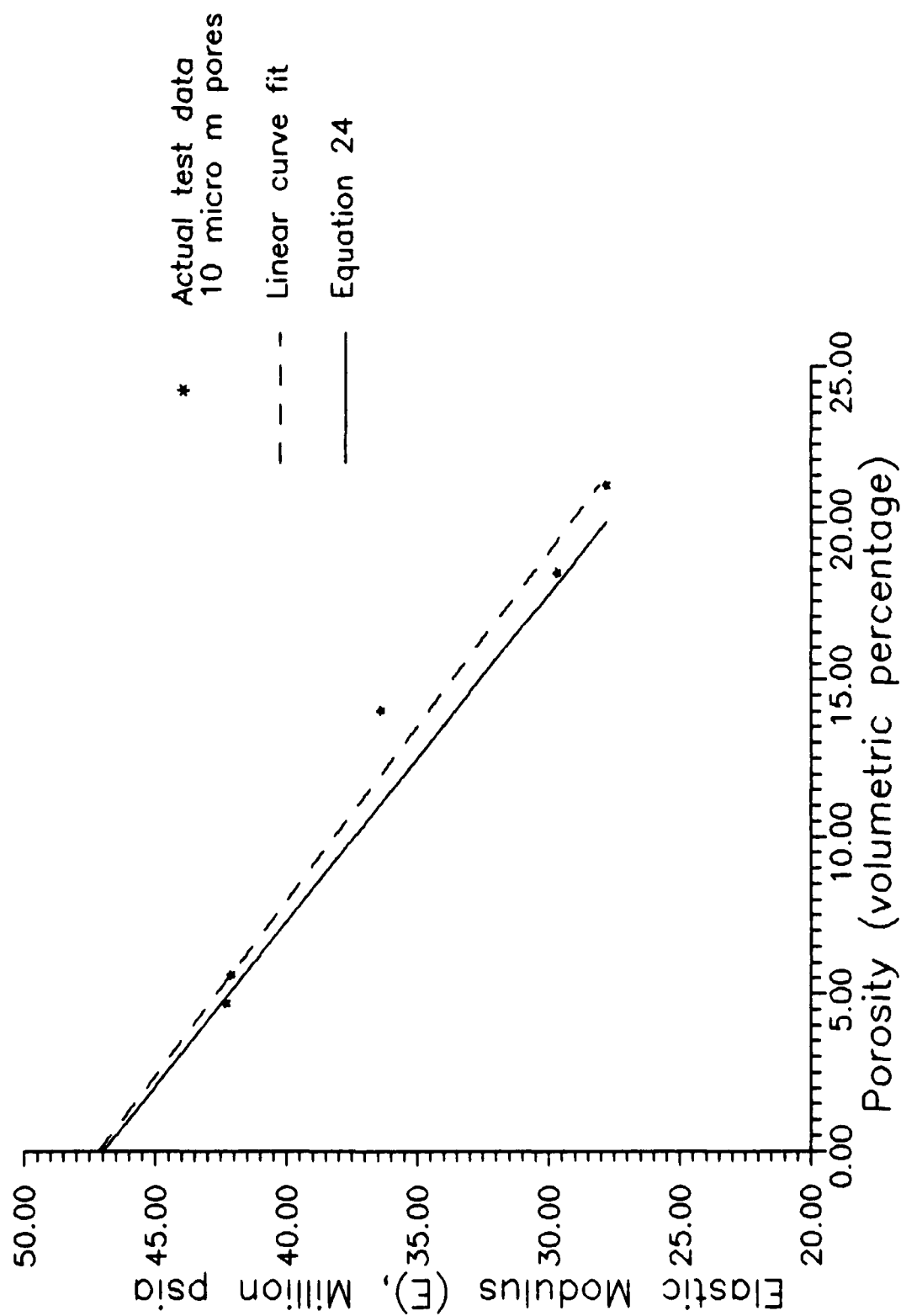


Figure 40 Aluminum Nitride Elastic Modulus vs Porosity

Plasticity

The computer model used did not take into account any plasticity. It only used macroscopic linear elastic theory, and yet gave good results even though a microscopic phenomena was being modeled. No microscopic phenomena, grain size, plastic deformations, etc., was considered. The accuracy is partially due to the fact that at room temperature, all three materials (cobalt oxide, silicon carbide, and aluminum nitride) actually are linearly elastic and break before yielding. Figure 41, using data from Larson and Adams (13:152), shows this fact on a macroscopic stress strain curve for silicon carbide. It also shows that temperatures must rise above 1000 °C for the elastic modulus to change significantly from it's room temperature value.

I was unable to find any data on porosity (pore size and distribution) for a plastic or visco-elastic material. Future work using this paper's computer model on such a material is encouraged.

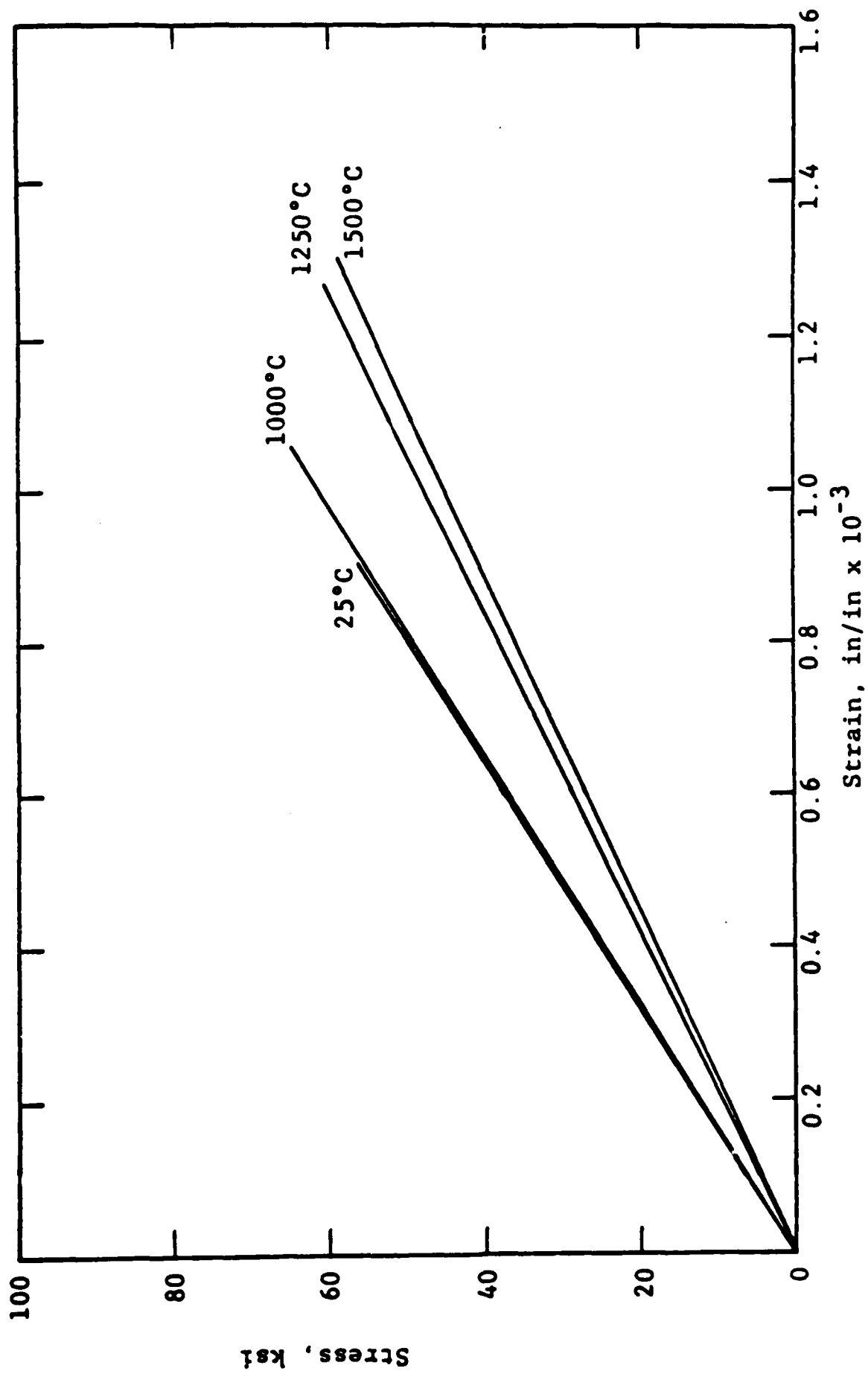


Figure 41 Stress-strain behavior of Kyocera SC-201 SiC (Ref 13)

VI. Conclusion

Up until now a material designer had to actually make and test several samples of a material, and curve fit the results to one of several empirical formulas (linear, exponential empirical, semi-empirical, or the power empirical equation) to see how a material behaved as porosity changed. This thesis has presented an alternative to actual production and testing.

Materials that are linearly elastic, like most ceramics, with average pore sizes between about 1 and 20 micrometers could be evaluated using Equation 24. This equation gives valuable information showing how pore size and porosity effect the materials elastic modulus. For example; a designer who wants to change the pore size of a ceramic material by using a cheaper manufacturing technique can get a good idea of what the new materials strength will be using Equation 24.

A closer prediction of actual material behavior can be obtained through the application of the finite element model (Appendix A). This computer model gave accurate results, with only slight error when compared to lab test results, for all materials analyzed. This finite element model can also be applied to materials that are not linear elastic using other MSC/NASTRAN solution algorithms, if the stress-strain curve for the zero porosity material is known.

The computer model was also useful in showing stress concentrations

about non-circular pores. Especially interesting was the effects on stress near areas where several pores meet and join. As the number of touching pores rose, both the stress and the stress gradient on the material near the pore rose dramatically, but the stress even one pore diameter away changed very slowly, if at all.

The computer model has clearly shown the relationship between pore size and how this effected elastic moduli as the porosity increased (i.e. the slope of the elastic modulus versus porosity line). As the pore size became larger, the slope decreased. The slope also decreased for very small pores, about one micrometer small. This information was used to generate Equation 24.

The finite element model used in this thesis represented a very small portion of material. If a designer wanted to model a real porous structure they could take this model, define it as one primary superelement, and then using imaging superelements, make copies of this primary superelement and put them together like building blocks to make a much larger structure.

In summary, the strength of a porous material depends primarily on volumetric porosity, and pore size. This thesis has developed an equation and a computer model which can be used to predict a material's elastic modulus using only pore size, porosity, and zero porosity elastic modulus.

Appendix A: Sample MSC/NASTRAN Input Data File
for the Computer Model (PORE2-9)

```

ASSIGN OUTPUT2=PORE2-9.OP2, STATUS=NEW, UNIT=12
SOL 101          $ SUPERELEMENT STATICS
TIME 50
DIAG 64
CEND
TITLE = POROUS CUBE OF COBOLT OXIDE
LOAD = 10
SPC = 20
SEALL = ALL
DISP = ALL
STRESS = ALL
BEGIN BULK
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 CHEXA, 20, 30, 411, 511, 512, 412, 2002, 2009
 , 2012, 2003
 CHEXA, 21, 30, 511, 521, 522, 512, 2010, 2013
 , 2014, 2011
 CHEXA, 22, 30, 521, 421, 422, 522, 2016, 2005
 , 2006, 2015
 CHEXA, 23, 30, 421, 411, 412, 422, 2008, 2001
 , 2004, 2007
 CHEXA, 24, 30, 412, 512, 522, 422, 2003, 2012
 , 2015, 2006
 CHEXA, 25, 30, 411, 511, 521, 421, 2002, 2009
 , 2016, 2005
 CHEXA, 26, 30, 2002, 2009, 2012, 2003, 2005, 2016
 , 2015, 2006
 CHEXA, 27, 31, 2001, 2002, 2003, 2004, 2008, 2005
 , 2006, 2007
 CHEXA, 28, 31, 2009, 2010, 2011, 2012, 2016, 2013

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 CPENTA, 29. 30. 411. 2001. 2002. 412. 2004. 2003
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 CPENTA, 33. 30. 512. 2011. 2012. 522. 2014. 2015
 CPENTA, 34. 30. 412. 2004. 2003. 422. 2007. 2006
 CPENTA, 35. 30. 411. 2001. 2002. 421. 2008. 2005
 CPENTA, 36. 30. 511. 2010. 2009. 521. 2013. 2016

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\$ SINGLE CELL. THREE PORES

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 CHEXA, 40, 30, 113, 213, 214, 114, 3111, 3311
 , 3313, 3113
 CHEXA, 41, 30, 213, 223, 224, 214, 3311, 3331
 , 3333, 3313
 CHEXA, 42, 30, 223, 123, 124, 224, 3331, 3131
 , 3133, 3333
 CHEXA, 43, 30, 123, 113, 114, 124, 3131, 3111
 , 3113, 3133
 CHEXA, 44, 30, 114, 214, 224, 124, 3113, 3313
 , 3333, 3133
 CHEXA, 45, 30, 113, 213, 223, 123, 3111, 3311
 , 3331, 3131
 CHEXA, 46, 31, 3111, 3211. 3221. 3121. 3112. 3212

, 3222, 3122
CHEXA, 47, 30, 3211, 3311, 3321, 3221, 3212, 3312
, 3322, 3222
CHEXA, 48, 30, 3121, 3221, 3231, 3131, 3122, 3222
, 3232, 3132
CHEXA, 49, 30, 3221, 3321, 3331, 3231, 3222, 3322
, 3332, 3232
CHEXA, 50, 30, 3112, 3212, 3222, 3122, 3113, 3213
, 3223, 3123
CHEXA, 51, 31, 3212, 3312, 3322, 3222, 3213, 3313
, 3323, 3223
CHEXA, 52, 31, 3122, 3222, 3232, 3132, 3123, 3223
, 3233, 3133
CHEXA, 53, 30, 3222, 3322, 3332, 3232, 3223, 3323
, 3333, 3233
SESET, 33, 3111, THRU, 3333
\$ *****
\$ SINGLE CELL, FOUR PORES
GRID, 4111, , 2.0, 2.0, 62.0
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GRID, 4332, , 18.0, 18.0, 70.0
GRID, 4333, , 18.0, 18.0, 78.0
CHEXA, 60, 30, 114, 214, 215, 115, 4111, 4311
, 4313, 4113
CHEXA, 61, 30, 214, 224, 225, 215, 4311, 4331
, 4333, 4313
CHEXA, 62, 30, 224, 124, 125, 225, 4331, 4131
, 4133, 4333

CHEXA, 63, 30, 124, 114, 115, 125, 4131, 4111
 , 4113, 4133
 CHEXA, 64, 30, 115, 215, 225, 125, 4113, 4313
 , 4333, 4133
 CHEXA, 65, 30, 114, 214, 224, 124, 4111, 4311
 , 4331, 4131
 CHEXA, 66, 31, 4111, 4211, 4221, 4121, 4112, 4212
 , 4222, 4122
 CHEXA, 67, 30, 4211, 4311, 4321, 4221, 4212, 4312
 , 4322, 4222
 CHEXA, 68, 30, 4121, 4221, 4231, 4131, 4122, 4222
 , 4232, 4132
 CHEXA, 69, 31, 4221, 4321, 4331, 4231, 4222, 4322
 , 4332, 4232
 CHEXA, 54, 30, 4112, 4212, 4222, 4122, 4113, 4213
 , 4223, 4123
 CHEXA, 55, 31, 4212, 4312, 4322, 4222, 4213, 4313
 , 4323, 4223
 CHEXA, 56, 31, 4122, 4222, 4232, 4132, 4123, 4223
 , 4233, 4133
 CHEXA, 57, 30, 4222, 4322, 4332, 4232, 4223, 4323
 , 4333, 4233
 SESET, 49, 4111, THRU, 4333

\$ *****

\$ SINGLE CELL, FIVE PORES

GRID, 5111, , 2.0, 62.0, 42.0
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 GRID, 5332, , 18.0, 78.0, 50.0

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 CHEXA, 70, 30, 143, 243, 244, 144, 5111, 5311
 , 5313, 5113
 CHEXA, 71, 30, 243, 253, 254, 244, 5311, 5331
 , 5333, 5313
 CHEXA, 72, 30, 253, 153, 154, 254, 5331, 5131
 , 5133, 5333
 CHEXA, 73, 30, 153, 143, 144, 154, 5131, 5111
 , 5113, 5133
 CHEXA, 74, 30, 144, 244, 254, 154, 5113, 5313
 , 5333, 5133
 CHEXA, 75, 30, 143, 243, 253, 153, 5111, 5311
 , 5331, 5131
 CHEXA, 76, 31, 5111, 5211, 5221, 5121, 5112, 5212
 , 5222, 5122
 CHEXA, 77, 31, 5211, 5311, 5321, 5221, 5212, 5312
 , 5322, 5222
 CHEXA, 78, 30, 5121, 5221, 5231, 5131, 5122, 5222
 , 5232, 5132
 CHEXA, 79, 31, 5221, 5321, 5331, 5231, 5222, 5322
 , 5332, 5232
 CHEXA, 80, 30, 5112, 5212, 5222, 5122, 5113, 5213
 , 5223, 5123
 CHEXA, 81, 31, 5212, 5312, 5322, 5222, 5213, 5313
 , 5323, 5223
 CHEXA, 82, 31, 5122, 5222, 5232, 5132, 5123, 5223
 , 5233, 5133
 CHEXA, 83, 30, 5222, 5322, 5332, 5232, 5223, 5323
 , 5333, 5233
 SESET, 45, 5111, THRU, 5333

\$ *****

\$ SINGLE CELL, SIX PORES

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 GRID, 6332, , 78.0, 18.0, 50.0
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 CHEXA, 90, 30, 413. 513. 514. 414. 6111. 6311
 , 6313, 6113
 CHEXA, 91, 30, 513. 523. 524. 514. 6311. 6331
 , 6333, 6313
 CHEXA, 92, 30, 523. 423. 424. 524. 6331. 6131
 , 6133, 6333
 CHEXA, 93, 30, 423. 413. 414. 424. 6131. 6111
 , 6113, 6133
 CHEXA, 94, 30, 414, 514, 524, 424, 6113, 6313
 , 6333, 6133
 CHEXA, 95, 30, 413, 513, 523, 423, 6111, 6311
 , 6331, 6131
 CHEXA, 96, 31, 6111, 6211, 6221, 6121, 6112, 6212
 , 6222, 6122
 CHEXA, 97, 31, 6211, 6311, 6321, 6221, 6212, 6312
 , 6322, 6222
 CHEXA, 98, 30, 6121, 6221, 6231, 6131, 6122, 6222
 , 6232, 6132
 CHEXA, 99, 31, 6221, 6321, 6331, 6231, 6222, 6322
 , 6332, 6232
 CHEXA, 84, 31, 6112, 6212, 6222, 6122, 6113, 6213
 , 6223, 6123
 CHEXA, 85, 30, 6212, 6312, 6322, 6222, 6213, 6313
 , 6323, 6223
 CHEXA, 86, 31, 6122, 6222, 6232, 6132, 6123, 6223
 , 6233, 6133
 CHEXA, 87, 30, 6222, 6322, 6332, 6232, 6223, 6323
 , 6333, 6233
 SESET, 36, 6111, THRU, 6333
 \$ *****
 \$ SINGLE CELL, SEVEN PORES
 GRID, 7111, , 2.0, 2.0, 22.0
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 GRID, 7233, , 10.0, 18.0, 38.0
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 GRID, 7333, , 18.0, 18.0, 38.0
 CHEXA, 170, 30, 112, 212, 213, 113, 7111, 7311
 , 7313, 7113
 CHEXA, 171, 30, 212, 222, 223, 213, 7311, 7331
 , 7333, 7313
 CHEXA, 172, 30, 222, 122, 123, 223, 7331, 7131
 , 7133, 7333
 CHEXA, 173, 30, 122, 112, 113, 123, 7131, 7111
 , 7113, 7133
 CHEXA, 174, 30, 113, 213, 223, 123, 7113, 7313
 , 7333, 7133
 CHEXA, 175, 30, 112, 212, 222, 122, 7111, 7311
 , 7331, 7131
 CHEXA, 176, 31, 7111, 7211, 7221, 7121, 7112, 7212
 , 7222, 7122
 CHEXA, 177, 31, 7211, 7311, 7321, 7221, 7212, 7312
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 CHEXA, 178, 31, 7121, 7221, 7231, 7131, 7122, 7222
 , 7232, 7132
 CHEXA, 179, 31, 7221, 7321, 7331, 7231, 7222, 7322
 , 7332, 7232
 CHEXA, 180, 30, 7112, 7212, 7222, 7122, 7113, 7213
 , 7223, 7123
 CHEXA, 181, 31, 7212, 7312, 7322, 7222, 7213, 7313
 , 7323, 7223
 CHEXA, 182, 31, 7122, 7222, 7232, 7132, 7123, 7223
 , 7233, 7133
 CHEXA, 183, 31, 7222, 7322, 7332, 7232, 7223, 7323
 , 7333, 7233
 SESET, 17, 7111, THRU, 7333
 \$ *****
 \$ SINGLE CELL, EIGHT PORE CONGLOMERATE
 GRID, 8111, , 22.0, 22.0, 22.0
 GRID, 8113, , 22.0, 22.0, 38.0
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 GRID, 8333, , 38.0, 38.0, 38.0
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 , 8313, 8113
 CHEXA, 9, 30, 322, 332, 333, 323, 8311, 8331
 , 8333, 8313
 CHEXA, 18, 30, 332, 232, 233, 333, 8331, 8131
 , 8133, 8333
 CHEXA, 38, 30, 232, 222, 223, 233, 8131, 8111
 , 8113, 8133
 CHEXA, 58, 30, 223, 323, 333, 233, 8113, 8313
 , 8333, 8133
 CHEXA, 88, 30, 222, 322, 332, 232, 8111, 8311
 , 8331, 8131
 CHEXA, 10, 31, 8111, 8311, 8313, 8113, 8131, 8331
 , 8333, 8133
 SESET, 38, 8111, THRU, 8333
 \$ **** COPIED SUPERELEMENTS
 CSUPER, 2, 45, 211, 212, 221, 222, 311, 312
 , 321, 322
 CSUPER, 3, 4, 311, 312, 321, 322, 411, 412
 , 421, 422
 CSUPER, 5, 49, 121, 122, 131, 132, 221, 222
 , 231, 232
 CSUPER, 6, 33, 221, 222, 231, 232, 321, 322
 , 331, 332
 CSUPER, 7, 4, 321, 322, 331, 332, 421, 422
 , 431, 432
 CSUPER, 8, 36, 421, 422, 431, 432, 521, 522
 , 531, 532
 CSUPER, 9, 33, 131, 132, 141, 142, 231, 232
 , 241, 242
 CSUPER, 10, 4, 231, 232, 241, 242, 331, 332
 , 341, 342
 CSUPER, 11, 33, 331, 332, 341, 342, 431, 432
 , 441, 442
 CSUPER, 12, 49, 431, 432, 441, 442, 531, 532
 , 541, 542
 CSUPER, 13, 33, 141, 142, 151, 152, 241, 242
 , 251, 252
 CSUPER, 14, 49, 241, 242, 251, 252, 341, 342
 , 351, 352
 CSUPER, 15, 4, 341, 342, 351, 352, 441, 442
 , 451, 452
 CSUPER, 16, 33, 441, 442, 451, 452, 541, 542
 , 551, 552
 CSUPER, 18, 4, 212, 213, 222, 223, 312, 313
 , 322, 323
 CSUPER, 19, 49, 312, 313, 322, 323, 412, 413
 , 422, 423

CSUPER, 21, 4, 122, 123, 132, 133, 222, 223
, 232, 233
CSUPER, 22, 33, 222, 223, 232, 233, 322, 323
, 332, 333
CSUPER, 23, 49, 322, 323, 332, 333, 422, 423
, 432, 433
CSUPER, 24, 4, 422, 423, 432, 433, 522, 523
, 532, 533
CSUPER, 25, 33, 132, 133, 142, 143, 232, 233
, 242, 243
CSUPER, 26, 33, 232, 233, 242, 243, 332, 333
, 342, 343
CSUPER, 27, 1, 332, 333, 342, 343, 432, 433
, 442, 443
CSUPER, 28, 45, 432, 433, 442, 443, 532, 533
, 542, 543
CSUPER, 29, 4, 142, 143, 152, 153, 242, 243
, 252, 253
CSUPER, 30, 33, 242, 243, 252, 253, 342, 343
, 352, 353
CSUPER, 31, 45, 342, 343, 352, 353, 442, 443
, 452, 453
CSUPER, 32, 49, 442, 443, 452, 453, 542, 543
, 552, 553
CSUPER, 34, 49, 213, 214, 223, 224, 313, 314
, 323, 324
CSUPER, 35, 49, 313, 314, 323, 324, 413, 414
, 423, 424
CSUPER, 37, 45, 123, 124, 133, 134, 223, 224
, 233, 234
CSUPER, 39, 36, 323, 324, 333, 334, 423, 424
, 433, 434
CSUPER, 40, 4, 423, 424, 433, 434, 523, 524
, 533, 534
CSUPER, 41, 45, 133, 134, 143, 144, 233, 234
, 243, 244
CSUPER, 42, 4, 233, 234, 243, 244, 333, 334
, 343, 344
CSUPER, 43, 49, 333, 334, 343, 344, 433, 434
, 443, 444
CSUPER, 44, 4, 433, 434, 443, 444, 533, 534
, 543, 544
CSUPER, 46, 17, 243, 244, 253, 254, 343, 344
, 353, 354
CSUPER, 47, 33, 343, 344, 353, 354, 443, 444
, 453, 454
CSUPER, 48, 49, 443, 444, 453, 454, 543, 544
, 553, 554
CSUPER, 50, 45, 214, 215, 224, 225, 314, 315
, 324, 325
CSUPER, 51, 1, 314, 315, 324, 325, 414, 415

, 424, 425
CSUPER, 52, 45, 414, 415, 424, 425, 514, 515
, 524, 525
CSUPER, 53, 4, 124, 125, 134, 135, 224, 225
, 234, 235
CSUPER, 54, 36, 224, 225, 234, 235, 324, 325
, 334, 335
CSUPER, 55, 33, 324, 325, 334, 335, 424, 425
, 434, 435
CSUPER, 56, 1, 424, 425, 434, 435, 524, 525
, 534, 535
CSUPER, 57, 49, 134, 135, 144, 145, 234, 235
, 244, 245
CSUPER, 58, 33, 234, 235, 244, 245, 334, 335
, 344, 345
CSUPER, 59, 33, 334, 335, 344, 345, 434, 435
, 444, 445
CSUPER, 60, 1, 434, 435, 444, 445, 534, 535
, 544, 545
CSUPER, 61, 45, 144, 145, 154, 155, 244, 245
, 254, 255
CSUPER, 62, 4, 244, 245, 254, 255, 344, 345
, 354, 355
CSUPER, 63, 33, 344, 345, 354, 355, 444, 445
, 454, 455
CSUPER, 64, 1, 444, 445, 454, 455, 544, 545
, 554, 555
CSUPER, 65, 33, 115, 116, 125, 126, 215, 216
, 225, 226
CSUPER, 66, 20, 215, 216, 225, 226, 315, 316
, 325, 326
CSUPER, 67, 49, 315, 316, 325, 326, 415, 416
, 425, 426
CSUPER, 68, 33, 415, 416, 425, 426, 515, 516
, 525, 526
CSUPER, 69, 49, 125, 126, 135, 136, 225, 226
, 235, 236
CSUPER, 70, 1, 225, 226, 235, 236, 325, 326
, 335, 336
CSUPER, 71, 33, 325, 326, 335, 336, 425, 426
, 435, 436
CSUPER, 72, 4, 425, 426, 435, 436, 525, 526
, 535, 536
CSUPER, 73, 33, 135, 136, 145, 146, 235, 236
, 245, 246
CSUPER, 74, 20, 235, 236, 245, 246, 335, 336
, 345, 346
CSUPER, 75, 4, 335, 336, 345, 346, 435, 436
, 445, 446
CSUPER, 76, 49, 435, 436, 445, 446, 535, 536
, 545, 546

CSUPER, 77, 33, 145, 146, 155, 156, 245, 246
 , 255, 256
 CSUPER, 78, 1, 245, 246, 255, 256, 345, 346
 , 355, 356
 CSUPER, 79, 4, 345, 346, 355, 356, 445, 446
 , 455, 456
 CSUPER, 80, 49, 445, 446, 455, 456, 545, 546
 , 555, 556
 \$ *****
 \$ GLOBAL DUMMY ELEMENT
 CHEXA, 999, 31, 111, 511, 551, 151, 116, 516
 , 556, 156
 PSOLID, 31, 41
 MAT1, 41, 3.0+6, , 0.0, 1.0-8
 \$ *****
 \$ GLOBAL RESTRAINTS AND FORCES
 SPC, 20, 111, 123456
 SPC, 20, 121, 3
 SPC, 20, 131, 3
 SPC, 20, 141, 3
 SPC, 20, 151, 3
 SPC, 20, 211, 3
 SPC, 20, 221, 3
 SPC, 20, 231, 3
 SPC, 20, 241, 3
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 FORCE, 10, 236, , 8.0, , , 1.0

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FORCE, 10, 346, , 8.0, , , 1.0
FORCE, 10, 356, , 4.0, , , 1.0
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FORCE, 10, 436, , 8.0, , , 1.0
FORCE, 10, 446, , 8.0, , , 1.0
FORCE, 10, 456, , 4.0, , , 1.0
FORCE, 10, 516, , 2.0, , , 1.0
FORCE, 10, 526, , 4.0, , , 1.0
FORCE, 10, 536, , 4.0, , , 1.0
FORCE, 10, 546, , 4.0, , , 1.0
FORCE, 10, 556, , 2.0, , , 1.0
ENDDATA

Appendix B: Computer Model Test Runs, Input Data and Results

Tests: PORE1-

Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

$E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

Applied force, upward: 128×10^{-11} Newtons

Evenly distributed pores (one per cell)

Simulated material: Porous Cobalt Oxide (CoO)

Data:

<u>Test Name</u>	<u>Avg Disp</u> <u>($\times 10^{-12}$)</u>	<u>Calculated</u> <u>E (10^{11} Pa)</u>	<u>% Porosity</u>	<u>Pore Size</u>
Pore1-1	9.9	2.0200	0	N/A
Pore1-2	10.7	1.8692	6.4	8 units
Pore1-3	11.1	1.8018	9.1	9 units
Pore1-4	12.36	1.6181	16.6	11 units
Pore1-5	13.3	1.5138	21.6	12 units

Tests: PORE1- B

Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

$E_0 = 33.2 \times 10^6$ psia, 2.29×10^{11} Pascals

Applied force, upward: 128×10^{-11} Newtons

Evenly distributed pores (one per cell)

Simulated material: Porous Cobolt Oxide (CoO)

Data:

<u>Test Name</u>	<u>Avg Disp (x 10^{-12})</u>	<u>Calculated E (10^{11} Pa)</u>	<u>% Porosity</u>	<u>Pore Size</u>
Pore1-1B	8.7336	2.2900	0	N/A
Pore1-2B	9.4574	2.1147	6.4	8 units
Pore1-3B	9.8110	2.0385	9.1	9 units
Pore1-4B	10.9318	1.8295	16.6	11 units
Pore1-5B	11.7966	1.6954	21.6	12 units

Test name: PORE2-1Unit length = 1 micrometer (10^{-6} m)80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

8.88 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobolt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	23	22	21	10	3	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.275	1.243	1.204	1.205	1.224
1.221	1.214	1.196	1.200	1.222
1.174	1.200	1.211	1.203	1.203
1.154	1.188	1.206	1.202	1.203
1.131	1.160	1.173	1.191	1.216

Results:average displacement = 12.181×10^{-12} unitscalculated value for E = 1.642×10^{11} Pascals= 23.813×10^6 Psia**Test name: PORE2-2**Unit length = 1 micrometer (10^{-6} m)80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

8.00 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobolt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.285	1.237	1.183	1.173	1.181
1.221	1.198	1.167	1.159	1.170
1.166	1.177	1.174	1.154	1.143
1.143	1.163	1.166	1.149	1.137
1.118	1.132	1.131	1.134	1.146

Results:average displacement = 11.684×10^{-12} unitscalculated value for E = 1.711×10^{11} Pascals= 24.8×10^6 Psia

Test name: PORE2-3

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals
 10.08 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobalt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	19	21	22	12	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.578	1.393	1.233	1.203	1.179
1.392	1.293	1.234	1.203	1.175
1.244	1.250	1.244	1.207	1.172
1.294	1.263	1.217	1.191	1.167
1.462	1.254	1.171	1.159	1.151

Results:

average displacement = 12.53×10^{-12} units
 calculated value for E = 1.596×10^{11} Pascals
 = 23.15×10^6 Psia

Test name: PORE2-4

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals
 11.52 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobalt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	15	19	23	15	6	2			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.690	1.526	1.454	1.459	1.335
1.459	1.380	1.367	1.334	1.274
1.274	1.297	1.300	1.265	1.229
1.294	1.275	1.240	1.217	1.195
1.434	1.236	1.161	1.155	1.149

Results:

average displacement = 1.320×10^{-12} units
 calculated value for E = 1.515×10^{11} Pascals
 = 21.975×10^6 Psia

Test name: PORE2-5Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

14.16 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobolt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	10	15	23	18	9	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.713	1.560	1.514	1.592	1.532
1.503	1.434	1.440	1.446	1.426
1.424	1.412	1.381	1.358	1.338
1.600	1.437	1.339	1.316	1.289
1.660	1.391	1.284	1.265	1.241

Results:average displacement = 14.933×10^{-12} unitscalculated value for E = 1.339×10^{11} Pascals= 19.42×10^6 Psia**Test name: PORE2-6**Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

17.12 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobolt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	6	11	21	20	13	6	2	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.712	1.618	1.633	1.741	1.660
1.528	1.526	1.593	1.613	1.580
1.462	1.517	1.546	1.550	1.538
1.643	1.531	1.487	1.503	1.504
1.707	1.476	1.408	1.423	1.427

Results:average displacement = 15.571×10^{-12} unitscalculated value for E = 1.28×10^{11} Pascals= 18.63×10^6 Psia

Test name: PORE2-7

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals
 18.08 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	5	10	20	20	14	7	3	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.730	1.653	1.686	1.812	1.746
1.536	1.547	1.630	1.671	1.653
1.456	1.555	1.599	1.588	1.598
1.629	1.584	1.548	1.559	1.540
1.693	1.471	1.488	1.497	1.448

Results:

average displacement = 15.79×10^{-12} units
 calculated value for E = 1.266×10^{11} Pascals
 = 18.36×10^6 Psia

Test name: PORE2-8

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals
 19.04 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	4	9	19	20	15	8	4	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.725	1.796	1.854	1.832	1.771
1.534	1.597	1.684	1.694	1.681
1.461	1.561	1.613	1.612	1.627
1.633	1.597	1.568	1.620	1.664
1.693	1.483	1.508	1.660	1.682

Results:

average displacement = 16.46×10^{-12} units
 calculated value for E = 1.215×10^{11} Pascals
 = 17.62×10^6 Psia

Test name: PORE2-9

Unit length = 1 micrometer (10^{-6} m)

80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units

$E_0 = 29.3 \times 10^6$ psia, 2.02×10^{11} Pascals

20.56 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobalt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	3	8	17	20	16	9	4	2	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.876	1.911	1.937	2.001	2.070
1.730	1.741	1.777	1.797	1.790
1.627	1.686	1.693	1.668	1.674
1.709	1.656	1.609	1.650	1.685
1.711	1.490	1.508	1.655	1.669

Results:

average displacement = 17.33×10^{-12} units

calculated value for E = 1.154×10^{11} Pascals
= 16.73×10^6 Psia

Test name: PORE2-2;2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 26.1 \times 10^6$ psia, 1.8×10^{11} Pascals
 8.00 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y				
1.442	1.388	1.328	1.316	1.325	
1.370	1.344	1.309	1.301	1.312	
1.309	1.322	1.317	1.295	1.283	
1.283	1.305	1.309	1.289	1.276	
1.255	1.270	1.269	1.273	1.286	

Results:

average displacement = 13.112×10^{-12} units
 calculated value for E = 1.525×10^{11} Pascals
 = 22.12×10^6 Psia

Test name: PORE2-3;2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 26.1 \times 10^6$ psia, 1.8×10^{11} Pascals
 10.08 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	19	21	22	12	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y				
1.771	1.563	1.383	1.350	1.324	
1.562	1.451	1.385	1.350	1.319	
1.396	1.402	1.396	1.355	1.315	
1.452	1.417	1.366	1.336	1.309	
1.641	1.407	1.314	1.301	1.292	

Results:

average displacement = 14.064×10^{-12} units
 calculated value for E = 1.422×10^{11} Pascals
 = 20.62×10^6 Psia

Test name: PORE2-5;2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 26.1 \times 10^6$ psia, 1.8×10^{11} Pascals
 14.16 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	10	15	23	28	9	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.923	1.751	1.699	1.787	1.719
1.687	1.609	1.616	1.623	1.600
1.598	1.584	1.550	1.525	1.501
1.796	1.612	1.503	1.477	1.447
1.863	1.561	1.441	1.419	1.393

Results:

average displacement = 16.113×10^{-12} units
 calculated value for E = 1.241×10^{11} Pascals
 = 18.00×10^6 Psia

Test name: PORE2-6;2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 26.1 \times 10^6$ psia, 1.8×10^{11} Pascals
 17.12 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Cobolt Oxide (CoO)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	6	11	21	20	13	6	2	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.921	1.815	1.833	1.954	1.863
1.714	1.713	1.787	1.810	1.773
1.640	1.703	1.735	1.739	1.727
1.844	1.718	1.669	1.687	1.687
1.915	1.656	1.580	1.579	1.601

Results:

average displacement = 17.474×10^{-12} units
 calculated value for E = 1.145×10^{11} Pascals
 = 16.60×10^6 Psia

Test name: PORE2-8;2Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 26.1 \times 10^6$ psia, 1.8×10^{11} Pascals

19.04 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Cobalt Oxide (CoO)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	4	9	19	20	15	8	4	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X -> Y

1.936	2.016	2.081	1.056	1.988
1.721	1.793	1.889	1.901	1.886
1.639	1.752	1.811	1.809	1.825
1.832	1.792	1.760	1.818	1.868
1.900	1.664	1.692	1.863	1.888

Results:average displacement = 18.472×10^{-12} unitscalculated value for E = 1.0827×10^{11} Pascals
= 15.70×10^6 Psia

Test name: PORE3-1Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

8.0 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
5.918	5.692	5.452	5.409	5.448
5.618	5.505	5.363	5.335	5.390
5.376	5.414	5.395	5.307	5.265
5.273	5.348	5.364	5.285	5.233
5.163	5.216	5.206	5.222	5.278

Results:average displacement = 5.379×10^{-12} unitscalculated value for E = 3.7181×10^{11} Pascals= 53.925×10^6 Psia**Test name: PORE3-2**Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

10.08 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	19	21	22	12	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.248	6.410	5.682	5.551	5.444
6.405	5.946	5.667	5.536	5.423
5.740	5.743	5.712	5.553	5.402
5.974	5.814	5.598	5.479	5.372
6.747	5.792	5.394	5.335	5.296

Results:average displacement = 5.771×10^{-12} unitscalculated value for E = 3.466×10^{11} Pascals= 50.264×10^6 Psia

Test name: PORE3-3Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

14.16 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	10	15	23	28	9	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.901	7.197	6.984	7.353	7.088
6.945	6.605	6.627	6.661	6.591
6.585	6.499	6.343	6.252	6.180
7.383	6.618	6.165	6.061	5.952
7.660	6.429	5.930	5.839	5.734

Results:average displacement = 6.623×10^{-12} units

calculated value for E = 3.019×10^{11} Pascals
 = 43.8×10^6 Psia

Test name: PORE3-4Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

20.56 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	3	8	17	20	16	9	4	2	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
8.691	8.856	8.973	9.266	9.594
7.997	8.037	8.193	8.294	8.291
7.525	7.762	7.783	7.681	7.732
7.884	7.638	7.423	7.614	7.784
7.884	6.890	6.974	7.643	7.705

Results:average displacement = 8.005×10^{-12} units

calculated value for E = 2.498×10^{11} Pascals
 = 36.24×10^6 Psia

Test name: PORE3-5Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

0.7 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	72	7	1						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y				
4.572	4.562	4.556	4.563	4.568	
4.590	4.577	4.567	4.565	4.561	
4.603	4.617	4.602	4.566	4.555	
4.621	4.633	4.616	4.577	4.563	
4.639	4.620	4.602	4.591	4.576	

Results:average displacement = 4.586×10^{-12} unitscalculated value for E = 4.361×10^{11} Pascals= 63.244×10^6 Psi**Test name: PORE3-6**Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

0.96 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	69	10	1						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y				
4.558	4.558	4.562	4.575	4.587	
4.581	4.577	4.577	4.582	4.584	
4.599	4.621	4.616	4.588	4.583	
4.622	4.643	4.635	4.605	4.600	
4.645	4.635	4.626	4.625	4.618	

Results:average displacement = 4.600×10^{-12} unitscalculated value for E = 4.3477×10^{11} Pascals= 63.056×10^6 Psia

Test name: PORE3-7Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

1.2 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	67	11	2						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

4.576	4.573	4.574	4.586	4.597
4.594	4.589	4.588	4.591	4.592
4.648	4.647	4.621	4.592	4.588
4.669	4.664	4.652	4.621	4.592
4.643	4.630	4.662	4.658	4.604

Results:average displacement = 4.614×10^{-12} unitscalculated value for E = 4.335×10^{11} Pascals= 62.866×10^6 Psia**Test name: PORE3-8**Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

2.16 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	59	16	4	1					

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

4.941	4.822	4.705	4.664	4.645
4.835	4.755	4.704	4.672	4.625
4.769	4.738	4.707	4.662	4.618
4.732	4.718	4.699	4.663	4.627
4.670	4.654	4.692	4.691	4.629

Results:average displacement = 4.7055×10^{-12} unitscalculated value for E = 4.250×10^{11} Pascals= 61.64×10^6 Psia

Test name: PORE3-9

Unit length = .25 micrometer (10^{-6} m)

80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units

$E_0 = 63.8 \times 10^6$ psia, 4.4×10^{11} Pascals

2.688 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poisson distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	50	10	8	2					

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X -> Y

4.925	4.835	4.742	4.721	4.725
4.843	4.799	4.780	4.778	4.772
4.802	4.794	4.793	4.842	4.909
4.762	4.805	4.813	4.835	4.910
4.683	4.783	4.834	4.771	4.758

Results:

average displacement = 4.8006×10^{-12} units

calculated value for E = 4.166×10^{11} Pascals
= 60.422×10^6 Psia

Test name: PORE3-1;2Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

8.0 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
6.127	5.893	5.645	5.600	5.640
5.817	5.699	5.553	5.523	5.580
5.565	5.605	5.586	5.494	5.451
5.459	5.537	5.553	5.471	5.418
5.345	5.400	5.390	5.406	5.464

Results:

average displacement = 5.569×10^{-12} units
 calculated value for E = 3.590×10^{11} Pascals
 = 52.08×10^6 Psia

Test name: PORE3-2;2Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

10.08 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	19	21	22	12	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.504	6.636	5.883	5.747	5.636
6.631	6.156	5.870	5.732	5.615
5.943	5.946	5.914	5.749	5.593
6.184	6.019	5.796	5.673	5.562
6.985	5.997	5.584	5.524	5.483

Results:

average displacement = 5.974×10^{-12} units
 calculated value for E = 3.3475×10^{11} Pascals
 = 48.55×10^6 Psia

Test name: PORE3-3;2Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

14.16 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	10	15	23	18	9	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

8.180	7.451	7.231	7.612	7.339
7.190	6.838	6.861	6.897	6.823
6.817	6.729	6.567	6.473	6.398
7.643	6.852	6.382	6.275	6.162
7.930	6.655	6.139	6.045	5.936

Results:average displacement = 6.857×10^{-12} unitscalculated value for E = 2.917×10^{11} Pascals= 42.3×10^6 Psia**Test name: PORE3-4;2**Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

20.56 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	3	8	17	20	16	9	4	2	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

8.997	9.168	9.289	9.593	9.933
8.279	8.321	8.482	8.587	8.583
7.790	8.036	8.058	7.953	8.005
8.163	7.907	7.685	7.883	8.059
8.162	7.133	7.221	7.913	7.977

Results:average displacement = 8.287×10^{-12} unitscalculated value for E = 2.413×10^{11} Pascals= 35.00×10^6 Psia

Test name: PORE3-11Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

0.7 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	77	7	1						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
4.733	4.723	4.717	4.723	4.729
4.752	4.739	4.728	4.726	4.722
4.765	4.780	4.764	4.728	4.716
4.784	4.796	4.778	4.738	4.724
4.802	4.783	4.764	4.753	4.737

Results:average displacement = 4.748×10^{-12} unitscalculated value for E = 4.212×10^{11} Pascals= 61.088×10^6 Psia**Test name: PORE3-12**Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

1.2 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	67	11	2						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
4.738	4.734	4.736	4.749	4.759
4.756	4.751	4.750	4.752	4.754
4.812	4.811	4.784	4.754	4.749
4.834	4.828	4.816	4.784	4.754
4.807	4.794	4.827	4.823	4.766

Results:average displacement = 4.777×10^{-12} unitscalculated value for E = 4.187×10^{11} Pascals= 60.72×10^6 Psia

Test name: PORE3-13Unit length = .25 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 61.64 \times 10^6$ psia, 4.25×10^{11} Pascals

2.16 % porosity

Pore size: cubes, 8 units on each side, i.e. 2 micrometers

Simulated material: Porous Silicon Carbide (SiC)

Pores follow a poisson distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	59	16	4	1					

Data:listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	-> Y				
5.115	4.992	4.871	4.829	4.809	
5.005	4.923	4.870	4.837	4.788	
4.937	4.906	4.873	4.826	4.781	
4.899	4.884	4.864	4.828	4.790	
4.835	4.819	4.858	4.856	4.792	

Results:average displacement = 4.873×10^{-12} unitscalculated value for E = 4.104×10^{11} Pascals
= 59.52×10^6 Psia

Test name: PORE4-1

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 8.00 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

7.643	7.538	7.414	7.509	7.667
7.264	7.304	7.297	7.406	7.596
6.968	7.201	7.342	7.363	7.434
6.883	7.150	7.319	7.345	7.405
6.776	7.002	7.128	7.278	7.484

Results:

average displacement = 7.3086×10^{-12} units
 calculated value for E = 2.7365×10^{11} Pascals
 = 39.688×10^6 Psia

Test name: PORE4-2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 10.08 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	19	21	22	12	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

8.444	7.614	6.904	7.026	7.317
7.560	7.263	7.436	7.946	8.296
6.930	7.292	7.898	8.423	8.738
7.556	7.715	7.815	8.166	8.586
8.936	7.971	7.817	8.185	8.615

Results:

average displacement = 7.858×10^{-12} units
 calculated value for E = 2.545×10^{11} Pascals
 = 36.91×10^6 Psia

Test name: PORE4-3Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

11.52 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	15	19	23	15	6	2			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
8.110	8.051	8.389	8.672	8.161
7.676	7.848	8.185	8.253	8.136
7.311	7.752	8.141	8.229	8.215
7.704	8.160	8.350	8.501	9.020
8.905	8.825	8.300	8.855	9.724

Results:average displacement = 8.181×10^{-12} unitscalculated value for E = 2.444×10^{11} Pascals= 35.455×10^6 Psia**Test name: PORE4-4**Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

14.16 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	10	15	23	18	9	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
8.395	8.452	9.005	9.721	9.614
8.251	8.380	8.739	9.044	9.223
8.567	8.619	8.746	8.928	8.874
9.700	8.936	8.797	9.053	9.372
10.325	8.935	8.486	9.192	10.035

Results:average displacement = 9.016×10^{-12} unitscalculated value for E = 2.218×10^{11} Pascals= 32.173×10^6 Psia

Test name: PORE4-5

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 17.12 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	6	11	21	20	13	6	2	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

9.425	9.462	9.837	10.060	9.314
8.987	9.274	10.046	10.009	9.340
9.092	9.410	9.862	9.909	9.667
10.269	9.614	9.384	9.858	10.512
10.772	9.383	9.071	9.974	10.965

Results:

average displacement = 9.740×10^{-12} units
 calculated value for E = 2.054×10^{11} Pascals
 = 29.78×10^6 Psia

Test name: PORE4-6

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 19.04 % porosity
 Pore size: cubes, 8 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	4	9	19	20	15	8	4	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

8.944	9.913	11.361	11.319	11.084
9.175	9.837	10.328	10.559	10.652
9.876	9.870	10.271	10.418	10.482
10.029	9.904	10.162	10.832	11.113
10.520	9.309	9.610	10.997	11.674

Results:

average displacement = 10.330×10^{-12} units
 calculated value for E = 1.936×10^{11} Pascals
 = 28.08×10^6 Psia

Test name: PORE4-7

Unit length = 1 micrometer (10^{-6} m)

80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units

$E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

20.56 % porosity

Pore size: cubes, 8 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poisson distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	3	8	17	20	16	9	4	2	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X -> Y

1.183	1.206	1.222	1.262	1.306
1.089	1.096	1.118	1.132	1.129
1.025	1.060	1.064	1.049	1.053
1.075	1.042	1.013	1.039	1.061
1.076	0.938	0.949	1.042	1.051

Results:

average displacement = 10.013×10^{-12} units

calculated value for E = 1.8326×10^{11} Pascals
= 26.58×10^6 Psia

Test name: PORE5-1

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 15.4 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	26	23	20	8	2	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
10.364	9.881	9.477	9.787	10.241
8.980	8.982	8.956	9.267	9.802
7.952	8.476	8.808	8.912	9.142
7.591	8.221	8.617	8.713	8.887
7.183	7.737	8.065	8.435	8.929

Results:

average displacement = 8.856×10^{-12} units
 calculated value for E = 2.258×10^{11} Pascals
 = 32.753×10^6 Psia

Test name: PORE5-2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 3.6 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	61	15	4						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
6.310	6.352	6.398	6.396	6.389
6.500	6.449	6.484	6.582	6.604
6.569	6.507	6.512	6.595	6.619
6.466	6.546	6.543	6.480	6.482
6.445	6.600	6.616	6.493	6.483

Results:

average displacement = 6.497×10^{-12} units
 calculated value for E = 3.078×10^{11} Pascals
 = 44.647×10^6 Psia

Test name: PORE5-3

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 5.156 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	55	18	6	1					

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.643	7.332	6.985	6.675	6.443
7.488	7.151	6.885	6.752	6.576
7.156	6.908	6.756	6.695	6.564
6.754	6.723	6.646	6.503	6.395
6.515	6.584	6.563	6.406	6.321

Results:

average displacement = 6.777×10^{-12} units
 calculated value for E = 2.951×10^{11} Pascals
 = 42.825×10^6 Psia

Test name: PORE5-4

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 6.56 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	50	20	8	2					

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.644	7.494	7.270	7.041	6.895
7.445	7.259	7.097	7.059	6.993
7.088	6.955	7.026	7.075	6.910
6.673	6.751	6.897	6.856	6.703
6.419	6.617	6.677	6.604	6.636

Results:

average displacement = 6.963×10^{-12} units
 calculated value for E = 2.872×10^{11} Pascals
 = 41.656×10^6 Psia

Test name: PORE5-5

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 10.47 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	38	23	14	4	1				

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
9.622	8.709	8.042	8.789	10.420
8.506	8.061	7.756	8.045	8.714
7.336	7.327	7.455	7.500	7.449
7.884	7.303	7.007	7.108	7.133
	7.721	6.621	6.833	7.135

Results:

average displacement = 7.392×10^{-12} units
 calculated value for E = 2.706×10^{11} Pascals
 = 39.242×10^6 Psia

Test name: PORE5-6

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 14.06 % porosity
 Pore size: cubes, 10 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	29	23	19	7	2				

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
10.477	9.249	8.664	10.741	
9.105	8.559	8.281	9.100	10.300
7.758	7.845	8.013	8.034	8.004
8.405	7.879	7.626	7.765	7.837
	8.383	7.316	7.569	7.915

Results:

average displacement = 8.277×10^{-12} units
 calculated value for E = 2.416×10^{11} Pascals
 = 35.046×10^6 Psia

Test name: PORE5-7Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

19.06 % porosity

Pore size: cubes, 10 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	20	21	22	12	4	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.120	1.049	1.055	1.321	
0.991	0.972	0.987	1.111	1.246
0.870	0.947	0.999	0.972	1.004
1.041	1.032	0.944	0.931	0.967
	1.059	0.862	0.889	0.935

Results:average displacement = 10.132×10^{-12} units

calculated value for E = 1.974×10^{11} Pascals
= 28.638×10^6 Psia

Test name: PORE5-8Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

16.875 % porosity

Pore size: cubes, 10 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	24	22	21	9	3	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

11.901	10.875	10.560	12.969	
10.013	9.684	9.749	10.847	12.039
8.166	8.437	8.863	9.092	9.154
8.299	7.878	7.705	8.017	8.249
	7.779	6.717	6.959	7.354

Results:average displacement = 9.187×10^{-12} units

calculated value for E = 2.177×10^{11} Pascals
= 31.74×10^6 Psia

Test name: PORE5-9

Unit length = 1 micrometer (10^{-6} m)

80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units

$E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

22.03 % porosity

Pore size: cubes, 10 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	16	19	23	14	6	2			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.102	1.072	1.118	1.429	
0.996	1.022	1.083	1.253	1.432
0.895	1.033	1.133	1.137	1.208
1.083	1.133	1.084	1.098	1.170
	1.148	0.988	1.050	1.128

Results:

average displacement = 10.844×10^{-12} units

calculated value for E = 1.844×10^{11} Pascals
= 26.748×10^6 Psia

Test name: PORE6-1

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 3.574% porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	9	14	23	19	10	4	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	-> Y				
6.720	6.731	6.678	6.573	6.567	
6.718	6.711	6.657	6.658	6.731	
6.702	6.730	6.706	6.741	6.864	
6.729	6.793	6.767	6.712	6.743	
6.784	6.807	6.740	6.638	6.605	

Results:

average displacement = 6.712×10^{-12} units
 calculated value for E = 2.980×10^{11} Pascals
 = 43.214×10^6 Psia

Test name: PORE6-2

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 5.41% porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	2	6	15	20	17	11	6	2	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	-> Y				
7.162	7.205	7.148	7.135	7.241	
7.156	7.128	7.104	7.153	7.173	
7.035	7.033	7.095	7.128	7.094	
6.943	6.984	6.972	6.947	7.000	
6.965	6.909	6.765	6.777	6.671	

Results:

average displacement = 7.045×10^{-12} units
 calculated value for E = 2.839×10^{11} Pascals
 = 41.17×10^6 Psia

Test name: PORE6-3

Unit length = 1 micrometer (10^{-6} m)
 80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 6.97 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8	9	10
Number of cells:	1	2	8	15	17	15	11	6	3	1	1

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

7.163	7.205	7.157	7.161	7.275
7.183	7.221	7.212	7.228	7.259
7.213	7.221	7.253	7.255	7.235
7.180	7.204	7.210	7.193	7.196
7.142	7.153	7.138	7.139	7.132

Results:

average displacement = 7.197×10^{-12} units
 calculated value for E = 2.779×10^{11} Pascals
 = 40.303×10^6 Psia

Test name: PORE6-4

Unit length = 1 micrometer (10^{-6} m)
 80 cells, $4 \times 4 \times 5$, each with dimension $20 \times 20 \times 20$ units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 8.46 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8	9	10
Number of cells:	0	1	4	9	14	15	14	10	7	4	2

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

7.322	7.370	7.376	7.315	7.289
7.329	7.348	7.354	7.334	7.287
7.349	7.328	7.328	7.305	7.273
7.304	7.304	7.292	7.257	7.211
7.255	7.249	7.223	7.209	7.184

Results:

average displacement = 7.296×10^{-12} units
 calculated value for E = 2.741×10^{11} Pascals
 = 39.757×10^6 Psia

Test name: PORE6-5Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

10.61 % porosity

Pore size: cubes, 5 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8	9	10
Number of cells:	0	0	1	3	7	11	13	14	13	11	7

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

7.492	7.585	7.611	7.536	7.497
7.539	7.560	7.573	7.547	7.484
7.524	7.537	7.538	7.503	7.472
7.414	7.451	7.469	7.458	7.447
7.332	7.337	7.344	7.405	7.450

Results:average displacement = 7.484×10^{-12} unitscalculated value for E = 2.672×10^{11} Pascals= 38.757×10^6 Psia**Test name: PORE6-6**Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

12.46 % porosity

Pore size: cubes, 5 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8	9	10
Number of cells:	0	0	0	1	2	5	8	12	16	18	18

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X → Y

7.446	7.498	7.499	7.446	7.403
7.538	7.536	7.517	7.465	7.403
7.575	7.561	7.519	7.469	7.432
7.535	7.538	7.512	7.468	7.444
7.510	7.493	7.484	7.456	7.416

Results:average displacement = 7.487×10^{-12} unitscalculated value for E = 2.671×10^{11} Pascals= 38.744×10^6 Psia

Test name: PORE7-1Unit length = 1 micrometer (10^{-6} m)80 cells, $4 \times 4 \times 5$, each with dimension $12 \times 12 \times 12$ units $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

5.43 % porosity

Pore size: cubes, 5 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	41	23	12	3	1				

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.254	1.205	1.184	1.190	1.183
1.202	1.171	1.153	1.159	1.164
1.124	1.123	1.119	1.139	1.169
1.079	1.092	1.111	1.133	1.153
1.075	1.087	1.114	1.118	1.107

Results:average displacement = 11.44×10^{-12} unitscalculated value for E = 2.914×10^{11} Pascals= 42.267×10^6 Psia**Test name: PORE7-2**Unit length = 1 micrometer (10^{-6} m)80 cells, $4 \times 4 \times 5$, each with dimension $12 \times 12 \times 12$ units $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

8.41 % porosity

Pore size: cubes, 5 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	28	23	29	8	2				

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.334	1.325	1.292	1.252	1.229
1.269	1.256	1.244	1.246	1.246
1.173	1.221	1.238	1.231	1.269
1.115	1.199	1.230	1.228	1.251
1.120	1.141	1.211	1.243	1.222

Results:average displacement = 12.31×10^{-12} unitscalculated value for E = 2.708×10^{11} Pascals= 39.277×10^6 Psia

Test name: PORE7-3

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 12x12x12 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 11.75 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	18	20	23	13	5	1			

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.509	1.590	1.587	1.421	1.350
1.413	1.435	1.402	1.362	1.323
1.288	1.328	1.339	1.306	1.307
1.287	1.365	1.339	1.331	1.397
1.554	1.349	1.337	1.441	1.468

Results:

average displacement = 13.93×10^{-12} units
 calculated value for E = 2.394×10^{11} Pascals
 = 34.72×10^6 Psia

Test name: PORE7-4

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 12x12x12 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 14.65 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	12	17	24	16	7	3	1		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X	→ Y			
1.578	1.640	1.628	1.446	1.359
1.538	1.533	1.481	1.435	1.391
1.493	1.484	1.448	1.400	1.391
1.548	1.561	1.472	1.425	1.465
1.828	1.559	1.487	1.540	1.530

Results:

average displacement = 15.06×10^{-12} units
 calculated value for E = 2.214×10^{11} Pascals
 = 32.11×10^6 Psia

Test name: PORE7-5

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 12x12x12 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 18.17 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	7	12	22	20	12	5	2		

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.967	1.925	1.730	1.615	1.771
1.856	1.777	1.649	1.598	1.555
1.872	1.762	1.687	1.585	1.464
1.947	1.830	1.702	1.608	1.551
1.999	1.734	1.646	1.674	1.644

Results:

average displacement = 17.259×10^{-12} units
 calculated value for E = 1.932×10^{11} Pascals
 = 28.02×10^6 Psia

Test name: PORE7-6

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 12x12x12 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 20.44 % porosity
 Pore size: cubes, 5 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	5	10	20	20	14	7	3	1	

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-11}$ units).

↓ X → Y

1.976	1.933	1.733	1.619	1.771
1.904	1.826	1.695	1.683	1.788
1.955	1.848	1.775	1.728	1.621
2.057	1.949	1.831	1.742	1.691
2.145	1.888	1.812	1.852	1.831

Results:

average displacement = 18.263×10^{-12} units
 calculated value for E = 1.826×10^{11} Pascals
 = 26.48×10^6 Psia

Test name: PORE8-1Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

5.2734 % porosity

Pore size: cubes, 15 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	71	8	1						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
6.528	6.488	6.428	6.338	6.197
6.649	6.677	6.672	6.512	6.304
7.232	6.964	6.732	6.576	6.376
7.372	6.985	6.868	6.754	6.393
6.943	6.823	7.184	7.074	6.419

Results:average displacement = 6.700×10^{-12} unitscalculated value for E = 2.985×10^{11} Pascals= 43.295×10^6 Psia**Test name: PORE8-2**Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

7.910 % porosity

Pore size: cubes, 15 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	67	11	2						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
6.412	6.427	6.409	6.348	6.253
6.654	6.749	6.810	6.717	6.580
7.345	7.153	7.015	6.995	6.928
7.571	7.275	7.264	7.291	7.079
7.230	7.210	7.667	7.619	7.073

Results:average displacement = 6.963×10^{-12} unitscalculated value for E = 2.872×10^{11} Pascals= 41.657×10^6 Psia

Test name: PORE3-3

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 11.074 % porosity
 Pore size: cubes, 15 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	62	15	3						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
7.587	7.376	7.158	7.093	7.069
7.355	7.340	7.318	7.194	7.039
7.589	7.392	7.198	7.518	8.304
7.543	7.243	7.171	7.562	8.052
6.968	6.921	7.375	7.162	6.470

Results:

average displacement = 7.320×10^{-12} units
 calculated value for E = 2.732×10^{11} Pascals
 = 39.626×10^6 Psia

Test name: PORE3-4

Unit length = 1 micrometer (10^{-6} m)
 80 cells, 4x4x5, each with dimension 20x20x20 units
 $E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals
 14.238 % porosity
 Pore size: cubes, 15 units on each side
 Simulated material: Porous Aluminium Nitride (AlN)
 Pores follow a poison distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	59	17	5						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X	→ Y			
8.076	7.886	8.808	8.426	6.917
7.728	7.751	8.017	7.745	7.112
7.886	7.687	7.427	7.756	8.607
7.783	7.452	7.388	7.857	8.438
7.146	7.093	7.552	7.366	6.702

Results:

average displacement = 7.704×10^{-12} units
 calculated value for E = 2.596×10^{11} Pascals
 = 37.649×10^6 Psia

Test name: PORE3-5

Unit length = 1 micrometer (10^{-6} m)

80 cells, 4x4x5, each with dimension 20x20x20 units

$E_0 = 46.7 \times 10^6$ psia, 3.22×10^{11} Pascals

18.984 % porosity

Pore size: cubes, 15 units on each side

Simulated material: Porous Aluminium Nitride (AlN)

Pores follow a poisson distribution:

Pores per cell:	0	1	2	3	4	5	6	7	8
Number of cells:	53	20	8						

Data:

listed below are the displacements of the 25 node points on the top of the model (each $\times 10^{-12}$ units).

↓ X -> Y

7.920	7.869	9.062	10.081	9.847
7.606	8.100	8.711	8.802	8.936
7.817	8.293	8.212	8.453	9.513
7.790	7.837	8.008	8.621	9.403
7.104	7.362	8.067	8.101	7.648

Results:

average displacement = 8.3667×10^{-12} units

calculated value for E = 2.3904×10^{11} Pascals
= 34.669×10^6 Psia

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A FINITE ELEMENT ANALYSIS OF POROSITY EFFECTS ON MATERIALS

This paper examines the effects of porosity using a three dimensional finite element model. Three previously used analytical equations (linear, empirical exponential, and Hassin's semi-empirical equation) are presented. These analytical models all require that the effects of porosity on material properties be determined experimentally for several cases, then extrapolated for other cases. This limits accurate prediction before a material is produced. In this paper a finite element model is developed that can predict the material modulus of elasticity, Young's modulus, using the limited information from one material sample. The model is shown to correspond with several known test results. The same finite element model is used to demonstrate the effects of pore size and distribution on material behavior.