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This report summarizes the results of a study on the control and stabilization of linear and nonlinear distributed systems covering the period from April, 1986 through December, 1988. The specific areas of this study consist of (i) stability enhancement of distributed systems describable by abstract linear evolution equations, (ii) a translation invariant approach to stability and stabilizability, (iii) stabilizability of bilinear systems, (iv) application of computer vision in static shape estimation, control, and failure detection in elastic systems, and (v) stabilization and control of distributed systems with application to aeroelastic systems with extendible lifting surfaces.

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CONTENTS

ABSTRACT

This report summarizes the results of a study on the control and stabilization of linear and nonlinear distributed systems covering the period from April,1986 through December,1988. The specific areas of this study consist of (i) stability enhancement of distributed systems describable by abstract linear evolution equations, (ii) a translation invariant approach to stability and stabilizability, (iii) stabilizability of bilinear systems, (iv) application of computer vision in static shape estimation, control, and failure detection in elastic systems, and (iv) stabilization and control of distributed systems with time-dependent spatial domains with application to aeroelastic systems with extendible lifting surfaces.

1. INTRODUCTION

The use of feedback controls for stabilizing the motions of distributed systems has been investigated in recent years. Despite the extensive results in this area, it is still not completely clear how to find stabilizing feedback controls for distributed systems, particularly when they are nonlinear. In this study, we focus our attention on the stabilization and control of certain classes of distributed systems describable by abstract linear evolution equations, bilinear systems, and systems involving time-dependent spatial domains. The latter type of systems arise in aeroelastic systems with extendible lifting surfaces, and also in large space structures with deployable structural members. Another aspect of this study pertains to the use of computer vision for shape estimation, control and failure detection of deformable bodies such as large elastic structures. The main objective of this study is to develop applicable mathematical theories and at the same time study specific systems arising from realistic aerospace applications.

2. RESEARCH SUMMARY

This report covers the period from April,1986 through December,1988. The results will be described categorically below:

2.1 Stability Enhancement by State Feedback:

Let (A,B) denote the distributed system described by the abstract equation:

$$\dot{x} = Ax + Bu, \tag{1}$$

where A is the generator of a strongly continuous semigroup T(t), $t \geq 0$, i.e., of the class C_o, of bounded linear operators over a Hilbert space H, and B is a bounded linear operator from a second Hilbert space U to H. Let F: $H \rightarrow U$, $u = Fx$, be a bounded linear feedback so that (1) becomes:

$$\dot{x} = (A + BF)x. \tag{2}$$

Then, since BF is bounded linear, A + BF also generates a C_o semigroup S(t), $t \geq 0$, (say), over H.

Now, if the open-loop system (A,B) is not stable and if a state feedback F can be found so that the closed-loop system (2) is stable, then we say that (A,B) is stabilizable by F. This is the well-known stabilizability problem in Control Theory. Our investigation here is somewhat more general. It was motivated by the fact that many physical systems are inherently stable. But their response may be too "slow". Thus we wish to use state feedbacks, not only for stabilizability, but also for "speeding up" the response of a system.

We began with the class of conservative systems, e.g., those described by the abstract wave equation. For this class, the semigroup $T(t)$, $t \geq 0$, is norm preserving: $\|T(t)x\| = \|x\|$ for all x in H. Thus a conservative system is not s(strongly)-stable on all the state space H: $T(t)x \to 0$, $t \to \infty$, for each x in H. It is also not e(exponentially)-stable, i.e., there are no constants $M \geq 1$ and $\alpha > 0$ so that: $\|T(t)\| \leq M \exp(-\alpha t)$, $t \geq 0$. Thus, the natural question to ask is when does a norm preserving semigroup $T(t)$, $t \geq 0$, w(weakly)-stable: $[T(t)x,y] \to 0$, $t \to \infty$, for x and y in H?

Necessary and sufficient conditions for a C_o semigroup to be ε-stable were given by Datko. They are indeed extension of the well-known Lyapunov Theorem for stability of matrices. However, as far as we know, there are no Lyapunov type theorems for weak and for strong stabilities of Hilbert space semigroups.

Since a norm preserving semigroup belongs to the class of contraction semigroups: $\|T(t)\| \leq 1$, $t \geq 0$, we began in [R1] by obtaining a Lyapunov type condition for w-stability of contraction semigroups. Namely, for a contraction semigroup $T(t)$, $t \geq 0$, with generator A in H, there always exists a non-negative operator P such that: $2\mathrm{Re}\,[PAx,x] \leq 0$ for x in the domain $\mathcal{D}(A)$ of A. In particular, if P is positive then the semigroup is weakly stable. Moreover, if the resolvent of A is compact -- which is the case in most physical systems -- then our conditions also result in strong stability of the semigroup.

Returning to the case of a conservative system. The key idea is to use a feedback so that the closed-loop semigroup $S(t)$, $t \geq 0$, is a contraction semigroup. One such feedback is $-B^*$, or more generally, $-Q^2B^*$ where Q is a self-adjoint operator on U. The latter has the advantage that if B is not compact, we can still have a compact feedback by requiring Q to be compact. Using our results stated above, we were able to show that if A has compact resolvent, and if 0 is not in the spectrum of BB^*, then the feedback $-B^*$ will strongly enhance the system stability. Stabilizability of conservative systems was also studied, using the same Lyapunov type results.

A rather interesting outcome of our work is the fact that if the resolvent of the generator A of a conservative system is compact, then it is always weakly unstable on all of its state space. This explains why, for such a system, controllability is required on all the state space before stabilizability or stability enhancement can be achieved. Behavior of the modes of a conservative system under the influence of a dissipative feedback was also studied. Conditions for the modes of the closed-loop system to be orthogonal were also obtained.

In order to develop Lyapunov type results for weak and strong stabilities, we studied in [R2] the operator equation:

$$[PAx,x] + [x,PAx] = -[Rx,x] \quad \text{for } x \in \mathcal{D}(A). \tag{3}$$

Here the operator R is required only to be nonnegative. This equation is, of course, a generalization of the Lyapunov equation -- in which R is either the identity operator or is strictly positive, i.e., $R \geq aI$ for some $a > 0$. Necessary and sufficient conditions for the existence of a unique non-negative solution P were obtained. The important case in which $R = BB^*$ was studied. Sufficient conditions for weak stability of a uniformly bounded C_o semigroup were then obtained. These conditions, as far as we can tell, have not been given before. It is of interest to note that when $R = BB^*$ our conditions becomes: (i) equation (3) admits a unique solution $P \geq 0$ if and only if the integral $\int_0^\infty \|B^*T(t)x\|^2\,dt$ converges for every x, and (ii) if the pair (A^*,B) is controllable then P is positive and the uniformly bounded semigroup is weakly stable. Condition (i) is clearly a Lyapunov type condition, while (ii) is needed for the positivity of P. Thus, unlike exponential stability, controllability is required in this case. These two conditions were used in [R2]. for stabilizability of uniformly bounded semigroups. Note that if the uniformly boundedness condition is not required, then the above conditions imply that the semigroup is a quasi-affine transform of a contraction semigroup. Hence we can show that it is only weakly stable on a dense subspace. This is precisely the case when one uses the feedback $-B^*P$ where $P \geq 0$ satisfies the steady-state Riccati equation: $[PAx,x] + [x,PAx] - [PBB^*Px,x] + [Rx,x] = 0$ for x in the domain of A, and for some $R \geq 0$. The above suggests a general relationship between Lyapunov type stability on the one hand and quasi-affine transforms of contraction semigroups on the other hand.

Results on Lyapunov type strong stability were obtained in [R3]. The approach here is again via equation (3). It turns out that if the solution P of the equation is strictly positive, then the semigroup is strongly stable. Several conditions for strict positivity of P were obtained. We must note that exponential stabilizability is the most desirable property, but this unfortunately is not always possible! This fact emphasizes the need for feedback controls which will either strongly or weakly stabilize or enhance the stability of a system. These were studied in [R4]. Much of prior work in stabilizing controls for distributed-parameter systems has emphasized exponential stabilizability. Compared with exponential stability, weak and strong stabilities are less desirable properties. However, there are situations under which infinite dimensional systems cannot be exponentially stabilized. In such cases, we propose weak and strong stabilities. One approach to the stabilization of finite dimensional systems and exponential stabilization of infinite dimensional systems has been the use of Lyapunov type functionals. This is one technique which we developed and extended here, to provide new conditions for strong and weak stabilities. We presented a new functional in order to obtain conditions for a certain semigroup to be strongly stable. This functional also suggested an inequality relation which, if satisfied will guarantee weak stability of uniformly bounded semigroups. Since stable semigroups are uniformly bounded and since this condition is important in verifying stability, we examined this phenomenon. Some new conditions were found for a uniformly bounded semigroup to remain uniformly bounded under suitable bounded perturbations.

In [R5], strong stability of distributed systems was studied by characterizing strongly stable semigroups of operators associated with distributed systems. Main emphasis is on contractive systems. Three different approaches to characterization of strongly stable contractive semigroups are developed. The first one is an operator theoretical approach. Using the theory of dilations, it was shown that every strongly stable contractive semigroup is related to the left shift semigroup on an L^2 space. Then, a decomposition for the state space which identifies strongly stable and unstable states was introduced. Based on this decomposition, conditions for a contractive semigroup to be strongly stable were obtained. Finally, extensions of Lyapunov equation for distributed systems are investigated. Sufficient conditions for weak and strong stabilities of uniformly bounded semigroups were obtained by relaxing the equivalent norm conditions on the right-hand-side of the Lyapunov equation.

These characterizations are then applied to the problem of feedback stabilization. First, it was shown via the state space decomposition that under certain conditions a contractive system (A,B) can be strongly stabilized by the feedback $-B^*$. Then, application of the extensions of the Lyapunov equation results in sufficient conditions for weak, strong and exponential stabilization of contractive systems by the same feedback $-B^*$. Finally, it was shown that for a contractive system $\dot{x} = Ax + Bu$, where B is any linear bounded operator, there is a related linear quadratic regulator problem and a corresponding steady-state Riccati equation which always has a bounded nonnegative solution.

2.2 A Translation Invariant Approach to Stability and Stabilizability:

This study was motivated by the following observations:

Let A be the generator of a contractive semigroup T(y), $t \geq 0$, over a Hilbert space H. Then for x in the domain $D(A)$ of A:

$$[Ax,x] + [x,Ax] \leq 0, \tag{4}$$

and

$$\|x\|^2 - \lim_{t\to\infty} \|T(t)x\|^2 = \int_0^\infty -2\mathrm{Re}\ [AT(t)x,T(t)x]\ dt, \tag{5}$$

where the limit always exists, since the function $\|T(t)x\|$ is nonincreasing. Define on $D(A)$ a new norm $\|\cdot\|_n$ by

$$\|x\|^2 = \int_0^\infty \|T(t)x\|_n^2\ dt \quad \text{for } x \in D(A). \tag{6}$$

Let K be the completion of $D(A)$ in the new norm -- modulo the zero vectors. Then (6) simply means that, for x in $D(A)$: T(t)x belongs to the space $L^2([0,\infty);K)$. Define an operator V: $D(A) \to L^2([0,\infty);K)$ by

$$(Vx)(t) = T(t)x \quad \text{for } t \geq 0.$$

Then it is plain from (6) that V is an isometry on $D(A)$. Therefore, since $D(A)$ is dense in H, we can extend V by continuity to all of H. More is true. We have, for x in H:

$$VT(t)x = T(\tau)T(t)x = T(t+\tau)x \quad \text{for } t,\tau \geq 0.$$

Therefore

$$VT(\tau)x = L(\tau)T(t)x = L(\tau)x \quad \text{for } x \text{ in } H, \tag{7}$$

where $L(\tau)$, $\tau \geq 0$, is the Left Translation semigroup over the space $L^2([0,\infty);K)$. We therefore conclude that the closed subspace VH is invariant for the semigroup $L(\tau)$, $\tau \geq 0$, and the strongly stable contraction semigroup $T(\tau)$, $t \geq 0$, to its invariant subspace VH. It is clear that if a semigroup is unitarily equivalent to a part, i.e., the restriction to an invariant subspace, of the Left Translation semigroup, then it is strongly stable.

We have in the above obtained a Translation Invariant Representation for a strongly stable contraction semigroup.

In [R6], we were able to obtain a general Translation Invariant Representation for a contraction semigroup by using the fact that the contractions $T(t)^*T(t)$, $t\geq 0$, converge in the strong operator topology to a positive contraction C^2 (say). Therefore, from (5), for x in $D(A)$:

$$\|x\|^2 - \|Cx\|^2 = \int_0^\infty \|T(t)x\|_n^2 \, dt. \tag{8}$$

Therefore, for x in $D(A)$:

$$\|x\|^2 = \int_0^\infty \|T(t)x\|_n^2 \, dt + \|Cx\|^2.$$

From this we can show that $T(t)$, $t \geq 0$, is unitarily equivalent to the restriction of the semigroup $L(t) \oplus W(t)$, $t \geq 0$, to its invariant subspace VH, where V is now the isometry from H to the space $L^2([0,\infty);K) \oplus \overline{CH}$ defined by $(Vx)(t) = T(t)x \oplus W(t)Cx$, and $W(t)$, $t \geq 0$, is an isometric semigroup over $\overline{CH}$ defined by $W(t)Cx = CT(t)x$. What interesting is that in the process of deriving the Translation Invariant Representation we obtain yet, another Lyapunov type result for w-stability of contraction semigroups. This results directly from (8) by defining $P = I - C^2$. Then (8) becomes

$$[Px,x] = \int_0^\infty \|T(t)x\|_n^2 \, dt \qquad \text{for } x \in D(A). \tag{9}$$

From which it is easy to see that the semigroup is w-stable as soon as P is positive.

The Translation Invariant Representation is not only limited to the class of contraction semigroups. We were able to show that a class of exponentially stable C_o semigroups also admits such a representation. To see this we only have to note that if $T(t)$, $t \geq 0$, is exponentially stable, then there exists a positive operator P such that, for every x:

$$[Px,x] = \int_0^\infty \|T(t)x\|^2 \, dt. \tag{10}$$

This, as in the above, shows that $T(t)x$ belongs to the space $L^2([0,\infty);H)$. Now define a new norm $\|\cdot\|_P$ by: $\|x\|_P^2 = [Px,x]$, and suppose that it is equivalent to the original norm $\|\cdot\|$. Then it is plain from (10) and from the above discussion that the semigroup is unitarily equivalent to the left shift semigroup over the space $L^2([0,\infty);H)$. Conditions for the two norms $\|\cdot\|_P$ and $\|\cdot\|$ to be equivalent have been given by Pazy. It turns out that they are equivalent for the "hyperbolic" case, but not for the "parabolic" one. We must note that if the generator A is bounded and the semigroup $\exp(At)$, $t \geq 0$, is exponentially stable, then the two norms are equivalent. This immediately shows that stability, stabilizability and stability enhancement in finite dimensional case can all be studied from the Translation Invariant Representation viewpoint.

2.3 Stabilizability of Bilinear Systems:

Consider the bilinear system

$$\dot{x}(t) = Ax(t) + B(x(t),u(t)), \quad t \geq 0,$$

where A is the generator of a linear C_o semigroup over a Hilbert space H, and $B(\cdot,\cdot)$ is a bilinear form from the product space $H \times U$ to H. Our problem here is to stabilize such a system by state feedback.

In [R7], we studied the class of "dissipative" bilinear systems, i.e., those for which the generator A is dissipative. The idea here is to use a state feedback so that the closed-loop system is also dissipative. We studied the case in which $B(\cdot,\cdot)$ is of the form $B(x(t),u(t)) = u(t)Qx(t)$, where Q is a bounded linear operator on H. A suitable feedback was obtained, and the system is weakly stabilizable as soon as it is "weakly controllable", i.e., $[B^*T(t)x,T(t)x] = 0$, $t \geq 0$, $\Rightarrow x = 0$.

Next, we consider the class of Semilinear Dissipative Systems:

$$\dot{x}(t) = Ax(t) + B(x(t))v(t).$$

Sufficient conditions for this class of systems to be strongly stabilizable by the feedback $-B^*$ were obtained. Namely,

$$\{x: B^*(T(t)x)T(t)x = 0, \ t \geq 0\} = \{0\}.$$

This is the exact analog of the weak controllability" condition above. However, in this case, stronger results was obtained. This requires the knowledge of the "isometric" subspace of the contraction semigroup generated by A, which is also the case for the class of linear dissipative systems.

2.4 Application of Computer Vision in Static Shape Estimation, Control and Failure Detection in Elastic Systems:

This study was motivated by the observation that a computer-vision system is capable of capturing the instantaneous shape of a deformable solid body over its entire spatial domain. Moreover, the sole dependence of vision data on the local surface properties of the body eliminates the need of a precise mathematical model of the system under consideration. These features make the vision system particularly attractive for the feedback control of elastic systems involving deformable solid surfaces. Moreover, computer-vision is capable of detecting large deformations and abnormalities in an elastic structure. This capability may be used to detect failures and abnormal conditions in elastic structures. The main objective of this study is to develop applicable mathematical theories for shape estimation, control and failure detection of elastic systems arising in aerospace applications.

In order to gain an understanding of the basic problems and issues in the application of computer-vision systems for shape estimation, we began in 1986 with an experimental study on static shape estimation using a small scale model of a spaceborne antenna consisting of two flexible ribs interconnected by elastic strings. A crude CCD camera with binary grey levels was used in the experiment. This study led to the formulation of physically meaningful mathematical problems in shape estimation. The preliminary results of this study were presented at the 1986 IFAC Symposium on the Control of Distributed-Parameter Systems [R8].

During the period 1986-1987, with the collaboration of the Jet Propulsion Laboratory (JPL) in Pasadena, we studied the application of computer vision in static shape estimation for elastic systems with the aid of the a reduced scale model of a large spaceborne antenna at the JPL Ground Antenna Test Facility. This model consists of twelve flexible ribs attached to a rigid hub which is connected to another rigid ring through a gimbal system. The entire antenna model has two degrees of freedom associated with the rigid body. The ribs, hub and the rings were coated with light reflecting paint so they could be viewed by means of a computer vision system. Our initial objective was to estimate the displacement of each rib relative to a suitable hub coordinate system.

The shape estimation problem can be decomposed into three subproblems: (P1) estimation of hub attitude relative to a specified inertial frame; (P2) estimation of rib displacement relative to the hub coordinate system, and (P3) reconstruction of the reflector surface from the estimated rib shape.

Each of the foregoing problems can be considered as a special case of the following general mathematical problem:

Let Γ be a nonempty compact subset of the 3-dimensional real coordinate space R^3 representing the deformable, stationary solid object under observation (eg. antenna hub, rib or reflector). The boundary of Γ is denoted by $\partial\Gamma$. To view $\partial\Gamma$ by means of a computer vision system, we cover $\partial\Gamma$ with a light-reflecting coating or introduce appropriate light-reflecting markings over a subset $\partial\Gamma_M$ of $\partial\Gamma$. We assume that $\partial\Gamma$ or its markings are illuminated by an appropriate light source. The reflected light is collected by the lens system of the camera and projected into an image plane. This image formation process can be described in abstract mathematical terms as follows:

Let $\mathcal{F}$ be a family of planes Π_i in R^3. The camera's image plane corresponding to a particular camera orientation is represented by a compact subset $\mathcal{P}_i$ of Π_i for some i. The domain of the image of $\partial\Gamma$ (resp. $\partial\Gamma_M$),denoted by $\mathcal{I}_i(\partial\Gamma)$ (resp. $\mathcal{I}_i(\partial\Gamma_M)$) is specified by $P_i(\partial\Gamma) \cap \mathcal{P}_i$ (resp. $P_i(\partial\Gamma_M) \cap \mathcal{P}_i$), where P_i represents the image projection operator from $R^3 \rightarrow \Pi_i$. Note that for perspective image projection, P_i is nonlinear. However, by using the "homogeneous coordinates", P_i can be represented by a linear transformation P_i defined on the augmented 4-dimensional space R^4. The light intensity of the image $\mathcal{I}_i(\partial\Gamma)$ (resp. $\mathcal{I}_i(\partial\Gamma_M)$) can be described by a real-valued nonnegative function f_i defined on $\mathcal{I}_i(\partial\Gamma)$ (resp. $\mathcal{I}_i(\partial\Gamma_M)$). The form of f_i depends on the light reflecting properties of $\partial\Gamma$ (or its markings $\partial\Gamma_M$), and the viewing geometry.

In setting up the computer vision system for determining the shape of the object Γ, it is of importance to choose the camera position and orientation such that the object's surface $\partial\Gamma$ can be determined from its image $\mathcal{I}_i(\partial\Gamma)$. This problem can be stated as follows:

(P1) **Camera Positioning Problem**: Given $\mathcal{F}$ and Γ, find a plane $\Pi_i \in \mathcal{F}$ such that $\partial\Gamma$ can be determined completely from its image $\mathcal{I}_i(\partial\Gamma)$. In the case of a dual-camera system, we wish to find a pair (Π_i,Π_j), $i \neq j$, Π_i, $\Pi_j \in \mathcal{F}$, such that $\partial\Gamma$ can be determined completely from the corresponding image pair $(\mathcal{I}_i(\partial\Gamma),\mathcal{I}_j(\partial\Gamma))$.

The foregoing problem is a purely geometric one. Suppose that this problem has been solved. We can now state a shape estimation problem under the assumption that a perfect image of the object is available. In what follows, we shall drop the subscript i with the understanding that suitable camera position and orientation have been selected.

(P2) Idealized Shape Estimation Problem (Single Camera Vision System): Given an image intensity function f defined on $\mathcal{I}(\partial\Gamma)$ corresponding to a specified camera orientation,find a set $\partial\hat{\Gamma} \subset R^3$ which best approximates $\partial\Gamma$ in some sense.

In an actual computer vision system, the actual image of $\partial\Gamma$ is formed through both spatial discretization and intensity quantization processes. Let the image plane $\mathcal{P}$ be partitioned into compact subsets Δ_j (pixels) such that $\mathcal{P} = \cup_j \Delta_j$. Let the characteristic function of Δ_j be denoted by $\tilde{S}_j$. Then the spatially discretized image intensity function is given by $f = \Sigma_j fS_j$ defined on $\mathcal{P}$. Let $L_N = \{\ell_1,\ldots,\ell_N\} \subset R$ denote the finite set of quantized light intensity levels. We introduce an intensity quantization rule specified by the function Q which maps each $f_j = fS_j$ into L_N. Thus, the quantized image intensity corresponding to pixel Δ_j is given by $\hat{f}_j = QfS_j$. Now, a discretized version of the shape estimation problem can be stated as follows:

(P3) Discrete Shape Estimation Problem : Given a spatially discretized intensity-quantized image of $\partial\Gamma$ represented by the intensity sequence $\{f_j\}$, find a set $\partial\hat{\Gamma} \subset R^3$ which best approximates $\partial\Gamma$ in some sense.

In physical situations, the image of the object of interest may be corrupted by noise induced by spatial discretization and intensity quantization, and by undesirable reflected light. Moreover, the total image may also contain images of background objects. For example, in estimating the rib shape of the JPL antenna model by means of computer vision, the hub and inner ring may also appear in the image. It is necessary to extract the rib-image data from the total image so that only relevant data are used in rib-shape estimation. To reduce the effect of noise, some form of image preprocessing may be introduced. The extraction of the image of the object of interest from the total image involves image segmentation and object recognition. In the JPL antenna experiment, this problem is solved by introducing appropriate localized illumination of the object.

During the period covered by this report, explicit forms for various mappings (i.e. P_i, f_i etc.) associated with problems (P1)-(P3) were obtained. Algorithms for shape estimation based on least-square criteria were derived and implemented. These algorithms were tested using real data obtained from an experiment performed at the JPL Ground Antenna Test Facility during the summer of 1987. The results were described in a UCLA Engineering report [R10].

After completion of the aforementioned experimental work, we extended our study to the detection of failures and abnormal conditions in large spaceborne antenna systems. The basic idea is as follows:

Let $\partial\Gamma^N$ and $\partial\Gamma^N_M$ (resp. $\partial\Gamma^A$ and $\partial\Gamma^A_M$) denote respectively the boundary surface of the light-reflecting markings of the spatial domain of the antenna system in its normal state (resp. actual state). The images of $\partial\Gamma^i$ and $\partial\Gamma^i_M$, i= N or A, under the projection operator P in the camera's image plane $\mathcal{P}$ are denoted by $\mathcal{I}(\partial\Gamma^i) = P(\partial\Gamma^i) \cap \mathcal{P}$ and $\mathcal{I}(\partial\Gamma^i_M) = P(\partial\Gamma^i_M) \cap \mathcal{P}$ respectively. The projection operator P depends on the camera parameters and orientation which are assumed to be fixed at all times. To fix ideas, we assume that the camera has binary intensity levels. Thus, the images $\mathcal{I}(\partial\Gamma^N_M)$ and $\mathcal{I}(\partial\Gamma^A_M)$ are compact subsets of $\mathcal{P} \subset R^2$.

Now, the basic approach for detection of failure and abnormal condition is as follows: First, a set of reference images $\mathcal{I}_j(\partial\Gamma^N_M), j=1,\ldots,K$, corresponding to the normal state of the antenna structure are obtained by means of a computer vision system. Then, the state of the antenna structure is monitored periodically by the same computer vision system. The resulting images are compared with their corresponding reference images and their deviations are measured in terms of a suitable index. Here, the deviation between two images $\mathcal{I}_j(\partial\Gamma^N_M)$ and $\mathcal{I}_j(\partial\Gamma^A_M)$ is taken to be the Hausdorff metric defined by

$$\rho(\mathcal{I}_j(\partial\Gamma^N_M),\mathcal{I}_j(\partial\Gamma^A_M)) \stackrel{\Delta}{=} \max\{\max\{\rho(r,\mathcal{I}_j(\partial\Gamma^N_M)),\max\{\rho(r',\mathcal{I}_j(\partial\Gamma^A_M))\}\},$$

where $\rho(r,A) = \inf\{\|r - r'\|,\ r' \in A\}$. The antenna is said to be in the "failed" or "abnormal" state when the deviation between the reference and actual images exceeds a predetermined threshold level. We may also introduce various weights in the Hausdorff metric to emphasize the relative importance of deformations in different parts of the antenna structure. Alternatively, the same approach may be used to monitor the state of certain critical portions of the antenna by localizing the camera image field to these portions of the antenna. Thus, the nature of the failure can be diagnosed. An obvious advantage of the foregoing approach is that the Hausdorff metric is capable of taking into account failures due to breakage of the structural members and punctures of the reflector surface due to meteorite impact. However, for a given antenna structure, there may exist structural deformations and movements which leave the image invariant. This difficulty may be remedied partially by using a multi-camera system. In physical situations, it is of importance to determine the sensitivity and robustness of the proposed failure detection scheme with respect to variations in threshold and illumination levels, and camera parameters. Mathematically, this task requires the computation of the Gateaux differentials of the Hausdorff metric with respect to certain parameters associated with the projection operator P.

The results of this study were presented at the Fifth IFAC Symposium on Control of Distributed Parameter Systems held at Perpignan, France in June, 1989. [R12]. These results also constitute a part of the Ph.D. dissertation of *Nai-jian Wang* [R13].

2.5. Stabilization and Control of Distributed Systems with Time-Dependent Spatial Domains:

Distributed system with time-dependent spatial domains arise naturally in many physical situations. Often, it is desirable to control the dynamic behavior of such systems by varying their spatial domains. For example, in large space structures with deployable elastic components, it is of interest to control the deployment of these components so that the induced vibrations will be below a specified level.

In this study, the problem of stabilization and control of distributed systems with time-dependent spatial domains was considered. Here, the evolution of the spatial domain with time is described by a finite dimensional system of ordinary differential equations, while the distributed systems are described by first and second-order linear evolution equations defined on appropriate Hilbert spaces. The controls affect the system domain motion through the ordinary differential equation.

First, results pertaining to the existence and uniqueness of solutions of the system equations were obtained. For the case with first-order evolution equations, the existence result was proved in a constructive way using finite dimensional Galerkin approximations with an appropriate time-dependent basis. This approach is useful in numerical computations. For the case of second-order evolution equations, we did not succeed to prove the existence of solutions using the same approach.

Next, we considered various optimal control problems associated with this class of systems. Sufficient conditions in the form of variational inequalities for which the optimal controls must satisfy were established. We also considered the feedback stabilization of such systems. Stabilizing control laws based on energy inequalities were derived. The application of the results were illustrated by simple examples.

The results of this study were described in a technical report [R-11], which has been accepted for publication in the *Journal of Optimization Theory and Applications*.

2.6 Stabilization and Control of Aeroelastic Systems with Extendible Lifting Surfaces:

This study was motivated from the problems associated with trans-atmospheric vehicles which are capable of multi-regime flights (i.e. subsonic,supersonic and hypersonic regimes). In these vehicles, movable lifting and control surfaces may be introduced to provide the desired lifting and maneuverability characteristics. The extension or contraction of these surfaces during flight could induce undesirable aeroelastic vibrations which must be stabilized so that their effectiveness is not impaired. In this study, we considered an aeroelastic cantilever wing which may extend or contract during flight.

The wing was modelled by a coupled system of partial differential equations defined on a time-dependent spatial domain describing the bending and torsional motions of the variable length wing. Attention was focused on the following questions:

(a) How does the extending or contracting motion of the wing affect the aeroelastic vibrations?

(b) Is it possible to stabilize the aeroelastic vibrations by introducing appropriate feedback controls?

(c) How do we introduce suitable finite dimensional approximations for the numerical solution of the system equations?

It was shown that when the wing extends with constant velocity, the vibrational energy increases monotonically with time. This implies that the extending motion has a destabilizing effect on the aeroelastic vibrations. On the other hand, when the wing contracts with uniform velocity, the vibrational energy decreases with time implying a stabilizing effect. Although a wing cannot extend indefinitely during flight, it is of importance that the amplitude of the aeroelastic vibrations induced by wing extension does not build up to a high level during the extension period. Therefore it is of interest to stabilize the motion-induced instability by introducing appropriate feedback controls. It was shown by introducing a simple modification of the proportional-plus-rate feedback control law depending on the wing-tip vibration velocity for controlling the extending motion of the wing, the motion-induced instability can be stabilized. The resulting feedback system is in the form of a partial differential equation with a free boundary. It was also shown that a useful finite dimensional Galerkin approximation can be obtained by introducing a suitable time-dependent basis corresponding to the eigenfunctions of the biharmonic and elliptic operators associated with the wing at any fixed time.

The results of this study were presented at the First International Conference in Industrial Applied Mathematics [P1]. The details were first described in a technical report [R9] which was published later in the journal *Dynamics and Stability of Systems* in 1988.

3. PAPERS, REPORTS AND THESES:

[R1]* N.Levan, "On Stabilizability of Conservative Systems,"*Proc.4-th IFAC Symposium on Control of Distributed-Parameter Systems,* Los Angeles, California, June 30-July 2,1986.

[R2]* N.Levan, "Stability, Stabilizability and the Equation [PAx,x] + [x,PAx] = $-\|Bx\|^2$," *Advances in Communication and Control Theory,* R.E.Kalman *et al*, Editors, Optimization Software Inc. Publications Div. New York, N.Y. 1987, pp.214-226.

[R3]* W.M.Miyaji, "A Lyapunov Strong Stability Theorem in Hilbert Space," *Proc.Intern.Symp.on the Mathematics of Networks & Systems,* June 15-19, 1987, Phoenix, Arizona, C.I.Byrnes *et al* (Editors), North-Holland, N.Y. 1988, pp.383-390.

[R4]* W.M.Miyaji, "Strong and Weak Stabilizability: Lyapunov Type Approach," Ph.D. Dissertation, Electrical Engr.Department, UCLA, 1988.

[R5] T. Cataltepe, "A study of Strong Stability of Distributed Systems," Ph.D. Dissertation, Electrical Engineering Department, UCLA, in preparation.

[R6]* N.Levan, "The Left Shift Semigroup Approach to Stability of Distributed Systems", *J.Math.Anal.Appls.* (to appear).

[R7] V.N.Do, "Stabilization of Distributed Semilinear Systems", *Proc. 1988 Int. Conf. on Advances in Communication and Control Systems*, W.A.Porter *et al* (Editors), Electrical and Computer Engineering Department, Louisiana State University, Baton Rouge, LA, Oct.19-21, 1988, pp.1400-1409.

[R8]* P.K.C.Wang, "Stabilization of Elastic Systems by Means of Computer Vision Feedback", *Proc.4-th IFAC Symposium on Control of Distributed-Parameter Systems,* Los Angeles, California, June 30-July 2,1986.

[R9]* P.K.C.Wang, "Feedback Control of Vibrations in an Extendible Cantilever Wing", *Dynamics and Stability of Systems*, Vol.3, Nos.3 and 4, 1988, pp.109-133. (UCLA Engr.Rpt. No.87-36, Sept.1987).

[R10]* P.K.C.Wang and Nai-jian Wang,"Static Shape Estimation in JPL/RPL Antenna Model via Computer Vision", UCLA Engr. Rpt.87-37, Sept.1987.

[R11]* P.K.C.Wang, "Stabilization and Control of Distributed Systems with Time-dependent Spatial Domains", UCLA Engr. Rpt.88-27, August,1988 (Accepted for publication in *J.Optimization Theory and Applications*).

[R12]* P.K.C.Wang and Nai-jian Wang, "Static Shape Estimation and Failure Detection in Large Spaceborne Antennas via Computer Vision",*Proc.5-th IFAC Symposium on Control of Distributed-Parameter Systems*,Perpignan, France, June 26-29,1989, pp.335-341.

[R13] Nai-jian Wang, "Shape Estimation and Failure Detection in Large Space Structures via Computer Vision", Ph.D. Dissertation, E.E.Dept. UCLA, in preparation.

4. PRESENTATIONS

[P1] P.K.C.Wang, "Application of Computer Vision in JPL/RPL Antenna Experiment," Jet Propulsion Laboratory,Pasadena, August 15,1986.

[P2] P.K.C.Wang, "Vibration Control of Elastic and Aeroelastic Systems with Time-varying Spatial Domains," Presented at the First Intl. Conf. on Industrial Applied Math.,Paris,June,1987.

[P3] N.Levan, "Stability and Stabilizability of Dissipative Systems," national Workshop on Stabilization of Flexible Structures, Montpellier, France, Dec.11-15, 1987.

5. PERSONNEL

Principal Investigators: N. Levan and P.K.C. Wang.

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*Items which have been sent to AFOSR earlier.