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Existing second order accurate finite difference methods have been evaluated for application to finite deformation transient dynamic response problems in solids. A new second order accurate finite difference technique has been developed. This is a two-step technique that can be used with deformable Lagrangian meshes. The technique has been developed by using the concepts of contour differences and MacCormack methods. The second order techniques have been applied to study many problems including dynamic poynting effect problems, interaction of stress waves and cracks, beams with nonstructured masses and stiffened shells.

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SECOND ORDER ACCURATE FINITE DIFFERENCE METHODS FOR FINITE DEFORMATION TRANSIENT SOLID DYNAMIC PROBLEMS

by

S. Hanagud

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e. "On the Interaction of Stress Waves and Cracks by Second Order Accurate Finite Difference Methods" (with R. L. Latham) (to be published).

f. "Contour Differences and Finite Element Methods" (to be published).

g. "Fluid-Moving Stiffened Shell Interaction" (to be published), accepted for presentation at the International Conference on Theoretical and Applied Mechanics.

h. "On Nonreflecting Boundaries as Solid Dynamic Problems" (to be published).

3. Presentations

a. "Finite Difference Schemes and Second Piola-Kirchhoff Formulation," U.S. National Congress of Applied Mechanics, June, 1982, Cornell University.

b. "A Comparison of Finite Difference Methods for Second Piola-Kirchhoff Formulation Third International Symposium on Numerical Methods in Engineering, Paris, France, 1983.

c. "Second Order Accurate Numerical Methods for Dynamic Response of Locking Materials," XI Southeastern Conference of Theoretical and Applied Mechanics, Huntsville, Alabama, 1982.

d. "A Comparison of Two Numerical Schemes for the Solution of Transient Dynamic Response Problems," XII Southeastern Conference of Theoretical and Applied Mechanics, Callaway Gardens, Georgia, 1984.

e. "Fluid-Moving Stiffened Shell Interaction" to be presented at the XVI International Congress of Theoretical and Applied Mechanics, Lyngby, Denmark, 1984.

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solution in the fluid. The numerical solution procedure for the shell has been supported by this grant.

The problem of homogeneous wave equation with boundary conditions prescribed on a rigid stationary surface is mathematically equivalent to Kirchhoff's retarded potential integral representation (13-15). Such a representation transforms the domain of integration from three dimensions to two dimensions of the surface on which the boundary conditions are prescribed. The retarded potential approach has been applied to the fluid-shell interaction problem to obtain stresses in shells, submerged in water, and impacted by a plane wave (16). As a result of the impact, the shell acquires a velocity which is a function of time. Kirchhoff's retarded potential representation is valid for a stationary surface and is not valid for a surface that is in motion or undergoing deformations.

There have been several attempts to obtain retarded potential solutions to wave equation with boundary conditions prescribed on a moving surface. The first attempt, by Morgan (17) in 1930, was later shown to be mathematically inconsistent. Kromov, in 1963, (18) presented another solution. His solution was also in error. The correct solution to the problem was presented by Ffowc-Williams and Hawkings (19) in 1969. The solution of (19) was for an aeroacoustic problem. The solution is in a form that is not directly applicable to the fluid-shell structure interaction problem. In this paper the retarded potential solution to the problem of wave equation with moving boundaries has been presented in a form that is suitable for fluid-shell structure interaction problem. There, the retarded potential representation has been applied to a specific problem involving a plane acoustic wave in a fluid and a stiffened cylindrical shell that can undergo large elastic-plastic deformations. The numerical values of stresses in the shell has been compared with available experimental results. Also, possible errors that may result in ignoring the effect of the motion of the moving surface has been discussed. The results of the work have been summarized. The paper has been accepted for presentation at the XVI International Congress of Theoretical and Applied Mechanics, August 19-25, 1984, Lyngby, Denmark.

VII. Beams with Nonstructured Masses

The second order accurate finite difference schemes have also been used to study the problem of none propagation in beams with nonstructured masses and branching. Large deflections have been considered. Few cases of beam have been considered. Some differential equations and use a deformable Lagrangian mesh work, has been developed. The method is an explicit method and is optimally stable. The technique has been developed by using the concept of contour differences and MacCormack methods.

Since the elastic-plastic finite deformation problem is always path-dependent or time-dependent, the response of this problem is most accurately calculated numerically by a step-by-step incremental analysis. A Cauchy stress and an updated Lagrangian approach are chosen to formulate such a problem. In such a formulation, the initially well-arranged meshes distort with increasing time as the body is subjected to finite deformations. Thus, the conventional finite difference schemes for spatial derivatives are no longer suitable. By defining some new contour difference operators, a new optimally stable and second order accurate numerical technique has been developed from the Lax-Wendroff scheme, the modified version of Strang's method due to Morris and Gottlieb, and the MacCormack two-step scheme. This finite difference method is suitable for solving problems where the grid system is not fixed but distorts with time.

In order to test the accuracy, this new method has been applied to solve some elastic and elastic-plastic problems. The numerical results have been compared with those obtained by a staggered leap-frog scheme with artificial viscosity. On the basis of the size of the time steps, the sharpness of the wave fronts, the phenomena of oscillations and overshoots behind the wave fronts, it has been concluded that the numerical procedure developed in this thesis is an improvement over the staggered leap-frog scheme for solving transient dynamic response problems in solids.

The results are being summarized in two technical papers. The papers will be submitted for publication. Parts of this work were supported on the grant. Other parts including some applications were supported by different funding sources. The research work has also been used for the Ph.D. thesis of Hsin-Piao Chen, Ph.D., (1983).

VI. Fluid-Moving Stiffened Shell Interaction Problem

In this phase of the work, second order accurate finite difference techniques have been used to solve the equations of a stiffened cylindrical shell that can undergo large elastic plastic deformations. This forms a part of the solution procedure for the fluid-shell interaction problem. A retarded potential method has been developed for the

IV. Interaction of Stress Waves and Cracks

The problem of stress wave-crack interaction problem has been solved by using a hybrid procedure that uses the concept of a singular element for the solution in a region around the crack tip while the finite difference procedure is used for the solution in the rest of the domain of interest. The complete solution involves the coupling of two computational domains.

The equations of motion have been expressed as a system of first order hyperbolic differential equations to allow the application of two-step forms of Lax-Wendroff scheme, namely Richtmyer and MacCormack schemes. The one-dimensional forms of these difference schemes are used with a Strang type of time splitting procedure to solve two and three-dimensional problems. Such a numerical technique has the advantage of the optimal stability and greater efficiency than the two- or three-dimensional conventional schemes. A nonreflecting boundary condition has been developed and implemented. Two- and three-dimensional problems have been solved and the results have been compared with the existing solutions, including a comparison with the results of a finite element solution. Finally, the technique has been applied to a two-dimensional plane stress problem with a crack. The interaction problem of the crack and stress waves travelling in the plane stress domain has been solved by using the hybrid procedure.

The results are being summarized in three papers. First of these papers entitled "A Comparison of Two Numerical Schemes for the Solution of Transient Dynamic Response Problem" has been presented at the Twelfth Southeastern Conference on Theoretical and Applied Mechanics. The paper is published in the proceedings SECTAM XII, Vol. II, p. 81. (Callaway Gardens, 1984). The other two papers will be submitted for publication.

The results of this phase of the investigation also constituted a part of the Ph.D. thesis of Ralph L. Latham (Ph.D., 1982).

V. Development of a New Second Order Accurate Finite Difference Method and Its Application to Finite Deformation Solid Mechanics Problems

This phase of the study is related to the second objective of the project. A second order accurate finite difference technique, that can be used to solve hyperbolic

(8). Hirao and Sugimoto (9) have applied the derivative expansion method to obtain the equations governing dispersive torsional waves in a circular rod. The equations have been solved for coupling effects in torsional and longitudinal waves. Antman and Liu (10) have studied travelling waves in hyperelastic rods. Haddow et al (1) have used a quasilinear hyperbolic form to obtain solutions for torsional and longitudinal waves. Similarity solutions have been obtained for a semi-infinite rod with step function loading conditions. Coupling effects have been examined.

The similarity solution for the wave propagation problem in a rod is obtainable only for the semi-infinite length of the rod with a step function type of boundary conditions. For a finite rod with different types of loading and boundary conditions, a numerical solution by the method of characteristics, although possible, is difficult and cumbersome. The order of difficulty increases for two dimensional problems. A solution by using a finite difference method for the coupled torsional and longitudinal wave propagation problem is presented in this paper. Finite difference techniques provide reasonably accurate solutions for different boundary conditions. The finite difference techniques, such as those discussed in the paper, have been extensively used in the field of fluid mechanics in the last decade. The references (11) and (12) contain a comprehensive bibliography of these finite difference methods. Several schemes to solve the system of hyperbolic partial differential equations are currently in use. In the present analysis, an explicit Lax-Wendroff second order accurate scheme with a two-step MacCormack forward-backward variant has been used. This method is well suited for shock capturing problems. In this paper, results obtained by the finite difference method have been compared with those of reference (1). In addition, the problem of a finite rod has been solved. Also a different type of loading, other than a step function, has been incorporated in the numerical solution.

Results of this work have been summarized in the form of a paper entitled "A Numerical Scheme for the Study of Poynting Effect in Wave Propagation Problems with Finite Boundaries". The paper has been submitted and accepted for publication in the International Journal of Nonlinear Mechanics. The paper will be published in late 1984 or early 1985.

The paper has also been published in the proceedings of the Third International Symposium on Numerical Methods in Engineering GAMNI, ISINA, (Edited by P. Lascaux, Vol. 2, P. 647, 1983.

III. A Numerical Scheme for the Study of Poynting Effects in Wave Propagation Problems

Most of the second order accurate finite difference schemes have been developed and applied to solve fluid mechanics problems. A test of these numerical techniques and their applicability to solid dynamic problems involving finite deformations need the consideration of problems that involve features that are encounted in solid mechanics (and not in fluid mechanics). One such problem is that of a cylindrical rod subjected to a finite twist. A second order effect, involving a change of length or an axial force, is present in hyperelastic cylindrical rods subjected to a finite twist. This second order effect leads to a coupling of torsional and longitudinal waves in such rods when they are subjected to finite deformations. In this work, the effects of such a coupling have been studied for cylindrical rods of finite length. This coupling, also known as Poynting effect, has been studied and documented for static problems. In reference (1), the dynamic Poynting effect problem has been formulated and solved by similarity methods for neo-Hookeon materials.

The neo-Hookeon material is an idealized material. Very few real materials experimentally conform to such an idealization. However, the resulting simple constitutive relationship provides an opportunity to understand and explore the transient dynamic response of such a class of nonlinear materials. Future extension of the study to more realistic materials will be useful in characterizing the dynamic response of materials used for damping of large space structures and "crush zone" or energy absorbing materials that are used in crashworthy structures (2, 3). In addition, a study of the idealized material has direct applications to some polymer structures (4, 5).

Wave propagation studies in hyperelastic materials have been reported by several other authors (6-9). Collins (6) has studied incompressible heat conducting materials. He has included the effects of entropy changes across discontinuities. The case of a sudden application of shearing stresses on a given elastic half space has been reported in (8). Jeffrey (7) has discussed nonlinear effects in wave propagation problems. Effects of a variable cross-section on the wave propagation in hyperelastic rods have been reported in

I. Introduction

This is the final report on the research work conducted under the Air Force Research Grant AFOSR-81-0224. The research project had two objectives. The first objective was to evaluate and apply the existing second order accurate finite difference techniques to transient dynamic response problems in the field of solid mechanics. A second objective was to develop new second order accurate finite difference techniques to solve transient dynamic response problems in the field of solid mechanics. A special emphasis on the new developments was to consider nonlinear problems. Nonlinear effects were not restricted to material nonlinearity. The objectives included the effects of finite deformation.

During the tenure of the research grant, three graduate students were partly supported. Two of the three graduate students have received their Ph.D. degree from the School of Aerospace Engineering. Their theses have been partly supported on the grant. The third graduate student is expected to complete his requirements for the Ph.D. degree during the Fall Quarter 1985. Some of the specific accomplishments are summarized in the following sections.

II. A Comparison of Finite Difference Schemes for Second Piola-Kirchhoff Formulation

In this work, five different second order accurate finite difference schemes have been used to solve transient dynamic response problems of solids involving finite deformations. The problems have been formulated by using Kirchhoff-Treftz stresses or second Piola-Kirchhoff stresses. A total Lagrangian approach has been used. The constitutive relationships considered include an elastic behavior, an elastic-plastic behavior and an elastic-plastic--nonlinearly compressible behavior. The resulting nonlinear hyperbolic differential equations have been numerically solved by using five different second order accurate schemes. The numerical results have been used to compare the accuracy and dispersion errors.

The results of work were summarized in the form of a paper entitled "A Comparison of the Finite Difference Schemes for Second Piola-Kirchhoff Formulation". The paper was presented at the Third International Symposium on Numerical Methods in Engineering.

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SECOND ORDER ACCURATE FINITE DIFFERENCE METHODS FOR FINITE DEFORMATION TRANSIENT SOLID DYNAMIC PROBLEMS

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