CORRELATION-INDUCED CHANGES OF SPECTRA

Professor Emil Wolf

Report on some research carried out under the ARO-URI program
at the University of Rochester

December, 1989

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) ARO 24626.119-PH-UIR
6a. NAME OF PERFORMING ORGANIZATION University of Rochester 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office
6c. ADDRESS (City, State, and ZIP Code) Rochester, NY 24627	7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U. S. Army Research Office 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER DAAL03-86-K-0173
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Correlation-Induced Changes of Spectra

12. PERSONAL AUTHOR(S)
Emil Wolf

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical	FROM ____ TO ____	December 1989	

16. SUPPLEMENTARY NOTATION The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Spectrum of Light, Doppler Effect, Communication Techniques, Spectro-Radiometric Scales, Laser Beams, Turbulence Theories, Spectral Lines

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

It is generally taken for granted that the spectrum of light does not change on propagation. This belief is implicit in all spectroscopy. In the last few years it was discovered that while such spectral invariance holds for radiation from commonly used sources it does not hold, in general, because the spectrum of emitted radiation depends not only on the source spectrum

(CONT'D ON BACK)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code) | 22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

but also on the coherence properties of the source. It was found that in some cases the induced changes may be appreciable, possibly resulting in frequency shifts of spectral lines. It has also been demonstrated that similar spectral changes may be produced by scattering on random media whose dielectric susceptibility fluctuations are appropriately correlated.

Very recently it was shown that under certain circumstances the frequency shifts generated by dynamic scattering may imitate the Doppler effect, even though the source, the scatterer and the observer are all at rest relative to each other.

The results appear to have considerable potential for both scientific and technological applications, for example in the following areas:

(1) Development of new communication techniques, some of which could be employed for covert operations.

(2) Improvements in the accuracy in standards determination for spectro-radiometric scales.

(3) Improvements in the accuracy in tracing of satellites by laser beams.

(4) Testing of theories of turbulence.

(5) Astronomical applications, especially in connection with interpretation of some puzzling features observed in the spectra of various astronomical sources, particularly quasars.

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1. INTRODUCTION

It is generally taken for granted that the spectrum of light does not change on propagation. This belief is implicit in all spectroscopy. In the last few years it was discovered that while such spectral invariance holds for radiation from commonly used sources it does not hold, in general, because the spectrum of emitted radiation depends not only on the source spectrum but also on the coherence properties of the source. It was found that in some cases the induced changes may be appreciable, possibly resulting in frequency shifts of spectral lines. It has also been demonstrated that similar spectral changes may be produced by scattering on random media whose dielectric susceptibility fluctuations are appropriately correlated.

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The latest paper entitled "Correlation-induced Doppler-like frequency shifts of spectral lines" by E. Wolf, published in the November 13, 1989 issue of *Physical Review Letters* (Vol. 63, 220-223) has attracted considerable attention, as is evident from reports that have since then appeared in various magazines in both the USA and abroad. A selection of them follows.

In this booklet we also include a list of references to all the scientific papers which have been published on this subject so far, as well as copies of papers which describe experimental verification of the theory. Most of the theoretical research was carried out under the sponsorship of the ARO-URI program at the University of Rochester, by Professor Emil Wolf and his students and other collaborators.

2. SOME RECENT PRESS REPORTS

PUBLISHED BY THE UNIVERSITY OF ROCHESTER

CURRENTS

VOL. 17 NO. 36
MONDAY, NOVEMBER 13. 1989

Physicist Emil Wolf extends theory of spectral shifts

A University professor has extended his theory challenging a traditional tenet of modern physics —the cause of spectral shifts. The work by Emil Wolf, Wilson Professor of Optical Physics in the College of Arts and Science, could have repercussions in many areas of science and suggests our view of the universe may need to change. Already, other scientists across the globe are applying his theory to long-standing problems of physics.

Wolf's latest paper, "Correlation-Induced Doppler-Like Frequency Shifts of Spectral Lines," appears in today's issue of *Physical Review Letters*. It represents an expansion of a principle he described in earlier work. In the new paper, Wolf shows how something other than gravity or the Doppler Effect can produce dramatic changes in the behavior of light or sound; in fact, the mechanism Wolf has discovered can *completely* imitate the Doppler Effect to any magnitude.

Scientists have known for decades that light from different sources has different spectral patterns and that these patterns can help identify the nature of a source. By studying the spectral patterns emanating from a star, for example, astronomers can determine its chemical composition.

Scientists have also known that light spectra can be shifted: The spectral pattern that is characteristic of a certain chemical at a certain wavelength, for example, can be shifted to shorter (bluer) or longer (redder) wavelengths. Scientists have long assumed that only

Three years ago Wolf theorized a new cause for spectral shifts. What has come to be known as the "Wolf Effect" or "Wolf Shift" has since been confirmed several times experimentally. In his latest paper, Wolf shows that the Wolf Effect can shift spectral lines as completely as the Doppler Shift. In addition, the shift is consistent —each line within a spectrum is shifted in the same proportion.

"This mechanism can exactly imitate the Doppler Shift, using known principles of physics and optics," Wolf says. "The redshift can be as large or as small as you want — in principle, it can have any magnitude."

Wolf's theory could have consequences in a wide range of areas, including astronomy, standards specification, coding of signals, and satellite tracking.

Previously, Wolf showed that the manner in which atoms in a light source are ordered (source correlation) affects the way these atoms emit light and the way this light travels through space. When the fluctuations, or light emissions, from these atoms are neither fully ordered, nor coherent (as in a laser), nor fully incoherent (as in a light bulb or candle flame), shifts can occur. Spectral shifts occur in the realm of partial coherence.

To explain partial coherence, Wolf uses this analogy: "Picture a contingent of 1,000 soldiers walking across a bridge," Wolf says. "If all the soldiers are walking in step, that's an example of coherence. If they are all drunk and wander randomly across the bridge, that's an example of incoherence. But if some walked in step, and some wandered across, that's an example of partial coherence."

Despite confirmations in the laboratory, it is not known whether and under what conditions the Wolf Effect occurs in nature. Wolf and several associates are continuing their investigations. A graduate student, Daniel James, has developed a model of a medium giving rise to such shifts. Whether such media exist in nature is now being studied by Wolf, James, and University astronomer Malcolm Savedoff.

Newly Discovered Property of Light Spectra May Resolve Major Dispute in Astronomy

By KIM A. McDONALD

An unusual effect of light discovered by a University of Rochester physicist may help astronomers resolve the puzzle of why some objects in the universe that are connected appear to be traveling at different speeds.

In a paper published in this week's issue of the journal *Physical Review Letters*, Emil Wolf, a professor of optical physics at Rochester, contends that the newly discovered effect, which he predicted three years ago, may exist on a much larger scale than has been observed in laboratory experiments.

If so, it may explain a major puzzle that has plagued modern astronomers for two decades.

Emil Wolf: His prediction would support astronomers who believe quasars are receding neither as rapidly nor as far from Earth as their red shifts indicate.

Measuring Light's Characteristics

The problem concerns quasars, unusually bright, compact sources of radiation located in the centers of ancient galaxies, that are thought to be among the most distant objects in the universe. By measuring the spectral characteristics of the light emitted by quasars, astronomers have deduced that many quasars are near the edge of the expanding universe, traveling outward at incredible velocities.

Those deductions have been based on the fact that the spectra from quasars are typically shifted far to the red end of the light spectrum. Scientists know that the greater this "red shift," the greater the distance and speed of the object receding from Earth. This is because of the Doppler effect, a property of sound and light demonstrated by the sound of a train whistle as it travels past an observer—becoming higher-pitched as it moves toward the observer and lower pitched (or shifting toward the

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other end of the frequency spectrum) as it moves away.

Physicists have long known that the Doppler effect and, to a lesser extent, gravity, can produce red shifts in light spectra. In recent years, researchers in the United States, India, and Italy have confirmed experimentally that the effect predicted by Mr. Wolf, now known as the "Wolf effect," also can affect red shifts. But because the Wolf effect, like gravity, is so small, astronomers have generally discounted it in their calculations.

In his paper, however, Mr. Wolf shows how the Wolf effect could produce a red shift as large as the Doppler effect for objects such as quasars. He said in an interview that some physicists had already begun testing his theory to see if it holds up experimentally.

If true, Mr. Wolf's newest prediction would bolster the claims of astronomers who believe quasars are receding neither as rapidly nor as far from Earth as their red shifts appear to indicate.

It would also suggest that astronomers may have miscalculated the age of quasars and their distance from Earth—long considered an important yardstick for measuring the size of the universe.

The question of whether red shifts reflect real cosmological distances has been one of the central issues of debate among astronomers for the past 20 years.

The debate has become particularly heated since the discovery of quasars connected to galaxies, in which the quasars appear to be traveling away from Earth at a much faster rate than the galaxies,

The question of whether red shifts reflect real cosmological distances has been debated by astronomers for the past 20 years.

an incongruity that astronomers have been unable to explain.

"That's the conflict that has brought us to this point," said Geoffrey R. Burbidge, a professor of physics at the University of California at San Diego who is one of the main proponents of the non-cosmological explanation of red shifts. "The data keep piling up and people keep arguing about the dilemma."

Mr. Wolf said the Wolf effect might explain the dilemma, since recent calculations led him to conclude that larger red shifts than have been observed experimentally could be produced when a certain type of light passes through a secondary medium, such as the atmosphere that surrounds a quasar.

In the laboratory, the Wolf effect produces a slight red shift in light that is partially coherent. Such light is produced when the fluctuations, or light emissions, from the atoms in a light source are neither fully ordered—or coherent, as in a laser—nor fully incoherent, as in a light bulb, candle flame, or star.

Source of Radiation Is Unclear

Although Mr. Wolf's model assumes the light from quasars is not stellar and probably partially coherent, astronomers say the source of radiation from quasars is not well understood. But if the light is partially coherent, Mr. Wolf said, as the fluctuations from the secondary source become greater, so too could the red shift, until it approximated the Doppler effect.

"If one provides a good-enough model for the atmosphere around quasars, one could quantify it," he said.

Mr. Burbidge, who has examined Mr. Wolf's paper, said he believed the concept was worthy of further examination and testing.

"My feeling is that he may be on the way to helping us understand these red shifts," he said.

PHYSICS

Stellar 'yardstick' disputed

By David L. Chandler
GLOBE STAFF

New work by a respected physicist could make the almost-sacred yardstick by which the universe has been measured seem rubbery and unreliable, forcing a reexamination of some central beliefs of modern astronomy.

In a paper being published today in the journal Physical Review Letters, Emil Wolf of the University of Rochester in New York casts doubt on the accepted view that changes in the color of light are an absolute and accurate indicator of astronomical distances.

The new "Wolf effect" could help to explain some strange observations involving distant, highly energetic objects called quasars, Wolf said. But some other physicists and astronomers say the effect, while probably real in principle, is likely to be irrelevant in the real world of astronomy.

The paper strikes to the heart of one of astronomy's most cherished laws, a principle laid down by astronomer Edwin Hubble in 1929. Hubble discovered the expansion of the universe – the fact that all the distant objects we see in the sky seem to be racing away from us.

Hubble's law says that the farther away a galaxy is, the *faster* it is going. Therefore, its speed can be used as a reliable yardstick to measure its distance from Earth.

And speed is a relatively easy thing to measure for distant, luminous objects like galaxies – which Hubble proved to be island universes containing billions of stars each. To measure their speed, astronomers need only spread out their light through a prism or other device to separate out the rainbow colors that make up white light.

This produces a spectrum containing characteristic lines of much brighter light. These lines, produced by gases in the stars that emit light of particular colors, have distinctive spacings like the barcodes on supermarket products – unique "fingerprints" that

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Astronomers cherished 'yardstick' challenged

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reveal the identity of the gases that produced them.

But if the objects producing the light are moving away from us, the lines are shifted toward the red end of the spectrum. The distinctive spacing of the lines stays the same, but their positions relative to the rainbow of colors is changed. This "red shift" is produced by the same effect – the Doppler effect – used by police radar guns, and it serves the same function for astronomers: It gives them a way to check the speeds of distant objects. The distance that the lines are shifted is directly related to the speed of the galaxy from which the light came.

If cars could be equipped with special shields that would artificially produce a Doppler shift, sending absurdly high or low speed readings to the radar guns, it would throw enforcement of speed limits into confusion. The Wolf Effect could have much the same confusing effect on astronomers' measurements of the size and age of the universe.

The effect, Wolf said in an interview last week, means that if light passes through certain kinds of matter it could end up with a redshift indistinguishable from that produced by great speed – even if the object emitting the light was not moving at all.

Wolf is cautious about making any sweeping claims. While convinced the effect is a real one, he said it probably only applies to a few unusual objects, not to all the galaxies we can see. He suspects, though, that it might help explain some odd objects that are otherwise difficult to account for.

"I don't want to get involved in these arguments about the size of the universe," he said. "At the moment, I'm only interested in these small effects."

In particular, there are many quasars – believed to be highly energetic, very distant galaxy-sized objects – that seem connected to nearby galaxies, but whose redshifts say they are many times farther away. In some cases, the two objects even appear linked by thin "bridges" of matter. Astronomer Halton Arp and a few others have accumulated an impressive, and highly controversial, catalog of such objects.

Wolf believes that layers of material connected with the quasar itself could produce the redshift that suggests it is farther away than its attached galaxy, and that the objects really are connected. But most astronomers believe such pairs are simply chance alignments and the objects are unrelated.

John Huchra, an astronomer at the Harvard-Smithsonian Center for Astrophysics, said that while the distances of quasars was once a controversial question, most astronomers now feel it has been solved. Whereas 20 years ago astronomers were probably split 50-50 as to whether quasars were nearby or at galactic distances (as indicated by their redshifts), he estimates that today 99.5 percent of astronomers believe that quasars are at their redshift distances.

Huchra said that evidence shows most quasars are clearly at the "right" distances as indicated by their redshifts. "If you can show that a whole bunch of them are" at the right distances, he said, "then it's probably true of the rest of them." He said the Wolf Effect is "an interesting idea, it probably operates somewhere, but it doesn't relate to the quasar situation."

Astronomer Geoffrey Burbidge of the University of California/San Diego is one of those who think the odd quasar-galaxy pairs undermine conventional views of the universe. He said Wolf's work is "good physics," but "whether or not it will really explain all of these things we are finding, I don't know. It's certainly a step in the right direction. To me, it looks promising."

And it could undermine much of the conventional view of the universe, he said. Virtually all of the most distant visible objects are "strange objects" that may not follow the same principles as conventional, nearby ones. If most of the apparently distant objects are being redshifted by the Wolf effect, that would "shrink the visible universe," he said. Instead of seeing almost to the edges of the visible universe, we may only be looking at a relatively nearby neighborhood.

Astrophysicist John Bahcall of Princeton dismisses the whole idea, saying the supposed quasar-galaxy associations are meaningless and the Wolf Effect probably has no application to the real universe. "It's a case of irrelevant physics used to explain bad statistics," he said.

But Wolf, who co-authored what is considered a classic book on optical theory and is respected even by those who dispute the relevance of his latest work, is convinced the effect will be found to have significance somewhere.

Citing the amazing and unexpected variety of planet and moon surfaces discovered by recent spacecraft as a demonstration of nature's capacity to surprise, he said, "Amongst all those billions of stars, to me it would be a miracle if there were none that showed this effect."

Expanding a theory for shifting starlight

If Oscar Peterson were running alongside a speeding Ella Fitzgerald as she belted out an A, he would hear an A. But if the pianist took a breather while singing Ella raced on, the Doppler effect would make the note sound lower to him.

For close to a century, scientists have known that the wavelengths of light from a rapidly receding source in outer space also get Doppler-shifted to lower frequencies and so appear redder than they would if their source remained stationary. Moreover, researchers have assumed that this so-called redshift can result only from Doppler motion or from a gravity-based mechanism also uncovered early this century.

Not so, contends physicist Emil Wolf of the University of Rochester (N.Y.). In 1986, he published a theoretical sketch of a third mechanism that could account for part of the redshift of light from exotic cosmic objects such as quasars. Several researchers have since reported laboratory confirmations of the process, which some of them refer to as the "Wolf shift" (SN: 9/13/86, p.166; 7/11/87, p.22).

Now Wolf reports an important extension of his theory, suggesting the process could account for arbitrarily large shifts toward either the red or blue end of the electromagnetic spectrum.

In its earlier form, the theory proposed a mechanism that could produce small redshifts in the spectra of light emitted from certain exotic sources. Wolf suggested at the time that the shifting mechanism could emerge physically from partially synchronized, or coherent, fluctuations in the wavelengths of light emitted from the countless individual atomic and molecular "microlamps" that make up such a source. As these emissions travel through space, their original spectrum would appear to shift in the same way as Doppler-shifted light.

Though its astronomical consequences remain unknown, the theory could change estimates of the size of the universe and help explain some anomalous astronomical observations, Wolf says. However, astronomers have not rushed to adopt it. No known light source has components that display the required correlated fluctuations, Wolf notes. He also blames the theory's unorthodoxy and its arcane mathematical formulation for its limited consideration by astronomers.

In the updated theory, described in the Nov. 13 PHYSICAL REVIEW LETTERS, Wolf outlines a more general – and perhaps physically more plausible – mechanism that could imitate Doppler shifts of any magnitude. Instead of requiring the microlamps in the source to fluctuate in some correlated fashion, he now proposes that a complex "scattering medium," such as the electrically charged and frenetic atmosphere thought to surround quasars, might serve as an unusual lens that restructures incoming light to have redshifting or blueshifting correlations upon leaving the medium. "A scattering medium of the right type between the source and an observer should produce these effects," Wolf told SCIENCE NEWS.

Wolf concedes that astronomers have never reported such a scattering medium and notes that he used simplifying assumptions in both the original and updated theories. Nonetheless, the expansion of the theory strengthens the case for a third physical mechanism underlying spectral shifts even in light from stationary sources. University of Rochester astronomer Malcolm P. Savedoff says the soundness of Wolf's theory demands that scientists take it seriously. In a paper submitted to ASTROPHYSICAL JOURNAL, Savedoff, Wolf and graduate student Daniel F.V. James have mathematically modeled scattering media that are consistent with typical models of the environment near quasars. – *I. Amato*

New Scientist 25 November 1989

Novel red shifts

UNUSUAL effects in which light is scattered might be causing large shifts in the wavelengths of light in the optical spectra of distant galaxies and quasars. This could mimic the effects of the expansion of the Universe, which astronomers believe causes the light in distant objects to be greatly red shifted. The phenomenon might explain some puzzling associations of quasars with galaxies which have much smaller red shifts (*Physical Review Letters*, vol 63, p 2220).

Emil Wolf, an optical theorist at the University of Rochester in New York State, has carried out calculations together with graduate student Daniel James. They find that large shifts in wavelength are possible when light is scattered from media with optical properties that change randomly in space and time. Such media may occur in the vicinity of the "engines" that drive quasars, violent galaxies that astronomers do not understand well at all. The shift would be similar at all wavelengths, thus mimicking the shift caused by the expansion of the Universe.

Wolf first showed, a few years ago, that small shifts could occur in light that is partially coherent, when viewed from different angles (*Physical Review Letters*, vol 58, p 2646). Researchers have since seen such changes in the laboratory. Until his discovery, physicists thought that such shifts in wavelength could be caused only by the motion of the light source, or by strong gravitational fields.

The large shifts, which Wolf and James predict, have yet to be demonstrated. So far, astronomers are unconvinced. "Even if they can be produced in the laboratory, they may not exist in nature," says Malcolm Savedeff, an astronomer at the University of Rochester. □

THE SUNDAY CORRESPONDENT NOVEMBER 26 1989 (London, England)

Cosmic theory gets red light

By Robert Matthews
Science Correspondent

RESEARCH by a distinguished professor of physics is likely to start a major controversy about the origin of the universe.

The work, published in an influential journal of physics, challenges one of the fundamental tenets of cosmology: that light from distant galaxies reveals how fast the universe is expanding.

According to current thinking, the universe began in a cataclysmic explosion — the Big Bang — about 15 billion years ago and has been expanding ever since.

The proof for this assertion dates from the 1920s, when the American astronomer Edwin Hubble discovered that galaxies were racing away from one another, with their speed increasing as distance increased. By measuring this rate of expansion of the universe, it is possible to work out the date of the Big Bang.

But Hubble's observations rely crucially on the so-called "red shift" effect. This is similar to the change in pitch heard from a police car siren as it passes by. Light coming from a receding galaxy is shifted towards longer, redder, wavelengths of light. By measuring the "red shift" of a galaxy it is possible to estimate its speed.

The fundamental assumption is that only movement away from an observer can cause a red shift. But now Emil Wolf, professor of optical physics at the university of Rochester, New York State, and graduate student Daniel James have found another cause of red shifts, which has nothing to do with movement.

It casts doubt on one of the few things cosmologists thought they knew for certain: that red shifts can be relied on to work out the age and fate of the universe.

Professor Wolf has discovered that it is possible to mimic a red shift when light coming from a source bounces off material with certain optical properties that change with both time and location relative to the source.

Details of the research are published in the November 13 issue of *Physical Review Letters*.

The process can account for a red shift of any size, including those associated with quasars, the incredibly bright centres of galaxies, thought to be the most distant objects in the universe.

Backing for Professor Wolf's work may have already come from a long-standing controversy concerning these objects.

Photographs taken with the world's largest telescopes by Dr Halton Arp, an American astronomer, appear to show physical links between some quasars and galaxies. Yet the red shifts of the two objects are vastly different, suggesting they are billions of light-years apart.

The standard explanation is that the two objects were lying in the same part of the sky, and were superimposed on each other.

Professor Wolf believes his new research may mean that Arp's contention that the photographs prove that there is something wrong with the red shift idea will have to be reexamined. Others think not.

Professor Martin Rees, a leading astrophysicist at the Institute of Astronomy in Cambridge, said there was plenty of evidence supporting the conventional interpretation of red shifts.

SCIENTIFIC AMERICAN *January 1990*

The Redshift Blues

New redshift theory challenges both physicists and cosmologists

Redshifts are to astronomy what tape measures are to carpentry: a well-understood tool whose validity hardly seems open to question. Modern cosmology is rooted in the belief that essentially all observed redshifts are caused by the Doppler effect, whereby light emitted by a receding object is "stretched out" and so shifted toward the red end of the spectrum. Emil Wolf of the University of Rochester, writing in *Physical Review Letters*, now questions this traditional interpretation by proposing a previously unrecognized mechanism that, he claims, can completely mimic the Doppler effect.

Wolf suggests that light can be redshifted if it passes through a scattering medium in which the index of refraction varies randomly over both space and time. If suitable correlations exist within the medium, the light will change frequencies even though the overall source is at rest. His calculations show that the result can in principle be a Doppler-like shift across the entire spectrum.

Wolf's ideas have generated considerable interest and controversy among his colleagues, in part because the big-bang theory rests on observations of redshifts that are interpreted as evidence that the universe is expanding. Wolf hesitates to suggest that the big-bang concept might be wrong. "It is something that should be looked at," he says cautiously.

More plausible, Wolf believes, is the possibility that his mechanism may explain long-disputed quasar-galaxy pairings. In some instances, quasars and galaxies that appear to be physically associated have vastly different redshifts, which indicates, by conventional reckoning, that they are billions of light-years apart. Many astronomers—including Christopher L. Carilli of Harvard University, who with two colleagues found the most recent such association—dismiss these associations as chance line-of-sight pairings. A few workers, among them Jack W. Sulentic of the University of Alabama at Tuscaloosa, maintain that they reveal either anomalies unexplained by the big-bang theory or else the existence of some kind of non-Dopplerian redshift.

Naturally, Wolf favors the latter interpretation. Matter surrounding quasars is probably anisotropic (not the same in all directions) because of turbulence or the powerful jets of matter that quasars often emit. Such anisotropy could produce the sort of correlation mechanism that Wolf has studied, generating redshifts that have hitherto been attributed to the quasars' great distance from the earth. If confirmed, this finding could solve another quasar mystery: some quasar jets appear to be expanding faster than the speed of light. If quasars were significantly less distant than generally believed, the jets would be much smaller and their rate of expansion within the cosmic speed limit.

Wolf is searching for more down-to-earth applications, such as methods for correcting satellite-tracking signals and for improving reference standards. He also reports interest from the Department of Defense, which is intrigued by the possibility that artificially induced spectral modulation could provide the ultimate in coded communications. —*Corey S. Powell*

3. LIST OF REFERENCE TO PAPERS DEALING WITH CORRELATION-INDUCED CHANGES OF SPECTRA

PUBLICATIONS DEALING WITH CORRELATION-INDUCED CHANGES OF SPECTRA

(a) *Spectral Changes Arising from Source Correlations*

(i) *Theoretical*

E. Wolf, "Invariance of spectrum of light on propagation", *Phys. Rev. Lett.* **56**,1370-1372 (1986).

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A. Gamliel and N. George, "Radiated spectrum from two partially correlated dipoles", *J. Opt. Soc.* A, **6**, 1150-1155 (1989).

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(ii) *Experimental*

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4. SELECTION OF BASIC THEORETICAL PAPERS

 4.1

Invariance of the Spectrum of Light on Propagation

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(Received 27 January 1986)

The question is raised as to whether the normalized spectrum of light remains unchanged on propagation through free space. It is shown that for sources of a certain class that includes the usual thermal sources, the normalized spectrum will, in general, depend on the location of the observation point unless the degree of spectral coherence of the light across the source obeys a certain scaling law. Possible implications of the analysis for astrophysics are mentioned.

PACS numbers: 42.10.Mg, 07.65.−b, 42.68.Hf

Measurements of the spectrum of light are generally made some distance away from its sources and in many cases, as for example in astronomy, they are made exceedingly far away. It is taken for granted that the normalized spectral distribution of the light incident on a detector after propagation from the source through free space is the same as that of the light in the source region. I will refer to this assumption as the assumption of *invariance of the spectrum on propagation*. This assumption, which is implicit in all of spectroscopy, does not appear to have been previously questioned, probably because with light from traditional sources one has never encountered any problems with it. However, with the gradual development of rather unconventional light sources and with the relatively frequent discoveries of stellar objects of an unfamiliar kind, it is obviously desirable to understand whether all such sources generate light whose spectrum is invariant on propagation, and if so, what the reasons for it are. Actually it is not difficult to conceive of sources that generate light whose spectrum is not invariant on propagation. In this note I will show what are the characteristics of a certain class of sources that generate light whose spectrum is invariant, at least in the far zone.

From the standpoint of optical coherence theory, invariance of the spectrum of light on propagation from conventional sources is a rather remarkable fact, as can be seen from the following simple argument. Consider an optical field generated by a stationary source in free space. The basic field variable, say the electric field strength at the space-time point $(\mathbf{r},t)$, may be represented by its complex analytic signal[1,2] $E(\mathbf{r},t)$. According to the Wiener-Khintchine theorem[3] the spectral density of the light at the point $\mathbf{r}$ is then represented by the Fourier transform,

$$S(\mathbf{r},\omega)=\int_{-\infty}^{\infty}\Gamma(\mathbf{r},\tau)e^{i\omega\tau}\,d\tau, \tag{1}$$

of the autocorrelation function (known in the optical context as the *self-coherence function*) of the field variable. It is defined as

$$\Gamma(\mathbf{r},\tau)=\langle E^*(\mathbf{r},t)E(\mathbf{r},t+\tau)\rangle, \tag{2}$$

where the angular brackets denote the ensemble average. Now the spectral density and the self-coherence function are the "diagonal elements" $(\mathbf{r}_2=\mathbf{r}_1=\mathbf{r})$ of two basic optical correlation functions, viz., the cross-spectral density

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega)=\int_{-\infty}^{\infty}\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)e^{i\omega\tau}\,d\tau, \tag{3}$$

and the mutual coherence function

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)=\langle E^*(\mathbf{r}_1,t)E(\mathbf{r}_2,t+\tau)\rangle. \tag{4}$$

It is well known that both the mutual coherence function and the cross-spectral density obey precise propagation laws. For example, in free space[4]

$$(\nabla_j^2+k^2)W(\mathbf{r}_1,\mathbf{r}_2,\omega)=0 \quad (j=1,2), \tag{5}$$

where

$$k=\omega/c, \tag{6}$$

with c being the speed of light *in vacuo* and ∇_j^2 being the Laplacian operator acting with respect to the variable $\mathbf{r}_j$. Consequently, both the mutual coherence function and the cross-spectral density and, in fact, also their normalized values change appreciably on propagation. For example, for a spatially incoherent planar source $W(\mathbf{r}_1,\mathbf{r}_2,\omega)$ and $\Gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)$ will be essentially δ correlated with respect to $\mathbf{r}_1$ and $\mathbf{r}_2$ at the source plane but will have nonzero values for widely separated pairs of points which are sufficiently far away from the source. This is the essence of the well known van Cittert–Zernike theorem (Ref. 1, Sect. 10.4.2). In physical terms, the correlation in the field generated by a spatially incoherent source may be shown to have its origin in the process of superposition. We thus have the following rather strange situation: The correlations of the light may change drastically on propagation; yet, under commonly occurring circumstances, their (suitably normalized) diagonal elements, which represent the spectrum of the light or its Fourier transform, remain unchanged.

To obtain some insight into this problem we consider light generated by a very simple model source; namely, a planar source occupying a finite domain D of a plane $z=0$ and radiating into the half space $z>0$, which has the same spectral distribution $S^{(0)}(\omega)$ at each source point $P(\rho)$ and whose degree of spectral coherence[5] $\mu^{(0)}(\rho_1,\rho_2,\omega)$ is statistically homogeneous, i.e., has the functional form $\mu^{(0)}(\rho_2-\rho_1,\omega)$. The cross-spectral density of the light across the source plane is then given by

$$W^{(0)}(\rho_1,\rho_2,\omega) = \epsilon(\rho_1)\epsilon(\rho_2)S^{(0)}(\omega)\mu^{(0)}(\rho_2-\rho_1,\omega), \quad (7)$$

where $\epsilon(\rho)=1$ or 0 according to whether the point $P(\rho)$ is located within or outside the source area D in the plane $z=0$.

We will also assume that at each effective frequency ω present in the source spectrum, the linear dimensions of the source are much larger than the spectral correlation length [the effective width Δ of $|\mu^{(0)}(\rho',\omega)|$]. Sources of this kind belong to the class of so-called *quasihomogeneous sources*,[6] which have been extensively studied in coherence theory in recent years. Most of the usual thermal sources are of this kind.

The radiant intensity $J_\omega(\mathbf{u})$, i.e., the rate at which energy is radiated at frequency ω per unit solid angle around a direction specified by a unit vector $\mathbf{u}$, is given by the expression [cf. Ref. 6, Eq. (4.8)]

$$J_\omega(\mathbf{u}) = k^2 A S^{(0)}(\omega)\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega)\cos^2\theta. \quad (8)$$

In this formula, A is the area of the source,

$$\tilde{\mu}^{(0)}(\mathbf{f},\omega) = \frac{1}{(2\pi)^2}\int \mu^{(0)}(\rho',\omega)e^{-i\mathbf{f}\cdot\rho'}d^2\rho' \quad (9)$$

is the two-dimensional spatial Fourier transform of the degree of spectral coherence, $\mathbf{u}_\perp$ is the transverse part of the unit vector $\mathbf{u}$, i.e., the component of $\mathbf{u}$ (considered as a two-dimensional vector) perpendicular to the z axis, and θ is the angle between the $\mathbf{u}$ and the z directions (see Fig. 1). Evidently the normalized spectral density $S^{(\infty)}(\mathbf{u},\omega)$ at a point in the far zone, in the direction specified by the unit vector $\mathbf{u}$, is given by

$$S^{(\infty)}(\mathbf{u},\omega) = J_\omega(u)\Big/\int J_\omega(\mathbf{u})d\omega. \quad (10)$$

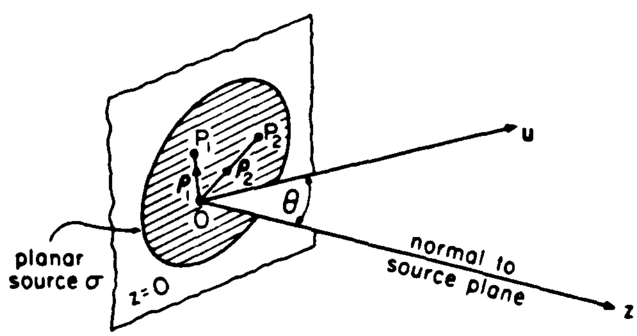

FIG. 1. Illustration of the notation.

On substituting Eq. (8) into Eq. (10) we obtain for the normalized spectrum in the far zone the expression

$$S^{(\infty)}(\mathbf{u},\omega) = \frac{k^2 S^{(0)}(\omega)\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega)}{\int k^2 S^{(0)}(\omega)\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega)d\omega}. \quad (11)$$

It is clear from Eq. (11) that the normalized spectrum of the light depends on the direction $\mathbf{u}$; i.e., it is in general not invariant throughout the far zone. However, it is seen at once from Eq. (11) that it will be invariant throughout the far zone if the Fourier transform of the degree of spectral coherence of the light in the source plane is the product of a function of frequency and a function of direction, i.e., it is of the form

$$\tilde{\mu}^{(0)}(k\mathbf{u}_\perp,\omega) = F(\omega)\tilde{H}(\mathbf{u}_\perp). \quad (12)$$

In this case Eq. (11) reduces to

$$S^{(\infty)}(\mathbf{u},\omega) = \frac{k^2 S^{(0)}(\omega)F(\omega)}{\int k^2 S^{(0)}(\omega)F(\omega)d\omega}, \quad (13)$$

and the expression on the right is independent of the direction $\mathbf{u}$.

I will now show that the condition (12) has some interesting implications, which follow from the fact that $\mu^{(0)}$ is a correlation coefficient. Before doing this we note that since $\mathbf{u}$ is a unit vector, $|\mathbf{u}_\perp|<1$. However, we will now assume that the factorization condition (12) holds for all two-dimensional vectors $\mathbf{u}_\perp$ $(0 \le |\mathbf{u}_\perp| < \infty)$. This assumption will be trivially satisfied if the degree of spectral coherence $\mu^{(0)}(\rho',\omega)$ is, at each effective temporal frequency ω, band limited in the spatial frequency plane to a circle of radius k about the origin; in more physical terms this condition means that $\mu^{(0)}(\rho',\omega)$ does not vary appreciably over distances of the order of the wavelength $\lambda = 2\pi c/\omega$. With this being understood let us take the Fourier transform of Eq. (12). We then find at once that

$$\mu^{(0)}(\rho',\omega) = F(\omega)\int \tilde{H}(\mathbf{u}_\perp)\exp(ik\mathbf{u}_\perp\cdot\rho')\,d^2(k\mathbf{u}_\perp), \quad (14)$$

i.e.,

$$\mu^{(0)}(\rho',\omega) = k^2 F(\omega)H(k\rho'), \quad (15)$$

where H is, of course, the two-dimensional Fourier transform of $\tilde{H}$. Since $\mu^{(0)}(\rho',\omega)$ is a correlation coefficient it has the value unity when $\rho'=0$, i.e.,

$$\mu^{(0)}(0,\omega) = 1, \text{ for all } \omega, \quad (16)$$

and hence Eq. (15) implies that

$$k^2 F(\omega) = [H(0)]^{-1}. \quad (17)$$

Since the left-hand side of Eq. (17) depends on the frequency but the right-hand side is independent of it, each side must be a constant (α say) and consequently

$$F(\omega)=\alpha/k^2. \tag{18}$$

Two important conclusions follow at once from these results. If we substitute Eq. (18) into Eq. (13) we obtain the following expression for the normalized spectrum of light in the far zone:

$$S^{(\infty)}(\mathbf{u},\omega)=S^{(\infty)}(\omega)=\frac{S^{(0)}(\omega)}{\int S^{(0)}(\omega)d\omega}. \tag{19}$$

This formula shows that not only is the normalized spectrum of the light now the same throughout the far zone, but it is also equal to the normalized spectrum of the light at each source point.

Next we substitute Eq. (18) into Eq. (15) and set $\alpha H=h$, $\rho'=\rho_2-\rho_1$. We then obtain for $\mu^{(0)}$ the expression

$$\mu^{(0)}(\rho_2-\rho_1,\omega)=h[k(\rho_2-\rho_1)] \quad (k=\omega/c); \tag{20}$$

i.e., the complex degree of spectral coherence is a function of the variable $\xi=k(\rho_2-\rho_1)$ only. We will refer to Eq. (20) as the *scaling law*. Obviously for a source that satisfies this law, the knowledge of the degree of spectral coherence of the light in the source plane at any particular frequency ω specifies it for all frequencies.

The scaling law (20), which ensures that for sources of the class that we are considering the normalized spectrum of the light is the same throughout the far zone and is equal to the normalized spectrum of the light at each source point [Eq. (19)], is the main result of this note.

It is natural to inquire whether sources are known that obey this scaling law. The answer is affirmative. Many of the commonly occurring sources, including blackbody sources, obey Lambert's radiation law [Ref. 1, Sect. 4.8.1]. It is known[7] that all quasi-homogeneous Lambertian sources have the same degree of spectral coherence, viz.

$$\mu^{(0)}(\rho_2-\rho_1,\omega)=\sin(k|\rho_2-\rho_1|)/k|\rho_2-\rho_1|, \tag{21}$$

which is seen to satisfy the scaling law (20). According to the preceding analysis such sources will generate light whose normalized spectrum is the same throughout the far zone and is equal to the normalized spectrum at each source point. This fact is undoubtedly largely responsible for the commonly held, but nevertheless incorrect, belief that spectral invariance is a general property of light.

This Letter has dealt with what is probably the simplest problem regarding spectral invariance on propagation. It would seem that some significant questions in this area might be profitably studied. Among them are the elucidation of the physical origin of the scaling law, spectral properties of light from a broader class of sources than considered here, the relation between the scaling law and Mandel's results regarding cross-spectrally pure light,[8,9] and relativistic effects. Applications of the results to problems of astrophysics might be of particular interest; at this stage one might only speculate whether source correlations may perhaps not give rise to differences between the spectrum of the emitted light and the spectrum of the detected light that originates in some stellar sources.

It is a pleasure to acknowledge stimulating discussions with Professor Leonard Mandel about the subject matter of this note. This research was supported by the National Science Foundation and by the Air Force Geophysics Laboratory under Air Force Office of Scientific Research Task No. 2310G1.

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$$\mu^{(0)}(\rho_1,\rho_2,\omega)=\frac{W^{(0)}(\rho_1,\rho_2,\omega)}{[W^{(0)}(\rho_1,\rho_1,\omega)]^{1/2}[W^{(0)}(\rho_2,\rho_2,\omega)]^{1/2}}.$$

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Reprinted from Nature, Vol. 326, No. 6111, pp. 363-365, 26 March 1987

Non-cosmological redshifts of spectral lines

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We showed in a recent report[1] (see also refs 2–4) that the normalized spectrum of light will, in general, change on propagation in free space. We also showed that the normalized spectrum of light emitted by a source of a well-defined class will, however, be the same throughout the far zone if the degree of spectral coherence of the source satisfies a certain scaling law. The usual thermal sources appear to be of this kind. These theoretical predictions were subsequently verified by experiments[5]. Here, we demonstrate that under certain circumstances the modification of the normalized spectrum of the emitted light caused by the correlations between the source fluctuations within the source region can produce redshifts of spectral lines in the emitted light. Our results suggest a possible explanation of various puzzling features of the spectra of some stellar objects, particularly quasars.

To explain why source correlations influence the spectrum of the emitted light consider a very simple example. Suppose that two point sources P_1 and P_2 have identical spectra $S_Q(\omega)$ and that measurements on the emitted field are made at some point P. The sources are assumed at rest relative to an observer at P. Assuming that the source fluctuations can be described by a stationary ensemble, the field at P may be characterized by an ensemble $\{V(P,\omega)\}$ of frequency-dependent realizations[6], each of the form

$$V(P,\omega)=Q(P_1,\omega)\frac{e^{ikR_1}}{R_1}+Q(P_2,\omega)\frac{e^{ikR_2}}{R_2} \qquad (1)$$

where $\{Q(P_j,\omega)\}$, $(j=1,2)$, characterize the strengths of the two fluctuating point sources, R_1 and R_2 are the distances from P_1 to P and from P_2 to P respectively (see Fig. 1) and $k=\omega/c$, c being the speed of light *in vacuo*. For simplicity polarization effects are ignored and hence V and Q are taken to be scalars. The spectrum of the light at P is then given by

$$S_V(P,\omega)=\langle V^*(P,\omega)V(P,\omega)\rangle \qquad (2)$$

where the asterisk denotes the complex conjugate and the angular brackets denote the ensemble average. On substituting from equation (1) into equation (2) and using the fact that

$$\langle Q^*(P_1,\omega)Q(P_1,\omega)\rangle=\langle Q^*(P_2,\omega)Q(P_2,\omega)\rangle=S_Q(\omega) \qquad (3)$$

the following expression is obtained for the spectrum of the emitted light at P:

$$S_V(P,\omega)=\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}\right)S_Q(\omega)+\left[W_Q(P_1,P_2,\omega)\frac{e^{ik(R_2-R_1)}}{R_1R_2}+\text{c.c.}\right] \qquad (4)$$

Here

$$W_Q(P_1,P_2,\omega)=\langle Q^*(P_1,\omega)Q(P_2,\omega)\rangle \qquad (5)$$

is the so-called cross-spectral density of the source fluctuations and c.c. denotes the complex conjugate.

The formula (4) shows that the spectrum $S_V(P,\omega)$ is, in general, not just proportional to $S_Q(\omega)$ but is modified by the correlation, characterized by $W_Q(P_1,P_2,\omega)$, between the fluctuations of the two source strengths $Q(P_1,\omega)$ and $Q(P_2,\omega)$. Only in some very special cases, for example, when the source fluctuations are uncorrelated $[W_Q(P_1,P_2,\omega)=0]$ will $S_V(P,\omega)$ be proportional to $S_Q(\omega)$. Hence, in general, the spectrum of the light generated by two point sources depends not only on their spectra but also on the correlation between the fluctuations of their strengths.

A generalization of the elementary formula (4) for radiation from three-dimensional steady-state (that is, statistically stationary) sources of any state of coherence is known[7]. Of special interest in the present context is the form that the formula takes when the source has the same normalized spectrum $s_Q(\omega)$, $(\int_0^\infty s_Q(\omega)\,d\omega=1)$ at each point in the source region and has a degree of spectral coherence[3] (appropriately normalized cross-spectral density) $\mu_Q(\mathbf{r}_1,\mathbf{r}_2,\omega)$ that depends on the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of any source points P_1 and P_2 only through their difference $\mathbf{r}_2-\mathbf{r}_1$. If, in addition, for each frequency that significantly contributes to the source spectrum, the spectral correlation length [the effective spatial width $|\Delta\mathbf{r}'|$ of $|\mu(\mathbf{r}',\omega)|$] is small compared to the linear dimensions of the source, the normalized spectrum $s_V^{(\infty)}(\mathbf{u},\omega)$ of the emitted light in the far zone, in a direction specified by a unit vector $\mathbf{u}$, becomes (see equation (3.11) of ref. 8)

$$s_V^{(\infty)}(\mathbf{u},\omega)=\frac{s_Q(\omega)\tilde{\mu}_Q(k\mathbf{u},\omega)}{\int s_Q(\omega)\tilde{\mu}_Q(k\mathbf{u},\omega)\,d\omega} \qquad (6)$$

where $\tilde{\mu}_Q(\mathbf{K},\omega)$ is the three-dimensional spatial Fourier transform of the degree of spectral coherence $\mu_Q(\mathbf{r}',\omega)\equiv\mu_Q(\mathbf{r}_2-\mathbf{r}_1,\omega)$.

Let us now choose as the normalized source spectrum $s_Q(\omega)$ a spectral line with a gaussian profile

$$s_Q(\omega)=\frac{1}{\delta\sqrt{2\pi}}\exp[-(\omega-\omega_0)^2/2\delta^2] \quad (\delta\ll\omega_0) \qquad (7)$$

and suppose that at each effective frequency ω, the source correlation decreases with the separation $|\mathbf{r}'|=|\mathbf{r}_2-\mathbf{r}_1|$ of any two source points in a gaussian manner, that is

$$\mu_Q(\mathbf{r}',\omega)=\exp[-r'^2/2\sigma_\mu^2(\omega)] \qquad (8)$$

On taking the Fourier transform of equation (8) and substituting the resulting expression into equation (6) we obtain the following expression for the normalized spectrum of the emitted light in the far zone (see equation (3.21) of ref. 8)

$$s_V^{(\infty)}(\omega)=\frac{s_Q(\omega)\sigma_\mu^3(\omega)\exp\{-\frac{1}{2}[k\sigma_\mu(\omega)]^2\}}{\int_0^\infty s_Q(\omega)\sigma_\mu^3(\omega)\exp\{-\frac{1}{2}[k\sigma_\mu(\omega)]^2\}\,d\omega} \qquad (9)$$

Here, $s_V^{(\infty)}(\omega)$ is written in place of $s_V^{(\infty)}(\mathbf{u},\omega)$, because the spectrum of the far field is now independent of $\mathbf{u}$, as a consequence of the assumed isotropy of μ_Q (see equation (8)).

The formula (9) shows that the spectrum of the emitted light in the far zone depends both on the spectrum of the source fluctuations and on the manner in which the effective source correlation length $\sigma_\mu(\omega)$ depends on the frequency ω.

Let us consider two particular cases. (1) Suppose first that $\sigma_\mu(\omega)$ is independent of ω. Letting ζ denote the (now constant) value of σ_μ and with $s_Q(\omega)$ given by equation (7), one can readily evaluate the integral in the denominator on the right of equation (9) and one then finds that

$$s_V^{(\infty)}(\omega)\approx\frac{\alpha}{\delta\sqrt{2\pi}}\exp\left[-\left(\omega-\frac{\omega_0}{\alpha^2}\right)^2\Big/2(\delta/\alpha)^2\right] \qquad (10)$$

where

$$\alpha^2=1+\left(\frac{\delta}{\Delta}\right)^2 \qquad (11a)$$

and

$$\frac{1}{\Delta}=\frac{\zeta}{c} \qquad (11b)$$

When the source is effectively spatially incoherent, $\zeta\to0$. Then according to equation (11) $\Delta\to\infty$ and $\alpha\to1$ and it follows from equations (10) and (7) that in this case

$$s_V^{(\infty)}(\omega)\to s_Q(\omega) \qquad (12)$$

Hence, in the limiting case of a completely incoherent source of the class that is considered here, the normalized spectrum of the emitted light in the far zone is identical with the normalized

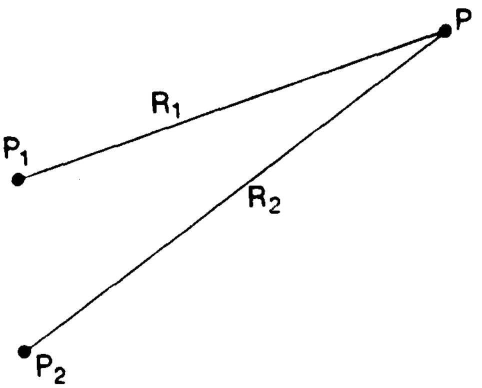

Fig. 1 Illustrating the notation relating to derivation of the formula (4).

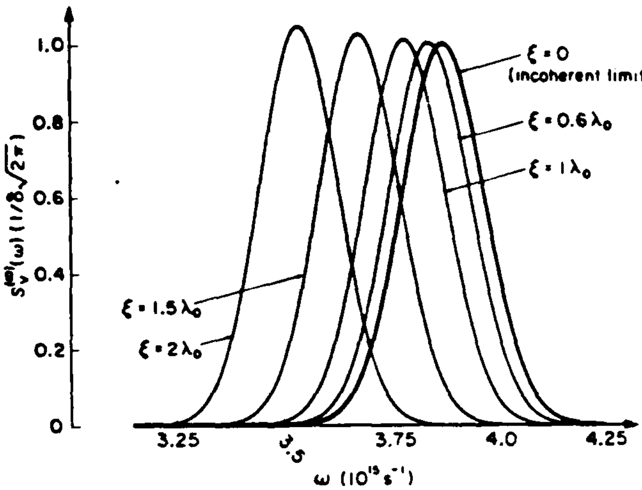

Fig. 2 Spectra $s_V^{(\infty)}(\omega)$ of the far field from sources with spectrum $s_Q(\omega) = (\delta\sqrt{2\pi})^{-1}\exp[-(\omega-\omega_0)^2/2\delta^2]$ and degree of spectral coherence $\mu_Q(r',\omega) = \exp(-r'^2/2\zeta^2)$, with $\omega_0 = 3.887\times10^{15}\,s^{-1}$ ($\lambda_0 = 4{,}861$ Å) and $\delta = 9.57\times10^{13}\,s^{-1}$, for several selected values of the effective source-correlation length ζ. The solid curve ($\zeta \to 0$) also represents the source spectrum $s_Q(\omega)$.

spectrum of the source fluctuations.

However, when the source fluctuations are correlated over an effective distance $\zeta > 0$, equation (10) shows that the spectrum $s_V^{(\infty)}(\omega)$, although it is also a line with a gaussian profile, is centred at a lower frequency $\omega_0' = \omega_0/\alpha^2 < \omega_0$. Hence the source correlations give rise to a spectral line $s_V^{(\infty)}(\omega)$ that is redshifted with respect to the spectral line produced by the completely spatially incoherent source with the source spectrum $s_Q(\omega)$. The shifted line is narrower, having root-mean-square width $\delta' = \delta/\alpha < \delta$ and has α-times greater height. Examples of spectra of light in the far zone, produced by several sources which emit the same spectral line but which have different correlation lengths are shown in Fig. 2. From the formula (10) one can readily deduce that the relative shift of the line, namely,

$$z \equiv \frac{\lambda_0 - \lambda_0'}{\lambda_0} = -\frac{\omega_0 - \omega_0'}{\omega_0'} \tag{13}$$

($\lambda_0 = 2\pi c/\omega_0$, $\lambda_0' = 2\pi c/\omega_0'$) is given by

$$z = \left(\frac{\delta}{\Delta}\right)^2 = \left(\frac{\delta}{c}\right)^2 \zeta^2 \tag{14}$$

which shows that in this case the redshift increases quadratically with the spectral source-correlation length ζ. (2) Next consider the situation when $\sigma_\mu(\omega) = a/\omega$ where a is a positive constant. The expression (9) for the normalized spectrum of the emitted light in the far zone now reduces to

$$s_V^{(\infty)}(\omega) = \frac{s_Q(\omega)/\omega^3}{\int_0^\infty [s_Q(\omega)/\omega^3]\,d\omega} \tag{15}$$

Note that this expression is independent of the value of the constant a.

When $s_Q(\omega)$ is a line with a gaussian profile, given by equation (7), the spectrum $s_V^{(\infty)}(\omega)$, given by equation (15) is *no longer* strictly gaussian but it can be closely approximated by a gaussian and can be shown to be redshifted with respect to $s_Q(\omega)$ by the relative amount

$$z \approx 3\left(\frac{\delta}{\omega_0}\right)^2. \tag{16}$$

An example of this situation is illustrated in Fig. 3.

This case $[\sigma_\mu(\omega) = a/\omega]$ is of special interest because, according to equation (8), the degree of spectral coherence is now given by

$$\mu_Q(r',\omega) = \exp[-(kr')^2/2(a/c)^2], \tag{17}$$

that is, it has the functional form

$$\mu_Q(r',\omega) = f(kr') \quad (k = \omega/c = 2\pi/\lambda) \tag{18}$$

Thus the degree of spectral coherence of the source distribution now satisfies the three-dimensional analogue of a requirement (called the scaling law) derived in ref. 1, as a sufficient condition for the spectrum of the light emitted by a planar secondary source of a well-defined class to have certain invariance properties on propagation. It will be shown in another publication (J. T. Foley and E. Wolf, in preparation) that for three-dimensional primary sources of an analogous class, whose degree of coherence satisfies this law, the spectrum of the emitted light has similar invariance properties. We conjecture that the usual thermal sources obey such a scaling law.

Now briefly consider the question of a physical mechanism for producing source correlations. Such correlations must clearly be manifestations of some cooperative phenomena. At the atomic level possible candidates may perhaps be superradiance and superfluorescence[9]. An effect of this kind was first predicted by Dicke in 1954 when he showed[10] that under certain circumstances energy from excited atoms may be released cooperatively in a much shorter time than the natural lifetime of the excited states of the atoms and with much larger emission intensity than would be obtained were the atoms radiating independently.

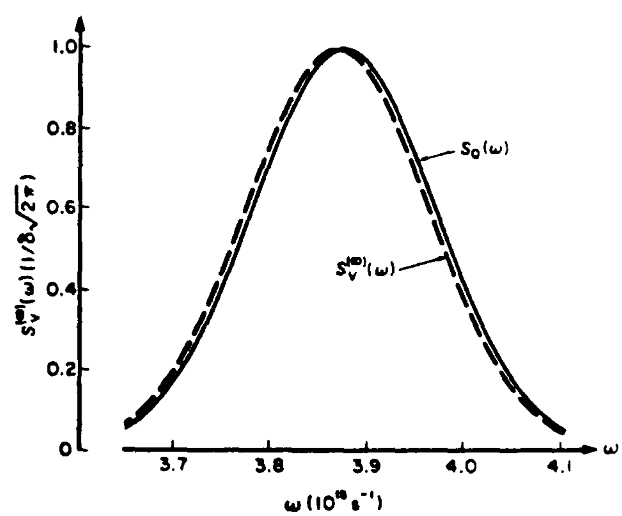

Fig. 3 The spectrum $s_V^{(\infty)}(\omega)$ of the far field from a source with source spectrum $s_Q(\omega) = (\delta\sqrt{2\pi})^{-1}\exp[-(\omega-\omega_0)^2/2\delta^2]$ and degree of spectral coherence $\mu_Q(r',\omega) = \exp[-(kr')^2/2(a/c)^2]$ (a = an arbitrary constant), with $\omega_0 = 3.887\times10^{15}\,s^{-1}$ ($\lambda_0 = 4{,}861$ Å) and $\delta = 9.57\times10^{13}\,s^{-1}$. The source spectrum $s_Q(\omega)$ is shown for comparison. Note that $\mu_Q(r',\omega)$ now obeys the scaling law.

However not enough is known at present about the coherence properties of large three-dimensional systems of this kind to make it possible to determine whether superradiance and superfluorescence might involve correlations that could give rise to spectral line shifts.

There is, however, quite a different mechanism, which can be described at the macroscopic level, and which can imitate effects of source correlations; namely effects of correlations between the refractive index at pairs of points in a spatially random but statistically homogeneous, time-invariant medium. If a wave illuminates such a medium, say a dilute gas, then, as is well known, the medium acts as a secondary source, namely as a set of oscillating charges set in motion by the incident wave. The secondary waves produced by the oscillating charges then combine with each other and with the incident wave and generate the scattered field. If the gas is not too dilute the collective response of the microscopic charges to the incident field can be described by macroscopic parameters such as the dielectric susceptibility or the refractive index. Now within the accuracy of the first Born approximation the basic equation for scattering is of the same form as the basic equation for radiation from primary sources, the 'equivalent source' for scattering being the product of the scattering potential (which is a simple function of the refractive index) and of the incident wave. This correspondence clearly implies that our results regarding the effects of source correlations on the spectrum of the emitted light must have analogues regarding the effects of a spatially random medium with correlated refractive index distribution on the spectrum of the light that is scattered by it. This topic will be discussed elsewhere.

Let us now consider some implications of this analysis. Using equation (14), the spectral line in Fig. 2, produced by the source whose correlation length $\zeta = \lambda_0$ is readily found to have a redshift given by $z = 0.0241$ with respect to the source spectrum. It is of interest to note that if an observer detected such a redshift unaware of its true origin and interpreted it on the basis of the Doppler shift formula $v/c = \Delta\lambda/\lambda_0 = z$ he would incorrectly conclude that the source was receeding from him with a speed $v = 0.0241c \approx 7{,}230\ \mathrm{km\ s^{-1}}$.

It seems worthwhile to note that there is a maximum line shift that can be produced by source correlations. This can be seen from the basic formula (6) which indicates that $s_V^{(\infty)}(\mathbf{u}, \omega) = 0$ when $s_Q(\omega) = 0$, implying that the spectrum of the far field can only contain those frequencies that are already present in the source spectrum. Consequently the maximum attainable frequency shift of the line cannot exceed its effective frequency range. However, any frequency contribution from the source spectrum to the normalized spectrum of the far field can be greatly magnified or greatly reduced, as is evident from equation (6) and from Fig. 2.

We have mainly considered effects of source correlations under circumstances when the source spectrum consists of a single line and when the degree of spectral coherence μ_Q that characterizes the source correlations depends on a single parameter. Preliminary calculations show that with a suitably chosen μ_Q which depends on a larger number of parameters, redshifts of several lines may be produced, all of which will have approximately the same z-values.

In this article we have considered redshifts of spectral lines. However, it is not difficult to specify source correlations which will produce blueshifts. Examples of this kind are given in a forthcoming publication[11].

It seems plausible that the mechanism discussed in this article may be responsible for some of the so far unexplained features of quasar spectra, including line asymmetries and small differences in the observed redshifts of different lines. In this connection it is of interest to recall that the role of coherence in the emission of radiation from quasars was stressed by Hoyle, Burbidge and Sargent in a well-known article[12].

I thank Mr A. Gamliel and Mr K. Kim for carrying out computations relating to the analysis presented in this article. The fact that scattering can also produce shifts of spectral lines was noted independently by Professor Franco Gori, who informed me of this result when commenting on an early version of the manuscript of this article. This investigation was supported by the NSF and by the US Air Force Geophysical Laboratory.

Received 10 September 1986, accepted 3 February 1987

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Red Shifts and Blue Shifts of Spectral Lines Emitted by Two Correlated Sources

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It has recently been shown theoretically that correlations between fluctuations of the source distribution at different source points can produce red shifts or blue shifts of emitted spectral lines. To facilitate experimental demonstration of this effect a simple example is analyzed. It involves only two small appropriately correlated sources and brings out the essential physical features of this new phenomenon.

PACS numbers: 42.68.Hf, 07.65.−b, 42.10.Mg

I showed not long ago that the spectrum of light produced by a fluctuating source depends not only on the source spectrum but also on the correlation that may exist between the source fluctuations at different points within the domain occupied by the source.[1] This result was recently confirmed experimentally.[2] I also showed that under certain circumstances source correlations may produce red shifts or blue shifts of spectral lines in the emitted radiation.[3,4] This prediction has obviously important implications, particularly for astronomy, and it is therefore desirable to verify it also by experiment.

In this Letter I analyze theoretically one of the simplest systems that will generate spectral shifts by this mechanism; namely, two small correlated sources, with identical spectra consisting of a single line of Gaussian profile. I show that with an appropriate choice of the correlation, the spectrum of the emitted radiation will also consist of a single line with a Gaussian profile; however, this emitted line will be red shifted or blue shifted with respect to the spectral line that would be produced if the sources were uncorrelated, the nature of the shift depending on the choice of one of the parameters that specifies the exact form of the correlation coefficient. The main features of this theoretical prediction have been confirmed by Bocko, Douglass, and Knox, using acoustical rather than optical sources. An account of their experiments is given in the accompanying Letter.[5]

Let us consider two small fluctuating sources located at points P_1 and P_2. I assume that the fluctuations are statistically stationary. Let $\{Q(P_1,\omega)\}$ and $\{Q(P_2,\omega)\}$ be the ensembles that represent the source fluctuations[6] at frequency ω. Furthermore, let $\{U(P,\omega)\}$ be the ensemble that represents the field at point P generated by the two sources (Fig. 1). Each realization $U(P,\omega)$ may then be expressed in the form[7]

$$U(P,\omega)=Q(P_1,\omega)\frac{e^{ikR_1}}{R_1}+Q(P_2,\omega)\frac{e^{ikR_2}}{R_2}, \tag{1}$$

where R_1 and R_2 are the distances from P_1 to P and from P_2 to P, respectively, and $k=\omega/c$, c being the speed of light in free space. The spectrum of the field at the point P is given by

$$S_U(P,\omega)=\langle U^*(P,\omega)U(P,\omega)\rangle, \tag{2}$$

where the angular brackets denote ensemble average. On substitution from Eq. (1) into Eq. (2), we find that

$$S_U(P,\omega)=(1/R_1^2+1/R_2^2)S_Q(\omega)+[W_Q(P_1,P_2,\omega)e^{ik(R_2-R_1)}/R_1R_2+\text{c.c.}]. \tag{3}$$

Here

$$S_Q(\omega)=\langle Q^*(P_1,\omega)Q(P_1,\omega)\rangle=\langle Q^*(P_2,\omega)Q(P_2,\omega)\rangle \tag{4}$$

is the spectrum (assumed to be the same) of each of the two source distributions,

$$W_Q(P_1,P_2,\omega)=\langle Q^*(P_1,\omega)Q(P_2,\omega)\rangle \tag{5}$$

is the cross-spectral density of the source fluctuations [first paper of Ref. 6, Eqs. (3.3) and (5.9)], and c.c. denotes the complex conjugate.

The degree of spectral coherence at frequency ω, which is a measure of correlation that may exist between the two fluctuating sources, is given by the formula[8]

$$\mu_Q(P_1,P_2,\omega)=W_Q(P_1,P_2,\omega)/S_Q(\omega). \tag{6}$$

The normalization in Eq. (6) ensures that $0\le|\mu_Q(P_1,P_2,\omega)|\le 1$. The extreme value $|\mu_Q|=1$ characterizes complete correlation (complete spatial coherence) at frequency ω. The other extreme value, $\mu=0$, characterizes complete absence of correlations (complete spatial incoherence).

On substituting for W_Q from Eq. (6) into Eq. (3), we find that

$$S_U(P,\omega)=S_Q(\omega)\{1/R_1^2+1/R_2^2+[\mu_Q(\omega)e^{ik(R_2-R_1)}/R_1R_2+\text{c.c.}]\}, \tag{7}$$

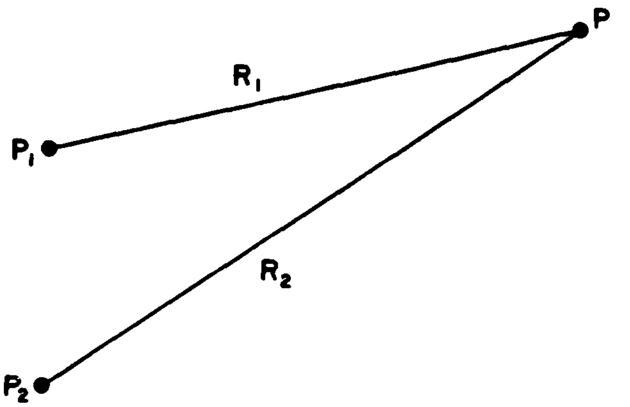

FIG. 1. Geometry and notation relating to the determination of the spectrum $S_U(P,\omega)$ of the field at P produced by two small sources with identical spectra $S_Q(\omega)$ located at P_1 and P_2.

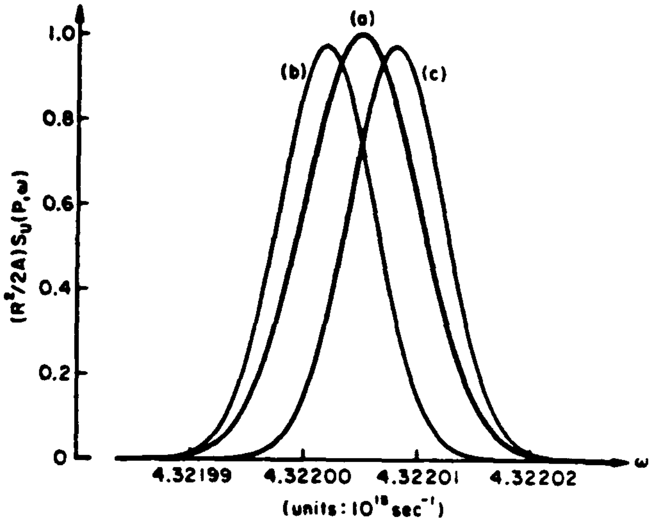

FIG. 2. Red shifts and blue shifts of spectral lines as predicted by formula (12). The spectrum $S_Q(\omega)$ of each of the two source distributions is a line with a Gaussian profile given by Eq. (9) with $A=1$, $\omega_0=4.32201\times10^{15}$ sec^{-1} (Hg line $\lambda=4358.33$ Å), $\delta_0=5\times10^9$ sec^{-1}. (a) The field spectrum $S_U(P,\omega)$ at P when the two sources are uncorrelated ($\mu_Q\equiv0$). (b),(c) The field spectra at P when the two sources are correlated in accordance with Eq. (10), with $a=1.8$, $\delta_1=7.5\times10^9$ sec^{-1}, and (b) $\omega_1=\omega_0-2\delta_0$ (red-shifted line), (c) $\omega_1=\omega_0+2\delta_0$ (blue-shifted line).

where I have omitted the arguments P_1 and P_2 in μ. For the sake of simplicity, let us choose the field point P to lie on the perpendicular bisector of the line joining P_1 and P_2. Then $R_1=R_2$ ($=R$, say) and formula (7) reduces to

$$S_U(P,\omega)=(2/R^2)S_Q(\omega)[1+\mathrm{Re}\,\mu_Q(\omega)], \tag{8}$$

where Re denotes the real part.

We note in passing that when either $\mu_Q(\omega)\equiv0$ (mutually completely uncorrelated sources) or when $\mu_Q(\omega)\equiv1$ (mutually completely correlated sources), the spectrum $S_U(P,\omega)$ of the field at the point P will be proportional to the spectrum $S_Q(\omega)$ of the source fluctuations. However, in general this will not be the case. In fact, it is clear from formula (8) that the field spectrum may differ drastically from the source spectrum, the difference depending on the behavior of the correlation coefficient $\mu_Q(\omega)$ as a function of frequency.

Suppose now that the spectrum of each of the two sources consists of a single line of the same Gaussian profile,

$$S_Q(\omega)=Ae^{-(\omega-\omega_0)^2/2\delta_0^2}, \tag{9}$$

where A, ω_0, and δ_0 ($\ll\omega_0$) are positive constants. Suppose further that the correlation between the two sources is characterized by the degree of spectral coherence

$$\mu_Q(\omega)=ae^{-(\omega-\omega_1)^2/2\delta_1^2}-1, \tag{10}$$

where a, ω, and δ ($\ll\omega_1$) are also positive constants. In order that expression (10) is a degree of spectral coherence, I must also demand that $a\leq2$. On substituting from Eqs. (9) and (10) into Eq. (8), I obtain the following expression for the spectrum of the field at the point P:

$$S_U(P,\omega)=\frac{2Aa}{R^2}e^{-(\omega-\omega_0)^2/2\delta_0^2}e^{-(\omega-\omega_1)^2/2\delta_1^2}. \tag{11}$$

By straightforward calculation one can show that this expression may be rewritten in the form

$$S_U(P,\omega)=A'e^{-(\omega-\omega_0')^2/2\delta_0'^2}, \tag{12}$$

where

$$A'=(2Aa/R^2)e^{-(\omega_1-\omega_0)^2/2(\delta_0^2+\delta_1^2)}, \tag{13}$$

$$\omega_0'=(\delta_1^2\omega_0+\delta_0^2\omega_1)/(\delta_0^2+\delta_1^2), \tag{14}$$

and

$$1/\delta_0'^2=1/\delta_0^2+1/\delta_1^2. \tag{15}$$

On the other hand, were the two sources uncorrelated, the correlation coefficient μ_Q would have zero value and we would then have, according to Eqs. (8) and (9),

$$[S_U(P,\omega)]_{\mathrm{uncorr}}=(2A/R^2)e^{-(\omega-\omega_0)^2/2\delta_0^2}. \tag{16}$$

Comparison of Eq. (12) with Eq. (16) shows that although both the spectral lines have Gaussian profiles, they differ from each other. Since according to Eq. (15) $\delta_0'<\delta_0$, the spectral line from the correlated sources is narrower than the spectral line from the uncorrelated sources. Further, we can readily deduce from Eq. (14) that

$$\omega_0'\lessgtr\omega_0$$

according as

$$\omega_1 \lessgtr \omega_0.$$

Hence if $\omega_1 < \omega_0$ the spectral line (12) produced by the correlated sources is centered on a lower frequency than the spectral line (16) from two uncorrelated sources, i.e., it is *red shifted* with respect to it; and if $\omega_1 > \omega_0$ the spectral line (12) is *blue shifted* with respect to the spectral line (16). Figure 2 illustrates these results by simple examples.

The preceding considerations show clearly the possibility of generating, by means of correlations between source fluctuations, either red shifts or blue shifts of lines in the spectrum of radiation emitted by sources that are stationary with respect to an observer.

I am obliged to Mr. A. Gamliel for carrying out the computations relating to Fig. 2. This research was supported by the U.S. National Science Foundation and by the U.S. Air Force Geophysics Laboratory under Air Force Office of Scientific Research Task No. 2310G1.

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Volume 65, number 2 OPTICS COMMUNICATIONS 15 January 1988

SPECTRAL MODULATION BY CONTROL OF SOURCE CORRELATIONS *

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Received 16 September 1987

It has recently been demonstrated both theoretically and experimentally that the spectrum of light emitted by a source can be modified by changing the correlation properties of the source. We examine the possibility of using this mechanism to modulate spectra in a desired manner. A simple configuration is considered, consisting of two small sources, emitting identical spectral lines and we show that by suitably correlating the sources one can control the width, the amplitude and the frequency of the emitted light. The possibility of generating several lines from a single one is also considered.

1. Introduction

It has been shown theoretically not long ago [1] that the spectrum of light emitted by a source depends not only on the source spectrum but also on the correlation properties of the source. This prediction has been verified by experiments [2a]. The spectral changes that may arise from source correlations may be such as to produce frequency shifts of spectral lines [3–5] either towards lower frequencies (red shifts) or towards higher frequencies (blue shifts). Such shifts have recently been demonstrated by experiments with optical [2b] and also with acoustical [6] sources.

In the present note we show that source-correlations can give rise to other interesting modifications of spectra. We consider only a simple arrangement, consisting of two small sources which generate radiation of identical spectra, and we analyze the effects of correlation between the two sources on the spectrum of the emitted radiation. We show that spectral lines can be frequency shifted, made narrower or broader and that several lines may be generated from a single line by this mechanism. These results suggest that it might be possible to develop a new technique for modulating spectra in a desired manner by control of source correlations.

2. Formulation of the problem

Consider light generated by two small fluctuating sources located at points P_1 and P_2. We assume that the fluctuations are statistically stationary. Let $\{Q(P_1, \omega)\}$ and $\{Q(P_2, \omega)\}$ be the ensembles that represent the source fluctuations at frequency ω. Further let $\{U(P, \omega)\}$ be the ensemble that represents the field at P, generated by the two sources (fig. 1). We assume that the spectra of the two source-distributions are identical and we denote them by $S_Q(\omega)$. More specifically

$$S_Q(\omega) = \langle Q^*(P_1, \omega)\, Q(P_1, \omega)\rangle = \langle Q^*(P_2, \omega)\, Q(P_2, \omega)\rangle,$$

where the angular brackets denote ensemble average.

* Research supported by the National Science Foundation and by the U.S. Army Research Office.
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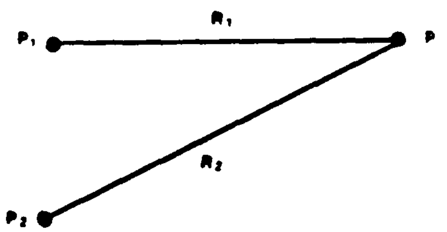

Fig. 1. Illustrating the configuration and notation.

It has been shown in ref. [5] that the spectrum of the field at a point P is given by

$$S_U(P,\omega)=S_Q(\omega)$$

$$\times\left\{\frac{1}{R_1^2}+\frac{1}{R_2^2}+2\mathscr{R}\left[\mu_Q(\omega)\frac{\exp[ik(R_2-R_1)]}{R_1R_2}\right]\right\}. \tag{2.1}$$

Here R_1 and R_2 denote the distances between P_1 and P and P_2 and P respectively; $\mathscr{R}$ denotes the real part and $\mu_Q(\omega)$, known as the degree of spectral coherence, characterizes the correlation between the two fluctuating sources. Explicitly,

$$\mu_Q(\omega)=[\langle Q^*(P_1,\omega)\,Q^*(P_2,\omega)\rangle]/S_Q(\omega). \tag{2.2}$$

The degree of spectral coherence satisfies the inequality

$$|\mu_Q(\omega)|\leqslant 1 \tag{2.3}$$

for all frequencies.

For simplicity we will only consider the spectrum of the emitted radiation at points located on the perpendicular bisector, which we will refer to as the axis, of the line joining the two sources. In this case $R_2=R_1(\equiv R$ say) and eq. (2.1) reduces to

$$S_U(P,\omega)=(2/R^2)\,S_Q(\omega)\,[1+\mathscr{R}\mu_Q(\omega)]. \tag{2.4}$$

We note that when $\mathscr{R}\mu_Q(\omega)$ is independent of ω, the spectrum of the field at all axial points will be proportional to the source spectrum $S_Q(\omega)$. This includes the case where the two sources are mutually incoherent at each frequency $[\mu_Q(\omega)\equiv 0]$. It also includes the case when $\mathscr{R}\mu_Q(\omega)\equiv 1$; the two sources are then mutually fully coherent at each frequency. These are, however, exceptional cases. In general, $\mathscr{R}\mu_Q(\omega)$ wil be frequency-dependent and eq. (2.4) shows that the field spectrum $S_U(\omega)$ will then be no longer proportional to the source spectrum $S_Q(\omega)$. Hence, *in general, not only the source spectrum but also the correlation between the two sources will determine the spectrum of the emitted light.*

Not long ago it was shown theoretically [3–5] that in some cases sources-correlations can produce shifts of spectral lines either towards lower frequencies (red shifts), or towards higher frequencies (blue shifts). This prediction has recently been verified experimentally with optical [2b] as well as acoustical [6] sources. In this note we will show that with suitable choices of the correlation other types of modulations of the spectrum of the emitted radiation may be produced.

3. A realizability condition

It will be convenient to set

$$S_U(\omega)\equiv(R^2/2)\,S_U(P,\omega). \tag{3.1}$$

We will refer to $S_U(\omega)$ as the *reduced* field spectrum. For the sake of simplicity we will consider source-correlations that are characterized by a real degree of coherence. Then eq. (2.4) becomes

$$S_U(\omega)=S_Q(\omega)\,[1+\mu_Q(\omega)]. \tag{3.2}$$

It follows at once from this formula, that in terms of S_U and S_Q,

$$\mu_Q(\omega)=[S_U(\omega)/S_Q(\omega)]-1. \tag{3.3}$$

From the inequality (2.3) and from eq. (3.3) it follows that only reduced field spectra $S_U(\omega)$ can be generated for which

$$S_U(\omega)\leqslant 2S_Q(\omega). \tag{3.4}$$

$S_U(\omega)$ and $S_Q(\omega)$ are, of course, necessarily nonnegative.

Conversely, when the inequality (3.4) is satisfied one finds at once from eq. (3.3) that

$$-1\leqslant\mu_Q(\omega)\leqslant 1. \tag{3.5}$$

Since the inequality (3.5) is the only constraint that the degree of spectral coherence $\mu_Q(\omega)$ must satisfy, we see from eq. (3.4) that *any reduced field spectrum $S_U(\omega)$ which does not exceed twice the magnitude of the source spectrum $S_Q(\omega)$ can, in principle, be generated by this mechanism.*

We will now consider a few examples.

4. Change of linewidth

4.1. Lorentzian line shapes

Let us assume first that the source spectrum consists of a spectral line of lorentzian profile, viz.,

$$S_Q(\omega) = A_0/[(\omega-\omega_0)^2+\Gamma_0^2] \tag{4.1}$$

(ω_0, Γ_0, A_0 are positive constants and $\Gamma_0 \ll \omega_0$). Suppose that we wish to produce a reduced field spectrum that consists also of a line of lorentzian profile centered on the same frequency ω_0, but is of a different width and of different strength, say

$$S_{\mathcal{U}}(\omega) = A_1/[(\omega-\omega_0)^2+\Gamma_1^2] \tag{4.2}$$

(Γ_1, A_1 are positive constants and $\Gamma_1 \ll \omega_0$). On substituting from eqs. (4.1) and (4.2) into the inequality (3.4) we find that we must have

$$(A_1/A_0)[f_0(\omega)]_{max} \leqslant 2\,, \tag{4.3}$$

where $[f_0(\omega)]_{max}$ is the maximum value, in the range $0<\omega<\infty$, of the function

$$f_0(\omega) = [(\omega-\omega_0)^2+\Gamma_0^2]/[(\omega-\omega_0)^2+\Gamma_1^2]\,. \tag{4.4}$$

Straightforward calculation shows that for all (positive) frequencies ω,

$$(\Gamma_0/\Gamma_1)^2 \leqslant f_0(\omega) < 1\,, \quad \text{if } \Gamma_1 > \Gamma_0\,, \tag{4.5a}$$

$$1 < f_0(\omega) \leqslant (\Gamma_0/\Gamma_1)^2\,, \quad \text{if } \Gamma_1 < \Gamma_0\,. \tag{4.5b}$$

Using these inequalities we deduce at once from (4.3) that we must have

$$A_1/A_0 < 2\,, \quad \text{when } \Gamma_1 > \Gamma_0\,, \tag{4.6a}$$

and

$$A_1/A_0 \leqslant 2(\Gamma_1/\Gamma_0)^2\,, \quad \text{when } \Gamma_1 < \Gamma_0\,. \tag{4.6b}$$

In the first case ($\Gamma_1 > \Gamma_0$) the emitted (reduced) spectral line is broader than the spectral line of the source; in the second case ($\Gamma_1 < \Gamma_0$) it is narrower.

With the conditions (4.6) assumed to be satisfied, the degree of spectral coherence that will produce the reduced field spectrum (4.2) from the source spectrum (4.1) is obtained at once on substituting from these equations into the formula (3.3). One then finds that the required degree of spectral coherence is given by

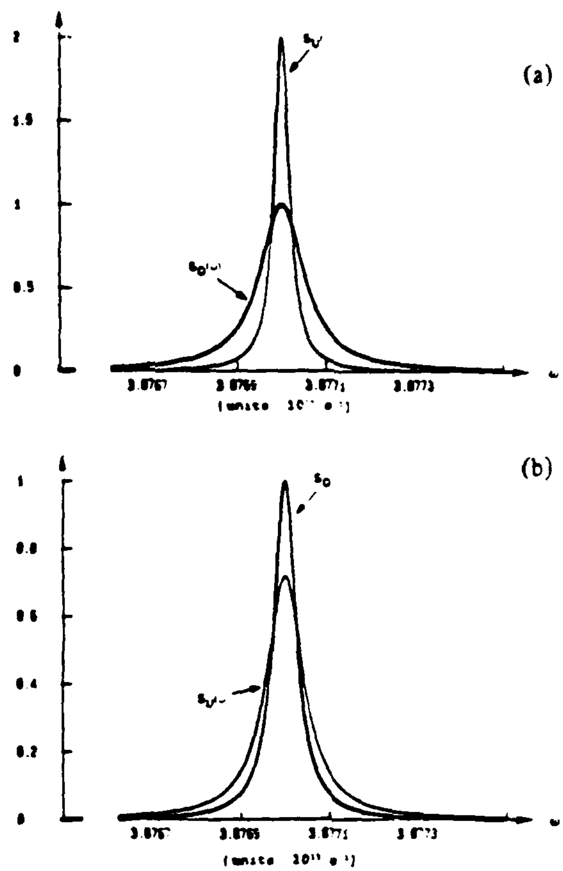

Fig. 2. Line narrowing and line broadening by source correlations. The source spectrum and the reduced field spectrum are lines of lorentzian profiles [eqs. (4.1) and (4.2)] with $\omega_0 = 3.877\times10^{15}\ s^{-1}$, $\Gamma_0 = 6\times10^{10}\ s^{-1}$, $\Gamma_1 = 2\times10^{10}\ s^{-1}$, $A_1/A_0 = 0.22$ (a), $\Gamma_0 = 3\times10^{10}\ s^{-1}$, $\Gamma_1 = 5\times10^{10}\ s^{-1}$, $A_1/A_0 = 2$ (b). The curves are normalized so that the source spectrum has the value unity at the center frequency ω_0.

$$\mu_Q(\omega) = \frac{A_1}{A_0}\,\frac{(\omega-\omega_0)^2+\Gamma_0^2}{(\omega-\omega_0)^2+\Gamma_1^2} - 1\,. \tag{4.7}$$

These results are illustrated in figs. 2.

4.2. Gaussian line shapes

Next let us consider a source spectrum that consists of a line of gaussian profile viz.,

$$S_Q(\omega) = A_0 \exp[-(\omega-\omega_0)^2/2\delta_0^2] \tag{4.8}$$

(ω_0, δ_0, A_0 are positive constants and $\delta_0 \ll \omega_0$). We examine the possibility of producing a reduced field spectrum that also consists of a line of gaussian profile, centered at the same frequency ω_0 but of different width and different strength, say

$$S_I(\omega) = A_1 \exp[-(\omega-\omega_0)^2/2\delta_1^2] \tag{4.9}$$

(δ_1, A_1 are positive constants and $\delta_1 \ll \omega_0$). On substituting from eqs. (4.8) and (4.9) into the inequality (3.4) we deduce at once that the following condition must be satisfied:

$$(A_1/A_0)\,[g(\omega)]_{max} \leqslant 2\,, \tag{4.10}$$

where $[g(\omega)]_{max}$ is the maximum value, in the range $0<\omega<\infty$, of the function

$$g(\omega) = \exp[-(\omega-\omega_0)^2/2\Delta]\,, \tag{4.11}$$

with

$$1/\Delta = (1/\delta_1^2) - (1/\delta_0^2)\,. \tag{4.12}$$

Consider first the case when $\delta_1 < \delta_0$ (line narrowing). Then $\Delta > 0$ and evidently $[g(\omega)]_{max} = g(\omega_0)) = 1$. Hence the realizability condition (4.10) becomes

$$A_1/A_0 \leqslant 2\,. \tag{4.13}$$

On the other hand when $\delta_1 > \delta_0$, Δ becomes negative and $g(\omega)$ has then no upper bound in the range $0<\omega<\infty$. Hence a broader line of gaussian profile centered at the same frequency ω_0 cannot be produced by this mechanism. However, in practice one is unlikely to be interested in situations where the spectra $S_Q(\omega)$ and $S_I(\omega)$ have gaussian forms for all frequencies. If one requires that the reduced field spectrum has gaussian shape only over a finite range around ω_0, say

$$\omega_0 - \alpha \leqslant \omega \leqslant \omega_0 + \beta\,, \tag{4.14}$$

where α and β are positive constants, the inequality (4.10) needs only to be satisfied when the maximum of $g(\omega)$ is taken over the restricted range (4.14). Instead of the inequality (4.10) we then have the constraint

$$(A_1/A_0)\exp(\gamma^2/2|\Delta|) \leqslant 2\,, \tag{4.15}$$

where γ is the largest of the constants α, β.

Let us return to the first case ($\delta_1 < \delta_0$). The degree of spectral coherence needed to achieve this modification of the spectral line is according to eqs. (3.3), (4.8) and (4.9) given by

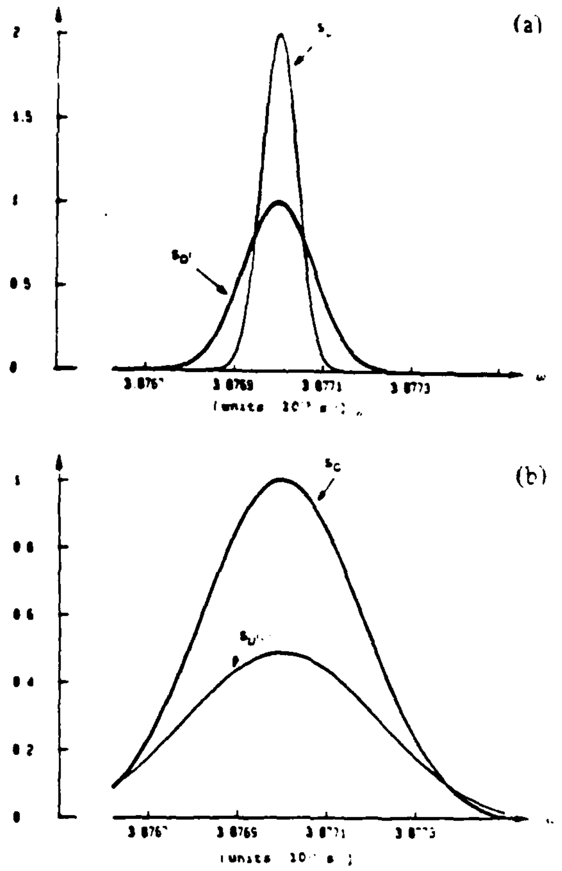

Fig. 3. Line narrowing and line broadening by source correlations. The source spectrum and the reduced field spectrum are lines with gaussian profiles [eqs. (4.8) and (4.9)] over the frequency range shown in the figures, with $\omega_0 = 3.877 \times 10^{15}\ s^{-1}$, $\delta_0 = 6\times10^{12}\ s^{-1}$; $\delta_1 = 2\times10^{12}\ s^{-1}$, $A_1/A_0 = 2$ (a), and $\delta_1 = 18\times10^{12}\ s^{-1}$, $A_1/A_0 = 0.5$ (b). The normalization is the same as in figs. 2.

$$\mu_Q(\omega) = (A_1/A_0)\exp[-(\omega-\omega_0)^2/2\Delta] - 1\,, \tag{4.16}$$

where Δ is defined by eq. (4.12). In the second case ($\delta_1 > \delta_0$) the degree of spectral coherence is given by eq. (4.16) only for frequencies that are within the range (4.14); for frequencies outside this range the degree of spectral coherence can take on arbitrary values, subject to the constraint expressed by eq. (3.5).

Some computed curves illustrating these results are shown in fig. 3.

5. Change of width of a lorentzian line accompanied by shift of the center frequency

Next let us consider the possibility of changing not only the width of the line but also its center frequency, from ω_0 to ω_1 say. Suppose that the source spectrum $S_Q(\omega)$ is again the single spectral line (4.1) of lorentzian profile, but that the reduced field spectrum, whilst also a line of lorentzian profile, is centered at a different frequency $\omega_1 \neq \omega_0$, i.e. that

$$S_U(\omega) = A_1/[(\omega-\omega_1)^2+\Gamma_1^2] \tag{5.1}$$

(ω_1, Γ_1, A_1 are positive constants, $\Gamma_1 \ll \omega_1$). On substituting from eqs. (4.1) and (5.1) into eq. (3.4) we deduce at once that the following inequality must now be satisfied:

$$(A_1/A_0)\,[f_1(\omega)]_{\max} \leq 2\,. \tag{5.2}$$

Here $[f_1(\omega)]_{\max}$ is the maximum value in the range $0<\omega<\infty$ of the function

$$f_1(\omega) = [(\omega-\omega_0)^2+\Gamma_0^2]/[(\omega-\omega_0)^2+\Gamma_1^2]\,. \tag{5.3}$$

Unlike in the case considered in sec. 4.1 (when $\omega_1=\omega_0$) an explicit expression for the maximum value of this function cannot readily be obtained. However it seems that with suitable choices of the constants that specify the reduced field spectrum (5.1) the inequality (5.2) can be satisfied for all non-negative frequencies ω. The degree of spectral coherence which gives rise to the reduced field spectrum (5.1) is then obtained on substituting from eqs. (5.1) and (4.1) into eq. (3.3). The resulting expression is

$$\mu_Q(\omega) = \frac{A_1}{A_0}\,\frac{(\omega-\omega_0)^2+\Gamma_0^2}{(\omega-\omega_1)^2+\Gamma_1^2} - 1\,. \tag{5.4}$$

An example of spectral changes in which both the line width and the center frequency are modified by source correlations of this type is presented in fig.4.

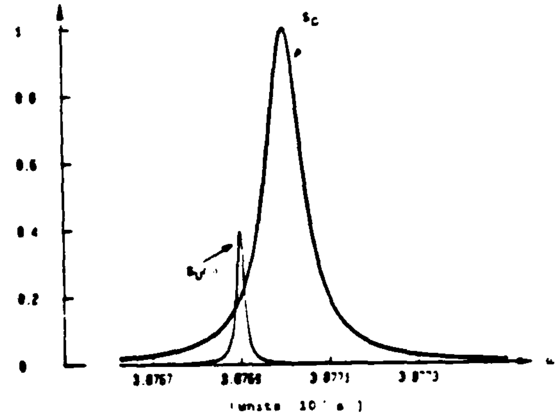

Fig. 4. Frequency shift by source correlations. The source spectrum and the field spectrum are lines with lorentzian profiles with $\omega_0 = 3.877\times10^{[illegible]}\,s^{-1}$, $\Gamma_0 = 5\times10^{[illegible]}\,s^{-1}$, $\Gamma_1 = 1\times10^{[illegible]}\,s^{-1}$, $A_1/A_0 = 1.588\times10^{-2}$. The normalization is the same as in figs 2.

6. Generation of several lines from a single line

As a last example we consider the possibility of generating from a source spectrum that consists of a single line of lorentzian profile a field spectrum that consists of several lines of lorentzian form. More specifically, with two sources that have identical spectra given by eq. (4.1) we wish to generate a field whose reduced spectrum has the form

$$S_U(\omega) = \sum_{n=1}^{N} \frac{A_n}{(\omega-\omega_n)^2+\Gamma_n^2}\,, \tag{6.1}$$

where N, A_n, ω_n, and Γ_n are positive constants and $\Gamma_n \ll \omega_n$ ($1 \leq n \leq N$). For this to be possible, the following condition obtained on substituting eqs. (6.1) and (4.1) into eq. (3.4), must be satisfied:

$$\sum_{n=1}^{N} (A_n/A_0)\,[f_n(\omega)]_{\max} \leq 2\,. \tag{6.2}$$

Here $[f_n(\omega)]_{\max}$ is the maximum value, in the range $0 \leq \omega < \infty$, of the function

$$f_n(\omega) = [(\omega-\omega_0)^2+\Gamma_0^2]/[(\omega-\omega_n)^2+\Gamma_n^2]\,. \tag{6.3}$$

Assuming that the constraint (6.2) is satisfied, the degree of spectral coherence needed for generating the reduced field spectrum (6.1) is obtained at once on substituting from eqs. (6.1) and (4.1) into eq. (3.3). One then finds that

$$\mu_Q(\omega) = \left(\sum_{n=1}^{N} \frac{A_n}{A_0}\,\frac{(\omega-\omega_0)^2+\Gamma_0^2}{(\omega-\omega_n)^2+\Gamma_n^2}\right) - 1\,. \tag{6.4}$$

An example of the generation of a field spectrum consisting of three lines from a source spectrum consisting of a single line is illustrated in fig. 5.

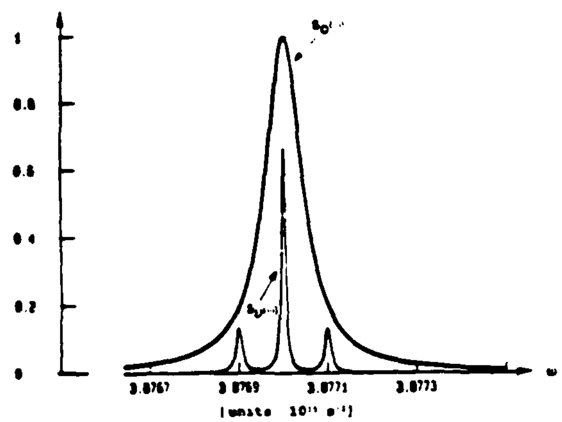

Fig. 5. Generation of three spectral lines from a single spectral line, all of lorentzian profiles, with $\omega_0 = 3.877\times10^{15}\,s^{-1}$, $\Gamma_0 = 5\times10^{11}\,s^{-1}$. The parameters that specify the three lines are $\omega_1 = 3.8769\times10^{15}\,s^{-1}$, $\Gamma_1 = 8\times10^{10}\,s^{-1}$, $A_1/A_0 = 3.4\times10^{-3}$; $\omega_2 = 3.8770\times10^{15}\,s^{-1}$, $\Gamma_2 = 5\times10^{10}\,s^{-1}$, $A_2/A_0 = 6.1\times10^{-3}$; $\omega_3 = 3.8771\times10^{15}\,s^{-1}$, $\Gamma_3 = 8\times10^{10}\,s^{-1}$, $A_3/A_0 = 3.4\times10^{-3}$. The normalization is the same as in figs. 2.

7. Discussion

We have considered in this note a very simple radiating system consisting of two small sources with identical spectra and we showed that by appropriately correlating the two sources, the spectrum of the emitted radiation can take on may different forms. With systems consisting of a larger number of radiating sources one could, of course, produce more diverse spectral changes than those considered in this note.

The basic question that we have not addressed is how to produce the prescribed correlations. Several methods for generating and modifying source correlations, at least for secondary sources, have been developed in recent years. They include the use of scattering by liquid crystals under the influence of an external d.c. field [7], the use of rotating ground glass plates [8] and of holographic filters [9], interaction of light with ultrasonic waves [10], imaging and lensless feedback systems [11] and use of achromatic Fourier transform lenses [2]. Unfortunately none of these techniques appear to be flexible enough to produce the kind of correlations that are needed to modulate spectra in the manner that we discussed in this note. However, the fact that controlled correlations have been produced, at least to a limited extent, seems to be promising. It might be possible to control correlations of sources of electromagnetic radiation more easily at lower frequencies, e.g. in the microwave region. Correlations between acoustical sources could probably also be controled more easily [6]. Such techniques, when utilized to generate spectral modulation, might perhaps find potential applications in communications systems.

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Reprinted from SPIE Vol. 976—Statistical Optics

New Method For Spectral Modulation

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ABSTRACT

A new method for modulating spectra by correlating source fluctuations is discussed. Potential advantages and limitations of this method are considered for both scalar and electromagnetic sources.

1. INTRODUCTION

Current methods for modulating spectra may be divided into two main types. The first is *active* modulation in which the spectrum is modified by controlling the gain or feedback of the system. The second type is *passive* modulation where the source emits a very broad spectrum and the modulation is accomplished by filtering out or attenuating specific spectral intervals.

In this paper we discuss a new method for spectral modulation that is based on controlling source correlations. We consider the effects of correlating the fluctuations of two small scalar sources. We review some of the possible spectral modifications and examine the advantages and limitations of this approach. Next we present an example in which the sources are two partially-correlated linear dipoles; it demonstrates correlation effects on the spectrum and on the radiation pattern.

2. SCALAR SOURCES

Consider two small scalar sources located at points P_1 and P_2 (see Fig. 1). We assume that the source fluctuations are characterized by stationary ensembles $\{Q(P_1,\omega)\}$ and $\{Q(P_2,\omega)\}$ that give rise to field fluctuations represented by an ensemble $\{V(P,\omega)\}$. We further assume that both sources have identical spectra, i.e. that

$$S_Q(\omega) = \left\langle Q^*(P_1,\omega)Q(P_1,\omega)\right\rangle = \left\langle Q^*(P_2,\omega)Q(P_2,\omega)\right\rangle , \tag{1}$$

where the asterisk denotes complex conjugation and the angular brackets denote ensemble averaging.

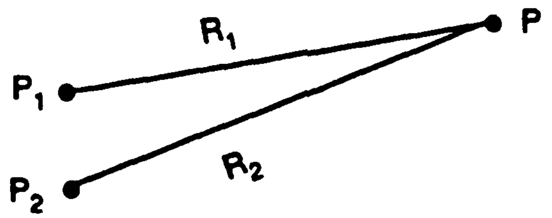

Fig. 1. Notation relating to the calculation of the field spectrum produced by two scalar sources.

In this notation the spectrum of the field at a point P is expressed by a similar formula

$$S_V(P,\omega) = \left\langle V^*(P,\omega)V(P,\omega)\right\rangle , \tag{2}$$

and the degree of correlation $\mu_Q(\omega)$ is given by[1]

$$\mu_Q(\omega) = \frac{\left\langle Q^*(P_1,\omega)Q(P_2,\omega)\right\rangle}{S_Q(\omega)} . \tag{3}$$

It has been shown that under the conditions stated, the spectrum of the field at any point P is given by[2, 3]

$$S_V(\omega) = S_Q(\omega)\left\{\frac{1}{R_1^2} + \frac{1}{R_2^2} + 2\Re\left[\mu_Q(\omega)\frac{e^{ik(R_2-R_1)}}{R_1R_2}\right]\right\} , \tag{4}$$

where $\Re$ denotes the real part and $k=\omega/c$ is the wavenumber associated with frequency ω.

In order to simplify the equations and illustrate their physical implications it is instructive to consider first the spectrum at observation points located on the axis (a perpendicular bisector to the line adjoining the two sources). At such points $R_2=R_1=R$ and the formula (4) for the field spectrum reduces to

$$S_V(P,\omega) = \frac{2}{R^2}S_Q(\omega)\left[1+\Re\mu_Q(\omega)\right] . \tag{5}$$

Since $-1 \le \Re\mu_Q(\omega) \le 1$, it is apparent from Eq. (5) that changes of the source correlations, represented by $\mu_Q(\omega)$, give rise to modulation of the field spectrum $S_V(\omega)$, subject to the constraint

$$0 \le S_V(\omega) \le \frac{4}{R^2}S_Q(\omega) . \tag{6}$$

It follows from this inequality that the field spectrum $S_V(\omega)$ must vanish at every frequency for which $S_Q(\omega)$ vanishes. In other words, *this mechanism does not produce new frequency components.*

Both limits in Eq. (6) occur when the sources are fully correlated. Other choices of source correlations can produce field spectra that are significantly different from their corresponding source spectra. To illustrate the effect of source correlations let $S_Q(\omega)$ consist of a line of Lorentzian shape, centered at frequency ω_0 and of width δ_0, i.e.,

$$S_Q(\omega) = \frac{1}{1+(\omega-\omega_0)^2/\delta_0^2} , \tag{7}$$

and with degree of correlation $\mu_Q(\omega)$ given by

$$\mu_Q(\omega) = \frac{A_1}{1+(\omega-\omega_1)^2/\delta_1^2} + \frac{A_2}{1+(\omega-\omega_2)^2/\delta_2^2} - 1 . \tag{8}$$

In Eqs. (7) and (8) A_1, A_2 are positive constants chosen so that $|\mu_Q(\omega)| \le 1$ throughout the frequency interval of interest. ω_0, ω_1, ω_2, δ_0, δ_1, δ_2 are positive constants of dimension sec^{-1}. For the sake of simplicity the dimensions are not indicated in Figs. 2, 4-6. The resulting field spectrum $S_V(\omega)$ at points on axis is obtained on substituting from Eqs. (7) and (8) into Eq. (5).

Examples of field spectra computed from these formulas are shown in Fig. 2. We see that while for fully correlated sources ($|\mu_Q(\omega)|= 1$) and uncorrelated sources ($|\mu_Q(\omega)|= 0$), the spectrum of the field is proportional to the spectrum of each source, appreciable modifications of the emitted spectrum are achieved with appropriate choice of the degree of correlation.

In general, given a source spectrum $S_Q(\omega)$ and a desired field spectrum $S_V(\omega)$ the appropriate choice of the degree of correlation is obtained from Eq. (5) as

$$\mu_Q(\omega) = \frac{R^2}{2}\frac{S_V(\omega)}{S_Q(\omega)} - 1 . \tag{9}$$

Of course only those field spectra $S_V(\omega)$ can be generated which are consistent with the constraint $|\mu_Q| \leq 1$. A limitation of this method is that there is an upper bound, for each frequency ω, for the strength of the emitted field spectrum. This upper bound is given by the spectrum that is produced when the two sources are fully correlated [cf. Eq. (6)].

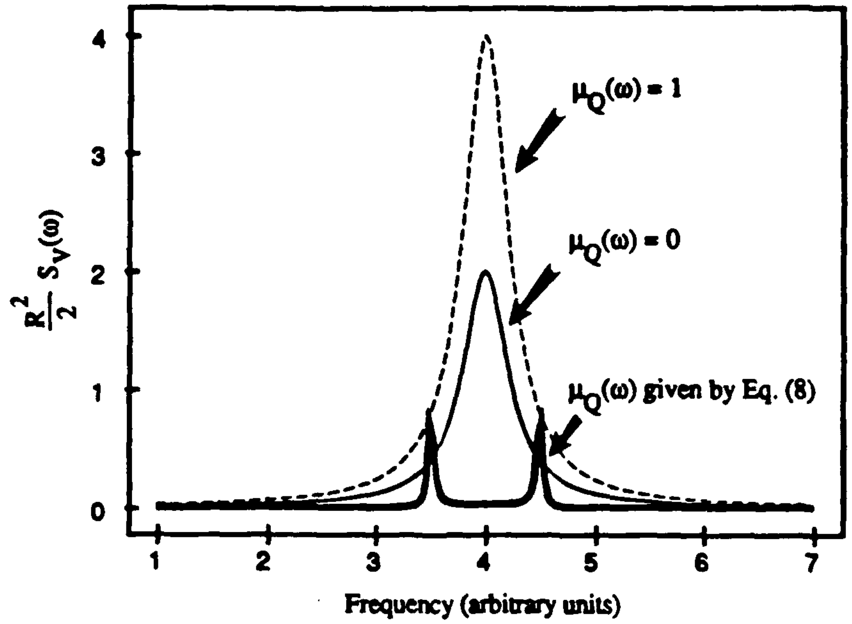

Fig. 2. Field spectra on axis for different types of source correlations, calculated from Eqs (5), (7) and (8). The constants were chosen as follows: $\omega_0 = 4$, $\delta_0 = 0.25$, $A_1 = 1.95$, $\omega_1 = 3.5$, $\delta_1 = 0.03$, $A_2 = 2$, $\omega_2 = 4.5$, and $\delta_2 = 0.03$.

Returning to Eq. (4) we note that the field spectrum also depends on the direction of observation because of the presence of R_1 and R_2. We postpone the discussion of the angular dependence and of complex-valued degree of correlation to the electromagnetic case that we will now consider.

3. LINEAR DIPOLES

Consider two linear dipoles located at distances y_0 from the origin and oscillating in the z direction as shown in Fig. 3. Let the electric polarization vectors of the dipoles be given by

$$\mathbf{P}_1(\mathbf{r},\omega) = p_1(\omega)\delta(\mathbf{r} - y_0\hat{\mathbf{y}})\,\hat{\mathbf{z}}, \qquad \mathbf{P}_2(\mathbf{r},\omega) = p_2(\omega)\delta(\mathbf{r} + y_0\hat{\mathbf{y}})\hat{\mathbf{z}}. \tag{10}$$

Here $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are unit vectors, δ is the Dirac delta function and $p_i(\omega)$, $(i=1,2)$ represent the polarization fluctuations as a function of frequency. If we assume that these fluctuations have the same spectrum, i.e. that

$$\left\langle |p_1(\omega)|^2 \right\rangle = \left\langle |p_2(\omega)|^2 \right\rangle \equiv S_Q(\omega) \ , \tag{11}$$

it can be shown that the spectrum of the emitted field in the far zone in the direction specified by a unit vector $\mathbf{u}$ is[4]

$$S(R\mathbf{u},\omega) = \frac{C}{R^2} k^4 \, S_Q(\omega) \sin^2\theta \, \left\{1 + \Re\left[\mu_Q(\omega)\exp(2iky_0 \sin\theta \, \sin\phi)\right]\right\} \ . \tag{12}$$

Here C is a constant depending on the choice of units and (θ,ϕ) are the spherical polar coordinates of the unit vector $\mathbf{u}$, with the polar axis taken along the z direction.

To simplify formula (12) we express the degree of correlation in the form

$$\mu_Q(\omega) = |\mu_Q(\omega)|\ e^{i2\psi(\omega)} , \tag{13}$$

where $\psi(\omega)$ is a real valued function[5] of ω. It follows that when the two dipoles are fully correlated $|\mu_Q(\omega)| = 1$ the spectrum of the field in the far-zone is given by

$$[S(Ru,\omega)]_{corr.} = 2\frac{C}{R^2}k^4\ S_Q(\omega)\ \sin^2\theta\ \cos^2\left[ky_0\sin\theta\ \sin\phi + \psi(\omega)\right] . \tag{14}$$

Similarly when the dipoles are uncorrelated $\mu_Q(\omega) = 0$ and the far-field spectrum, obtained from Eq. (12) is

$$[S(Ru,\omega)]_{uncorr.} = \frac{C}{R^2}k^4\ S_Q(\omega)\ \sin^2\theta . \tag{15}$$

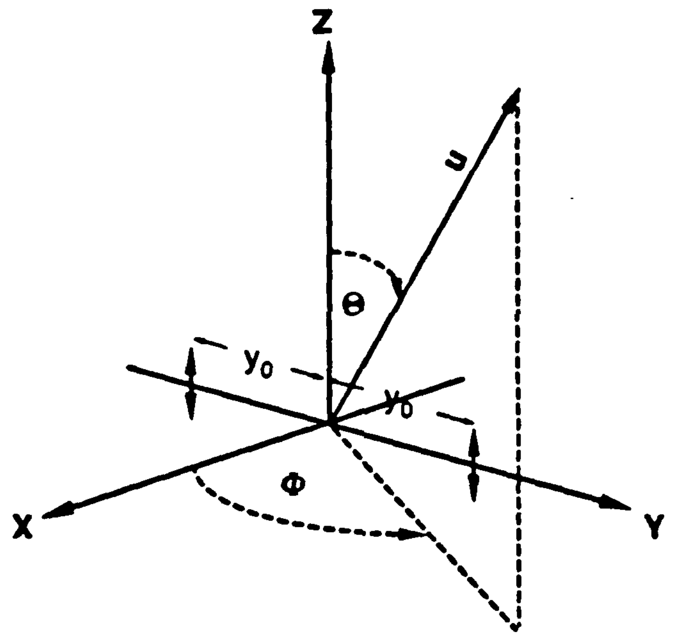

Fig. 3. Notation relating to the calculation of the far-field spectrum produced by two linear dipoles P_1 and P_2.

For every degree of correlation $\mu_Q(\omega) \neq 0$ the frequency-dependence of the far field spectrum is coupled to the angular variables through the product $ky_0\sin\theta\sin\phi$. To illustrate this remark consider observation points in the x, y-plane. The spectrum is then a function of frequency ω and of the azimuthal angle ϕ only, viz.,

$$S(R,\ \theta=\frac{\pi}{2},\ \phi;\omega) = \frac{C}{R^2}k^4\ S_Q(\omega)\left\{1 + \left|\mu_Q(\omega)\right|\ \cos\left[2ky_0\sin\phi + 2\psi(\omega)\right]\right\} . \tag{16}$$

Let us consider some implications of Eq. (16). Suppose the spectrum $S_Q(\omega)$ of each dipole consists of a single Lorentzian line [Eq.(7)]. We now compare the angular distribution of the far field spectrum when the degree of correlation is given by Eq. (8) and when the dipoles are fully correlated. Both cases are examined for fixed frequencies and for a fixed value of y_0. Figures 4 and 5 show the angular distributions of the spectra for two selected frequencies. It is to be noted that larger values of y_0 give rise to multiple lobes in the radiation pattern (see Fig. 6), but it can be shown that such systems will radiate relatively little power.

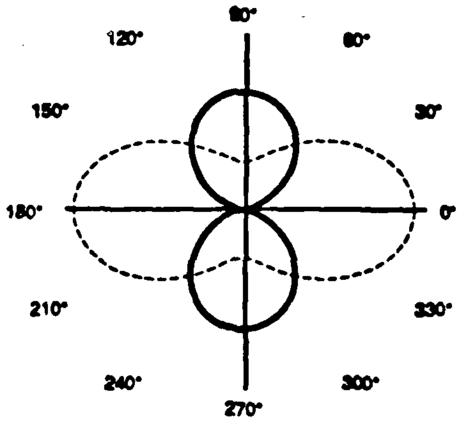

Fig. 4. Angular distribution of the far field spectra $S(R,\theta=\pi/2,\phi;\omega)$ of fully correlated sources (dashed line) and for μ_Q given by Eq. (8) (solid line). Both curves are for a fixed frequency $\omega = 4$ and $k_0y_0=1$.

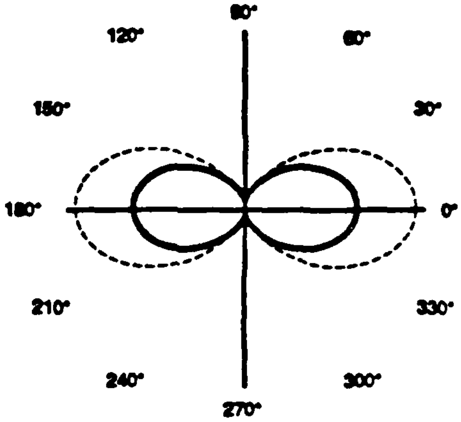

Fig. 5. Angular distribution of the far field spectra $S(R,\theta=\pi/2,\phi;\omega)$ of fully correlated sources (dashed line) and for μ_Q given by Eq. (8) (solid line). Both curves are for a fixed frequency $\omega = 3.515$ and $k_0y_0=1$.

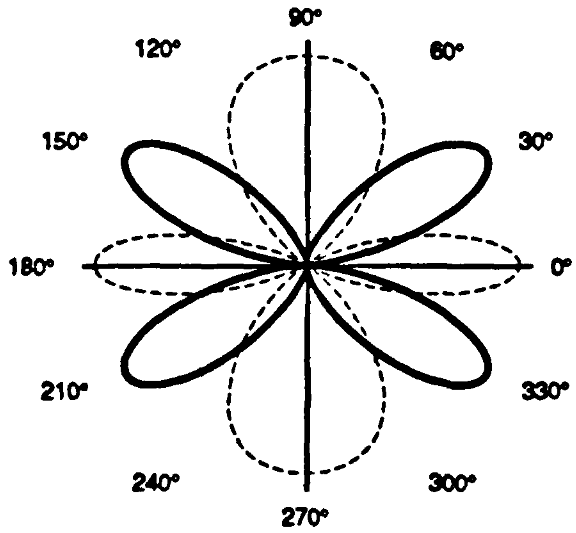

Fig. 6 Angular distribution of the far field spectra $S(R,\theta=\pi/2,\phi;\omega)$ of fully correlated sources (dashed line) and and for μ_Q given by Eq. (8) (solid line). Both curves are for a fixed frequency $\omega= 4$ and $k_0y_0=3$.

Another consequence of Eq. (16) is that when the observation points are located along the x-axis ($\sin\phi=0$) the far zone spectrum of the electromagnetic field exhibits the same frequency dependence as we found before for the scalar case [Eq. (5)]. For other directions of observation the far zone spectrum has, in general, a more complicated form due to the presence of the factor $\cos[2ky_0\sin\phi+2\psi(\omega)]$. With the same choices of the source spectrum and of the degree of correlation as before, we show in Figs. 7 and 8 the far field spectrum along the x-and the y axes.

On comparing Fig 7 with the corresponding figure 2 for the scalar case, we note that the curves in Fig. 7 show asymmetry not present in Fig. 2. This difference is due to the presence of the factor k^4 in Eq. (16) relating to the electromagnetic case.

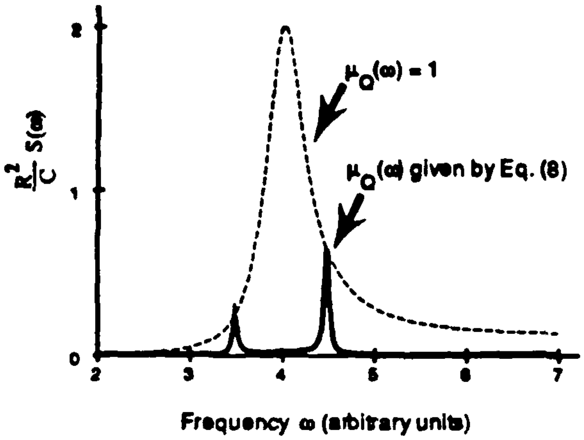

Fig. 7 Far zone field spectrum for points located on the x-axis.

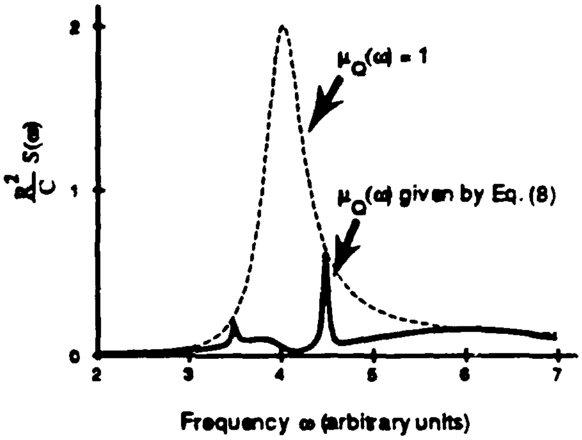

Fig. 8. Far zone field spectrum for points located on the y-axis.

DISCUSSION

We have shown in this note how spectral modulation can be achieved by controlling correlations between source fluctuations. Both scalar sources and electromagnetic sources were considered and they illustrate typical spectral features that may be produced by relatively simple choices of the functional forms of the correlation coefficient. The main limitation of this method is the requirement that all spectral components of the desired spectrum must be present in the source spectrum.

The practical difficulty of inducing the desired correlation in scalar or electromagnetic sources has been overcome in some special cases.[6-8] At present it seems that more complicated functional forms of the degree of correlation are easier to achieve with microwaves than in the optical region of the electromagnetic spectrum.

ACKNOWLEDGEMENTS

The author thanks Professor E. Wolf for assistance and guidance throughout this research. This research was supported in part by the New York State Science and Technology Foundation, by the National Science Foundation and by the Army Research Office.

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4.6

Radiated spectrum from two partially correlated dipoles

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Received November 21, 1988; accepted April 4, 1989

We examine the spatial directivity and the temporal spectrum of the field radiated by two partially correlated linear dipoles. The two dipoles are parallel and have identical spectra in their source excitation. Expressions are derived for the radiated spectrum, for the total emitted power, and for the directivity. We illustrate these results by calculating the radiated spectrum for a particular choice of the frequency-dependent correlation function. Results for the limiting cases of fully correlated dipoles and for uncorrelated dipoles are also derived.

INTRODUCTION

Recent advances in coherence theory include investigations of the effects of source correlations on the spectrum of the radiated field.[1-7] The type of source correlation that gives rise to line shifts in the observed spectrum was considered in a number of those studies. Some of the theoretical predictions have since been verified experimentally by several groups.[5-7] One of the possible applications of the present theory is a method for modulating field spectra that is based on the controlling of source correlations. The effect of spectral modulation was illustrated in an earlier publication in which a simple configuration of two scalar sources was analyzed for observation points that are located on the axis.[4]

In the present paper we are interested in spectral effects that are also associated with the direction of observation. In our treatment we consider a system of two partially correlated linear dipoles. First we derive expressions for the observed spectrum at an arbitrary point in the far zone; then we examine how the degree of correlation affects the spectrum of the field observed in particular directions and its effects on the angular distribution of the radiant intensity for fixed frequencies. In Sections 3 and 4 expressions are derived for the total radiated power and the directivity of the system as a function of the degree of correlation and the spatial separation between the two dipoles.

Throughout the analysis, we compare our results for the partially correlated dipoles with the results in the well-known limiting cases of fully correlated and uncorrelated dipoles. This comparison gives a valuable measure for the range of possible spatial and spectral modulation effects that can be produced by controlling source correlations.

1. FAR-ZONE SPECTRUM OF PARTIALLY CORRELATED DIPOLES

Consider two linear dipoles situated at points $\pm y_0$ and vibrating in the z direction as shown in Fig. 1. Let

$$\mathbf{P}_1(\mathbf{r}, t) = p_1(t)\delta(\mathbf{r} - y_0\hat{y})\hat{z}, \tag{1.1a}$$

$$\mathbf{P}_2(\mathbf{r}, t) = p_2(t)\delta(\mathbf{r} + y_0\hat{y})\hat{z} \tag{1.1b}$$

be the electric polarization vectors of the two dipoles. Here $p_j(t)$ $(j = 1, 2)$ characterize the polarization fluctuations of the dipoles as a function of time, and $\hat{y}$ and $\hat{z}$ are unit vectors in the positive y and z directions, respectively. We assume that $p_j(t)$ are random functions of time, characterized by stationary ensembles.

In the space–frequency representation,[8] the electric Hertz potential of the field produced by the dipoles is given by

$$\mathbf{\Pi}_e(\mathbf{r}, \omega) = \hat{z}\left[\tilde{p}_1(\omega)\frac{\exp(ikR_1)}{R_1} + \tilde{p}_2(\omega)\frac{\exp(ikR_2)}{R_2}\right], \tag{1.2}$$

in which $R_1 = |\mathbf{r} - y_0\hat{y}|$, $r_2 = |\mathbf{r} + y_0\hat{y}|$ and

$$\tilde{p}_j(\omega) = \int_{-\infty}^{\infty} p_j(t)e^{i\omega t}dt \qquad (j = 1, 2). \tag{1.3}$$

In order to calculate the radiant intensity in the far zone, it is sufficient to evaluate the magnetic field only; this is given by the expression

$$\mathbf{B}(\mathbf{r}, \omega) = -ik\nabla \times \mathbf{\Pi}_e(\mathbf{r}, \omega). \tag{1.4}$$

Since $\Pi_e(\mathbf{r}, \omega)$ is a vector along the z direction, it follows that

$$\nabla \times \mathbf{\Pi}_e(\mathbf{r}, \omega) = \left(\frac{\partial}{\partial y}\hat{x} - \frac{\partial}{\partial x}\hat{y}\right)\Pi_e(\mathbf{r}, \omega). \tag{1.5}$$

On substituting Eq. (1.2) into Eq. (1.5), we obtain the formula

$$\nabla \times \mathbf{\Pi}_e(\mathbf{r}, \omega) = \tilde{p}_1(\omega)\frac{\exp(ikR_1)}{R_1}\left(ik - \frac{1}{R_1}\right)\left[\frac{(y - y_0)}{R_1}\hat{x} - \frac{x}{R_1}\hat{y}\right] + \tilde{p}_2(\omega)\frac{\exp(ikR_2)}{R_2}\left(ik - \frac{1}{R_2}\right)\left[\frac{(y + y_0)}{R_2}\hat{x} - \frac{x}{R_2}\hat{y}\right]. \tag{1.6}$$

For field points $\mathbf{r} = r\hat{u}$ in the far zone $kR_i \gg 1$ $(i = 1, 2)$, we have

$$R_1 \sim r - y_0\hat{u}\cdot y, \qquad R_2 \sim r + y_0\hat{u}\cdot\hat{y}, \tag{1.7}$$

in which $\hat{u}$ is a unit vector for the polar radius $\mathbf{r}$.

Using Eq. (1.6) and expressions (1.7), we may express the far zone B field in the form

$$\mathbf{B}(r\hat{u}, \omega) \sim -k^2\frac{e^{ikr}}{r}\sin\theta[\tilde{p}_1(\omega)\exp(-iky_0\hat{u}\cdot\hat{y}) + \tilde{p}_2(\omega)\exp(iky_0\hat{u}\cdot\hat{y})]\hat{\Phi} \qquad \text{as } kr \to \infty. \tag{1.8}$$

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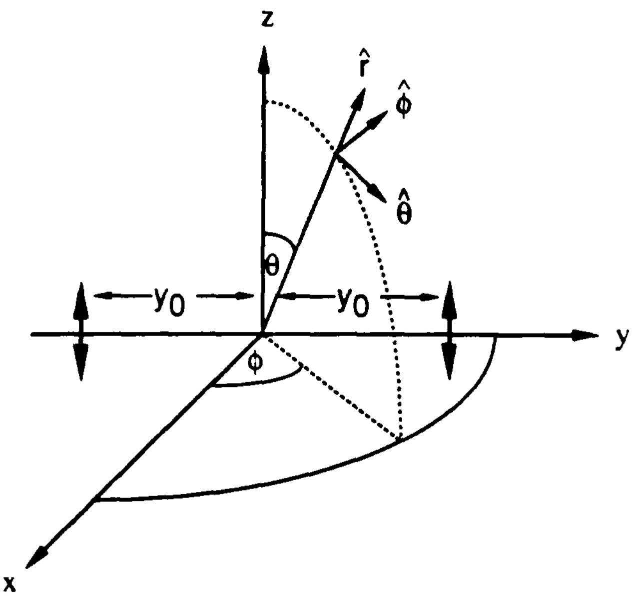

Fig. 1. Illustration of the configuration and notation used in this study.

Here, we have used the spherical polar coordinates (r, θ, ϕ) with the polar axis along the z direction and with $\hat{\Phi} = \sin\theta(\cos\phi\hat{y} - \sin\phi\hat{x})$.

The radiant intensity $J(\hat{u}, \omega)$, i.e., the power per unit frequency at frequency ω, per unit solid angle centered about direction specified by the unit vector $\hat{u}$, is given by

$$J(\hat{u}, \omega) = \lim_{kr\to\infty} r^2\hat{u}\cdot\langle \mathbf{S}^{(\infty)}(r\hat{u}, \omega)\rangle, \tag{1.9}$$

where $\mathbf{S}^{(\infty)}(r\hat{u}, \omega)$ is the Poynting vector in the far zone and the angle brackets denote the ensemble average. In terms of the B field, we have

$$J(\hat{u}, \omega) = \lim_{kr\to\infty} r^2 \frac{c}{8\pi}\mathcal{R}\langle \hat{u}\cdot[\mathbf{B}(r\hat{u}, \omega)\times\hat{u}]^* \times \mathbf{B}(r\hat{u}, \omega)\rangle, \tag{1.10}$$

where $\mathcal{R}$ denotes the real part. Using the identity

$$\hat{u}\cdot(\mathbf{B}\times\hat{u})^*\times\mathbf{B} = |\mathbf{B}\times\hat{u}|^2, \tag{1.11}$$

we simplify Eq. (1.10) for the radiant intensity to the form

$$J(\hat{u}, \omega) = \lim_{kr\to\infty} r^2\frac{c}{8\pi}\langle|\mathbf{B}(r\hat{u}, \omega)|^2\rangle. \tag{1.12}$$

Let us now assume that both dipoles have the same spectrum, i.e., that

$$\langle|p_1(\omega)|^2\rangle = \langle|p^2(\omega)|^2 \equiv S_p(\omega). \tag{1.13}$$

We also introduce the complex degree of spatial coherence at frequency ω that characterizes the correlation between the two dipoles by the formula[9]

$$\mu_p(\omega) = \frac{\langle p_1^*(\omega)p_2(\omega)\rangle}{S_p(\omega)}. \tag{1.14}$$

On substituting from expression (1.8) and Eqs. (1.13) and (1.14) into Eq. (1.12), we obtain the following expression for the radiant intensity produced by the two partially correlated dipoles:

$$J(\hat{u}, \omega) = \frac{ck^4}{4\pi}S_p(\omega)\sin^2\theta \times \{1 + \mathcal{R}[\mu_p(\omega)\exp(2iky_0\sin\theta\sin\phi)]\}. \tag{1.15}$$

If we express the degree of correlation in the form

$$\mu_p(\omega) = |\mu_p(\omega)|\exp[i2\psi(\omega)], \tag{1.16}$$

in which the phase $2\psi(\omega)$ corresponds to the steering angle in the fully coherent case,[10] we find that the radiant intensity of the two partially correlated linear dipoles is given by

$$J(\hat{u}, \omega) = \frac{ck^4}{4\pi}S_p(\omega)\sin^2\theta\{1 + |\mu_p(\omega)| \times \cos[2ky_0\sin\theta\sin\phi + 2\psi(\omega)]\}. \tag{1.17}$$

2. EFFECTS OF SOURCE CORRELATION ON THE RADIATED SPECTRUM

We now examine some special cases that help to illustrate Eq. (1.17). First let us consider two uncorrelated dipoles. In this case $\mu_p(\omega) \equiv 0$, and Eq. (1.17) reduces to

$$[J(\hat{u}, \omega)]_{uncorr} = \frac{ck^4}{4\pi}S_p(\omega)\sin^2\theta. \tag{2.1}$$

As one might expect, the same expression is obtained if the radiating spectrum originated from a single dipole located at the origin,[11] whose strength equals the sum of the strength of the two uncorrelated dipoles. Similarly, when the dipoles are fully correlated, i.e., when $|\mu_p(\omega)| = 1$, Eq. (1.17) for the radiant intensity reduces to

$$[J(\hat{u}, \omega)]_{corr} = \frac{ck^4}{2\pi}S_p(\omega)\sin^2\theta\cos^2[ky_0\sin\theta\sin\phi + \psi(\omega)]. \tag{2.2}$$

We observe that for every frequency ω, the phase angle $\psi(\omega)$ and the product ky_0 completely determine the angular distribution of the radiated power.

Returning to the general case in which the two dipoles are partially correlated, we note that when the point of observation is on the x axis ($\theta = \pi/2$, $\phi = 0$), the radiant intensity according to Eq. (1.17) is given by

$$J(\hat{x}, \omega) = \frac{ck^4}{4\pi}S_p(\omega)\{1 + \mathcal{R}[\mu_p(\omega)]\}. \tag{2.3}$$

The result for this special case is in the same form as the corresponding expression for the radiant intensity from two small partially correlated scalar sources [cf. Ref. 4, Eq. (2.4)]. The only significant difference is in the factor k^4 appearing in Eq. (2.3), in which the product $k^4S_p(\omega)$ is shifted to higher frequencies relative to $S_p(\omega)$.

The radiation pattern produced by the partially correlated dipoles differs from those produced in the two limiting cases of fully correlated and completely uncorrelated dipoles in several ways. If we denote the direction for which the radiant intensity is a maximum by $\hat{u}_m$, it can readily be shown that the maximum possible radiant intensity $[J(\hat{u}_m,$

$\omega)]_{max}$ from the two dipoles is obtained when they are fully correlated, and it is given by

$$[J(\hat{u}_m, \omega)]_{max} = \frac{ck^4}{2\pi} S_p(\omega). \tag{2.4}$$

On the other hand, when the dipoles are partially correlated the maximum radiant intensity is smaller by a factor $\frac{1}{2}[1 + |\mu_p(\omega)|]$.

In the two limiting cases of fully correlated dipoles and uncorrelated dipoles, the nulls of the radiant intensity distribution are determined by the factor $\sin^2\theta$ in Eq. (2.1) and by the factor $\sin^2\theta \cos^2[ky_0 \sin\theta \sin\phi + \psi(\omega)]$ in Eq. (2.2). By contrast, it follows from Eq. (1.17) that when the dipoles are partially correlated ($0 < |\mu_p(\omega)| < 1$) there are no nulls of the radiant intensity outside the plane $\theta = 0$. This fact is significant in connection with the theorems regarding the approximation of desired radiation patterns by arrays of such sources.[12]

For a numerical illustration, let the dipole spectrum be a Lorentzian line of width δ_0 centered at frequency ω_0, i.e.,

$$S_p(\omega) = \frac{1}{1 + (\omega - \omega_0)^2/\delta_0^2}. \tag{2.5}$$

We choose a real-valued degree of correlation given by

$$\mu_p(\omega) = A_1 \exp\left[-\frac{(\omega - \omega_1)^2}{2\delta_1^2}\right] + A_2 \exp\left[-\frac{(\omega - \omega_2)^2}{2\delta_2^2}\right] - 1. \tag{2.6}$$

Here ω_1, ω_2, δ_1, and δ_2 are positive constants, and we select real-valued A_1 and A_2 so that $\mu_p(\omega)$ satisfies the constraint

$$|\mu_p(\omega)| \le 1 \tag{2.7}$$

throughout the frequency range of interest. We note that, if we choose a real-valued degree of correlation, the steering angle is set to zero [i.e., $2\psi(\omega) \equiv 0$] in the following examples.

Figures 2 and 3 show the angular distributions of the radiant intensity in the (x, y) plane ($\theta = \pi/2$) for two fully correlated dipoles and for partially correlated dipoles whose degree of correlation is given by Eq. (2.6). In Fig. 2 the angular distribution is calculated for the center frequency $\omega = \omega_0$ of the polarization spectrum; Fig. 3 shows the angular distribution at a different frequency. When the separation between the dipoles increases, the lobe structure becomes more complicated. In Figs. 4 and 5 we show the angular distributions of the radiant intensity for dipole separation $k_0y_0 = 3$. Comparing the angular distributions for dipole separation $k_0y_0 = 1$ (Figs. 2 and 3) and $k_0y_0 = 3$ (Figs. 4 and 5), we note that at each frequency the number of lobes of the two choices of correlations is identical, although their angular distributions are somewhat different.

At a fixed direction in space, we can compare the spectrum produced by the partially correlated dipoles with the spectrum produced by fully correlated dipoles. In Fig. 6 we show the spectra for two choices of correlation when the observation is along the x-direction. Although it is not shown in the figure, we remark that the spectrum produced by the fully correlated dipoles differs from that of the uncorrelated dipoles by a factor of 2, whereas the spectrum produced by the partially correlated dipoles displays features that are not present in the polarization spectra. We also point out that the spectrum of the field differs from the Lorentzian line shape of the polarization spectrum because of the factor of k^4 in Eq. (1.17). A similar comparison of spectra for observation points along the y direction is shown in Fig. 7. Here we note that the spectrum produced by fully correlated dipoles has a lower magntiude than the spectrum produced by the

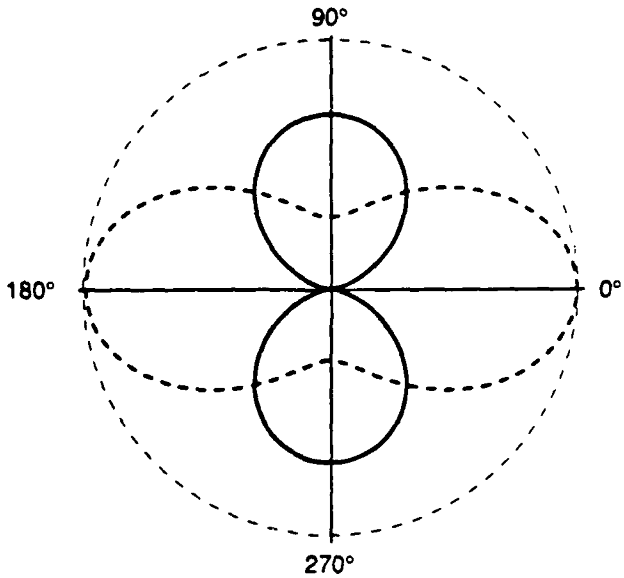

Fig. 2. Angular distributions of the radiant intensity at an offset frequency $\omega = 4\ \mathrm{sec}^{-1}$ and a dipole separation $k_0y_0 = 1$ when the two dipoles are partially correlated (solid curves) and fully correlated (dashed curves). The constants used in the calculations are (in reciprocal seconds) $\omega_0 = 4$, $\omega_1 = 3.5$, $\omega_2 = 4.5$, $\delta_0 = 0.25$, and $\delta_1 = \delta_2 = 0.03$. These simple numerical constants can be scaled to any frequency band that is being considered.

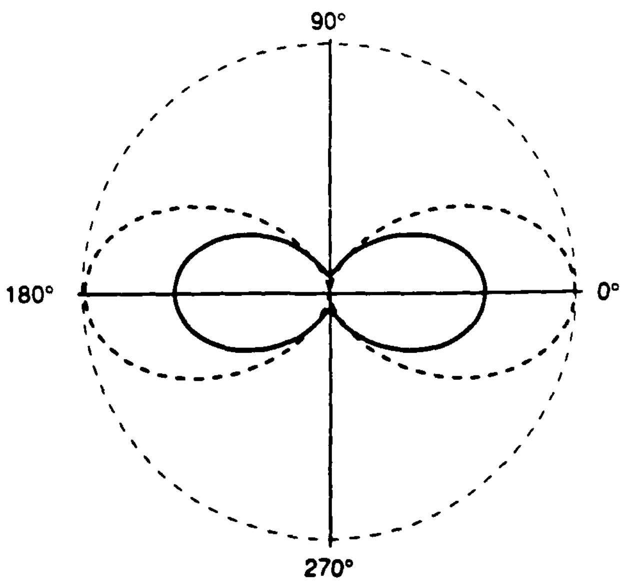

Fig. 3. Angular distributions of the radiant intensity at an offset frequency $\omega = 3.52\ \mathrm{sec}^{-1}$ and a dipole separation $k_0y_0 = 1$ when the two dipoles are partially correlated (solid curves) and fully correlated (dashed curves). The constants used in the calculation are the same as in Fig.2.

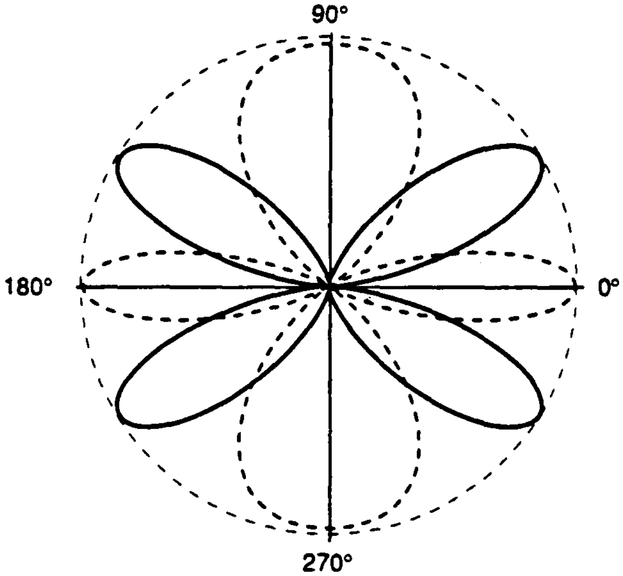

Fig. 4. Angular distribution of the radiant intensity at an offset frequency $\omega = 4\ \mathrm{sec}^{-1}$ and a dipole separation $k_0 y_0 = 3$.

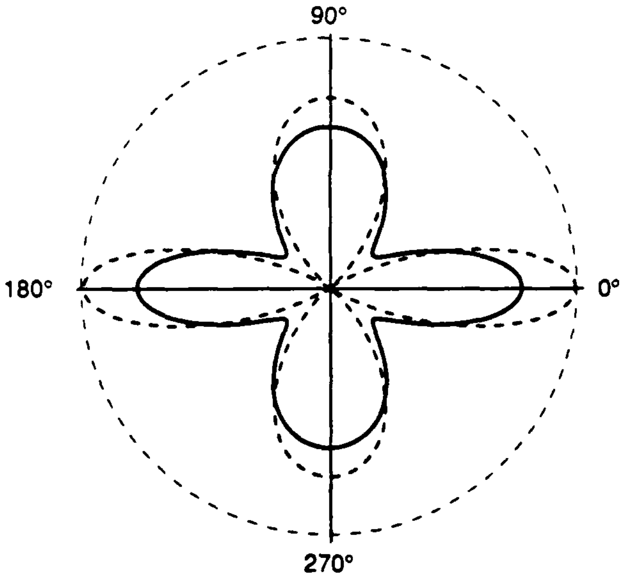

Fig. 5. Angular distribution of the radiant intensity at an offset frequency $\omega = 3.52\ \mathrm{sec}^{-1}$ and a dipole separation $k_0 y_0 = 3$.

partially correlated dipoles. This fact may be explained by Fig. 2, for example, in which the radiant intensity in the y direction is higher for the partially correlated dipoles. Figures 6 and 7 demonstrate that the spectral features of the partially correlated dipoles vary considerably, depending on the direction of observation, whereas the spectrum produced by the fully correlated dipoles remains essentially unchanged.

The two limiting cases of fully correlated and uncorrelated dipoles constitute the boundaries of possible modification of spectra. In Fig. 8 we illustrate the range of modulation that can be achieved at every frequency ω, by the variation of the magnitude of the degree of correlation $|\mu_p(\omega)|$ and the phase $2\psi(\omega)$. The figure shows four concentric circles, (i)–(iv), representing the relative ranges of the angular distributions of the radiant intensity in the (x, y) plane. The external circle (i) corresponds to the limit of fully correlated dipoles. As one may observe from Eq. (2.2), the radiant intensity of two correlated dipoles may attain any value inside the circle of radius $ck^4 S_p(\omega)/2\pi$. The location of the peak is controlled by the choice of the steering angle $2\psi(\omega)$. Similarly, it follows from Eq. (2.1) that, when the dipoles are uncorrelated, the radiant intensity is constrained to values on a circle (iii) of radius $ck^4 S_p(\omega)/4\pi$. When the dipoles are partially correlated, we see from Eq. (1.17) that the radiant intensity can have any value in an annular domain bounded by circles (ii) and (iv). It is also apparent from this representation that

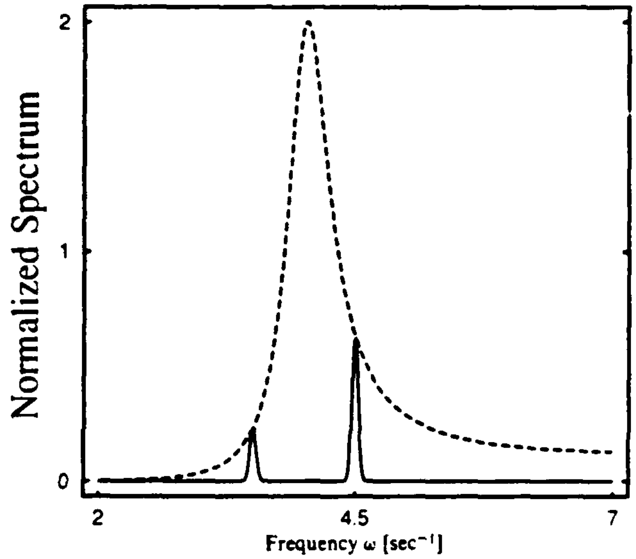

Fig. 6. Comparison of spectra along the x direction when the two dipoles are partially correlated (solid curves) and fully correlated (dashed curves). The constants used in the calculation are the same as in Fig.2.

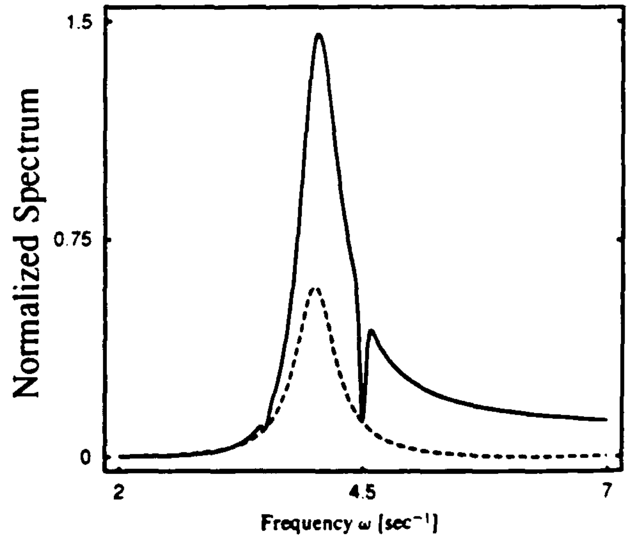

Fig. 7. Comparison of spectra along the y direction when the two dipoles are partially correlated (solid curves) and fully correlated (dashed curves).

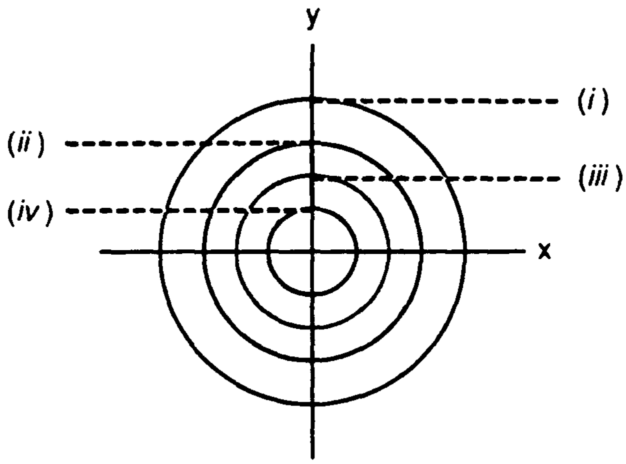

Fig. 8. Range of control over angular distribution of the radiant intensity. The shaded area indicates the region in which the maxima and minima of the radiant intensity for partially correlated dipoles are found.

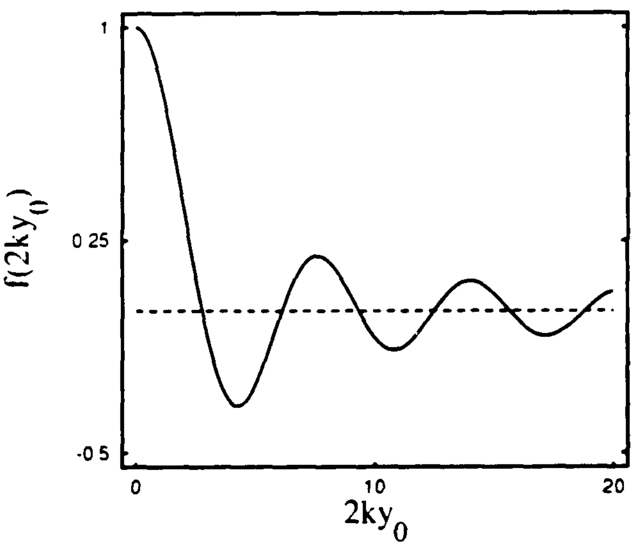

Fig. 9. The behavior of the function f in Eq. (3.3) as a function of the product $2ky_0$.

unless the two dipoles are fully correlated there are no nulls of the radiant intensity outside the plane $\theta = 0$.

3. TOTAL EMITTED POWER

The total power $P_{tot}(\omega)$ radiated by the system at frequency ω is defined by

$$P_{tot}(\omega) = \int_{(4\pi)} J(\hat{u}, \omega)d\Omega, \tag{3.1}$$

in which the integration extends over the whole 4π solid angle. On substituting Eq. (1.17) into Eq. (3.1) and performing the integration, we find that

$$P_{tot}(\omega) = {}^2\!/\!_3 ck^4 S_p(\omega)\{1 + f(2ky_0)\mathcal{R}[\mu_p(\omega)]\}, \tag{3.2}$$

in which

$$f(z) = {}^3\!/\!_2[j_0(z) - j_1(z)/z] \tag{3.3}$$

and $j_0(z)$ and $j_1(z)$ are spherical Bessel functions. The maximum of the function $f(z)$ can be shown to occur when $z = 0$, and it has the value $f(0) = 1$ (see Fig. 9). It follows that the upper bound for the total radiated power is

$$[P(\omega)]_{max} = {}^4\!/\!_3 ck^4 S_p(\omega). \tag{3.4}$$

When the two dipoles are uncorrelated, the total radiated power is given by

$$[P(\omega)]_{uncorr} = {}^2\!/\!_3 ck^4 S_p(\omega). \tag{3.5}$$

On comparing the total powers of two partially correlated dipoles $P(\omega)$ and uncorrelated dipoles $[P(\omega)]_{uncorr}$, we see from Eqs. (3.2) and (3.5) that

$$\frac{P(\omega)}{[P(\omega)]_{uncorr}} = 1 + f(2ky_0)\mathcal{R}[\mu_p(\omega)]. \tag{3.6}$$

Since the function $f(z)$ given by Eq. (3.3) decreases rapidly with increasing z, it is evident that the total radiated power from two dipoles that are separated by a distance much larger than a wavelength is equivalent to the total radiated power from two uncorrelated dipoles.

4. DIRECTIVITY

The directivity $D(\hat{u}, \omega)$ of a radiating system is defined by the ratio[13]

$$D(\hat{u}, \omega) = \frac{4\pi J(u, \omega)}{P_{tot}(\omega)}, \tag{4.1}$$

where $\hat{u}$ is the direction of observation. Here we are particularly interested in the maximum directivity $D(\omega) \equiv D(\hat{u}_m, \omega)$ that occurs in a particular direction $\hat{u}_m$. It follows from Eqs. (1.17) and (3.2) that the directivity of the two partially correlated dipoles is given by

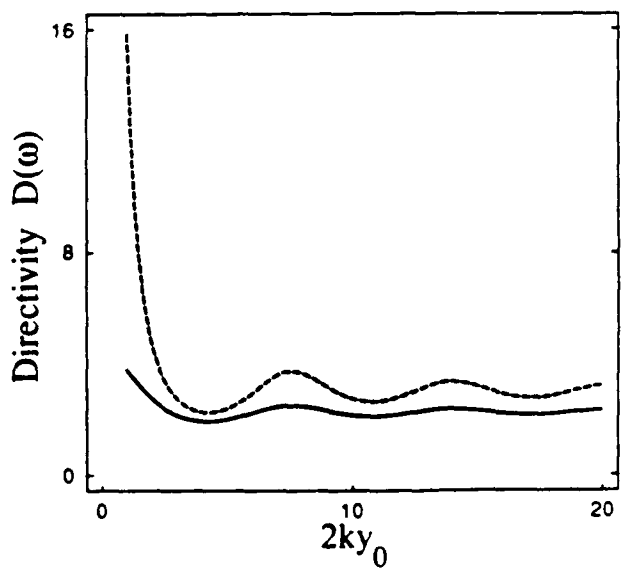

Fig. 10. Comparison of the directivity $D(\omega)$ for fully correlated dipoles (dashed curve) and uncorrelated dipoles (solid curve) as a function of the dipole separation $2ky_0$. In both cases the value of the steering angle was $2\psi(\omega) = \pi$.

$$D(\omega) = \frac{3}{2}\frac{1 + |\mu_p(\omega)|}{|1 + f(2ky_0)\mathcal{R}[\mu_p(\omega)]|}. \tag{4.2}$$

It is straightforward to show that, when the dipoles are uncorrelated, $[D(\omega)]_{\text{uncorr}} = 3/2$. Similarly, when the dipoles are fully correlated the directivity is given by

$$D(\omega) = \frac{3}{|1 + f(2ky_0)\cos[2\psi(\omega)]|}, \tag{4.3}$$

in which $2\psi(\omega)$ is the phase of $\mu_p(\omega)$.

Equations (4.2) and (4.3) also indicate that, when the two dipoles are separated by a distance that is much larger than a wavelength, the directivity of partially correlated dipoles approaches the value $D(\omega) \sim 3[1 + |\mu_p(\omega)|]/2$, whereas the directivity of fully correlated dipoles approaches the value $[D(\omega)]_{\text{corr}} \sim 3$ (see Fig. 10). This result is consistent with our earlier remarks regarding the total power radiated from such dipoles. In addition, if we use Eq. (1.16) in Eq. (4.2) and differentiate with respect to $|\mu_p(\omega)|$, it is straightforward to show that the directivity is maximized for a fixed value of $f(2ky_0)\cos[2\psi(\omega)]$ when $|\mu_p(\omega)| = 1$, i.e. when the dipoles are fully correlated.

5. DISCUSSION

Although the properties of the fields produced by linear dipoles have been known for many years, most of the previous studies were confined to the limits of fully correlated and uncorrelated dipoles. In this paper we described the spectrum of the field and the angular distribution of the radiant intensity produced by two linear dipoles with polarization fluctuations that are subject to an arbitrary choice of frequency-dependent correlations.

In Section 2 we used a simple choice of frequency-dependent correlation function to illustrate its effects on the angular distribution of the radiant intensity for selected values of the frequency. We showed that the angular distribution for different frequencies changes significantly despite the fact that we used a fixed steering angle $2\psi(\omega) \equiv 0$ in all cases. We also showed that the spectrum observed in one direction may have distinct features that are not present in other directions.

In all the examples that we considered, the precise spectral and angular details depend on the choice of the correlation function. We believe that the essential facets of the theory are illustrated well by our simple choice of correlation function [Eq. (2.6)]. Additional choices of the correlation function are discussed in Ref. 4.

In Sections 3 and 4 we examined the effects of the correlation between the dipoles on the total radiated power and the directivity of the two dipole system. In both cases we showed that the degree of the correlation was coupled with a function of the separation constant k_0y_0. With increasing separation between the two dipoles, the total radiated power becomes independent of the correlation, and as a result the directivity becomes bounded by a constant value.

ACKNOWLEDGMENT

This research was supported in part by the New York State Science and Technology Foundation.

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Frequency shifts of spectral lines produced by scattering from spatially random media

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Received July 29, 1988; accepted February 23, 1989

Scattering of polychromatic light by a medium whose dielectric susceptibility is a random function of position is considered within the accuracy of the first Born approximation. It is shown, in particular, that if the two-point spatial correlation function of the dielectric susceptibility has Gaussian form and the spectrum of the incident light has a Gaussian profile, the spectrum of the scattered light may be shifted toward the shorter or the longer wavelengths, depending on the angle of scattering. The results are analogous to those derived recently in connection with radiation from partially coherent sources [Nature (London) 326, 363 (1987)].

I. INTRODUCTION

It was predicted theoretically not long ago, that, in general, correlations between source fluctuations produce changes in the spectrum of the emitted light.[1,2] For radiation from planar secondary sources this prediction was subsequently verified by experiment.[3] It was also shown theoretically that the changes may be such as to produce red shifts or blue shifts of spectral lines,[4-6] and this prediction too has been verified.[7-9] A similar effect can also be expected to arise with acoustical waves and was, in fact, observed not long ago.[10] Some other modifications of spectra due to source correlations have also been discussed.[11]

Because of the well-known analogy that exists between the processes of radiation and scattering, one might expect that similar phenomena will arise when a polychromatic wave is scattered by a medium whose dielectric susceptibility is a random function of position. We show, in the present paper, that this indeed is the case. First we derive an expression, valid within the accuracy of the first Born approximation, for the spectrum of the scattered light in terms of the spectrum of the incident light and the two-point spatial correlation function of the dielectric susceptibility of the medium. We then show that if the spectrum of the incident light consists of a single line of Gaussian profile and the correlation function of the dielectric susceptibility is also a Gaussian function, the spectrum of the scattered field will consist of a line that has approximately a Gaussian profile. However, this line is, in general, red shifted or blue shifted with respect to that of the incident light, depending on the angle of scattering. This result may appear, at first sight, to contradict the well-known fact that there is no frequency change in linear scattering on a time-invariant medium and that different frequency components of the incident and also of the scattered light are uncorrelated. The resolution of this apparent paradox is briefly discussed in the concluding section.

2. EXPRESSION FOR THE SPECTRUM OF THE SCATTERED FIELD

Let us consider a field incident upon a scattering medium occupying a finite volume V. Suppose that the incident field propagates in a direction specified by a real unit vector $\mathbf{s}_0$. We do *not* assume, however, that the field is monochromatic; rather we consider it to fluctuate at each point, generally in a random manner, characterized by an ensemble that is statistically stationary. The cross-spectral density $W^{(i)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ of the incident field at points whose location is specified by position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ may be expressed in the form[12]

$$W^{(i)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^{(i)*}(\mathbf{r}_1, \omega) U^{(i)}(\mathbf{r}_2, \omega)\rangle, \tag{2.1}$$

where $\{U^{(i)}(\mathbf{r}, \omega)\}$ represents a statistical ensemble of random functions, all of the form

$$U^{(i)}(\mathbf{r}, \omega) = a(\omega)\exp(ik\mathbf{s}_0 \cdot \mathbf{r}), \tag{2.2}$$

with

$$k = \frac{\omega}{c}, \tag{2.3}$$

c being the speed of light *in vacuo*. In Eq. (2.2) $a(\omega)$ are (generally complex) frequency-dependent random variables, and the angle brackets in Eq. (2.1) denote the expectation value, taken over the ensemble of the incident field.

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The spectrum

$$S^{(i)}(\mathbf{r}, \omega) \equiv W^{(i)}(\mathbf{r}, \mathbf{r}, \omega) = \langle U^{(i)*}(\mathbf{r}, \omega) U^{(i)}(\mathbf{r}, \omega) \rangle \tag{2.4}$$

of the incident field is then, according to Eq. (2.2), given by

$$S^{(i)}(\omega) \equiv S^{(i)}(\mathbf{r}, \omega) = \langle a^*(\omega) a(\omega) \rangle \tag{2.5}$$

and is seen to be independent of position.

We assume that the scatterer is weak in the sense that the amplitude of the scattered field $U^{(s)}(\mathbf{r}, \omega)$ is small compared with the amplitude of the incident field $[|U^{(s)}| \ll |U^{(i)}|]$. We may then calculate $U^{(s)}$ on the basis of the first Born approximation, which yields[13]

$$U^{(s)}(\mathbf{r}, \omega) = a(\omega) \int_V F(\mathbf{r}', \omega) G(\mathbf{r}, \mathbf{r}', \omega) \exp(ik\hat{\mathbf{s}}_0 \cdot \mathbf{r}') d^3 r', \tag{2.6}$$

where

$$F(\mathbf{r}, \omega) = (\omega/c)^2 \eta(\mathbf{r}, \omega), \tag{2.7}$$

$\eta(\mathbf{r}, \omega) = [n^2(\mathbf{r}, \omega) - 1]/4\pi$ being the dielectric susceptibility of the scattering medium, $n(\mathbf{r}, \omega)$ its refractive index, and

$$G(\mathbf{r}, \mathbf{r}', \omega) = \frac{\exp[ik(\mathbf{r} - \mathbf{r}')]}{|\mathbf{r} - \mathbf{r}'|} \tag{2.8}$$

the outgoing free-space Green's function.

We will consider only the scattered field in the far zone. If we set $\mathbf{r} = r\hat{\mathbf{s}}$, $(\hat{s}^2 = 1)$, the Green's function may then be approximated by its asymptotic form (see Fig. 1)

$$G(\mathbf{r}, \mathbf{r}', \omega) \sim \frac{e^{ikr}}{r} \exp(-ik\hat{\mathbf{s}} \cdot \mathbf{r}') \qquad (kr \to \infty). \tag{2.9}$$

On substituting from the formula (2.9) into Eq. (2.6), we find at once that

$$U^{(\infty)}(r\hat{\mathbf{s}}, \omega) = a(\omega) \frac{e^{ikr}}{r} \int_V F(\mathbf{r}', \omega) \exp[-ik(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0) \cdot \mathbf{r}'] d^3 r', \tag{2.10}$$

where we have written $U^{(\infty)}$ rather than $U^{(s)}$ to stress that the expression on the right-hand side of Eq. (2.10) represents the scattered field in the far zone. If we introduce the Fourier transform of the scattering potential by the formula

$$\tilde{F}(\mathbf{K}, \omega) = \int_V F(\mathbf{r}', \omega) \exp(-i\mathbf{K} \cdot \mathbf{r}') d^3 r', \tag{2.11}$$

Eq. (2.10) becomes

$$U^{(\infty)}(r\hat{\mathbf{s}}, \omega) = a(\omega) \tilde{F}[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] \frac{e^{ikr}}{r}, \tag{2.12}$$

or, using Eq. (2.7),

$$U^{(\infty)}(r\hat{\mathbf{s}}, \omega) = a(\omega) \left(\frac{\omega}{c}\right)^2 \tilde{\eta}[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] \frac{e^{ikr}}{r}, \tag{2.12a}$$

where $\tilde{\eta}(\mathbf{K}, \omega)$ is the Fourier transform, defined by a formula of the form (2.11), of the dielectric susceptibility $\eta(\mathbf{r}', \omega)$.

Let us next consider the spectrum of the scattered field in the far zone. It can be determined from the following formula analogous to Eq. (2.4), which is

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \langle U^{(\infty)*}(r\hat{\mathbf{s}}, \omega) U^{(\infty)}(r\hat{\mathbf{s}}, \omega) \rangle. \tag{2.13}$$

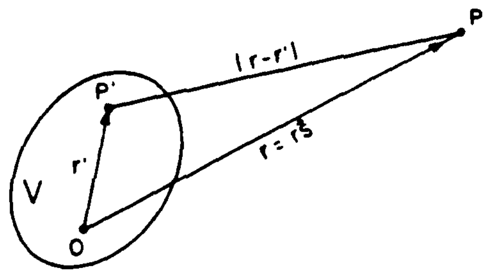

Fig. 1. Illustrating the notation relating to the asymptotic approximation (2.9).

On substituting from Eq. (2.12a) into Eq. (2.13) and on using Eq. (2.5), we find at once that

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{1}{r^2} \left(\frac{\omega}{c}\right)^4 \tilde{\eta}^*[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] \tilde{\eta}[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] S^{(i)}(\omega). \tag{2.14}$$

Up to now we have assumed that the scattering medium was deterministic. Let us now assume, instead, that its physical properties are characterized by random functions of position but are independent of time. An example of such a medium would be the atmosphere under conditions when its temporal fluctuations are slow enough to be ignored. Then, for each ω, the dielectric susceptibility $\eta(\mathbf{r}, \omega)$ will be a random function of $\mathbf{r}$, and consequently $\tilde{\eta}(\mathbf{K}, \omega)$ will be a random function of $\mathbf{K}$. A meaningful measure of the spectrum of the scattered field in the far zone is then the expectation value of the right-hand side of Eq. (2.14), taken over an ensemble of different realizations of the scattering medium; i.e., it is given by the expression

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{1}{r^2} \left(\frac{\omega}{c}\right)^4 \langle \tilde{\eta}^*[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] \tilde{\eta}[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0), \omega] \rangle_\eta S^{(i)}(\omega). \tag{2.15}$$

Here the angle brackets, with the subscript η, denote the average value taken over this ensemble. It is to be noted that the expression (2.15) involves two averaging procedures, one over the ensemble of the incident field [cf. Eq. (2.5)] and the other over an ensemble of scatterers. We implicitly assume here that these two sources of randomness are mutually independent.

Although the formula (2.15) for the far-field spectrum involves an average, obtained from experiments with macroscopically similar but microscopically different scatters, one can in some cases deduce the value of this average, at least to a good approximation, from experiments performed with a single scatterer. For example, often the necessarily finite size of the detector aperture will provide spatial averaging, which is essentially equivalent to ensemble averaging.[14,15] The value of the ensemble average may also be obtained sometimes by the use of a moving aperture in front of the scatterer.[16]

If we express $\tilde{\eta}$ in terms of its Fourier inverse η and interchange the order of averaging and integrations, we find that

$$\langle \tilde{\eta}^*(\mathbf{K}_1, \omega) \tilde{\eta}(\mathbf{K}_2, \omega) \rangle_\eta = \int_V \int_V C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega) \times \exp[-i(\mathbf{K}_2 \cdot \mathbf{r}_2' - \mathbf{K}_1 \cdot \mathbf{r}_1')] d^3 r_1' d^3 r_2', \tag{2.16}$$

where

$$C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle \eta^*(\mathbf{r}_1', \omega)\eta(\mathbf{r}_2', \omega)\rangle_\eta \qquad (2.17)$$

is the spatial correlation function of the dielectric susceptibility. If we introduce its six-dimensional Fourier transform

$$\tilde{C}_\eta(\mathbf{K}_1, \mathbf{K}_2, \omega) = \int_V \int_V C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega) \times \exp[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')]\mathrm{d}^3r_1'\mathrm{d}^3r_2', \qquad (2.18)$$

Eq. (2.16) becomes

$$\langle \tilde{\eta}^*(\mathbf{K}_1, \omega)\tilde{\eta}(\mathbf{K}_2, \omega)\rangle_\eta = \tilde{C}_\eta(-\mathbf{K}_1, \mathbf{K}_2, \omega). \qquad (2.19)$$

Finally, on substituting from Eq. (2.19) into Eq. (2.15), we obtain the following expression for the spectrum of the scattered field in the far zone:

$$S^{(\infty)}(r\hat{s}, \omega) = \frac{1}{r^2}(\omega/c)^4\tilde{C}_\eta[-k(\hat{s} - \hat{s}_0), k(\hat{s} - \hat{s}_0), \omega]S^{(i)}(\omega). \qquad (2.20)$$

The formula (2.20) shows that the spectrum of the scattered field in the far zone differs, in general, from the spectrum $S^{(i)}(\omega)$ of the incident field by the effect of two multiplicative factors, namely, a factor proportional to ω^4 and the factor $\tilde{C}_\eta$. The ω^4 factor is a reflection of the fact that on the microscopic level the medium responds to the incident field as a set of dipole oscillators [cf. Ref. 17, Secs. 2.2.1 and 2.2.3]. The other factor, $\tilde{C}_\eta$, may be regarded as representing the correlation that exists between them. Because $\tilde{C}_\eta$ in Eq. (2.20) depends on the momentum transfer vector $\mathbf{K} = k(\hat{s} - \hat{s}_0)$, it is a function of the angle of scattering, and hence, in general, the far-field spectrum will be different in different directions of observation.

The situation is somewhat different in the idealized special case when the dielectric susceptibility of the scattering medium is completely uncorrelated, i.e., when

$$\begin{aligned} C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega) &= I(\omega)\delta^{(3)}(\mathbf{r}_2' - \mathbf{r}_1') \qquad &\text{when } \mathbf{r}_1' \in V,\ \mathbf{r}_2' \in V \\ &= 0 &\text{otherwise,} \end{aligned} \qquad (2.21)$$

where $\delta^{(3)}(\mathbf{r}')$ is the three-dimensional Dirac delta function and $I(\omega)$ is a nonnegative function of frequency. We then have, according to Eqs. (2.18) and (2.21),

$$\tilde{C}_\eta[-k(\hat{s} - \hat{s}_0), k(\hat{s} - \hat{s}_0), \omega] = I(\omega)V, \qquad (2.22)$$

and the formula (2.20) reduces to

$$[S^{(\infty)}(r\hat{s}, \omega)]_{\text{uncorr}} = \frac{V}{r^2}\left(\frac{\omega}{c}\right)^4 I(\omega)S^{(i)}(\omega). \qquad (2.23)$$

We see that in this special case the spectrum of the scattered field is the same for all angles of scattering and that it differs from the source spectrum $I(\omega)$ only by a multiplicative factor that is proportional to the dipole term ω^4.

It seems worthwhile to stress that in deriving Eq. (2.20) no assumption was made regarding homogeneity or isotropy of the scatterer. When the scatterer is statistically homogeneous, as is often the case, the formula (2.20) considerably simplifies, as we will now show.

3. SPECTRUM OF THE FIELD SCATTERED BY A STATISTICALLY HOMOGENEOUS MEDIUM

When the medium is statistically homogeneous, the correlation function $C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega)$ will depend on $\mathbf{r}_1'$ and $\mathbf{r}_2'$ only through the difference $\mathbf{r}_2' - \mathbf{r}_1'$, and we will then write

$$\begin{aligned} C_\eta(\mathbf{r}_1', \mathbf{r}_2', \omega) &= C_\eta(\mathbf{r}_2' - \mathbf{r}_1', \omega) \qquad &\text{when } \mathbf{r}_1' \in V,\ \mathbf{r}_2' \in V \\ &= 0 &\text{otherwise.} \end{aligned} \qquad (3.1)$$

In this case Eq. (2.18) becomes

$$\tilde{C}_\eta(\mathbf{K}_1, \mathbf{K}_2, \omega) = \int_V \int_V C_\eta(\cancel{\mathbf{r}_1' - \mathbf{r}_1'}, \omega)\exp[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')]\mathrm{d}^3r_1'\mathrm{d}^3r_2', \quad \mathbf{r}_2' - \mathbf{r}_1' \qquad (3.2)$$

and hence

$$\tilde{C}_\eta[-k(\hat{s} - \hat{s}_0), k(\hat{s} - \hat{s}_0), \omega] = \int_V \int_V C_\eta(\mathbf{r}_2' - \mathbf{r}_1', \omega)\exp[-ik(\hat{s} - \hat{s}_0) \cdot (\mathbf{r}_2' - \mathbf{r}_1')]\mathrm{d}^3r_1'\mathrm{d}^3r_2'. \qquad (3.3)$$

If we change the variables of integration from $\mathbf{r}_1'$, $\mathbf{r}_2'$ to $\mathbf{r}$, $\mathbf{r}'$ by setting

$$\mathbf{r} = (\mathbf{r}_1' + \mathbf{r}_2')/2, \qquad \mathbf{r}' = (\mathbf{r}_2' - \mathbf{r}_1') \qquad (3.4)$$

and assume that the linear dimensions of the scattering volume are large compared both with the correlation length of the dielectric susceptibility [the effective $\mathbf{r}'$ range of $|C_\eta(\mathbf{r}', \omega)|$] and with the wavelength $\lambda = 2\pi c/\omega$, Eq. (3.3) gives

$$\tilde{C}_\eta[-k(\hat{s} - \hat{s}_0), k(\hat{s} - \hat{s}_0), \omega] \approx V\tilde{C}_\eta[k(\hat{s} - \hat{s}_0), \omega], \qquad (3.5)$$

where $\tilde{C}_\eta(\mathbf{K}, \omega)$ is the three-dimensional Fourier transform of $C_\eta(\mathbf{r}', \omega)$, viz.,

$$\tilde{C}_\eta(\mathbf{K}, \omega) = \int_V C_\eta(\mathbf{r}', \omega)\exp(-i\mathbf{K} \cdot \mathbf{r}')\mathrm{d}^3r'. \qquad (3.6)$$

On substituting from formula (3.5) into the general formula (2.20) we obtain the following simpler formula for the spectrum of the scattered field:

$$S^{(\infty)}(r\hat{s}, \omega) \approx \frac{V}{r^2}\cancel{\left(\frac{\omega}{c}\right)}\tilde{C}_\eta[k(\hat{s} - \hat{s}_0), \omega]S^{(i)}(\omega). \qquad (3.7) \quad \left(\frac{\omega}{c}\right)^4$$

This formula again shows that the spectrum of the incident field is, in general, modified by interaction with the scattering medium. The modification arises from a dipole contribution that is proportional to the fourth power of the frequency and by the influence of the dielectric susceptibility correlations, characterized by the correlation function $C_\eta(\mathbf{r}', \omega)$.

4. FREQUENCY SHIFTS GENERATED BY SCATTERING

To illustrate the effect of the dielectric susceptibility correlations of the scattering medium, let us consider the situation when the correlation function is a three-dimensional Gaussian distribution, i.e., when

$$C_\eta(\mathbf{r}', \omega) = \frac{A}{(2\pi\sigma^2)^{3/2}} \exp(-r'^2/2\sigma^2), \tag{4.1}$$

where A and σ are positive constants, i.e., they are independent of both $\mathbf{r}'$ and ω.

The three-dimensional Fourier transform [defined by Eq. (3.6)] of the expression (4.1) is

$$\tilde{C}_\eta(\mathbf{K}, \omega) = A \exp[-(K\sigma)^2/2]. \tag{4.2}$$

Now with $\mathbf{K} = k(\hat{s} - \hat{s}_0)$ we have, since $\hat{s}$ and $\hat{s}_0$ are unit vectors and $k = \omega/c$,

$$K^2 = 4(\omega/c)^2 \sin^2(\theta/2), \tag{4.3}$$

where θ is the scattering angle ($\hat{s} \cdot \hat{s}_0 = \cos\theta$). Hence it follows that, in this case,

$$\tilde{C}_\eta[k(\hat{s} - \hat{s}_0), \omega] = A \exp\left[-2\left(\frac{\omega}{c}\right)^2 \sigma^2 \sin^2\left(\frac{\theta}{2}\right)\right], \tag{4.4}$$

and the expression (3.7) for the spectrum of the scattered field becomes

$$S^{(\infty)}(r\hat{s}, \omega) = \frac{AV}{r^2}\left(\frac{\omega}{c}\right)^4 \exp\left[-2\left(\frac{\omega}{c}\right)^2 \sigma^2 \sin^2\left(\frac{\theta}{2}\right)\right] S^{(i)}(\omega). \tag{4.5}$$

Suppose that the spectrum of the incident field is a Gaussian function of rms width Γ_0, centered at frequency ω_0, i.e., that

$$S^{(i)}(\omega) = B \exp\left[\frac{-(\omega - \omega_0)^2}{2\Gamma_0^2}\right], \tag{4.6}$$

where B, ω_0 and Γ_0 are positive constants. On substituting from Eq. (4.6) into Eq. (4.5), we see that the right-hand side contains the product of two Gaussian functions. With the help of a product theorem for Gaussian functions established in Appendix A, we show in Appendix B that the resulting expression for the spectrum of the scattered field can be written in the form

$$S^{(\infty)}(r, \theta; \omega) = N(r) H(\theta) \omega^4 \exp\left\{-\frac{1}{2}\left[\frac{\omega - \tilde{\omega}(\theta)}{\tilde{\Gamma}(\theta)}\right]^2\right\}, \tag{4.7}$$

where we have now written $S^{(\infty)}(r, \theta; \omega)$ rather than $S^{(\infty)}(r\hat{s}, \omega)$. The various quantities that appear on the right-hand side of Eq. (4.7) are defined by the following formulas:

$$N(r) = \frac{VAB}{c^4 r^2}, \tag{4.8a}$$

$$H(\theta) = \exp\left[-\frac{2}{\alpha^2(\theta)}\left(\frac{\sigma}{\lambda_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right)\right], \tag{4.8b}$$

$$\tilde{\omega}(\theta) = \frac{\omega_0}{\alpha^2(\theta)}, \tag{4.8c}$$

$$\tilde{\Gamma}(\theta) = \frac{\Gamma_0}{\alpha(\theta)}, \tag{4.8d}$$

where

$$\alpha(\theta) = \left\{1 + \left[2\left(\frac{\sigma}{\lambda_0}\right)\left(\frac{\Gamma_0}{\omega_0}\right) \sin\left(\frac{\theta}{2}\right)\right]^2\right\}^{1/2} \tag{4.8e}$$

and

$$\lambda_0 = \frac{\lambda_0}{2\pi}. \tag{4.8f}$$

We see from Eq. (4.8e) that $\alpha(\theta) \geq 1$ and that the equality holds only [i.e., that $\alpha(\theta) = 1$] when either $\sigma = 0$ (completely uncorrelated scatterer) or $\Gamma_0 = 0$ (monochromatic light) or when $\theta = 0$ (forward scattering). Hence except in these special cases

$$\tilde{\omega}(\theta) < \omega_0, \qquad \tilde{\Gamma}(\theta) < \Gamma_0, \tag{4.9}$$

implying that the Gaussian function in Eq. (4.7) is centered on a frequency that is lower than the central frequency ω_0 of the incident light and its rms width is smaller. The fact that $\tilde{\omega}(\theta) < \omega_0$ implies that the Gaussian function in the expression (4.7) is centered at a lower frequency (i.e., is red shifted) with respect to the Gaussian spectral line of the incident light, the magnitude of the shift depending on the angle of scattering,[18] θ. On the other hand, the factor ω^4 in the expression (4.7) is an increasing function of the frequency and hence will produce a shift toward the higher frequencies (i.e., a blue shift). Consequently the spectrum of the scattered field will be either red shifted or blue shifted with respect to the spectrum of the incident light, depending on the magnitudes of these two contributions. It is shown in Appendix C that the maximum of the spectrum [Eq. (4.7)] of the scattered field at scattering angle θ occurs at frequency $\omega = \omega_0'(\theta)$, where

$$\omega_0'(\theta) = \frac{\omega_0}{2\alpha^2(\theta)}\left\{1 + \left[1 + \alpha^2(\theta)\left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}\right\}. \tag{4.10}$$

It is customary, especially in astronomy, to specify frequency shifts by the quantity

$$z = \frac{\lambda_0' - \lambda_0}{\lambda_0} = \frac{\omega_0 - \omega_0'}{\omega_0'}, \tag{4.11}$$

where $\lambda_0 = 2\pi c/\omega_0$ is the original wavelength and $\lambda_0' = 2\pi c/\omega_0'$ is the corresponding shifted wavelength. Evidently $z > 0$ when the line is red shifted and $z < 0$ when it is blue shifted. On substituting from Eq. (4.10) into Eq. (4.11) we readily find that in the present case

$$z(\theta) = \frac{2\alpha^2(\theta) - \left\{1 + \left[1 + \alpha^2(\theta)\left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}\right\}}{1 + \left[1 + \alpha^2(\theta)\left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}}. \tag{4.12}$$

Were the scatterer completely uncorrelated ($\sigma = 0$), we would have, according to Eq. (4.8c), $\alpha = 1$ for all values of θ, and Eq. (4.12) would reduce to

$$z_{\text{uncorr}} = \frac{1 - \left[1 + \left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}}{1 + \left[1 + \left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}}. \tag{4.13}$$

In Figs. 2–4 our main results are illustrated by a number of computed curves. Figure 2 shows spectra of the scattered field at different angles of scattering for some selected values of the parameters. Figure 3 illustrates the behavior of the height factor $H(\theta)$ defined by Eq. (4.8b). The behavior of

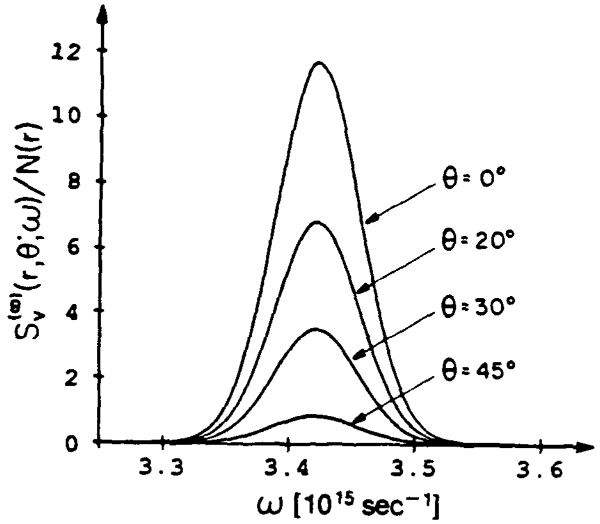

Fig. 2. Spectrum [in units of $N(r)$] of light scattered at various angles θ, from a Gaussian correlated medium with rms width $\sigma = 3\lambda_0$, $\lambda_0 = 5500\text{Å}$ ($\omega_0 = 3.427 \times 10^{15}\ \text{sec}^{-1}$). The spectrum of the incident light is a line with Gaussian profile of rms width $\Gamma_0 = 10^{-2}\ \omega_0$). The curve labeled $\theta = 0$ also represents the spectrum [in units of $N(r)$] for the case when the scatterer is completely uncorrelated ($\sigma = 0$) [see Eq. (4.4)].

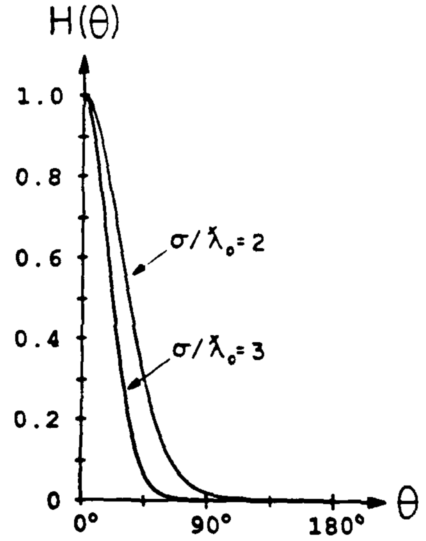

Fig. 3. Height factor $[H(\theta)]$ [Eq. (4.8b)], with $\Gamma_0/\omega_0 = 10^{-2}$. The values of the correlation parameter σ/λ_0 are indicated on the curves.

the relative frequency shift as a function of the angle of scattering is shown in Fig. 4. We see that for a certain range of directions around the forward direction $\theta = 0$, the line is blue shifted ($z < 0$), but the magnitude of the shift decreases with increasing θ, and eventually red-shifted lines ($z > 0$) are produced. The lower curves in Fig. 4 show the difference between the values pertaining to scattering from a spatially random medium of finite (nonzero) correlation length ($\sigma > 0$) and from one that is completely uncorrelated ($\sigma = 0$). This difference is seen to be positive for all angles of scattering $\theta \neq 0$, indicating that when the scatterer has a finite (nonzero) correlation length the spectrum is always red shifted with respect to the spectrum that is produced with a completely uncorrelated scatterer.

In the case when

$$(4\Gamma_0/\omega_0)^2 \ll 1, \qquad \sigma/\lambda_0 = O(1) \tag{4.14}$$

(conditions that were satisfied in connection with the computed curves), $z(\theta)$ as given by Eq. (4.12) can be approximated by an expression that clearly indicates the roles of the physical parameters. One finds, after a straightforward calculation, that

$$\begin{aligned} z(\theta) &\approx 4\left(\frac{\Gamma_0}{\omega_0}\right)^2\left[\left(\frac{\sigma}{\lambda_0}\right)^2 \sin^2(\theta/2) - 1\right] \\ &= 4\left[\left(\frac{\sigma}{L_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right) - \left(\frac{\Gamma_0}{\omega_0}\right)^2\right], \end{aligned} \tag{4.15}$$

where $L_0 = c/\Gamma_0$ is the coherence length of the incident light (cf. Note 18). Further, when the conditions (4.14) are fulfilled, the fractional shift of the center frequency of the Gaussian factor in Eq. (4.7), defined by the expression

$$z_G(\theta) \equiv \frac{\omega_0 - \bar{\omega}(\theta)}{\bar{\omega}(\theta)}, \tag{4.16}$$

is readily found to be approximately equal to

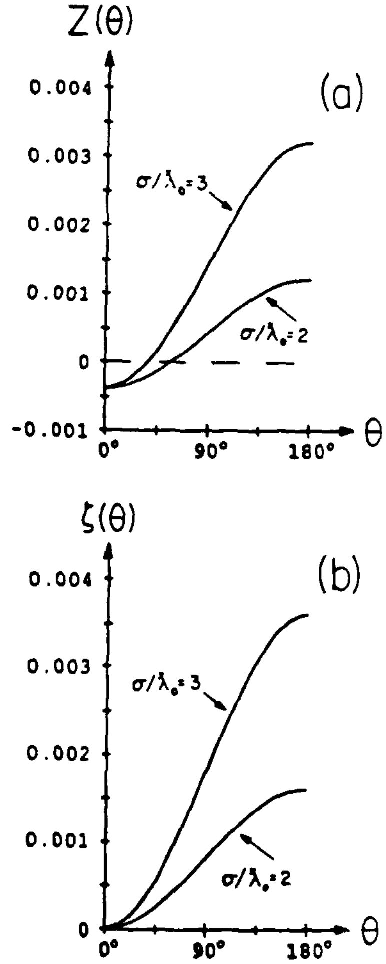

Fig. 4. Relative frequency shift $z(\theta)$, (a), and the difference $\zeta(\theta) = z(\theta) - z_{\text{uncorr}}$, (b), of the spectral line of light scattered at various angles θ from a Gaussian-correlated medium with rms width σ. The spectrum of the incident light is a line with a Gaussian profile of rms width $\Gamma_0 = 10^{-2}\ \omega_0$.

$$z_G(\theta) \approx 4\left(\frac{\sigma}{L_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right). \tag{4.17}$$

Hence *the first term in the formula (4.15) is a red shift due to the spatial correlations in the physical properties of the medium. The second term is a blue shift due to the dipole factor* ω^4.

5. CONCLUDING REMARKS

We have considered in this paper the effect of a random medium whose physical properties do not change in time on the spectrum of the light scattered by it. It is commonly stated that under these circumstances the frequency of the incident light will not change by the process of scattering. How is it possible then that our analysis indicates that such scattering may produce frequency shifts of the spectral lines? The resolution of this apparent paradox lies in the fact that most previous work was concerned with the scattering of essentially *monochromatic* light, of well-defined frequency, ω_0, say. The spectrum of the scattered light will then necessarily also be essentially monochromatic and of the same frequency ω_0. However, when the incident light is *polychromatic*, the different frequency components of the incident light, which are scattered in any particular direction, will be scattered with different strengths. As a result, the spectrum of the scattered light will differ from that of the incident light, even though the different frequency components are uncorrelated. As we showed, the difference may manifest itself as a frequency shift. This possibility of generating frequency shifts by scattering is analogous to that discovered not long ago in connection with effects of source correlations on the spectrum of the emitted light.[4-9] The only difference is that we now deal with secondary sources, namely, with the polarization induced in the scattering medium by the incident wave. The induced polarization will, in general, be correlated over finite distances of the scattering medium and will thus imitate correlations in primary sources. It was this analogy between scattering and radiation that, in fact, led to the present analysis.

Finally we might mention that our results suggest a new method for determining correlation properties of refractive-index distribution in spatially random media, based on the analysis of the spectral changes produced by a spatially random medium when illuminated by polychromatic light of known spectral composition.

APPENDIX A: PRODUCT THEOREM FOR GAUSSIAN FUNCTIONS

In this appendix we will establish the following theorem.

Theorem: If

$$G(\omega - \omega_j; \Gamma_j) = \exp[-(\omega - \omega_j)^2/2\Gamma_j^2] \qquad (j = 1, 2), \tag{A1}$$

then

$$G(\omega - \omega_1; \Gamma_1)G(\omega - \omega_2; \Gamma_2) = G[\omega_1 - \omega_2; (\Gamma_1^2 + \Gamma_2^2)^{1/2}]G(\omega - \tilde{\omega}; \tilde{\Gamma}), \tag{A2}$$

where

$$\tilde{\omega} = \frac{\omega_1\Gamma_2^2 + \omega_2\Gamma_1^2}{\Gamma_1^2 + \Gamma_2^2}, \tag{A3}$$

$$\frac{1}{\tilde{\Gamma}^2} = \frac{1}{\Gamma_1^2} + \frac{1}{\Gamma_2^2}. \tag{A4}$$

To establish this theorem we multiply together the two formulas (A1) and express the product in the form

$$G(\omega - \omega_1; \Gamma_1)G(\omega - \omega_2; \Gamma_2) = \exp[-g(\omega)], \tag{A5}$$

where

$$\begin{aligned} g(\omega) &= \frac{1}{2\Gamma_1^2\Gamma_2^2}[\Gamma_2^2(\omega - \omega_1)^2 + \Gamma_1^2(\omega - \omega_2)^2] \\ &= \frac{1}{2\Gamma_1^2\Gamma_2^2}(a^2\omega^2 - 2b\omega + c) \end{aligned} \tag{A6}$$

and

$$a^2 = \Gamma_1^2 + \Gamma_2^2, \tag{A7a}$$

$$b = \omega_1\Gamma_2^2 + \omega_2\Gamma_1^2, \tag{A7b}$$

$$c = \omega_1^2\Gamma_2^2 + \omega_2^2\Gamma_1^2. \tag{A7c}$$

On completing the square in Eq. (A6), we find that

$$\begin{aligned} g(\omega) &= \frac{a^2}{2\Gamma_1^2\Gamma_2^2}\left(\omega - \frac{b}{a^2}\right)^2 + \frac{1}{2\Gamma_1^2\Gamma_2^2}\left(c - \frac{b^2}{a^2}\right) \\ &= \frac{1}{2\tilde{\Gamma}^2}(\omega - \tilde{\omega})^2 + \frac{1}{2\Gamma_1^2\Gamma_2^2}\left(c - \frac{b^2}{a^2}\right), \end{aligned} \tag{A8}$$

where

$$\frac{1}{\tilde{\Gamma}^2} = \frac{a^2}{\Gamma_1^2\Gamma_2^2} = \frac{1}{\Gamma_1^2} + \frac{1}{\Gamma_2^2}, \tag{A9}$$

$$\tilde{\omega} = \frac{b}{a^2} = \frac{\omega_1\Gamma_2^2 + \omega_2\Gamma_1^2}{\Gamma_1^2 + \Gamma_2^2}. \tag{A10}$$

By using Eqs. (A7a)–(A7c), one can readily show that

$$c - \frac{b^2}{a^2} = \frac{\Gamma_1^2\Gamma_2^2}{\Gamma_1^2 + \Gamma_2^2}(\omega_1 - \omega_2)^2. \tag{A11}$$

On substituting from Eq. (A11) into Eq. (A8) and from Eq. (A8) into Eq. (A5), we find that

$$\begin{aligned} G(\omega - \omega_1; \Gamma_1)G(\omega - \omega_2; \Gamma_2) &= \exp\left[-\frac{(\omega_1 - \omega_2)^2}{2(\Gamma_1^2 + \Gamma_2^2)}\right]\exp\left[-\frac{(\omega - \tilde{\omega})^2}{2\tilde{\Gamma}^2}\right] \\ &= G[\omega_1 - \omega_2; (\Gamma_1^2 + \Gamma_2^2)^{1/2}]G(\omega - \tilde{\omega}; \tilde{\Gamma}), \end{aligned} \tag{A12}$$

where the last step follows from Eq. (A1). This completes the proof of the theorem.

Some remarks about the implications of this theorem are in order. Equation (A2) shows that *the product of two Gaussian functions is proportional to a new Gaussian function*, $G(\omega - \tilde{\omega}; \tilde{\Gamma})$, the constant of proportionality being $G[\omega_1 - \omega_2; (\Gamma_1^2 + \Gamma_2^2)^{1/2}]$. The center frequency and width of the new Gaussian function are $\tilde{\omega}$ and $\tilde{\Gamma}$, given by the formulas (A10) and (A9), respectively.

The relationship between $\tilde{\omega}$ and the two original center frequencies ω_1 and ω_2 can be expressed in a different form. It follows from Eq. (A3) that

$$\tilde{\omega} = f_1\omega_1 + f_2\omega_2, \tag{A13}$$

where

$$f_1 = \frac{\Gamma_2^2}{\Gamma_1^2 + \Gamma_2^2}, \qquad f_2 = \frac{\Gamma_1^2}{\Gamma_1^2 + \Gamma_2^2}. \tag{A14}$$

Since $f_1 + f_2 = 1$, $\bar{\omega}$ *is a weighted average of the two original center frequencies.* In particular, if $0 < \omega_1 < \omega_2$, and neither Γ_1 nor Γ_2 is zero,

$$\omega_1 < \bar{\omega} < \omega_2. \tag{A15}$$

The relationship between $\bar{\Gamma}$ and the original widths, Γ_1 and Γ_2, can be deduced from Eq. (A4). If neither Γ_1 nor Γ_2 is zero, we find that *the new Gaussian is narrower than either of the two original Gaussians*, i.e., that

$$\bar{\Gamma} < \Gamma_1, \qquad \bar{\Gamma} < \Gamma_2. \tag{A16}$$

Finally, it follows directly from Eq. (A12) that the maximum value (as a function of ω) of the product $G(\omega - \omega_1; \Gamma_1)G(\omega - \omega_2; \Gamma_2)$ occurs at the frequency $\omega = \bar{\omega}$ and is equal to $G[(\omega_1 - \omega_2; (\Gamma_1^2 + \Gamma_2^2)^{1/2}]$, whereas the maximum values of each of the original Gaussians occur at $\omega = \omega_j$ and are each equal to unity. Therefore, as long as $\omega_1 \neq \omega_2$, *the maximum value of the product of two Gaussians is smaller than the maximum value of either of the two original Gaussians.*

APPENDIX B: DERIVATION OF EXPRESSION (4.7) FOR SPECTRUM OF THE SCATTERED FIELD

We have, according to Eqs. (4.5) and (4.6),

$$S^{(\infty)}(r\hat{s}, \omega) = \frac{AV}{r^2}\left(\frac{\omega}{c}\right)^4 \exp\left(-\frac{\omega^2}{2\Gamma_1^2}\right) B \exp\left[-\frac{(\omega - \omega_0)^2}{2\Gamma_0^2}\right], \tag{B1}$$

where[19]

$$\frac{1}{\Gamma_1^2} = \frac{4}{c^2}\sigma^2 \sin^2\left(\frac{\theta}{2}\right). \tag{B2}$$

If we set

$$G(\omega - \omega_j; \Gamma_j) = \exp\left[-\frac{(\omega - \omega_j)^2}{2\Gamma_j^2}\right], \tag{B3}$$

$$N = \frac{VAB}{c^4 r^2}, \tag{B4}$$

the expression (B1) becomes

$$S^{(\infty)}(r\hat{s}, \omega) = N\omega^4 G(\omega; \Gamma_1)G(\omega - \omega_0; \Gamma_0). \tag{B5}$$

Now according to the product theorem for Gaussian functions established in Appendix A,

$$G(\omega; \Gamma_1)G(\omega - \omega_0; \Gamma_0) = G[\omega_0; (\Gamma_1^2 + \Gamma_2^2)^{1/2}]G(\omega - \bar{\omega}; \bar{\Gamma}), \tag{B6}$$

where

$$\bar{\omega} = \frac{\omega_0 \Gamma_1^2}{\Gamma_0^2 + \Gamma_1^2} \tag{B7}$$

and

$$\frac{1}{\bar{\Gamma}^2} = \frac{1}{\Gamma_0^2} + \frac{1}{\Gamma_1^2}. \tag{B8}$$

It will be convenient to rewrite Eqs. (B7) and (B8) as

$$\frac{\bar{\omega}}{\omega_0} = \frac{1}{1 + (\Gamma_0/\Gamma_1)^2}, \tag{B9}$$

$$\frac{\bar{\Gamma}}{\Gamma_0} = \frac{1}{[1 + (\Gamma_0/\Gamma_1)^2]^{1/2}}. \tag{B10}$$

Now from Eq. (B2) it follows that

$$\frac{\Gamma_0}{\Gamma_1} = \frac{2}{c}\sigma\Gamma_0 \sin\left(\frac{\theta}{2}\right), \tag{B11}$$

or, when the relation $\lambda_0\omega_0 = c$, ($\lambda_0 = \lambda_0/2\pi$) is used, Eq. (B11) gives

$$\frac{\Gamma_0}{\Gamma_1} = 2\left(\frac{\sigma}{\lambda_0}\right)\left(\frac{\Gamma_0}{\omega_0}\right)\sin\left(\frac{\theta}{2}\right). \tag{B12}$$

On substituting from Eq. (B12) into Eqs. (B9) and (B10) we find that

$$\frac{\bar{\omega}}{\omega_0} = \frac{1}{\alpha^2}, \tag{B13}$$

$$\frac{\bar{\Gamma}}{\Gamma_0} = \frac{1}{\alpha}, \tag{B14}$$

where

$$\alpha = \left\{1 + \left[2\left(\frac{\sigma}{\lambda_0}\right)\left(\frac{\Gamma_0}{\omega_0}\right)\sin\left(\frac{\theta}{2}\right)\right]^2\right\}^{1/2}. \tag{B15}$$

Further we have, if we use Eqs. (B10),

$$\Gamma_0^2 + \Gamma_1^2 = \Gamma_1^2\left[1 + \left(\frac{\Gamma_0}{\Gamma_1}\right)^2\right] = \Gamma_1^2\left(\frac{\Gamma_0}{\bar{\Gamma}}\right)^2$$

or, if we also use Eq. (B14),

$$\Gamma_0^2 + \Gamma_1^2 = \alpha^2\Gamma_1^2. \tag{B16}$$

Hence the first factor on the right-hand side of the identity (B6) has the explicit form

$$G[\omega_0; (\Gamma_0^2 + \Gamma_1^2)^{1/2}] = \exp\left[-\frac{\omega_0^2}{2\alpha^2\Gamma_1^2}\right]$$

or, if we make use of Eq. (B12),

$$G[\omega_0; (\Gamma_0^2 + \Gamma_1^2)^{1/2}] = \exp\left[-\frac{2}{\alpha^2}\left(\frac{\sigma}{\lambda_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right)\right]. \tag{B17}$$

On substituting from Eq. (B17) into Eq. (B6) we find that

$$G(\omega; \Gamma_1)G(\omega - \omega_0; \Gamma_0) = \exp\left[-\frac{2}{\alpha^2}\left(\frac{\sigma}{\lambda_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right)\right]G(\omega - \bar{\omega}; \bar{\Gamma}). \tag{B18}$$

Finally, on substituting from Eq. (B18) into the expression (B5) we obtain the following expression for the spectrum of the scattered field:

$$S^{(\infty)}(r\hat{s}, \omega) = N(r)H(\theta)\omega^4 \exp\left\{-\frac{1}{2}\left[\frac{\omega - \bar{\omega}(\theta)}{\bar{\Gamma}(\theta)}\right]^2\right\}, \tag{B19}$$

where

$$H(\theta) = \exp\left[-\frac{2}{\alpha^2(\theta)}\left(\frac{\sigma}{\lambda_0}\right)^2 \sin^2\left(\frac{\theta}{2}\right)\right]. \tag{B20}$$

The quantities $\tilde{\omega}(\theta)$ and $\tilde{\Gamma}(\theta)$, which appear on the right-hand side of Eq. (B19), are given by the formulas (B13) and (B14), respectively, with $\alpha(\theta)$ given by Eq. (B15). The factor $N(r)$ is defined by the formula (B4).

APPENDIX C: DERIVATION OF THE FORMULA (4.10) FOR THE FREQUENCY AT WHICH THE SHIFTED LINE ATTAINS ITS MAXIMUM

The formula (4.7) for the spectrum of the scattered field may be written in the form

$$S^{(\infty)}(r, \theta; \omega) = N(r)H(\theta)f(\omega, \theta), \tag{C1}$$

where

$$f(\omega, \theta) = \omega^4 \exp\left\{-\frac{1}{2}\left[\frac{\omega - \tilde{\omega}(\theta)}{\tilde{\Gamma}(\theta)}\right]^2\right\}, \tag{C2}$$

with $\tilde{\omega}(\theta)$ and $\tilde{\Gamma}(\theta)$ given by the formulas (4.8c) and (4.8d), respectively.

To determine the maximum of $f(\omega, \theta)$ as a function of ω (with θ fixed), we differentiate Eq. (C2) with respect to ω. We find that

$$\frac{\partial f(\omega, \theta)}{\partial \omega} = \left[4\omega^3 - \frac{\omega - \tilde{\omega}}{\tilde{\Gamma}^2}\omega^4\right]\exp\left[-\frac{1}{2}\left(\frac{\omega - \tilde{\omega}}{\tilde{\Gamma}}\right)^2\right]. \tag{C3}$$

Obviously

$$\frac{\partial f(\omega, \theta)}{\partial \omega} = 0$$

when $\omega = 0$ or when $\omega = \omega_\pm$, where

$$4 - \frac{\omega_\pm - \tilde{\omega}}{\tilde{\Gamma}^2}\omega_\pm = 0,$$

i.e., when

$$\omega_\pm = \frac{1}{2}\tilde{\omega}\left\{1 \pm \left[1 + \left(\frac{4\tilde{\Gamma}}{\tilde{\omega}}\right)^2\right]^{1/2}\right\}. \tag{C4}$$

If we make use of Eqs. (4.8c) and (4.8d), it follows that

$$\omega_\pm = \frac{\omega_0}{2\alpha^2}\left\{1 \pm \left[1 + \alpha^2\left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}\right\}. \tag{C5}$$

Both of these roots can readily be shown to be frequencies at which $f(\omega, \theta)$ and consequently $S^{(\infty)}(r, \theta; \omega)$ attain maximum values. Only the one with the positive sign is relevant because of our use of the complex analytic signal representation of the field[12] [see also Ref. 17, Sec. 10.2]. Denoting this root by $\omega_0'(\theta)$, we have

$$\omega_0'(\theta) = \frac{\omega_0}{2\alpha^2}\left\{1 + \left[1 + \alpha^2\left(\frac{4\Gamma_0}{\omega_0}\right)^2\right]^{1/2}\right\}. \tag{C6}$$

ACKNOWLEDGMENTS

We wish to express our thanks to Avshalom Gamliel for assistance with computations relating to Figs. 2–4.

This research was supported by the National Science Foundation and by the U.S. Army Research Office.

E. Wolf is also with the Institute of Optics, University of Rochester.

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18. It follows from Eqs. (4.8e) and (4.8c) that for a given value of θ the magnitude of this red shift depends on the product of the two ratios σ/λ_0 (the correlation length of the medium in units of λ_0) and Γ_0/ω_0 (the linewidth of the incident light in units of ω_0). Since the linewidth of the incident light is Γ_0, its coherence length, L_0, may be defined by the expression $L_0 = c/\Gamma_0$. Therefore $(\sigma/\lambda_0)(\Gamma_0/\omega_0) = \sigma/L_0$, and the expression (4.8e) may be rewritten as
$$\alpha(\theta) = \left\{1 + \left[2\left(\frac{\sigma}{L_0}\right)\sin\left(\frac{\theta}{2}\right)\right]^2\right\}^{1/2}.$$
Hence, for fixed θ, $\alpha(\theta)$ is a monotonically increasing function of the ratio σ/L_0; consequently Eq. (4.8c) implies that the center frequency $\tilde{\omega}$ of the Gaussian factor in Eq. (4.7) is a monotonically decreasing function of this ratio.
19. For the sake of simplicity we do not show explicitly the dependence of Γ_1 (nor of the quantities $\tilde{\omega}$, $\tilde{\Gamma}$, and α defined below) on θ throughout the main part of this appendix.

VOLUME 63, NUMBER 20 PHYSICAL REVIEW LETTERS 13 NOVEMBER 1989

Correlation-Induced Doppler-Like Frequency Shifts of Spectral Lines

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(Received 7 February 1989)

The question is examined whether there may be scattering media which could generate spectral frequency shifts in radiation from sources that are at rest relative to the observer and yet would imitate Doppler shifts. Scattering kernels of media whose physical properties fluctuate randomly in both space and time are presented which achieve this to a good approximation. The frequency shifts produced in this manner are not necessarily small. The results might be of particular interest in connection with the long-standing controversy about the origin of some discrepancies observed in the spectra of quasars.

PACS numbers: 42.50.Ar, 03.80.+r, 05.40.+j, 98.50.−v

It has been predicted theoretically[1-3] and confirmed experimentally[4-7] not long ago that correlations in the fluctuations of a source distribution can give rise to frequency shifts of lines in the spectrum of the emitted radiation, even when the source is at rest relative to the observer. Because of the close analogy that exists between the processes of radiation and scattering, a similar effect may be expected to occur when radiation is scattered by a "static" medium, i.e., a medium whose constitutive parameters (e.g., the dielectric susceptibility) are independent of time but are random functions of position, with appropriate correlation properties. Indeed, a very recent theoretical analysis[8] has shown this to be the case.

The possibility of generating frequency shifts of spectral lines by a mechanism other than the motion of the source relative to the observer or by gravitation might be of particular interest for astronomy, especially in connection with the long-standing controversy surrounding quasars (see, for example, Refs. 9–14). However, source correlations and correlations produced by static scattering do not imitate, in all respects, frequency shifts produced by the motion of a source relative to the observer (i.e., Doppler shifts). The relative frequency shifts[15]

$$z=\frac{\bar{\lambda}'-\bar{\lambda}}{\bar{\lambda}}=\frac{\bar{\omega}-\bar{\omega}'}{\bar{\omega}'} \tag{1}$$

are, in general, frequency dependent when they are generated by such correlations, whereas they are frequency independent when they are manifestations of the Doppler effect. Moreover, frequency shifts which are generated by source correlations or by correlations in the constitutive parameters of static scatterers are restricted in magnitude to values that are of the order of or smaller than the effective widths of the spectral lines[16] [see the discussion following Eq. (10) below].

In the present Letter we investigate whether a more general correlation mechanism, involving dynamic scattering, could give rise to relative frequency shifts that are essentially the same for all the lines in the spectrum of radiation originating in a (stationary) source and whose magnitude could be arbitrarily large.

Consider a linearly polarized plane electromagnetic wave of spectral profile $S^{(i)}(\omega)$ that propagates in the direction specified by a unit vector $\mathbf{u}$, incident on a linear medium, localized in space, whose dielectric susceptibility[17] $\tilde{\eta}(\mathbf{r},t;\omega)$ is a random function of both position ($\mathbf{r}$) and time (t). We assume that the ensemble which characterizes the statistical behavior of the medium is homogeneous, isotropic, and stationary, at least in the wide sense.[18] It has recently been shown that, within the accuracy of the first Born approximation, the spectrum of the scattered field at a point $\mathbf{r}=r\mathbf{u}'$ ($|\mathbf{u}'|=1$) in the far zone of the scatterer is given by[19]

$$S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=A\omega'^4\int_{-\infty}^{\infty}\mathscr{S}\left[\frac{\omega'}{c}\mathbf{u}'-\frac{\omega}{c}\mathbf{u},\omega'-\omega;\omega\right]S^{(i)}(\omega)d\omega\,, \tag{2}$$

where

$$A=\frac{(2\pi)^3V\sin^2\psi}{c^4r^2}. \tag{3}$$

In Eq. (2)

$$\mathscr{S}(\mathbf{K},\Omega;\omega)=\frac{1}{(2\pi)^4}\int_V d^3R\int_{-\infty}^{\infty}dT\,G(\mathbf{R},T;\omega)e^{-i(\mathbf{K}\cdot\mathbf{R}-\Omega T)} \tag{4}$$

is the generalized structure function of the scattering medium, being the four-dimensional Fourier transform of the correlation function[20]

$$G(\mathbf{R},T;\omega)=\langle\tilde{\eta}^*(\mathbf{r},t;\omega)\tilde{\eta}(\mathbf{r}+\mathbf{R},t+T;\omega)\rangle \tag{5}$$

of the dielectric susceptibility and the angular brackets denote the average, taken over the ensemble of the random medium. In Eq. (3) V represents the volume of the scatterer (whose linear dimensions are assumed to be large compared to the correlation distance of $\tilde{\eta}$), ψ denotes the angle between the direction of the electric vector of the incident wave and the direction $\mathbf{u}'$ of scattering, and c is the speed of light *in vacuo*.

For our purpose it is convenient to rewrite Eq. (2) in the form

$$S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=A\omega'^4\int_{-\infty}^{\infty}\mathcal{K}(\omega',\omega;\mathbf{u}',\mathbf{u})S^{(i)}(\omega)d\omega\,, \tag{6}$$

where

$$\mathcal{K}(\omega',\omega;\mathbf{u}',\mathbf{u})=\tilde{G}\left[\frac{\omega'}{c}\mathbf{u}'-\frac{\omega}{c}\mathbf{u},\omega'-\omega;\omega\right]. \tag{7}$$

We will refer to the function $\mathcal{K}(\omega',\omega;\mathbf{u}',\mathbf{u})$ as the *scattering kernel*.

In the special case of static scattering the correlation function $G(\mathbf{R},T;\omega)$ will be independent of the temporal argument T and we will then denote it by $g(\mathbf{R};\omega)$. In this case Eqs. (4) and (7) imply that the scattering kernel is given by

$$\mathcal{K}(\omega',\omega;\mathbf{u}',\mathbf{u})=\tilde{g}\left[\frac{\omega'}{c}\mathbf{u}'-\frac{\omega}{c}\mathbf{u},\omega\right]\delta(\omega'-\omega)\,, \tag{8}$$

where

$$\tilde{g}(\mathbf{K};\omega)=\frac{1}{(2\pi)^3}\int_V g(\mathbf{R};\omega)e^{-i\mathbf{K}\cdot\mathbf{R}}d^3R \tag{9}$$

is the three-dimensional spatial Fourier transform of $g(\mathbf{R},\omega)$ and δ is the Dirac delta function. On substituting from Eq. (8) into Eq. (6) we see that the spectrum of the scattered radiation is given by

$$S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=A\omega'^4\tilde{g}\left[\frac{\omega'}{c}(\mathbf{u}'-\mathbf{u});\omega'\right]S^{(i)}(\omega'). \tag{10}$$

This expression, which is the electromagnetic analog of the main result of Ref. 8 relating to scattering of scalar waves by static media, shows that spatial correlation of the dielectric susceptibility, represented by the two-point correlation function $g(\mathbf{R};\omega)$, modifies the spectrum of the radiation incident on the scatterer. However, because the influence of the correlations is manifested in Eq. (10) only through a multiplicative factor, no new frequency components are generated by static scattering [i.e., $S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=0$ whenever $S^{(i)}(\omega')=0$]. This conclusion is, of course, an immediate consequence of the presence of the factor $\delta(\omega'-\omega)$ in the expression (8), which implies that each frequency component ω of the incident radiation gives rise to one and only one frequency ω' in the scattered radiation and that, moreover, $\omega'=\omega$.

Let us now turn to the more general case of a medium whose dielectric susceptibility varies randomly not only in space but also in time. If $S^{(i)}(\omega)=\delta(\omega-\omega_0)$, Eq. (6) gives

$$S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=A\omega'^4\mathcal{K}(\omega',\omega_0;\mathbf{u}',\mathbf{u})\,. \tag{11}$$

This formula implies that a frequency component $\omega=\omega_0$ of the incident radiation will give rise to a frequency component ω' in the scattered radiation and that, in general, $\omega'\neq\omega_0$; and, moreover, there may be several values of ω', possibly a continuous range of them, associated with any particular value of ω_0. An example of the latter situation is provided by Brillouin scattering from a simple fluid under equilibrium conditions. The scattering kernel is then proportional to the sum of three Lorentzian distributions, centered at frequencies $\omega'=\omega_0$, $\omega_0[1+2(v/c)\sin(\theta/2)]$, and $\omega_0[1-2(v/c)\sin(\theta/2)]$, where v is the speed of sound in the fluid and θ is the angle of scattering. Another example is discussed in Ref. 21.

As one of the simplest generalizations of the formula (8) to media which vary randomly both in space and in time let us suppose that the scattering kernel has the form

$$\mathcal{K}(\omega',\omega;\mathbf{u}',\mathbf{u})=f(\omega',\omega;\mathbf{u}',\mathbf{u})\delta(a\omega'-\omega)\,, \tag{12}$$

where a is independent of the frequencies but may depend on $\mathbf{u}$ and $\mathbf{u}'$. In this case the formula (6) gives the following expression for the spectrum of the scattered radiation:

$$S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})=A\omega'^4f(\omega',a\omega';\mathbf{u}',\mathbf{u})S^{(i)}(a\omega'). \tag{13}$$

Suppose further that the spectrum $S^{(i)}(\omega)$ of the incident radiation consists of a single line centered on the frequency $\omega=\bar{\omega}$ and that over the width of the line the factor $\omega'^4f(\omega',a\omega';\mathbf{u},\mathbf{u}')$ does not change appreciably with ω'. Then, according to Eq. (13) the spectrum $S^{(\infty)}(\omega';r\mathbf{u}',\mathbf{u})$ of the scattered radiation will also consist of a single line, but this line will be centered close to the frequency $\omega'=\bar{\omega}'$, where

$$\bar{\omega}'=\bar{\omega}/a\,. \tag{14}$$

On substituting from Eq. (14) into the formula (1) we see that the spectral line has been shifted in frequency by the relative amount

$$z=\frac{\bar{\omega}-\bar{\omega}/a}{\bar{\omega}/a}=a-1\,. \tag{15}$$

Thus apart from a small contribution arising from the factor $\omega'^4f(\omega',a\omega';\mathbf{u}',\mathbf{u})$, *the relative frequency shift is independent of the central frequency $\bar{\omega}$ of the spectral line of the incident radiation and of the width of the line.*

Evidently, if instead of a single line the spectrum of the incident radiation consisted of several lines, each would be shifted by essentially the same amount $z=a-1$, thus imitating a Doppler shift. The shift will be towards the lower frequencies (redshift) if $a>1$ and towards higher frequencies (blueshift) if $a<1$ and can, in principle, have any magnitude.

Next let us now consider a more general kernel, of the form

$$\mathcal{K}(\omega',\omega;u',u)=f(\omega',\omega;u',u)e^{-(a\omega'-\omega)^2/2\sigma^2}, \qquad (16)$$

where a and σ are positive quantities that are independent of the frequencies. Suppose that the spectrum of the incident radiation is the line

$$S^{(i)}(\omega)=Be^{-(\omega-\bar{\omega})^2/2\Gamma^2}, \qquad (17)$$

where B, $\bar{\omega}$, and Γ are positive constants. On substituting from Eqs. (16) and (17) into the formula (6) and assuming that the factor $f(\omega',\omega;u',u)$ does not vary appreciably with ω over the effective width of the spectral line (17), we obtain for $S^{(\infty)}(\omega';ru',u)$ the expression

$$S^{(\infty)}(\omega';ru',u)=AB\omega'^4 f(\omega',\bar{\omega};u',u)\int_{-\infty}^{\infty} e^{-(a\omega'-\omega)^2/2\sigma^2}e^{-(\omega-\bar{\omega})^2/2\Gamma^2}d\omega . \qquad (18)$$

The integral in Eq. (18) may be readily evaluated [most simply by making use of the so-called product theorem for Gaussian functions (cf. Appendix A of Ref. 8)] and one then finds that

$$S^{(\infty)}(\omega';ru',u)=AB\bar{\Gamma}(2\pi)^{1/2}\omega'^4 f(\omega',\bar{\omega};u',u)\times e^{-(\omega'-\bar{\omega}/a)^2/2\Gamma'^2}, \qquad (19)$$

where

$$\Gamma'=\frac{1}{a}(\sigma^2+\Gamma^2)^{1/2},\quad \bar{\Gamma}=\left[\frac{1}{\sigma^2}+\frac{1}{\Gamma^2}\right]^{-1/2}. \qquad (20)$$

The formula (19) shows that the spectrum $S^{(\infty)}(\omega';ru',u)$ of the scattered radiation is proportional to the product of the factor $\omega'^4 f(\omega',\bar{\omega};u',u)$ and a Gaussian function centered at the frequency $\omega'=\bar{\omega}/a$ and of rms width Γ'. This Gaussian function is shifted with respect to the Gaussian function (17) by an amount whose relative value is again given by the expression (15) and is, therefore, independent of the mean frequency $\bar{\omega}$ of the incident radiation. The widths of the two lines are not the same, however. Moreover, the factor $\omega'^4 f(\omega',\bar{\omega};u',u)$ in Eq. (19) will produce a distortion of the line and possibly an additional frequency shift, whose relative value may or may not be small compared to the value $z=a-1$, depending on the exact form of the function $f(\omega'\omega;u',u)$.

It is evident from the preceding analysis that there is a possibility that scattering from some media whose macroscopic properties fluctuate randomly both in space and in time may generate frequency shifts of spectral lines which closely resemble Doppler shifts. As mentioned earlier, this possibility might be of particular interest in connection with the long-standing quasar controversy. However, in order to apply the present theory to quasar problems it would be necessary to have a reliable model available for the medium through which radiation originating in these astronomical sources passes, and no models that have been proposed so far can be accepted with full confidence. Moreover, the very process of generation of the radiation that reaches us from quasars is not currently understood. One should also bear in mind that throughout the preceding analysis the incident field was assumed to be a plane wave. For applications to quasars and possibly other astronomical objects, additional averaging over a range of directions of incidence would also have to be performed. Moreover, extension of the analysis beyond the first Born approximation and also to anisotropic media might be required.[22]

There is a widely held opinion that no other mechanism except the Doppler effect and gravitation exist within the framework of present-day physics, which can account even for parts of the redshifts observed in the spectra of radiation that reaches us from astronomical sources. The analysis presented in this Letter raises a question about the correctness of this opinion.

This research was supported by the National Science Foundation and by the Army Research Office. I wish to thank Professor Jack W. Sulentic for making available to me a preprint of his paper (cited in Ref. 16) prior to publication and for stimulating discussions about quasar astronomy.

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5. SELECTION OF EXPERIMENTAL PAPERS

Volume 62, number 1 OPTICS COMMUNICATIONS 1 April 1987

EFFECTS OF SOURCE CORRELATION ON THE SPECTRUM OF LIGHT

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Received 17 November 1986

Experiments that illustrate the influence of source correlations on the spectrum of light detected in the far field are reported. It is demonstrated that source correlations which violate the scaling law [E. Wolf, Phys. Rev. Lett. 56 (1986) 1370] produce a normalized spectrum in the far field that is different from the normalized spectrum of the light at the source.

1. Introduction

In a recent Letter by Wolf [1], it was shown that the normalized spectrum of light is the same throughout the far zone and is equal to the normalized spectrum of light at the source provided that the complex degree of spectral coherence is a function of the variable $k(\rho_2-\rho_1)$ only, where $(\rho_2-\rho_1)$ denotes the (vectorial) distance between two points on the source, $k=\omega/c$, ω represents the angular frequency and c is the speed of the light. If the complex degree of spectral coherence depends on frequency only through the variable $k(\rho_2-\rho_1)$, the source is said to satisfy the *scaling law*.

It is well known that the correlation properties of light can be controlled by application of the Van Cittert–Zernike theorem [2–6]. To date, using conventional sources and optical systems, the secondary sources with controlled correlation that have been produced all satisfy the scaling law.

In this article an experimental arrangement that provides a secondary source with a complex degree of coherence that is independent of the wavenumber k (i.e. violates the scaling law) is described. In accord with theoretical predictions, when the scaling law is violated the spectrum in the far zone is not equal to the spectrum of light at the secondary source and it also depends on the observation point. The experimental measurements of the spectrum in the far zone are found to be in good agreement with theory.

2. Coherence in linear optical systems

Consider a general linear system with an impulse-response function given by $h(x,y,\xi,\eta,\omega)$, as illustrated in fig. 1. With a given transverse component of the electric field, we associate a complex analytic signal $u(\xi,\eta,t)$ and define the spectral amplitude $U(\xi,\eta,\omega)$ to be the time-truncated Fourier transform of $u(\xi,\eta,t)$,

$$U(\xi,\eta,\omega)=\int_{-T}^{T} u(\xi,\eta,t)\exp(+i\omega t)\,dt\,. \qquad (1)$$

Using this definition one can express the spectral amplitude of the system output $U_{II}(x,y,\omega)$ in terms of the input spectral amplitude $U_I(\xi,\eta,\omega)$ as follows

$$U_{II}(x,y,\omega) = \iint U_I(\xi,\eta,\omega)\,h(x,y,\xi,\eta,\omega)\,d\xi\,d\eta\,. \qquad (2)$$

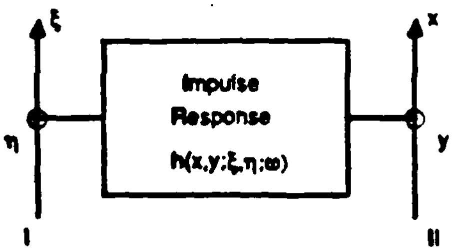

Fig. 1. Linear system with frequency-dependent impulse response $h(x,y,\xi,\eta,\omega)$.

In eq. (2) the spatial coordinates in the input and output planes are denoted by (ξ, η) and (x, y), respectively, and $\omega = 2\pi c/\lambda$, where λ is the wavelength.

When one considers the operation of an optical system with broadband illumination, by far the most convenient quantity to calculate is the cross-spectral density function defined [7] as

$$W(x_1, y_1; x_2, y_2; \omega) = \lim_{T\to\infty} \frac{\langle U(x_1, y_1; \omega)\, U^*(x_2, y_2; \omega)\rangle_s}{2T}, \quad (3)$$

in which $\langle ... \rangle_s$ denotes an average over an ensemble of sources. Note that the time-truncated field is used since, in general, the noiselike field $u(x, y, t)$ is not square integrable.

A quantity of particular interest in this article is the spectral intensity

$$S(x, y, \omega) \equiv W(x, y, x, y, \omega), \quad (4)$$

which represents the intensity at point (x, y) and frequency ω. Note that the total intensity at point (x, y) is obtained simply by integrating $S(x, y, \omega)$ over all temporal frequencies.

If an input object $t_1(\xi, \eta, \omega)$ is inserted at plane I and illuminated by a field with spectral amplitude $U_{in}(\xi, \eta, \omega)$, then the spectral amplitude $U_1(\xi, \eta, \omega)$ leaving plane I is given approximately by

$$U_1(\xi, \eta, \omega) = U_{in}(\xi, \eta, \omega)\, t_1(\xi, \eta, \omega). \quad (5)$$

Using eqs. (2), (3) and (5), the cross-spectral density function $W_{II}(x_1, y_1; x_2, y_2; \omega)$ in plane II is written as

$$\begin{aligned} W_{II}(x_1, y_1; x_2, y_2; \omega) = &\iiiint t_1(\xi, \eta, \omega) \\ &\times t_1^*(\xi', \eta'; \omega)\, W_{in}(\xi, \eta, \xi', \eta'; \omega) \\ &\times h(x_1, y_1; \xi, \eta, \omega)\, h^*(x_2, y_2; \xi', \eta'; \omega) \\ &\times d\xi\, d\eta\, d\xi'\, d\eta'. \end{aligned} \quad (6)$$

Eq. (6) is a general expression relating the cross-spectral densities in planes I and II.

It is useful to define a normalized form of the cross-spectral density function given as follows:

$$\mu_{12}(\omega) = \frac{W(x_1, y_1; x_2, y_2; \omega)}{[S(x_1, y_1; \omega)\, S(x_2, y_2; \omega)]^{1/2}}, \quad (7)$$

in which $|\mu_{12}(\omega)|$ can assume values in the range $0 \le |\mu_{12}(\omega)| \le 1$. $\mu_{12}(\omega)$ is known as the *complex degree of spectral coherence* [8].

3. Control of spatial coherence

Diagrams of the optical systems used in the experiments are shown in fig. 2. In both system configurations, a primary source illuminates a circular aperture of radius a_1 in plane I. It is assumed that illumination is provided by a quasi-homogeneous thermal source [6] located directly behind the aperture in plane I. The impulse-response function connecting planes I and II is used to produce a secondary source with controlled spatial coherence at plane II. The spectrum of light is measured at the secondary source, plane II, and in the far field of the secondary source, plane III.

A theoretical expression for the complex degree of spectral coherence in the secondary source plane II can be calculated using eqs. (6) and (7). For the system shown in fig. 2(a), plane I is located in the front focal plane of an achromatic telescope objective of focal length F. Plane II is located in the back focal plane of the objective. For this case the impulse-

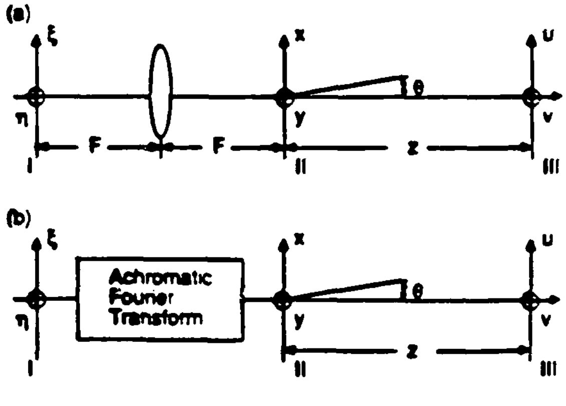

Fig. 2. Experimental configurations for realization of secondary sources with controlled spatial coherence that (a) satisfy the scaling law, and (b) violate the scaling law. The aperture in plane I is illuminated using a broadband, spatially incoherent source. A secondary source with a specified complex degree of spectral coherence is formed in plane II through application of the Van Cittert–Zernike theorem. Measurements of the spectral intensity are made in the secondary source plane II and in the far field of the secondary source, plane III.

response function connecting planes I and II is given by

$$h^{\rm conv}(x,y;\xi,\eta;\omega)$$
$$=(-i\omega/2\pi cF)\exp[(-i\omega/cF)(x\xi+y\eta)]\,; \quad (8)$$

this is the conventional impulse response for a F-to-F Fourier-transform system [9].

In the system arrangement shown in fig. 2(b), an achromatic-Fourier-transform lens [10–12] is inserted between planes I and II. With spatially coherent light the effect of this *Fourier achromat* is to produce an optical Fourier transform in plane II in which the transform size is independent of wavelength. The impulse response of the Fourier achromat is given by

$$h^{\rm afl}(x,y;\xi,\eta;\omega)$$
$$=(-i\omega_0/2\pi cF)\exp[(-i\omega_0/cF)(x\xi+y\eta)]\,, \quad (9)$$

in which ω_0 is a constant and corresponds to a particular design frequency.

The effect of the impulse-response functions given by eqs. (8) and (9) is illustrated in fig. 3; these photographs are optical transform patterns recorded in plane II of the systems in figs. 2(a) and 2(b), respectively. In each case, a 400-μm-diameter, circular aperture is inserted in plane I and illuminated with spatially coherent, broadband light. The spectral bandwidth of the light is 200 nm. In fig. 3(a) one notes that the size of the transform pattern scales linearly with the wavelength of illumination. The scaling of the pattern with wavelength can be traced directly to the frequency dependence of $h^{\rm conv}(x,y;\xi,\eta;\omega)$ in eq. (8). The system in fig. 2(a) produces a secondary source in plane II that obeys the scaling law. In fig. 3(b) it is seen that the scale size of the optical transform pattern produced by the Fourier achromat is independent of the illumination wavelength. The Fourier achromat of fig. 2(b) produces a secondary source in plane II such that the complex degree of spectral coherence is independent of the illumination wavelength, and therefore, does not satisfy the scaling law.

Using eqs. (6)–(8) gives the following expressions for the spectral intensity and complex degree of spectral coherence in plane II for the system in fig. 2(a):

$$S_{\rm II}^{\rm conv}(\omega)=\kappa_0 S^{(0)}(\omega)\,, \quad (10)$$

$$\mu_{12}^{\rm conv}(\omega)=J_1(\chi)/\chi\,, \quad (11)$$

where $S^{(0)}(\omega)$ denotes the spectrum at the primary source, $J_1(\chi)$ is the Bessel function of order one, $\chi=(\omega a_1/cF)[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$, and κ_0 is a constant. We will refer to sources that have a complex degree of spectral coherence with the functional form of eq. (11) as *Bessel-correlated* sources.

By using eqs. (6), (7) and (9) one can calculate expressions for spectral intensity and complex degree of spectral coherence in plane II of fig. 2(b), and one finds that

$$S_{\rm II}^{\rm afl}(\omega)=\kappa_1\,(\omega_0/\omega)^2\,T(\omega)\,S^{(0)}(\omega)\,, \quad (12)$$

$$\mu_{12}^{\rm afl}(\omega)=J_1(\chi')/\chi'\,, \quad (13)$$

where $T(\omega)$ is introduced to account for transmission losses through the Fourier achromat, $\chi'=(\omega_0 a_1/cF)[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$, and κ_1 is a constant. Note that the only difference in the expressions for the complex degree of spectral coherence in eqs. (11) and (13) is that ω in eq. (11) has been replaced by ω_0 in eq. (13). A detailed calculation for the spectral intensity shows that $S_{\rm II}^{\rm afl}(r;\omega)$ is independent of the spatial location r only in a region whose size depends on the spectral bandwidth. In the experiments reported below, a circular aperture is inserted in plane II to ensure that $S_{\rm II}^{\rm afl}(r;\omega)=S_{\rm II}^{\rm afl}(\omega)$ for all points within the aperture.

4. Spectrum of light in the far field

The spectrum of light in the far field of the secondary source located in plane II can be calculated using the above linear system formalism to describe the propagation between planes II and III of fig. 2. The spectral intensity in plane III is readily found to be given by the following formula:

$$S_{\rm III}(u,v,\omega)$$
$$=\iiiint t_{\rm II}(x_1,y_1;\omega)\,t_{\rm II}^*(x_2,y_2;\omega)$$
$$\times W_{\rm II}(x_1,y_1;x_2,y_2;\omega)\,h^{(\infty)}(u,v;x_1,y_1;\omega)$$
$$\times h^{(\infty)*}(u,v;x_2,y_2;\omega)\,dx_1\,dy_1\,dx_2\,dy_2\,, \quad (14)$$

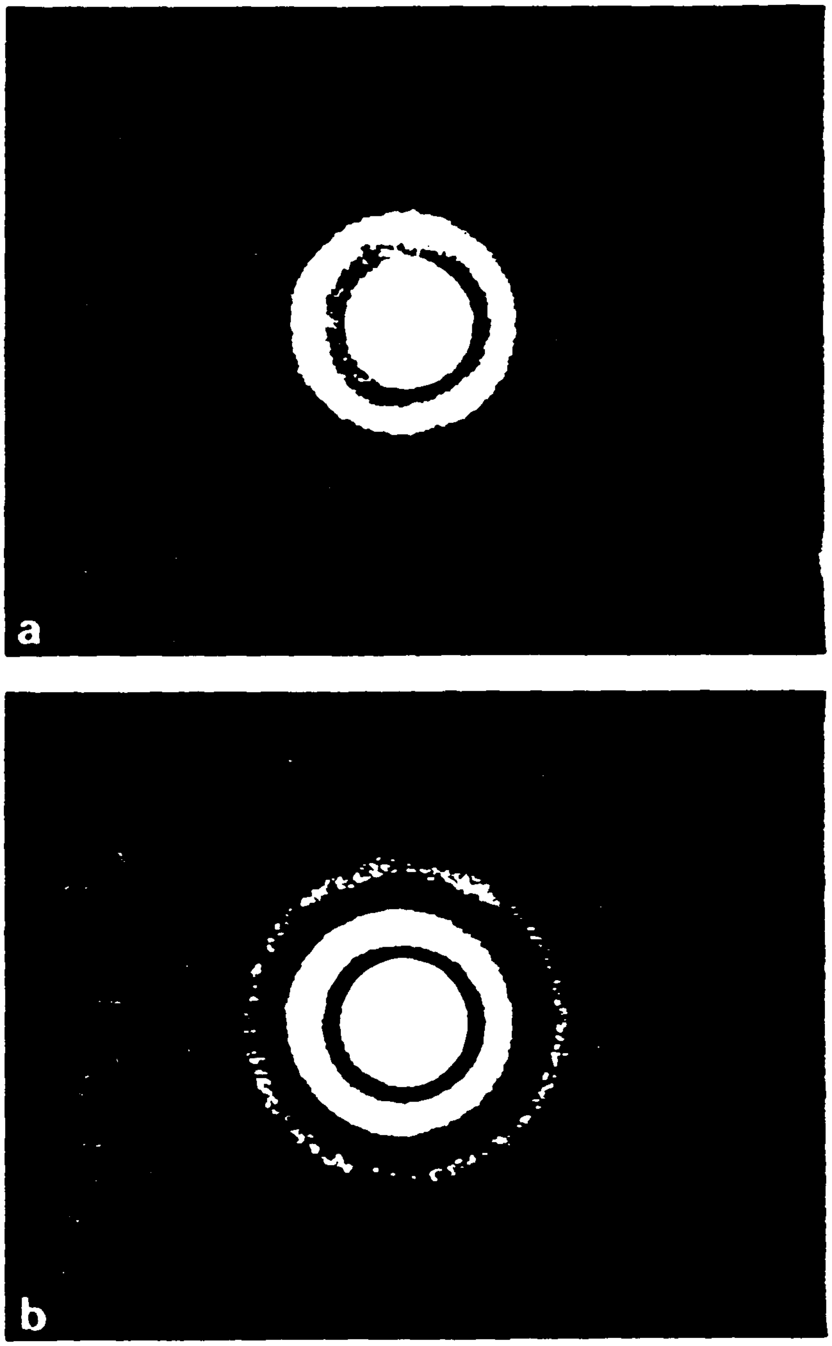

Fig. 3. Airy-disk diffraction pattern recorded in the back focal plane, plane II, of (a) an achromatic telescope objective lens, and (b) an achromatic-Fourier-transform lens. These photographs were produced by illuminating a circular aperture located in plane I with broad-band, spatially coherent light. In (a) the diffraction pattern scales linearly with wavelength, and in (b) the size of the diffraction pattern is independent of the wavelength.

where $t_{II}(x, y; \omega)$ represents a transmission function in plane II, and

$$h^{(\infty)}(u, v; x, y; \omega) = (-i\omega/2\pi cz)$$
$$\times \exp[(+i\omega/2cz)(u^2+v^2)]$$
$$\times \exp[(-i\omega/cz)(ux+vy)] . \quad (15)$$

Taking $t_{II}(x, y; \omega) = 1$ and using eqs. (7), (10), (11) and (14), one finds that

$$S_{III}^{conv}(r, \omega) = \kappa_2 S_{II}^{conv}(\omega) , \quad r \leq a_1 z/F ,$$
$$= 0 , \quad r > a_1 z/F , \quad (16)$$

where $r = (u^2+v^2)^{1/2}$ and κ_2 is a constant.

Using eqs. (7) and (12)–(14) one obtains the following expression for the spectrum of the field in the far zone of the achromatized secondary source of fig. 2(b):

$$S_{III}^{afl}(r, \omega) = \kappa_3(\omega/\omega_0)^4 S_{II}^{afl}(\omega) , \quad r \leq \omega_0 a_1 z/\omega F ,$$
$$= 0 , \quad r > \omega_0 a_1 z/\omega F , \quad (17)$$

where κ_3 is a constant.

5. Experimental results

In the experiments a 100-watt tungsten-halogen lamp is used as the primary source and is located in close proximity behind a circular aperture of radius $a_1 = 2.5$ mm in plane I [see fig. 2]. The focal length F is 180 mm. Using these values the coherence interval at the secondary source plane II is 24 μm at $\lambda = \lambda_0 = 550$ nm.

A circular aperture of radius $a_2 = 0.5$ mm is located in plane II. The far-field condition is satisfied when $z \gg a_2^2/\lambda = 0.5$ m at $\lambda = 500$ nm. To obtain an acceptable SNR in our detection system, the distance z is taken to be one meter. The aperture in plane II is also used to ensure that the spectrum of the secondary source in the experimental arrangement of fig. 2(b) is independent of the spatial location within the aperture [see eq. (12)].

The spectral intensity of the light at the various observation points is measured using a grating monochromator.

Fig. 4(a) shows curves for the spectral intensity measured at the source plane II and at various locations in the far field, plane III, for the system configuration of fig. 2(a). As expected, since this source obeys the scaling law, the normalized spectrum is indeed invariant on propagation.

In the experiments involving the system configuration of fig. 2(b), an all-glass Fourier achromat [11]

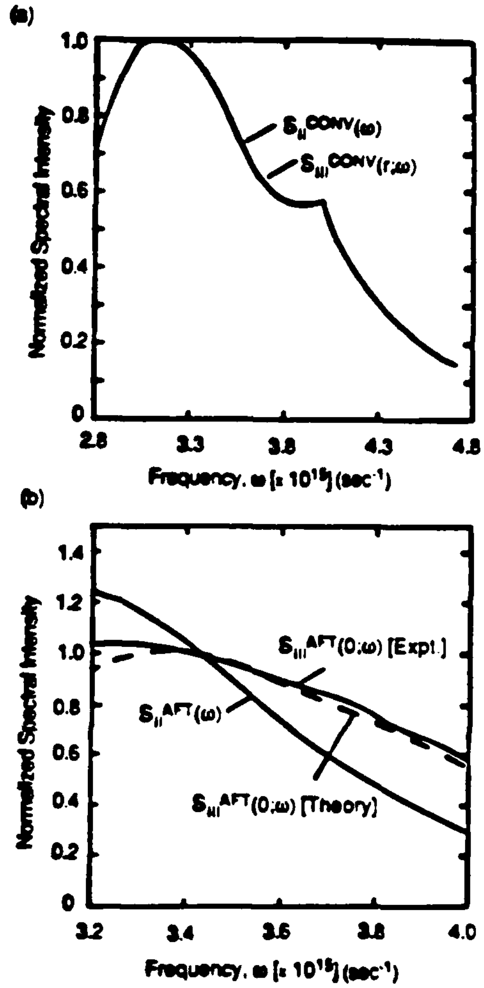

Fig. 4. Normalized spectral intensity at the secondary source plane II and in the far field of the secondary source, plane III, for (a) a source that obeys the scaling law, and (b) a source in which the complex degree of spectral coherence is independent of the illumination frequency (i.e., violates the scaling law). In (a) the spectral intensity is normalized to the peak value of the spectral intensity in the respective planes. In (b) the spectral intensity in planes II and III are scaled so that $S_{II}^{afl}(\omega_0) = S_{III}^{afl}(0; \omega_0) = 1$.

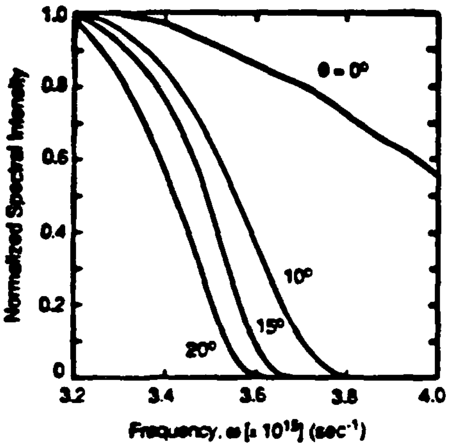

Fig. 5. Measured values of the normalized spectral intensity in the far field, plane III, for case (b) (i.e. when the source does not obey the scaling law). The spectral intensity is measured at off-axis angles $\theta = 0°, 10°, 15°, 20°$. Note that the spectral intensity versus ω narrows as the off-axis angle θ increases. The intensity data are normalized with respect to $S_{III}(0;\omega_1)$, where $\omega_1 = 3.2\times10^{15}\ s^{-1}$.

is used. The lens consists of three widely spaced lens groups. The lens is capable of forming 250 adjacent, achromatized Airy disks across a 7-mm diameter in the optical transform plane. Achromatization is well corrected for wavelengths ranging from approximately 470 nm to 590 nm [$4.0\times10^{15} > \omega > 3.2\times10^{15}\ s^{-1}$].

The spectral intensity measured on-axis at planes II and III for the Fourier-achromat-system configuration is shown in fig. 4(b) – the solid curves are plotted from laboratory measurements and the dashed curve is the theoretical prediction for the spectral intensity as given in eq. (17).

In fig. 5, measurements of the spectral intensity at various off-axis points in plane III of fig. 2(b) are given. Note that the width of the spectral intensity versus ω narrows as the angle of observation increases. For the off-axis points, the measured values of the spectral intensity are somewhat lower than that predicted by eq. (17). This can be explained by the fact that the far-field condition is only approximately satisfied. For the calculation of the spectral intensity at off-axis points in plane III, one should actually use the impulse response that corresponds to a Fresnel-diffraction geometry. However, for the sake of brevity, this calculation is omitted in the present treatment.

6. Discussion

As observed in the experiments and consistent with Wolf's theoretical predictions [1], the spectrum of light in the far field depends on the correlation properties of the light at the source. If the optical field at the source obeys the scaling law, the spectrum of light on propagation is invariant. Departures from the scaling law produce changes in the spectrum that depend on the correlation properties of the source and on the location of the observation point.

In our experiments a circular aperture is inserted in plane I of fig. 2 to produce a secondary source at plane II that is Bessel-correlated [see eqs. (11) and (13)]. A Fourier achromat [see fig. 2(b)] is used to produce a field correlation in plane II that is independent of the illumination frequency (a departure from the scaling law). In this case the spectrum in the far field is different from the spectrum measured at the source and changes for different observation points in the far field [see eq. (17) and figs. 4 and 5]. While a Bessel-correlated source was used in these experiments, it is noted that virtually any allowed functional form for the complex degree of spectral coherence can be produced through application of the Van Cittert-Zernike theorem. For example, a gaussian-correlated source can be produced by inserting a gaussian transmission function in plane I. A Fourier achromat inserted between planes I and II eliminates the wavelength dependence in the complex degree of spectral coherence in plane II. A different frequency dependence for the complex degree of spectral coherence could be produced by a modification of the design rules used for the Fourier achromat.

The effects of source correlation on the spectrum is to be distinguished from changes in the spectrum due to diffraction of fully or partially coherent light by an aperture located in the source plane. Although both effects can modify the spectrum of light on propagation, their physical origin is different. For example, suppose that one placed a diffraction grating in the secondary-source plane II. Using eq. (14) and the appropriate expression for the transmission function $t_{II}(x,y,\omega)$ of a diffraction grating, one can write an expression for the spectrum $S_{III}(u, v; \omega)$ in the far field. The transmission function $t_{II}(x, y, \omega)$ will affect $S_{III}(u, v; \omega)$ only if the coherence interval, associated with the cross-spectral density $W_{II}(x_1, y_1; x_2, y_2; \omega)$, is on the order of or greater than the grating period, i.e. the light is spatially coherent over at least a portion of the grating structure. Similiar reasoning can be used for the case in which a refractive element, such as a prism, is placed in the secondary-source plane.

In our experiments, a 1-mm-diameter circular aperture is inserted in plane II. The purpose of this aperture is to ensure that the far-field condition is satisfied (approximately) in the measurements of $S_{III}(u, v; \omega)$. As stated above, the coherence interval at plane II is 24 μm. Since the coherence interval is much less than the object structure (aperture diameter), the illumination in plane II is essentially incoherent and the aperture does not modify the spectrum of light in the far field. In our experiments, changes in the spectrum on propagation are due to the correlation properties of the secondary source.

Acknowledgements

Helpful discussions with E. Wolf are gratefully acknowledged. This research was supported in part by the New York State Center for Advanced Optical Technology.

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Observation of Frequency Shifts of Spectral Lines Due to Source Correlations

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(Received 24 February 1987)

Wolf has recently shown that the spectrum of radiation from an extended source changes on propagation unless a certain scaling condition is obeyed by the degree of spectral coherence across the source. For a large class of source-coherence functions, the change may be such as to produce red shifts or blue shifts of spectral lines. We have performed an acoustic experiment with two small partially correlated sources and demonstrated Wolf's prediction of frequency shifts of spectral lines by this mechanism.

PACS numbers: 42.68.Hf, 42.10.Mg, 43.20.+g

Wolf[1] has shown theoretically that the spectral density of radiation in the far zone of an extended source may be different from the spectral density of the individual radiating source points if there is some spatial coherence between the source fluctuations at different points in the source region. Morris and Faklis subsequently confirmed experimentally the noninvariance of the spectrum for radiation from a particular optical source.[2] In later publications Wolf[3,4] has shown that the center frequency of a spectral line could be shifted from the center frequency of the same line as seen at the source. This shift could be either a red shift or a blue shift depending on the specific form of the source-correlation function.

Wolf's main result is the following expression for the spectral density of the field at frequency ω, at a point $R\mathbf{u}$ ($\mathbf{u}^2=1$) in the far zone, emitted by sources of a wide class:

$$S_U(R\mathbf{u}) \propto S_Q(\omega)\tilde{\mu}_Q(k\mathbf{u},\omega). \qquad (1)$$

Here $S_Q(\omega)$ is the spectral density at each source point, $\tilde{\mu}_Q(k\mathbf{u},\omega)$ is the spatial Fourier transform of the degree of spectral coherence $\mu_Q(\mathbf{r}_1-\mathbf{r}_2,\omega)$ of the source fluctuations at two typical source points, and $k=\omega/c$, c being the velocity of propagation of the radiation. Formula (1) shows that, in general, the spectrum of the far field will not be equal to the source spectrum $S_Q(\omega)$ but will be modified by the source correlations.

Wolf's analysis was based on correlation theory of scalar fields and hence must also apply to acoustic fields. To test experimentally the dependence of the spectrum of the emitted field on the correlation properties of the source, we considered a very simple radiating system, consisting of two small correlated acoustical sources with the same source spectrum $S_Q(\omega)$ and we measured the spectrum $S_U(\omega)$ of the emitted field in the far zone. The correlation between the two sources can be characterized by a correlation coefficient $\mu_Q(\omega)$ which is a function of the frequency. For our case one finds that Eq. (1) reduces to the following expression for the spectral intensity $S_U(\omega)$ at the observation point:

$$S_U(\omega) \propto S_Q(\omega)[1+\mathrm{Re}\,\mu_Q(\omega)]. \qquad (2)$$

This formula is derived and $\mu_Q(\omega)$ is defined in the accompanying Letter [Wolf,[5] Eq. (8)], where it is also shown that the factor $[1+\mathrm{Re}\,\mu_Q(\omega)]$ in Eq. (2) can produce a field spectrum $S_U(\omega)$ whose central frequency is shifted either towards the lower frequencies (producing a red shift) or towards the higher frequencies (producing a blue shift), compared with the central frequency of the source spectrum $S_Q(\omega)$.

We have demonstrated this "Wolf shift" with a two-point acoustic source which has a frequency-dependent cross correlation of the two source points. We have observed both red shifts and blue shifts.

Our experiment is illustrated in Fig. 1. Two loudspeakers of diameter $d=9$ cm are separated by a distance $D=40$ cm and located at distances R_1 and R_2 from a microphone with an aperture of 1.5 cm. We chose $R_1=R_2=3$ m. None of these dimensions appeared to be critical and were not expected to be. An acoustic spectral line, i.e., band-limited random noise, with center frequency ω_0 (wavelength λ_0) and width $\Delta\omega$ was broadcast from each speaker.

In Fig. 2 we show the detected spectral density of the field from source 1, labeled A, and source 2, labeled B. These spectra were measured at the microphone one at a time, with the other source switched off, and had the same shape as the spectra at the source which we verified by also measuring the spectrum of the voltage wave form which drove each speaker. The intensities of the two sources at each frequency were equal to within 5%; the center frequencies were both at 1180 Hz and the linewidth at half maximum amplitude was ~300 Hz. The spectral density of the sound at the microphone with

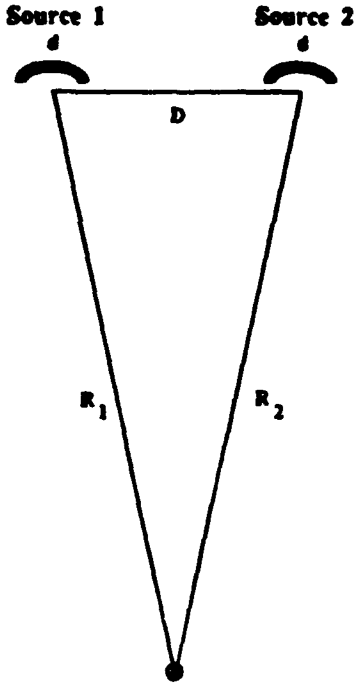

FIG. 1. The configuration of our experiment. Two acoustic sources each with diameter d are located distance D apart. The detector is a microphone at distances R_1 and R_2 from the two sources.

both sources switched on is shown by the curve labeled C in Fig. 2. It has a peak at 1020 Hz exhibiting a red shift of 160 Hz. We will now explain how this was achieved.

Although both sources radiated band-limited random noise with the same spectral density, they were, in fact, partially correlated. Each spectrum had two parts, an anticorrelated component, a broad line centered at 1180 Hz, and a correlated component centered at 1020 Hz. The anticorrelated component of each line was prepared by modulating the phase of a Hewlett-Packard 3325A frequency synthesizer with low-pass filtered white noise which created a 300-Hz-wide spectral line centered at the synthesizer's set frequency. This signal was passed through a L-C notch filter set at 1020 Hz then split into two equal parts, and one part was inverted to yield two anticorrelated random signals $x(t)$ and $-x(t)$. The correlated components were generated by passing the output of a second Hewlett-Packard 3325A synthesizer, which was set to the same frequency as the first and phase modulated by an independent source of random noise, through a L-C band-pass filter tuned to the same frequency as the notch filter. This signal was divided into two equal parts, each denoted by $y(t)$, and added to the uncorrelated signals to give the two signals $x(t)+y(t)$ and $-x(t)+y(t)$ which drove the speakers.

Assuming $x(t)$ and $y(t)$ to be independent random processes, one can calculate the degree of spectral coher-

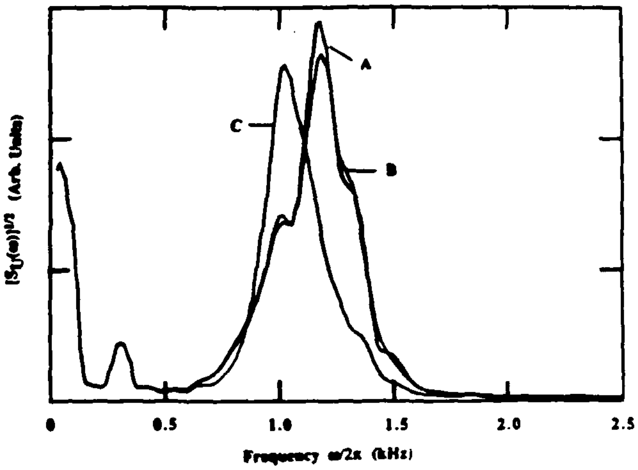

FIG. 2. Square root of the spectral density of the acoustic spectral lines in three cases. The curves labeled A and B are the results of measurements at the microphone when one source at a time is switched on. The curve labeled C represents the red-shifted acoustic spectral line when both sources are switched on to act as a correlated extended source.

ence $\mu_Q(\omega)$ and we find that

$$\mu_Q(\omega) = [2S_y(\omega)/S_Q(\omega)] - 1, \tag{3}$$

where $S_y(\omega)$ is the spectrum of $y(t)$. Using Eq. (2) we find the spectral density at the observation point is

$$S_U(\omega) \propto S_y(\omega), \tag{4}$$

i.e., it is proportional to the spectrum of the correlated components of the signals which drove the two speakers.

We also produced a blue shift of a spectral line by moving the center frequency of the anticorrelated component of the source spectra to 840 Hz keeping the correlated components the same. With both sources

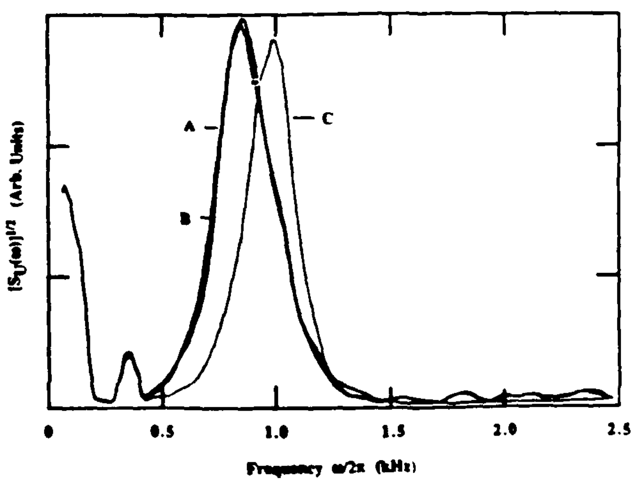

FIG. 3. The curves labeled A and B have the same meaning as in Fig. 2. The curve labeled C now represents a blue-shifted line.

switched on, the peak in the spectrum at the detector was at 1000 Hz, thus exhibiting a 160-Hz blue shift, as seen in Fig. 3.

There were a few other features in the spectra which we now wish to explain. The weak line present at 300 Hz and the feature at very low frequency were both direct electrical pickup from the power lines and had no effect on the experiment. The jagged features in the spectra in Fig. 2 seem to be an effect of the room in which the experiments were performed. The rectangular room was not totally anechoic and the room resonances made it impossible to simultaneously obtain smooth profiles for all the lines.

In summary, we have confirmed experimentally the theoretical prediction that lines in the spectrum of radiation from a partially coherent source may indeed be shifted with respect to their locations in the emitted spectra of the individual source points. The significance of this effect goes, however, beyond the present illustration of principle. It demonstrates a new mechanism by which spectral lines from extended sources may be frequency shifted. This effect may be important in astrophysics. The effect of spatial correlations between emitting elements of an extended astronomical source have not previously been considered (with the possible exception of astronomical masers). Specifically, the cosmological interpretation of the quasar red shifts has been questioned[6,7] and Wolf has suggested that this new mechanism may provide contributions to the red shifts of quasers.[3] However, there has not yet been offered any firm suggestion for the origin of possible correlations among radiating atoms in astronomical sources but the correlation lengths required for producing observable red shifts need only be of the order of a wavelength,[3] which seems a relatively modest requirement.

We would like to thank Emil Wolf for several useful discussions as well as for his valuable suggestions on this Letter. We also thank Dennis Fleisher for discussions and for providing the use of the acoustically shielded room. Finally, we wish to thank Jeffrey Baxter for his assistance with the experiments. This research was supported in part by the Office of Naval Research.

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Volume 67, number 1 OPTICS COMMUNICATIONS 1 June 1988

OBSERVATION OF OPTICAL REDSHIFTS AND BLUESHIFTS PRODUCED BY SOURCE CORRELATIONS

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Received 29 January 1988

Frequency shifts of the optical spectra emitted by a pair of suitably correlated small sources have been recently predicted by Wolf and demonstrated experimentally in the acoustic domain by Bocko, Douglass and Knox. We show that a small modification in the form of the correlation functions used by Wolf leads to a model of the correlated pair that can be easily synthesized in the optical domain. The results of experimental tests exhibiting redshifts and blueshifts are presented.

The existence of spatial correlations within a source affects the way in which the spectral properties of the light emitted by the source propagate [1–4]. Furthermore, it has been proved by Wolf that a suitably correlated source can give rise to redshifts or blueshifts of the emitted light with respect to the case of an uncorrelated source [5,6]. In view of the importance of this prediction, in particular for astronomical problems, experimental proofs of it are very significant. In order to facilitiate experiments, Wolf offered what is perhaps the simplest model of a source exhibiting redshifts and blueshifts of the above type [7]. It consists of a pair of small sources suitably correlated. Bocko, Douglass and Knox [8] succeeded in synthesizing an acoustic source of this type and found a complete agreement with the theoretical predictions.

In spite of its simplicity, the form of the correlation functions used in Wolf's paper is not very easy to synthesize in the optical domain. To overcome this, we propose a small modification of the form of the correlation functions to be used in the source model. We will show that in this way redshifts and blueshifts can still be exhibited while making the experimental synthesis of the source quite easy. We shall begin by recalling a few essential results from the paper of Wolf [7]. Two small fluctuating sources, partially correlated, are located at certain points, say P_1 and P_2. The space–frequency description of the correlation between the source fluctuations is given by the cross-spectral density [9] $W_Q(P_1, P_2, \omega)$, where ω is the angular frequency. The normalized version of the cross-spectral density is the degree of spectral coherence [9]

$$\mu_Q(P_1, P_2, \omega) = W_Q(P_1, P_2, \omega)/S_Q(\omega)\,, \tag{1}$$

where $S_Q(\omega)$ is the spectrum (assumed to be the same) of each of the two source distributions. Suppose that the spectrum of the field produced by the two sources is observed at any point P on the perpendicular bisector of the line joining P_1 and P_2. Wolf showed that such a spectrum, say $S_U(P, \omega)$, is given by

$$S_U(P, \omega) = (2/R^2)\, S_Q(\omega)\, [1 + \mathrm{Re}\,\mu_Q(\omega)]\,, \tag{2}$$

where Re denotes the real part and R is the common distance of the point P from the two sources. Hereafter, the explicit dependence of both μ_Q and W_Q on

P_1 and P_2 is dropped. Eq.(2) clearly shows that the field spectrum S_U at the point P is different from the source spectrum S_Q unless the degree of spectral coherence is independent from ω (in particular, for either uncorrelated sources, i.e. for $\mu_Q(\omega)=0$, or perfectly correlated sources, i.e. for $\mu_Q(\omega)=1$). In certain cases, the field spectrum can be redshifted or blueshifted with respect to the spectrum of the source fluctuations. This can be seen through a suitable choice of $W'_Q(\omega)$ and $S_Q(\omega)$. We shall first consider the form chosen by Wolf in ref. [7]. In order to facilitate the comparison with the modified form to be seen shortly, we shall quote the Wolf's formulas in the following manner, slightly different (but of course equivalent) to the original one

$$W'_Q(\omega)=B\exp[-(\omega-\omega'_0)^2/2\delta'^2_0] -A\exp[-(\omega-\omega_0)^2/2\delta_0^2], \quad (3)$$

$$S_Q(\omega)=A\exp[-(\omega-\omega_0)^2/2\delta_0^2], \quad (4)$$

where A, ω_0, ω'_0, δ_0 and δ'_0 are all positive constants with the same meaning as in ref. [7]. B is also a positive constant and must satisfy the inequality $B/A<2$ (see ref. [7] for further details). On inserting from eqs. (3) and (4) into eqs.(1) and (2), the following expression is obtained for the spectrum of the field at the point P

$$S_U(P,\omega)=(2B/R^2)\exp[-(\omega-\omega'_0)^2/2\delta'^2_0]. \quad (5)$$

Hence, a redshift is produced if $\omega'_0<\omega_0$ and a blueshift in the opposite case.

We now modify W'_Q and S_Q in the following way

$$W'_Q(\omega)=A_\alpha\exp[-(\omega-\omega_\alpha)^2/2\delta_\alpha^2] -A_\beta\exp[-(\omega-\omega_\beta)^2/2\delta_\beta^2], \quad (6)$$

$$S_Q(\omega)=A_\alpha\exp[-(\omega-\omega_\alpha)^2/2\delta_\alpha^2] +A_\beta\exp[-(\omega-\omega_\beta)^2/2\delta_\beta^2], \quad (7)$$

where A_α, A_β, ω_α, ω_β, δ_α and δ_β are positive constants. For the moment, no constraint is imposed on A_α/A_β. On comparing eqs.(6), (7) with eqs.(3), (4), we see that the present case differs from the one considered by Wolf mainly for the form of $S_Q(\omega)$. However, see eq.(1), the normalized version of the correlation function $\mu_Q(\omega)$ is also modified. Proceeding as before, we easily obtain

$$S_U(\omega)=(4A_\alpha/R^2)\exp[-(\omega-\omega_\alpha)^2/2\delta_\alpha^2]. \quad (8)$$

In order to see that eq.(8) can entail a redshift or a blueshift, let us refer to the case $A_\alpha=A_\beta$, $\delta_\alpha=\delta_\beta$ and $|\omega_\alpha-\omega_\beta|<2\delta_\alpha$. In this case, the overlapping of the two gaussian profiles given by eq.(7) gives rise to a curve in which the two component lines are not resolved and the source spectrum has a maximum at $\omega_M=(\omega_\alpha+\omega_\beta)/2$. Accordingly, the field spectrum, see eq.(8), is redshifted if $\omega_\alpha<\omega_M$ and blueshifted in the opposite case. An example is shown in fig. 1, where the numerical values are approximately the same as in the experiment to follow. Furthermore, it can be easily seen that the shift phenomena appear even if the conditions on the parameters of eqs.(6) and (7) are somewhat relaxed.

The main virtue of the present source model, as compared to the previous one, is that an experimental synthesis of it is easier to produce.

In order to see this, we begin by noting that both the cross-spectral density and the power spectrum described by eqs.(6), (7) are the sum of two terms with weighting factors A_α and A_β. Were $A_\beta=0$, the remaining terms in eqs.(6) and (7) would represent perfectly correlated source fluctuations at P_1 and P_2 with a gaussian power spectrum centered at ω_α. In fact, on evaluating the degree of spectral coherence we would find $\mu_Q(\omega)=1$. On the other hand, were

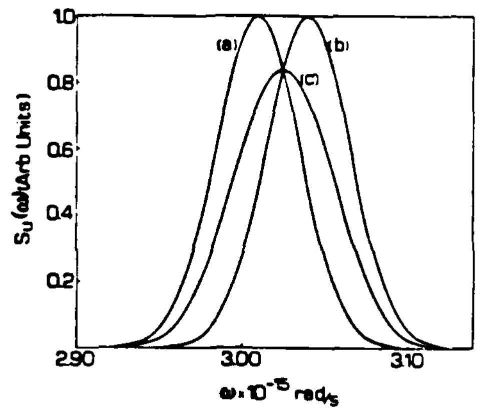

Fig. 1. Redshift and blueshift produced by two sources correlated as in eqs.(6) and (7). Curve (a): spectrum obtained when the sources radiate simultaneously with the following parameters: $\omega_\alpha=3.01\times10^{15}$ rad/s; $\omega_\beta=3.04\times10^{15}$ rad/s; $\delta_\alpha=\delta_\beta=2.5\times10^{13}$ rad/s; $A_\alpha=A_\beta=1$ (arbitrary units). Curve (b): same as curve (a) except that $\omega_\alpha=3.04\times10^{15}$ rad/s and $\omega_\beta=3.01\times10^{15}$ rad/s. Curve (c): field spectrum (multiplied by a factor 2) produced at the observation point by a single source.

$A_\alpha=0$, we would find $\mu_Q(\omega)=-1$, evidentiating that the second term alone in eqs.(6) and (7) accounts for perfectly anticorrelated fluctuations of the sources. For $A_\alpha\neq 0$ and $A_\beta\neq 0$, the source correlation has both a correlated and an anticorrelated component. These two components are mutually independent as implied by the fact that they are summed together in eqs.(6) and (7).

Starting from the previous interpretation it is not difficult to devise possible schemes for synthesizing two small sources describable by eqs.(6) and (7). As an example, let us refer to the scheme of fig. 2. The two sources are realized as small holes, centered at P_1 and P_2, pierced in an opaque mask M. Each hole is illuminated by two different beams of light by means of a beam-splitter BS. One beam, say B_α, provides a spatially coherent and cophasal illumination at P_1 and P_2. Its power spectrum is described by eq.(7) letting $A_\beta=0$. The other beam, say B_β, whose power spectrum is obtained by eq.(7) letting $A_\alpha=0$, gives a spatially coherent but anticorrelated (i.e. with a π dephasing) illumination at P_1 and P_2. The anticorrelation condition can be easily satisfied acting on the angular position of BS in order to produce a suitable tilting of the wavefronts of B_β with respect to M. The spectrum of the field produced by the two sources at a point P located at the same distance R from both sources is then examined by a spectrum analyzer SA.

Of course, the two sources synthesized in this way are secondary instead of primary sources, but this does not change the basic mechanism leading to the redshift or blueshift phenomenon.

We performed an experimental test of the synthesis procedure using a scheme derived from that of fig. 2, with some unessential changes (such as replacing holes by rectilinear slits to improve the luminosity of the system). Each of the two beams B_α and B_β, spatially coherent across the distance from P_1 to P_2, was prepared in a standard way by collimating the light emerging from a sufficiently narrow slit illuminated by a primary halogen lamp. Filters were inserted across each beam in order to produce the required spectral distribution. In a first experiment, aimed at evidentiating the redshift phenomenon, the spectrum of B_α was centered at a wavelength of 526 nm (giving $\omega_\alpha=3.01\times10^{15}$ rad/s) with a linewidth of 8 nm, whereas the spectrum of B_β was centered at 520 nm ($\omega_\beta=3.04\times10^{15}$ rad/s) with a linewidth of 10 nm. In fig. 3, curve (c) gives the spectrum produced at P by a single source (at either P_1 or P_2). Curve (a) gives the spectrum obtained at P when both sources are turned on. A shift of the spectral maximum towards lower frequencies of about 1×10^{13} rad/s is revealed. In order to produce a blueshift, the spectra of B_α and B_β were interchanged ($\omega_\alpha=3.04\times10^{15}$ rad/s; $\omega_\beta=3.01\times10^{15}$ rad/s). The system was adjusted in such a way that the spectrum produced at P by a single source is still represented by the curve (c). The spectrum obtained in this new situation when both sources are radiating is given by curve (b). A blueshift of about 2×10^{13} rad/s is apparent. The asymmetry between redshift and blueshift in the present experiment is to

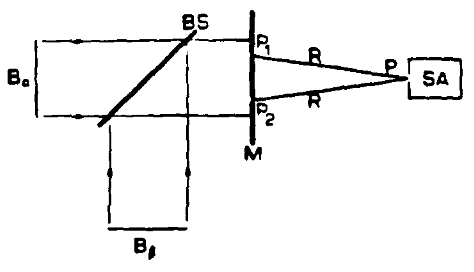

Fig. 2. Experimental setup.

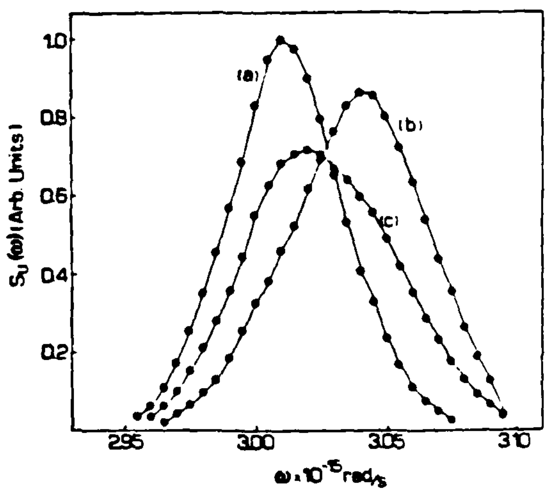

Fig. 3. Experimental results (heavy dots) of the measurement of field spectrum for the redshifting (curve (a)) and the blueshifting (curve (b)). Curve (c) represents the spectrum (multiplied by a factor 2) produced by a single source. For the sake of clarity, the experimental points have been connected by lines.

be ascribed to differences of the two spectral components (both in linewidth and in weight).

While this paper was managed for publication, the results of a similar experiment were presented by W.H. Knox and R.S. Knox at the 1987 Annual Meeting of the Optical Society of America.

This work was supported by Ministero Pubblica Istruzione (Italy).

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Volume 73, number 3 OPTICS COMMUNICATIONS 1 October 1989

WOLF SHIFT AND ITS APPLICATION IN SPECTRORADIOMETRY

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Received 24 April 1989; revised manuscript received 20 June 1989

Shifts in spectral lines, due to source correlation and violation of scaling law, have been discussed with reference to spectroradiometric measurements. The large scatter found in the intercomparison of spectroradiometric scales maintained by standards laboratories is attributed to the spectral shifts.

1. Introduction

Recently Wolf [1–3] has shown that statistical correlation of light emitted by a partially coherent source can produce a frequency shift in the spectrum observed in the far field. This shift is produced only when the correlation function of the emitted radiation does not satisfy a certain scaling law. A few experiments [4–8] have been performed to prove this theoretical prediction which has far reaching consequences in the field of optical measurements. The effect can be directly observed in spectroradiometric measurements where the spectral power of a source is measured as power emanating/incident per unit area per unit wavelength. To measure the spectral power distribution, different apertures are placed in the optical path between the source and the monochromator. These apertures make the incoherent light partially coherent and a source correlation is introduced which along with the optics involved violates certain scaling law thereby modifying the spectral distribution of the source in the far zone. While discussing the sources of errors in spectroradiometry, Moore [9] has stated emphatically that the spectral transmission characteristics of monochromators are notoriously non-uniform. No suitable explanation of this non-uniform behaviour of monochromator transmission was given. On the basis of our experimental observations, it is felt that shifts in the spectral lines due to source correlation [2,3] may have some bearing with the unusual behaviour of the monochromator transmission.

In spectroradiometric measurements it is usual to place an integrating sphere before the entrance slit of the monochromator so that radiation enters the monochromator only after undergoing multiple reflections within the sphere. Accurately measured small apertures of mm order dimensions are put on the surface of the integrating sphere. These apertures and the optics involved modify the degree of spectral coherence of the light incident on the monochromator and the monochromator behaves in an erratic manner.

This communication describes the effects of apertures placed in the optical path and the observed shifts in the source spectrum. We have also endeavoured to provide an explanation for the unexpected nonuniform behaviour of the monochromator.

2. Theory

Consider a quasi-homogeneous source whose spectrum is a line having gaussian profile

$$S_Q(\omega)=(1/\delta\sqrt{2\pi}) \times \exp[-(\omega-\omega_0)^2/2\delta^2]\,, \quad \delta \ll \omega_0\,, \tag{1}$$

and whose degree of spectral coherence is also gaussian and independent of ω viz.

$$\mu_Q(r)=\exp(-r^2/2\xi^2)\,, \tag{2a}$$

and

$$\tilde{\mu}_Q(k)=(\xi\sqrt{2\pi})^3\exp(-k^2\xi^2/2)\,, \tag{2b}$$

where ξ is called effective coherence length and $\tilde{\mu}_Q(\boldsymbol{k})$ is a three-dimensional Fourier transform of $\mu_Q(\boldsymbol{r})$. Wolf has shown that such a source will emit light whose spectrum in the far zone will be red shifted if the degree of spectral coherence and coherence length is independent of frequency ω. In general, the spectral profile and degree of coherence may not be true gaussian. In such cases a shift may occur on either side of the source spectrum.

The spectrum of light in the far zone generated by a source represented by eqs. (1) and (2) is given by

$$S_{\xi}^{\infty}(\omega)=(\alpha/\delta\sqrt{2\pi}) \times \exp[-(\omega-\omega_0^2/\alpha^2)/2(\delta/\alpha)^2] , \qquad (3)$$

where $\alpha^2=1+(\delta\xi/c)^2$, c being the velocity of light

Eq. (3) shows that the emitted light in the far zone has a gaussian profile. The frequency is not centred at ω_0 but is shifted.

It is thus evident that if some correlation exists in the light emitted by a source, the spectrum in the far zone is shifted with respect to the source spectrum and the amount of shift depends on the effective correlation length.

3. Experimental results

To study the effect of apertures placed in between the source and monochromator vis-a-vis the correlation properties of the source, the experimental set up (fig. 1) used was an integrating sphere of 0.5 m diameter coated with $BaSO_4$ paint. A 450 W tungsten halogen lamp was placed at the centre of the sphere. An aperture of 0.24 cm diameter was put on the surface of the sphere. After multiple reflections, the light falls on this aperture. (The aperture plane is assigned as source plane I.) Different filter–lens and filter–mirror combinations were used to produce a secondary source at the focus of the combination (considered as secondary plane II). Apertures of varying sizes from 0.24 cm to 1.0 cm were put at the secondary plane II. The spectrum of light in far zone i.e. in plane III, was recorded with the help of a 0.5 m Jarrell–Ash grating monochromator without and with the apertures. It was observed that without the apertures, the spectrum of light in plane III was independent of the location of the monochromator. Insertion of an aperture in plane II made the spectrum shift. Observations with varying aperture sizes were made and it was observed that after a critical size of the aperture the shift disappeared. The observations were repetitive and reproducible and are presented in table 1.

The circular aperture placed at plane I produced a secondary source at plane II. Various combinations of filter–lens/mirror were used to eliminate the wavelength dependence of the complex degree of spectral coherence which is produced through application of Van Cittert–Zernike theorem. The illumination thus produced in plane II violated the scaling law. Introduction of a circular aperture in plane II helped in modifying the spectrum in the far zone.

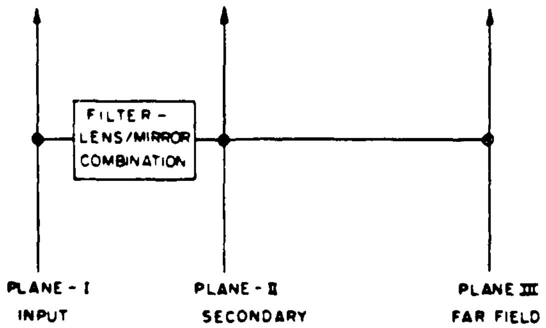

Fig. 1. Experimental arrangement for realizing secondary source with varying degree of spectral coherence.

Table 1
Spectral shift variation with different aperture

Peak transmission wavelength	Half band-width (nm)	Shifted peak transmission wavelength (nm) with secondary aperture	
		dia 2.4 mm	dia 10 mm
422.0	9	421.0	422.0
484.1	9	483.6	484.1
512.4	5	514.1	512.4
566.1	13	564.1	566.1
609.1	8	610.3	609.1
652.0	8	653.2	652.0

4. Discussion

The above observations reveal that if the size of the aperture is very small the shift in the spectral line is large and when the aperture size is increased the shift reduces. In all spectroradiometric measurements it is a common practice that small apertures of the size of mm are used on the integrating sphere. To the best of our knowledge no work has been reported where the partial coherent nature introduced by propagation through the optical system and rendered to the light emitted from the aperture of an integrating sphere is taken into consideration. The experimental evidence [8] as well as the theoretical stimulations have, however, indicated that due weight has to be given to the correlation properties of light emitted by the apertures and the effect of correlation property on the spectrum of the emitted light. If these above factors are considered, it is expected that partially coherent light when viewed through interposed optics and monochromator will show a spectral shift. The nature and magnitude of the spectral shift will depend on the optical path and the violation of scaling law for the spectral degree of coherence. This qualitatively explains the unexpected behaviour in the measurement through a monochromator which was earlier ignored or was taken as measurement uncertainty. Far reaching consequences of this effect can be expected throughout the entire electromagnetic spectrum. For example the large scatter in the 1974 intercomparison [9] of the spectral irradiance scale of eight standard laboratories could, to some extent, be attributed to the correlation property of the light emitted from the integrating sphere and also due to different optical set ups used by different national laboratories.

In a series of papers, Wolf has shown that if the spectral degree of coherence follows scaling law, the far field has the same spectrum as that of the non coherent light. It means that for a source which is effectively spatially incoherent i.e. $\zeta \to 0$ and $\tilde{\mu}_{ij}(k) \to 0$, no shift will occur. The above prediction has been verified by us. It is also evident from the theory that for a source with $|\mu_0(r)| = 1$, this shift will be very small but for a partially coherent light which violates scaling law, the shift is appreciable. In our observation we have seen that if the size of the secondary apertures is less than the size of the coherence length produced by the primary aperture, the shift is less and if a third aperture with a size less than the coherence length produced by the secondary aperture is placed at plane III, no shift is observed. With increase in the size of the aperture at the secondary plane we have observed that the shift increases and that after a certain size of the aperture the shift disappears. The observations are consistent with the theoretical findings of Wolf.

In order to establish that the size of an aperture placed before a radiometer plays a significant role in spectral power measurements, we have taken observations by placing a filter having a $V(\lambda)$ response before the aperture. Fig. 2 shows the shift in the spectral distribution of a lamp–$V(\lambda)$ filter combination in presence and in absence of a secondary aperture. The observed spectral shift was 10 nm. In the case of interference filters, the shifts observed were of the order of their half bandwidth. For $V(\lambda)$ filter, however, the shift did not correspond to its large half bandwidth but was relatively very small. We attribute the smaller shift with the $V(\lambda)$ filter to a partial violation of the scaling law. It was also found that, if the size of secondary aperture is more than some critical size, $V(\lambda)$ spectral distribution does not change during propagation.

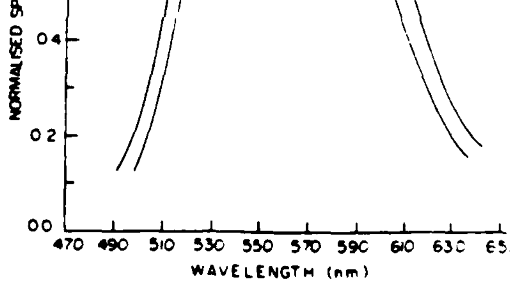

Fig. 2. Spectral distribution of $V(\lambda)$ filter using lens optics (A) without secondary aperture and (B) with secondary aperture.

It is, therefore, concluded that mere placing an aperture of any arbitrary value makes the spectroradiometric and radiometric measurements erroneous. It should be ensured before taking any measurement that the spectral distribution of the source and the filters or monochromator combination should not change by the optics involved in the path of the light beam. Otherwise the measurements will not be true measurements and may result in large deviations from the true values.

Acknowledgement

The authors wish to thank the Director, National Physical Laboratory, New Delhi, for permission to publish this paper.

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