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MADALINE RULE II: AN INVESTIGATION

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF ELECTRICAL ENGINEERING
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

By
Rodney G. Winter
January 1989

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Bernard Widrow
(Principal Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Joseph W. Goodman

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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Abstract

Madaline Rule II, MRII, is a supervised learning algorithm for training layered feed-forward networks of Adalines. An Adaline is a basic neuron-like processing element which forms a binary output based upon a weighted sum of its inputs. The algorithm was developed based on the minimal disturbance principle. This principle states that changes made to the network to correct errors should disturb it as little as possible. A method to insure all processing elements share responsibility for forming the global solution was introduced and called usage. Details of the algorithm were developed with consideration for hardware implementation.

Simulation results of MRII are presented. The algorithm exhibits interesting generalization properties. Generalization is the network's ability to make correct responses to inputs not included in the training set. Networks that contained more Adalines than necessary to solve a given training problem exhibited good generalization when trained by MRII. The algorithm was found to not always converge to known solutions. A failure mode of the algorithm was identified and is detailed in this report.

The sensitivity of Madaline networks to random changes in weights and errors in inputs was investigated. Simple formulas were found to predict the average change of the input/output mapping due to these effects. These results apply to networks where the individual weights are selected independently from a random distribution.

Preface

The guidance and direction of Dr. Bernard Widrow during the conduct of this research is gratefully acknowledged. His insistence for simplicity and clarity forced me to arrive at simple results. He is largely responsible for any utility the results presented here have.

The support of Ben Passarelli of Alliant Computer Systems Corporation is also greatly appreciated. Ben made available his company's equipment for performing many of the simulations presented in this report. More importantly, he gave selflessly of his time to teach me how to best use the equipment and make it available on weekends.

My colleagues in the "zoo" have been the source of much inspiration. Special thanks goes to Maryhelen Stevenson who proofread much of this report before the reading commitee. Her competence saved me some embarrassment.

During the period of this research, the author was an active duty officer in the United States Air Force. He was assigned to Stanford University under the Civilian Institutes Program of the Air Force Institute of Technology. No information or statement contained in this report should be construed to be supported by or representative of the policies or views of the United States Air Force.

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Chapter 1

Introduction

Research in the field of neural networks has seen a resurgence of interest in the past several years. Neural networks offer the hope of being able to teach a machine how to perform a given task by showing it examples of sample behavior. Since the machine learns by experience, the task at hand does not need to be decomposed into an algorithm that can be programmed into the machine. This dissertation investigates a method by which a particular type of neural network can be trained.

The neural network to be trained is a layered feed-forward Adaline network. The term Adaline, which stands for "adaptive linear neuron," was coined by Widrow in 1959 [1]. The original work presented in this report builds on the work done by Widrow and many of his graduate students in the early 1960s. This introduction will review some of this past work on Adalines and simple networks of Adalines, called Madalines for "many Adalines." The concepts behind Madaline Rule II, which is a method for training more complicated networks of Adalines, will also be presented.

1.1 Contributions

This dissertation presents two major contributions. The first is the Madaline Rule II algorithm, MRII. The second is the sensitivity results presented in Chapter 6. There are some minor contributions within these two major ones.

The Madaline Rule II, as the "II" suggests, is the result of continuing work started by others. This introduction chapter will outline this previous work. First the Adaline will be presented. Then a simple network of Adalines called a Madaline will be presented. This early Madaline has a much simpler structure than that addressed by Madaline Rule II. The procedures for training these two simple structures are grounded in a concept called minimal disturbance. Minimal disturbance is a basic concept behind MRII and is the author's inheritance from the early researchers. The last section of this chapter explains a concept for training a complex Madaline using a sequence of trial adaptations. The trials least disturbing to the network are to be performed first, in accord with the minimal disturbance principle. This idea is presented by the author in greater detail than previously published by other researchers in the last section of this chapter. Though more detailed, the ideas presented there follow directly from previous work and the author claims no contribution for them.

The author has taken the ideas left behind by the early researchers and made them work. In formulating MRII, some changes to the minimal disturbance principle were required. These changes are detailed in Chapter 3 and are the author's contributions. Real neural networks need to be built in circuitry. This requirement molded the choices of which trial adaptations to perform. Minimal disturbance also needed modification to insure all units shared responsibility for arriving at a network solution. This was implemented by the author as "usage" in the MRII algorithm. After discovering the need for and implementing usage, the author discovered a similar idea had been proposed by another. This minor contribution by the author appears to be independent but not original. The author's experimental results with the algorithm are offered as a contribution as well as the several heuristics presented for using the algorithm.

In working with MRII, the author discovered a failure mode of the algorithm. This mode prevents the algorithm from coming to a solution even when a known solution exists. This mode takes on the form of a limit cycle and is detailed in Chapter 5. The author presents this as a contribution since an understanding of this phenomenon is essential to improving the algorithm. An attempt to improve MRII by finding an escape to the failure mode led the author to his second major contribution.

The sensitivity of the input/output response of Madalines to changes in the weights of the system as well as to errors in the inputs to the system were analyzed. Approximations were made to reduce the analytic results to useful form. These approximations were verified for accuracy by experiment. The result is a useful and relatively easy to use set of equations that accurately predict the sensitivity of "random" Madalines. These results should be a worst case situation for Madalines that have been trained. The results should also be a useful tool for further research into improving the MRII algorithm.

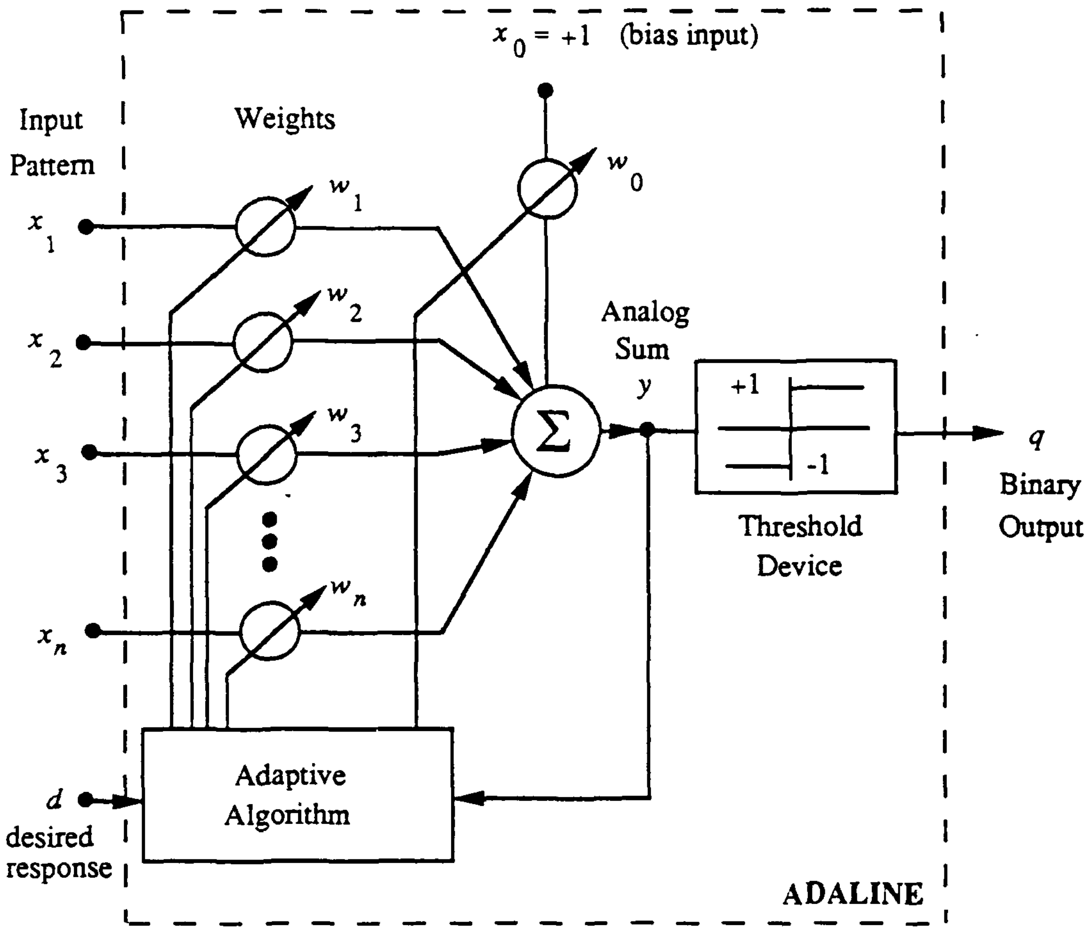


Figure 1.1: The adaptive linear neuron, or Adaline.

1.2 The Adaline

The Adaline is an adaptive threshold logic unit and is depicted in Figure 1.1. The Adaline performs a weighted sum of its inputs and makes a binary decision based on this sum. The input to the Adaline has n variable components and compose what will be called the input pattern. The input pattern along with the constant +1 bias input form the $(n+1)$-dimensional input vector $\vec{X} = [x_0 = +1, x_1, \ldots, x_n]^T$. The input **vector** is the bias augmented input **pattern**, a semantic distinction which will be followed throughout this paper. The weighting vector is $\vec{W} = [w_0, w_1, \ldots, w_n]^T$. The weighted sum, referred to as the Adaline's analog sum or analog output, is the dot product of these two vectors, $y = \vec{X}^T\vec{W}$. The Adaline's binary output is the result of passing this analog sum through a threshold device, $q = \text{sgn}(y)$, where $\text{sgn}(\cdot)$ is $+1$ for positive argument, and -1 otherwise.

The individual components of the input vector could be any real analog values but often are restricted to being binary. Except as noted, the input component values will be either $+1$ or -1 in this paper. There are two reasons for making this restriction. First, this will be the case when an Adaline is receiving its inputs from the outputs of other Adalines. Secondly, with this restriction on the inputs, the magnitude of the input vector, $|\vec{X}|$, will be a constant $\sqrt{n+1}$. This will simplify the mathematical analysis to be presented.

The weights are allowed to be continuous real valued numbers. From Figure 1.1 it is noted that these weights are adjustable by an adaptive algorithm. The actual hardware implementation of adjustable analog valued weights will not be specifically addressed in this paper. This issue has been addressed in the past and resulted in the invention of a device called a memistor [2]. Solid state implementations of adjustable weights are the focus of much current research. It will be assumed here that such weights are available for the eventual hardware realization of neural networks. The issue of how close these weights have to be to their nominal values will be addressed later.

The Adaline is capable of making a binary decision based upon its inputs. What is the nature of this decision making capability? This can be determined by examining the Adaline's governing equation at its decision threshold, that is, when the analog sum y is zero. For the case of an Adaline with two variable inputs this equation is:

$$y = \vec{X}^T\vec{W} = w_0 + x_1w_1 + x_2w_2 = 0\,.$$

This can be rewritten as:

$$x_2 = -\frac{w_1}{w_2}x_1 - \frac{w_0}{w_2}\,.$$

This is a straight line in the x_1–x_2 input space with a slope $-w_1/w_2$ and x_2-intercept of $-w_0/w_2$. An example plot of this equation is shown in Figure 1.2. The inputs to the right of the separating line cause the analog sum to be positive, resulting in a $+1$ decision by the Adaline. To the left of the line, inputs result in a -1 decision. The Adaline thus performs a mapping from its multi-dimensional input space to its one-dimensional binary output space. The input/output mapping effected by this example Adaline is represented by:

$$\begin{aligned}(+1,+1) &\mapsto +1\\(+1,-1) &\mapsto +1\\(-1,-1) &\mapsto +1\end{aligned} \tag{1.1}$$

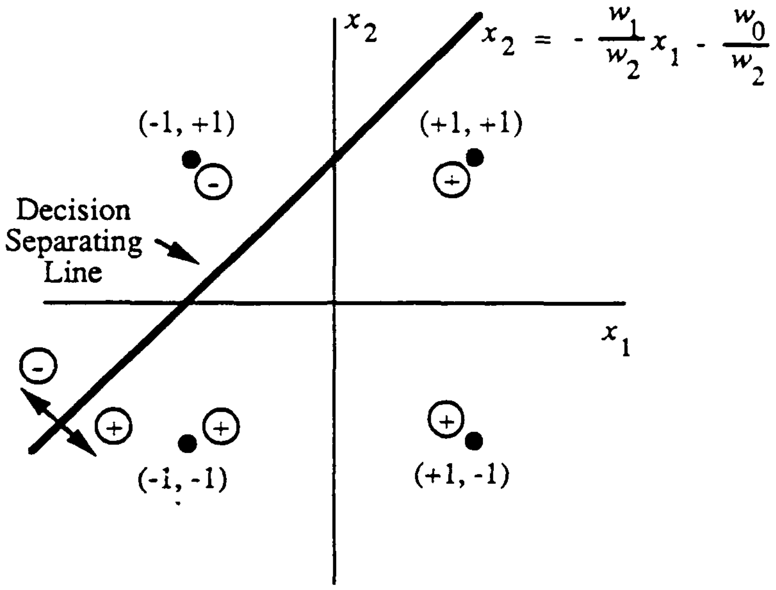


Figure 1.2: Graph of Adaline decision separating line in input 2-space.

$$(-1,+1) \mapsto -1$$

By adjusting the weights, the orientation of the line in Figure 1.2 can be changed as well as which side of the line results in a positive decision. This then can cause a change in the input/output mapping of the Adaline. Methods for adapting the weights of a single Adaline to achieve a desired input/output mapping were developed in the early 1960's.

The method for changing the weights is represented by the "adaptive algorithm" block in Figure 1.1. The algorithm has as inputs the desired response, d, for the pattern being presented and the actual analog response of the Adaline. Mays [3] summarizes three adaptation procedures for the single Adaline. All three of these procedures will produce a set of weights to provide the desired input/output mapping, if such weights exist, in a finite number of adaptations. An example of an input/output mapping that cannot be achieved by a single Adaline will be shown later.

Mays' formulation of the adaptation procedures included a concept called the deadzone. In any hardware implementation of an Adaline, a real thresholding element will be used. If the analog sum y is very close to zero, it is possible the thresholder could flip output states in an erratic fashion due to noise. Manufacturing tolerances might also cause the actual threshold to be different from zero. For these reasons it is desired that the magnitude of the analog sum be greater than a deadzone value $\delta > 0$. The magnitude of the analog sum is referred to as the confidence level. It is a measure of how sure the Adaline is about its decision. During training then, not only must the binary decision be correct, but it must be made with a minimum confidence level equal to the deadzone.

All of the Adaline adaptation procedures can be summarized as follows. There exists a set of patterns and associated desired responses that the Adaline is to learn. This set is called the training set. Present a pattern and its associated desired response from the training set to the Adaline. If the binary response is correct and the confidence level is greater than the deadzone, go on to the next pattern. If the response is incorrect or not confident enough, make a change in the weights. The weights will be changed by adding or subtracting some portion of the input. The weight changing method for the modified relaxation procedure is detailed here.

$$\begin{aligned} \vec{\mathrm{W}}(k+1) &= \vec{\mathrm{W}}(k) && \text{for } d\vec{\mathrm{X}}^T\vec{\mathrm{W}}(k) \geq \delta \\ &= \vec{\mathrm{W}}(k) + \frac{\eta d}{n+1}\vec{\mathrm{X}}[L - d\vec{\mathrm{X}}^T\vec{\mathrm{W}}(k)] && \text{for } d\vec{\mathrm{X}}^T\vec{\mathrm{W}}(k) < \delta \end{aligned} \tag{1.2}$$

Here k is an adaptation counter so that $\vec{\mathrm{W}}(k+1)$ are the weights after the kth adaptation. As before, d is the desired binary response associated with the input vector $\vec{\mathrm{X}}$. The adaptation constant, η, must have value $0 < \eta \leq 2$ to insure convergence. The deadzone can be selected $0 < \delta < 1$. The quantity L is called the adaptation level and is selected $\delta < L$. The adaptation level can be thought of as a target confidence level when $\eta = 1$. To see this, compute the resulting analog sum after adaptation by premultiplying both sides of Equation 1.2 by $\vec{\mathrm{X}}^T$. Remembering that $y = \vec{\mathrm{X}}^T\vec{\mathrm{W}}$, and $\vec{\mathrm{X}}^T\vec{\mathrm{X}} = n + 1$:

$$\begin{aligned} y(k+1) &= y(k) + \eta d[L - dy(k)] \\ &= y(k)[1-\eta] + \eta dL \\ &= dL \qquad \text{for } \eta = 1. \end{aligned}$$

After adaptation, the Adaline will provide the correct response with confidence equal to the adaptation level when $\eta = 1$ is used in the modified relaxation scheme.

1.3 Early Madalines

There are some input/output mappings that a single Adaline cannot realize. The class of functions that can be realized by a single Adaline are called linearly separable. From Figure 1.2 it can be seen that inputs requiring plus decisions must be separable from those requiring a minus decision by a straight line. In higher dimensional input spaces this requirement generalizes to requiring the separation being done by a hyperplane. An input/output mapping that is not linearly separable for the two variable input case is the exclusive-or function represented by:

$$\begin{aligned} (+1,+1) &\mapsto -1 \\ (-1,+1) &\mapsto +1 \\ (-1,-1) &\mapsto -1 \\ (+1,-1) &\mapsto +1 \end{aligned} \tag{1.3}$$

The number of binary functions of n variable inputs is 2^{2^n}. For $n = 2$, 14 of the 16 functions are linearly separable. While no general formula exists for determining how many of the possible functions of n variables are linearly separable for general n, it is known that the fraction becomes very small for even moderate values of n. For example, at $n = 5$ only 94,572 of the possible 4.3 x 10^9 binary functions are linearly separable [4]. Thus, the single Adaline's ability to realize arbitrary input/output mappings in high dimension input spaces is very limited.

To combat this limitation of the Adaline, Ridgway [4] used simple networks of Adalines which were called Madalines. The form of Madaline investigated by Ridgway is shown in Figure 1.3. It consists of a layer of Adalines whose outputs feed into a fixed logic element. All of the Adalines receive the input vector $\vec{X}$ as an input. The Madaline's response is taken at the output of the fixed logic unit and is compared with the desired response associated with a particular $\vec{X}$ during training.

Some of the fixed logic units used by Ridgway are the AND, OR and majority vote taker elements. All of these logic units can be realized by an Adaline with fixed weights as shown in Figure 1.4. The weights shown in this figure are not unique but do realize the required logic function. Thus Ridgway's Madalines can be thought of as the simplest of 2-layer feed-forward Adaline networks, the second layer being restricted to a single nonadaptive Adaline.

Ridgway developed a method for training Madalines of the type in Figure 1.3. Because the logic element is fixed, it is possible to determine which Adaline(s) are contributing to any output errors during training. This determination of which elements in a network are contributing correctly to the networks overall output is commonly referred to as the credit

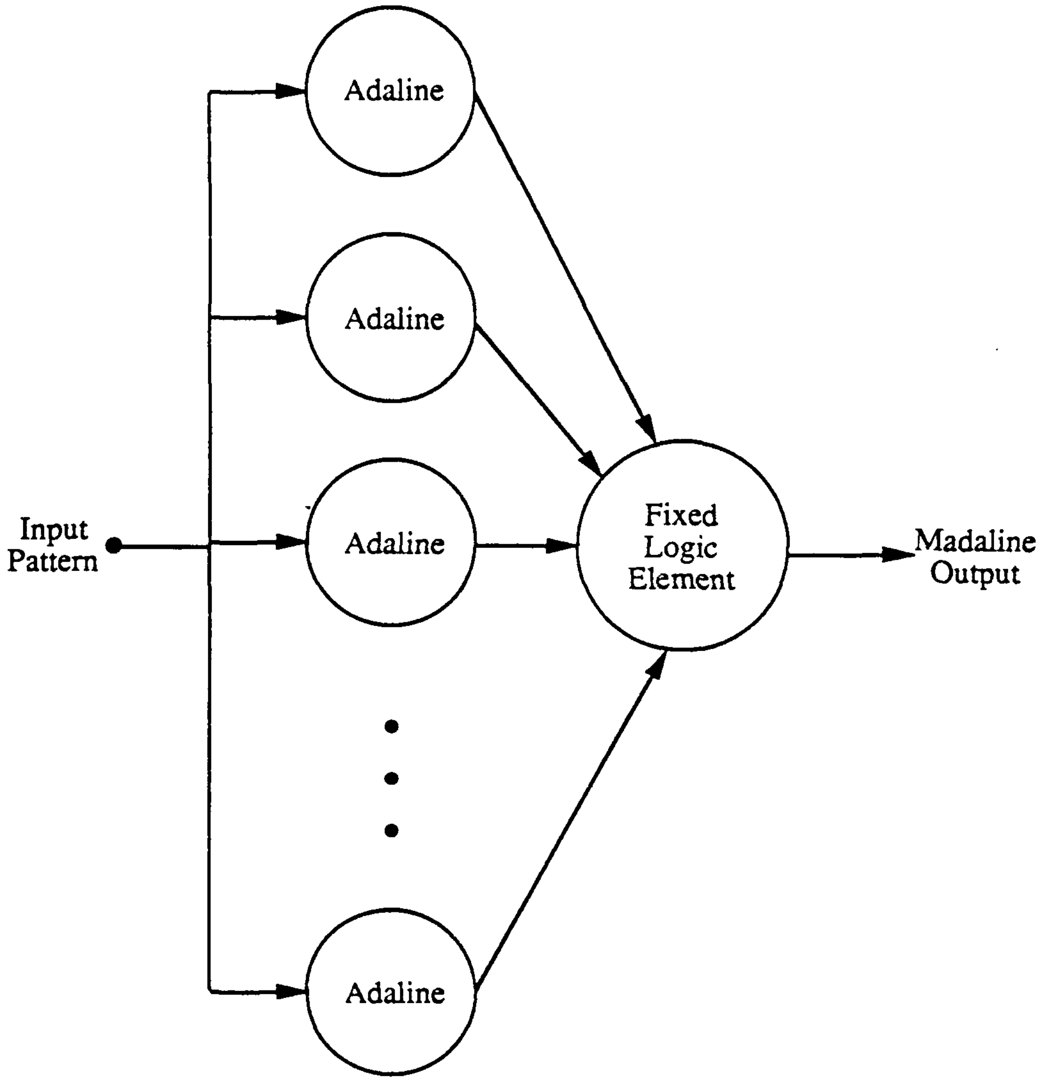


Figure 1.3: Structure of the Ridgway Madaline.

assignment problem.

Consider the case where the logic element is an OR. A multi-input OR element makes a $+1$ decision whenever one or more of its inputs is $+1$. It makes a -1 decision only when all of its inputs are -1. Suppose a pattern is presented and its desired response is -1 but the Madaline's actual response is $+1$. All those Adalines with $+1$ outputs need to be adapted to provide a minus response. These adaptations can be done using the methods outlined by Mays. Suppose instead the desired response is $+1$ but the actual response is -1. This means all of the Adalines are responding with -1. One or more of these Adalines need to

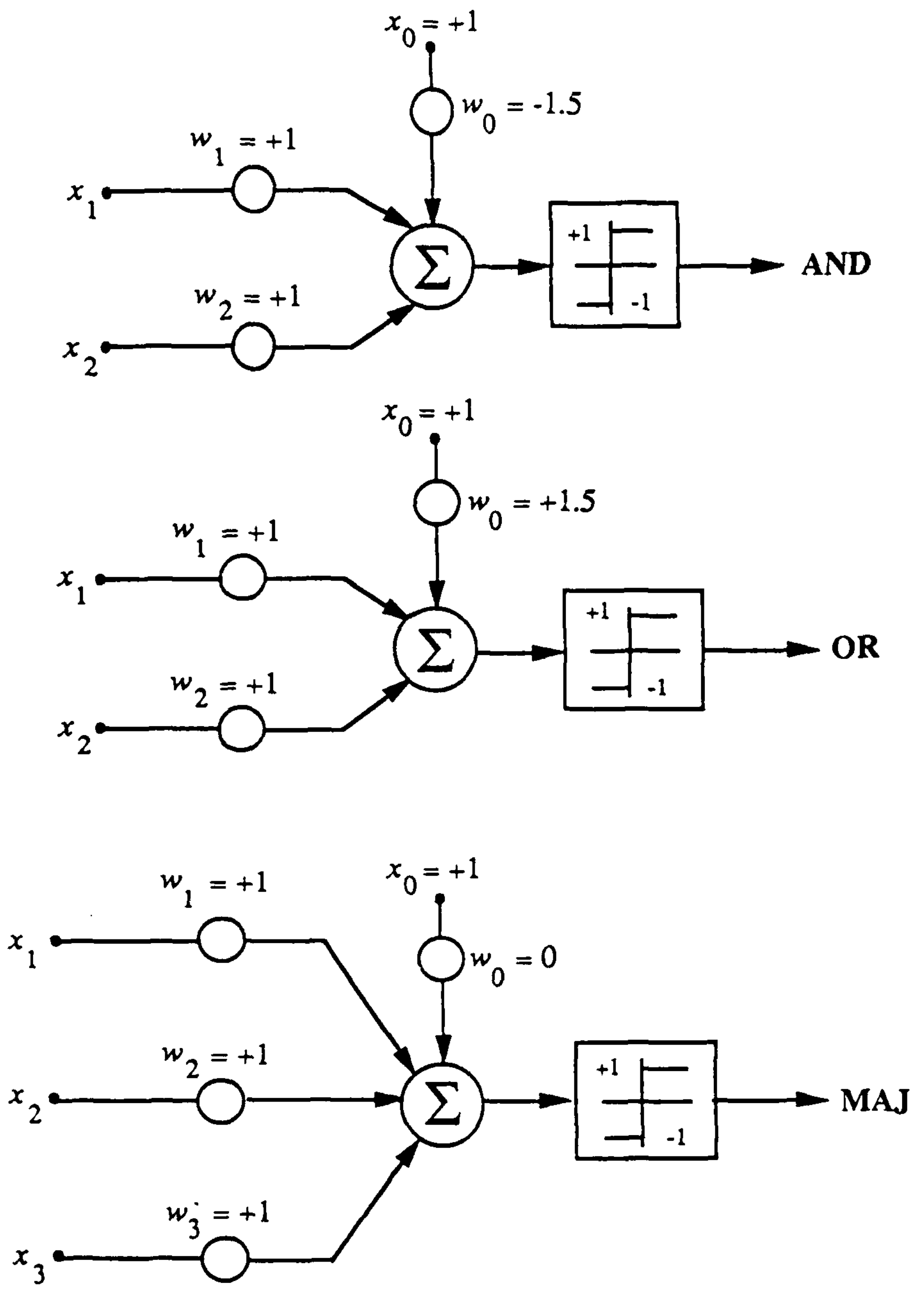


Figure 1.4: Implementation of the AND, OR and MAJority logic units using Adalines with fixed weights.

be adapted to respond with +1 to correct the Madaline's overall response. Ridgway's rule says to adapt only one Adaline and to choose the one with the lowest confidence, that is, the one whose analog sum is closest to zero. Of course, if the actual response is correct, no changes need be made.

Ridgway's algorithm for training the Madaline of Figure 1.3 will be called Madaline Rule I throughout this paper. It can be summarized as follows.

- Present a pattern to the Madaline. If the Madaline output and the desired response match, go on to the next pattern. Make no adaptations.

- If an error occurs, use knowledge about the fixed logic device to determine which Adaline(s) are contributing to the erroneous output. Select a minimum number of these such that if their outputs reverse state, the Madaline will respond correctly. If this minimum number does not include all of those contributing to the error condition, select those with the lowest confidence levels to be adapted. Use one of Mays' procedures to adapt the selected Adaline(s) in such a way as to reverse their outputs. (Note: Mays' procedures will not necessarily cause an Adaline to reverse state. The Adaline's analog response will be changed in a direction to provide the new response. Depending upon the adaptation constant, this change may not be large enough to actually change the Adaline's binary response.) Go on to the next pattern.

- Repeat this procedure until all patterns are responded to correctly.

Ridgway also points out that the pattern presentation sequence should be random. He found that cyclic presentation of the patterns could lead to cycles of adaptation. These cycles would cause the weights of the entire Madaline to cycle, preventing convergence.

Adaptations are being performed to correct the weights for the pattern being presented at that time. It is not obvious these weight changes will contribute correctly to a global solution for the entire training set. Ridgway presents an argument for convergence of his procedure. His argument is of a probabilistic nature. He shows that good corrections will outnumber bad corrections on the average. Thus a global solution will be reached after enough time, probably.

The exclusive-or problem represented in Equation 1.3 can be solved by the Ridgway Madaline shown in Figure 1.5. Here the fixed logic element is the OR from Figure 1.4. Figure 1.6 is a graphical depiction of how this network works. The outputs of the two Adalines in Figure 1.5 have been labeled q_1 and q_2. The Adalines map the input $x_1 - x_2$

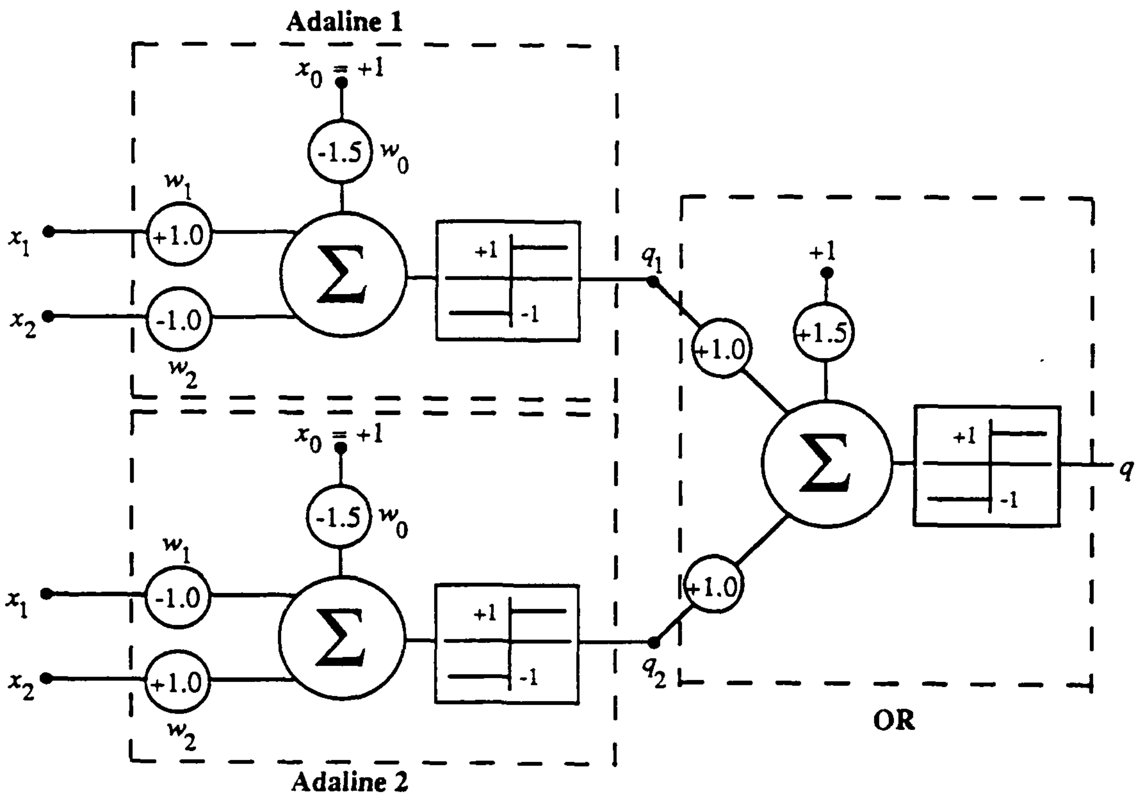


Figure 1.5: Ridgway Madaline realization of the exclusive-or function.

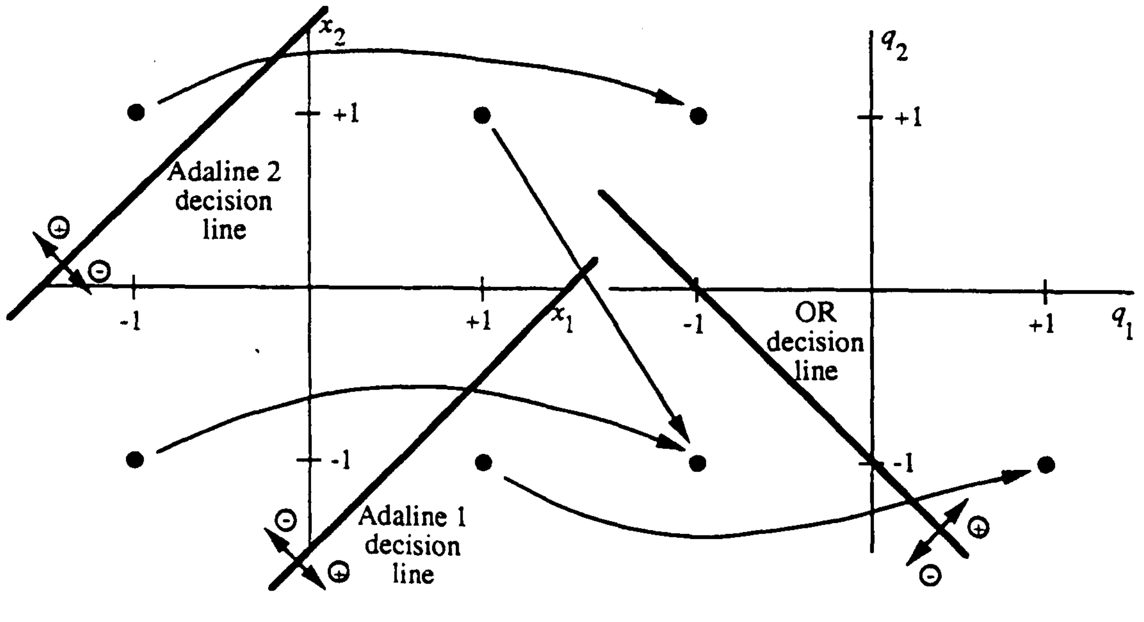


Figure 1.6: Graphical presentation of the Madaline of Figure 1.5. The Adalines map the input space into an intermediate space separable by the OR unit.

space to an intermediate $q_1 - q_2$ space that the OR element can separate as required. This mapping of the input space to an intermediate space that is then linearly separable is the essence of how layered Adaline networks operate.

The exclusive-or function can be written in the Boolean algebra sum of products form as:

$$x_1\overline{x_2} + \overline{x_1}x_2 .$$

Figure 1.4 shows how to realize the AND function when the inputs are uncomplemented. To realize a general Boolean product term, weight uncomplemented inputs by +1, complemented inputs by −1, and set the threshold weight $w_0 = -(n - \frac{1}{2})$. This is how the weights for the Adalines of Figure 1.5 were determined. Since any Boolean logic function can be written in the sum of products form, it follows that the Ridgway Madaline with the OR fixed logic unit has the capability of realizing it. It is only necessary to provide a sufficient number of Adalines. Ridgway notes this number is as high as 2^{n-1} for the n-input case.

1.4 Concept of Madaline Rule II — Minimal Disturbance

Madaline Rule II, or MRII, is a training algorithm for Madalines more complicated than those of Ridgway's. The general Madaline will have multiple layers. Each layer will have an arbitrary number of adaptive Adalines. Figure 1.7 shows an example of a 3-layer Madaline. This work will present experimental results of training 2-layer Madalines with MRII. The procedure generalizes to Madalines having more than two layers of Adalines, though no results of training such Madalines will be presented.

Figure 1.7 introduces some terminology to describe the general Madaline. Let l be the number of layers in the network. In the case of Figure 1.7, $l = 3$. To remain consistent with previous notation, the input vector $\vec{X}$ will have n variable components plus a constant bias component and will be the input to all the first-layer Adalines. Since the outputs of the first-layer Adalines are the variable inputs of the second-layer Adalines, let n_1 be the number of Adalines in the first layer. The outputs of the first-layer Adalines represent an intermediate pattern in the input/output pattern mapping scheme done by the Madaline. The current literature often refers to this pattern as being "hidden." Therefore, let $\vec{H}^1 = [h_0^1 = +1, h_1^1, \ldots, h_{n_1}^1]^T$ be the input vector to the second-layer Adalines. The constant bias input to the second layer is not pictured and is assumed to be provided internally to the Adalines as in the general Adaline of Figure 1.1. Similarly, $\vec{H}^2$ with n_2 variable inputs

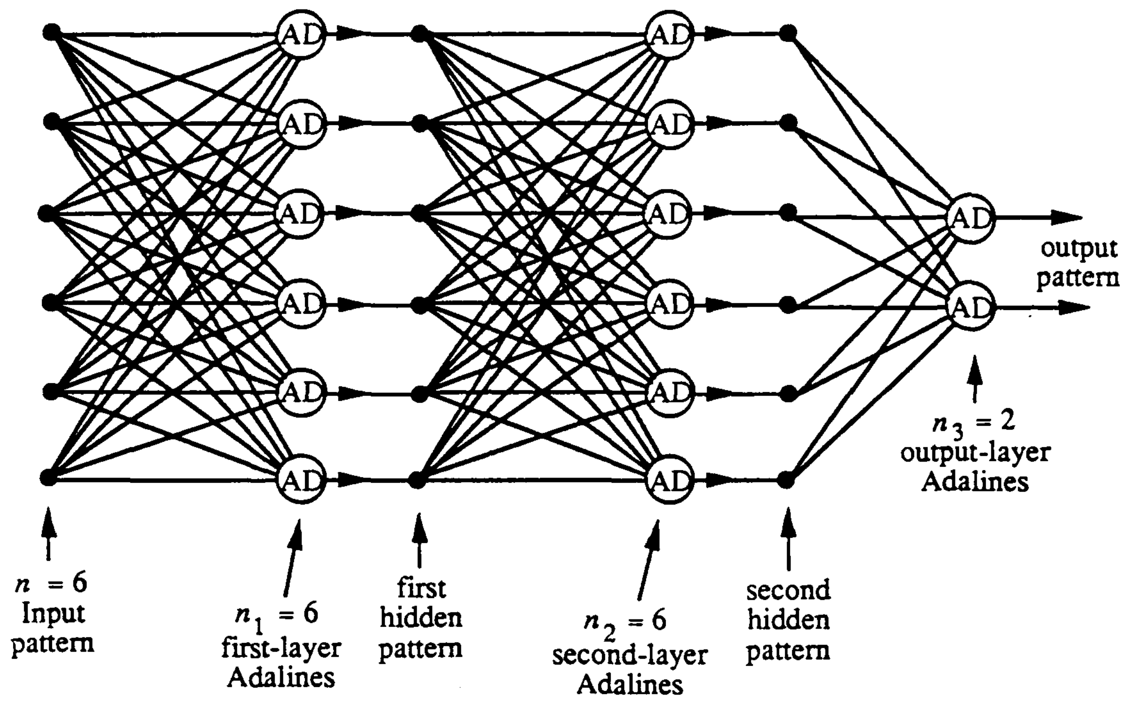

AD
output pattern
$n_3 = 2$ output-layer Adalines
$n = 6$ Input pattern
$n_1 = 6$ first-layer Adalines
first hidden pattern
$n_2 = 6$ second-layer Adalines
second hidden pattern

Figure 1.7: A three layer example of the general Madaline

plus a constant bias input will be the input vector to the third layer. The final layer of Adalines will be called the output layer as the response of these Adalines will constitute the Madaline's response. The desired and actual responses will from now on be considered vectors. The actual output vector will be $\vec{O} = [o_1, \ldots, o_{n_l}]^T$. The desired response vector will be designated $\vec{D}$. The components of these vectors will be called bits, being the actual or desired binary response for a particular output-layer Adaline. The configuration of a particular Madaline will be referred to as an n_1-feed-n_2-...-feed-n_l network with n inputs. Thus, for $\vec{X}$ in Figure 1.7 having 6 variable components, the network will be called a 6-feed-6-feed-2 Madaline with 6 inputs.

The credit assignment task for the general Madaline becomes much more complicated than for the Ridgway Madaline. Suppose a Madaline has three output Adalines and for a particular input, only one of the output Adalines' responses agrees with the corresponding desired response. How much is the output of any particular Adaline in the first layer helping or hurting the overall response of the network? One way to check is to reverse the output of an Adaline and let this change propagate through the network. In some instances the Madaline's output may not change at all if a given first-layer Adaline's response changes.

For other input patterns, changes in a first-layer Adaline's response may cause more of the output Adalines' responses to be correct, other times fewer may be correct, and still other times outputs will change but no net gain or loss of correct responses will occur. Faced with all these possibilities, how does one know which Adalines in a network need to be changed when errors occur during training? The adaptation techniques for the single Adaline and the Ridgway Madaline provide some insight.

All of the algorithms examined so far make no changes to the network if the current response matches the desired response (assuming no deadzone criterion is being used). When changes need to be made in the Ridgway Madaline, the fewest number of Adalines are changed and those of lowest confidence are selected. This is because the weights of the low confidence Adalines need to be changed least to change their outputs. To see this, suppose a generic weight update rule is used:

$$\vec{W}(k+1) = \vec{W} + \Delta\vec{W}$$

Premultiplying both sides by $\vec{X}^T$,

$$y(k+1) = y(k) + \vec{X}^T(\Delta\vec{W})$$

so that,

$$\begin{aligned}\Delta y &= y(k+1) - y(k)\\ &= \vec{X}^T(\Delta\vec{W})\\ &= |\vec{X}||\Delta\vec{W}|\cos\theta\end{aligned}$$

where θ is the angle between $\vec{X}$ and $\Delta\vec{W}$. The change in the Adaline's analog response is greatest for a given magnitude of weight change when $\Delta\vec{W}$ is selected aligned with $\vec{X}$. This is the type of weight update correction used by Mays' methods. Thus, the Adaline and Madaline procedures seen so far allow the current pattern presented to the system to be accommodated with least overall disturbance to the system. This is important because adaptations are being made based only upon the current input. By changing the overall system as little as possible, there is less likelihood of disturbing the response for other patterns in the training set.

This procedure of making corrections only when errors occur, and then making corrections that are least disturbing to the overall system is called the minimal disturbance principle. This idea can be used to formulate a strategy for training the general multi-layer, multiple output Madaline.

The procedure is as follows. Present a pattern from the training set to the network. Count how many of the output Adalines' responses do not match their desired responses. This number is actually the Hamming distance between the desired response vector and the actual output vector. Look now at the least confident Adaline in the first layer. Perform a trial adaptation by reversing the response of this Adaline. That is, if the selected Adaline had been responding $+1$, cause it to now respond -1 or vice versa. This change will propagate through the network and perhaps change the output Adalines' responses. If the Hamming distance between the new actual response and the desired response is reduced, accept the trial adaptation. If the number of errors is not reduced, return the trial adapted Adaline to its previous state. If errors in the output remain, perform a trial adaptation on the next least confident Adaline of the first layer. Again, accept or reject this trial adaptation depending upon whether the number of output errors are reduced. Continue in this fashion until all output Adalines respond correctly, or all single Adaline trial adaptations have been tried. If errors remain at this point, try trial adapting the first layer Adalines two at a time, beginning with the pair which are least confident. The criterion for acceptance of the trial is the same as before. If pairwise trials are exhausted, try trial adaptations involving three Adalines at a time, then four at a time, etc. until the output errors are zero. If all possible trial adaptations are exhausted without reducing the errors to zero, repeat the procedure using the Adalines of the second layer, then the third layer, etc. until one finally reaches the output layer. The output layer of course can be corrected to give the desired outputs by adapting each erroneous Adaline to give the correct response.

The basic philosophy is to give responsibility for corrections to those Adalines which can most easily assume it. That is, make the response for the current input correct with least overall change to the weights in the network. The details of how to perform trial adaptations, how confident an Adaline should be after it is adapted, how to choose the ordering of possible pairwise trial adaptations, etc. are deferred to Chapter 3, and are the author's specific contributions. The general procedure presented above is a specific way to implement a concept for adapting the general Madaline proposed by Widrow in 1962 [5].

Chapter 2

Mathematical Concepts

This chapter will cover some mathematical background that will be needed to analyze Madaline networks. The single Adaline is not a particularly simple element to analyze. The input and weight vectors introduced so far can have high dimensionality. The thresholding element of the Adaline introduces nonlinearity and discontinuity to complicate any analysis. Graphical techniques fail whenever the dimensionality exceeds two or three even for the single Adaline. Attempting graphical analysis for networks of even low dimensionality is virtually impossible.

The concepts of multi-dimensional geometry will provide the needed tools for analysis. Fortunately, many of the needed concepts follow intuitively from the two and three dimensional cases. Indeed, representations of the multi-dimensional situation can often be presented in terms of three dimensional drawings.

Analysis is often aided by making appropriate approximations. One such approximation which will be very useful is the Hoff hypersphere approximation [6].

2.1 Multi-Dimensional Geometry

The concept of angle between two vectors extends to higher dimension spaces through the dot product relation,

$$\vec{V}^T \vec{U} = |\vec{V}|\, |\vec{U}| \cos\theta \, .$$

Here $\vec{V}$ and $\vec{U}$ are just two generic vectors in n-space. Two such vectors define a two dimensional plane and θ is the smaller positive angle between them in this plane, $0 < \theta <$ 180 degrees. Two vectors are normal to each other when their dot product is zero.

The equation that describes the decision separating surface for an Adaline is a dot product relation, $\vec{X}^T\vec{W} = \vec{W}^T\vec{X} = 0$. The vectors are located in $(n+1)$-space. The equation specifies a normality condition. The symmetry of the dot product allows two perspectives in understanding the Adaline.

In the first perspective, consider the weight vector to be fixed in the input vector space. The decision surface is a hyperplane through the origin and perpendicular to the weight vector. This hyperplane is itself an n-dimensional space. It divides the input vector space in half. Those input vectors lying on the same side of this hyperplane as the weight vector will have a positive dot product and result in a $+1$ decision by the Adaline. Those input vectors on the opposite side of the hyperplane result in -1 decisions. This then is the perspective that the weight vector defines a division of the input space into decision regions.

The other perspective is to assume an input vector has been chosen in the weight space. An input vector will have associated with it a desired response. The hyperplane through the origin, perpendicular to the input vector will divide weight space in half. Weight vectors on only one side of this hyperplane will provide the correct desired response for this particular input vector. If now a second input vector is considered, its desired response will also define a half-space where the weight vector must lie to provide a correct response. To satisfy both input vectors, the weight vector must lie in the intersection of these two half-spaces. To solve an entire training set, the weight vector must lie in the intersection of the half-spaces defined by each input vector and its associated desired response. This intersection will be a convex cone emanating from the origin. Any weight vector lying in this cone will be a solution vector for the given training set. If the intersection of the half-spaces happens to be empty, the training set defines a not linearly separable function. There is no weight vector for the single Adaline that can solve this training set. This second perspective then is that of the input vectors defining a solution region for the weight vector.

At this point the reader may become concerned about some apparent contradictions. Above it was said the decision separating hyperplane passed through the origin. In all of the graphs shown in Chapter 1 though, none of the decision separating lines passed through the origin. This is because the graphs of Chapter 1 were drawn in the input pattern space not the input vector space. The relation between these two spaces is the following. The input pattern space is n-dimensional, having n variable components. The input vector space has an extra component x_0. This component however is not truly variable as it is constant at a value $+1$. The input pattern space lies in the hyperplane, $x_0 = +1$, of the input vector space. In the case of two-dimensional pattern spaces, as presented in Chapter 1, the situation is easily visualized. Consider all possible binary triples (x_0, x_1, x_2), $x_i \in \{-1, +1\}$. There are 8 of them. They are the vertices of a cube centered on the origin with edge length 2. With x_0 fixed as $+1$, four of them are actual allowable patterns and these are located on a face of the cube. This face lies in a two-dimensional plane. The plane perpendicular to the weight vector and passing through the origin intersects the plane containing the patterns in a line. This line does not necessarily pass through the origin of the pattern plane and in general will not.

From the above it is seen there is an option to do analysis in either the n-dimensional pattern space or the $(n + 1)$-dimensional input vector space. There are advantages and disadvantages to both approaches.

The primary advantage to working in the $(n + 1)$-dimensional space is that the hyperplanes perpendicular to the weight and input vectors all pass through the origin. As will be seen in Chapter 6, this simplifies analysis when one is concerned with deviations of the weights and inputs from nominal values. These deviations will take the form of angular deflections from the nominal direction. The disadvantage to doing analysis in this space is that the distribution of possible weight vectors and allowable pattern vectors is not the same. The weight vector can assume any direction in this space. The input vectors however are constrained to lie in the $x_0 = +1$ hyperplane. The input vectors have an orientation in the positive x_0 direction. This orientation destroys spherical symmetry for the input vectors. A consequence is the fact that only half of the possible binary vectors in this space are true patterns, those having $x_0 = -1$ being unadmissible.

In the n-dimensional pattern space, all binary component vectors are true patterns and they have spherical symmetry in the space. The bias weight, w_0, is no longer considered part of the weight vector. This changes the equation for the decision separating hyperplane to $\mathrm{X}^T\mathrm{W} = -w_0$. Here a notation change is used to distinguish weight and input patterns without bias components from those that do. In the n-dimensional weight space, this is an equation of a hyperplane perpendicular to W but offset from the origin by a distance $|w_0|/|\mathrm{W}|$. This offset complicates analysis. The decision separation now becomes dependent on the magnitude of the reduced dimension weight pattern instead of just its direction. Any change in the weights must then be resolved into changes in direction and offset from the origin.

The major analytical results presented in this paper deal with how the input/output mapping of a Madaline change when the weights are disturbed from their nominal values. The $(n + 1)$-dimensional space is much easier to use for this analysis.

2.2 Hoff Hypersphere Approximation

Hoff [6] is responsible for the very powerful hypersphere approximation that allowed most of the analytical results on Adalines and Madalines to date to be derived. Hoff formulated this approximation in the n-dimensional pattern space. This section will explain the hypersphere approximation and introduce some more multi-dimensional geometry concepts.

In three dimensional pattern space it is easy to visualize the locations of all the input patterns as being the vertices of a cube. In higher dimensions, these patterns are found at the vertices of a hypercube. The input patterns all have the same magnitude, $\sqrt{n}$. The hypercube can be inscribed then in a hypersphere of radius $\sqrt{n}$. The vertices of a hypercube have a regular arrangement symmetric about the origin. Hoff postulated that as n gets large, one could say the input patterns are uniformly distributed on the surface of the hypersphere. This immerses the case of binary input patterns into the continuous analog patterns case. He showed that this was a valid assumption in his doctoral dissertation.

The power of this assumption allows one to use probabilistic methods to analyze the Adaline. The probability of an input pattern lying in a particular region of space could be computed as the ratio of the area of the region on the hypersphere to the area of the whole hypersphere. Thus, instead of performing summations over discrete points, integrals over regions of the hypersphere could be used in many of the analyses that needed to be done. One use of these techniques was to prove that the capacity of an Adaline to store random patterns was equal to twice the number of weights in it [7].

Glanz [8] applied the hypersphere approximation to the $(n+1)$-dimensional input vector space. He argued that since the x_0 component could only take on half the values it could before, the hypersphere approximation would apply to half the hypersphere. The input vectors could be thought of as being uniformly distributed on the hemihypersphere in the positive x_0 half-space.

Most previous analyses could be performed assuming a unit hypersphere. In these analyses, only the direction of the input and weight vectors was important. In the analytical results to be presented in this paper, the magnitudes of vectors will be important and care must be taken to work with the proper sized hypersphere.

At this a point a few facts about hyperspheres will be introduced. Further information can be found in Kendall [9] or Sommerville [10].

- Strictly speaking the area of a hypersphere should be called its surface content but the term area will usually be used. The area of a hypersphere of radius r in n-dimensional space is given by:

$$\mathrm{A}_n = \mathrm{K}_n r^{n-1} \tag{2.1}$$

where,

$$\mathrm{K}_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}\,.$$

The expression for K_n can be written in terms of factorials instead of the gamma function if distinction is made for n even and odd:

$$\mathrm{K}_n = \frac{2\pi^m}{(m-1)!} \qquad \text{for } n = 2m \tag{2.2}$$

$$= \frac{2^{2m+1}\pi^m m!}{(2m)!} \qquad \text{for } n = 2m+1 \tag{2.3}$$

- The intersection of a hyperplane and a hypersphere in n-space is a hypersphere in $(n-1)$-space. For $n = 3$, this says a plane intersects a sphere in a circle. The center of the reduced dimension hypersphere is the projection of the center of the n-sphere onto the intersecting hyperplane.

- The differential element of area $d\mathrm{A}_n$, on a hypersphere of radius r as a function of one polar angle ϕ is given by:

$$d\mathrm{A}_n = \mathrm{K}_{n-1}\, r^{n-1} \sin^{n-2}\phi\, d\phi \tag{2.4}$$

  This can be understood by realizing that a particular value of ϕ defines a $n-1$ dimension hypersphere of radius $r \sin\phi$. The differential area element is the surface content of this reduced dimension hypersphere multiplied by a thickness $r\,d\phi$. A representation of this in three dimensions is shown in Figure 2.1.

- The hyperplane perpendicular to a vector emanating from the origin will divide a hypersphere centered on the origin into two hemihyperspheres. In similar fashion, a second vector will define a hyperplane that bisects the hypersphere. The surface con-

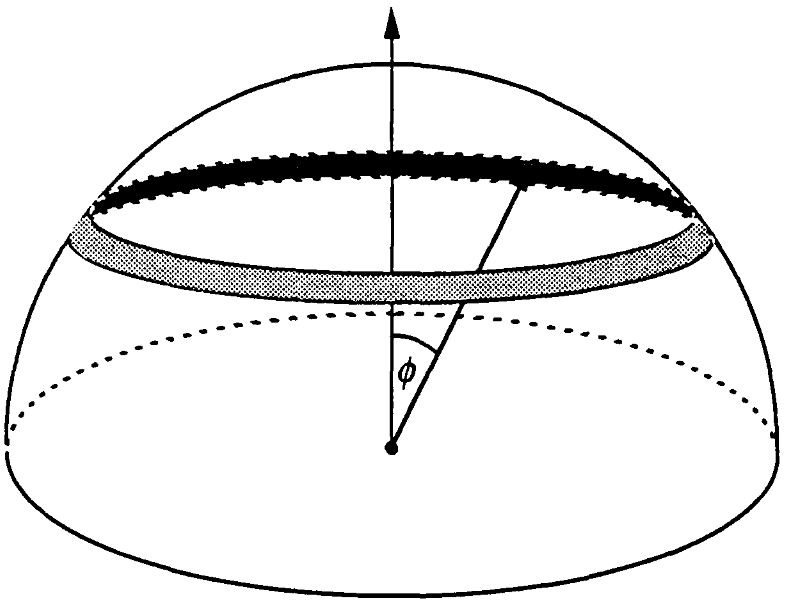


Figure 2.1: The differential area element of a hypersphere as a function of the polar angle ϕ.

tained between specific sides of two such hyperplanes is called a lune (see Figure 2.2). If the angle between the two vectors is θ, the surface content of the lune is given by:

$$\text{Area of lune } = \frac{\theta}{2\pi}\text{A}_n \tag{2.5}$$

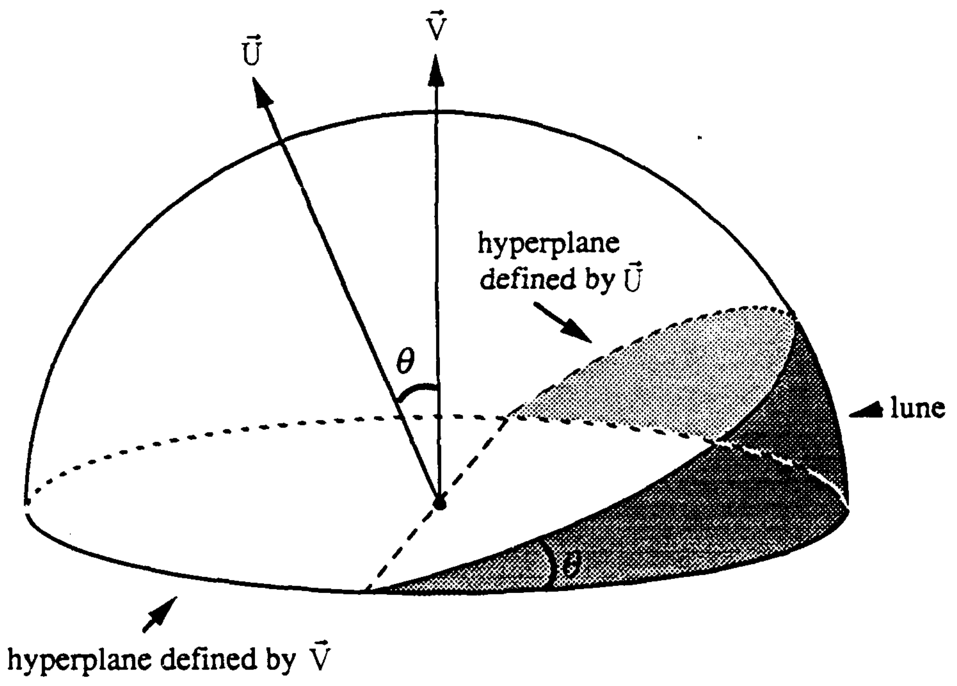

$\vec{U}$
$\vec{V}$
hyperplane defined by $\vec{U}$
θ
lune
θ
hyperplane defined by $\vec{V}$

Figure 2.2: A lune of angle θ formed by two bisecting hyperplanes.

Chapter 3

Details of Madaline Rule II

This chapter will present the actual details of how MRII works. The reader will be able to write computer simulation code after reading the chapter.

The chapter will begin by discussing some desirable features of a neural network training algorithm. These desirable features required modifying the minimal disturbance principle to develop a practical implementation. The MRII algorithm was developed empirically. As the algorithm evolved it was necessary to introduce a concept called "usage" that further modified the minimal disturbance principal. The usage concept, why it was needed, and its implementation will be covered.

3.1 Desired Features for a Training Algorithm

A neural network is a collection of relatively simple processing elements. They are connected together in such a way that the collection exhibits computational capabilities that a single element cannot perform. The specific computation the network performs is "trained in" by presenting a collection of sample inputs and desired responses to the network. It is reasonable to assume the network can be trained off-line. That is, the training data is available for presentation as many times as necessary via some external storage and presentation system. It will be assumed that training can take a relatively long but reasonable time and this amount of time is not a critical factor. Once trained, the neural network's utility is realized by being able to respond almost instantaneously to new inputs presented to it. This speed is due to the fact that the computation being performed is distributed among all the processing elements. In a layered feed-forward Madaline, the response time will be roughly the number of layers multiplied by the time for a single Adaline to perform its weighted sum and thresholding operation.

The hardware implementation of neural networks must contend with the problem of connectivity. The neural net relies on a high degree of connectivity to perform. The realization of the high fan-in and fan-out needed for neural networks is a definite hardware challenge. The issue to be made here is that the training algorithm not exacerbate this problem. The training algorithm must be implemented with a minimum of added connections.

For MRII, there will be a need for a master controller to direct the trial adaptations and decide which are accepted and rejected. Communications require hardware and time, two quantities to be minimized in a training implementation. Therefore, the controller should operate with a minimum of communication. Information about a specific Adaline's weight values or analog sum should not be needed by other units or the master controller.

While it has been assumed that training time is not a prime consideration, this time has to be reasonable. The question of how training time grows with network size cannot be completely ignored. Any training algorithm that requires an exponential or combinatoric increase of training time as the size of the network increases will probably not be acceptable.

The next section will show how to implement the concepts of minimal disturbance with the above considerations in mind.

3.2 Implementing MRII

This section will present a block diagram implementation of MRII. The purpose here is to present the implementation at a high level of abstraction, not to present a wiring diagram. The function of each block in the diagram will be explained but its hardware realization will not be addressed.

The basic scheme of MRII was presented in Chapter 1. Its implementation requires several things. The first is the ability to perform trial adaptations by layers. The first layer being trial adapted first and the final layer adapted only if all output errors could not be corrected by the previous layers. Within a layer there must be the ability to involve different numbers of Adalines in the trials. First, trials involving only a single Adaline will be done. Then, if necessary, pairwise trials or trials involving two Adalines at a time will be done. These will be followed by threewise, fourwise, etc., trials. Finally, it is desired to trial adapt the Adalines in order of increasing confidence. Those Adalines with analog responses closest to zero are to be trial adapted before the more confident ones.

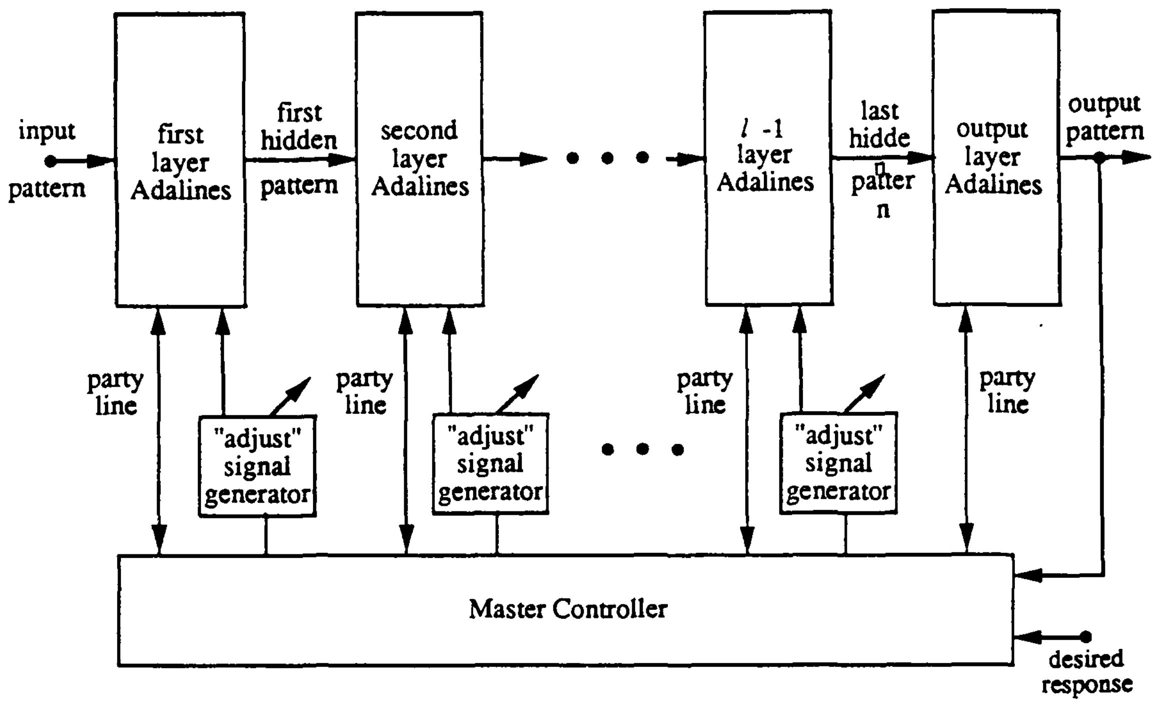


Figure 3.1: Block diagram implementation of a Madaline trained by MRII.

Figure 3.1 shows a Madaline at the highest level of abstraction. There is a master controller that communicates with each layer of Adalines in the network by means of a two-way "party line." The master controller also controls an "adjust" signal generator for each layer of the Madaline except the output layer. Figure 3.2 shows the structure of a single layer within the network. The party line and adjust signal line are connected in parallel to each Adaline of the layer.

The party line provides the communications link for command and control during trial adaptations. Communication on this party line will be structured so that at most one element is transmitting at a time. The important aspect of this party line is that if an Adaline transmits on it, all other Adalines receive this transmission, not just the master controller. This is why it is called a party line.

The minimal disturbance principle requires that trial adaptations begin with the least confident Adalines first. This implies that the Adalines on a particular layer will need to be sorted by the value of their analog response. To do this without actually communicating analog values around the network is the purpose of the adjust signal.

Figure 3.3 shows a block diagram of the Adalines needed to implement MRII. These

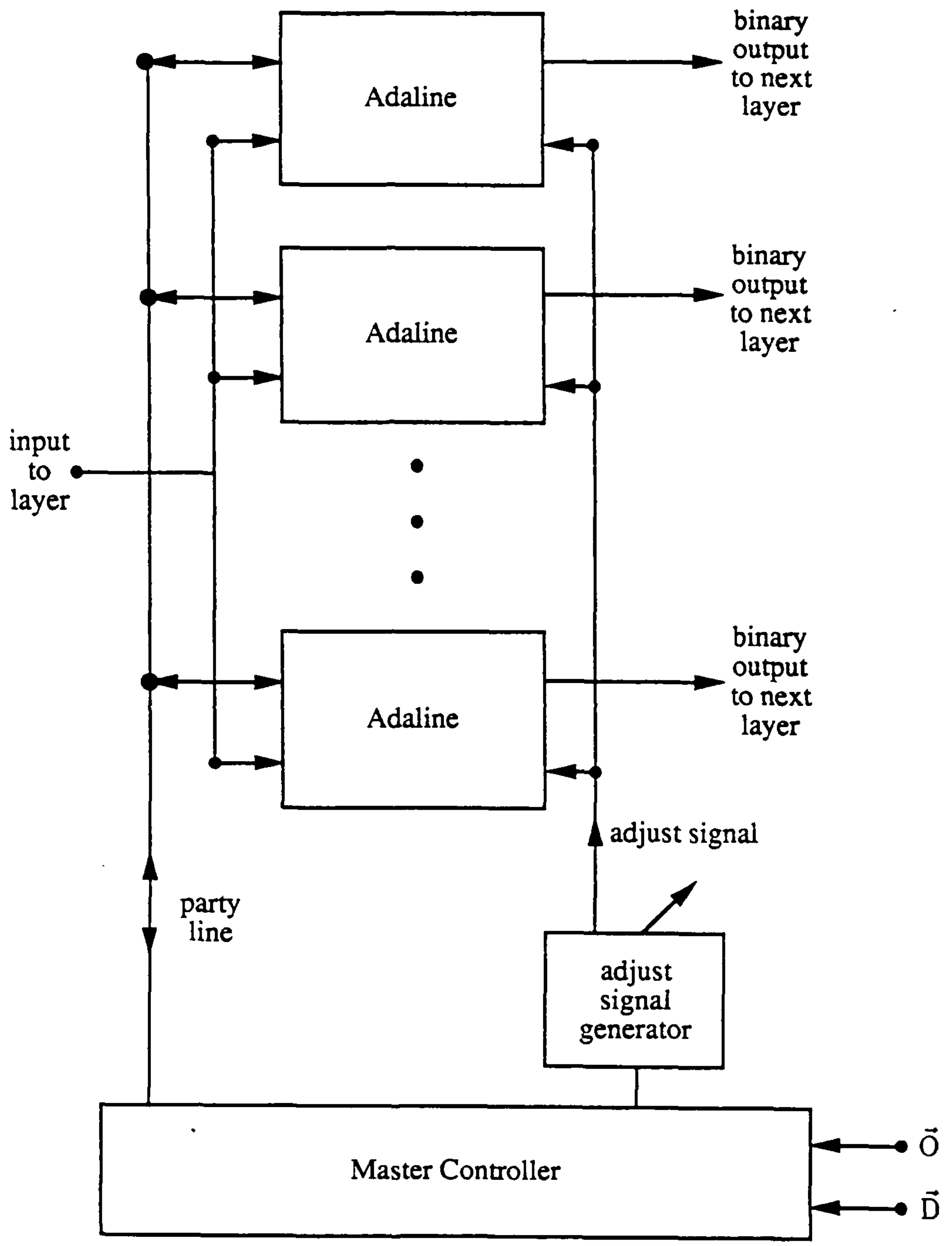


Figure 3.2: Structure of a single layer within the network of Figure 3.1.

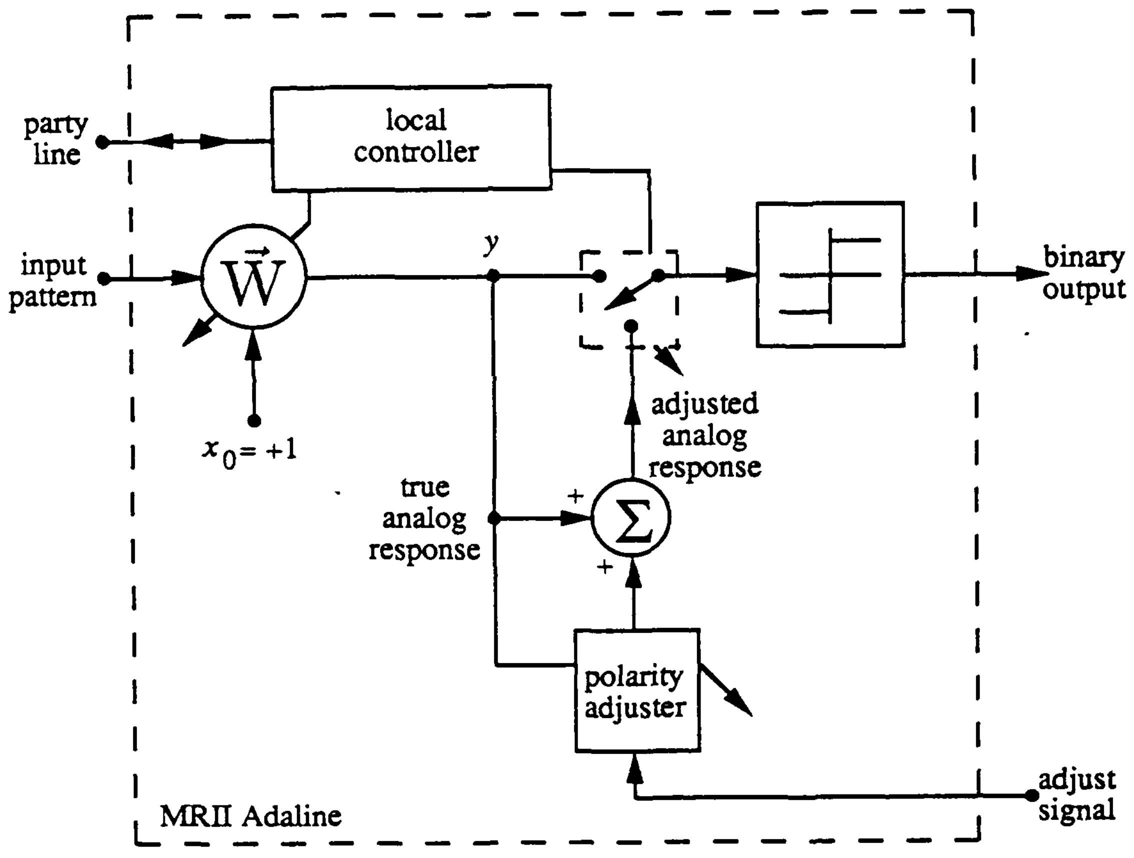


Figure 3.3: Block diagram of the Adalines for Figure 3.2.

Adalines use the adjust signal to modify their analog responses during trial adapting. The Adalines local controller controls an internal switch. This switch selects either the true analog response or an adjusted analog response as input to the thresholder. The polarity adjuster senses the polarity of the true analog response. It sets the polarity of the adjust signal to be opposite the polarity of the true analog response. As the adjust signal increases, it will drive down the apparent confidence of the Adaline. If the magnitude of the adjust signal gets large enough, the sign of the adjusted analog response will be opposite that of the true analog response. This will cause the binary response to flip or reverse from its previous output.

When the master controller wants to trial adapt a layer it will transmit on the layer's party line that it is to begin trial adaptation. The local controller of each Adaline on the adapting layer will throw its internal switch so that its output is modified by the adjust signal. The adjust signal is initially zero. The master controller then slowly increases the adjust signal. Each Adaline on the layer being adapted adds the adjust signal with proper polarity to its analog response in a way to reduce the apparent confidence of the Adaline. As the adjust signal increases, the adjusted analog response will eventually cross zero and take on a sign opposite to the true analog response. The binary response will reverse and the Adaline is said to be trial adapted. As the adjust signal increases, the reversal of outputs will occur in order from least confident Adaline to most confident Adaline. At the time of reversal, the Adaline transmits on the party line that it has reversed.

If trials involving one Adaline at a time are being performed, the master controller will stop increasing the adjust signal as soon as the first Adaline reverses. It will wait a sufficient time for changes to propagate through the net, and then check the actual response versus the desired response for improvement. If improvement occurs, the master controller will transmit an "adapt" command. The Adaline which was trial adapted will then change its weights to provide its new binary response. This weight change will be done by the modified relaxation rule. The parameters for the weight update will be detailed later but will be chosen such that the weights will change by a sufficient amount to actually reverse the Adaline's output. If the Hamming distance between the desired and actual responses had not decreased, the master controller would transmit a "reset" command. The trial adapted Adaline would then throw its internal switch to disconnect its output from influence by the adjust signal. The Madaline will return to its original state before trial adaptation. The master controller would then begin increasing the adjust signal until the next least confident Adaline reverses its output, etc.

If trials involving two Adalines at a time were being performed, the master controller would increase the adjust signal until a second Adaline transmits that it has reversed state. The master controller would then check the output performance. Again, if an improvement occurred, the master controller would transmit the adapt command. Now both of the Adalines participating in the trial would change their weights. If output performance had not improved, the master controller transmits reset. Now only the first Adaline of the pair that reversed its outputs disconnects from the adjust signal. This is why the Adalines need to be able to hear each other on the party line. They have to be able to keep track of their position in the sequence of output reversals. Only in this way can they know when a reset command applies to them. After the first Adaline resets, the master controller increases the adjust signal until another Adaline reverses output. At this point the pair of Adalines participating in the trial adaptation are the second and third least confident of those when the trial began. If no improvement in output performance is obtained, the reset command is given and the second least confident Adaline disconnects from the adjust signal. The adjust signal then increases until the third and fourth least confident Adalines are trial adapted, etc.

Three at a time trial adaptations are handled similarly. The adjust signal is increased until three Adalines reverse output. Then the output performance is checked. If the trial is rejected, the first Adaline of the three that reversed outputs disconnects from the adjust signal.

Any time a trial adaptation is accepted, the master controller will restart the adaptation procedure with trials involving one Adaline at a time. This is done because after an Adaline adapts its weights, its new analog response will cause it to have a confidence different from what it had before. Also, since the response of the layer has changed, some of the trial adaptations that were previously rejected may now be accepted. This may cause what really would have been an accepted pairwise adaptation to masquerade as two accepted single adaptations. In similar fashion, a three at a time adaptation may be accepted as a single and a pair. Experience simulating the algorithm shows that it is rarely necessary to consider more than three at a time trial adaptations when the layer has less than twenty Adalines. The number of accepted single trial adaptations outnumbers the accepted pairwise trials by a factor of ten. The accepted threewise trials are about one-fourth the number of pairwise acceptances. Due to this diminishing acceptance rate, performing trials involving more than three Adalines at a time is not really worth the time it takes to do them.

With this implementation, the least confident and third least confident Adalines will not be considered as a pair during pairwise trial adaptation. Thus, some possibly good trial adaptations will not be considered. The sum of the confidences of the first and third least confident Adalines will be less than that of the second and third least confident. Based on minimal disturbance only, the trial adaptation involving the first and third least confident Adalines should be considered first. The implementation modifies the minimal disturbance principle to tradeoff for practicality. This scheme insures the training time will not grow at a combinatoric rate as the number of Adalines on a layer increases. It would if all possible combinations of pairwise, three-wise, four-wise, etc., trial adaptations were considered. A more complicated implementation would also be needed to consider these other trials.

As mentioned earlier, the weights of an Adaline will be adapted using the modified relaxation method of Equation 1.2. It was shown in Chapter 1 that if η is chosen as 1, the confidence of the Adaline after adaptation would be equal to the adaptation level, L. The Adaline would also be responding with its new desired response. Experience indicates that

$\eta = 1$ and L = .2 are good parameters to use. The desired response d is either $+1$ or -1, the sign being the same as the current trial output of the Adaline. A deadzone value of zero will be used. This means that only those Adalines actually accepted in trial adaptations will have their weights changed. There could be Adalines of very low confidence that are not accepted during the trials. These Adalines will not be adapted to provide a minimum confidence. (Note: A nonzero deadzone could be used. The simulation results presented in this paper had a zero deadzone. To use a nonzero deadzone, the master controller would send a special command to the Adalines of a layer after the trials were completed. The low confidence Adalines would then adjust their weights to provide their current binary response with confidence equal to the deadzone. Do this by using $\eta = 1$ and L $= \delta$ in Equation 1.2.)

The minimal disturbance principle demands one final consideration. The confidence levels of some Adalines on a layer may be quite high for a given input pattern. If such an Adaline is adapted, it will require a great change in its weight vector. The minimal disturbance principle suggests it may be better to let the next layer assume responsibility for correcting output errors than to make large weight changes on an earlier layer. Experience with the algorithm confirms this idea.

There are two ways to limit the number of Adalines considered for trial adaptation on a layer. One way is to set a maximum value for the adjust signal. This will prevent Adalines above some threshold confidence from participating in trial adaptations. A second implementation is to allow only a set fraction of the total number of Adalines on a layer be trial adapted. Suppose a layer had ten Adalines. One might allow only the five least confident ones to participate in trials. This would set the fraction at one-half.

If a nonzero deadzone is used, the second approach is preferable. It is difficult to know ahead of time how large the weight vectors and, correspondingly, the confidence levels are going to have to be to insure a minimum confidence over a training set. This complicates the choice of a maximum adjust signal level. Experience indicates the fixed fraction idea provides an algorithm that works better over a larger set of problems. The value used in the simulations presented in this paper was one-half of the Adalines on a layer plus one if there were an even number of Adalines, and the big half if there were an odd number.

3.3 Usage

The implementation of MRII detailed in the previous section was simulated on a computer. The training often failed to converge to a solution even on training sets for which there were known solutions. It was found that the failures were typified by one particular Adaline always being accepted during trial adaptations. This Adaline will be called the "hinge" Adaline since the response of the Madaline as a whole was dependent on how this Adaline was adapted.

The training set would end up being divided into three subsets. The patterns in one subset would be responded to correctly independent of the response of the hinge Adaline. Patterns in the second subset would be responded to correctly when the hinge Adaline responded +1 while the third subset needed the hinge Adaline to respond −1 to provide a correct response. The hinge Adaline was the low confidence Adaline for patterns from either the second or third subsets. It also had the power to solve the Madaline's response whenever patterns from either set caused output errors. Thus, the hinge Adaline was always trial adapted first and always accepted for adaptation whenever an error occurred. Unfortunately, the second and third subsets were not linearly separable from each other. The dynamics of the algorithm are then exactly those of a single Adaline trying to separate a not linearly separable set.

This type of behavior was also noted to occur with the Ridgway Madaline. Glanz [8] reports that Hoff suggested using "activity levels" to force Adalines not being adapted to be adapted. Prior to discovering these notes by Glanz, this author came up with the concept of "usage." Usage is a way to modify the confidence levels of the Adalines.

A way to break the cycle of always adapting the hinge Adaline is to make its confidence not be the lowest when trial adaptations occur. This forces another Adaline to be trial adapted before the hinge Adaline. In this way, the rest of the Adalines on the layer can be forced to help the hinge Adaline separate the two problematic subsets. Usage modifies the minimal disturbance principle by requiring a spreading out of responsibility among all the Adalines.

To implement the usage concept, pass the analog response through a variable gain amplifier. Set the gain to a value proportional to the number of times that particular Adaline has been adapted. This can be done by the Adaline's local controller. A usage modified Adaline is shown in Figure 3.4.

Deciding exactly how to set the gain of the usage amplifier remains the most empirical

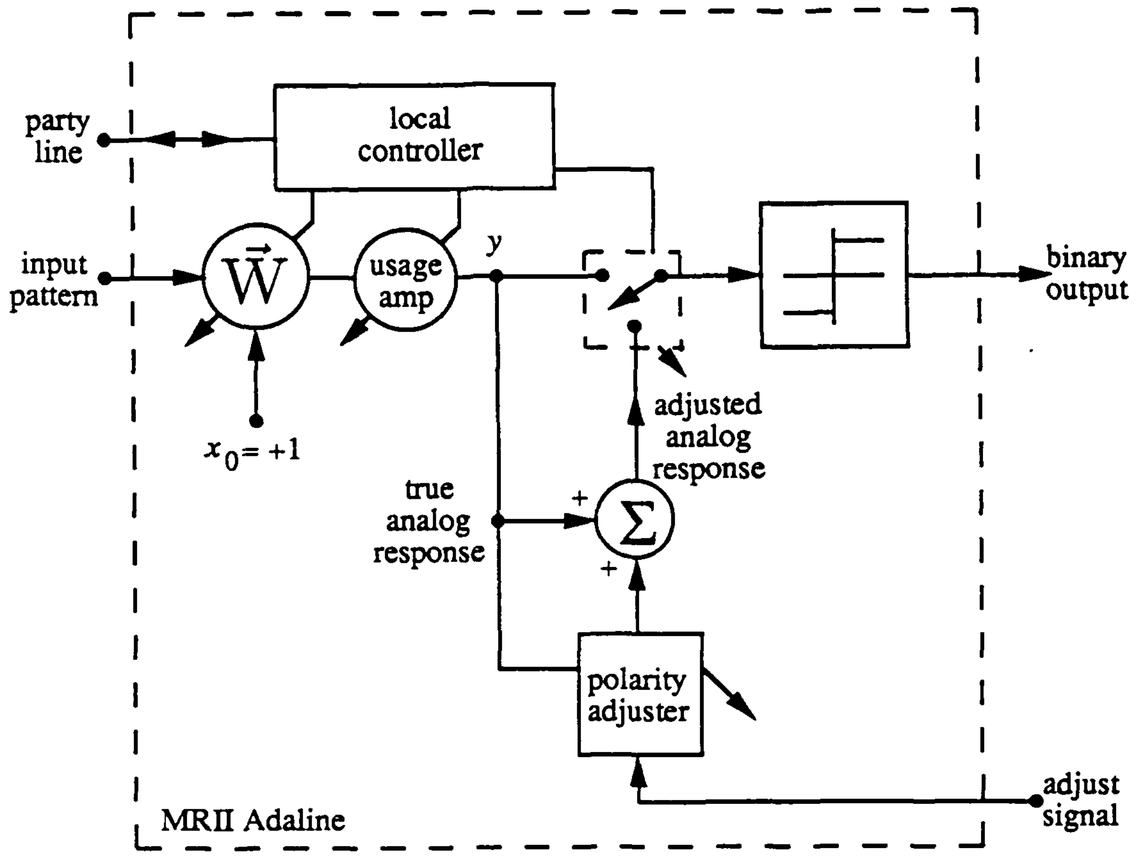


Figure 3.4: An Adaline modified to implement the usage concept.

part of the MRII algorithm. A formula which has worked well in simulations is:

$$\text{gain} = 1 + \frac{\text{adaptation count}}{\text{MULT} * N}$$

where N is the number of patterns in the training set and "MULT" is a multiplier value. Experience indicates MULT = 5 is a good choice. The idea here is to not let the usage gain get too large before the network really gets into a cycle. The next chapter shows some results of simulations using MRII.

Chapter 4

Simulation Results

This chapter will present the results of using MRII to train various Madaline networks. Madalines were used to solve several problems including an associative memory problem and an edge detection problem. An investigation of how well MRII could train networks to learn arbitrary input/output mappings was also conducted. This latter investigation also included a study of the generalizing properties of MRII. Results of these experiments will be presented following a presentation of the performance measures used to evaluate the algorithm.

4.1 Performance Measures

To evaluate a learning scheme there needs to be established some set of criteria by which to measure the performance. Of primary concern is whether the algorithm converges to a solution. If it converges, how fast does it converge. If it doesn't converge, does it in some way get close to a solution. In either case it is often desirable to know how well the resulting trained network responds to inputs that were not in the training set. This is the issue of generalization. This section will define the measures used to quantify the answers to these questions.

Before actually addressing the measures used, a few comments on the experimental technique used for the simulations is in order. The Madaline network begins training with some set of weight values already set. The simulations randomized these initial weight values. Each individual weight was selected independently from a distribution that was uniform on the interval $(-1, +1)$. The training patterns were presented in random order.

At each new presentation, each pattern was equally likely of being the one presented. The selection process then was like pulling one pattern out of a grab bag containing all the patterns and returning the pattern to the bag after each presentation. At regular intervals during training, all of the patterns would be presented to the network one after another to check how many of them were being responded to correctly. These "checkpoints" allow measuring the network's global performance over the entire training set. No adaptations are performed during these performance checks. If the response to all patterns was correct, then convergence was said to occur. Finally, the results are presented as the average of an ensemble of training attempts, each attempt beginning with a new set of randomized weights.

The first measure to decide upon is one that measures how long it takes the algorithm to reach a solution. For a Madaline implemented in hardware, the process of presenting a pattern and getting a response will take almost no time. If the response is correct the network can immediately go on to another pattern. If the response is incorrect though, the network will go through a series of trial adaptations until the output is corrected. The trial adaptation procedure will take up the majority of the training time. As convergence is reached, most pattern presentations will require no adaptations. Thus, the reasonable measure of training time will be the number of pattern presentations that require adaptations be performed. This will be a good measure of the time an actual hardware implementation would require for training. Furthermore, the network only learns from its mistakes. A pattern presentation that requires adaptations be made can be thought of as a learning opportunity. By measuring network performance against the number of learning opportunities, a measure of the algorithm's efficiency as it proceeds towards convergence can be obtained.

The Madalines that are of most interest are those with more than one output bit. Ultimately, convergence will require that all bits of all output patterns be correct. In measuring performance prior to convergence, one has two choices. One can count the number of patterns whose bit responses are all correct or one could count the number of bits that are correct. One would expect that the number of patterns that have a completely correct response would not increase until almost all the bits are correct. A distinction will be made then between pattern rates and bit rates. Since the problems which will be presented later have different size networks and different size training sets, these rates will be normalized by presenting them as percentages. The learning curves that will be presented will plot a training rate versus the number of pattern presentations that required adaptations. A "bit learning curve" will present the percentage of the total output bits for the training set that are correct as a function of the number of adaptation causing presentations. Similarly, pattern learning curves will plot the percentage of patterns whose response is completely correct versus the learning opportunities.

It is a fact that MRII does not always arrive at a solution. Sometimes it may take a very long time for a solution to occur from a particular set of initial weights. At other times the algorithm gets trapped in a limit cycle and will not proceed to a solution. In performing the simulations in the following sections, the problem to be solved was attempted many times starting from new initial weights each time. It was necessary to set some arbitrary limit to the number of patterns requiring adaptation that would be trained on before the particular attempt was abandoned. If convergence had not occurred by the time this limit was reached it did not necessarily mean that convergence would not have occurred if training had continued. This needs to be remembered when a convergence rate is reported. The convergence rate represents the number of training attempts that reached a solution within some limit of training time.

Those training attempts that converge can be used to generate learning curves that typify the algorithm's performance on that particular problem. What should be done though for the cases of nonconvergence? For these cases one might want to know how close to a solution the algorithm came. With some extra circuitry, it would be possible to build Madalines that could save their weights at intermediate stages during training. The learning exhibited by MRII is not a monotonic process. This is due to the fact weight changes are made based only upon the particular pattern being presented rather than on a global assessment of the effect of weight changes on the entire training set. Some adaptations will cause regression of global performance. With the ability to save weights at intermediate stages this effect can be minimized. One becomes interested then in the best performance exhibited at any stage of training. In the cases of nonconvergence, the average best performance will be presented as a measure of how close to a solution the network came.

4.2 An Associative Memory

An associative memory associates an input pattern with an output pattern. A need for such an application arose in the adaptive descrambler portion of a translation invariant pattern recognizer reported in Widrow, et al [11]. The basic structure of the system is shown in Figure 4.1. An retinal image containing a possibly translated figure is presented to an invariance box. The invariance box outputs a pattern that is unique to the figure presented and does not change when the figure is translated on the retina. The descrambler associates this translation invariant representation of the figure with the original figure in a standard position. The translation invariant input pattern then acts as a key that unlocks the required desired response. Most associative memories have the ability to perform well with incomplete keys or keys with errors in them. This was not expected of the application here. The requirement then is for the descrambler to act as a simple lookup table.

The structure used for this application is a 25 input, 25-feed-25 Madaline. There were 36 patterns in the training set. Three different training sets were used, each representing a different invariance box. The reason for making the descrambler adaptive is because the exact form of the outputs from the invariance box is dependent on the particular problem. The intent is to show the ability to associate patterns with different invariant representations.

The average learning curves for patterns and bits are presented in Figure 4.2. The pattern learning curve doesn't show an appreciable increase in performance until almost all the bits are learned. This is because a pattern is not considered correct until all its bits are correct.

The algorithm always reached convergence when training on this application. It also reaches a solution in a reasonable amount of training, only needing to adapt on each pattern about 15 times to insure convergence. The network is not being tasked heavily by this example. While the capacity of layered feed-forward Madalines is not known, one can suspect it exceeds 36 for the structure being trained. The example does show that MRII is capable of training networks with a relatively large number of inputs and units on both the first and output layers.

4.3 Edge Detector

Image analysis techniques often require extraction of certain features from a scene such as edges. A Madaline that could be used as a component of an edge detection system was simulated and trained using MRII. The problem is illustrated in Figure 4.3. The Madaline receives its input from a linear array of pixel elements. The array has a horizontal orientation. It is desired to detect the first "on" pixel scanning from left to right and output its position by a binary code. The array has eight elements and the case of no pixels being

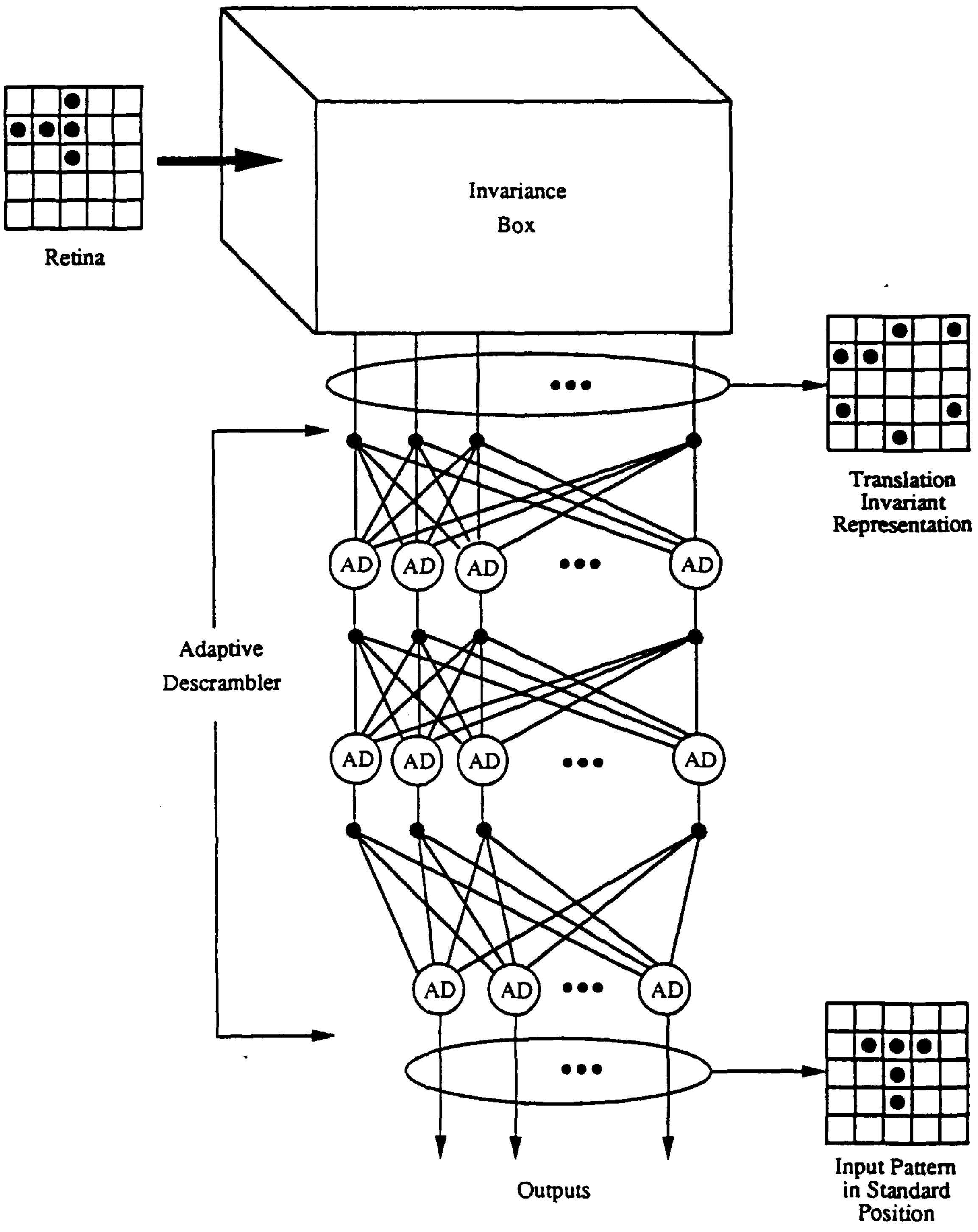
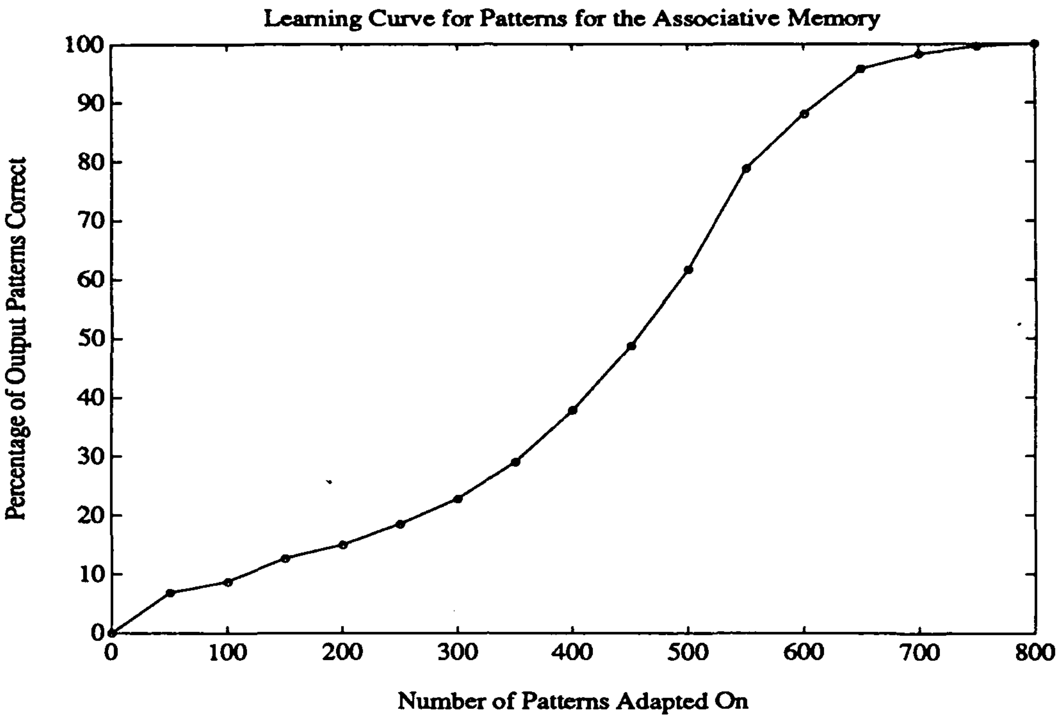

Invariance Box
Retina
Translation Invariant Representation
Adaptive Descrambler
AD
Outputs
Input Pattern in Standard Position

Figure 4.1: Diagram of a translation invariant pattern recognition system. The adaptive descrambler associates a translation invariant representation of a figure to its original representation in standard position.



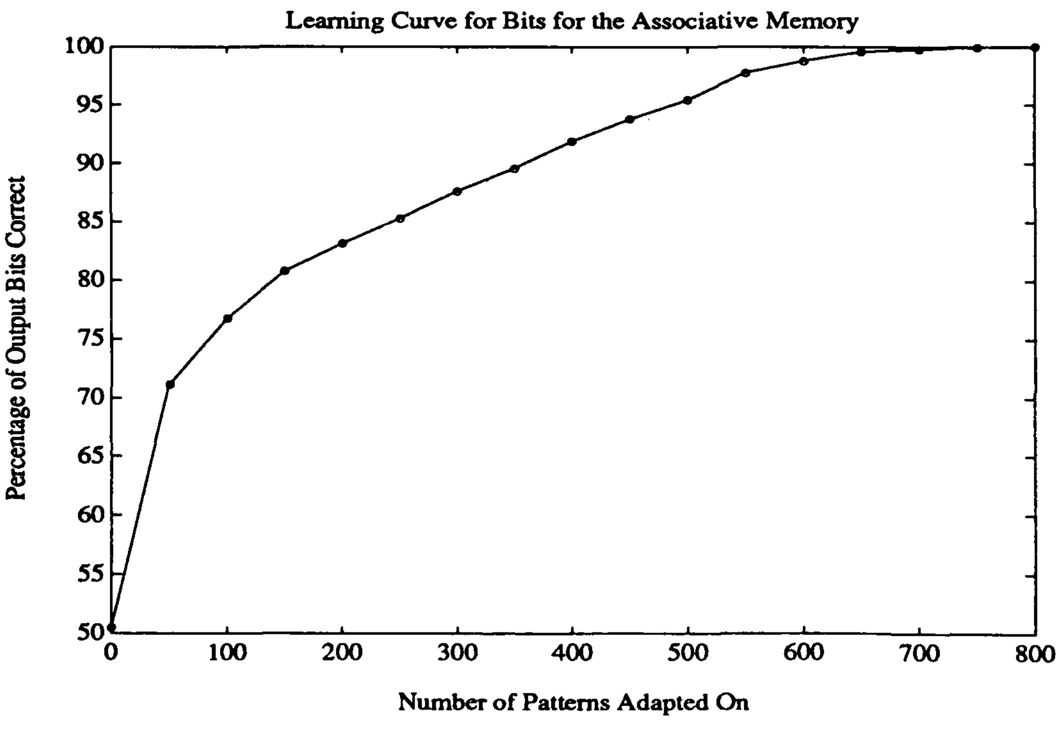


Figure 4.2: Learning curves for patterns and bits for the adaptive descrambler associative memory application. Average of training 100 nets. Data points are marked by "o".

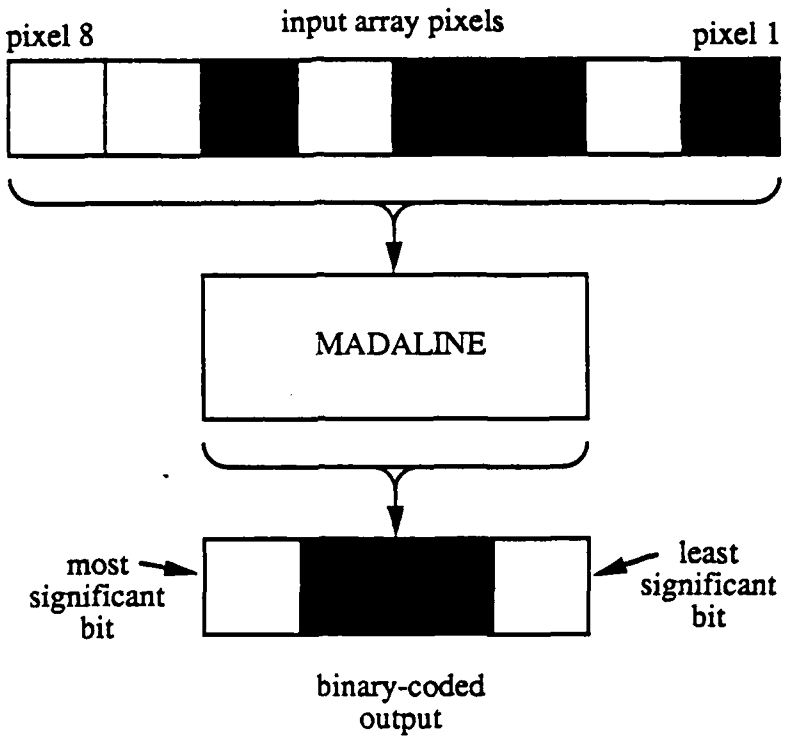


Figure 4.3: A component of an edge detector system. The Madaline outputs the position of the first black pixel in a binary code.

on must be allowed for. This means there are nine possible outputs for the Madaline, requiring four output Adalines to represent them. The minimum possible network then to solve the problem is an 8 input, 4-feed-4 Madaline.

The training set consisted of all 256 possible sequences of "on" and "off" pixels represented as sequences of $+1$s and -1s. Besides the minimal network, 8 input, 5-feed-4 and 6-feed-4 Madalines were also trained to solve the problem.

Table 4.1 lists the simulation results for the different size networks. One hundred nets starting from different initial conditions were trained for each size of network. Training was terminated after presenting 2000 patterns requiring adaptation. The convergence rate and average number of patterns adapted to reach convergence are listed for each architecture trained. For those cases where convergence was not reached, the average best performance of the network is listed. This best performance is the highest percentage of the total output bits that were correct at any of the intermediate checkpoints.

When the algorithm failed to converge, usually all but a couple of input patterns were

Edge Detector Performance			
network size	% convergence	ave patterns adapted to converge	ave best bit rate (%) for non-converge
4-feed-4	52	545	99.524
5-feed-4	61	422	99.775
6-feed-4	89	282	99.849

Table 4.1: Performance for the edge detector problem. Averages are for training on 100 nets beginning at random initial weights.

learned correctly. These unlearned patterns usually only had about one output bit per pattern in error. This is why the average best performance for bits even during nonconvergence was so good. The network had arrived at a solution that satisfied all but a few output bits out of the 1024 total. This failure to converge lead to an investigation of the algorithm dynamics to find out why convergence did not occur. The next chapter presents an example of a failure mode identified during that investigation. All failures of the algorithm to converge on this edge detector problem were of this type.

Further examination of the simulation results reveals a not unexpected result. There was a noticeable trend as the trained net became larger in size than absolutely necessary to solve the problem presented. The convergence rate went up and training time, as measured by the number of patterns adapted to converge, went down.

The use of a network larger than the minimum required will be referred to as "overarchitecturing." Overarchitecturing in the case of networks with two layers of adaptive units means using more first layer units than necessary. The number of output units is fixed by the problem definition. As one overarchitects, there are more ways to achieve a set of hidden patterns that are separable in the required ways by each of the output units. This makes a given problem easier to solve. Thus, it is expected convergence rate would increase and training required to reach convergence to decrease.

This example problem did not address the issue of generalization. Generalization is the ability of a network to respond correctly to patterns it has not been specifically trained on. In this example, the entire set of possible patterns were presented to the net during training. The issue of generalization and the effect of overarchitecturing on generalization performance will be addressed in the next section.

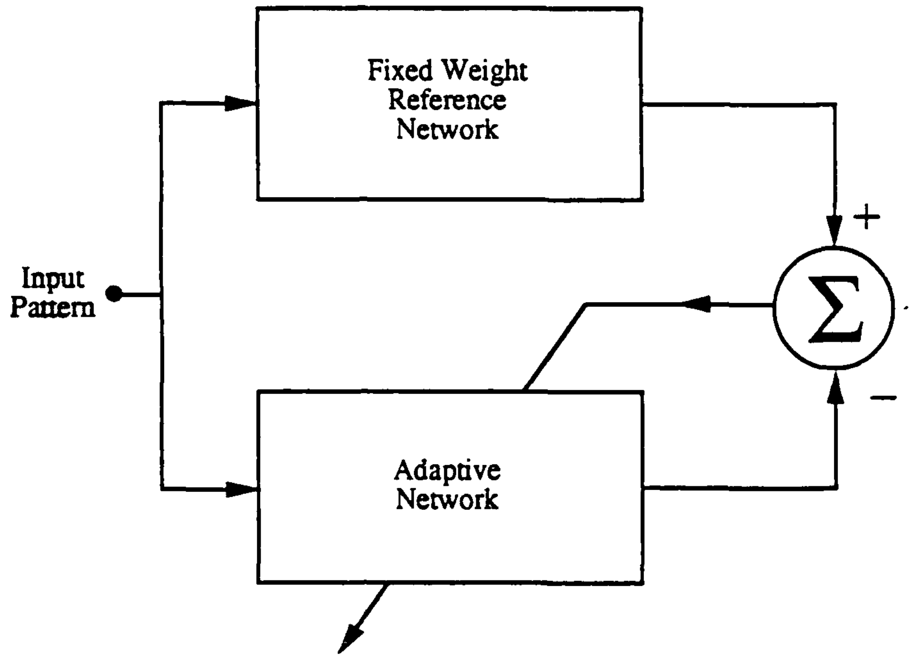


Figure 4.4: Training a network to emulate another. The fixed net provides desired responses for the adaptive net.

4.4 The Emulator Problem

One of the intended applications of neural networks is to learn the nonobvious relationships between the inputs to a decision process and the resulting decision. It is a relatively simple problem to write down a boolean logic function for the edge detector of the previous section. Each output bit is a relatively simple function of the eight input bits. The ease with which this can be done makes the problem a simple one to program on a computer. In similar fashion one could write logical relationships for each output bit of the associative memory problem addressed earlier. Due to the dimensionality of that problem, it would not be easy. Trying to program this problem as a relationship between input bits and output bits would be difficult. Thus, the function the Madaline performs in the associative memory case is nonobvious. What ability does MRII have to train networks to learn nonobvious, perhaps arbitrary, input/output relationships?

A structure to investigate this question is shown in Figure 4.4. The idea is to train an adaptive net to emulate a fixed net. The two nets will receive the same inputs and have the same number of output units. During training, the fixed net acts as a teacher by providing desired responses to the adaptive net. The fixed net can have its weights set randomly to provide a wide range of arbitrary input/output mappings for the adaptive net to learn. The adaptive net will have its weights initialized to some other random set of values. MRII will be used to train the adaptive net to respond just like the fixed net to a given input.

If the adaptive net has the same architecture as the fixed net, then at least one set of weights exist that will allow the adaptive net to emulate the fixed net perfectly, the set in the fixed net. With this structure, one can be assured the problem being presented to the adaptive net is solvable.

The issue of generalization is easily addressed by this structure. Training can be conducted on a subset of the possible input patterns. After training is complete, patterns not in the training set can be presented in parallel to the fixed and trained nets and their responses compared.

The issue of overarchitecturing can also be addressed. There is no need for the adaptive net to have the same number of first-layer units as the fixed net. It could have more. By zeroing the weights of the unneeded units, the perfect solution offered by the fixed net could still be obtained. The effect on generalization performance of extra units can be easily checked.

The example presented here is a 16 input, 3-feed-3 fixed net. Adaptive nets with 3, 6, and 9 first layer Adalines were trained. Two different size training sets were used. The smaller set had 650 patterns and the larger set had 1500 patterns. These patterns were selected randomly from the set of possible input patterns numbering 65,536. Training was allowed to continue until convergence or 100,000 patterns were adapted upon. After training was completed, generalization was checked on the balance of the possible input pattern set. The results are shown in Table 4.2.

As in the case of the edge detector, overarchitecturing improved training performance. The number of training attempts that reached convergence increased as extra units were added to the adaptive net. The number of patterns that had to be adapted to reach convergence decreased as the number of first layer Adalines increased. This is not surprising. The generalization results though are surprising.

Generalization is seen to decrease with overarchitecturing. This is expected. The amount of decrease is very slight though and this is surprising. Consider an adaptive net with nine first layer units versus one with only three. The net with nine units has a hidden pattern space with 512 patterns available in it. The minimal architecture can solve the problem with only eight. The fact that the larger net arrives at a solution that preserves generalization performance is counterintuitive.

Emulating a 16 input, 3-feed-3 Fixed Madaline					
adaptive net architecture	convergence rate (%)	ave patterns adapted	cnvgd. general.		nonconverged best bit trng rate (%)
			bits(%)	pats(%)	
650 Pattern Training Set					
3-feed-3	58	22940	98.11	95.98	89.28
6-feed-3	72	20860	96.70	92.20	88.84
9-feed-3	80	16120	95.43	88.66	90.68
1500 Pattern Training Set					
3-feed-3	39	50860	99.20	98.32	92.43
6-feed-3	65	46610	98.68	96.92	90.33
9-feed-3	65	37780	98.13	95.36	89.31

Table 4.2: Training and generalization performance for the emulator application. The fixed net was 16 input, 3-feed-3.

As one would expect, the smaller training set is learned quicker and with a greater convergence rate than the larger set. There is also the expected decrease in generalization performance. This decrease is slight. This indicates that 650 patterns are sufficient to define the type of problem the fixed net can present. The 650 patterns are sufficient to insure good generalization even from the highly overarchitectured 9-feed-3 adaptive net. This training set represents about one percent of the available patterns in the space.

Use of the smaller training set and larger adaptive nets can save a lot of training time. The 9-feed-3 net with the smaller training set converges twice as often with one third the number of adapted patterns as the 3-feed-3 net trained on 1500 patterns. The difference in generalization performance is modest.

Not all training attempts reached convergence. Table 4.2 reports the average best bit training rate for those cases that did not converge. Training set size and architecture of the adaptive net had little effect on this measure of performance. Generalization testing was performed on these nonconverged cases also. It was found that generalization performance was very close to the performance on the training set at the cessation of training. To check how well generalization tracks training performance, a generalization check was done at each of the training checkpoints. A typical result is plotted in Figure 4.5. The adaptive network is 9-feed-3, the one with the worst expected generalization performance, being trained on the 1500 pattern training set. The plot shows that generalization and training performance track each other closely at all levels of performance. This allows one to expect

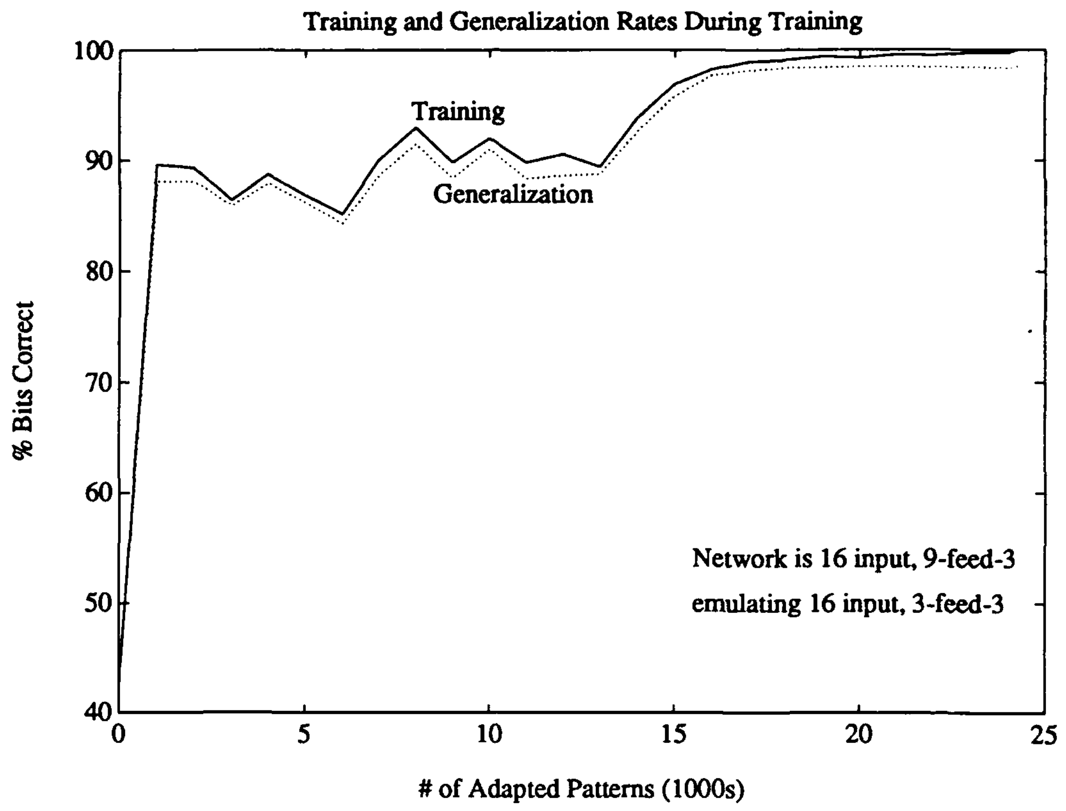


Figure 4.5: Tracking of generalization and training performances. Network is a 16 input 9-feed-3 emulating a 3-feed-3 fixed net.

that generalization performance will be good anytime one gets good training performance. The plot shows that little added generalization performance was obtained during the final 9000 adapted patterns that led to final convergence.

The learning curve plot in Figure 4.5 is typical. MRII's performance rises quickly and then levels off. Further improvements take a relatively long time to occur and do not happen monotonically.

For those cases of nonconvergence, the final improvement towards convergence did not happen in the limit set for training time. Unlike the failures to converge identified for the edge detector and characterized in the next chapter, there is no indication here that convergence wouldn't occur if enough time were allowed. The failure mode identified in the next chapter could be responsible but just not recognizable. Due to the larger dimensionality of this problem, specific reasons for nonconvergence could not be identified.

Chapter 5

A Failure Mode of MRII

The previous chapter showed the MRII algorithm does not always converge to a solution. Examination of some instances of nonconvergence revealed a failure mode of MRII. This chapter presents this failure mode by examining a particular instance of it. This particularly simple example will show that a modification of the algorithm will be required to handle this mode of failure. Some possible modifications are presented.

A secondary purpose of this chapter is to provide the reader some insight into the dynamics of the algorithm. The example used in this chapter is simple enough to represent in two dimensions, allowing presentation using graphical methods. The reader will be able to see what happens to the mappings from input pattern to hidden pattern to output pattern as trial adaptations and weight changes are made. While the chapter highlights a case where the algorithm fails to proceed to a solution, its study can lead to a better understanding of the methodology used by the algorithm.

5.1 The And/Xor Problem

The particular problem used for this example is called the and/xor problem. The challenge is to train a 2-input, 2-feed-2 network to give the logical "and" of its inputs as its first output, and the exclusive-or of its inputs as the second output. The desired input/output mapping is presented as a truth table in Table 5.1.

A solution for this problem is shown graphically in Figure 5.1. In the figure, the decision lines of the various Adalines have been identified by the weight vectors which define them. To allow easy reference to these weight vectors a double superscript is used on the weight

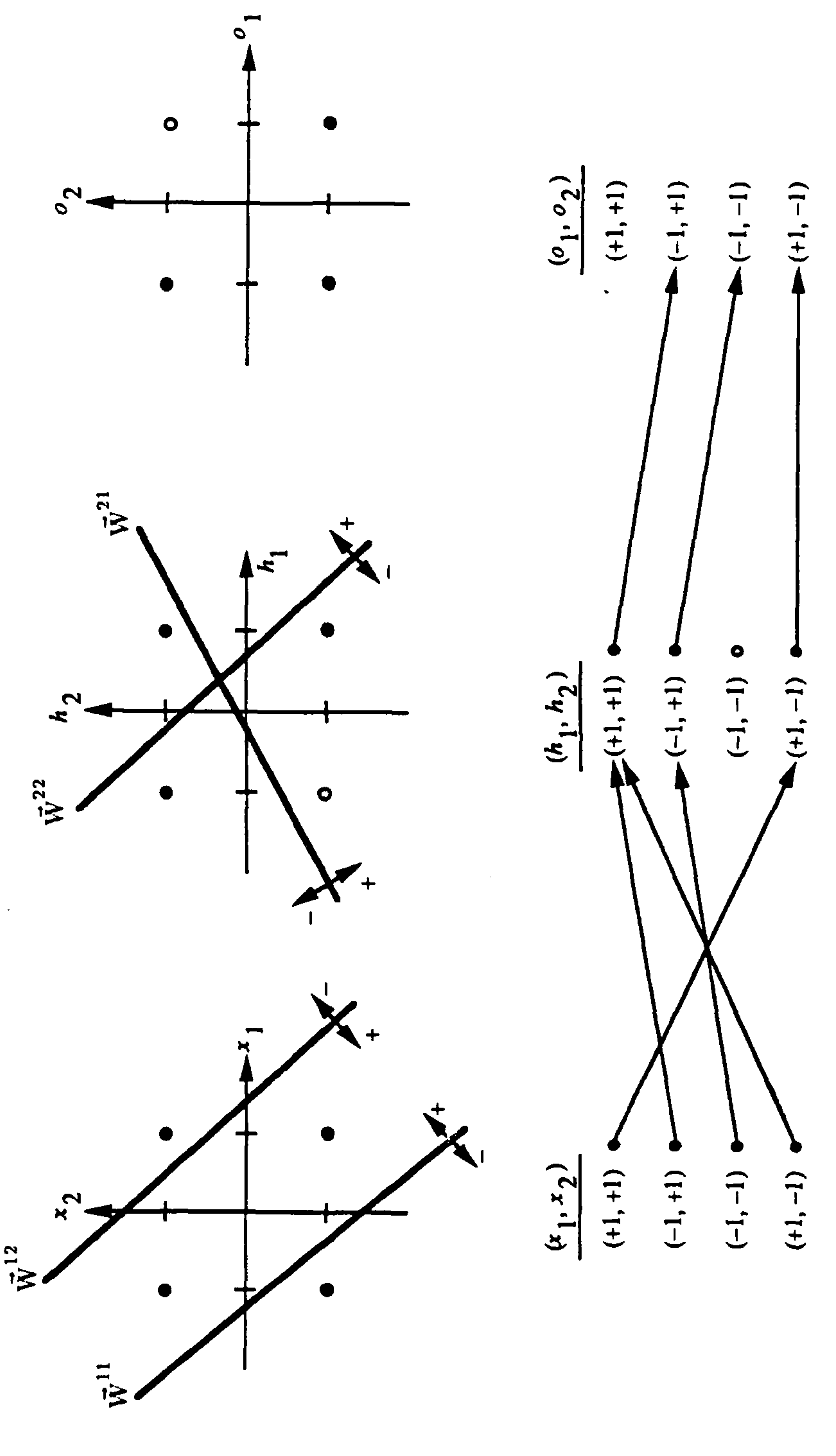


Figure 5.1: Graphical representation of a network that solves the and/xor problem.

TRUTH TABLE			
input pattern		desired response	
x_1	x_2	d_1	d_2
+1	+1	+1	−1
+1	−1	−1	+1
−1	+1	−1	+1
−1	−1	−1	−1

Table 5.1: Truth table for the and/xor problem.

vector symbol. For instance, $\vec{W}^{21}$ refers to the weight vector of the first Adaline on the second (output) layer. As there is only one hidden layer in this network, the superscript on $\vec{H}$ and its components has been dropped.

This particular problem forces the Adalines of the first layer to perform double duty. They must provide a hidden pattern set that can be separated by the second output Adaline to do the exclusive-or function. This hidden pattern set must also be separable by the first output Adaline to allow realization of the and function. Thus there is an implicit requirement for the output Adalines to cooperate with each other on settling upon a mutually satisfactory hidden pattern set. The whole goal of MRII is to implement a method to facilitate this cooperation.

For this particular problem and for some of the problems presented in Chapter 4, there are states for the network from which MRII cannot proceed to a solution. The next section will show an example of a limit cycle that MRII cannot escape.

5.2 A Limit Cycle

Consider a network with the following sets of weights in it:

$$\begin{aligned}
\vec{W}^{11} &= [.3,\ -.15,\ -.3]^T \\
\vec{W}^{12} &= [-.15,\ .05,\ -.05]^T \\
\vec{W}^{21} &= [.15,\ -.15,\ .15]^T \\
\vec{W}^{22} &= [-.15,\ .15,\ .15]^T
\end{aligned}$$

State 1					
input pattern		hidden pattern		output pattern	
x_1	x_2	h_1	h_2	o_1	o_2
+1	+1	−1	−1	+1	−1
+1	−1	+1	−1	−1	−1 *
−1	+1	+1	−1	−1	−1 *
−1	−1	+1	−1	−1	−1

Table 5.2: The input pattern to hidden pattern to output pattern mapping for **State 1.** Incorrect output responses are marked by *.

These weights were taken from an actual simulation result. They have been rounded to make easier the algebra and graphing to follow in this section. The decision lines and mappings for this network are shown graphically in Figure 5.2. The mapping information is also shown in tabular form in Table 5.2. In the table, incorrect output responses are indicated by an asterisk. This set of weights and mappings will be referred to as **State 1.**

The network in **State 1** makes an error for two of the four input patterns. The two patterns produce the same hidden pattern and output response and also have the same desired response. Therefore, it makes no difference which of these two patterns is considered first. Until one of these error producing patterns is presented, no adaptations will take place since the responses for the other two input patterns are correct. When one of the two input patterns that result in an error is presented to the network, MRII will attempt to correct the output by trial adapting the first layer Adalines.

At this point, it is important to note that o_1 is already correct. MRII will then accept a first-layer trial adaptation only if it completely corrects the output pattern. It will not accept a "trade" of errors in the bits of the output pattern. The Hamming distance between the output and desired response must actually be reduced.

For an error producing input, the first layer responds with the hidden pattern $h_1 = +1$, $h_2 = -1$. A trial adaptation can be thought of as moving this hidden pattern to another position in the hidden space and checking the decisions made by output Adalines on this new point. This concept is shown graphically in Figure 5.3. This same information is show in tabular form in Table 5.3. The table shows none of the possible trial adaptations will completely correct the output so they will all be rejected. Failing to find an acceptable first-layer adaptation, MRII will proceed by adapting the second output Adaline to provide

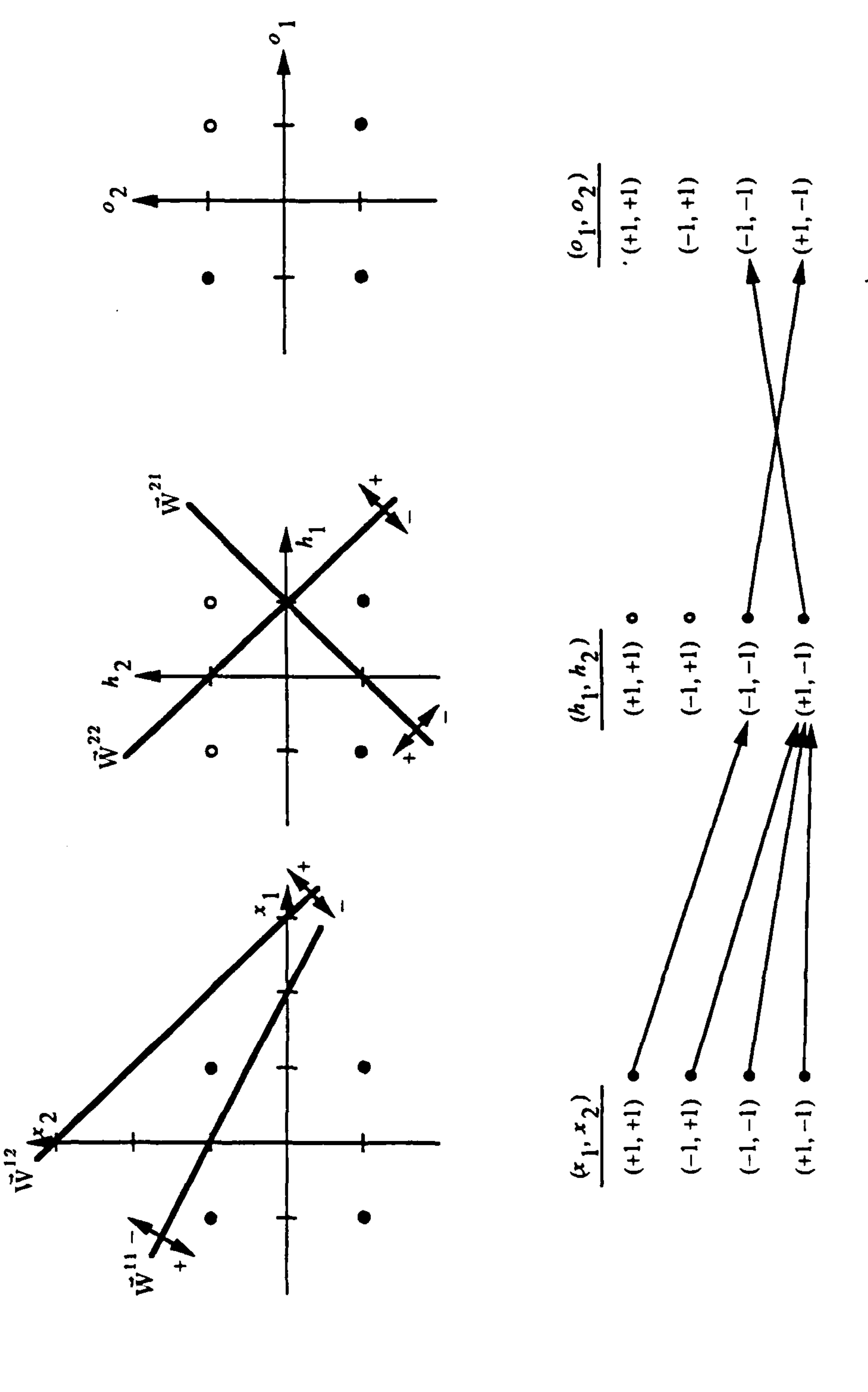


Figure 5.2: Graphical presentation of **State 1**.

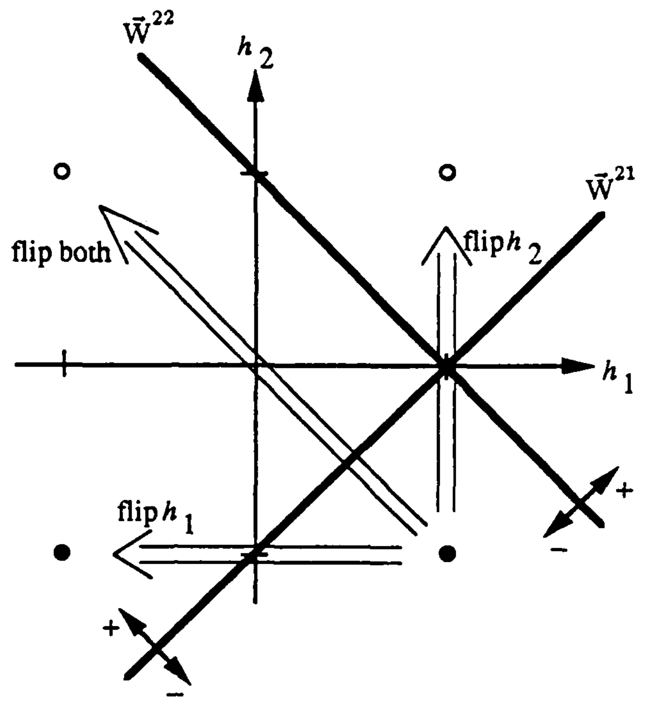


Figure 5.3: Graphical presentation of trial adaptations from **State 1**.

the desired response, +1, for its input, the hidden pattern $h_1 = +1$, $h_2 = -1$.

This brings the network to what will be referred to as **State 2**. The only change in the network has been to the second output Adaline. The hidden patterns used by the network have not changed. It is assumed the response to the hidden pattern $h_1 = -1$, $h_2 = -1$, has not been affected. The mapping information for the network in **State 2** is shown in Table 5.4. A repositioning of the second output Adaline's decision line in the hidden space to provide this mapping is shown in Figure 5.4.

As Table 5.4 shows, the network now makes an error for only one input pattern, $x_1 = -1$ and $x_2 = -1$. No adaptations will be made by the network until this pattern is presented. The possible trial adaptations that can be made in this case will have the same bad result that occurred from **State 1**. All trials will result in o_1 becoming incorrect. This is because the hidden pattern is still the same as that considered in **State 1** and the decision line of the first output Adaline has not been changed in getting to **State 2**. Therefore, all the trial adaptations will be rejected. The second output Adaline will again require adaptation. Its input is the same as it was when adapting out of **State 1**, but the error is now opposite to

Trial Adapting for State 1				
Trial Adapt	h_1	h_2	o_1	o_2
none	+1	−1	−1	−1 *
flip h_1	−1	−1	+1 *	−1 *
flip h_2	+1	+1	+1 *	+1
flip both	−1	+1	+1 *	−1 *

Table 5.3: The results of trial adaptations from **State 1**.

State 2					
input pattern		hidden pattern		output pattern	
x_1	x_2	h_1	h_2	o_1	o_2
+1	+1	−1	−1	+1	−1
+1	−1	+1	−1	−1	+1
−1	+1	+1	−1	−1	+1
−1	−1	+1	−1	−1	+1 *

Table 5.4: Mapping information for the network in **State 2**.

what it was. Thus, the adaptation will return the network to **State 1**, basically undoing the previous adaptation. The network is in a limit cycle.

The astute reader will now question whether **State 2** is the only possible result of adapting out of **State 1**. Suppose the second output Adaline's response to the hidden pattern $h_1 = -1$, $h_2 = -1$ had also changed with the adaptation out of **State 1**. Indeed, a plausible positioning for the second output Adaline's decision line is shown in Figure 5.5. This would result in a **State 3** whose mapping is detailed in Table 5.5.

An examination of Table 5.5 shows there are now two input patterns that result in errors. For random presentation order, the pattern $x_1 = +1$, $x_2 = +1$ will be presented prior to the other error producing input sometime **State 3** is visited. This input pattern maps to a different hidden pattern than considered previously. For this hidden pattern, the trial adaptation involving a flip of h_2 corrects the output completely and will be accepted. This trial adaptation is indicated on Figure 5.5. Thus, there exists a mechanism for escape from the claimed limit cycle *if* **State 3** can be reached. Unfortunately, **State 3** cannot be reached as a few lines of algebra will now show.

MRII uses the modified relaxation method for performing its weight adaptations. Given

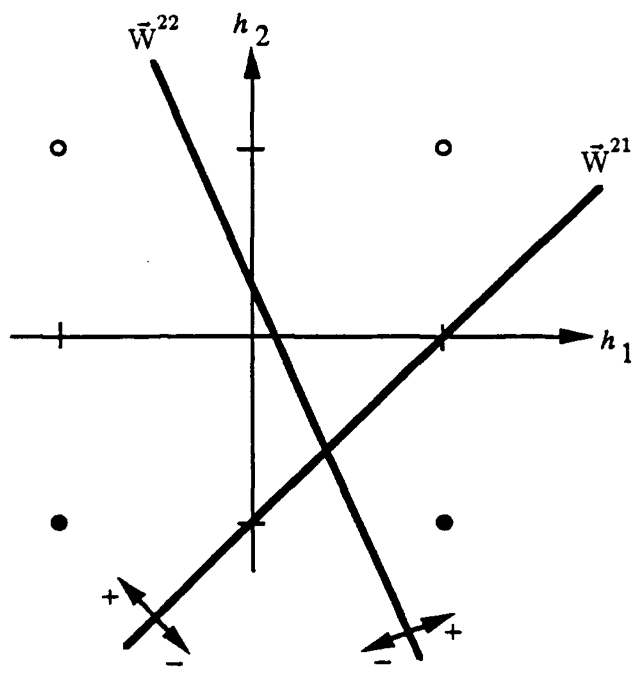


Figure 5.4: Separation of the hidden pattern space for **State 2**.

that it is desired to adapt only when there is a binary error present, all of the adaptation procedures proposed by Mays [3] have the same basic form. Using k as an adaptation counter, all the weight change formulas have the form,

$$\Delta\vec{W}_k = \delta_k d_k \vec{X}_k\,, \qquad \delta_k > 0.$$

Here it is emphasized that the size of the adaptation step is not necessarily constant from adaptation to adaptation by subscripting δ.

MRII adapts only when there is a binary decision error present. This is in accord with the minimal disturbance principle. When following this procedure, one will note that the sign of the desired response and the sign of the analog error defined as, $\epsilon_k = d_k - y_k$, will be the same. Thus, even the famous Widrow-Hoff least means squares (LMS) algorithm has the form above. The LMS algorithm is usually written, $\Delta\vec{W}_k = 2\mu\epsilon_k\vec{X}$. Here the magnitude of $2\mu\epsilon_k$ can be identified as δ_k. All the various weight update methods used to adapt Adalines differ only in how they choose δ_k. It will be shown that not only can **State 3** not be reached, but this non-reachability is not dependent on the weight update

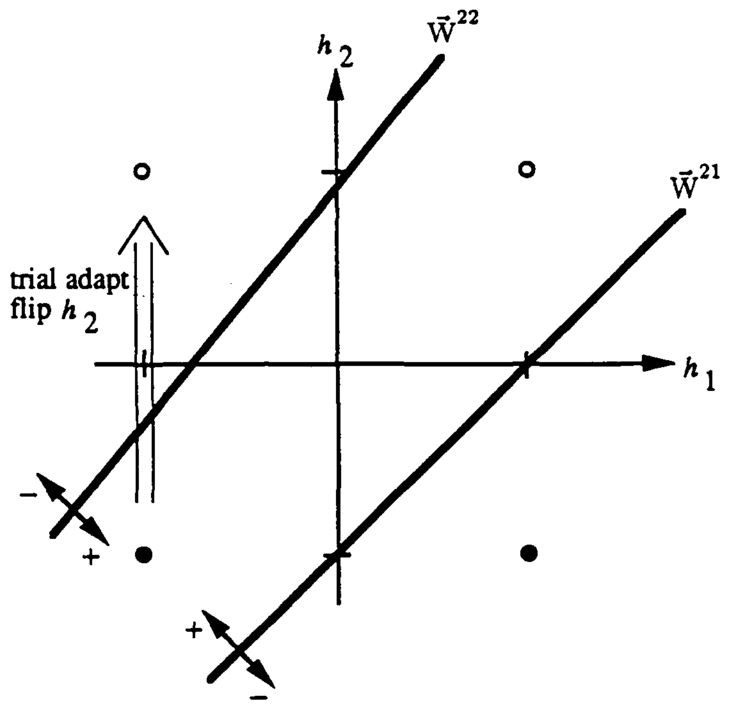


Figure 5.5: An output separation that would give **State 3**.

rule used.

The situation under consideration is the adaptation of the second output Adaline from **State 1**. Assuming k adaptations have occurred to this point, the situation is as follows: $\vec{H}_k = [+1, +1, -1]^T$, $o_2 = -1$, $d_2 = +1$, and $\vec{W}_k^{22} = [-.15, .15, .15]^T$, where the bias input component has been included in the hidden vector. For the moment, generalize the second Adaline's weight vector to $\vec{W}_k^{22} = \alpha[-1, +1, +1]^T$. This generalized weight vector implements the same decision line but with a generalized confidence.

The particular requirement is to adapt the second output Adaline to provide a +1 response. Note that if the weight update rule does not select a large enough weight adjustment to actually change the binary response of the second Adaline, the network will remain in **State 1**. The next presentation of a pattern producing an error will require further adaptation of the second output Adaline. This will continue until **State 2** is finally reached. The weight update actually used by MRII will put the network in **State 2** in one step. The point here is that there is no advantage to adapting using smaller steps in this case.

Postulated **State 3**					
input pattern		hidden pattern		output pattern	
x_1	x_2	h_1	h_2	o_1	o_2
+1	+1	−1	−1	+1	+1 *
+1	−1	+1	−1	−1	+1
−1	+1	+1	−1	−1	+1
−1	−1	+1	−1	−1	+1 *

Table 5.5: Mapping for the postulated **State 3**.

After reaching **State 2**, the weight vector for the second output Adaline will have the form, $W_{k+1}^{22} = [-\alpha + \delta_k,\ \alpha + \delta_k,\ \alpha - \delta_k]^T$. A minimum value of δ_k can be computed since in **State 2** it is required, $\vec{\mathrm{H}}_k^T \vec{\mathrm{W}}_{k+1}^{22} > 0$. The minimal value of δ_k works out to be $\alpha/3$.

For the adaptation from **State 1** to result in **State 3**, another minimal δ can be computed. The requirement to reach **State 3** is that the hidden pattern $h_1 = -1$, $h_2 = -1$, be responded to with +1 by the second output Adaline. This requires $\delta_k > 3\alpha$. Thus, to reach **State 3**, an adaptation step nine times larger than that needed to just reach **State 2** is required. This fact is independent of the value of α. Such steps may not be consistent with the minimal disturbance principle. Indeed, they may not be achievable by the particular weight update rule in use.

Consider what the response to $\vec{\mathrm{H}}_k$ will be if **State 3** is reached. This value is what the adaptation level, L, would have to have been in the MRII weight update rule when adapting out of **State 1**. Setting $\delta_k = 3\alpha$ in $\vec{\mathrm{W}}_{k+1}^{22}$ and taking the dot product with $\vec{\mathrm{H}}_k$ one gets 8α. For the particular example of this section, $\alpha = .15$. If **State 3** were the result of adapting from **State 1** the resulting response to the adapted on hidden pattern, $\vec{\mathrm{H}}_k$, would be greater than 1.2.

This resulting confidence is greater than the desired response for this hidden pattern. None of the weight update rules use step sizes that result in overshooting the desired response. Indeed, doing so is certainly not in accord with the minimal disturbance principle. MRII is based on the minimal disturbance principle. No weight update rule that is consistent with this principle will allow escape from the limit cycle that results in this example.

5.3 Discussion

The previous section illustrates an example of MRII caught in a limit cycle. Why did this cycle happen? The reasons are twofold. First, the first output Adaline reached a solution before the second output Adaline. Second, the hidden pattern set settled upon by the first output Adaline was unusable by the second output Adaline in reaching a solution. Any attempt to change the hidden pattern set by the second output Adaline was effectively vetoed by the first output Adaline. Any change to the hidden pattern set destroyed the solution arrived at by the first output Adaline. Thus, the system had entered a local error minimum. No change on the first layer could reduce output errors.

The second layer Adalines found themselves in a situation not unlike that which inspired the usage concept. There existed a nonseparable subset of hidden patterns that the second output Adaline was forced to try to separate. It was successful at performing the separation for a given presentation but could not come to a global solution. Unfortunately, usage is not a technique that can be employed at the output layer. When the output layer is forced to adapt, there is no freedom left to share responsibility for correcting an output.

At this point it is important to note that usage, even on the first layer, will not solve this particular example. In examining the possible trial adaptations from **State 1**, all were considered and rejected. Usage affects only which particular Adalines participate in trial adaptations when participation is limited to some number less than the total number of Adalines on the layer. It can have no effect when all Adalines are considered for participation.

In this particular example, the limit cycle is easily recognizable. It is characterized by a lack of accepted trial adaptations on the first layer, and by a single output Adaline being adapted over and over. The master controller could detect a lack of accepted trial adaptations. In this case, there is hope for implementing an escape mechanism since a developed limit cycle can be recognized.

For larger networks with larger training sets, the failure mode may not be so easily recognizable. It may be possible for output units to come to solutions on subsets of the training set. These units would then limit the allowable hidden patterns to a small set. This small set would not allow the other units to come to global solution but allow some first-layer trial adaptations to be accepted. The limit cycle would consist of a large number of specific states. By restricting the hidden patterns to a nonsolvable set, wide variations of training performance would be seen. This would effectively mask the failure mode. The author conjectures situations like this occurred in some of the failures to converge with the emulator problem.

Though not reported in the previous chapter, solving the n-bit parity problem was attempted by MRII. The network required is an n-input, n-feed-1 Madaline. This was generally successful but a failure mode seen there was quite similar to the one noted above. The parity problem has a single output, so there is no possibility of another output Adaline vetoing trial adaptations as in the and/xor problem. What happens instead is that the allowed trial adaptations are ineffective. The ones tried do not correct the output. This happens because of the somewhat arbitrary limit placed on the number of Adalines that will be considered for adaptation. The generally useful heuristic of considering the least confident half of the Adalines on a layer sometimes fails. In this case, it is easy for the master controller to increase the number of Adalines considered for adaptation and the number of Adalines at a time that are trial adapted. Unfortunately, this procedure is very heuristic. It does however work. With some experience, methods using this concept can be implemented to cause the n-bit parity problem to almost always converge. The fine tuning used to achieve good convergence for the n-bit parity problem will not always transfer to another problem, nor even to the $(n+2)$-bit parity problem so the details are omitted here. Sometimes with smaller nets, all possible trial adaptations would be tried and rejected. The cause for this was a large bias weight being initialized in the ouput Adaline. The bias determined the network's response and the first-layer Adalines weren't weighted sufficiently to have any effect. A simple solution to this is to initialize the weights in the output Adaline all equal. It then starts as a majority voter, giving every first layer Adaline equal say in the network response. In any event, these techniques will not allow escape from the and/xor limit cycle detailed above.

The specific and/xor example cited in this chapter resulted after several earlier adaptations. Starting with the same initial weights, a different pattern presentation sequence would avoid the limit cycle and instead converge to a solution quite quickly. The point is, there seems to be no way to characterize a set of initial weights as being a set that leads to a limit cycle. Indeed, a cursory look at the weights that were presented at the beginning of the chapter shows nothing remarkable about them. They could easily have occurred at initialization. Indeed, weights that define a limit cycle condition sometimes do occur at initialization. It seems unlikely that a method for avoiding limit cycles can be found while retaining the basic form of the algorithm. The only feasible strategy seems to be to detect the limit cycle and then implement some escape.

The minimal disturbance principle underlies the MRII algorithm. Any limit cycle escape mechanism will have to selectively depart from the minimal disturbance principle, perhaps in an extreme fashion. The size of the adaptation level needed to reach **State 3** is an example of how extreme this departure may have to be.

Various techniques have been used with varying degrees of success. Most are of a heuristic nature, and often need to be hand crafted for the particular problem being solved. Some of these techniques include: allowing the adaptation level of non-output units to be a random variable; modifying the output unit adaptation levels by their usage count; modifying the number of Adalines allowed to participate in trial adaptations by the average output unit usage count. None of these offer much hope of being generally applicable.

Probably the easiest escape that is generally applicable is to reinitialize the weights in the system and start the training over. Often this is not a bad thing to do. Experience indicates that if MRII is going to find a solution it will find it fairly quickly. If training performance levels off at an unacceptable level, after a reasonable time, start over. The determination of a reasonable time can be done by doing an ensemble of training attempts. A look at the average learning curve will reveal an average performance increase versus adaptations performed. This will allow extrapolation to the convergence point allowing one to set a reasonable limit to the number of adaptations that will be performed before reinitializing.

Reinitialization has the drawback of losing all benefit of training performed up to that point. Often the system will be somewhere near a solution and merely needs nudged out of a local minima. One way to do this is to occasionally add some random perturbations to the weights. This moves the system to a random place in weight space. The idea is to get away from the local minima but not so far away from the previous weight position as to negate all previous train: ıg.

It is hoped this weight perturbation method would have some generality to it. Some preliminary success has been achieved using this technique with the n-bit parity problem for n up to seven. The perturbation magnitude needed and a schedule for applying it were determined by experience. The magnitude of perturbations needed for a general problem might depend only on the particular architecture being trained. Thus, an analysis of the effect of weight perturbations on the input/output mapping of a Madaline system might be very helpful in generalizing this technique. This reasoning is the prime motivation for the sensitivity work presented in the next chapter.

Chapter 6

Sensitivity of Madalines to Weight Disturbances

6.1 Motivation

It's been shown the training performance of Madaline Rule II rises quickly and then levels off. For even moderately sized networks, the number of training attempts that result in convergence represent a smaller fraction of the total attempts than one would like. For some architectures, considerable refinement of the number of Adalines allowed to participate in trial adaptations and adjustment of the "MULT" factor of the usage gain is required to get performance levels like those reported in Chapter 4. The identified failure mode presented in the previous chapter may account for much of the problem. In larger networks this failure mode may be difficult to recognize. However, the problem may be more fundamental.

For the emulator problem, the weights in the fixed net were generated randomly. It may be the network is non-robust in the weights. The mapping from input patterns to outputs may change significantly for a very small change in the weights of the fixed net. If such is the case, how close do the weights in the trained adaptive net have to be to the perfect solution weights of the fixed net to get good performance? What is the rate of performance degradation as the weights deviate further and further from exactly right?

An investigation of the sensitivity of the input/output mapping to changes in the weights may lead to insightful improvements of the MRII algorithm. It was proposed at the end of the last chapter to use random weight perturbations to escape from the algorithm's failure mode. A thoughtful approach to this method requires some understanding of the weight

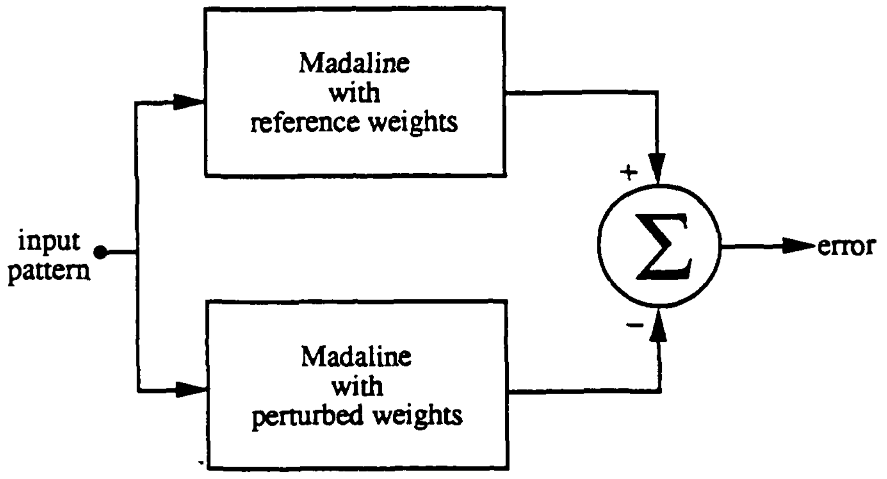


Figure 6.1: Comparing a network with a weights perturbed version of itself.

sensitivity issue. The study may also suggest modifications of the network architectures and input representations to minimize the effects of weight disturbances.

The investigation uses a structure similar to that of Figure 4.4. Instead of an adaptive network being compared to a fixed network, a perturbed version of a net is compared against the unperturbed net, called the reference net, as shown in Figure 6.1. The effect of weight changes, with known statistical distribution, on the input/output mapping of a single Adaline is done first.

6.2 Perturbing the Weights of a Single Adaline

Hoff [6] did an analysis of the effect of weight perturbations on the decision mapping of an Adaline using the hypersphere approximation. He did his analysis in the n-dimensional pattern space. As pointed out in Chapter 2, the bias weight introduces a complication to such an analysis. The bias weight is not considered part of the weight vector by Hoff. This causes the separating hyperplane to pass through the pattern hypersphere offset from the origin by a distance dependent on the bias weight, and the magnitude of the reduced dimension weight vector. Changes in the weights must then be resolved into components parallel and perpendicular to the weight vector. The final results were complicated and difficult to apply to answering the question of how close weights have to be to a known solution for good performance.

By using the hypersphere approximation in the $(n+1)$-dimensional weight space, simplifications occur. The input pattern is now considered to be of dimension $n+1$, but with the bias input, x_0, a constant $+1$. The set of all possible input patterns now lie on the positive x_0 half of a hypersphere of radius $\sqrt{n+1}$ centered at the origin of $(n+1)$-space. Glanz [8] maintained the hypersphere approximation could now be applied to this hemi-hypersphere. The patterns are now confined to an oriented half of the hypersphere but within this region they are assumed uniformly distributed. The major simplification is that now the separating hyperplane always passes through the origin. The decision of which input patterns are classified $+1$ and which are -1 is determined by how the hyperplane intersects the pattern hemihypersphere. Changes in the weights cause the orientation of the hyperplane to change. These changes can now be described by a single parameter, the angle between the original weight vector and the new weight vector.

In Figure 6.2 the positive x_0 hemihypersphere, a weight vector, a perturbed weight vector and the associated intersections of the decision hyperplanes are shown. The result of perturbing the weight vector is a reorientation of the decision hyperplane. For those patterns in the darker shaded region, the perturbed network will make a "decision error" relative to the reference network.

The darker shaded area consists of two partial lunes. By symmetry, one can see the area of the two partial lunes add to that of a lune of angle θ, the angle between the weight and perturbed weight vectors. Using the hypersphere approximation, the probability of a decision error due to a shift in the weight vector is:

$$\begin{aligned}\text{P[Decision Error]} &= \frac{\text{Area of shaded lune}}{\text{Area of hemihypersphere}} \\ &= \frac{\theta}{\pi}\end{aligned}$$

Taking the expectation of both sides,

$$\text{Ave P[Decision Errors]} = \frac{\text{E}\{\theta\}}{\pi} = \frac{\overline{\theta}}{\pi} \tag{6.1}$$

To determine $\overline{\theta}$, look at Figure 6.3. In $(n+1)$-space, $\vec{W}$ and $\Delta\vec{W}$ determine a plane and the angle between them in that plane, ϕ. The angle, θ, between $\vec{W}$ and $\vec{W}+\Delta\vec{W}$ can be found by plane geometry.

$$\theta = \tan^{-1}\frac{|\Delta\vec{W}|\sin\phi}{|\vec{W}|+|\Delta\vec{W}|\cos\phi}$$

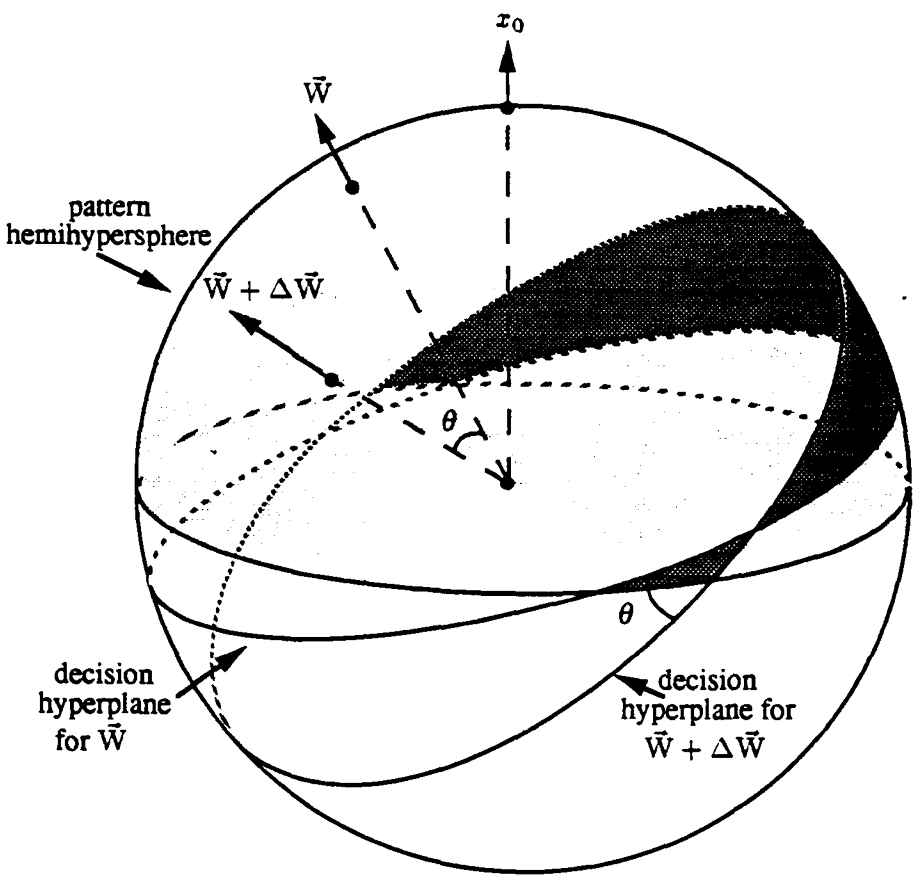

x_0
$\vec{W}$
pattern hemihypersphere
$\vec{W} + \Delta\vec{W}$
θ
θ
decision hyperplane for $\vec{W}$
decision hyperplane for $\vec{W} + \Delta\vec{W}$

Figure 6.2: Reorientation of the decision hyperplane due to a disturbance of the weight vector. Patterns lying in the darker shaded region change classification.

$$= \tan^{-1} \frac{\sin\phi}{\frac{|\vec{W}|}{|\Delta\vec{W}|} + \cos\phi}$$

$$\approx \tan^{-1}\left(\frac{|\Delta\vec{W}|}{|\vec{W}|}\sin\phi\right) \quad \text{for } \frac{|\vec{W}|}{|\Delta\vec{W}|} \gg 1$$

Using $\tan^{-1} x \approx x$ for small x,

$$\theta \approx \frac{|\Delta\vec{W}|}{|\vec{W}|}\sin\phi$$

Then,

$$\bar{\theta} \approx \frac{|\Delta\vec{W}|}{|\vec{W}|}\mathrm{E}\{\sin\phi\}$$

It has been assumed here that the ratio of the magnitude of the weight perturbation vector to the magnitude of the original weight vector is known and constant. If this is not the

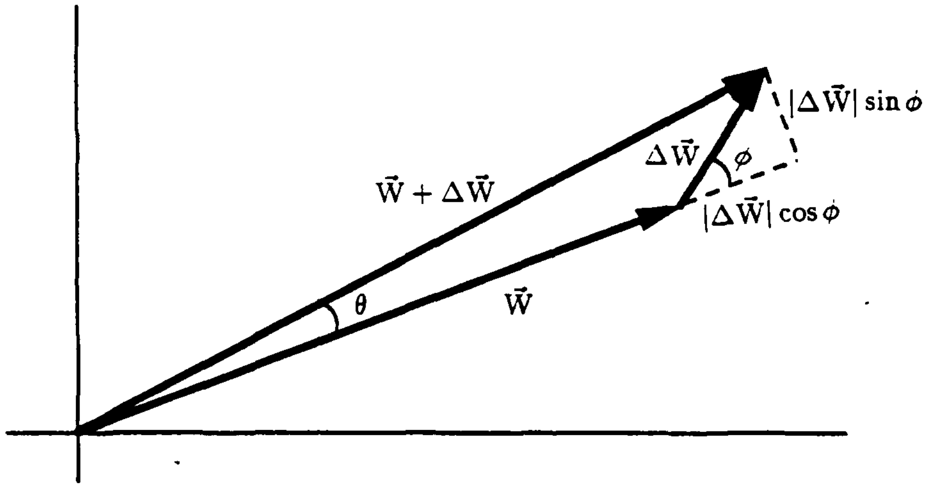


Figure 6.3: Geometry in the plane determined by $\vec{W}$ and $\Delta\vec{W}$.

case, but the perturbations are independent of each other and independent of the original weights, an average perturbation magnitude can be used.

To complete the derivation of the average decision errors a single Adaline makes due to perturbation of its weights, a formula for $E\{\sin\phi\}$ is needed. The vectors $\vec{W}$ and $\Delta\vec{W}$, or their extensions, will intersect a hypersphere of unit radius. Since only the directions of these vectors determine ϕ, the analysis can assume a unit radius sphere without loss of generality. Glanz [8] notes that the probability that the angle between two vectors, $\vec{V}_1$ and $\vec{V}_2$, has value between ϕ and $\phi + \Delta\phi$ is proportional to the differential area at angle ϕ (see Equation 2.4). Modifying his equation for use in $(n+1)$-space, the probability density function can be written,

$$p[\ \vec{V}_1 \text{ and } \vec{V}_2 \text{ form angle } \phi\ ] = \frac{K_n}{K_{n+1}} \sin^{n-1}\phi \tag{6.2}$$

Therefore,

$$\begin{aligned} E\{\sin\phi\} &= \int_0^{\pi} \sin\xi \, \frac{K_n}{K_{n+1}} \sin^{n-1}\xi \, d\xi \\ &= \frac{2K_n}{K_{n+1}} \int_0^{\frac{\pi}{2}} \sin^n\xi \, d\xi \\ &= \frac{2K_n}{K_{n+1}} \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma(\frac{n}{2}+1)} \end{aligned}$$

$$\begin{aligned}
&= \frac{2\pi^{n/2}}{\Gamma(\frac{n}{2})} \frac{\Gamma\left(\frac{n+1}{2}\right)}{2\pi^{\frac{n+1}{2}}} \sqrt{\pi} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma(\frac{n}{2}+1)} \\
&= \frac{\Gamma^2\left(\frac{n+1}{2}\right)}{\Gamma^2(\frac{n}{2})} \frac{2}{n} \\
&= \left(\frac{\mathrm{K}_n}{\mathrm{K}_{n+1}}\right)^2 \frac{2\pi}{n} \qquad (6.3)
\end{aligned}$$

This result can be further simplified by using Stirling's approximation for factorials and the expressions for K_n for n even and odd in Equations 2.2 and 2.3. Stirling's approximation is:

$$n! \approx \sqrt{2\pi n}\, n^n \mathrm{e}^{-n}.$$

For $n = 2m$,

$$\begin{aligned}
\frac{\mathrm{K}_n}{\mathrm{K}_{n+1}} &= \frac{2\pi^m}{(m-1)!} \frac{(2m)!}{\pi^m 2^{2m+1} m!} \\
&= \frac{(2m)!}{2^{2m}\, m!\,(m-1)!} \\
&= \frac{m\,(2m)!}{2^{2m}\,(m!)^2} \\
&\approx \frac{m\sqrt{4\pi m}\,(2m)^{2m}\mathrm{e}^{-2m}}{2^{2m}\, 2\pi m\, m^{2m}\mathrm{e}^{-2m}} \\
&\approx \sqrt{\frac{m}{\pi}} = \sqrt{\frac{n}{2\pi}} \qquad \text{for } n \text{ even.}
\end{aligned}$$

For n odd, $n = 2m+1$ and $n+1 = 2(m+1)$,

$$\begin{aligned}
\frac{\mathrm{K}_n}{\mathrm{K}_{n+1}} &= \frac{\pi^m 2^{2m+1}(m)!}{(2m)!} \frac{m!}{2\pi^{m+1}} \\
&= \frac{2^{2m}\,(m!)^2}{\pi\,(2m)!} \\
&\approx \frac{2^{2m}\left(\sqrt{2\pi m}\, m^m \mathrm{e}^{-m}\right)^2}{\pi\sqrt{2\pi 2m}\,(2m)^{2m}\mathrm{e}^{-2m}} \\
&\approx \frac{2\pi m}{\pi\sqrt{4\pi m}} \\
&\approx \sqrt{\frac{m}{\pi}} = \sqrt{\frac{n-1}{2\pi}} \qquad \text{for } n \text{ odd.}
\end{aligned}$$

For n sufficiently large, the even and odd cases become indistinguishable and it can be approximated,

$$\frac{\mathrm{K}_n}{\mathrm{K}_{n+1}} \approx \sqrt{\frac{n}{2\pi}} \qquad \text{for } n \text{ large} \tag{6.4}$$

With this result, Equation 6.3 becomes,

$$\begin{aligned} \mathrm{E}\{\sin\phi\} &\approx \left(\sqrt{\frac{n}{2\pi}}\right)^2 \frac{2\pi}{n} \\ &\approx 1 \qquad \text{for } n \text{ large} \end{aligned}$$

The final result for determining the change in the input/output mapping of an Adaline caused by a disturbance of its weights can now be formulated. Using the results so far,

$$\begin{aligned} \text{P[Decision Error]} &= \frac{\overline{\theta}}{\pi} \\ &\approx \frac{1}{\pi}\frac{|\Delta\vec{\mathrm{W}}|}{|\vec{\mathrm{W}}|}\left(\frac{\mathrm{K}_n}{\mathrm{K}_{n+1}}\right)^2 \frac{2\pi}{n} \qquad (6.5) \\ &\approx \frac{1}{\pi}\frac{|\Delta\vec{\mathrm{W}}|}{|\vec{\mathrm{W}}|} \qquad (6.6) \end{aligned}$$

The last two expressions represent different levels of approximation. Future reference will call the first the complicated approximation while the second will be called the simple approximation. The simple approximation ignores any dependency on the number of inputs and is good in the limit as n gets large.

6.3 Decision Errors Due to Input Errors

The above result can be used to predict the error rate of an Adaline in the first layer of a multilayer network. For a randomly generated reference network, it can be assumed that first-layer Adalines will behave independently of each other. Assume all Adalines of the first layer are affected by weight disturbances such that their weight perturbation ratios, $|\Delta\vec{\mathrm{W}}|/|\vec{\mathrm{W}}|$ are the same. They will all have, on average, the same probability of making an error. The number of errors in the output of this first layer of a perturbed network relative to the reference network will be binomially distributed. This distribution has as parameters the number of first-layer Adalines and the error rate of an Adaline.

The perturbed net's second layer will see inputs that are in error relative to those seen by the second layer of the reference network. Some of these erroneous patterns may cause decision errors at the network output even if there are no weight errors in the second and subsequent layers. The basic question to investigate then is, how do random input errors affect an Adaline's decision mapping?

In the binary case where the components of the input are either $+1$ or -1, an input error can be thought of as a "flipped" bit. If a pattern has one flipped bit, the result is a nearest neighbor, a pattern of Hamming distance one from the original. One way to think of flipping a bit is to add a $\Delta\vec{X}$ to $\vec{X}$. To generate a nearest neighbor of $\vec{X}$, add a vector to it that has all components equal to zero except for one, say the ith. Let the ith component, Δx_i be equal to $-2x_i$. This disturbance vector has a Euclidean length of two.

In general, you can generate a pattern vector which is a Hamming distance h from a given pattern $\vec{X}$, by flipping h of its bits. This can be done by adding a disturbance vector $\Delta\vec{X}$ having h nonzero components equal to -2 times the corresponding $\vec{X}$ component. This $\Delta\vec{X}$ will have a Euclidean length of $\sqrt{4h}$.

Note that all of $\vec{X}$'s pattern neighbors of Hamming distance h are at the same Euclidean distance from $\vec{X}$. Thus, the pattern neighbors of given Hamming distance from $\vec{X}$ lie on a hypersphere of radius $\sqrt{4h}$ with the tip of $\vec{X}$ as origin. The neighbors also lie on the pattern hemihypersphere on which $\vec{X}$ is located, since they are legitimate patterns in the input space. The possibility of the bias input being flipped is excluded. The intersection of these two hyperspheric regions is a hypersphere of dimension one less than the pattern hemihypersphere. Hoff's hypersphere approximation leads one to conjecture that the pattern neighbors of $\vec{X}$ are uniformly distributed on this reduced dimension surface. A visualization that supports this conjecture is shown in Figure 6.4. Consider the situation where $n = 3$ and there is no bias input. The eight patterns available in 3-space form the vertices of a cube which can be inscribed in a sphere (not shown in the figure). The vertex marked $\vec{X}$ has three nearest neighbors. They lie equally spaced on a circle which lies in a plane. Furthermore, the vector $\vec{X}$ intersects this plane at the center of the circle. For the following analysis, it will be assumed the conjecture of a pattern's neighbors being uniformly distributed in the reduced dimension space to be true.

If the weight vector components are chosen from a zero mean distribution, the weight vector will be orthogonal to the x_0 axis, on average. This is because the expected value of the dot product of the weight vector with the x_0 axis is the expected value of the bias weight, which is zero. This leads to the intuitive conclusion that an "average" Adaline will have equally likely $+1$ and -1 outputs. This average Adaline will be analyzed to determine the effect of random input errors on its input/output map.

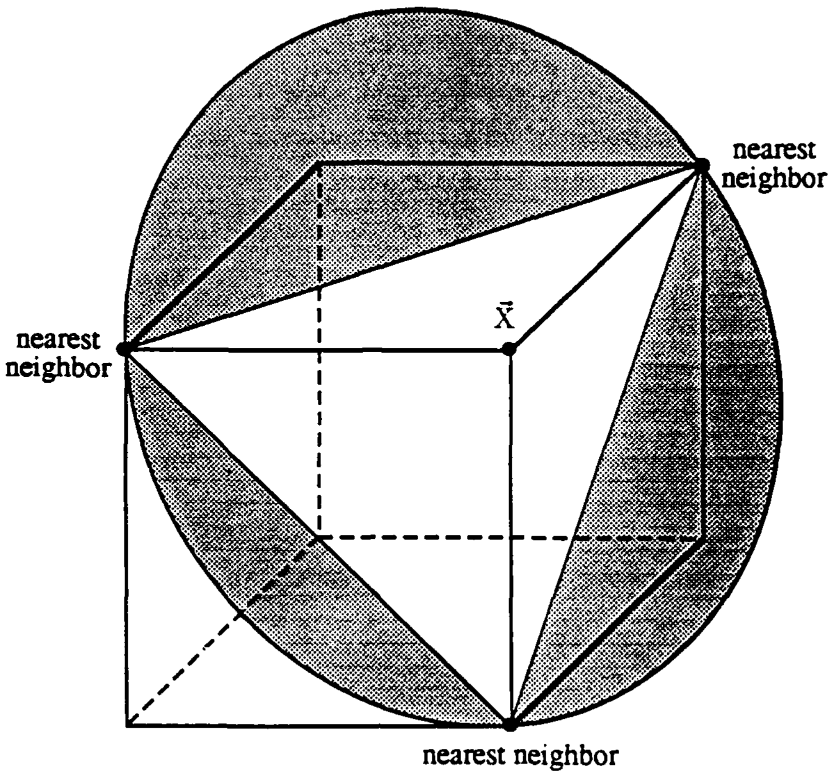


Figure 6.4: The nearest neighbors of $\vec{X}$ lie equally spaced on a reduced dimension hypersphere.

The idea of the analysis is shown in Figure 6.5. Without loss of generality, assume the pattern vector $\vec{X}$ of interest lies on the positive side of the decision hyperplane. Shown in the figure is a representation of the location of the pattern neighbors located Euclidean distance d from two different $\vec{X}$s. $\vec{X}_1$ is located sufficiently far from the decision hyperplane that all of its pattern neighbors distance d away will be classified positive. No errors will be made by this Adaline if any of these neighbors are input instead of the correct pattern $\vec{X}_1$. The pattern $\vec{X}_2$ is located closer to the decision hyperplane and some of its neighbors at distance d are on the opposite side of the decision hyperplane. Thus, a fraction of those times that a neighbor is presented instead of the correct $\vec{X}_2$, a decision error will be made by the Adaline. It will be determined here what fraction of these neighbor presentations will result in decision errors.

At this point, the notions of dimensionality need to be made more precise. The terminology employed by Sommerville [10] will be used. The input patterns lie on a hypersphere

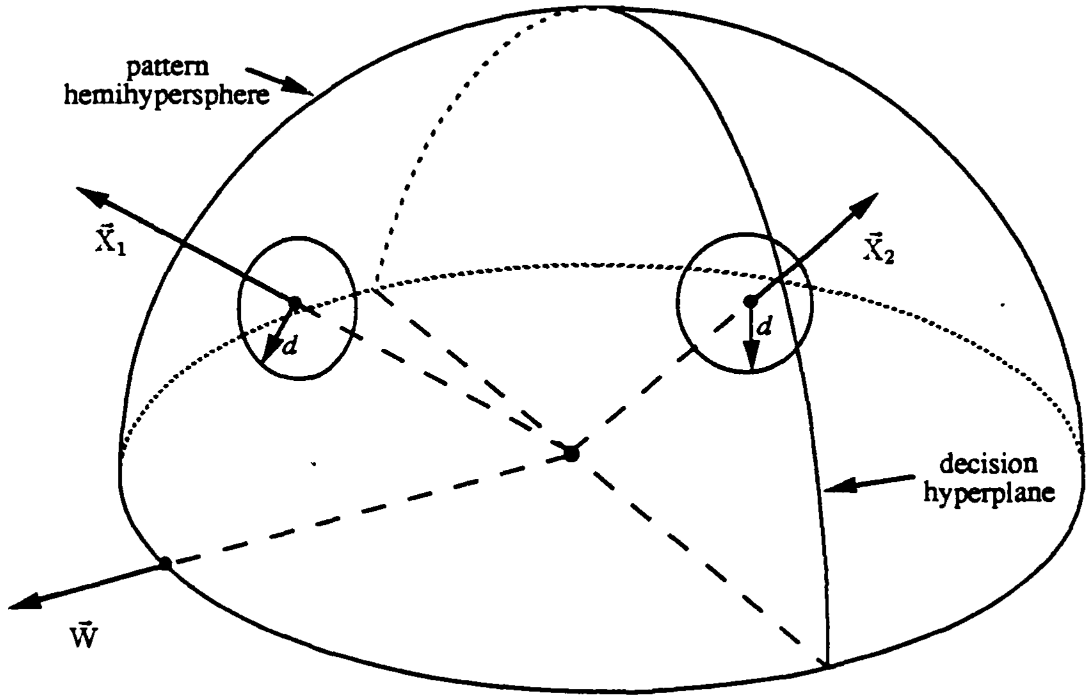


Figure 6.5: All the patterns at distance d from $\vec{X}_1$ will be classified the same as $\vec{X}_1$. Some of the patterns at distance d from $\vec{X}_2$ are classified different from $\vec{X}_2$.

of n dimensions in $(n+1)$-space. The hypersphere is of dimension n because only n components are needed to specify a point on this surface. The 2^n patterns considered to make up the input space of the Adaline are confined to the positive x_0 hemihypersphere. The bias input is supplied internally by the Adaline and is considered not subject to error. As has been pointed out before, the decision hyperplane of an Adaline described analytically by $\vec{X}^T\vec{W} = 0$, is a surface of n dimensions. Sommerville calls this an n-flat but will be referred to as an n-dimensional hyperplane here. The pattern neighbors of $\vec{X}$ at Euclidean distance d are located on a surface defined by the intersection of two hyperspheres of n dimensions in $(n+1)$-space. The first hypersphere is the one of radius $\sqrt{n+1}$ centered at the origin and the second is the hypersphere of radius $d = \sqrt{4h}$ centered about the tip of $\vec{X}$. This intersection is a hypersphere of dimension $n-1$. Furthermore, this intersection lies in an n-dimensional hyperplane. This can be seen as follows. Let $\vec{N}$ be a pattern neighbor. It is representable by $\vec{N} = \vec{X} + \Delta\vec{X}$ where $\Delta\vec{X}$ is of the form described earlier. Then,

$$\vec{X}^T\vec{N} \;=\; \vec{X}^T(\vec{X} + \Delta\vec{X})$$

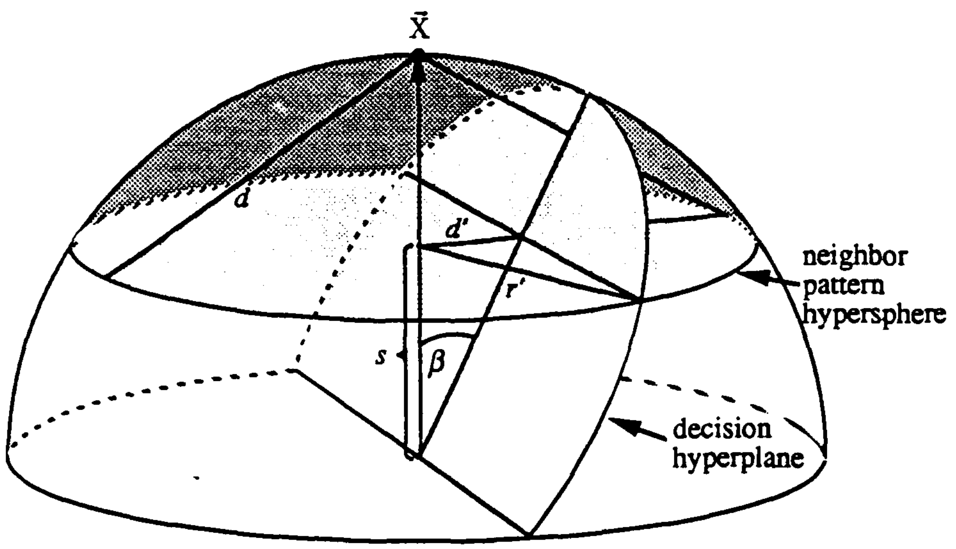


Figure 6.6: A representation of the location of the patterns at distance d from the input vector $\vec{X}$.

$$= \vec{X}^T\vec{X} + \vec{X}^T\Delta\vec{X}$$

$$= n + 1 + -2h = \text{a constant},$$

h being the Hamming distance of the neighbor. This is the equation of an n-dimensional hyperplane at a distance from the origin of $(n+1-2h)/\sqrt{n+1} = r - (2h/r)$ where $r = \sqrt{n+1}$ is the radius of the pattern hypersphere. Thus, the pattern neighbors of $\vec{X}$ at Euclidean distance d lie on a hypersphere of limension $n-1$ which is contained in an n-dimensional hyperplane.

The details of the geometric analysis are presented in Figures 6.6–6.8. Figure 6.6 shows a representation of the situation in the $(n+1)$-dimensional pattern/weight space. Here the perspective is rotated such that the true pattern vector $\vec{X}$ is pointing up. The pattern neighbors of $\vec{X}$ that lie at Euclidean distance d from $\vec{X}$ are represented by a circle which lies on the pattern sphere. This circle lies in a plane which cuts off a spherical cap. This cap is shaded in the figure to aid visualization. The figure is only an aid for visualization purposes. More precisely, the patterns are located on a hypersphere of n dimensions. The pattern neighbors are on a hypersphere of $n-1$ dimensions which lies in an n-flat which cuts off a hyperspherical cap from the pattern hypersphere. Because the perspective has been rotated, the hemisphere shown in the figure no longer represents the pattern hemihypersphere. The

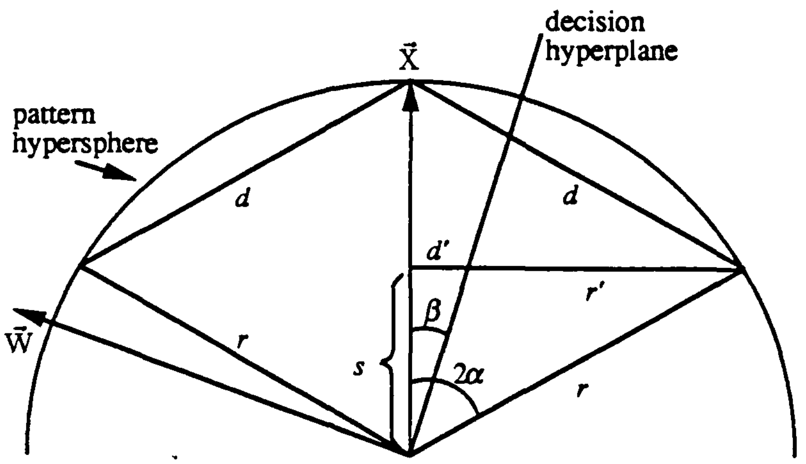


Figure 6.7: A representation of the geometry in the plane formed by $\vec{W}$ and $\vec{X}$.

pattern neighbors of $\vec{X}$ do lie on the pattern hemihypersphere, however, and it will be assumed that their distribution is unaffected by "edge" effects caused by the actual position of the pattern hemihypersphere. This assumption is good for the average Adaline that has not too many input errors in the pattern presented.

The pattern vector $\vec{X}$ and weight vector $\vec{W}$ define a plane. Figure 6.7 shows the geometry in this plane. This plane intersects the decision hyperplane in a line, and the pattern hypersphere in a circular arc. The plane intersects the hypersphere of $n-1$ dimensions containing the pattern neighbors in two points. Ordinary plane geometry can be used for analysis in this plane formed by $\vec{X}$ and $\vec{W}$.

A representation of the hyperplane containing the pattern neighbors is shown in Figure 6.8. This hyperplane forms the floor of the hyperspherical cap, and the circle represents the hypersphere in which the pattern neighbors lie. The center of this hypersphere is the intersection of the hyperplane containing the neighbor patterns and the vector $\vec{X}$. The decision hyperplane intersects the hyperplane containing the pattern neighbors in a hyperplane of dimension $n-1$ or an $(n-1)$-flat, represented by a line in the figure. The pattern neighbors on the opposite side of the decision hyperplane from the center of the pattern neighbor hypersphere will cause errors if they are presented to the Adaline instead of the true pattern $\vec{X}$. The Hoff hypersphere approximation says the probability of error will be the ratio of the area of the neighbor hypersphere on the opposite side of the decision hyperplane to the

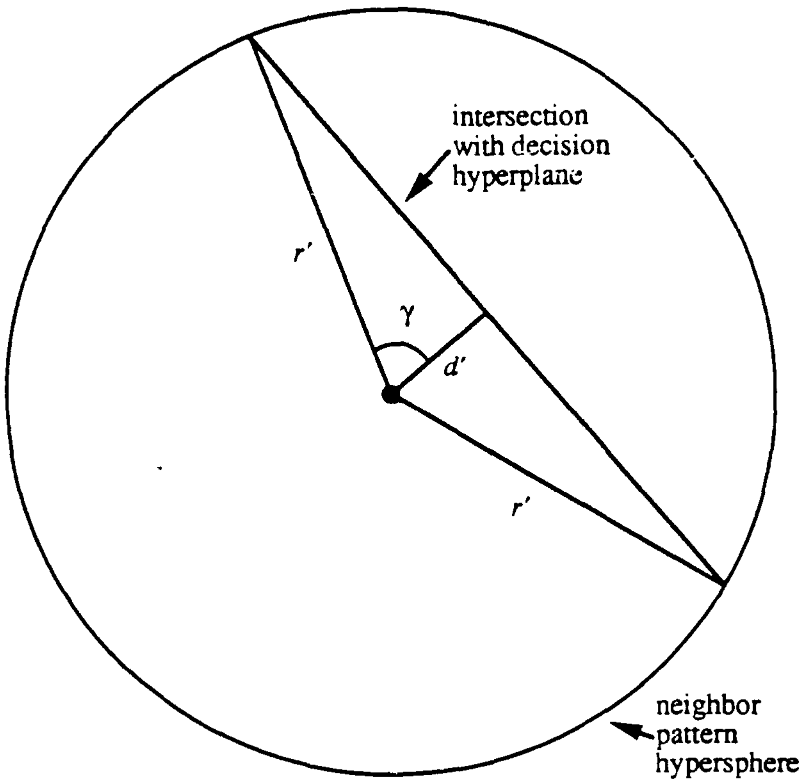

intersection with decision hyperplane
r'
γ
d'
r'
neighbor pattern hypersphere

Figure 6.8: Representation of the geometry in the hyperplane containing the patterns at distance d from $\vec{X}$. This hyperplane is the floor of the shaded cap in Figure 6.6.

total area of the neighbor hypersphere. The radius of the neighbor pattern hypersphere is designated r'. The distance from the center of this hypersphere to the intersection with the decision hyperplane is designated d'. Determining r' and d' allows the determination of the angle γ, which can be used to compute the required area on the opposite side of the decision hyperplane. The quantities r' and d' can be determined using plane geometry in the $\vec{X}$-$\vec{W}$ plane shown in Figure 6.7.

Referring back to Figure 6.7 an expression for the angle α is derived first.

$$\alpha = \sin^{-1}\left(\frac{d/2}{r}\right)$$

Now r' can be computed.

$$\frac{r'}{r} = \sin 2\alpha$$

$$r' = r \sin\left(2 \sin^{-1}\frac{d}{2r}\right)$$

$$\begin{aligned}
&= 2r\sin\left(\sin^{-1}\frac{d}{2r}\right)\cos\left(\sin^{-1}\frac{d}{2r}\right)\\
&= 2r\,\frac{d}{2r}\,\sqrt{1-\left(\frac{d}{2r}\right)^2}\\
&= d\sqrt{1-\left(\frac{d}{2r}\right)^2}
\end{aligned}$$

An intermediate result, s is now derived.

$$\frac{s}{r} = \cos 2\alpha$$

$$\begin{aligned}
s &= r\cos\left(2\sin^{-1}\frac{d}{2r}\right)\\
&= r\left(1-2\sin^2\left(\sin^{-1}\frac{d}{2r}\right)\right)\\
&= r\left(1-\frac{d^2}{2r^2}\right)\\
&= \frac{2r^2-d^2}{2r}
\end{aligned}$$

With an expression for s, d' can now be derived.

$$\frac{d'}{s} = \tan\beta$$

$$d' = \frac{2r^2-d^2}{2r}\tan\beta$$

Now γ can be derived by referring to Figure 6.8.

$$\begin{aligned}
\gamma &= \cos^{-1}\frac{d'}{r'}\\
&= \cos^{-1}\left(\frac{2r^2-d^2}{2r}\,\frac{\tan\beta}{d\sqrt{1-(\frac{d}{2r})^2}}\right)\\
&= \cos^{-1}\left(\frac{(2r^2-d^2)\tan\beta}{d\sqrt{4r^2-d^2}}\right)\\
&= \cos^{-1}(\rho\tan\beta) \qquad (6.7)
\end{aligned}$$

where ρ is defined,

$$\rho = \frac{2r^2 - d^2}{d\sqrt{4r^2 - d^2}}. \tag{6.8}$$

The area of the neighbor hypersphere on the opposite side of the decision hyperplane from the true input vector can be expressed by the integral,

$$\int_0^{\gamma} \mathrm{K}_{n-1} \left(r' \sin\phi\right)^{n-2} r' d\phi .$$

This integral sums up slices perpendicular to the line that d' lies on in Figure 6.8. Each slice is the area of a hypersphere of dimension $n-2$, and has a thickness of $r'\, d\phi$, where ϕ is an angle measured from the line d' lies on. The probability of error then when a neighbor pattern is presented instead of the true pattern is the ratio of the integral above to the total surface content of the neighbor pattern hypersphere,

$$\mathrm{P[error|input\ error]} = \frac{\mathrm{K}_{n-1}}{\mathrm{K}_n} \int_0^{\cos^{-1}(\rho\beta)} \sin^{n-2}\phi \, d\phi \tag{6.9}$$

This conditional probability is a function of β, the angle from $\vec{\mathrm{X}}$ to the decision hyperplane. For $\beta \approx 0$, the pattern $\vec{\mathrm{X}}$ lies very close to the decision hyperplane and one would expect the probability of an error to be one-half. Indeed, the reader can check that Equation 6.9 yields a value of one-half for $\beta = 0$. For $\beta > 2\alpha$, $\vec{\mathrm{X}}$ is sufficiently far from the decision hyperplane that none of the pattern neighbors produce errors. To get an average probability of error when the input is a pattern at distance d from the true input, the error rate above must be weighted by the probability density of finding the true pattern at angle β from the decision hyperplane. For positive patterns, β is $\pi/2$ less the angle between $\vec{\mathrm{X}}$ and $\vec{\mathrm{W}}$. Using a modification of Equation 6.2 and remembering the patterns exist on a hemihypersphere, the average probability of error for an average Adaline can be written,

$$\begin{aligned} \mathrm{Ave\ P[decision\ error]} &= \int_0^{2\alpha} \int_0^{\cos^{-1}(\rho\beta)} \frac{\mathrm{K}_{n-1}}{\mathrm{K}_n} \sin^{n-2}\phi \, d\phi \, \frac{\mathrm{K}_n}{\frac{1}{2}\mathrm{K}_{n+1}} \cos^{n-1}\beta \, d\beta \\ &= 2\frac{\mathrm{K}_{n-1}}{\mathrm{K}_{n+1}} \int_0^{2\sin^{-1}\frac{d}{2r}} \int_0^{\cos^{-1}(\rho\beta)} \sin^{n-2}\phi \cos^{n-1}\beta \, d\phi \, d\beta \quad (6.10) \end{aligned}$$

This double integral can be evaluated numerically on a computer. Because of the dependency of the upper limit of the inner integral on the outer variable of integration and the large range of integration of the inner integral, it is difficult to arrive at a closed form approximation. An approximation arrived at by a combination of intuition and accident is,

$$\mathrm{Ave\ P[decision\ error]} \approx \frac{1}{\pi}\frac{|\Delta\vec{\mathrm{X}}|}{|\vec{\mathrm{X}}|} . \tag{6.11}$$

This approximation is quite similar to the result obtained when decision errors due to weight errors were analyzed. The intuition was that there exists a basic symmetry when working in the $(n+1)$-dimensional pattern/weight space. No further justification of the approximation will be presented other than to show during the results to follow that it is accurate. The approximation is most accurate for small numbers of errors in the patterns but is not unusable for numbers of errors approaching half the number of inputs. The integral expression cannot be expected to hold for errors numbering more than this due to the assumption about "edge effects" not affecting the distribution of neighbor patterns about the true pattern.

At this point it should be noted that though the effects of weight errors and input errors are similar, their effects are independent. The above result for input errors is unaffected by weight errors that might be present. The result shows the effect of using a pattern's neighbors at a given distance away from it as inputs to the Adaline instead of the pattern itself. When looking over the entire input space, it really doesn't matter where the decision hyperplane is (as long as the Adaline is close to average). For a particular position of the decision hyperplane, there will be a set of patterns whose neighbors will cause errors if the neighbors are presented. Moving the position of the hyperplane by weight errors will change that set of patterns which will be involved in decision errors if their neighbors are presented, but over the input space, the number of such patterns will remain the same. Therefore, if errors in the weights are also present, these errors will, on average, affect only those patterns which are presented with no input errors.

6.4 Total Decision Errors for Adalines and Madalines

The results from the above two sections now allow a formulation of the total probability of an Adaline making a decision error due to the combined effects of weight errors and input errors. In networks where it can be assumed the Adalines are independent of each other, this result can be extended to predict the error rate of the entire network.

An introduction of new notation will facilitate writing the results compactly. Define $\mathcal{D}_h$ as the probability an Adaline makes a decision error given that its input has h errors. Define $\mathcal{E}$ as the total probability that an Adaline makes a decision error. Then,

$$\mathcal{E} = \sum_{h=0}^{n} \mathcal{D}_h \cdot \mathrm{P}[\text{input has } h \text{ errors}] \tag{6.12}$$

From Section 6.2 and the discussion above about the independence of weight errors and input errors it is concluded that,

$$\mathcal{D}_0 \approx \frac{1}{\pi}\frac{|\Delta\vec{W}|}{|\vec{W}|}\left(\frac{K_n}{K_{n+1}}\right)^2\frac{2\pi}{n} \tag{6.13}$$

$$\approx \frac{1}{\pi}\frac{|\Delta\vec{W}|}{|\vec{W}|} \tag{6.14}$$

$\mathcal{D}_h$ for h other than zero is obtained from Equation 6.10 or 6.11. At this point use the facts that $r = \sqrt{n+1}$ and $d = \sqrt{4h}$ in these two equations and the defining equation for ρ. As explicit functions of h and n they become,

$$\mathcal{D}_h = 2\frac{K_{n-1}}{K_{n+1}}\int_0^{2\sin^{-1}\sqrt{\frac{h}{n+1}}}\int_0^{\cos^{-1}(\rho\beta)}\sin^{n-2}\phi\,\cos^{n-1}\beta\,d\phi\,d\beta \tag{6.15}$$

$$\approx \frac{1}{\pi}\sqrt{\frac{4h}{n+1}} \tag{6.16}$$

and ρ as defined in Equation 6.8 is now given by,

$$\rho = \frac{n+1-2h}{2\sqrt{h(n+1-h)}} \tag{6.17}$$

The equations above allow one to predict the average decision errors that a weights perturbed Madaline will make relative to its unperturbed reference. The use of Equations 6.13 and 6.15 in Equation 6.12 will be referred to as the complicated approximation. The simple approximation will mean the use of the simpler Equations 6.14 and 6.16. Knowledge about the relative magnitudes of the weight disturbance vectors relative to the unperturbed weight vectors of the Madaline is needed.

When comparing a perturbed net against a reference, one must assume the input patterns to the first layer contain no errors. The reference will respond to what it sees as if it were the true input and the perturbed net's response is compared against the reference's output. The theory developed above however is more powerful. It can be used to factor in knowledge about the distribution of errors in the input patterns to a network to predict its performance under real conditions.

An analysis begins with the first layer. If only weight errors are present, Equation 6.13 or 6.14 will provide the error rate for each Adaline on the layer. The binomial distribution is used to predict the probability of the first hidden pattern having a given number of errors.

This then is the input error distribution for the second layer Adalines. This information is used in Equation 6.12 to predict the error rate for each Adaline in the next layer, etc.

The above derivations made use of some approximations. Most of these approximations required either the weight perturbations or the Hamming distance between true inputs and erroneous inputs to be small. The results, especially Equations 6.14 and 6.16 are quite simple in form. They cannot be expected to hold in cases of large weight perturbations or high probability of input errors. The next section will show the results of simulations to check the accuracy of the theory and the approximations of the theory made above.

6.5 Simulation Results

The system depicted in Figure 6.1 was used to test the theory developed in the previous sections. A reference Madaline system was generated randomly by selecting each weight in the system independently from a uniform distribution over the interval $(-1, +1)$. These weights were then copied to the perturbed system.

To the weight vectors of each Adaline in the perturbed system a weight perturbation vector $\Delta\vec{W}$ was added. This vector was generated by selecting each component independently from the same uniform distribution as before. The perturbation vector was then scaled so that a desired weight perturbation ratio, $|\Delta\vec{W}|/|\vec{W}|$, was obtained. Thus the assumptions of the derivation were maintained. The orientation of the perturbation weight vector was random and independent of the reference weight vector but had a fixed magnitude relative to the reference. This weight perturbation ratio is expressed as a percentage in the results that follow.

A large number of patterns from the input space were then presented to the reference and perturbed systems in parallel and their outputs compared. If any bit of the outputs differed, a decision error was said to have been made. The results present the number of decision errors as a percentage of the patterns tested. For any given architecture being checked, this procedure was repeated for several different reference systems. Several perturbations of each reference system were tested. The results presented are the average percent decision errors over the different references and their perturbations.

The first system investigated is that of a single Adaline. No input errors are allowed since the reference needs a true input to calculate a true result. The only term needed in Equation 6.12 then is the $\mathcal{D}_0$ term. The simulation performed used fifty reference Adalines. Each reference was compared with fifty different perturbations of itself and the results

Single Adaline Sensitivity Study			
$\frac{\lvert\Delta\vec{W}\rvert}{\lvert\vec{W}\rvert}$ (%)	% Decision Errors Simple Approximation	% Decision Errors Complicated Approximation	% Decision Errors Simulation Results
$n = 8$			
5	1.59	1.50	1.53
10	3.18	2.99	3.11
20	6.37	5.98	6.15
30	9.55	8.97	9.06
$n = 16$			
5	1.59	1.54	1.56
10	3.18	3.09	3.12
20	6.37	6.17	6.19
30	9.55	9.26	9.17
$n = 30$			
5	1.59	1.57	1.57
10	3.18	3.13	3.14
20	6.37	6.26	6.24
30	9.55	9.39	9.22
$n = 49$			
5	1.59	1.58	1.58
10	3.18	3.15	3.16
20	6.37	6.30	6.25
30	9.55	9.45	9.24

Table 6.1: Effects of perturbing the weights of a single Adaline. The predicted and simulated percentage decision errors for weight disturbance ratios of 5, 10, 20 and 30 percent are shown for Adalines with 8, 16, 30, and 49 inputs.

averaged. This was repeated at each value of weight perturbation ratio. The predictions by the simple and complicated theory approximations and the simulation averages are tabulated for different n and $\lvert\Delta\vec{W}\rvert/\lvert\vec{W}\rvert$ in Table 6.1.

The results show there is a weak dependence on n which is well predicted by the complicated theory approximation. The simple approximation is good in the limit as n grows large and is an upper limit. The simulation results generally lie between the predictions of the two formulas. To gain an appreciation of the weak dependence on n and how close the simple approximation is, the data and the simple approximation prediction are plotted in

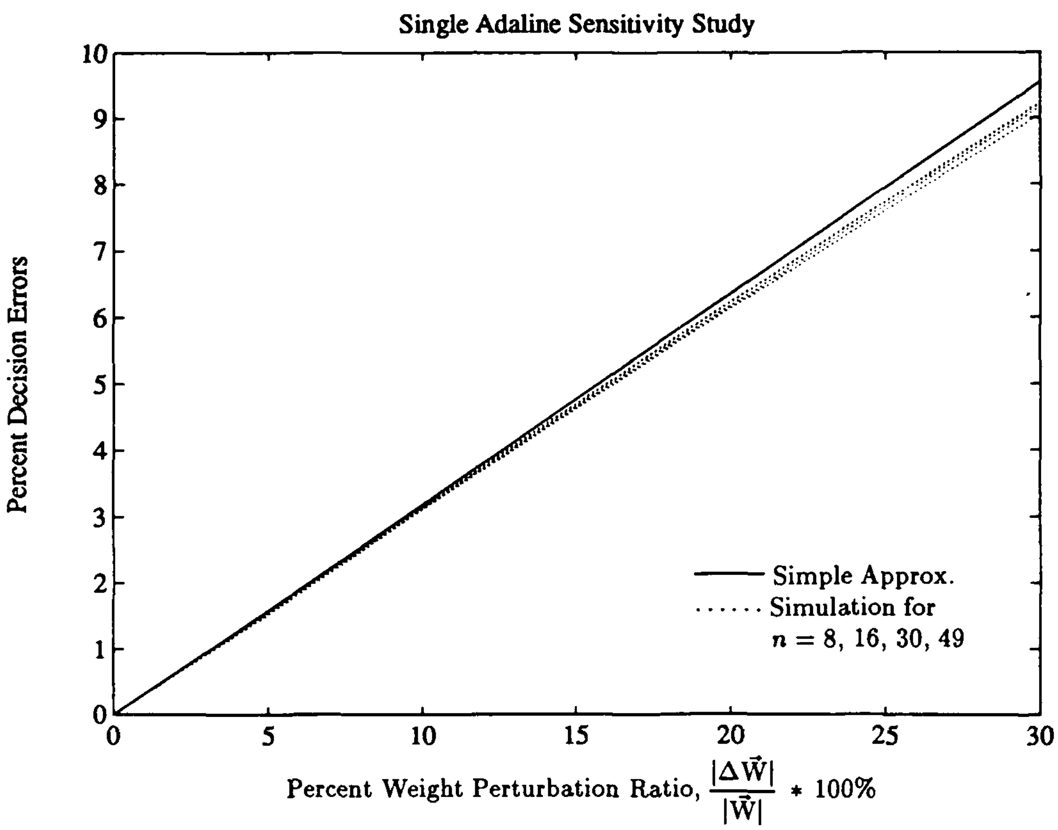


Figure 6.9: Sensitivity of the single Adaline. Dotted lines are simulation results for Adalines with 8, 16, 30, and 49 inputs. Solid line is the theory prediction by the simple approximation. Data is taken from Table 6.1.

Figure 6.9. The plots nearly lie on top of each other. The complicated approximation would lie on top of the simulation plots and is not shown. The important thing to note is the linear dependence on the weight perturbation ratio as predicted by the theory is borne out by the simulation results. It can be concluded that the theory is very accurate in predicting the effects of weight perturbations on a single Adaline.

For a network with randomly generated weights, the above validated theory can be applied independently to each Adaline in the first layer. The distribution of the Hamming distances between the hidden patterns of a reference and perturbed system can be predicted using the binomial distribution. This information can be used in Equation 6.12 along with the results derived for the effect of input errors on the decision mapping of an Adaline to predict the performance of second (output) layer Adalines. As before, the simple and complicated theory approximations will be presented.

Input Errors and Decision Errors for Output Adaline 49 input, 35-feed-1 Madaline, $\frac{\lvert\Delta W\rvert}{\lvert W\rvert} = .2$						
	frequency (%) of input errors			miss rate, $\mathcal{D}_h$ (%), of each input error		
errors	simple	complicated	observed	simple	complicated	observed
0	10.00	10.25	10.47	6.36	6.28	6.13
1	23.80	24.12	24.41	10.61	10.66	12.78
2	27.51	27.58	27.66	15.00	15.15	16.48
3	20.58	20.40	20.23	18.38	18.64	19.66
4	11.19	10.98	10.78	21.22	21.63	22.48
5	4.72	4.58	4.45	23.72	24.31	24.96
6	1.60	1.54	1.48	25.99	26.77	27.26
7	.45	.43	.40	28.07	29.07	29.52

$$\mathcal{E}_{\text{simple}} = 15.15\%$$
$$\mathcal{E}_{\text{complicated}} = 15.26\%$$
$$\mathcal{E}_{\text{observed}} = 16.39\%$$

Table 6.2: An output Adaline sees errors in its input relative to the reference network. Predicted and observed frequency of each number of input errors and predicted and observed error rates for each number of input errors are presented.

A Madaline with 49 inputs, 35 first-layer Adalines and 1 output Adaline was simulated in the configuration of Figure 6.1. The weight perturbation ratio was 20% for each Adaline in the system. The error rate for each first-layer Adaline is just $\mathcal{D}_0$ since the inputs are assumed to be without error. This rate is used in the binomial distribution to predict the distribution of input errors to the output Adaline that the perturbed system sees relative to the reference system. Since the simple and complicated theory approximations use different formulae for $\mathcal{D}_0$, the predicted frequency of each number of input errors to the output Adaline will be slightly different for the two approximations. The rate at which the output Adaline will miss each occurrence of a hidden pattern with a given number of errors in it are given by the formulas for $\mathcal{D}_h$ with the understanding that the output Adaline has 35 inputs. The total error rate for the output Adaline is computed using Equation 6.12. All this information for both the complicated and simple theory approximations as well as simulation results is presented in Table 6.2.

The table shows excellent agreement between the simple and complicated theory approximations. This particular case involved a weight perturbation ratio of 20%. At this level of disturbance the theory is underestimating the observed error rates for inputs with

Single-output Madaline Sensitivity Study			
$\frac{\lvert\Delta\vec{W}\rvert}{\lvert\vec{W}\rvert}$ (%)	% Decision Errors Simple Approximation	% Decision Errors Complicated Approximation	% Decision Errors Simulation Results
10	8.79	8.66	8.97
20	14.46	14.39	14.96
30	18.45	18.49	19.47

Table 6.3: Percent decision errors for a 16-input, 16-feed-1 Madaline with weight disturbance ratios of 10, 20, and 30 percent.

one error or more. The theory is closer at lower disturbance levels but remains useful at even higher weight perturbation ratios.

The next result shows how well the theory predicts output Adaline error rates over a range of weight disturbance ratios. The system simulated was an n-input n_1-feed-1 network. A typical result is presented here for $n = n_1 = 16$. In this simulation 15 reference nets were each perturbed 25 times for each value of the weight perturbation ratio. The average results are tabulated in Table 6.3 and plotted in Figure 6.10.

Theory and simulation are found to be acceptably close though diverging somewhat at the higher perturbation levels. The simple and complicated approximations are nearly identical over the whole range of perturbations. Examination of Table 6.2 may cause one to wonder why they can be so close. The output Adaline $\mathcal{D}_h$s are lower for the simple approximation than the complicated. The simple approximation however uses a higher error rate, the upper bound for n large, for the first-layer Adalines. This pushes the distribution of predicted input errors to the output Adaline to a higher average number of errors. The approximation errors balance each other out to provide an excellent match to the complicated approximation prediction.

To provide a test of the theories on a multioutput network, a 49-input 25-feed-3 network was simulated. Ten reference nets each perturbed 15 times for each weight perturbation ratio tested were averaged to get the results in Table 6.4. Here, weight perturbation ratios lower than any previously presented were tried to check the accuracy of the theory at smaller disturbance levels. A plot of the data alongside the simple and complicated theory approximations is shown in Figure 6.11. The excellent agreement between the two approximations of the theory is repeated in this example. Both provide very accurate predictions compared

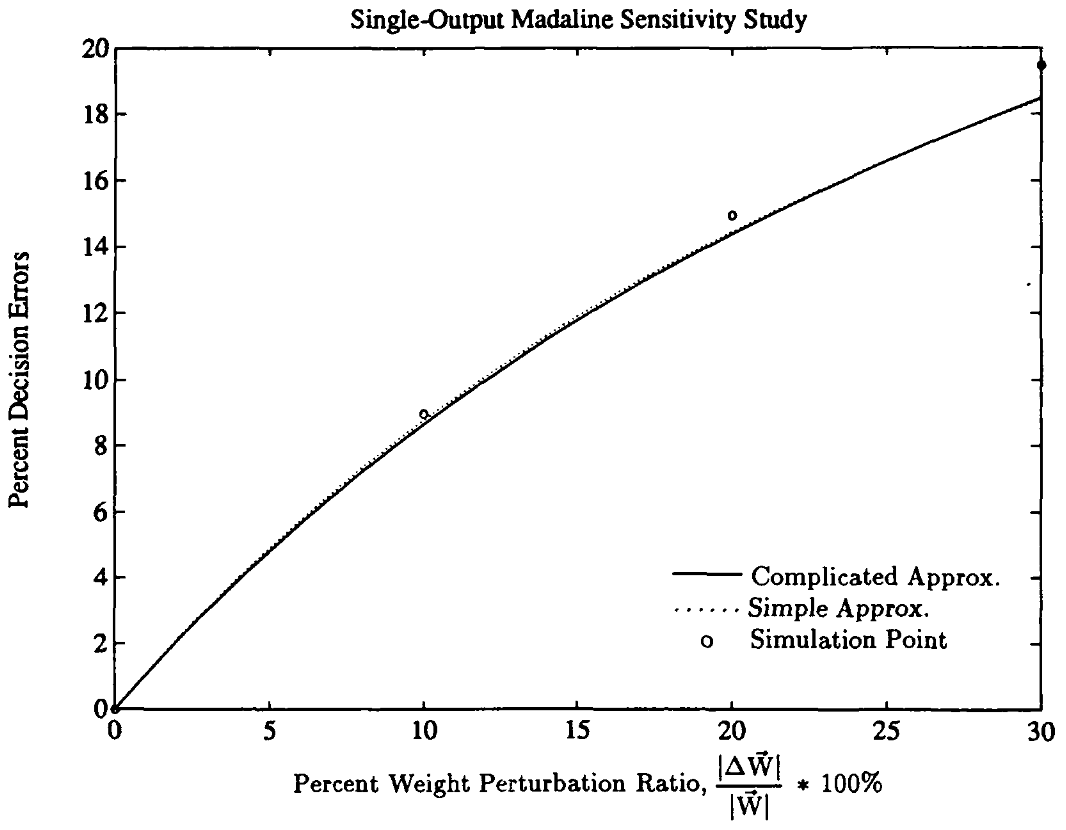


Figure 6.10: Percent decision errors versus percent weight perturbation. Network is a 16-input, 16-feed-1 Madaline. Theory predictions by two different levels of approximation are shown with simulation results. Data from Table 6.3.

Multioutput Madaline Sensitivity Study			
$\frac{\lvert\Delta\vec{W}\rvert}{\lvert\vec{W}\rvert}$ (%)	% Decision Errors Simple Approximation	% Decision Errors Complicated Approximation	% Decision Errors Simulation Results
1	3.75	3.73	3.60
5	15.69	15.64	15.44
10	25.93	25.93	26.11
20	38.42	38.65	39.95
30	46.17	46.66	49.12

Table 6.4: Simulation vs. theory for a multioutput Madaline. The network was a 49-input, 25-feed-3. A decision error occurs when any bit of the output is different from the reference.

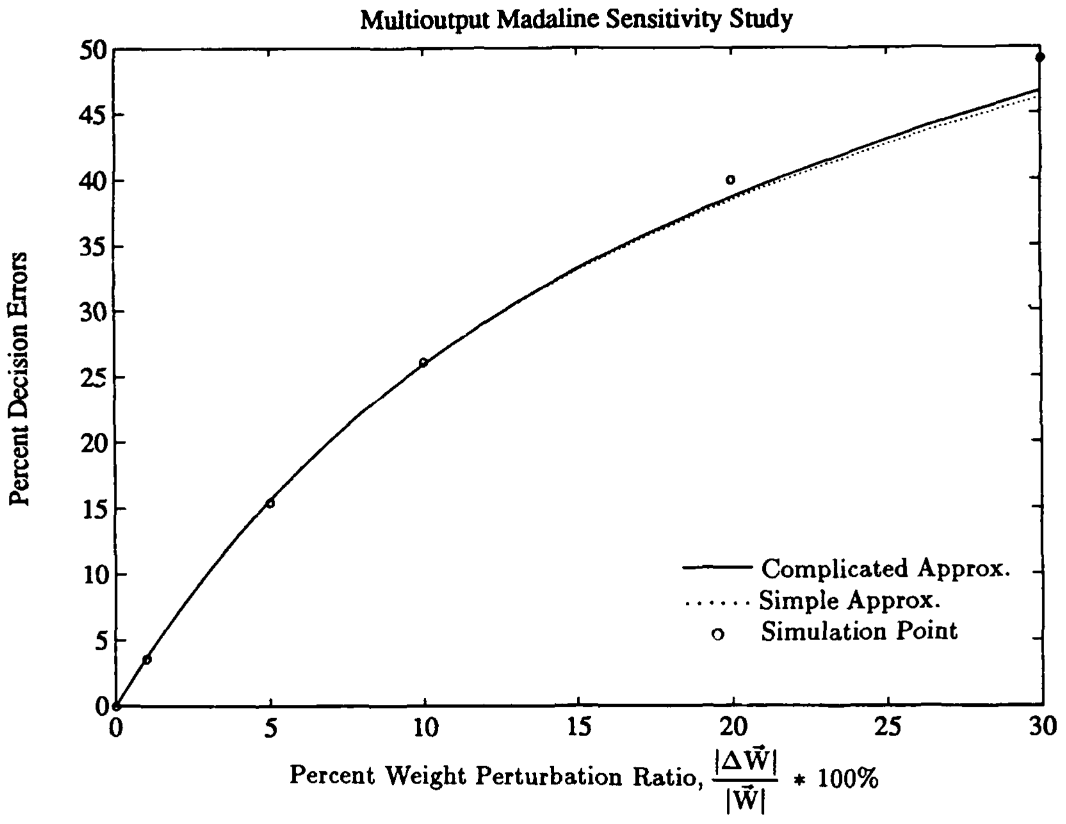


Figure 6.11: Decision Errors vs. Weight Perturbation Ratio for a multioutput Madaline. Network is 49-input, 25-feed-3. Data taken from Table 6.4.

to the simulation results.

6.6 Discussion

This chapter derives a fairly simple theory. This theory relates the change of the input/output mapping of a Madaline to the size of weight perturbations relative to a reference system. Two different levels of approximation have been addressed throughout the chapter. The approximations do not differ from each other by very much and closely predict results obtained by simulation. The results obtainable by the simple approximation make the added computation of the more complicated approximation unwarrantable.

The basic theory is valid for the single Adaline and was derived using two basic assumptions. It was assumed the weight perturbations would be independent of the reference weights and that knowledge of the ratio of the magnitude of the perturbation weight vector to the magnitude of the reference weight vector would be known. The second assumption was that the distribution of input errors would be known or computable and that errors would occur independently by bit.

This theory for the single Adaline can be applied to networks of Adalines if the Adalines can be considered independent of each other. This is the case for a randomly generated network. This type of network served as the fixed net for the emulator training experiments conducted in Chapter 4. The motivation for developing the theory was to gain an appreciation of how close to the reference solution a trained network would have to be to get good results.

This issue of closeness can now be addressed. One way to measure closeness between two weight vectors would be to consider the angle between them. The most basic result derived in this chapter, Equation 6.1, says performance degradation is directly proportional to this angle. Another result shows this angle is the same as the weight perturbation ratio when n is large (for small angle). A 10 percent weight perturbation ratio means the angle between the two weight vectors being compared is about 6 degrees (.1 radian). A single Adaline, with no input errors to aggravate the situation, will suffer a performance difference of about 3% ($10\%/\pi$). The performance degradation is also linear in the deviation angle or equivalently, the weight perturbation ratio. An angle of 12 degrees between two weight vectors would cause a 6% difference in the mappings of the two Adalines. This doesn't seem particularly bad.

Unfortunately, useful networks will require more than one Adaline. The binomial distribution is unforgiving. Suppose a network's first layer has 6 Adalines each of which are responding correctly to 94% of its inputs. The aggregate response of the layer will be completely correct only about 69% of the time if the Adalines make their errors independently of each other. If this layer is providing inputs to another layer, this can be disastrous. Or is it?

It will be remembered that the error rate of a single Adaline due to weight disturbances was only weakly dependent on the number of inputs and actually upper bounded by the simple theory approximation. How is the error rate of a second layer Adaline affected by the number of Adalines on the first layer? For the example shown in Table 6.2, about 90% of the hidden patterns presented to the output Adaline had an error in them. Only 15% of the output Adaline's responses were incorrect. Thus, the output Adaline "cleaned up" most of the errors presented to it. How is this ability affected by the number of Adalines on the first layer for a given weight perturbation ratio?

The theory was exercised and simulations to confirm the theory's predictions on this

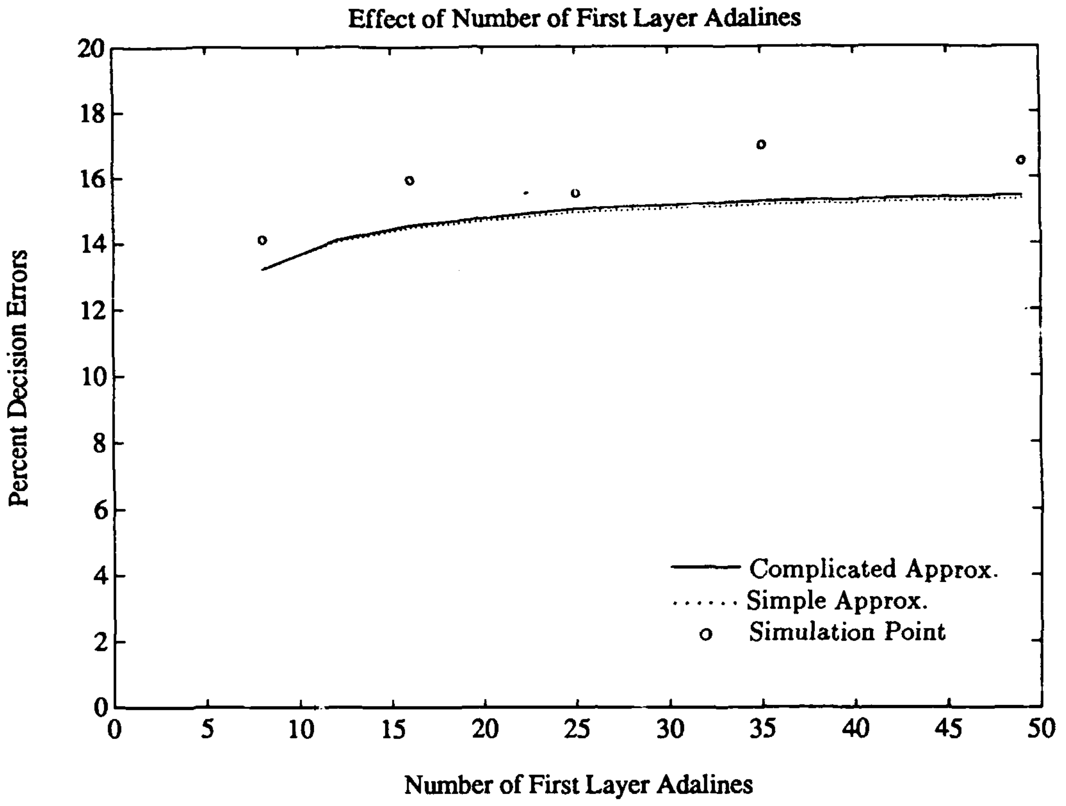


Figure 6.12: Sensitivity of networks as n_1 is varied. The net used had 49 inputs, 1 output Adaline and the percent weight perturbation ratio was 20%. Actual data points for the simulation results are marked by "o".

matter were performed. The results are shown in Figure 6.12. The networks all had 49 inputs and the same percent weight perturbation ratio of 20%. Networks with $n_1 = 8$, 16, 25, 35 and 49 were simulated. For n_1 greater than about 15, theory predicts and simulation bears out that output error rate is unaffected by the number of first-layer units. This is similar to the single Adaline with no input errors.

The error rate per output Adaline is somewhat immune then to the architecture of the network preceding it. The true curse of the binomial distribution is felt at the output layer. The ability of multioutput Madalines to produce a response with all bits correct is very dependent on how many output Adalines there are. A small per Adaline error rate in the output layer can cause large output pattern error rates in systems with only a few output Adalines. The 3 output Madaline graphed in Figure 6.11 has decision errors in 25% of its responses for only a ten percent weight perturbation error.

This sensitivity shown by randomly generated networks indicate they make poor teachers. To achieve good training performance with the fixed/adaptive emulator system of Chapter 4 requires very precise learning by the adaptive network. It becomes understandable that MRII had so much trouble learning the mapping presented by these random nets. They are hard problems from a sensitivity point of view.

It is expected that trained networks would be less sensitive to weight perturbations. This expectation results because no attention to optimal placement of decision hyperplanes is made when generating a random net. A trained net on the otherhand has undergone a series of adaptations. These adaptations sometimes help and sometimes hurt the global performance of the net on the training set. In the long run though, the adaptations that help will persist and the ones that hurt will be undone as the net approaches convergence. As decision hyperplanes are shifted and adjusted during adaptations, one would expect them to arrive at positions where their shifts affect fewer and fewer patterns in the training set as convergence is reached. Thus, the theory derived should represent a worst case situation for Madalines that have undergone training.

Chapter 7

Conclusions

7.1 Summary

Madaline Rule II was developed based on the principles of minimal disturbance. Some modifications to these principles were required. The issue of circuit implementation of the algorithm forced some compromises in the number and types of trial adaptations that would be performed if minimal disturbance was the only guiding principle. Minimal disturbance also led to a local minima phenomenon that required the implementation of a concept called usage. Usage implements the concept of responsibility sharing by forcing the participation of all hidden units in arriving at a hidden pattern representation of the input patterns.

MRII as presented in this report suffers from another kind of local minima phenomenon. Rather than afflicting the hidden units, this failure of the algorithm is the result of the output units. Certain output units will arrive at a solution to their part of the global problem and "freeze" the hidden pattern set. This set, unfortunately, is not separable by the rest of the output units in the manner they require. This situation is a most difficult one. A departure from the principles of minimal disturbance is required but it is difficult to know precisely how. A simple escape from this failure situation is to reinitialize the network with random weights and begin again. This seems wasteful of the previous training performed. The idea of using random changes of the weights to escape this failure was conceived. This led to a study of the sensitivity of the mapping of a random Adaline to weight changes and input errors.

The sensitivity analysis yielded a simple set of formulas. They predict quite well how the mapping of an Adaline with randomly generated weights will be affected by weight changes and input errors. In nets where the weights of the units are set independently of each other, the results can be applied to predict the sensitivity of the entire net. Such networks are found to be quite sensitive to changes in their weights. The input/output mappings that random networks generate can be very challenging for an adaptive network to learn.

In spite of failure modes and being tasked by difficult problems, MRII exhibits some good performance. It has shown an ability to train networks to perform a wide variety of tasks. It also exhibits interesting generalization properties. It was observed that training and generalization performance track each other very closely. Generalization was also found to be robust in networks that had many more units than the minimum necessary to solve a given problem. The use of oversized nets can lead to impressive improvements in training performance.

7.2 Suggestions for Further Research

The results of the sensitivity study have not been applied to the MRII failure mode. It was hoped the findings would allow a more thoughtful approach to using random noise in the weights for escaping the apparent local minima that MRII encounters. The sensitivity results are strictly applicable to random nets. The sensitivity of trained nets needs to be determined. Somewhere between these two extremes should lie the networks that are associated with the failure mode.

The generalization properties of MRII should be explored further. The ability to maintain good generalization when using networks larger than required is very significant. No specific mechanism for insuring this was included in MRII. Some investigation of why generalization is maintained might yield some useful information. It would be interesting to check how many hidden patterns are used by overarchitectured nets in arriving at their solutions. The geometric distances between hidden patterns mapped to the same output pattern versus those mapped to other outputs should be checked. A related point is how sensitivity is affected by overarchitecturing.

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