

DTIC FILE COPY

**NUSC Technical Report 8595
10 August 1989**



AD-A218 261

Operating Characteristics for Combiner with a Dead Zone in Each Channel

**Albert H. Nuttall
Surface ASW Directorate**

DTIC
ELECTE
FEB 21 1990
S D



**Naval Underwater Systems Center
Newport, Rhode Island / New London, Connecticut**

Approved for public release; distribution is unlimited.

90 02 20 047

Preface

This research was conducted under NUSC Project No. A75205, Subproject No. RR0000-N01, "Applications of Statistical Communication Theory to Acoustic Signal Processing," Principal Investigator Dr. Albert H. Nuttall (Code 304). This technical report was prepared with funds provided by the NUSC In-House Independent Research and Independent Exploratory Development Program, sponsored by the Office of Chief of Naval Research. Also, this research was conducted under Project No. PE6533N, "Surface Ship ASW Advanced Development," Principal Investigator Ira B. Cohen (Code 33A), Project Manager David M. Ashworth (Code 33A), sponsored by NAVSEA, Program Managers CDR L. Schneider and Eric Plummer (Code 63D).

The technical reviewer for this report was Ira B. Cohen (Code 33A).

Reviewed and Approved: 10 August 1989

A handwritten signature in dark ink, appearing to read "D. J. Sullivan" or similar, with a stylized flourish at the end.

**Daniel M. Viccione
Associate Technical Director
Research and Technology**

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
4. PERFORMING ORGANIZATION REPORT NUMBER(S) TR 8595			7a. NAME OF MONITORING ORGANIZATION		
6a. NAME OF PERFORMING ORGANIZATION Naval Underwater Systems Center		6b. OFFICE SYMBOL (If applicable) Code 304	7b. ADDRESS (City, State, and ZIP Code)		
6c. ADDRESS (City, State, and ZIP Code) New London Laboratory New London, CT 06320			9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Office of Chief of Naval Research		8b. OFFICE SYMBOL (If applicable)	10. SOURCE OF FUNDING NUMBERS		
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000		PROGRAM ELEMENT NO.	PROJECT NO. A75205	TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) OPERATING CHARACTERISTICS FOR COMBINER WITH A DEAD ZONE IN EACH CHANNEL					
12. PERSONAL AUTHOR(S) A. H. Nuttall					
13a. TYPE OF REPORT		13b. TIME COVERED FROM TO		14. DATE OF REPORT (Year, Month, Day) 1989 August 10	
				15. PAGE COUNT 60	
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Operating Characteristics Combiner		
			Receiver Performance Dead Zone		
			Matched Filters Detection Probability		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) The receiver operating characteristics, namely detection probability versus false alarm probability, for a combiner which employs nonlinearities with dead zones in each of its channels, are derived in closed form, as a function of: N, the number of channels; F, the fraction of data passed by the nonlinearity in each channel; R, the signal-to-noise ratio in each channel; and T, the system output threshold. Plots of these results for N = 1, 2, 4, 6, 8, 16, 32, 64 and F = 1, .1, .01, .001 reveal that inclusion of the dead zone does not significantly degrade performance, the typical loss being of the order of 1 dB for small N, and 3 dB for larger N, in the important operating ranges. The only limitation is that certain ranges of false alarm probabilities are unachievable; however, since these ranges generally correspond to undesirable operating conditions, the limitation is not too relevant. <i>Key words:</i>					
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL A. H. Nuttall			22b. TELEPHONE (Include Area Code) (203) 440-4618		22c. OFFICE SYMBOL Code 304

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

18. SUBJECT TERMS (Cont'd.)

False Alarm Probability

Breakpoint of Threshold. (AF) *R*

Nonlinearity

Multiple Channels

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	ii
LIST OF TABLES	iii
LIST OF SYMBOLS	iv
INTRODUCTION	1
PROCESSOR DESCRIPTION	3
ANALYSIS OF PERFORMANCE	5
EXPONENTIAL EXAMPLE	9
Statistics of Detector Output	9
Characteristic Function of Output z	11
Auxiliary Functions	12
Exceedance Distribution Function of Output z	13
Detection and False Alarm Probabilities	14
Special Cases	15
GRAPHICAL RESULTS	17
Achievable False Alarm Values	18
Erratic Behavior of Receiver Operating Characteristics	19
Observations	20
SUMMARY	51
APPENDIX. Program for Receiver Operating Characteristics	53
REFERENCES	59

Accession For	
NTIS CRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

LIST OF ILLUSTRATIONS

Figure	Page
1. Processor Block Diagram	4
2. Nonlinear Device Characteristics	4
3. ROC for $N = 1$, $F = 1$.	22
4. ROC for $N = 2$, $F = 1$.	23
5. ROC for $N = 2$, $F = .1$	24
6. ROC for $N = 2$, $F = .01$	25
7. ROC for $N = 2$, $F = .001$	26
8. ROC for $N = 4$, $F = 1$.	27
9. ROC for $N = 4$, $F = .1$	28
10. ROC for $N = 4$, $F = .01$	29
11. ROC for $N = 4$, $F = .001$	30
12. ROC for $N = 6$, $F = 1$.	31
13. ROC for $N = 6$, $F = .1$	32
14. ROC for $N = 6$, $F = .01$	33
15. ROC for $N = 6$, $F = .001$	34
16. ROC for $N = 8$, $F = 1$.	35
17. ROC for $N = 8$, $F = .1$	36
18. ROC for $N = 8$, $F = .01$	37
19. ROC for $N = 8$, $F = .001$	38
20. ROC for $N = 16$, $F = 1$.	39
21. ROC for $N = 16$, $F = .1$	40
22. ROC for $N = 16$, $F = .01$	41
23. ROC for $N = 16$, $F = .001$	42
24. ROC for $N = 32$, $F = 1$.	43

LIST OF ILLUSTRATIONS (cont'd)

Figure		Page
25.	ROC for $N = 32$, $F = .1$	44
26.	ROC for $N = 32$, $F = .01$	45
27.	ROC for $N = 32$, $F = .001$	46
28.	ROC for $N = 64$, $F = 1.$	47
29.	ROC for $N = 64$, $F = .1$	48
30.	ROC for $N = 64$, $F = .01$	49
31.	ROC for $N = 64$, $F = .001$	50

LIST OF TABLES

Table		Page
1.	Required Signal-to-Noise Ratio for $P_F = 1E-6$, $P_D = .5$	21
2.	Required Signal-to-Noise Ratio for $P_F = 1E-8$, $P_D = .9$	21

LIST OF SYMBOLS

N	number of channels, figure 1
F	fraction of data passed by nonlinearity, (3),(27)
R	signal-to-noise power ratio in each channel
T	system output threshold, figure 1
r_n	n -th input to system, figure 1
x_n	squared-envelope filter output, figure 1
L	breakpoint (threshold) of nonlinearity, figure 2,(28)
y_n	output of nonlinearity, figure 1
z	system output, figure 1
$p_x(u)$	probability density function of random variable x
$P_x(u)$	cumulative distribution function of x , (1),(13)
$Q_x(u)$	exceedance distribution function of x , (1),(13)
$p_y(u)$	probability density function of random variable y , (4)
$f_y(\xi)$	characteristic function of random variable y , (5),(14)
z	summer output, (6)
$f_z(\xi)$	characteristic function of random variable z , (7),(16)
P_F	false alarm probability, (8),(26),(32)
P_D	detection probability, (8),(25),(32)
a	auxiliary parameter, (12)
B	auxiliary parameter, (15)
$E(u,n)$	auxiliary function, (17)
$p_n(u)$	normalized probability density function, (18)
$f_n(\xi)$	normalized characteristic function, (19)
$Q_n(u)$	normalized exceedance distribution function, (20)

LIST OF SYMBOLS (cont'd)

$p_z(u)$	probability density function of output z , (22)
$Q_z(u)$	exceedance distribution function of z , (23)
$R(\text{dB})$	signal-to-noise ratio R in decibels, (33)
ROC	receiver operating characteristic

OPERATING CHARACTERISTICS FOR COMBINER
WITH A DEAD ZONE IN EACH CHANNEL

INTRODUCTION

Some data processing shortcuts are often required in order to keep the computational burden in today's detection and tracking systems within manageable limits. One strategem to accomplish this goal is to quantize the signal levels at various points in the receiver processing chain. Another is to reject low-level quantities, and retain only the larger terms, in the belief that only the latter will lead to statistically meaningful decisions on signal presence versus absence.

Here, we investigate ^a~~one~~ such technique, where all levels below a breakpoint or threshold value are rejected, that is, set to zero, while those signal levels above the breakpoint are retained in their full accuracy. In particular, this approach is employed in each branch of a combiner, as encountered in diversity or multiple ping transmission. The question to be addressed is the cost of this data reduction procedure, in terms of the additional signal-to-noise ratio required to maintain a desired level of performance, as measured by the false alarm and detection probabilities.

PROCESSOR DESCRIPTION

The processor of interest is depicted in figure 1. Received inputs $r_1, \dots, r_N$ are either composed of noise-only or they all contain signal plus noise. An example of this situation is afforded by a multiple ping transmission, with search on the range of a possible target. The received signal in each channel (if present) is match-filtered and square-law envelope-detected at the candidate time instant of suspected or hypothesized peak output.

At this point, instead of simply summing up these multiple outputs, and in an effort to reduce the amount of information sent on for further data processing, the squared envelope x_n in the n -th channel is subjected to the nonlinear operation depicted in figure 2. Namely, all input levels to the nonlinearity below breakpoint (threshold) value L are replaced by zero, whereas those levels above the breakpoint are kept as is. The breakpoint value L is chosen so that a specified fraction F of the input data to the nonlinearity is passed, when noise-alone is present at the inputs; the hope is that F can be chosen very small, without significant degradation in performance. Finally, the output of the summer in figure 1 is compared with output threshold T for purposes of deciding on signal presence versus absence.

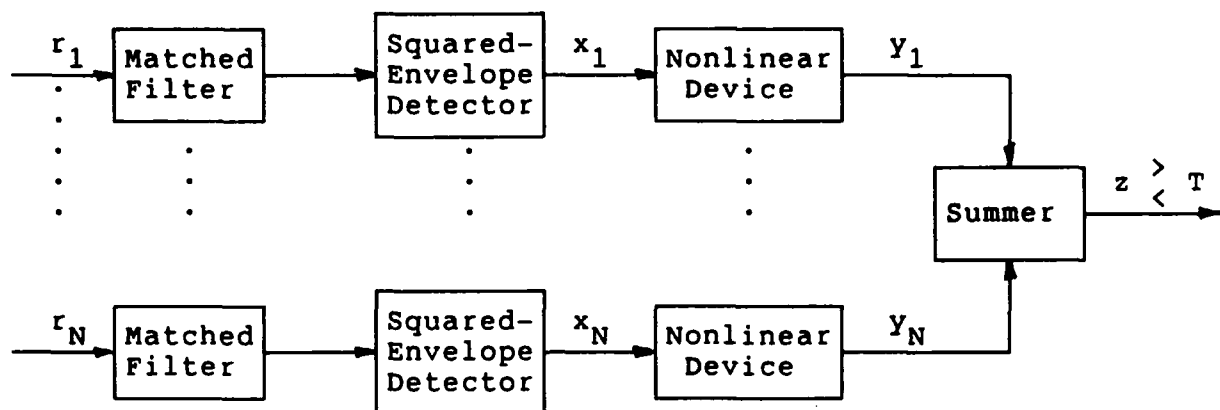


Figure 1. Processor Block Diagram

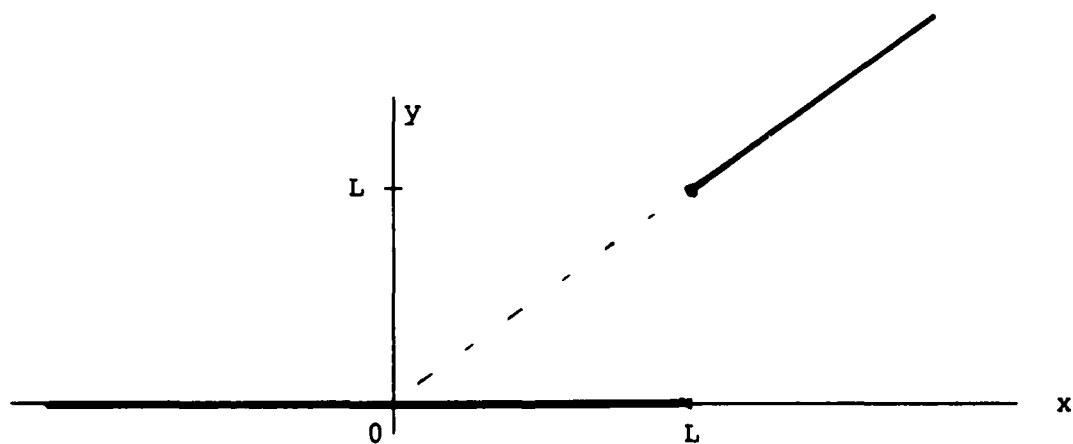


Figure 2. Nonlinear Device Characteristic

ANALYSIS OF PERFORMANCE

The inputs $\{r_n\}$ to figure 1 are presumed to be statistically independent of each other, whether signal is present or not. The squared-envelope outputs $\{x_n\}$ are, therefore, also statistically independent of each other, with probability density function $p_x(u)$, which is presumed known for both cases of signal present as well as signal absent. The corresponding cumulative distribution function and exceedance distribution function are, respectively,

$$P_x(u) = \int_{-\infty}^u dt p_x(t) = \text{Prob}(x \leq u),$$

$$Q_x(u) = \int_{u+}^{\infty} dt p_x(t) = \text{Prob}(x > u). \quad (1)$$

The nonlinear device in figures 1 and 2 is characterized mathematically by

$$y = \begin{cases} 0 & \text{for } x < L \\ x & \text{for } x \geq L \end{cases}; \quad L \geq 0. \quad (2)$$

Breakpoint L is presumed nonnegative, since the output of the squared-envelope detector in figure 1 can never be negative. The fraction of data passed by the nonlinearity is

$$F = \text{Prob}(x > L) = Q_x(L). \quad (3)$$

Since exceedance distribution function Q_x is known, this equation can be solved for the required breakpoint value L , once fraction

F is specified. This calculation is done for the noise-only case, since the breakpoint is desired to be set for this condition.

Inspection of figure 2 immediately reveals that the probability density function of random variable y_n is given by

$$p_y(u) = P_x(L) \delta(u) + p_x(u) U(u - L), \quad (4)$$

where δ and U are the delta function and the unit step function, respectively. Therefore, the characteristic function of random variable y_n is

$$\begin{aligned} f_y(\xi) &= \int_{-\infty}^{\infty} du \exp(i\xi u) p_y(u) = \\ &P_x(L) + \int_L^{\infty} du \exp(i\xi u) p_x(u). \end{aligned} \quad (5)$$

Finally, using the statistical independence of the system inputs, the summer output,

$$z = \sum_{n=1}^N y_n, \quad (6)$$

has characteristic function

$$f_z(\xi) = [f_y(\xi)]^N = \left[P_x(L) + \int_L^{\infty} du \exp(i\xi u) p_x(u) \right]^N. \quad (7)$$

For general given probability density function $p_x(u)$, the integral on u in (7) can be done efficiently by means of a fast Fourier transform. Then the numerical evaluation of the exceedance distribution function of z , namely $Q_z(u)$, can be accomplished by

the techniques utilized in [1, 2, 3]. This numerical approach would have to be carried out for both cases of signal absent and signal present, in order to get the false alarm probability P_F as well as the detection probability P_D . Specifically,

$$P_F = Q_z(T; \text{noise-only}),$$

$$P_D = Q_z(T; \text{signal-plus-noise}). \quad (8)$$

In essence, (7) characterizes the performance of the processor in figures 1 and 2. The remaining effort is the analytical and numerical manipulation of (7) into useful computer forms and evaluation.

EXPONENTIAL EXAMPLE

STATISTICS OF DETECTOR OUTPUT

If the inputs $\{r_n\}$ to the processor of figure 1 are Gaussian, then the squared-envelope detector outputs $\{x_n\}$ are exponentially distributed. We take the probability density function of x_n to be

$$p_x(u) = \begin{cases} 0 & \text{for } u < 0 \\ \exp(-u) & \text{for } u \geq 0 \end{cases} \quad \text{for noise-only.} \quad (9)$$

This corresponds to a mean value of

$$\bar{x} = \int_{-\infty}^{\infty} du \, u \, p_x(u) = 1 \quad \text{for noise-only.} \quad (10)$$

This choice of scaling at the detector output does not constitute any loss of generality, since absolute level obviously has no effect upon the receiver operating characteristics of the processor in figure 1.

For Gaussian signal also present at the system input, the probability density function of x_n is

$$p_x(u) = \begin{cases} 0 & \text{for } u < 0 \\ a \exp(-au) & \text{for } u \geq 0 \end{cases} \quad \text{for signal-present.} \quad (11)$$

Here,

$$a = \frac{1}{1 + R}, \quad (12)$$

where R is the signal-to-noise power ratio at the matched filter output. (If $R = 0$, then $a = 1$, and (11) reduces to (9).) Thus,

any signal processing gains associated with the filtering process are incorporated in the value of R . Observe that R is the signal-to-noise power ratio per channel or per ping, not the "total signal-to-noise ratio" at the system output.

Another signal model, which also leads to probability density function (11) for the detector output, is slow Rayleigh fading in the medium through which the transmitted pings traveled. That is, during a single ping duration, the medium attenuation is constant, but from ping to ping, the attenuation is statistically independent and governed by a Rayleigh probability density function on the received signal envelope.

CHARACTERISTIC FUNCTION OF OUTPUT z

We will determine the statistics of output z of figure 1 for the signal-present probability density function of x , as given by (11). The case for noise-only will then follow immediately by setting $a = 1$.

The cumulative distribution and exceedance distribution functions of x_n are given by substitution of (11) in (1), that is

$$\begin{aligned} P_x(u) &= \begin{cases} 0 & \text{for } u < 0 \\ 1 - \exp(-au) & \text{for } u \geq 0 \end{cases}, \\ Q_x(u) &= \begin{cases} 1 & \text{for } u < 0 \\ \exp(-au) & \text{for } u \geq 0 \end{cases}. \end{aligned} \quad (13)$$

The characteristic function of random variable y is obtained by substituting (11) and (13) in (5); thus

$$\begin{aligned} f_y(\xi) &= 1 - \exp(-aL) + \int_L^\infty du \, a \exp(i\xi u - au) = \\ &= 1 - B + B \frac{a \exp(i\xi L)}{a - i\xi}, \end{aligned} \quad (14)$$

where we define

$$B = \exp(-aL); \quad L \geq 0. \quad (15)$$

The characteristic function of output z is given by (7) as

$$\begin{aligned} f_y(\xi) &= \left[1 - B + B \frac{a \exp(i\xi L)}{a - i\xi} \right] = \\ &= \sum_{n=0}^N \binom{N}{n} (1-B)^{N-n} B^n \frac{\exp(i\xi Ln)}{(1-i\xi/a)^n}. \end{aligned} \quad (16)$$

AUXILIARY FUNCTIONS

Define the set of functions

$$E(u, n) = \begin{cases} 1 & \text{for } u < 0 \\ \int_u^\infty dt \frac{t^n \exp(-t)}{n!} = \exp(-u) e_n(u) & \text{for } u \geq 0 \end{cases} \quad (17)$$

$$e_n(u) = \exp(-u) \sum_{k=0}^n \frac{u^k}{k!}$$

for $n \geq 0$. Here, we used the partial-exponential notation $e_n(u)$ given in [4; 6.5.11]. The expansion of the integral in (17) may be verified by repeated integrations by parts.

Also, define the set of normalized probability density functions

$$p_n(u) = \begin{cases} 0 & \text{for } u < 0 \\ \frac{u^{n-1} \exp(-u)}{(n-1)!} & \text{for } u \geq 0 \end{cases} \quad \text{for } n \geq 1,$$

$$p_0(u) = \delta(u) \quad \text{for all } u. \quad (18)$$

The corresponding characteristic functions are

$$f_n(\xi) = \frac{1}{(1-i\xi)^n} \quad \text{for } n \geq 0, \quad (19)$$

while the exceedance distribution functions are

$$Q_n(u) = E(u, n-1) \quad \text{for all } u, n \geq 1,$$

$$Q_0(u) = \begin{cases} 1 & \text{for } u < 0 \\ 0 & \text{for } u \geq 0 \end{cases}. \quad (20)$$

EXCEEDANCE DISTRIBUTION FUNCTION OF OUTPUT z

Since $p_n(u)$ and $f_n(\xi)$ are a Fourier transform pair for $n \geq 0$, it follows that

$$\frac{1}{(1-i\xi/a)^n} \quad \text{and} \quad a p_n(au) \quad (21)$$

are a Fourier transform pair. Then (16) allows us to determine the probability density function of output random variable z as

$$p_z(u) = \sum_{n=0}^N \binom{N}{n} (1-B)^{N-n} B^n a p_n(a(u - Ln)) \quad \text{for all } u, \quad (22)$$

where the "shift factor" $u - Ln$ is due to the $\exp(i\xi Ln)$ term.

This is a useful expansion, even for large N , since all the terms are positive or zero; there is no cancellation, as there would be for an alternating series.

The exceedance distribution function of z follows immediately from (22) as

$$Q_z(u) = \sum_{n=0}^N \binom{N}{n} (1-B)^{N-n} B^n Q_n(a(u - Ln)) \quad \text{for all } u. \quad (23)$$

Again, this series has no negative terms. Also, the Q_n terms are sums of positive quantities, as may be seen by referring to (20) and (17). We will be interested only in $u \geq 0$ in the following; then the $n = 0$ term in (23) is, by use of (20),

$$(1 - B)^N Q_0(au) = 0 \quad \text{for } u \geq 0. \quad (24)$$

DETECTION AND FALSE ALARM PROBABILITIES

We now utilize (8), (23), (24), and (20) to obtain the detection probability as

$$P_D = \sum_{n=1}^N \binom{N}{n} (1-B)^{N-n} B^n E(a(T - Ln), n-1). \quad (25)$$

Here, B is given by (15), and a is given by (12).

The false alarm probability is obtained by setting $R = 0$, that is, $a = 1$:

$$P_F = \sum_{n=1}^N \binom{N}{n} (1-F)^{N-n} F^n E(T - Ln, n-1). \quad (26)$$

Here, we have utilized (3) et seq., (13), and the fact that B in (15) reduces, for $a = 1$, to

$$\exp(-L) = Q_x(L; \text{noise-only}) = F, \quad (27)$$

which is the fraction of data passed by the nonlinearity in figure 1, for noise-only. In fact, (27) allows us to solve explicitly for the required breakpoint value L , for this exponential example, as

$$L = -\ln(F). \quad (28)$$

To summarize, (25) and (26) give the detection and false alarm probabilities in terms of fundamental quantities

- N , number of channels,
 - F , fraction of data passed,
 - R , signal-to-noise power ratio per channel,
 - T , output threshold.
- (29)

The remaining variables in (25) and (26) are given by (28), (12), and (15) as

$$L = -\ln(F), \quad a = \frac{1}{1+R}, \quad B = \exp(-aL), \quad (30)$$

in order.

SPECIAL CASES

For $N = 1$, one channel, (25) and (26) reduce to

$$P_F = F Q_1(T-L) = \begin{cases} F & \text{for } 0 \leq T < L \\ \exp(-T) & \text{for } L \leq T \end{cases},$$

$$P_D = B Q_1(a(T-L)) = \begin{cases} F^a & \text{for } 0 \leq T < L \\ \exp(-aT) & \text{for } L \leq T \end{cases}. \quad (31)$$

That is, $P_D = P_F^a$ for $N = 1$, independent of the value of fraction F . This is obvious from (13) in this case.

Instead, if fraction $F = 1$, that is, no nonlinearity, then (28) and (15) yield $L = 0$, $B = 1$, and we find

$$\left. \begin{aligned} P_F &= E(T, N-1) \\ P_D &= E(aT, N-1) \end{aligned} \right\} \quad \text{for } F = 1. \quad (32)$$

These results agree with [5; (7) and (8)]. A program for the evaluation of general results (25) and (26), as well as the special case (32), is presented in the appendix.

GRAPHICAL RESULTS

In figure 3*, the receiver operating characteristic (ROC) is given for $N = 1$, $F = 1$. That is, there is one channel and the nonlinearity is not active. There is no need to consider values of F less than 1, according to the comment under (31); however, see the subsection below on achievable false alarm values. The curves in figure 3 are parameterized according to

$$R(\text{dB}) = 10 \log R. \quad (33)$$

The remaining fundamental quantity, threshold T in (29), has been eliminated, and P_D is plotted versus P_F on normal probability paper.

In figures 4, 5, 6, 7, the number of channels is kept at $N = 2$, while fraction

$$F = 1, .1, .01, .001, \quad (34)$$

respectively. Additional cases for

$$N = 4, 6, 8, 16, 32, 64, \quad (35)$$

in figures 8 through 31, complete the coverage in a similar fashion.

*Figures 3 through 31 are grouped at the end of this section.

ACHIEVABLE FALSE ALARM VALUES

Not all values of false alarm probability can be reached by the processing system of figure 1. Since the nonlinear device output y_n can only take on the values $y_n = 0$ and $y_n \geq L$, the sum z can only assume the values $z = 0$ and $z \geq L$. Also, since the probability of $z = 0$ is $(1 - F)^N$ for noise-only, where F is the fraction of data passed by the nonlinearity in each channel, then the probability of getting $z \geq L$ is $1 - (1 - F)^N$. Thus, the range of reachable false alarm probabilities is

$$P_F \leq 1 - (1 - F)^N. \quad (36)$$

This bound holds regardless of the form of the probability density for random variables $\{x_n\}$ in figure 1.

For the special case of $N = 1$, this rule yields $P_F \leq F$. Thus, the plot in figure 3 for $N = 1$, $F = 1$ must be modified for $F < 1$, to the extent that only the values for $P_F \leq F$ are achievable.

For $N > 1$, the rule in (36) first becomes obvious in figure 6 for $N = 2$, $F = .01$. Namely, (36) yields

$$P_F \leq 1 - (1 - .01)^2 = .0199. \quad (37)$$

Thus, the curves in figure 6 are terminated to the right of this value of the false alarm probability. This termination feature occurs in numerous other figures, always governed by (36).

ERRATIC BEHAVIOR OF RECEIVER OPERATING CHARACTERISTICS

Some of the curves develop significant kinks for larger values of the false alarm probability; see figure 31 for the most pronounced example in this set of results. This behavior is not due to computer round-off error; rather, it is due to the shifted components of probability density function (22) "kicking in" when the output threshold reaches various multiples of breakpoint L . Equivalently, the shifted E-function components of the detection probability and false alarm probability in (25) and (26) are activated at different threshold levels, reflecting the inherent abrupt change of behavior of these functions at zero argument. For example,

$$\begin{aligned}
 E(u,0) &= \begin{cases} 1 & \text{for } u < 0 \\ \exp(-u) & \text{for } u \geq 0 \end{cases}, \\
 E(u,1) &= \begin{cases} 1 & \text{for } u < 0 \\ \exp(-u)(1+u) & \text{for } u \geq 0 \end{cases}. \quad (38)
 \end{aligned}$$

Thus, $E(u,0)$ has a discontinuous slope at $u = 0$, while $E(u,1)$ has a discontinuous second derivative at $u = 0$.

OBSERVATIONS

The required signal-to-noise ratios for various values of N and F are presented in tables 1 and 2 for two different levels of performance, as read directly from figures 3 through 31. The overriding impression is that the degradation in performance is not severe, even for small values of F , the fraction of data passed by the nonlinearity. For example, from table 1, the decibel difference at $F = .001$ versus $F = 1$ is, for $N = 1, 2, 4, 6, 8, 16, 32, 64$, respectively, just

0, 0.3, 0.8, 1.0, 1.2, 1.7, 2.2, 2.7 dB.

For table 2, these differences are substantially the same:

0, 0.2, 0.5, 0.8, 1.0, 1.5, 2.0, 2.5 dB.

Thus, the losses increase from 0 dB at $N = 1$ channel, to less than 3 dB for $N = 64$ channels.

The situation is slightly worse for the lower-quality case of $P_F = 1E-3$, $P_D = .5$. Namely, as F is changed from 1 to .001, the required increment in signal-to-noise ratio is 0 dB for $N = 1$, whereas it is 3.4 dB for $N = 64$.

N	Required R(dB) for F =			
	1	.1	.01	.001
1	12.8	12.8	12.8	12.8
2	9.5	9.6	9.7	9.8
4	6.8	6.9	7.2	7.6
6	5.4	5.5	5.9	6.4
8	4.5	4.6	5.1	5.7
16	2.3	2.6	3.2	4.0
32	0.4	0.7	1.6	2.6
64	-1.4	-1.0	0.1	1.3

Table 1. Required Signal-to-Noise Ratio for $P_F = 1E-6$, $P_D = .5$

N	Required R(dB) for F =			
	1	.1	.01	.001
1	22.4	22.4	22.4	22.4
2	16.0	16.0	16.1	16.2
4	11.6	11.6	11.8	12.1
6	9.4	9.6	9.9	10.2
8	8.1	8.2	8.6	9.1
16	5.3	5.6	6.1	6.8
32	2.9	3.3	4.0	4.9
64	0.8	1.2	2.2	3.3

Table 2. Required Signal-to-Noise Ratio for $P_F = 1E-8$, $P_D = .9$

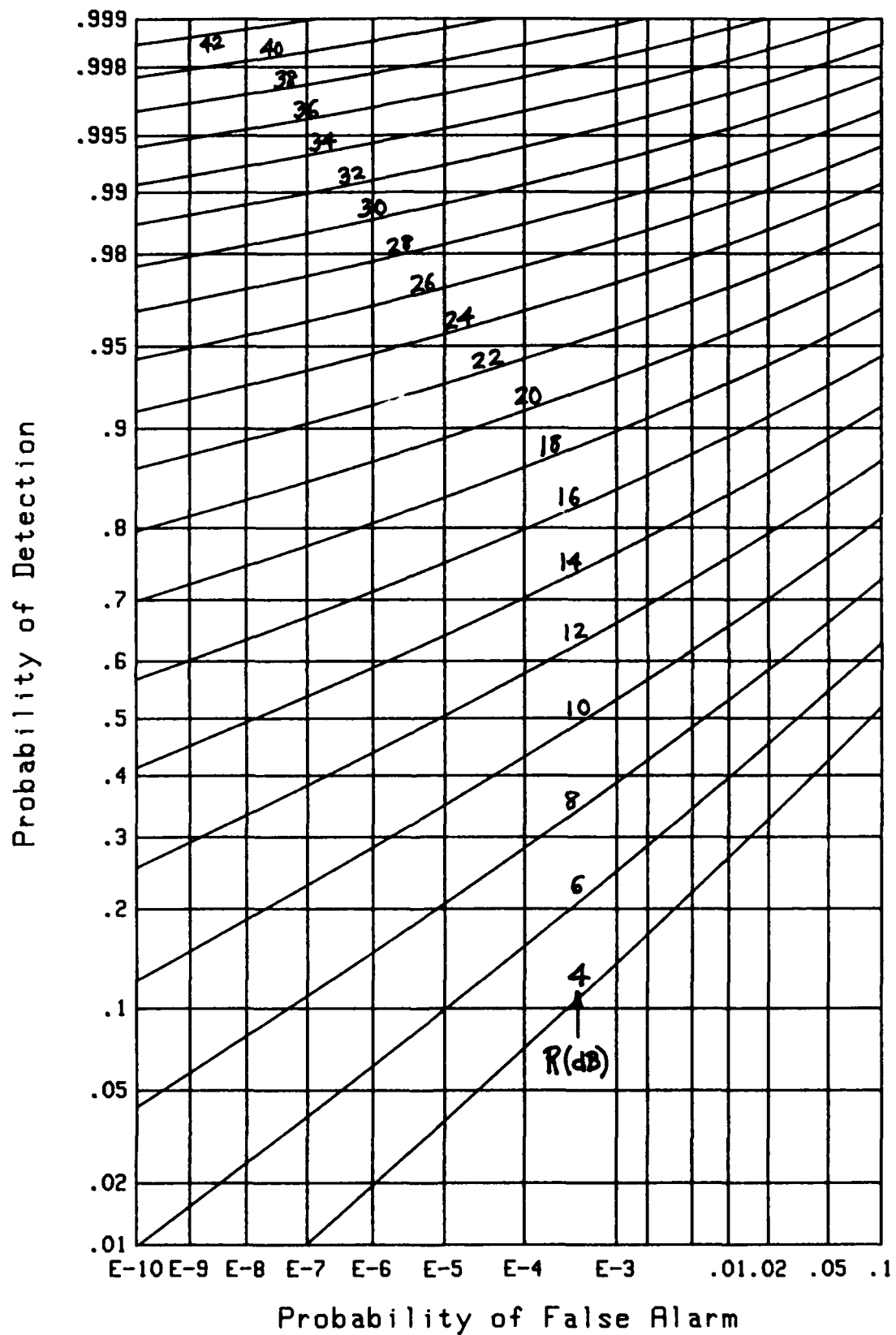
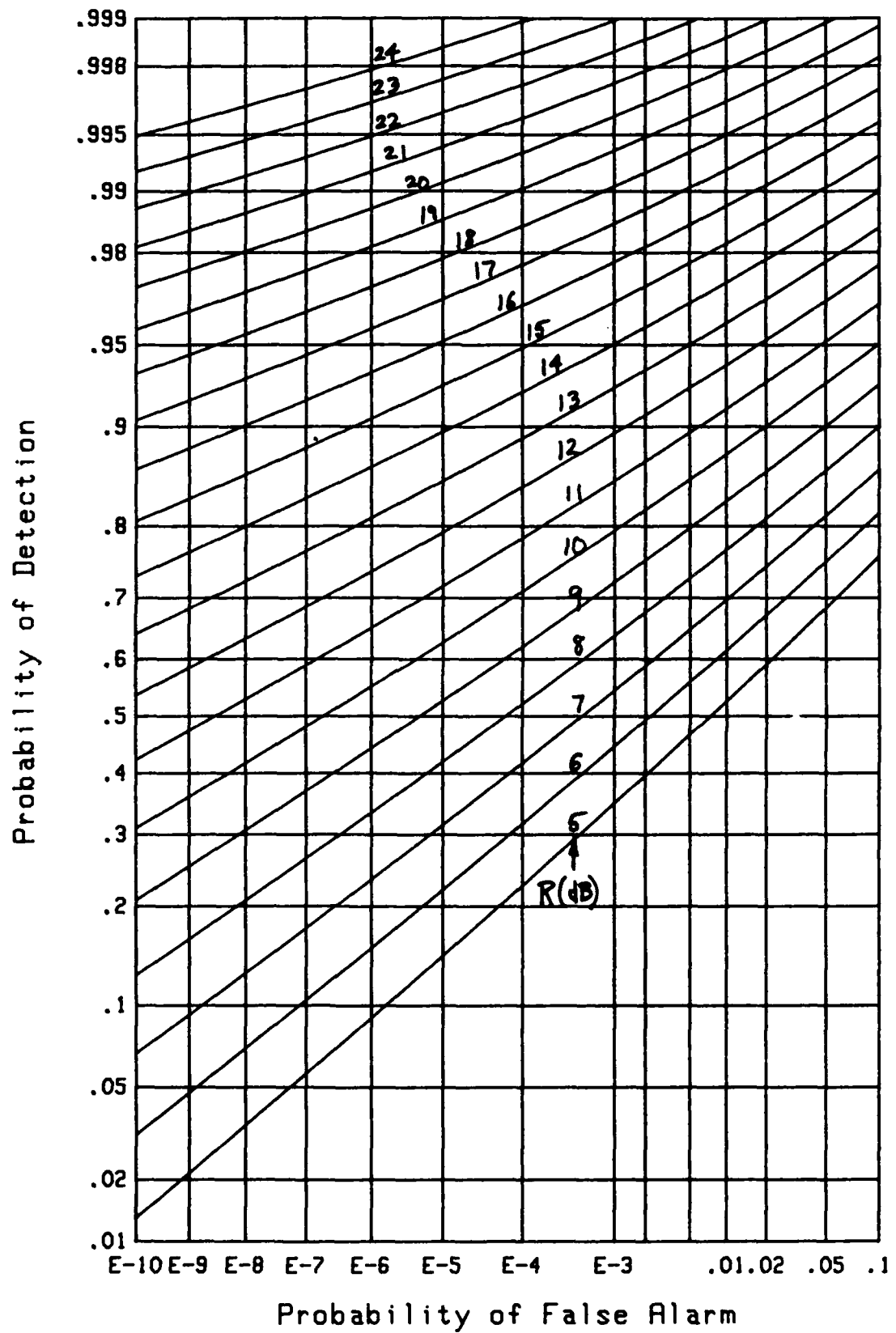
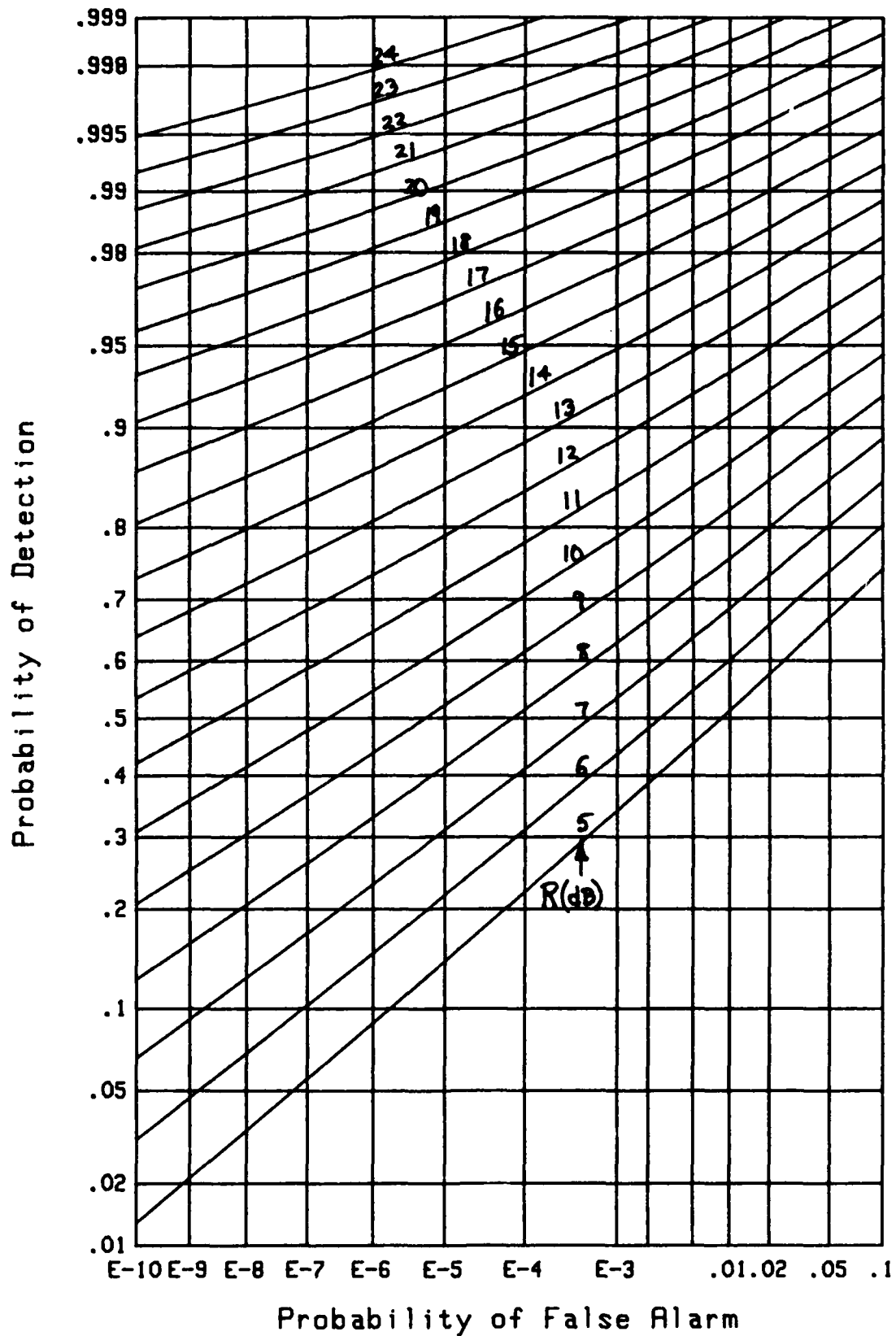


Figure 3. ROC for $N=1$, $F=1$.

Figure 4. ROC for $N=2$, $F=1$.

Figure 5. ROC for $N=2$, $F=.1$

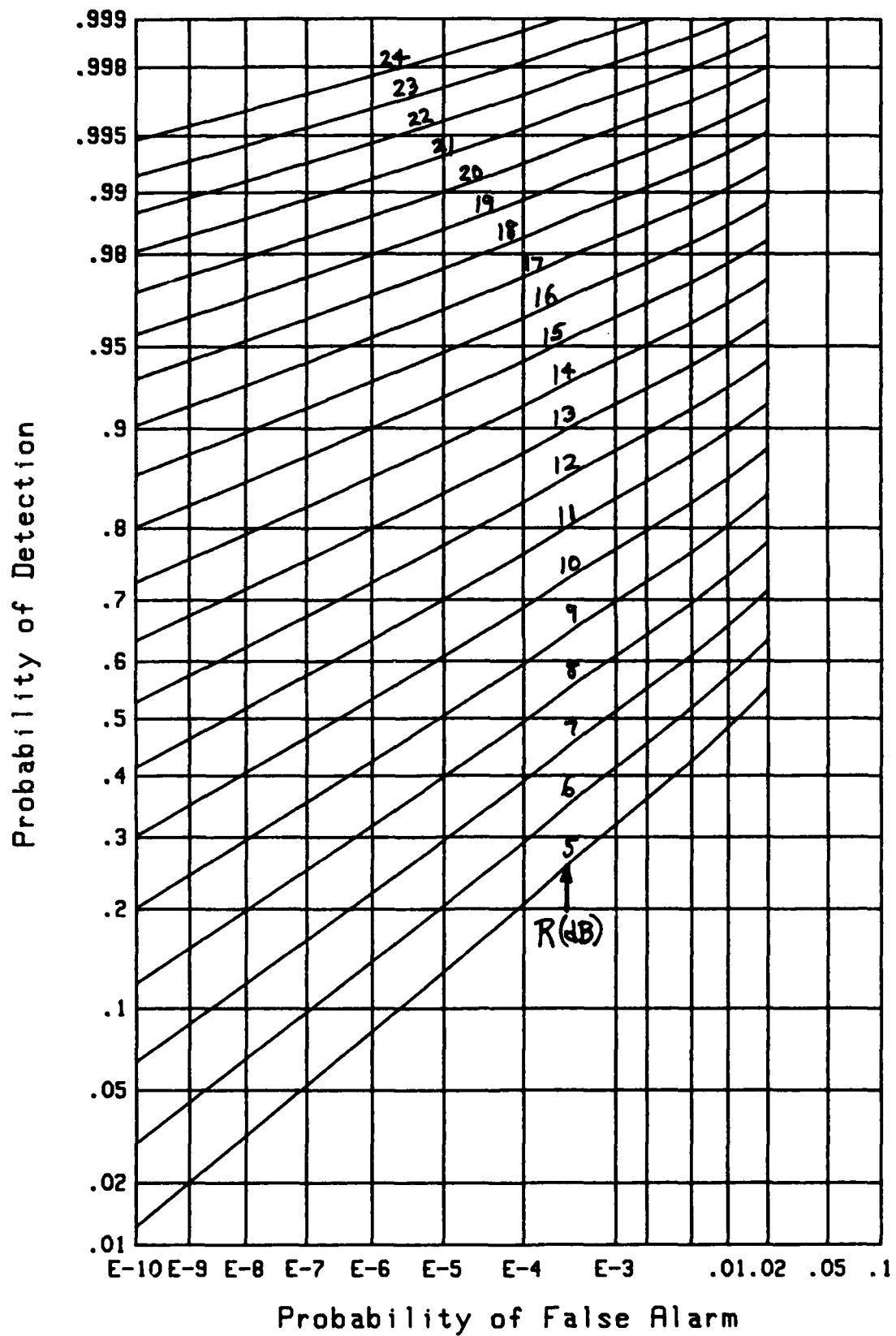
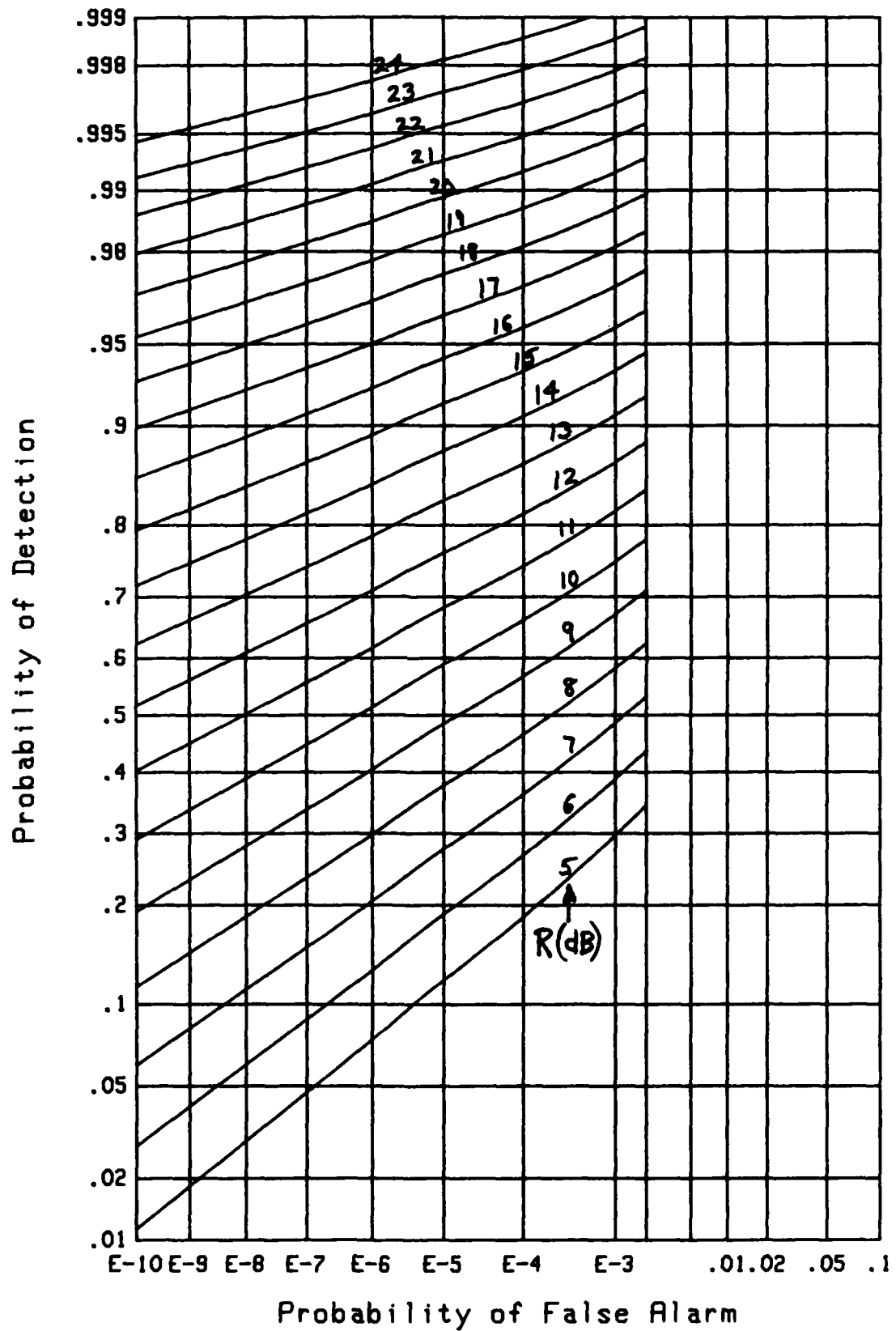
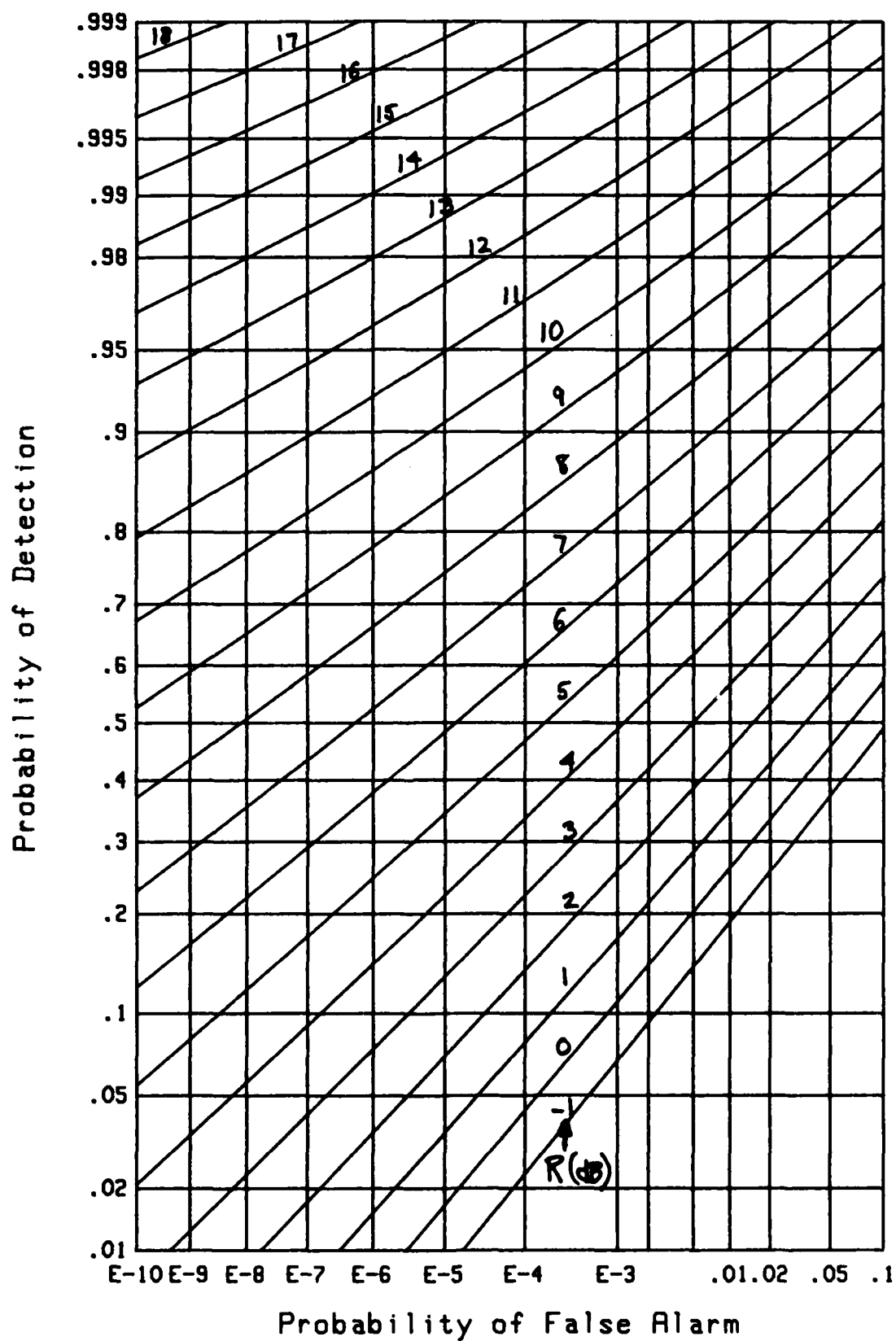
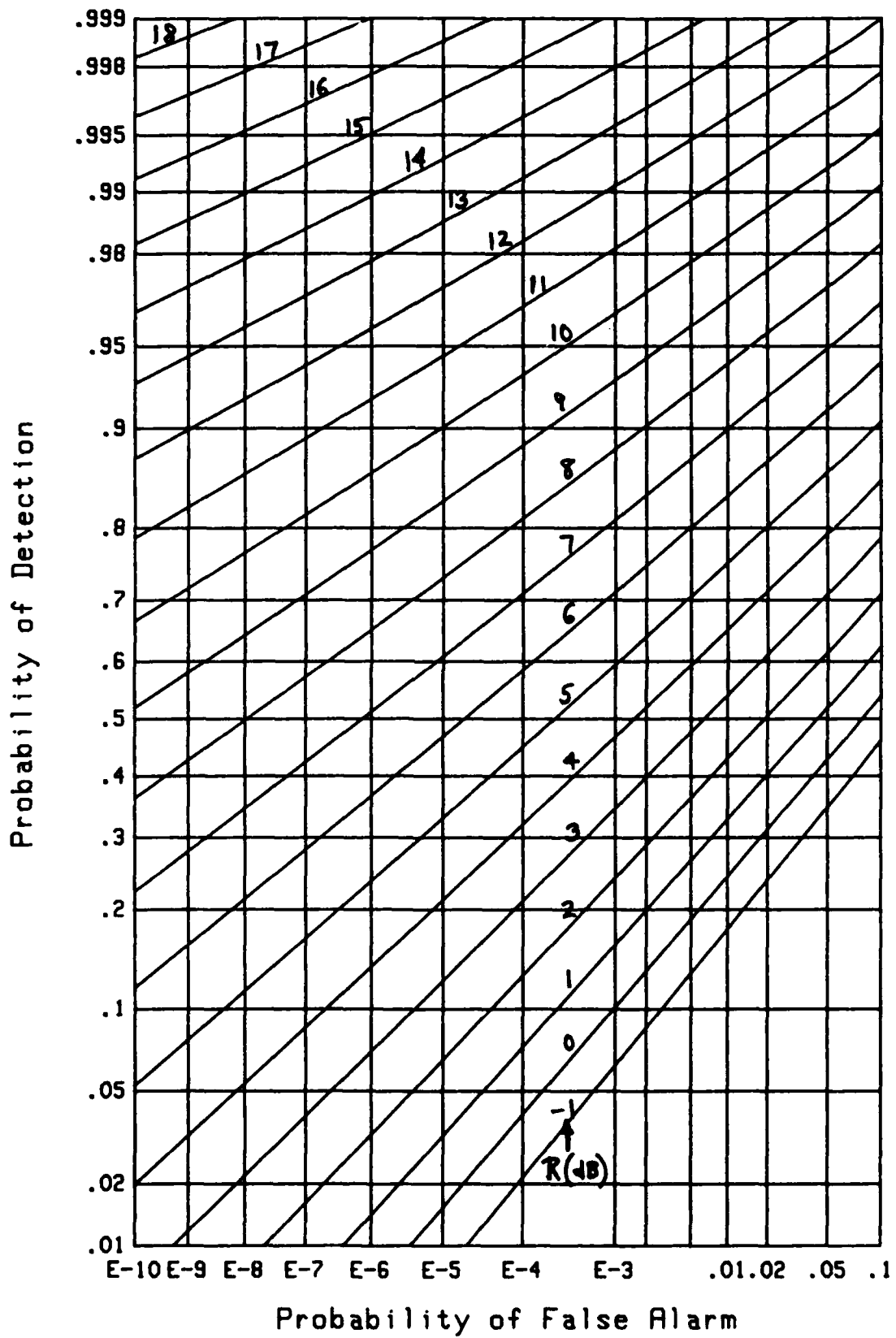


Figure 6. ROC for $N=2$, $F=.01$

Figure 7. ROC for $N=2$, $F=.001$

Figure 8. ROC for $N=4$, $F=1$.

Figure 9. ROC for $N=4$, $F=.1$

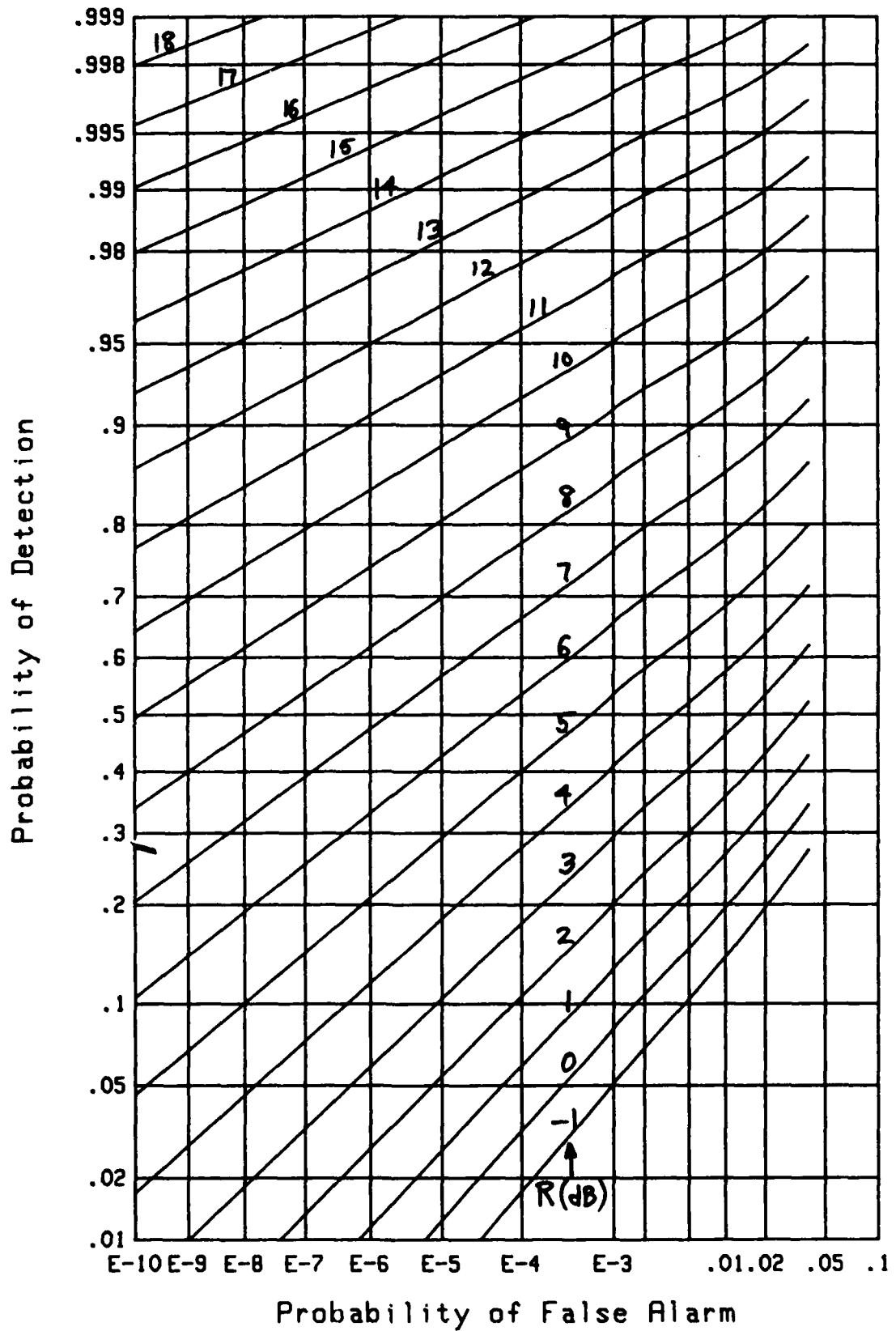
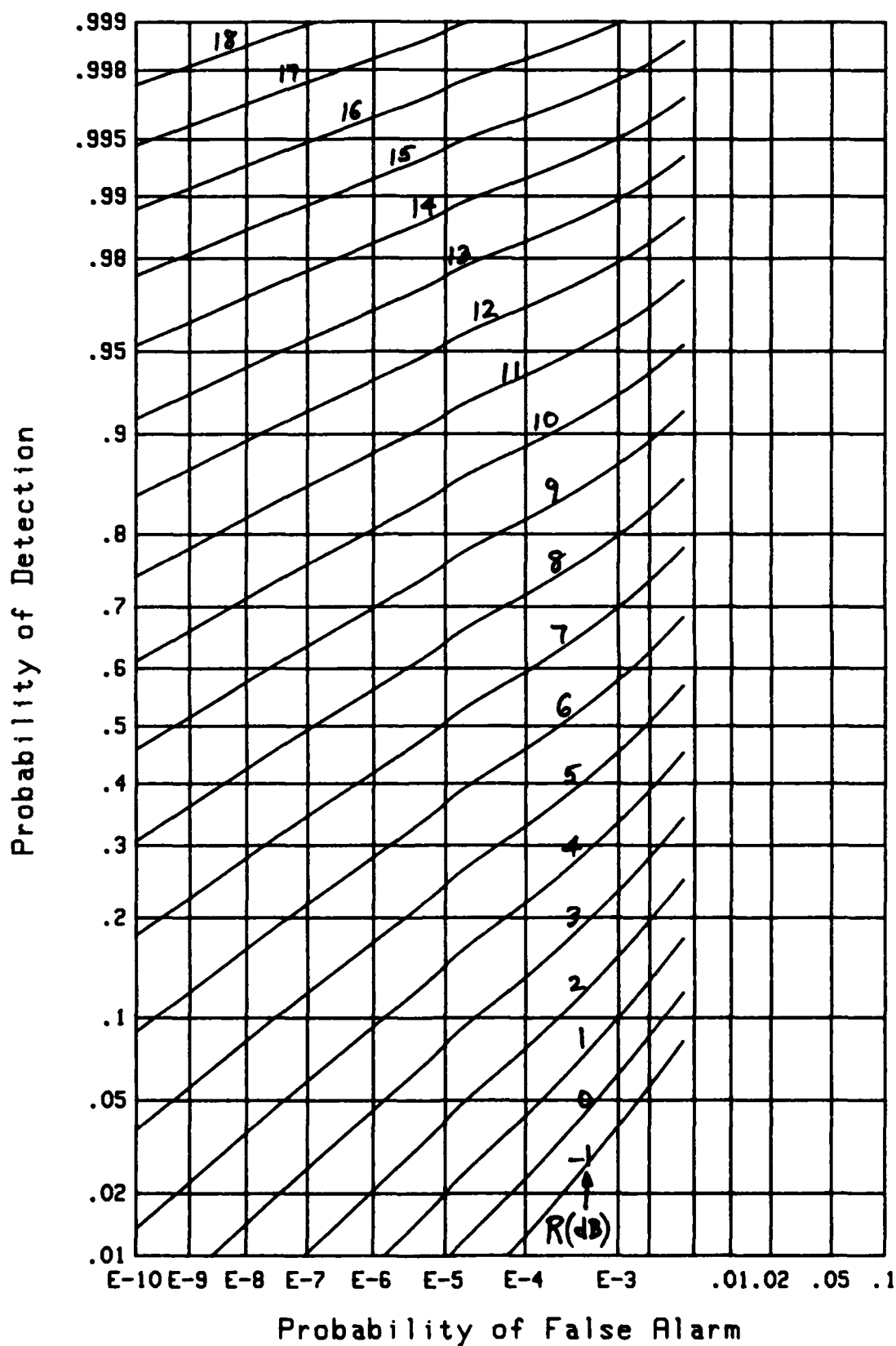
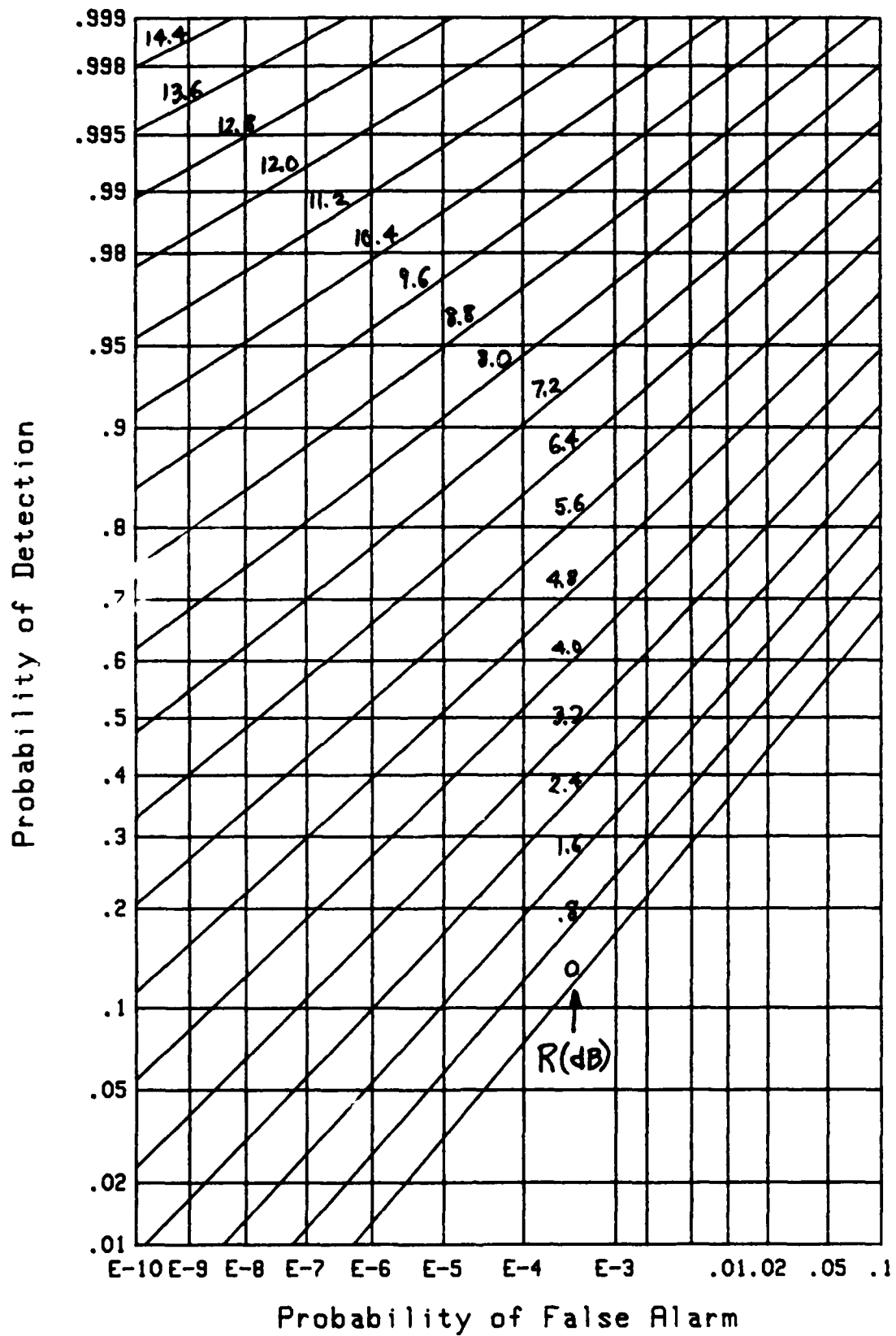


Figure 10. ROC for $N=4$, $F=.01$



Figure 12. ROC for $N=6$, $F=1$.

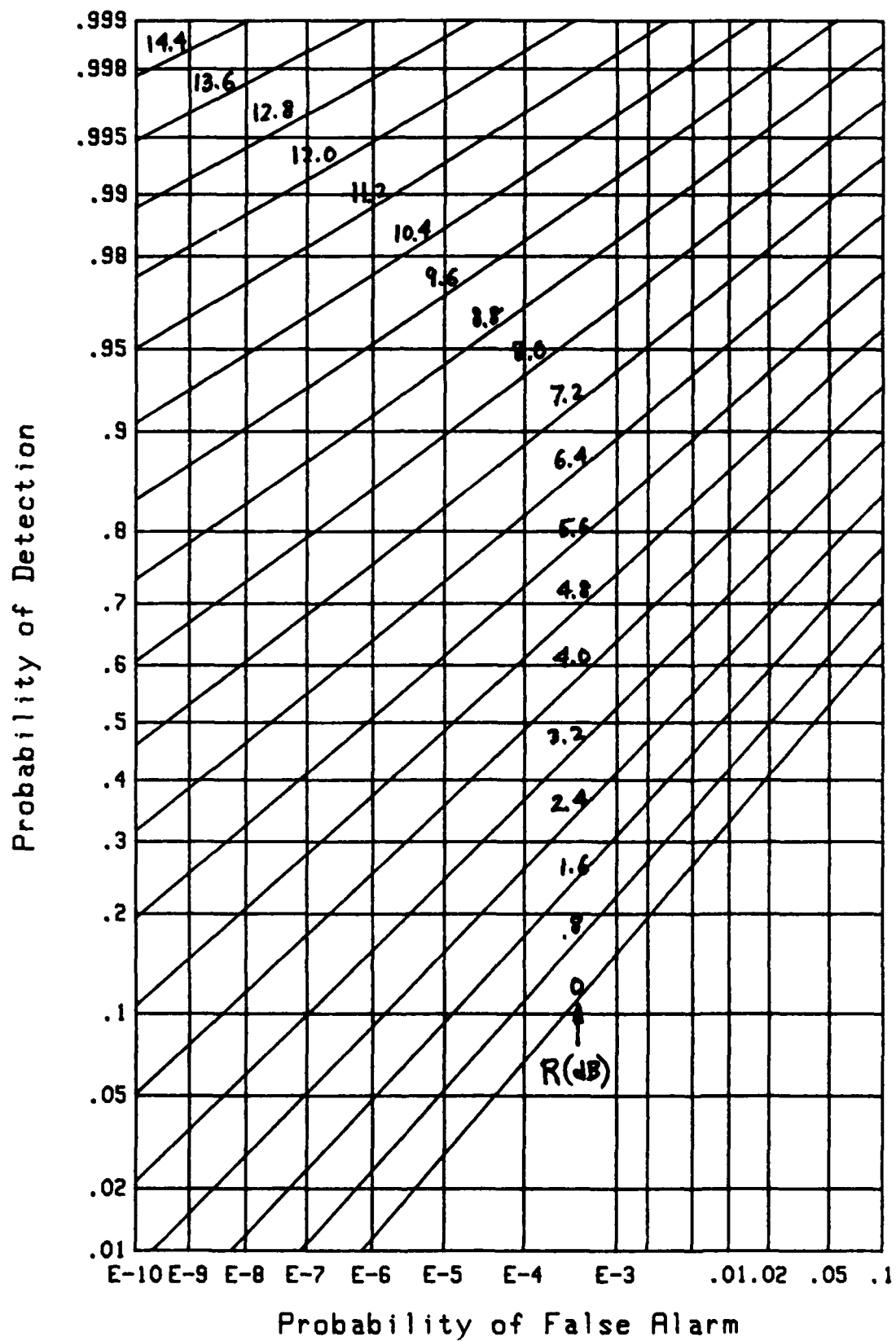
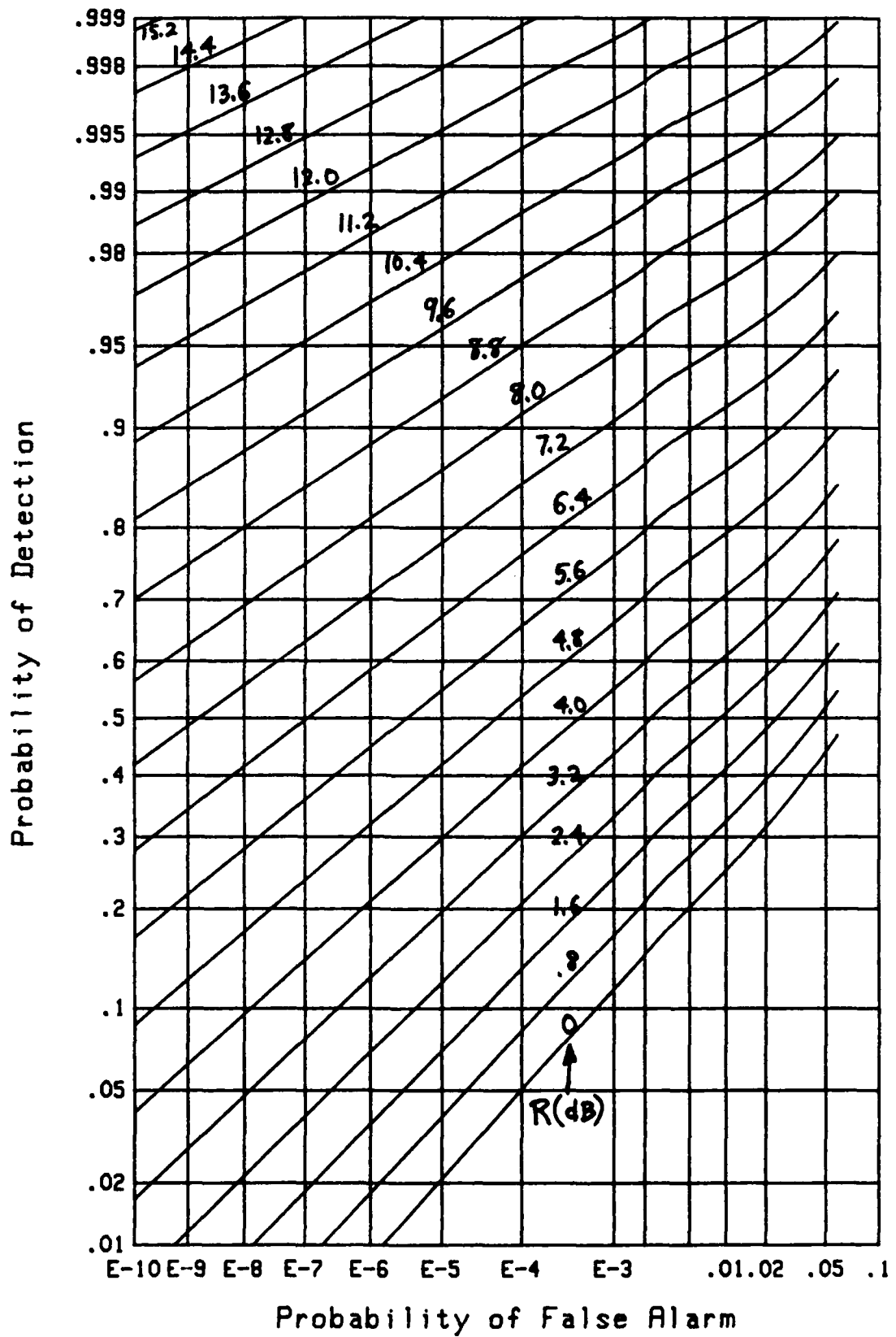


Figure 13. ROC for $N=6$, $F=.1$

Figure 14. ROC for $N=6$, $F=.01$

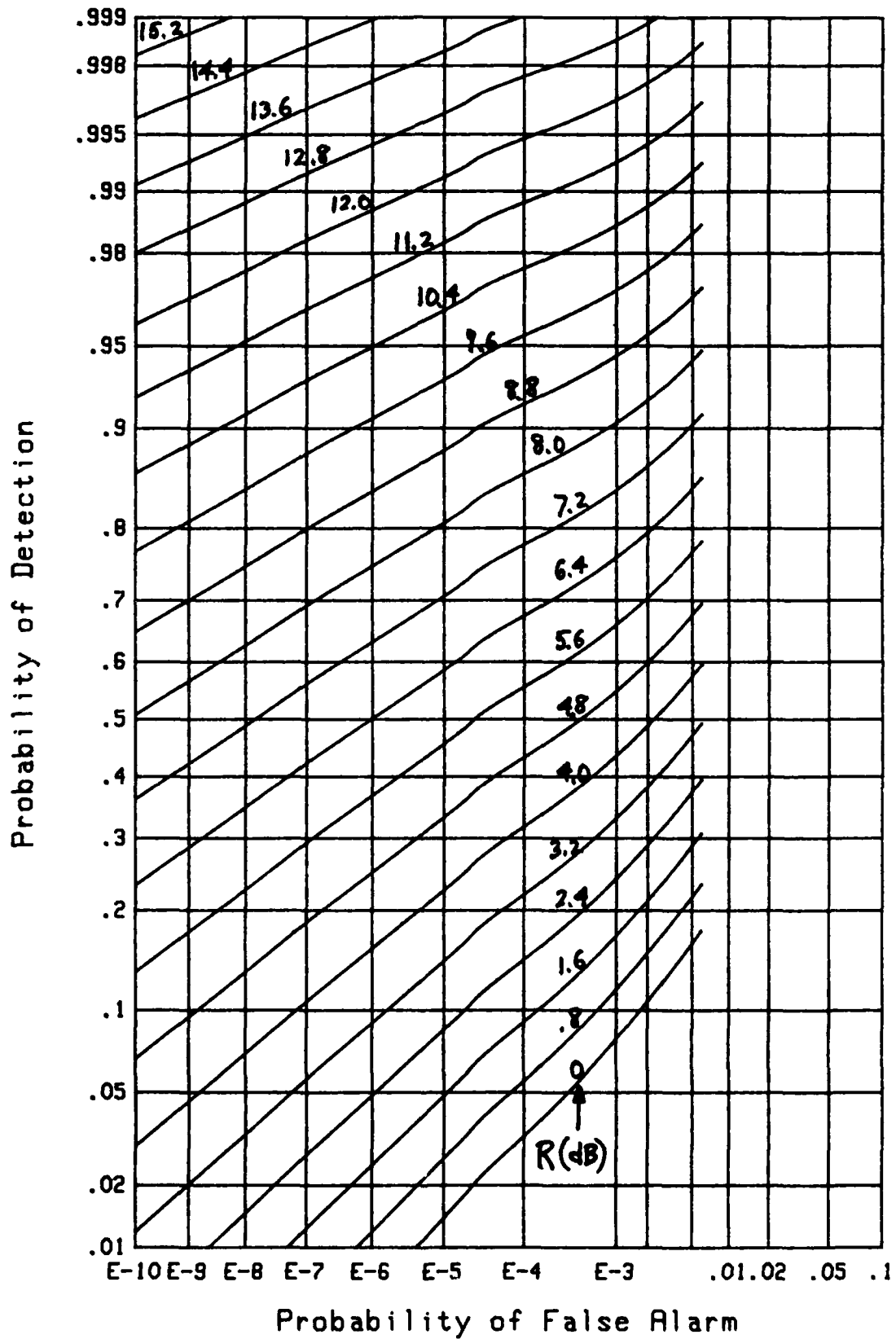
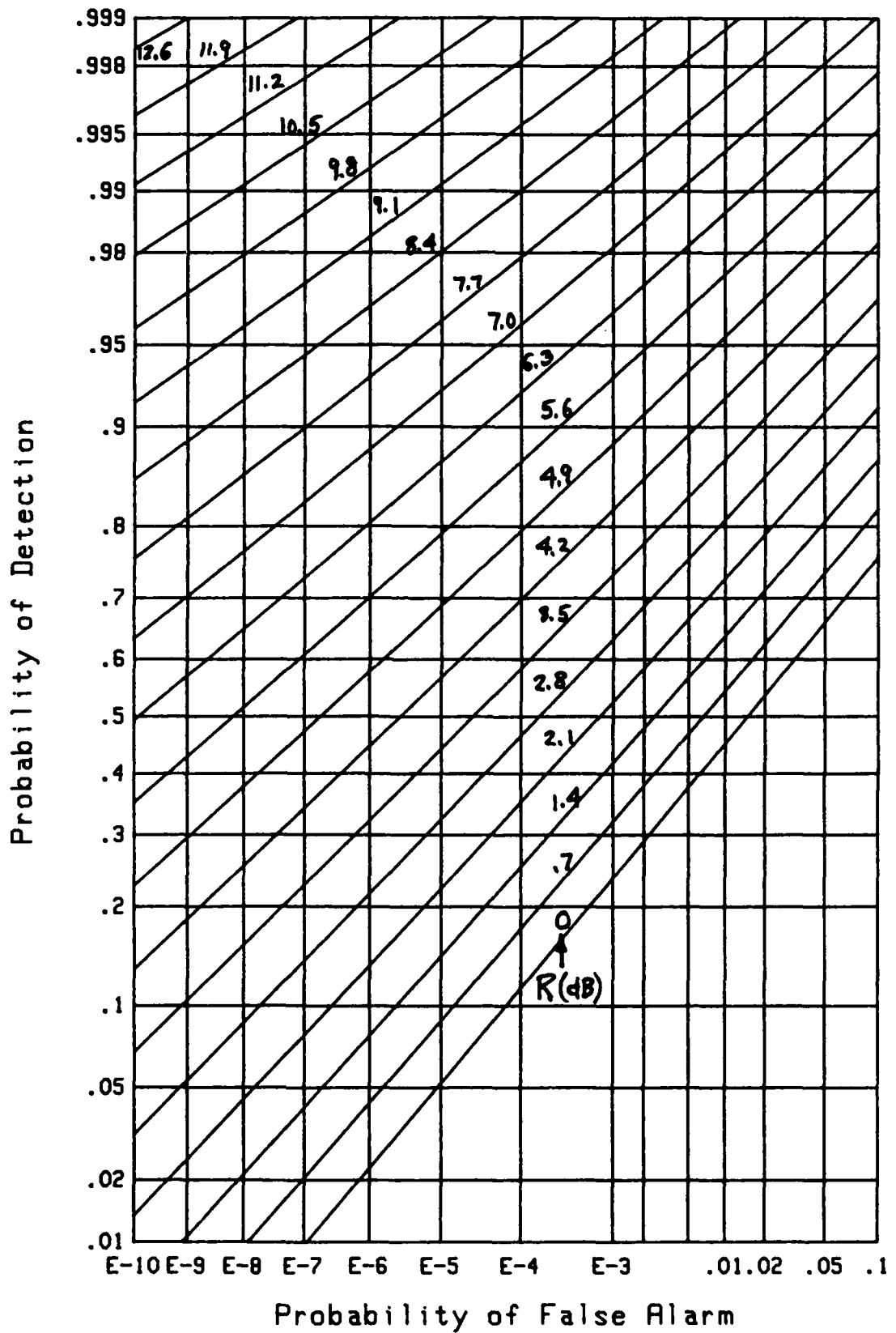
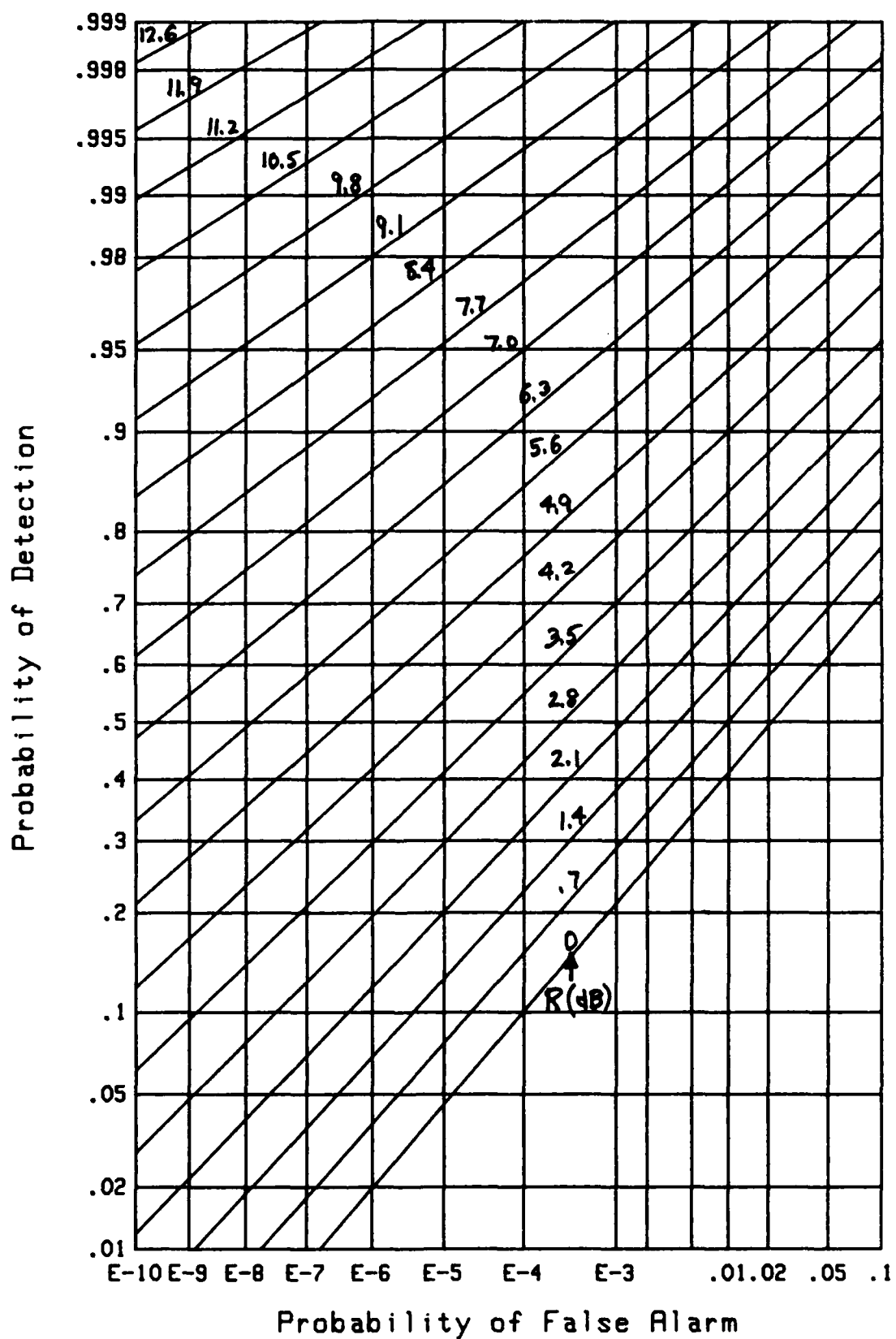
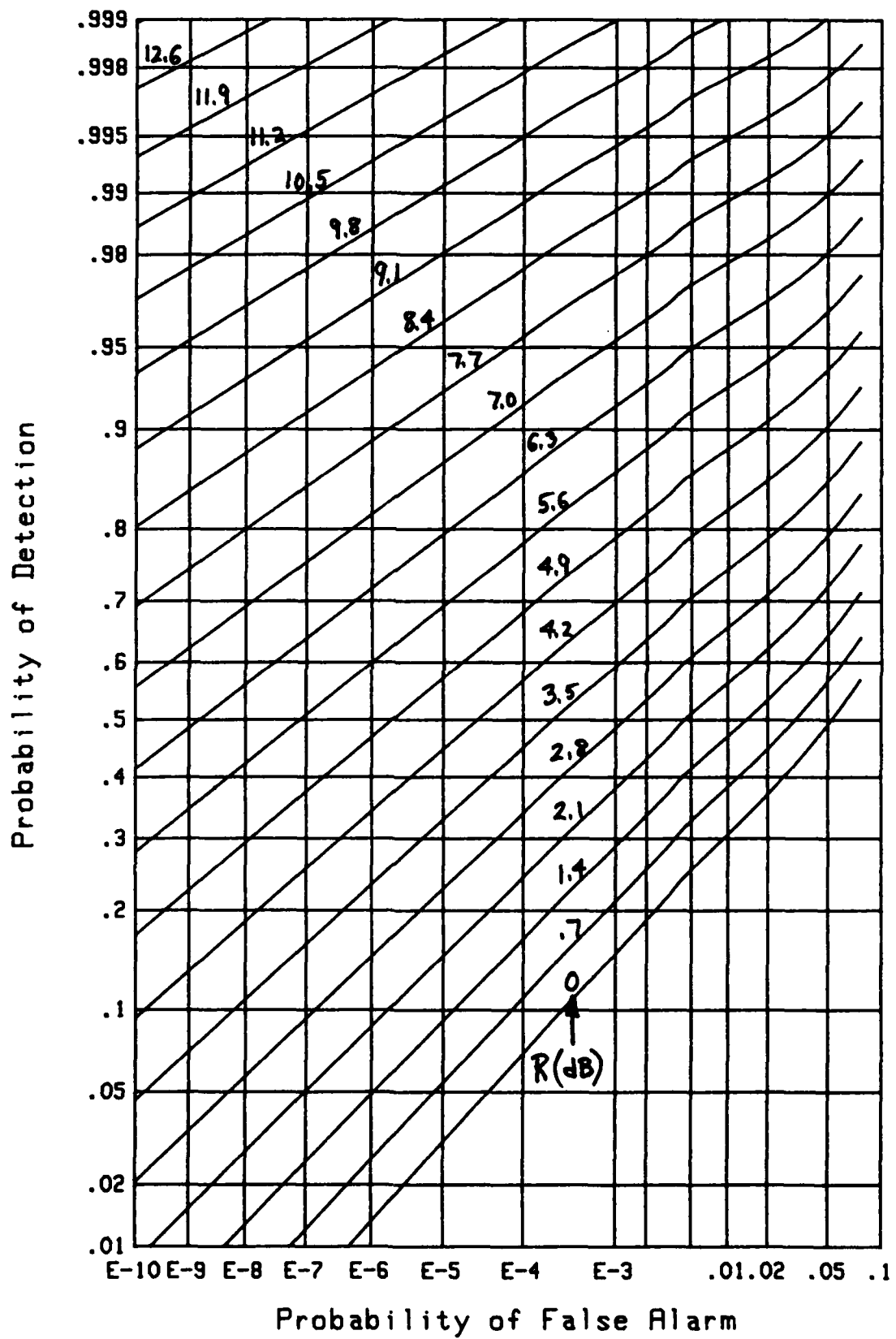
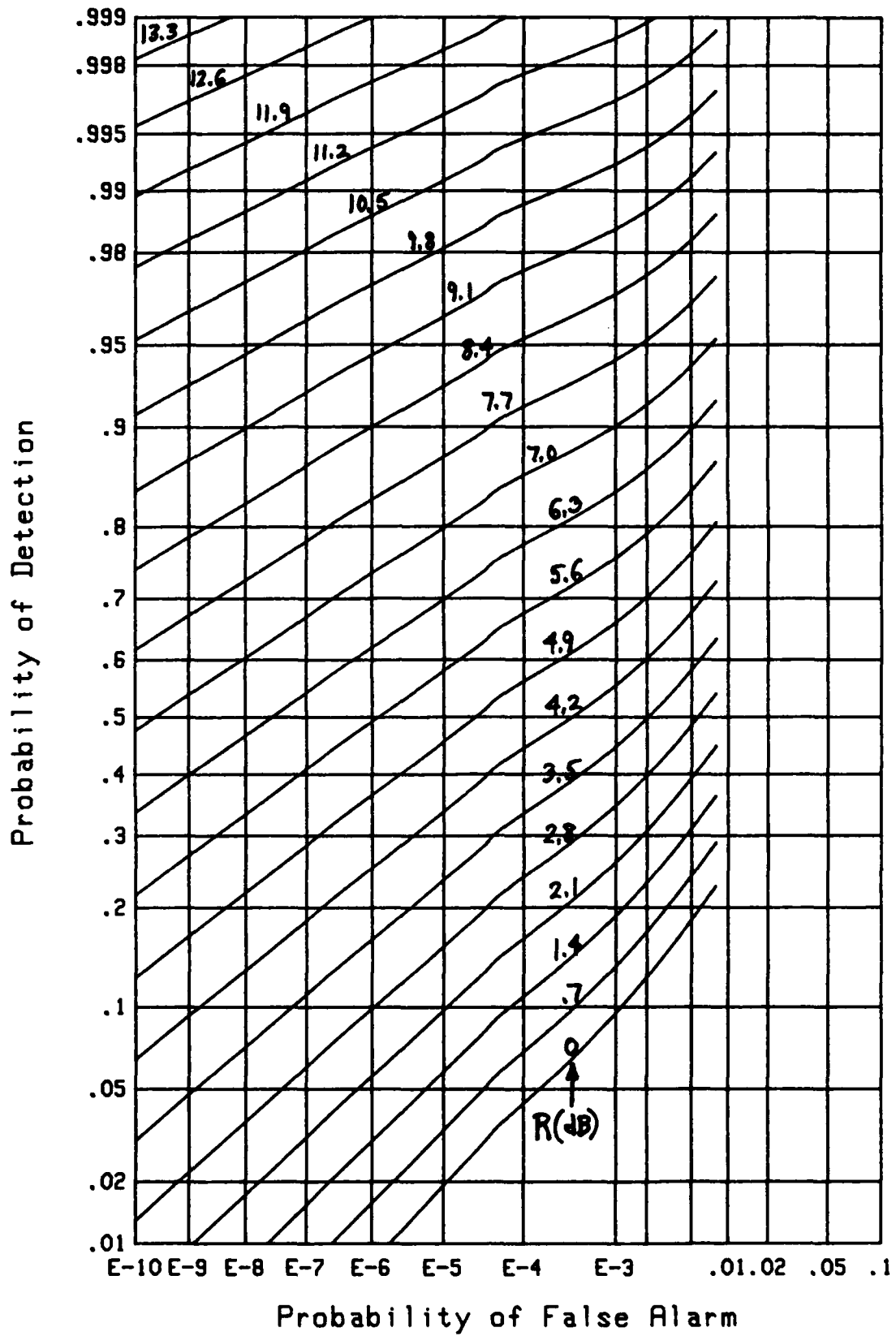


Figure 15. ROC for $N=6$, $F=.001$



Figure 17. ROC for $N=8$, $F=.1$

Figure 18. ROC for $N=8$, $F=.01$

Figure 19. ROC for $N=8$, $F=.001$

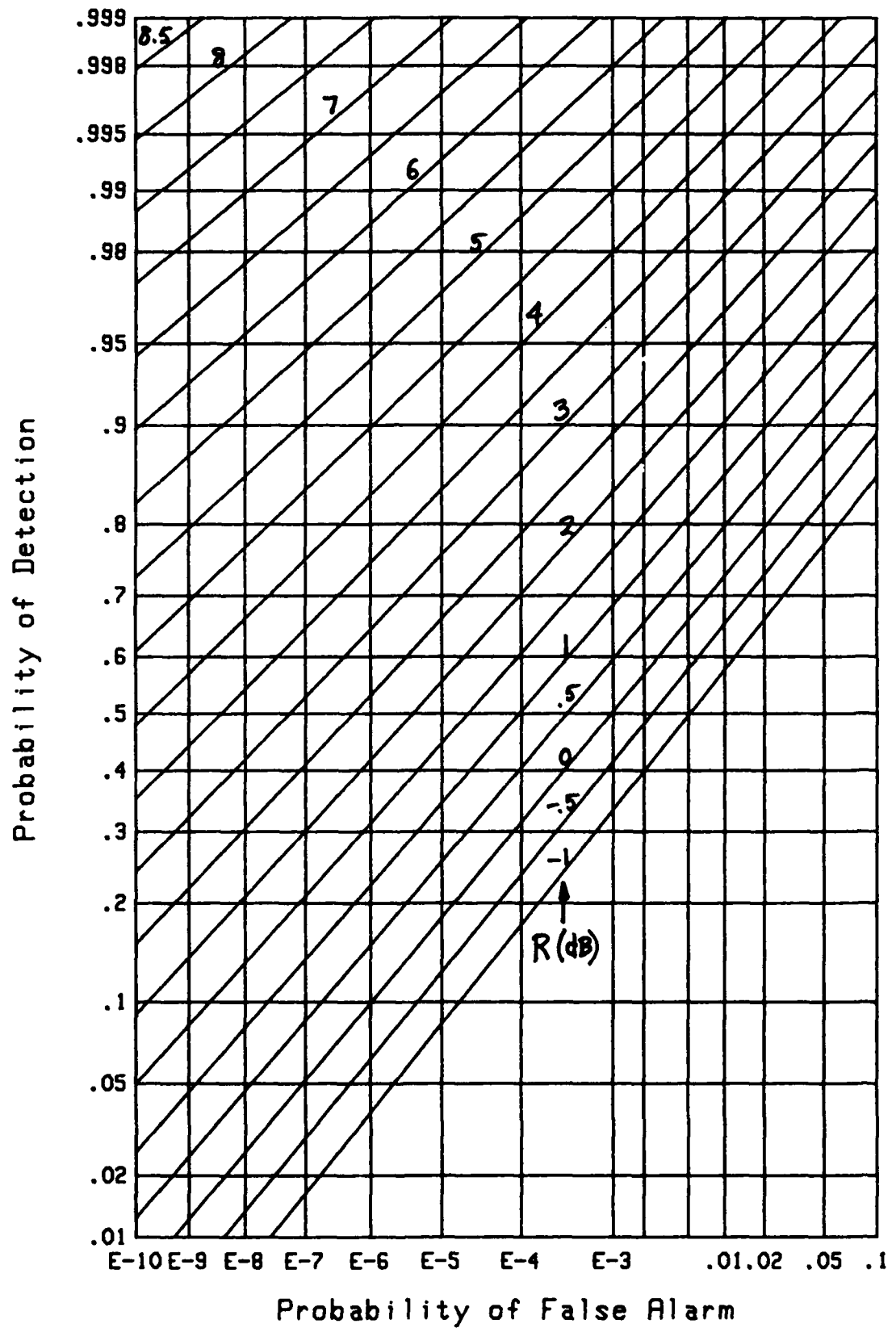


Figure 20. ROC for $N=16$, $F=1$.

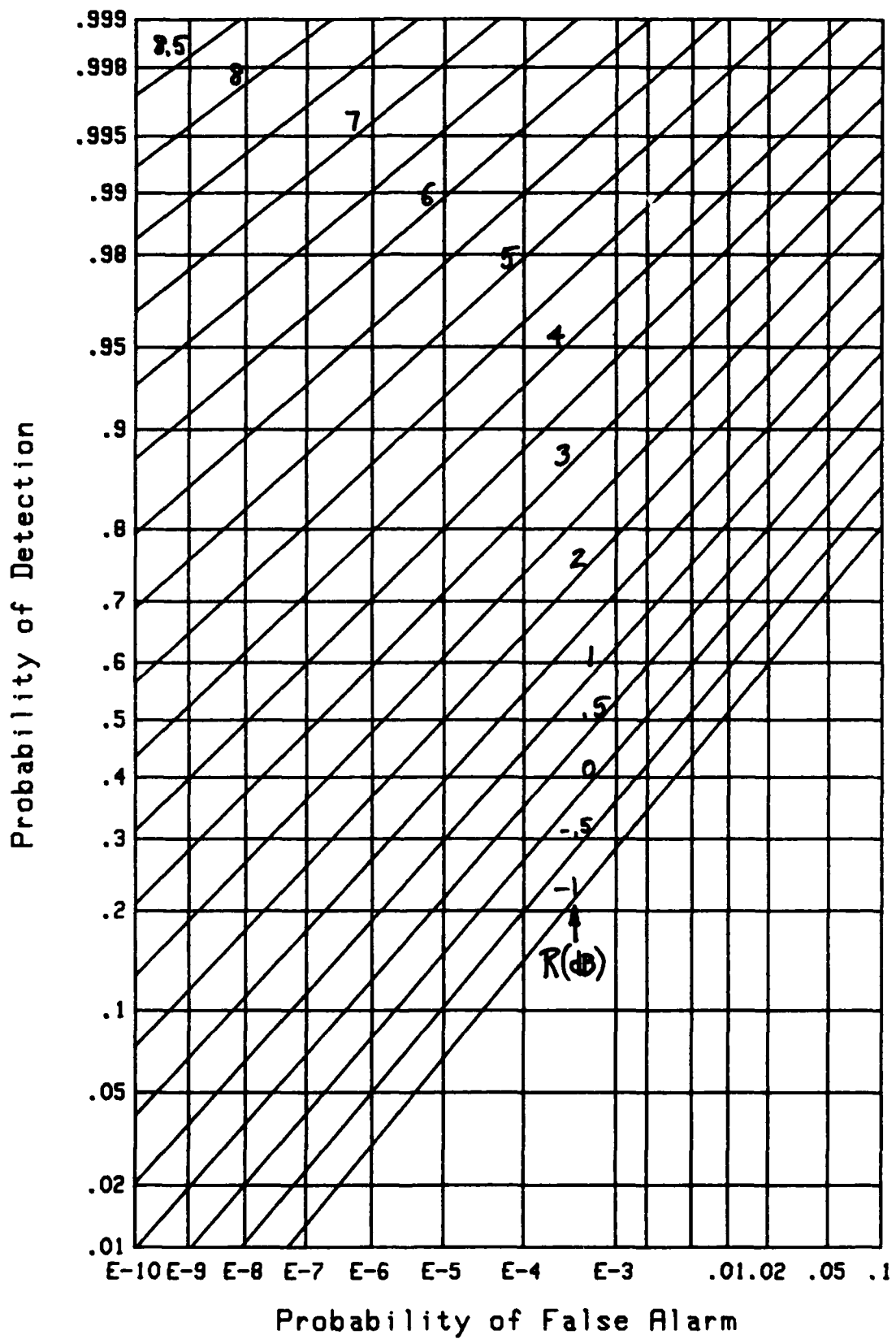
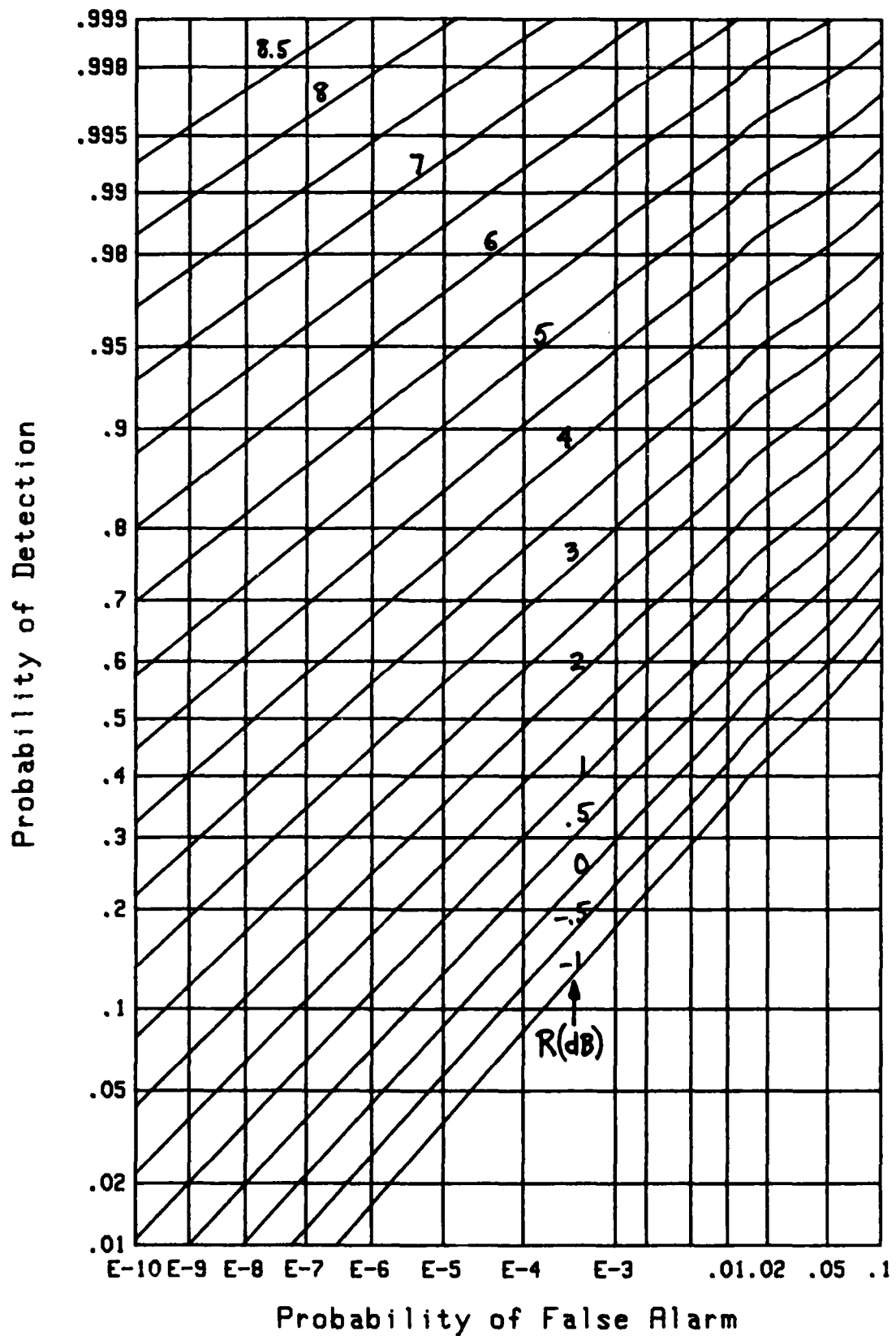
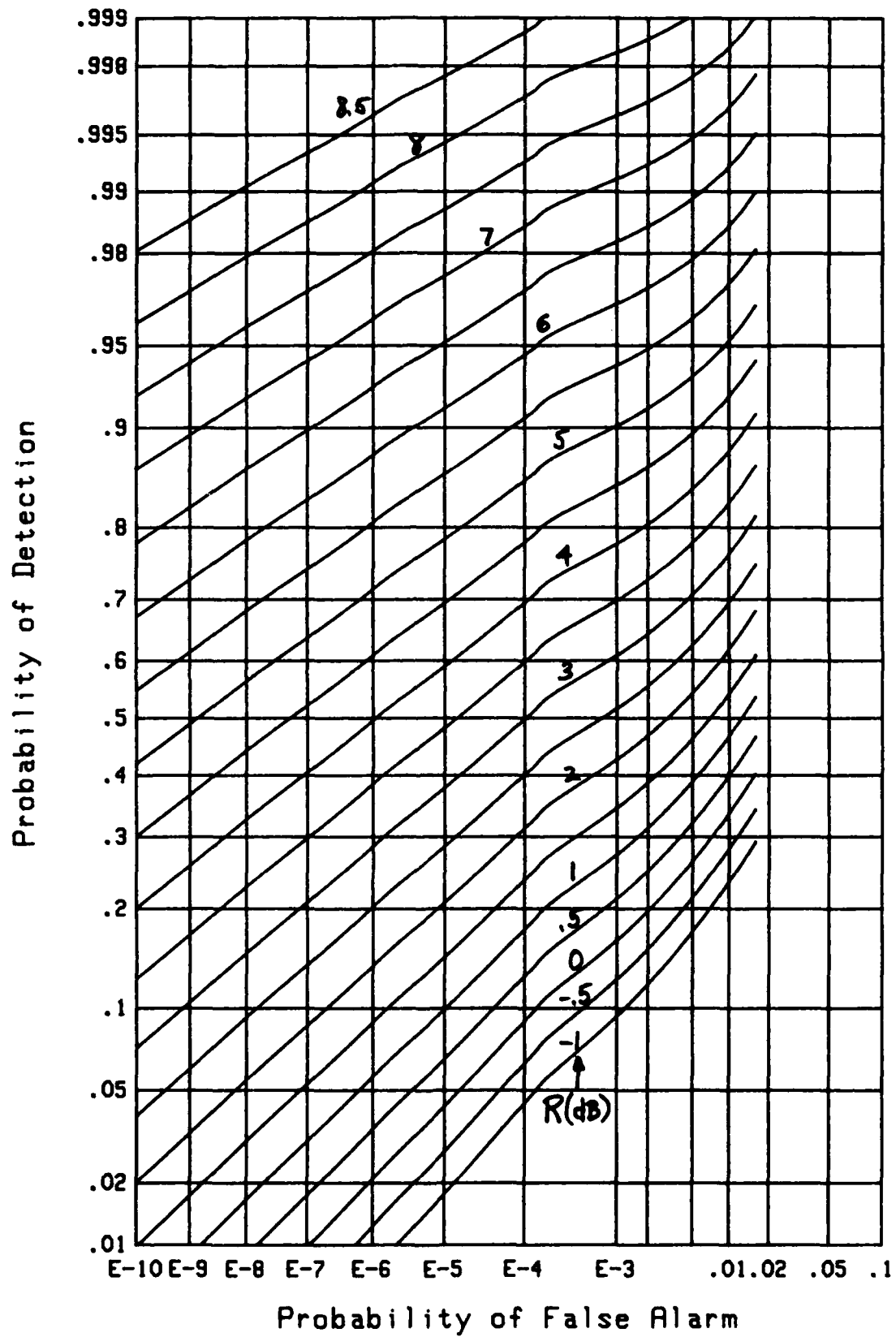


Figure 21. ROC for $N=16$, $F=.1$

Figure 22. ROC for $N=16$, $F=.01$

Figure 23. ROC for $N=16$, $F=.001$

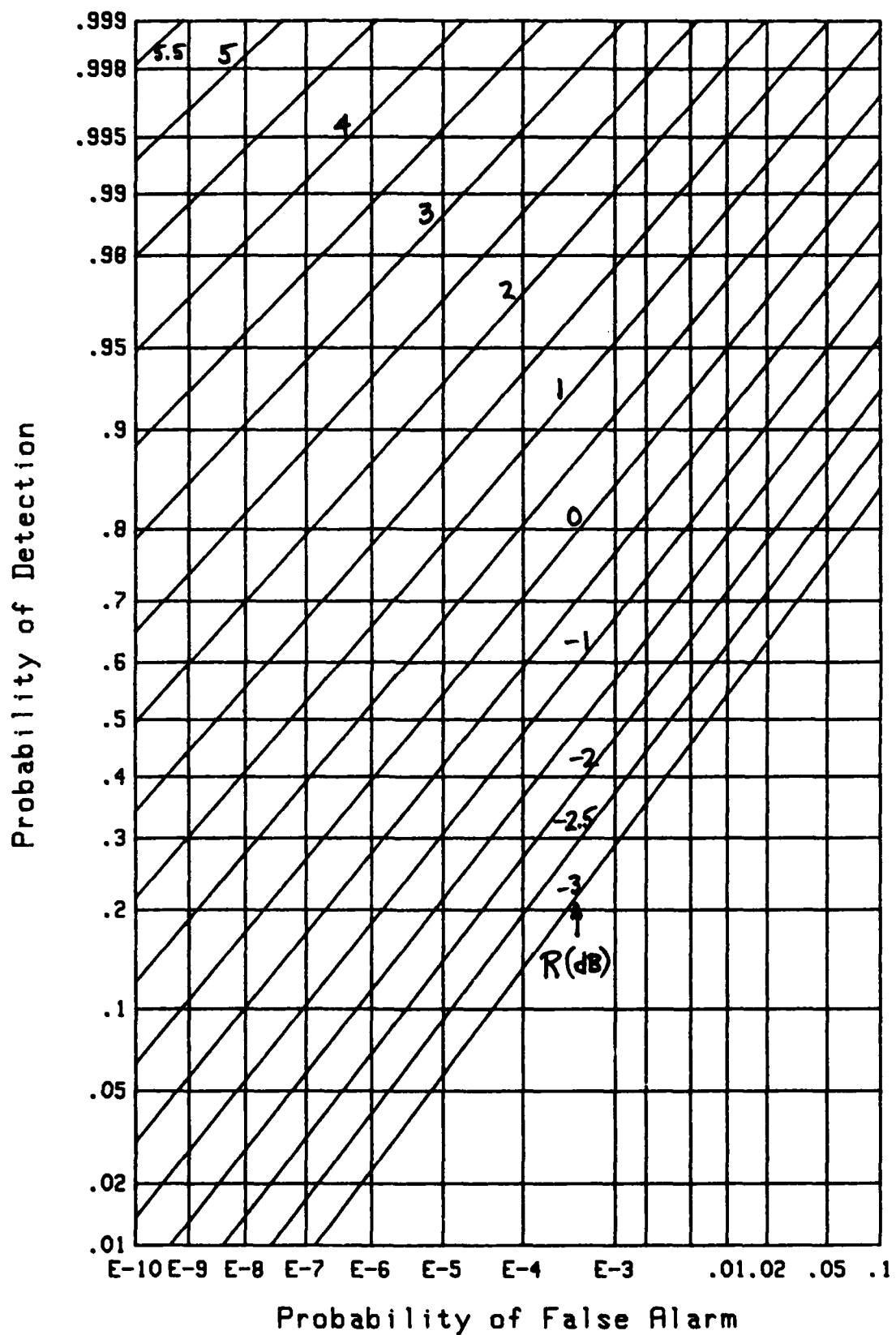


Figure 24. ROC for $N=32$, $F=1$.

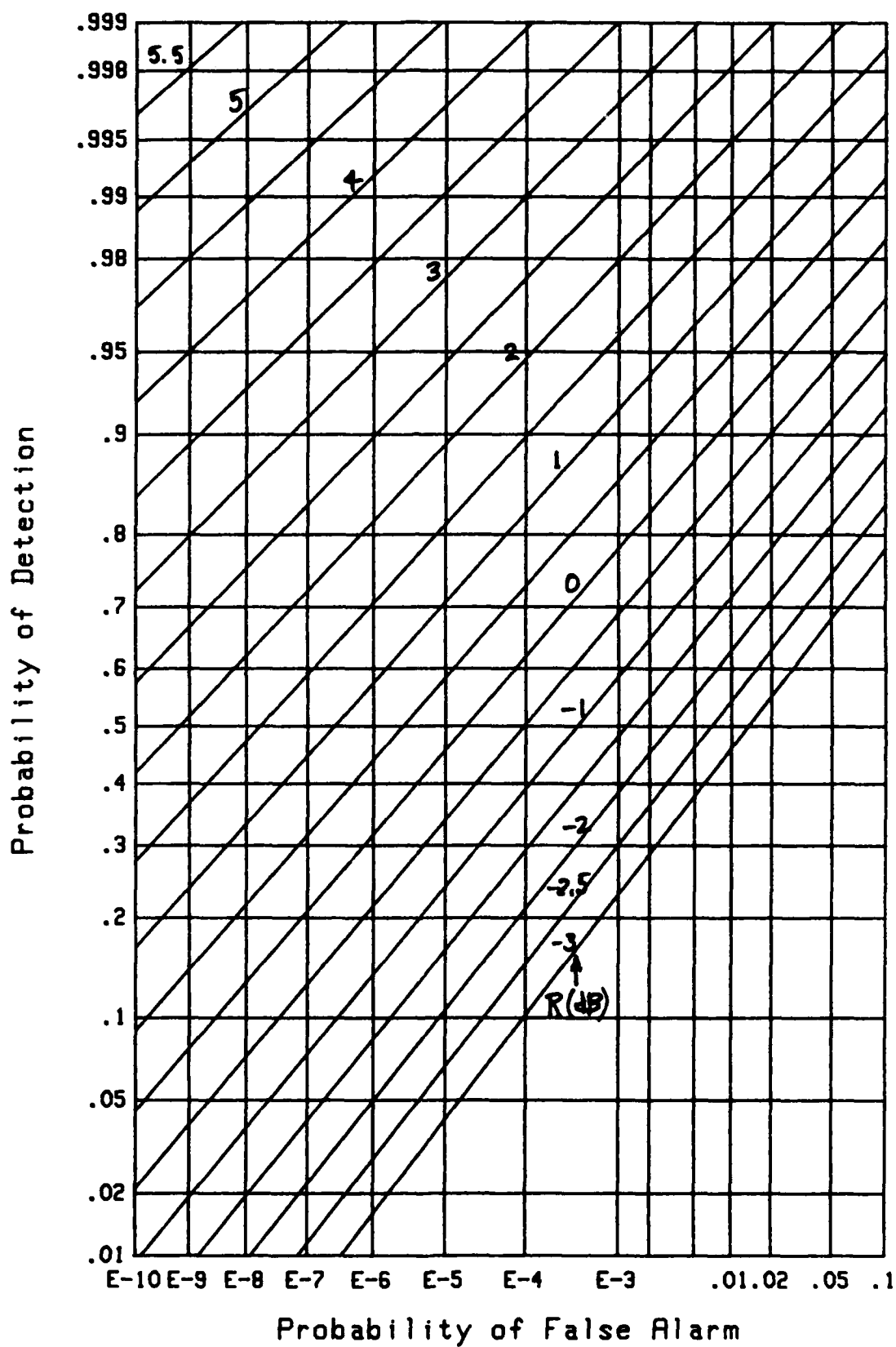
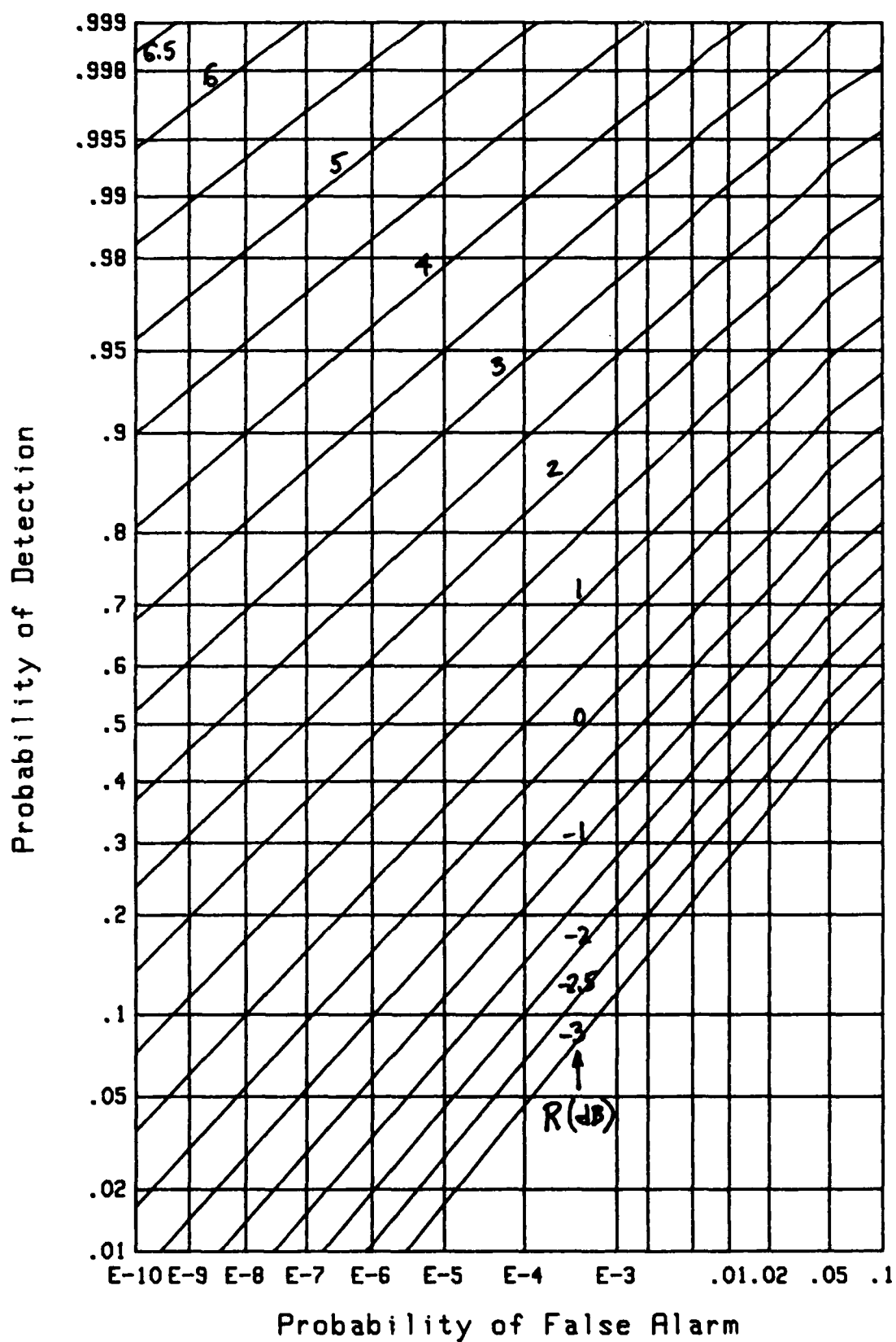
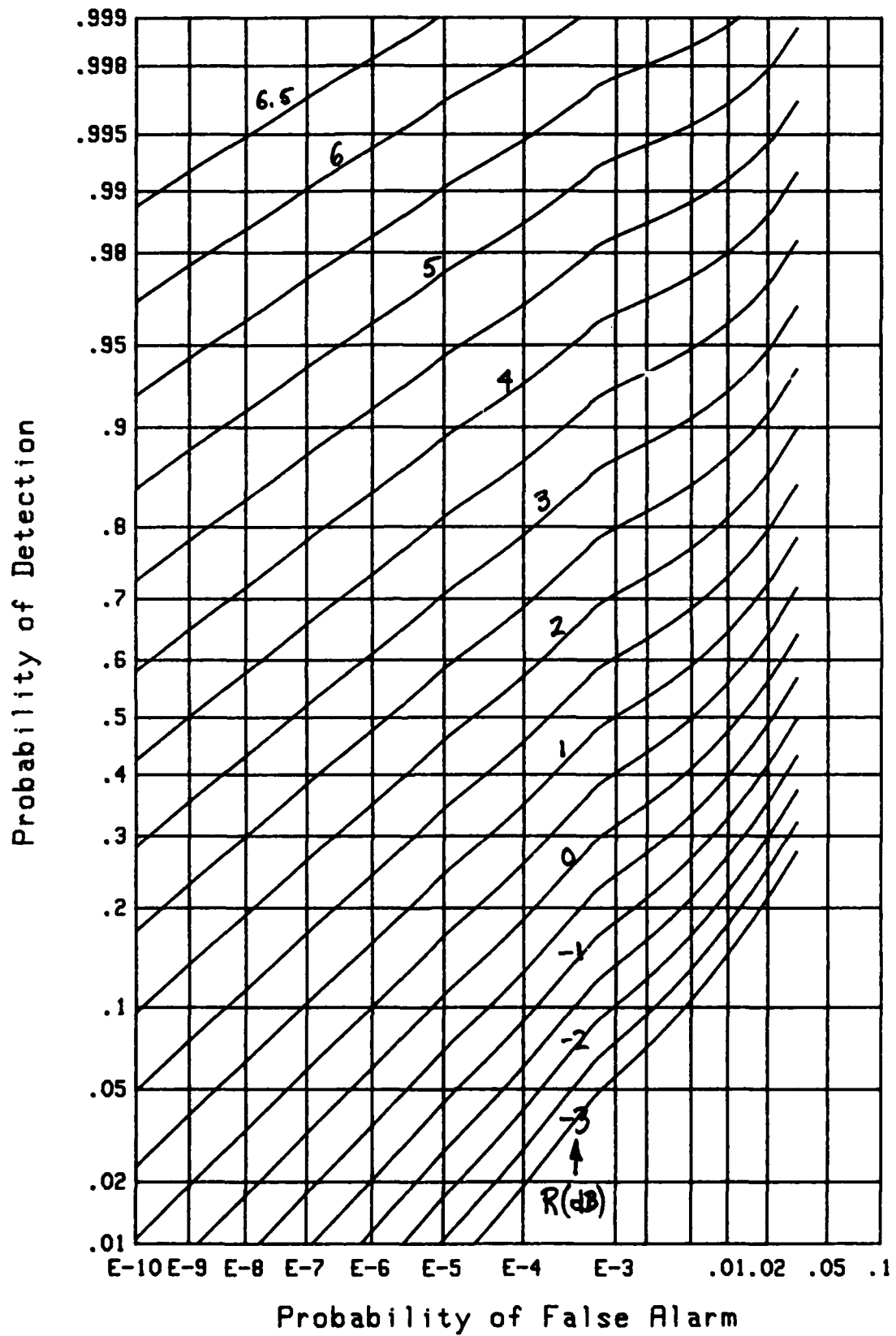
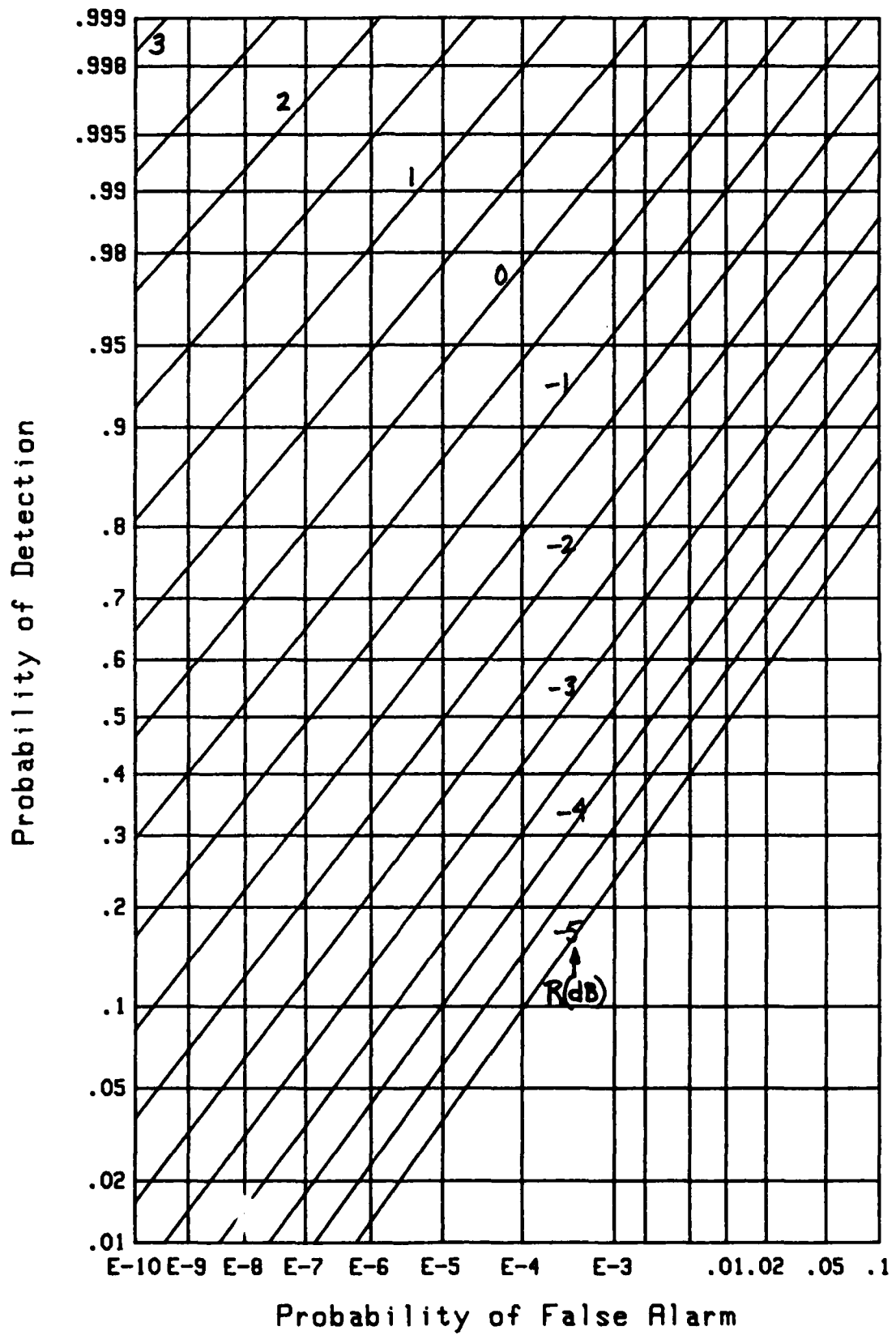
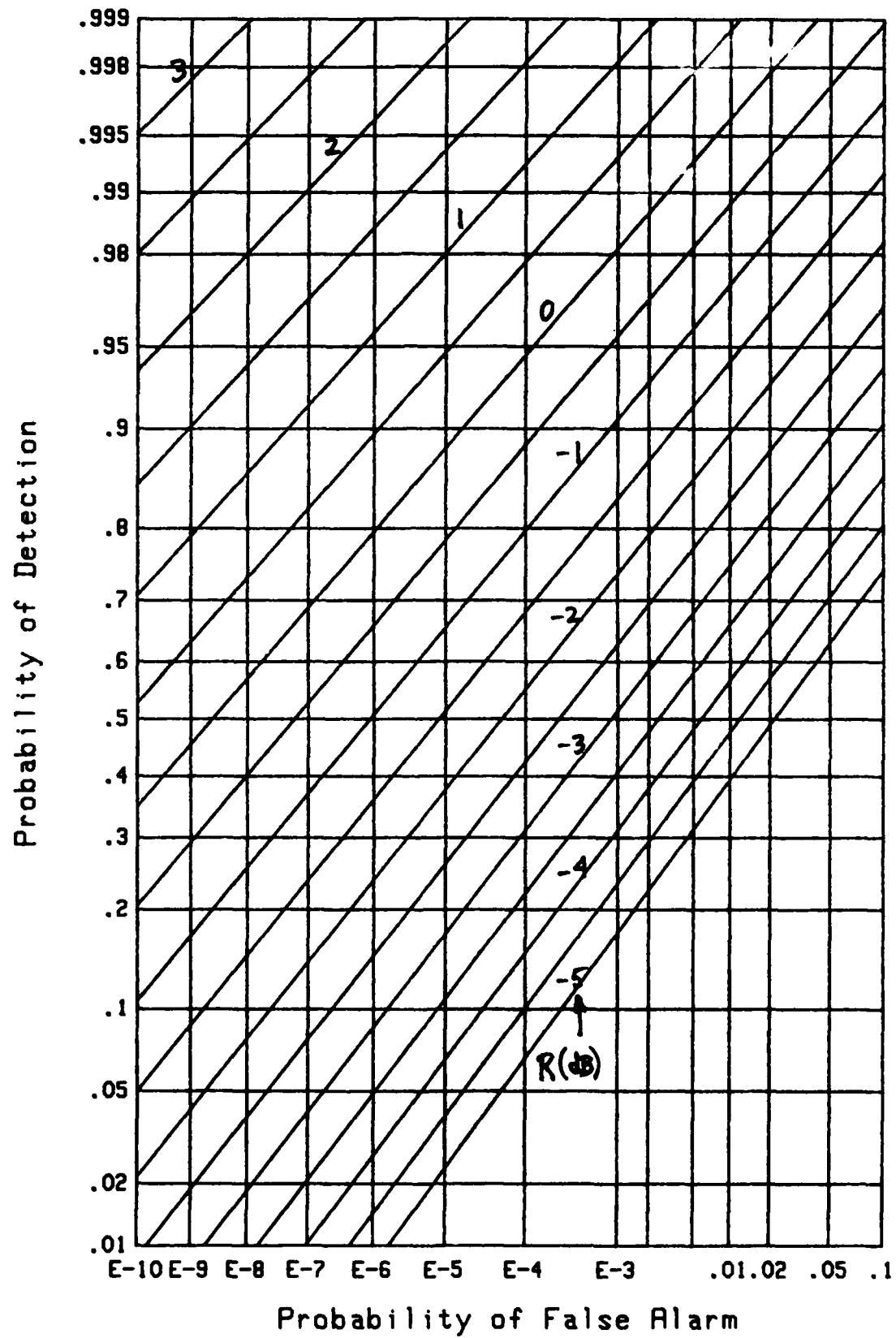


Figure 25. ROC for $N=32$, $F=.1$

Figure 26. ROC for $N=32$, $F=.01$

Figure 27. ROC for $N=32$, $F=.001$

Figure 28. ROC for $N=64$, $F=1$.

Figure 29. ROC for $N=64$, $F=.1$

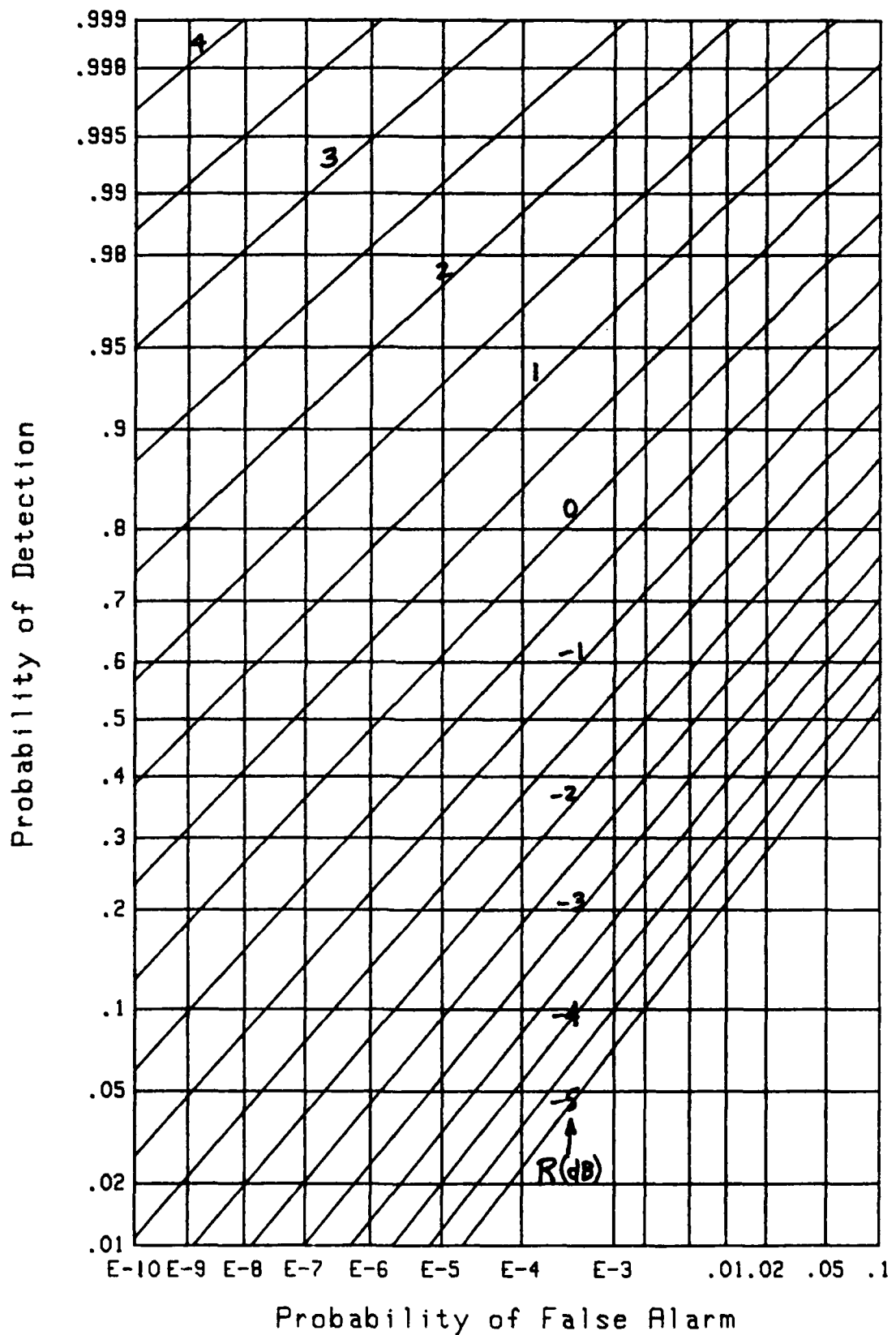
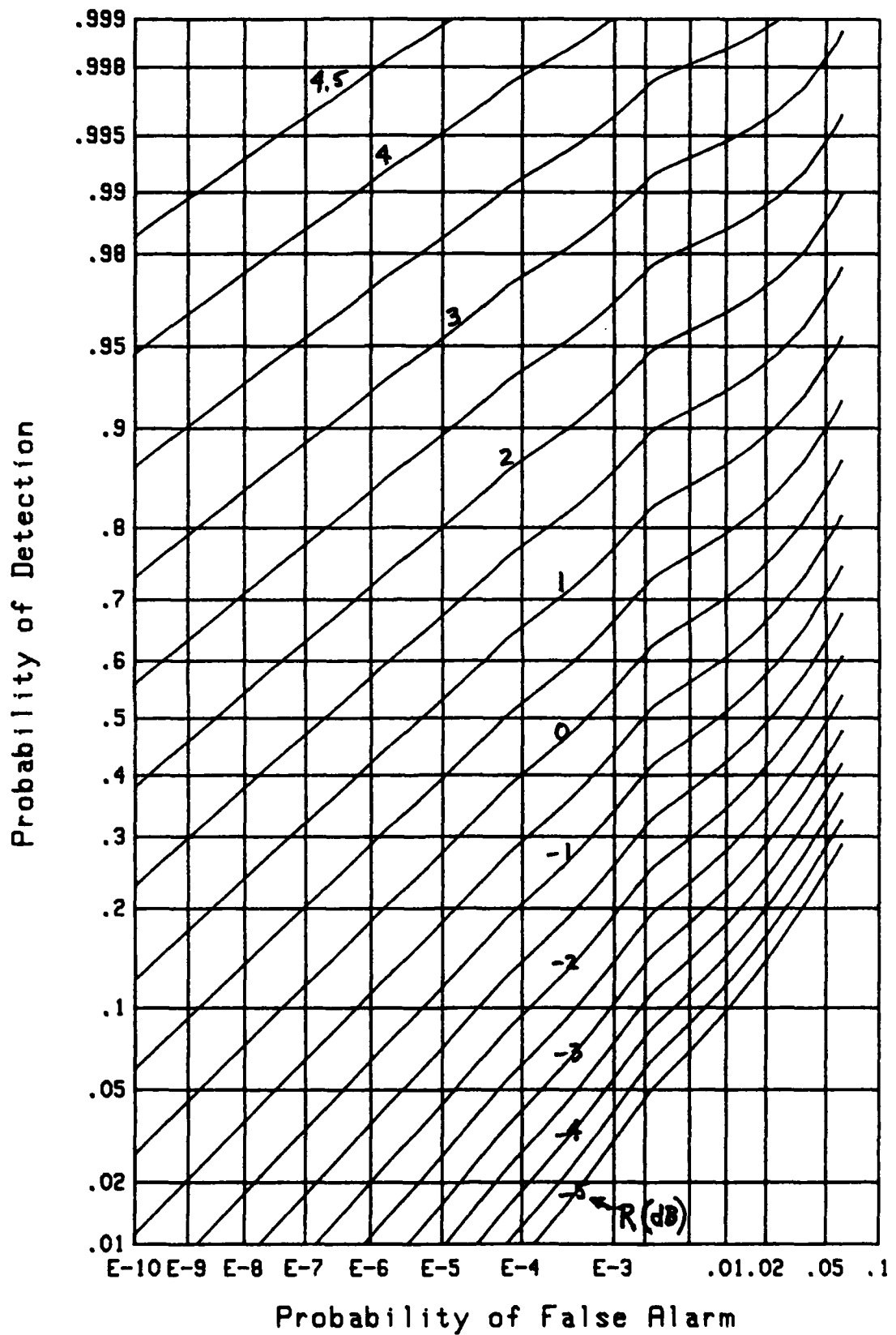


Figure 30. ROC for $N=64$, $F=.01$

Figure 31. ROC for $N=64$, $F=.001$

SUMMARY

The cost of suppressing the low-level outputs of the detected squared-envelopes is generally minimal, unless the number of channels becomes very large. This conclusion has been drawn only for the example where these squared-envelopes have an exponential probability density function for both the noise-only as well as the signal-plus-noise cases. It should also be checked out for other candidate forms of probability density functions besides exponential.

One line of reasoning that makes this conclusion more acceptable is that it is only the larger outputs from the detectors that are going to lead to positive statements about signal presence. Thus, suppression of the smaller outputs should be inconsequential, at least for few channels. However, for a large number of channels, the sum of many nonzero low-level quantities may add up to a significant value and affect an occasional detection decision.

APPENDIX A. PROGRAM FOR RECEIVER OPERATING CHARACTERISTICS

```

10 ! GENERATE PD VS PF; COMBINER WITH DEAD ZONE IN EACH CHANNEL, TR8595
20 N=64 ! NUMBER OF CHANNELS; N>=1
30 F=.001 ! FRACTION OF DATA PASSED; 0.<F<=1.
40 DIM T(100) ! THRESHOLD VALUES
50 COM Pf(100),Pd1(100),Pd2(100),Pd3(100),Pd4(100),Pd5(100)
60 COM Pd6(100),Pd7(100),Pd8(100),Pd9(100),Pd10(100),Pd11(100)
70 COM Pd12(100),Pd13(100),Pd14(100),Pd15(100),Pd16(100),Pd17(100)
80 COM Pd18(100),Pd19(100),Pd20(100)
90 DOUBLE N,I,J ! INTEGERS
100 T=.01
110 T=T+.01
120 Pf=FNPf(T,F,N)
130 IF Pf>.1 THEN 110
140 T1=MAX(T-.01,.01)
150 T=T+.01
160 Pf=FNPf(T,F,N)
170 IF Pf>1E-10 THEN 150
180 T2=T
190 Delt=(T2-T1)/100.
200 FOR I=0 TO 100
210 T=T1+Delt*I
220 T(I)=T ! THRESHOLD VALUES
230 Pf(I)=FNPf(T,F,N) ! FALSE ALARM PROBABILITIES
240 NEXT I
250 R1db=-5 ! STARTING SIGNAL-TO-NOISE RATIO (dB)
260 Delr=.5 ! INCREMENT IN SNR (dB)
270 FOR J=1 TO 20
280 Rdb=R1db+(J-1)*Delr ! SIGNAL-TO-NOISE RATIO PER CHANNEL (dB)
290 R=10.^(.1*Rdb) ! POWER RATIO
300 FOR I=0 TO 100
310 T=T(I)
320 Pd=FNPd(R,T,F,N) ! DETECTION PROBABILITIES
330 IF J=1 THEN Pd1(I)=Pd
340 IF J=2 THEN Pd2(I)=Pd
350 IF J=3 THEN Pd3(I)=Pd
360 IF J=4 THEN Pd4(I)=Pd
370 IF J=5 THEN Pd5(I)=Pd
380 IF J=6 THEN Pd6(I)=Pd
390 IF J=7 THEN Pd7(I)=Pd
400 IF J=8 THEN Pd8(I)=Pd
410 IF J=9 THEN Pd9(I)=Pd
420 IF J=10 THEN Pd10(I)=Pd
430 IF J=11 THEN Pd11(I)=Pd
440 IF J=12 THEN Pd12(I)=Pd
450 IF J=13 THEN Pd13(I)=Pd
460 IF J=14 THEN Pd14(I)=Pd
470 IF J=15 THEN Pd15(I)=Pd
480 IF J=16 THEN Pd16(I)=Pd
490 IF J=17 THEN Pd17(I)=Pd
500 IF J=18 THEN Pd18(I)=Pd
510 IF J=19 THEN Pd19(I)=Pd
520 IF J=20 THEN Pd20(I)=Pd
530 NEXT I
540 NEXT J

```

```

550   FOR I=0 TO 100
560   Pf(I)=FNInvphi(Pf(I))
570   Pd1(I)=FNInvphi(Pd1(I))
580   Pd2(I)=FNInvphi(Pd2(I))
590   Pd3(I)=FNInvphi(Pd3(I))
600   Pd4(I)=FNInvphi(Pd4(I))
610   Pd5(I)=FNInvphi(Pd5(I))
620   Pd6(I)=FNInvphi(Pd6(I))
630   Pd7(I)=FNInvphi(Pd7(I))
640   Pd8(I)=FNInvphi(Pd8(I))
650   Pd9(I)=FNInvphi(Pd9(I))
660   Pd10(I)=FNInvphi(Pd10(I))
670   Pd11(I)=FNInvphi(Pd11(I))
680   Pd12(I)=FNInvphi(Pd12(I))
690   Pd13(I)=FNInvphi(Pd13(I))
700   Pd14(I)=FNInvphi(Pd14(I))
710   Pd15(I)=FNInvphi(Pd15(I))
720   Pd16(I)=FNInvphi(Pd16(I))
730   Pd17(I)=FNInvphi(Pd17(I))
740   Pd18(I)=FNInvphi(Pd18(I))
750   Pd19(I)=FNInvphi(Pd19(I))
760   Pd20(I)=FNInvphi(Pd20(I))
770   NEXT I
780   CALL Plot
790   END
800   !
810   DEF FNInvphi(X)           ! AMS 55, 26.2.23
820   IF X=.5 THEN RETURN 0.
830   P=MIN(X,1.-X)
840   T=-LOG(P)
850   T=SQR(T+T)
860   P=1.+T*(1.432788+T*(.189269+T*.001308))
870   P=T-(2.515517+T*(.802853+T*.010328))/P
880   IF X<.5 THEN P=-P
890   RETURN P
900   FNEND
910   !
920   DEF FNE(U,DOUBLE N)      ! N>=0
930   DOUBLE K                  ! INTEGER
940   IF U<=0. THEN RETURN 1.
950   S=T=EXP(-U)
960   IF N=0 THEN RETURN S
970   FOR K=1 TO N
980   T=T*U/K
990   S=S+T
1000  NEXT K
1010  RETURN S
1020  FNEND
1030  !

```

```

1040 DEF FNPf(T,F,DOUBLE N) ! FALSE ALARM PROB. T>=0,0<F<=1,N>=1
1050 DOUBLE Ns,N1 ! INTEGERS
1060 IF F<1. THEN 1090
1070 Pf=FNE(T,N-1)
1080 RETURN Pf
1090 L=-LOG(F)
1100 N1=N+1
1110 F1=1.-F
1120 A=F/F1
1130 Tn=F1^N
1140 C=T
1150 Pf=0.
1160 FOR Ns=1 TO N
1170 C=C-L
1180 Tn=Tn*A*(N1-Ns)/Ns
1190 Pf=Pf+Tn*FNE(C,Ns-1)
1200 NEXT Ns
1210 RETURN Pf
1220 FNEND
1230 !
1240 DEF FNPd(R,T,F,DOUBLE N) ! DETECTION PROB. R>=0,T>=0,0<F<=1,N>=1
1250 DOUBLE Ns,N1 ! INTEGERS
1260 As=1./((1.+R) ! a
1270 IF F<1. THEN 1300
1280 Pd=FNE(As*T,N-1)
1290 RETURN Pd
1300 L=-LOG(F)
1310 N1=N+1
1320 B=EXP(-As*L)
1330 B1=1.-B
1340 A=B/B1
1350 Tn=B1^N
1360 C=T
1370 Pd=0.
1380 FOR Ns=1 TO N
1390 C=C-L
1400 Tn=Tn*A*(N1-Ns)/Ns
1410 Pd=Pd+Tn*FNE(As*C,Ns-1)
1420 NEXT Ns
1430 RETURN Pd
1440 FNEND
1450 !

```

```

1460 SUB Plot ! PLOT PD VS PF ON NORMAL PROBABILITY PAPER
1470 COM Pf(*),Pd1(*),Pd2(*),Pd3(*),Pd4(*),Pd5(*)
1480 COM Pd6(*),Pd7(*),Pd8(*),Pd9(*),Pd10(*),Pd11(*)
1490 COM Pd12(*),Pd13(*),Pd14(*),Pd15(*),Pd16(*),Pd17(*)
1500 COM Pd18(*),Pd19(*),Pd20(*)
1510 DIM A$(30),B$(30),C$(31)
1520 DIM Xlabel$(1:30),Ylabel$(1:30)
1530 DIM Xcoord(1:30),Ycoord(1:30)
1540 DIM Xgrid(1:30),Ygrid(1:30)
1550 DOUBLE N,Lx,Ly,Nx,Ny,I ! INTEGERS
1560 !
1570 A$="Probability of False Alarm"
1580 B$="Probability of Detection"
1590 C$="Figure ROC for N=64, F=.001"
1600 !
1610 Lx=12
1620 REDIM Xlabel$(1:Lx),Xcoord(1:Lx)
1630 DATA E-10,E-9,E-8,E-7,E-6,E-5,E-4,E-3,.01,.02,.05,.1
1640 READ Xlabel$(*)
1650 DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4,.001,.01,.02,.05,.1
1660 READ Xcoord(*)
1670 !
1680 Ly=18
1690 REDIM Ylabel$(1:Ly),Ycoord(1:Ly)
1700 DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1710 DATA .95,.98,.99,.995,.998,.999
1720 READ Ylabel$(*)
1730 DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1740 DATA .95,.98,.99,.995,.998,.999
1750 READ Ycoord(*)
1760 !
1770 Nx=14
1780 REDIM Xgrid(1:Nx)
1790 DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4
1800 DATA .001,.002,.005,.01,.02,.05,.1
1810 READ Xgrid(*)
1820 !
1830 Ny=18
1840 REDIM Ygrid(1:Ny)
1850 DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1860 DATA .95,.98,.99,.995,.998,.999
1870 READ Ygrid(*)
1880 !
1890 FOR I=1 TO Lx
1900 Xcoord(I)=FNInvphi(Xcoord(I))
1910 NEXT I
1920 FOR I=1 TO Ly
1930 Ycoord(I)=FNInvphi(Ycoord(I))
1940 NEXT I
1950 FOR I=1 TO Nx
1960 Xgrid(I)=FNInvphi(Xgrid(I))
1970 NEXT I
1980 FOR I=1 TO Ny
1990 Ygrid(I)=FNInvphi(Ygrid(I))
2000 NEXT I

```

```

2010  X1=Xgrid(1)
2020  X2=Xgrid(Nx)
2030  Y1=Ygrid(1)
2040  Y2=Ygrid(Ny)
2050  GINIT 200./260.                                !  VERTICAL PAPER
2060  PLOTTER IS 505,"HPGL"
2070  PRINTER IS 505
2080  PRINT "VS2"
2090  LIMIT PLOTTER 505,0.,200.,0.,260.                !  1 GDU = 2 mm
2100  VIEWPORT 22.,85.,19.,122.
2110  WINDOW X1,X2,Y1,Y2
2120  FOR I=1 TO Nx
2130  MOVE Xgrid(I),Y1
2140  DRAW Xgrid(I),Y2
2150  NEXT I
2160  FOR I=1 TO Ny
2170  MOVE X1,Ygrid(I)
2180  DRAW X2,Ygrid(I)
2190  NEXT I
2200  LDIR 0
2210  CSIZE 2.3,.5
2220  LORG 5
2230  Y=Y1-(Y2-Y1)*.02
2240  FOR I=1 TO Lx
2250  MOVE Xcoord(I),Y
2260  LABEL Xlabel$(I)
2270  NEXT I
2280  CSIZE 3.,.5
2290  MOVE .5*(X1+X2),Y1-.06*(Y2-Y1)
2300  LABEL A$
2310  MOVE .5*(X1+X2),Y1-.1*(Y2-Y1)
2320  LABEL C$
2330  CSIZE 2.3,.5
2340  LORG 8
2350  X=X1-(X2-X1)*.01
2360  FOR I=1 TO Ly
2370  MOVE X,Ycoord(I)
2380  LABEL Ylabel$(I)
2390  NEXT I
2400  LDIR PI/2.
2410  CSIZE 3.,.5
2420  LORG 5
2430  MOVE X1-.15*(X2-X1),.5*(Y1+Y2)
2440  LABEL B$
2450  PENUP

```

```
2460 PLOT Pf(*),Pd1(*)
2470 PENUP
2480 PLOT Pf(*),Pd2(*)
2490 PENUP
2500 PLOT Pf(*),Pd3(*)
2510 PENUP
2520 PLOT Pf(*),Pd4(*)
2530 PENUP
2540 PLOT Pf(*),Pd5(*)
2550 PENUP
2560 PLOT Pf(*),Pd6(*)
2570 PENUP
2580 PLOT Pf(*),Pd7(*)
2590 PENUP
2600 PLOT Pf(*),Pd8(*)
2610 PENUP
2620 PLOT Pf(*),Pd9(*)
2630 PENUP
2640 PLOT Pf(*),Pd10(*)
2650 PENUP
2660 PLOT Pf(*),Pd11(*)
2670 PENUP
2680 PLOT Pf(*),Pd12(*)
2690 PENUP
2700 PLOT Pf(*),Pd13(*)
2710 PENUP
2720 PLOT Pf(*),Pd14(*)
2730 PENUP
2740 PLOT Pf(*),Pd15(*)
2750 PENUP
2760 PLOT Pf(*),Pd16(*)
2770 PENUP
2780 PLOT Pf(*),Pd17(*)
2790 PENUP
2800 PLOT Pf(*),Pd18(*)
2810 PENUP
2820 PLOT Pf(*),Pd19(*)
2830 PENUP
2840 PLOT Pf(*),Pd20(*)
2850 PENUP
2860 BEEP 500,2
2870 PAUSE
2880 PRINTER IS CRT
2890 PLOTTER 505 IS TERMINATED
2900 SUBEND
```

REFERENCES

1. A. H. Nuttall, **Accurate Efficient Evaluation of Cumulative or Exceedance Probability Distributions Directly From Characteristic Functions**, NUSC Technical Report 7023, Naval Underwater Systems Center, New London, CT, 1 October 1983.
2. A. H. Nuttall, **Operating Characteristics of Cross-Correlator With or Without Sample Mean Removal**, NUSC Technical Report 7045, Naval Underwater Systems Center, New London, CT, 14 August 1984.
3. A. H. Nuttall, **Evaluation of Densities and Distributions via Hermite and Generalized Laguerre Series Employing High-Order Expansion Coefficients Determined Recursively via Moments or Cumulants**, NUSC Technical Report 7377, Naval Underwater Systems Center, New London, CT, 28 February 1985.
4. **Handbook of Mathematical Functions**, U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series No. 55, U. S. Government Printing Office, Washington, DC, June 1964.
5. A. H. Nuttall and A. F. Magaraci, **Signal-to-Noise Ratios Required for Short-Term Narrowband Detection of Gaussian Processes**, NUSC Technical Report 4417, Naval Underwater Systems Center, New London, CT, 20 October 1972.

A. H. Nuttall

INITIAL DISTRIBUTION LIST

Addressee	No. of Copies
ADMIRALTY RESEARCH ESTABLISHMENT, London, England (Dr. L. Lloyd)	1
ADMIRALTY UNDERWATER WEAPONS ESTABLISHMENT, Dorset, England	1
APPLIED PHYSICS LAB, JOHN HOPKINS (John C. Stapleton)	1
APPLIED PHYSICS LAB, U. WASHINGTON (C. Eggen)	1
APPLIED RESEARCH LAB, PENN STATE, (Frank W. Symons)	1
APPLIED RESEARCH LAB, U. TEXAS (Dr. M. Frazer)	1
APPLIED SEISMIC GROUP, Cambridge, MA (Richard Lacoss)	1
A & T, Stonington, Ct (H. Jarvis)	1
ASTRON RESEARCH & ENGINEERING, Santa Monica, CA (Dr. Allen Piersol)	1
BBN, Arlington, Va. (Dr. H. Cox)	1
BBN, Cambridge, MA (H. Gish)	1
BBN, New London, Ct. (Dr. P. Cable)	1
BELL COMMUNICATIONS RESEARCH, Morristown, NJ (J. Kaiser and D. Sunday (Library)	2
BENDAT, Julius Dr., Los Angeles, CA	1
BLEINSTEIN, Norman Dr., Denver, CO	1
BROWN UNIV, Providence, RI (Documents Library)	1
CANBERRA COLLEGE OF ADV. EDUC, BELCONNEN, A.C.T. Australia (P. Morgan)	1
COAST GUARD ACADEMY, New London, CT (Prof. J. Wolcin)	1
COAST GUARD R & D, Groton, CT (Library)	1
COGENT SYSTEMS, INC, (J. Costas)	1
COHEN, Leon Dr., Bronx, NY	1
CONCORDIA UNIVERSITY H-915-3, Montreal, Quebec Canada (Prof. Jeffrey Krolík)	1
CNO (NOP-098)	1
CNR-OCNR-00, 10, 12, 13, 20	5
DALHOUSIE UNIV., Halifax, Nova Scotia, Canada (Dr. B. Ruddick)	1
DAVID W. TAYLOR RESEARCH CNTR, Annapolis, MD (P. Prendergast, Code 2744)	1
DARPA, Arlington, VA (A. Ellinthorpe)	1
DEFENCE RESEARCH CENTER, Adelaide, Australia (Library)	1
DEFENCE RESEARCH ESTAB. ATLANTIC, Dartmouth, Nova Scotia (Library)	1
DEFENCE RESEARCH ESTAB. PACIFIC, Victoria, Canada (Dr. D. Thomson)	1
DEFENCE SCIENTIFIC ESTABLISHMENT, MINISTRY OF DEFENCE, Auckland, New Zealand (Dr. L. Hall)	1
DEFENSE SYSTEMS, INC, Mc Lean, VA (Dr. G. Sebestyen)	1
DIA	1
DIAGNOSTIC/RETRIEVAL SYSTEMS, INC, Tustin, CA. (J. Williams)	1
DTIC	1
DTRC	1
DREXEL UNIV, (Prof. S. Kesler)	1
EDO CORP, College Point, NY (M. Blanchard)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
EG&G, Manassas, VA (D. Frohman)	1
GENERAL ELECTRIC CO, Moorestown, NJ (Dr. Mark Allen 108-102)	1
GENERAL ELECTRIC CO., Philadelphia, PA (T. J. McFall)	1
GENERAL ELECTRIC CO, Pittsfield, MA (R. W. Race)	1
GENERAL ELECTRIC CO, Syracuse, NY (J. L. Rogers, Dr. A. M. Vural and D. Winfield)	3
HAHN, Wm, Wash, DC	1
HARRIS SCIENTIFIC SERVICES, Dobbs Ferry, NY (B. Harris)	1
HARVARD UNIVERSITY, Gordon McKay Library	1
HONEYWELL ENGR SERV CNTR, Poulsbro, WA (C. Schmid)	1
HUGHES AIRCRAFT, Fullerton, CA (S. Autrey)	1
HUGHES AIRCRAFT, Buena Park, CA (T. Posch)	1
IBM, Manassas, VA (G. Demuth)	1
INDIAN INSTITUTE OF TECHNOLOGY, Madras, India (Dr. K. M. M. Prabhu)	1
INTERSTATE ELECTRONICS CORP, Anaheim, CA (R. Nielsen, 8011)	1
JOHNS HOPKINS UNIV, Laurel, MD (J. C. Stapleton)	1
KILDARE CORP, New London, CT (Dr. R. Mellen)	1
LINCOM CORP., Northboro, MA (Dr. T. Schonhoff)	1
MAGNAVOX ELEC SYSTEMS CO, Ft. Wayne, IN (R. Kenefic)	1
MARINE BIOLOGICAL LAB, Woods Hole, MA	1
MARINE PHYSICAL LABORATORY SCRIPPS	1
MASS. INSTITUTE OF TECHNOLOGY (Prof. A. Baggaroer, Barker Engineering Library)	2
MBS SYSTEMS, Norwalk, CT (A. Winder)	1
MIDDLETON, DAVID, NY, NY	1
NADC (5041, M. Mele)	1
NAIR-03	1
NASH, Harold E., Quaker Hill, CT	1
NATIONAL RADIO ASTRONOMY OBSERVATORY, Charlottesville, VA (F. Schwab)	1
NATIONAL SECURITY AGENCY, FT. Meade, MD (Dr. James R. Maar, R51)	1
NATO SACLANT ASW RESEARCH CENTRE, APO NY, NY (Library, R. E. Sullivan and G. Tacconi)	3
NCSC	1
NEPRF	1
NORDA	1
NRL UND SOUND REF DET, Orlando, FL	1
NAVAL INTELLIGENCE COMMAND	1
NAVAL INTELLIGENCE SUPPORT CENTER	1
NAVAL OCEAN SYSTEMS CENTER, San Diego, CA (James M. Alsup, Code 635)	1
NAVAL OCEANOGRAPHY OFFICE	1
NAVAL SYSTEMS DIV., SIMRAD SUBSEA A/S, Norway (E. B. Lunde)	1
NICHOLS RESEARCH CORP., Wakefield, MA (T. Marzetta)	1
NORDA (Dr. B. Adams)	1
NORDA (Code 345) N STL Station, MS (R. Wagstaff)	1
NORTHEASTERN UNIV. Boston, MA (Prof. C. L. Nikias)	1
NORWEGIAN DEFENCE RESEARCH EST, Norway (Dr J. Glattetre)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
NOSC, (James M. Alsup, Code 635, C. Sturdevant; 73, J. Lockwood, F. Harris, 743, R. Smith; 62, R. Thuleen)	6
NPRDC	1
NPS, Monterey, CA (C. W. Therrien, Code 62 T1)	2
NRL, Washington, DC (Dr. J. Buccaro, Dr. E. Franchi, Dr. P. Abraham, Code 5132, A. A. Gerlach, W. Gabriel (Code 5370), and N. Yen (Code 5130)	6
NRL, Arlington, VA (N. L. Gerr, Code 1111)	1
NSWC	1
NSWC DET Ft. Lauderdale	1
NSWC WHITE OAK LAB	1
NUSC DET TUDOR HILL	1
NUSC DET WEST PALM BEACH (Dr. R. Kennedy Code 3802)	1
NWC	1
ORI CO, INC, New London, CT (G. Assard)	1
ORINCON CORP., Columbia, MD (S. Larry Marple)	1
PAPOUTSANIS, P. D., Athens, Greece	1
PENN STATE UNIV., State College, PA (F. Symons)	1
PIERSOLL ENGR CO, Woodland Hills, CA (Dr. Allen G. Piersoll)	1
POHLER, R., Austin, TX	1
POLETTI, Mark A., Acoustics Research Centre, School of Architecture, Univ. of Auckland, Auckland, New Zealand	1
PROMETHEUS, INC, Sharon, MA (Dr. J. Byrnes)	1
PROMETHEUS INC, Newport, RI (Michael J. Barrett)	1
PRICE, Robert Dr. Lexington, MA	1
PURDUE UNIV, West Lafayette, IN (N. Srinivasa)	1
RAISBECK, Dr. Gordon, Portland, ME	1
RAN RESEARCH LAB, Darlinghurst, Australia	1
RAYTHEON CO, Portsmouth, RI (J. Bartram, R. Connor) and S. S. Reese)	3
RICHTER, W., Annandale, VA.	1
ROCKWELL INTERNATIONAL CORP, Anaheim, CA (L. Einstein and Dr. D. Elliott)	2
ROYAL MILITARY COLLEGE OF CANADA, (Prof. Y. Chan)	1
RUTGERS UNIV., Piscataway, NJ (Prof. S. Orfanidis)	1
RCA CORP, Moorestown, NJ (H. Upkowitz)	1
SACLANT UNDERSEA RESEARCH CENTRE, APO NY NY (Dr. John Ianniello, Dr. S. Stergiopolous and Giorgio Tacconi, Library)	4
SAIC, Falls Church, VA (Dr. P. Mikhailevsky)	1
SAIC, New London, CT (Dr. F. Dinapoli)	1
SANDIA NATIONAL LABORATORY (J. Claasen)	1
SCHULKIN, Dr. Morris, Potomac, MD	1
SEA-00, 63, 63X	3
SIMON FRASER UNIV, British Columbia, Canada (Dr. Edgar Velez)	1
SONAR SURVEILLANCE GROUP, Darlinghurst, Australia	1
SOUTHEASTERN MASS. UNIV (Prof. C. H. Chen)	1
SPAWARS-00, 04, 005, PD-80 and PMW-181	5
SPERRY CORP, Great Neck, NY	1
STATE UNIV. OF NY AT STONY BROOK (Prof. M. Barkat)	1
TEL-AVIV UNIV, Tel-Aviv, Israel (Prof. E. Winstein)	1
TOYON RESEARCH CORP, Goleta, CA (M. Van Blaricum)	1
TRACOR, INC, Austin, TX (Dr. T Leih and J. Wilkinson)	2
TRW FEDERAL SYSTEMS GROUP, Fairfax, VA (R. Prager)	1
UNITED ENGINEERING CENTER, Engr. Societies Library, NY, NY	1
UNIV. OF AUCKLAND, New Zealand (Dr. Murray D. Johns)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
UNIV. OF ALBERTA, Edmonton, Alberta, CANADA (K. Yeung)	1
UNIV OF CA, San Diego, CA (Prof. C. Helstrom)	1
UNIV OF COLORADO, Boulder, CO (Prof. L. Scharf)	1
UNIV. OF CT, Storrs, CT. (Library and Prof. C. Knapp)	2
UNIV OF FLA, Gainesville, FL (D. Childers)	1
UNIV OF ILLINOIS, Urbana, IL 61801 (Dr. Douglas L. Jones)	1
UNIV OF MICHIGAN, Ann Arbor, MI (EECS Bldg. North Campus))	1
UNIV. OF MINN, Minneapolis, Mn (Prof. M. Kaveh)	1
UNIV. OF NEWCASTLE, Newcastle, NSW, Canada (Prof. A. Cantoni)	1
UNIV. OF QUEENSLAND, St. Lucia, Queensland 4067, Australia (Dr. Boualem Boashash)	1
UNIV. OF RI, Kingston, RI (Prof. G. F. Boudreaux-Bartels, Library, Prof. S. Kay, and Prof. D. Tufts)	4
UNIV. OF ROCHESTER, Rochester, NY (Prof. E. Titlebaum)	1
UNIV. OF SOUTHERN CA., LA. (Prof. William C. Lindsey, Dr. Andreas Polydoros, PHE 414)	2
UNIV. OF STRATHCLYDE, ROYAL COLLEGE, Glasgow, Scotland (Prof. T. Durran)	1
UNIV. OF TECHNOLOGY, Loughborough, Leicestershire, England (Prof. J. Griffiths)	1
UNIV. OF WASHINGTON, Seattle (Prof. D. Lytle)	1
URICK, ROBERT, Silver Springs, MD	1
US AIR FORCE, Maxwell AF Base, AL (Library)	1
VAN ASSELT, Henrik, USEA S.P.A., La Spezia, Italy	1
VILLANOVA UNIV, Villanova, PA (Prof. Moeness G. Amin)	1
WEAPONS SYSTEMS RESEARCH LAB, Adelaide, Australia	2
WERBNER, A., Medford, MA	1
WESTINGHOUSE ELEC. CORP, OCEANTIC DIV, Annapolis, MD (Dr. H. Newman and Dr. H. L. Price)	2
WESTINGHOUSE ELEC. CORP, Waltham, MA (D. Bennett)	1
WILSON JAMES H., San Clemente, CA	1
WOODS HOLE OCEANOGRAPHIC INSTITUTION (Dr. R. Spindel and Dr. E. Weinstein, Library)	3
YALE UNIV. (Library, Prof. P. Schultheiss and Prof. F. Tuteur)	3