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FINAL TECHNICAL REPORT

Material Instabilities in Solids
AFOSR-88-0200

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Abstract. The principal investigator considered a number of mathematical problems that may help to explain material failures in certain polymers and ductile metals. In particular, useful results were obtained concerning the static instability and surface cracking of a single hole and the formation of crazes in glassy polymers.

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SUMMARY

Experiments on elastomers and ductile metals by other researchers have indicated that a major failure mechanism in such materials is that of void formation and coalescence (as loads are applied small holes appear, grow larger, and combine to form cracks).

The principal investigator considered a number of mathematical problems that could help to explain the aforementioned material failures. In particular, useful results were obtained concerning: the static instability and surface cracking of a single void; and the formation of crazes in glassy polymers.

Static Instability of Voids. It was shown that a single spherical void is not the energy minimizer for a class of nonlinearly elastic materials. In particular, the formation of secondary cracks on the void surface should yield solutions of lower energy. The linking of these secondary cracks between two distinct voids may be the mechanism for void coalescence that causes final material failure. In obtaining this result a new test for the static stability of solutions for a homogeneous material was discovered. Roughly speaking this result says that in order for a solution to be stable with respect to new void formation each of the homogeneous deformations obtained by applying the deformation gradient at one point to the whole material must be stable with respect to new void formation. It is expected that this result will be useful in determining the stability of solutions that other researchers have constructed for fracture problems.

Crazing in Glassy Polymers. The introduction of long thin cylindrical voids was shown to be a viable method of reducing the energy of a material. A possible connection was made to the formation of crazes (cracks whose edges are connected by filaments of material) in polymers such as polycarbonate,

polymethyl methacrylate and (low molecular weight) polystyrene. In particular optimal energy reduction is obtained by an insertion of many voids that resemble observations of craze initiation.

Publications

1. The Formation of Filamentary Voids in Solids (with R.D. James). Submitted.
2. Remarks on $W^{1,p}$ -Quasiconvexity, Interpenetration of Matter, and Function Spaces for Elasticity (with R.D. James). Submitted.
3. Linear Deformations as Global Minimizers in Elasticity. Submitted.

Interactions.

1. "On Nonuniqueness for a Hyperbolic System: Cavitation in Nonlinear Elastodynamics." Presented to the Conference on Hyperbolic Systems of Conservation Laws held at the Mathematisches Forschungsinstitut Oberwolfach, West Germany during July 1988. Among those in attendance were H. Alber, P. Brenner, C. Dafermos, H. Engler, J. Greenberg, E. Horst, C. Klingenberg, M. Meier, B. Perthame, R. Racke, B. Scarpellini, M. Schatzman, D. Serre, M. Shearer, Y. Shibata, T. Sideris, M. Slemrod, T. Ting, A. Tzavaras, P. Vuillermot, W. von Wahl, and P. Weidemaier.
2. "The Formation of Filamentary Voids in Elastic Solids." Presented to the joint SES-ASME meeting held in Berkeley, CA during June 1988. Among those in attendance were S. Antman, P. Bauman, J. Blume, D. Carlson, C. Horgan, J. Knowles, G. MacSithigh, N. Owen, T. Pence, D. Phillips, P. Podio-Guidugli, E. Sternberg, and P. Sternberg.

3. "On Void Formation in Elastic Materials." Presented to the Department of Mathematics, University of Kentucky, during October 1988. Among those in attendance were M.F. Beatty, R. Gariepy, A. Heinricher, C.S. Man, and R. Molzon.
4. "Singular Minimizer and Singular Solutions in Elasticity." Presented to the Department of Mathematics, Virginia Tech. during November 1988. Among those in attendance were K. Hannsgen, J. Kim, M. Renardy, Y. Renardy, R. Rogers, and R. Wheeler.
5. "Surface Instabilities in Elastic Materials." Presented to the Department of Aerospace Engineering and Mechanics, University of Minnesota during February 1989. Among those in attendance were J. Ericksen, R. Fosdick, D. Kinderlehrer, R. James, P. Sethna, T. Tezduyar, I. Vardoulakis, and W. Warner.
6. "Surface Instabilities in Elastic Materials." Presented to the Center for Applied Mathematics, Purdue University, West Lafayette, IN during March 1989. Among those in attendance were P. Bauman, J. Douglis, B. Lucier, and D. Phillips.
7. "On Void Formation in Elastic Materials." Presented to the Mathematical Sciences Department, University of Bath, England during August 1989. Among those in attendance were E. Fraenkel, N. Owen, G. Parry, J. Sivaloganathan, and J. Toland.
8. "Surface Instabilities and the Complementing Condition." Presented to Euromech 252: Bifurcation Phenomena in Solids, which was held at the University of Glasgow during September 1989. Among those in attendance were J. Bielski, R. Billardon, J. Blachut, C. Calladine, A. Cimetiere, E. Duka, D. Durban, G. Galletly, J. Hancock, D. Haughton, G. Hunt, A. Kounadis, M. Koczak, J. Kruzelecki, Y. Leroy, C. Menken, K. Neale, A. Needleman, Nguyen quoc Son,

R. Ogden, N. Owen, H. Petryk, M. Potier-Ferry, A. Prinaris, D. Reynolds, J. Rhodes, J. Rotter, B. Schaeffer, M. Sewell, J. Sivaloganathan, J. Skrzypek, S. Sridharan, C. Stolz, H. Troger, and V. Tvergaard.

9. "Dynamic Void Formation in Elasticity." Presented to the ASME Winter Annual Meeting in San Francisco during December of 1989. Among those in attendance were S. Antman, R. Batra, M. Beatty, W. Feng, J. Haddow, T. Healey, C. Horgan, J. Jenkins, J. Knowles, G. MacSithigh, C. Man, J. Nohel, R. Rajagopal, D. Steigman, and A. Wineman.

Detailed Description of Results Obtained.

The Formation of Filamentary Voids in Solids

by

R.D. James & S.J. Spector

In 1958 Gent & Lindley observed a striking rupture phenomena in short rubber cylinders that were bonded at their ends to parallel steel plates and pulled in tension. At a load that was in many cases less than a fourth of the ultimate breaking load, they observed the appearance of small, approximately spherical holes in the interior of the test-piece.

An analysis of this phenomena was made in 1982 in a fundamental paper by Ball. He considered a finite ball of compressible elastic material, with no hole present initially, subject to a pure radial displacement of amount λ^{-1} at its boundary. He found that for appropriate materials and for λ sufficiently large the minimizer of the total stored energy among radial deformations contains a spherical cavity.

To our knowledge all analyses of the phenomenon of void formation in elastic materials have been concerned with the radial problem in which all deformations $f: \Omega \rightarrow \mathbb{R}^n$ that compete for a minimum are required to have the special form

$$f(x) = \frac{r(R)}{R}x, \quad R = |x|.$$

To realize the program of actually predicting the formation of voids at a stress concentration in an elastomer two questions naturally arise:

- 1) Are the radial solutions with holes minimizers of the energy when nonradial deformations are allowed to compete for a minimum?
- 2) What information from the analyses of homogeneous boundary value problems in which $f(x) = F_0 x$ on $\partial\Omega$ can be carried over to inhomogeneous deformations that happen to take on the deformation gradient F_0 at some point $x_0 \in \Omega$?

Regarding question 2 we have previously shown that if a void of any kind, not necessarily radially symmetric, reduces the energy in the boundary value problem with linear boundary conditions $f(x) = F_0 x$, $x \in \partial\Omega$, then one can find a closely related deformation that reduces the energy of an inhomogeneous deformation that assumes the deformation gradient F_0 at some interior point.

The answer to question 1 clearly depends on the stored energy function. In this paper we show that, for a large and realistic class of stored energy functions, radial solutions with holes are not minimizers of energy. Our method is based on the following ideas. Consider first the radial problem with boundary conditions $f(x) = \hat{\lambda}x$ at $|x| = 1$. The homogeneous deformation satisfying these boundary conditions $f(x) = \hat{\lambda}x$, $|x| \leq 1$, induces a large volume change when $\hat{\lambda}$ is large. A material may find it energetically unfavorable to undergo such a large change in

volume and will instead open a spherical hole at its center. That is, the typical radial deformation that creates a traction-free hole has principal stretches of the form

$$\alpha(R), \quad \lambda(R), \quad \lambda(R), \quad R = |x|,$$

with $\alpha\lambda^2$ bounded and $\lambda(R) \rightarrow +\infty$ as $R \rightarrow 0^+$. Our analysis involves the same idea carried one step further. A deformation with principal stretches of this form might also be judged energetically unfavorable because of the large change in area $\lambda(R)^2$ as $R \rightarrow 0^+$. We show that for a large class of stored energy functions the material would "prefer" to have principal stretches of the form

$$\alpha, \quad \lambda_1, \quad \lambda_2$$

with $\lambda_1\lambda_2 < \lambda^2(R)$. In order to realize a competitor with these principal stretches, we construct, just outside of the hole, a short but very thin filamentary void, which points in the radial direction.

The filamentary void provides another mechanism for energy reduction, different from cavitation. A striking example of a rupture process that produces something resembling a filamentary void is the phenomenon of crazing in glassy polymers. A weakness of our results, which prevents a quantitative comparison, is that we do not give critical conditions for the formation of a filamentary void, but we only say that if certain combinations of principal stretches are sufficiently large, then a filamentary void will reduce the energy. Furthermore, all polymers are to some extent viscoelastic, as is true of the elastomers studied by Gent & Lindley, and our thinking relies on the notion that most viscoelasticity theories have a Lyapunov functional which has the form of a nonlinear elastic energy.

Remarks on $W^{1,p}$ -Quasiconvexity, Interpenetration of Matter,
and Function Spaces for Elasticity

by

R.D. James & S.J. Spector

The aim of this paper is two-fold. First, we point out certain difficulties that arise when spaces of the type

$$\{f \in W^{1,p}(\Omega, \mathbb{R}^n) \cap L^{\infty}(\Omega, \mathbb{R}^n) : \det \nabla f > 0 \text{ a.e.}, \quad 1 \leq p < n\}$$

are used as the basic function spaces for the theory of nonlinear elasticity. These difficulties are illustrated by several examples which show that in such spaces and with mild restrictions on the energy function, a ball of material under severe compressive boundary conditions of the form

$$f(x) = \lambda x, \quad |x| = 1, \quad \lambda \ll 1,$$

will not suffer an expected contraction, $f(x) = \lambda x$, $|x| < 1$, but rather will reduce its energy by interpenetrating matter, that is, by failing to be one-to-one. These examples are foreshadowed by a theorem of Ball & Murat to the effect that for $1 \leq p < n$, $W(\nabla f) = \delta(\det \nabla f)$ is $W^{1,p}$ -quasiconvex at every $\nabla f \in \mathbb{R}^{n^2}$ if and only if δ is constant.

The second purpose of this paper is to prove (using elementary methods) a new $W^{1,p}$ -quasiconvexity theorem. The form of this theorem is ideally suited to analyses of the formation of voids in nonlinear elastic materials. We use this theorem in a forthcoming paper to show that under physically reasonable hypotheses on the energy function, some of the radial solutions found in the literature for the formation of spherical voids are in fact unstable relative to the formation of filamentary voids.

Linear Deformations as Global Minimizers in Nonlinear Elasticity

by

Scott J. Spector

In this paper we give constitutive hypotheses under which the linear deformations $f_0(x) := F_0 x$ are minimizers of the total energy among those deformations that are one-to-one and have the same boundary-values and orientation as f_0 . In particular we assume that

$$W(F) = g(F, \text{adj } F) + h(\det F),$$

where g and h are convex and $h'(H) = 0$, and show that

$$\int_B W(F_0 + \nabla \tilde{u}(x)) dx \geq \int_B W(F_0) dx \quad (1)$$

whenever $0 < \det F_0 \leq H$ and $\tilde{u} \in W_0^{1,2}(B)$ is such that the deformation $f_0 + \tilde{u}$ is one-to-one and satisfies the containment constraint $(f_0 + \tilde{u})(B) \subset f_0(B)$. Thus such materials will not exhibit any material instabilities in compression, but can exhibit cavitation or other such instabilities in tension.

Results of Ball & Murat and James & Spector show that the injectivity hypothesis is crucial to our result. If the growth of g is slow enough to permit cavitation then Ball & Murat have shown that h must be constant in order for (1) to be satisfied for all $\tilde{u} \in W_0^{1,2}(B)$ and all matrices F_0 , while James & Spector have shown that if instead $h(\det F) \rightarrow +\infty$ when $\det F \rightarrow 0^+$, as is expected from the physics, then (1) will not be satisfied, for all $\tilde{u} \in W_0^{1,1}(B)$, at any F_0 with sufficiently small determinant. The failure of (1) is due to the ability of the body to relieve severe compressive strains and lower the total energy by overlapping material.

In this paper we also investigate the containment constraint $f(B) \subset d(B)$. We note that each equivalence class $\tilde{f} \in W^{1,1}(B)$ has a representative $f \in \tilde{f}$ that is absolutely continuous on almost every line segment parallel to the coordinate axes. We show that a one-to-one function that satisfies such a continuity property and assumes the same boundary values as a continuous one-to-one function d will either satisfy $f(B-N) \subset d(B)$ or $f(B-N) \subset \mathbb{R}^3 - d(\bar{B})$ for some N with measure zero. Thus such a function must assume one of only two possible orientations.