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MULTIWAVE DIFFRACTION ANALYSIS OF
TRANSMISSION PHASE GRATINGS

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ROYAL SIGNALS AND RADAR ESTABLISHMENT

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Title: MULTIWAVE DIFFRACTION ANALYSIS OF TRANSMISSION PHASE GRATINGS
 Author: C W Slinger
 Date: September 1989

SUMMARY

A multiwave coupled wave analysis of phase modulated, transmission gratings is made. The aim is to calculate the first order diffraction efficiency as a function of the modulation and a volume parameter, for arbitrary phase profiles. Sinusoidal, square, triangular and sawtooth gratings are investigated in detail through the thin, multiwave and volume diffraction regimes, using analytic and numerical techniques. The relative merits of these profiles are discussed in terms of efficiency and ease of fabrication. High efficiencies (>90%) are found to be possible in the multiwave regime.



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MULTIWAVE DIFFRACTION ANALYSIS OF TRANSMISSION PHASE GRATINGS

C W Slinger

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- 3 RESULTS
- 4 CONCLUSIONS

1 INTRODUCTION

Grating structures are widely exploited in many technological disciplines. They find uses in diffractive optical elements, integrated optics, acousto optics, optical processing and computing, holography and other areas involving manipulation of electromagnetic radiation. A knowledge of the diffraction characteristics of gratings is essential, in order that they may be used effectively.

In this paper, the behaviour of lossless, planar (the grating is uniform throughout), phase modulated (diffraction is caused by a periodic variation in optical path length), unslanted (the grating fringes are normal to the boundaries), transmission gratings is analysed over a range of operating conditions. This, somewhat idealised, class of gratings is nevertheless a reasonable approximation in many practical situations. A prime aim of the analysis is the investigation of diffraction efficiency - in particular, the variation of efficiency with the amplitude and profile of the phase modulation, and with the degree of volume character possessed by the grating. The volume characteristics of a grating are often important in governing its application, as this property determines the number of diffraction orders present when the grating is illuminated. A volume grating can be defined as one in which, on replay, a maximum of only one diffraction order, in addition to the zero order (incident wave), is present. Moharam et al [1] discuss several criteria defining volume behaviour. The other extreme of grating behaviour - the thin grating regime - is characterised by a multitude of diffraction orders being significant.

The performance of gratings operating in the various diffraction regimes have been studies by several authors. Thin (Raman-Nath) grating behaviour [e.g 2,3] and volume behaviour [e.g 4,5] have been extensively covered, particularly for the case of sinusoidally modulated gratings. Grating behaviour between the thin and volume regimes - the multiwave regime - has been considered to a lesser extent [6 - 8], although the theoretical tools to do so are well developed [e.g 9,10]. In particular, multiwave coupled wave theory provides a powerful method for investigating diffraction in any of the three regimes [11,12]. The theory is intuitively pleasing and is the approach adopted here.

The general coupled wave equations will be derived in terms of dimensionless parameters. They will be applied to several different phase modulation profiles and the analytic solutions in the limiting cases of thin and volume behaviour discussed. Detailed numerical solutions of the equations over a range of grating parameters will then be

presented and some general conclusions drawn.

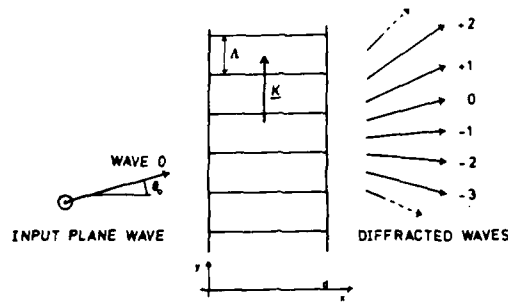


Figure 1 The grating system to be analysed

2 THEORETICAL ANALYSIS

Figure 1 depicts the grating system to be analysed. For simplicity, the grating is assumed to be purely phase modulated and lossless, although it is straightforward to include losses and absorption modulation. The grating's characteristics will generally vary as a function of y , but it is assumed to be locally plane. This is valid if the grating fringes (characterised by the grating vector) change little with x [13]. The periodic phase variation in the grating - its modulation profile - can be written as:

$$\epsilon(\underline{r}) = \epsilon'_0 + \sum_{i=1}^{\infty} \epsilon'_i \cos(i\mathbf{K} \cdot \underline{r} + \psi_i)$$

where $\underline{r}$ is a position vector

ϵ'_0 is the bulk relative permittivity (dimensionless) of the slab containing the grating,

ϵ'_i is the amplitude of the i th harmonic of the phase profile

$\mathbf{K}$ is the grating vector

with $|\mathbf{K}| = 2\pi/\Lambda$

where Λ is the grating period

ψ_i is a constant characterising the relative phases of the modulation terms.

A monochromatic plane wave, free space wavelength λ and propagating at an angle θ_0 , is incident upon the grating. The wave is polarised perpendicular to the x - y plane (H mode), although it is relatively simple to treat waves of arbitrary polarisation [11]. Inside the grating, the diffracted waves are assumed to be such that the electric field is

given by:

$$E = \sum_{m=-\infty}^{+\infty} A_m \exp(-jk_m z) \quad (2)$$

where A_m is the complex amplitude of the m th diffraction order having wave vector k_m . The relationship between the diffracted waves is taken in the 'k-vector closure' form:

$$k_m = k_0 + m K \quad (3)$$

H mode polarisation means that scalar wave equation can be used:

$$\nabla^2 E + \beta^2 (\epsilon/\epsilon_0) E = 0 \quad (4)$$

where

$$\beta = \frac{2\pi/\epsilon_0}{\lambda}$$

By combining equations (1) to (4), and equating coefficients of $\exp(-jk_m z)$, a set of differential equations is obtained:

For $m = -\infty, \dots, -1, 0, +1, \dots, +\infty$:

$$\begin{aligned} \frac{-\kappa_1}{\cos^2 \theta_0} \frac{1}{2j\beta} \frac{d^2 A_m}{dz^2} + \frac{dA_m}{dz} - jm\Omega(m+P)A_m \\ + j \sum_{i=1}^{\infty} \frac{\kappa_i}{\kappa_1} \left\{ \exp(j\psi_i) A_{m+i} + \exp(-j\psi_i) A_{m-i} \right\} = 0 \end{aligned} \quad (5)$$

where A_m is the amplitude of the m th diffraction order

$$\kappa_i = \frac{\beta \epsilon_i}{4 \epsilon_0}$$

$\xi = \kappa_1 x / \cos(\theta_0)$ is a modulation parameter,

$\Omega = K^2 / (2\beta\kappa_1)$ is a volume parameter,

$P = \sin(\theta_0) 2\beta/K$ is a bragg parameter.

This set of coupled wave equations is exact under the conditions specified. With appropriate matching at the planar grating boundaries (tangential electrical and magnetic fields), they accurately describe diffraction in gratings of arbitrary phase modulation. Variation of the phase modulation with depth can be accommodated by slicing the grating into thin sections - this technique is used for the majority of surface relief structures [14].

In the case of low modulation, such that $\epsilon'_i \ll \epsilon'_0$, a further simplification can be made by neglecting second derivatives. Generally speaking, this approximation is valid for $\epsilon'_i/\epsilon'_0 < \sim 0.2$, although the accuracy varies with the degree of volume nature of the grating [12]. The vast majority of bulk gratings satisfy this criterion, but nearly all surface relief do not. With this approximation, the equations reduce to the set, for $m = -\infty, \dots, -1, 0, +1, \dots, +\infty$:

$$\frac{dA_m}{d\xi} - jm\Omega(m + P)A_m +$$

$$j \sum_{i=1}^{\infty} \frac{\kappa_i}{\kappa_1} \left\{ \exp(jv_i) A_{m+i} + \exp(-jv_i) A_{m-i} \right\} = 0 \quad (6)$$

The grating boundary conditions simplify to become:

$$A_m(x = 0) = 0 \quad (m \neq 0)$$

$$\text{and } A_0(x = 0) = A_{00} \quad (7)$$

No backward diffracted waves are now possible. The coupled wave equations (6) obey power conservation and can be interpreted in a simple manner. The m th diffraction order is coupled to the $m+i$ and $m-i$ orders by the coupling coefficient κ_i . The coefficients of the central term are a measure of the mismatch in phase velocities of the diffracted orders.

For significant power transfer from one mode to the other, three conditions must be met [5]. Firstly there must be a coupling path between the two orders (this need not be a direct path [15,16]). Also the length of the interaction region must be correctly chosen. Finally, their phase velocities must be approximately equal. In equations (6), ξ governs the length of the interaction region. Ω is a measure of the dephasing between the relative orders [17]. This parameter has prime importance in governing the diffraction regime within which the grating will operate [18]. Phariseau [19] showed that the sum of the diffraction efficiencies of the higher orders is less than, or approximately equal to $1/\Omega^2$. Note that the Ω term is a function of the coupling constant κ_1 , but does not include the physical thickness of the grating. Thus gratings can be only one or two wavelengths of light thick, yet still exhibit true volume diffraction, a point misunderstood by many researchers. The form of the Ω parameter's independence of physical thickness is in contrast with the parameter $Q = K^2d/\beta$, used by many early workers as a diffraction regime criterion.

Diffraction Regimes

In general, the solution of the infinite set of equations (6) requires some truncation to a finite set of diffraction orders and numerical techniques. However, there are two limiting cases for which analytic solutions are possible. These are at the opposite extremes of transmission grating behaviour, and occur when the volume parameter Ω takes vanishingly small or very large values. The former case is referred to as thin grating behaviour (often termed Raman Nath Diffraction after Raman and Nath's thin, sinusoidal grating analysis [2]); the latter as volume (or thick) diffraction, and was treated, for the sinusoidal case, by Kogelnik [4].

As Ω tends to very small values, the dephasing term for each diffraction order becomes small. Large numbers of orders can have significant power in them. Equations (6) become, for $m = -\infty, \dots, -1, 0, +1, \dots, +\infty$:

$$\frac{dA_m}{dz} + j \sum_{i=1}^{\infty} \frac{\kappa_i}{\kappa_1} \left\{ \exp(j\psi_i) A_{m+i} + \exp(-j\psi_i) A_{m-i} \right\} = 0 \quad (8)$$

Analytic solutions to this equation exist for a variety of profiles (e.g [3]). Identical solutions can be obtained by other techniques such as the optical path method [20], and the classical transmittance methods. The thin grating regime is found, typically, for $\Omega < 0.01$. As Ω increases, equations (8) become less accurate, due to the dephasing terms in (6) becoming significant.

At large values of Ω , Bragg effects become dominant. Diffraction orders other than the one on-Bragg (i.e such that $P + m = 0$) have such a large mismatch in phase velocities that very little power is coupled into them. In the limiting case then, for replay of the grating in the vicinity of the $m = i$ on-Bragg condition, the infinite set of coupled wave equations (6) reduce to only two in number - provided 'i' is a harmonic of the grating profile:

$$\frac{dA_0}{dz} + j \frac{\kappa_i}{\kappa_1} A_i = 0 \quad (9)$$

$$\frac{dA_i}{dz} - j i \Omega (i + P) A_i + j \frac{\kappa_i}{\kappa_1} A_0 = 0 \quad (10)$$

Analytic solutions are possible. When fully on the i th Bragg condition, these reduce to:

$$\eta_0 = \cos^2(\xi \kappa_i / \kappa_1) \quad (11)$$

and

$$\eta_i = \sin^2(\xi \kappa_i / \kappa_1) \quad (12)$$

For the $i = 1$ case, the solutions of (9) and (10) agree with Kogelnik's analysis [4]. For large enough Ω then, 100% conversion into the i th order is possible. This occurs when replay is on-Bragg for the i th harmonic of the grating at:

$$\xi = (2n + 1)\pi \kappa_1 / 2\kappa_i \quad (n = 0, 1, 2, \dots) \quad (13)$$

The high efficiencies of these volume, phase gratings mean that they have many applications. The above results show that sinusoidal modulation is not a necessary requirement.

3 RESULTS

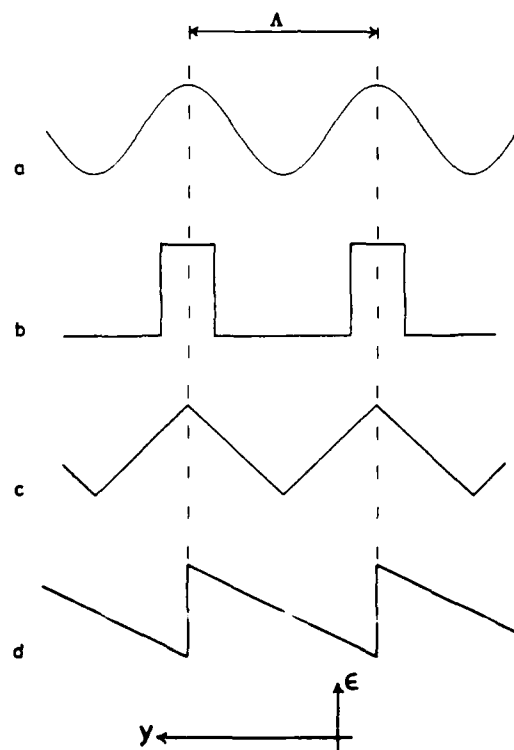


Figure 2 The sinusoidal, rectangular, triangular and sawtooth modulation profiles

Defining the modulation profile as the permittivity variation as a function of y , the gratings with the following profiles (figure 2) were investigated:

- a Sinusoidal
- b Rectangular
- c Triangular
- d Sawtooth

The sinusoidal profile is of much practical importance, as holographic (interferometric) recording techniques generally give rise to such a grating, at least to a first approximation. The modulation is given by:

$$\epsilon'_o = (\epsilon'_{\max} + \epsilon'_{\min})/2 \quad (14)$$

$$\epsilon'_i = \Delta\epsilon/2 \quad (15)$$

$$\psi'_i = 0 \quad (16)$$

where $\epsilon'_{\min}$ is the minimum and $\epsilon'_{\max}$ is the maximum relative dielectric constant of the grating profile and

$$\Delta\epsilon = \epsilon'_{\max} - \epsilon'_{\min}$$

The thin grating efficiency is given by the well known formula, as derived by Raman and Nath [2]:

$$\eta_m = \frac{|A_m|^2}{|A_{00}|^2} = J_m^2(2\zeta)$$

where $J_i(x)$ is a Bessel function of the first kind, of order i . A maximum of 33.9% occurs at $m = \pm 1$, $\zeta = 1.84$.

The other three grating profiles are usually obtained by exposure of a recording medium through a mask. This is a commonly used method of grating fabrication, particularly for computer generated holograms [21] and infra-red diffractive structures [22]. Binary masks are the simplest, and result in rectangular gratings. The modulation in such a case is given by:

$$\epsilon'_o = \epsilon'_{\min} + \Delta\epsilon \mu \quad (17)$$

$$\epsilon'_i = (2/i\pi)\Delta\epsilon \sin(i\mu\pi) \quad (19)$$

$$\psi'_i = 0 \quad (20)$$

μ is a fill (mark/space) parameter. A value of 0.5 corresponds to a square wave grating.

The thin rectangular grating diffraction efficiency is given by:

$$\eta_0 = 1 - 4(\mu - \mu^2) \cdot \sin^2 \left\{ \pi \zeta / \left[2 \left(1 - \cos(2\pi\mu) \right) \right]^{\frac{1}{2}} \right\} \quad (21)$$

$$\eta_m = \left\{ 2 / (m\pi)^2 \right\} \cdot \left\{ 1 - \cos(2\pi m\mu) \right\} \cdot \sin^2 \left\{ \pi \zeta / \left[2 \left(1 - \cos(2\pi\mu) \right) \right]^{\frac{1}{2}} \right\} \quad (22)$$

$$(m \neq 0)$$

Maximum efficiency for a square wave grating is 40.5% for the ± 1 orders at $\zeta = 1$. For a given material modulation then, a thin regime square wave grating gives higher efficiency and requires less physical thickness than an equivalent sinusoidal grating.

Grey scale masks are capable of producing many modulation profiles. The triangular profile is characterised by:

$$\epsilon'_0 = (\epsilon'_{\max} + \epsilon'_{\min})/2 \quad (23)$$

$$\epsilon'_i = \Delta \epsilon \left\{ 2 / (i\pi) \right\}^2 \quad (i \text{ odd}) \quad (24)$$

$$\epsilon'_i = 0 \quad (i \text{ even}) \quad (25)$$

$$\psi_i = 0 \quad (26)$$

The thin, triangular grating efficiency is given by:

$$\eta_m = \left\{ \zeta / \left[(\pi \zeta / 2)^2 - m^2 \right] \right\}^2 \cdot \sin^2(\pi^2 \zeta / 4) \quad (m \text{ even}) \quad (27)$$

$$\eta_m = \left\{ \zeta / \left[(\pi \zeta / 2)^2 - m^2 \right] \right\}^2 \cdot \cos^2(\pi^2 \zeta / 4) \quad (m \text{ odd}) \quad (28)$$

A maximum of 29.8% occurs for $m = \pm 1$ at $\zeta = 0.87$.

Finally, the sawtooth profile is described by:

$$\epsilon'_0 = (\epsilon'_{\max} + \epsilon'_{\min})/2 \quad (29)$$

$$\epsilon_i' = \Delta\epsilon/(i\pi) \quad (30)$$

$$\psi_i = \pi/2 \quad (31)$$

The thin sawtooth grating efficiency is:

$$\eta_m = \frac{\sin^2(\pi\zeta)}{\{\pi(\zeta - m)\}^2} \quad (32)$$

Perhaps surprisingly, a maximum efficiency of 100% is achievable. For example, this occurs at $m = +1$, $\zeta = 1$. This overturns the notion that thin gratings are necessarily inefficient. A simple qualitative explanation for the high performance of the sawtooth profile can be given [20]: 100% efficiency may be achieved for a particular order when the direction of that order coincides with the 'blazing'.

Numerical Results

The coupled wave equations (6), subject to the boundary conditions (7) were solved for the sinusoidal, square, triangular and sawtooth gratings, using a Runge Kutta technique [23]. In each case, the first order diffraction efficiency η_1 , for $m = 1$ on-Bragg replay, was calculated. In the sinusoidal, rectangular and triangular gratings ($\psi_i = 0$), η_{+1} around $m = +1$ is equal to η_{-1} around $m = -1$, due to symmetry considerations. However, in the case of the sawtooth grating, this is not so. η_{-1} at $m = -1$ was found to give comparatively poor performance, and is not shown here.

A sufficient number of diffraction orders were taken to give at least 1% accuracy. Typically, for the multiwave and thin regimes, this required orders up to ± 20 and ± 40 respectively. The results are shown in figures 3 to 6. Both contour and perspective plots are depicted to aid interpretation. Computation time was of the order of six hours per plot (running on a VAX 8650).

Several observations can be made from the plots:

- i) For low values of Ω ($\Omega \rightarrow 0.01$), in all cases, the results agree closely with the appropriate analytic expressions (equations 17, 21, 22, 27, 28, 32) for thin grating behaviour. This confirms that the Ω parameter is a useful guide to the onset of the thin diffraction regime, whatever the modulation profile.
- ii) At high values of Ω ($\Omega \rightarrow >10$), the diffraction efficiencies all converge to the same values, irrespective of grating profile. These values are those given by equations (11) and (12), for the $i=1$ case. This seems a reasonable result, and can be explained heuristically by noting that the diffraction orders, other than the $+1$, are so far off Bragg that there is no power coupled into them. Thus the behaviour of all grating profiles converge to the fundamental sinusoid under these conditions.

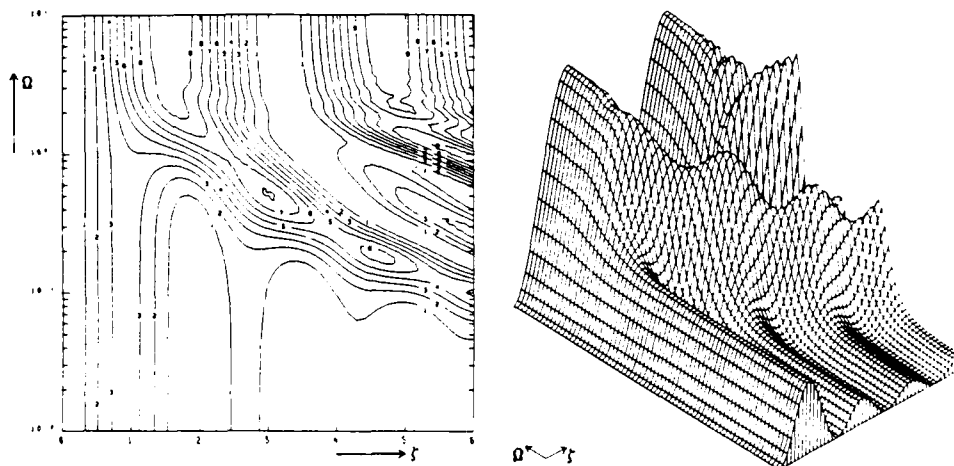


Figure 3 Sinusoidal grating. +1 diffraction efficiency, for $m = +1$ on-Bragg replay, as a function of the modulation parameter ξ and the volume parameter Ω . Note the logarithmic Ω scale. In the contour plot, contour 1 = 10% efficiency, 2 = 20% etc.

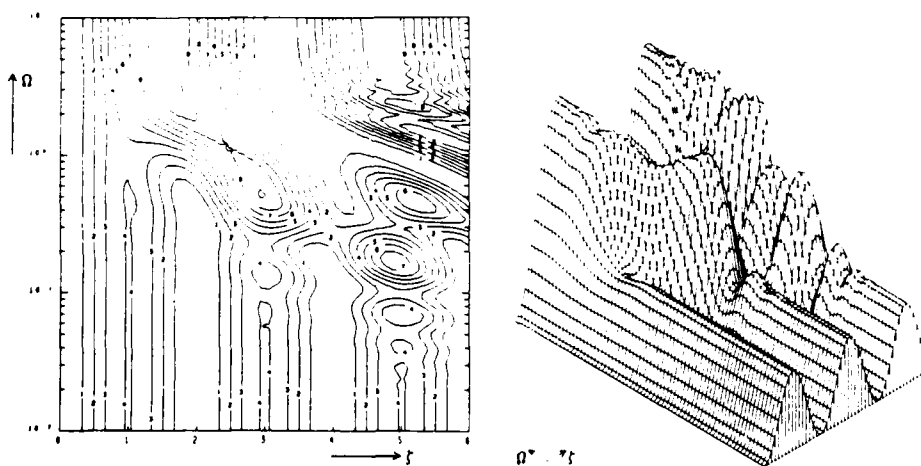


Figure 4 Square wave grating. Otherwise, as in figure 3.

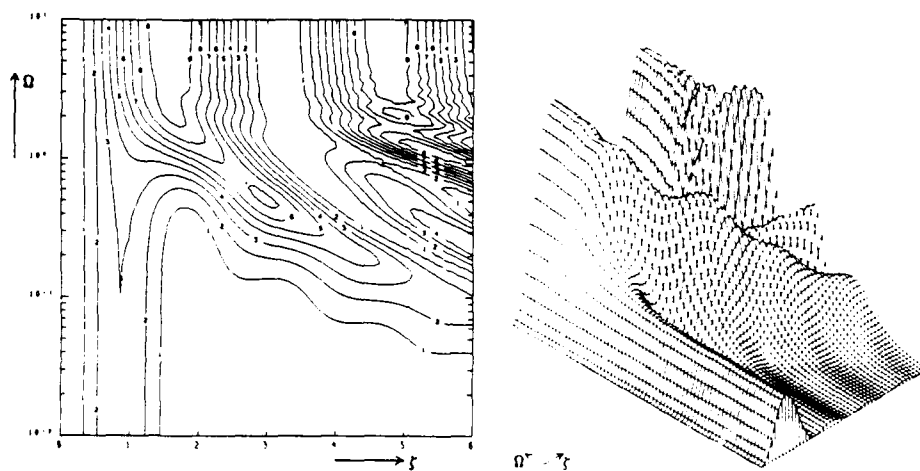


Figure 5 Triangular grating. Otherwise, as in figure 3.

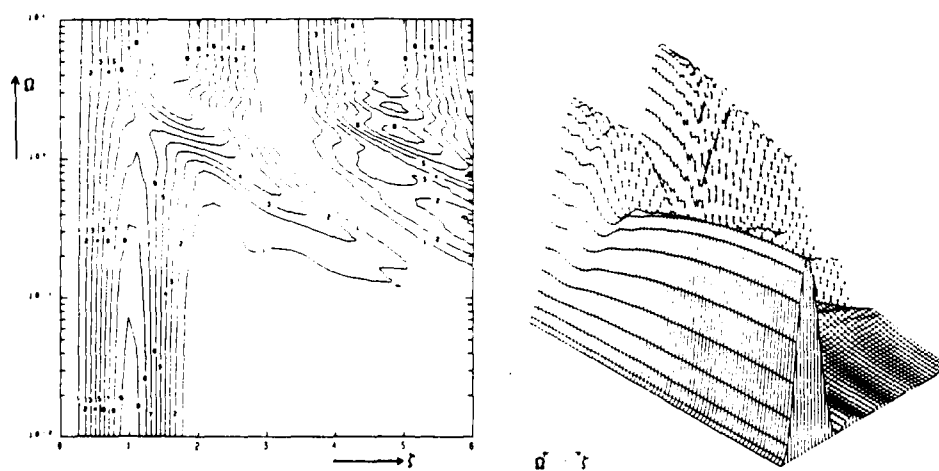


Figure 6 Sawtooth grating. Otherwise, as in figure 3

- iii) High efficiencies occur in several places in the multiwave regime - for example, over 81% at $(\xi, \Omega) = (3.0, 0.51)$ for the sinusoidal grating, over 91% at $(2.97, 0.5)$ for the square wave case and over 70% at $(3.0, 0.65)$ for the triangular profile. For rectangular gratings, the square wave ($\mu = 0.5$) is not

optimum in terms of multiwave efficiency [24]. A rectangular grating with $\mu = 0.4$ has over 97% efficiency at $(\xi, \Omega) = (1.46, 3.3)$, for example. Such regions may be important in some practical devices in that gratings can be fabricated which are not volume, yet are of high efficiency. This enables a much wider choice in grating thickness, modulation and grating vector. The lower the value of Ω required, for example, the less off-axis a transmissive diffractive element would have to be. In other situations, it may be difficult to make a volume grating, due to material considerations [e.g. 25]. Alternatively, it may be that the resolution (grating frequency) achievable by the mask system, or the recording material itself, is limited. Therefore operation in the multiwave regime at a high efficiency point can be beneficial.

Another, potentially valuable, advantage of operating in the multiwave regime may be in computer generated hologram (CGH) manufacture. Generally speaking, current technology for writing CGHs is limited by the resolution of the writing device. This means that non-volume gratings are invariably produced (although at longer wavelengths than visible, this need not be so). High efficiency CGHs can then be made holographically, by recording a volume grating copy from the master CGH. However, by operating in the multiwave regime, at one of the efficiency maxima, high efficiency can be achieved with a grating of relatively low spatial frequency. For example, a square wave grating operating at $(\xi, \Omega) = (2.97, 0.5)$ could be 91% efficient, yet has a grating period of ~ 5 times larger than the equivalent volume device. Thus, it may be possible for high efficiency computer generated holograms to be fabricated directly without the time consuming holographic copying step.

However, it must be borne in mind that whilst high efficiencies can be achieved for on-Bragg replay in the multiwave regime, off-Bragg replay of such gratings can produce several diffraction orders of significant amplitude. Figure 7 shows a typical example of replay of the square wave grating, with

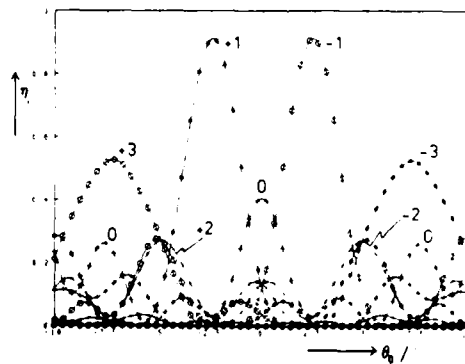


Figure 7 Replay of a square wave grating in the multiwave regime, around $(\xi, \Omega) = (2.97, 0.5)$. Variation of diffracted intensities in the significant orders as a function of the replay angle $-10^\circ < \theta < +10^\circ$. Other grating parameters: on-Bragg angles ($m = \pm 1$) = $\pm 2.41^\circ$;

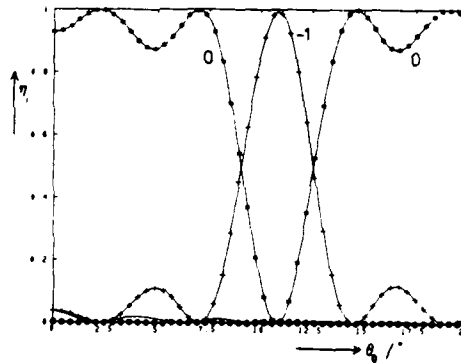


Figure 8 Replay of a square wave grating in the volume regime, around $(\xi, \Omega) = (\pi/2, 10.0)$. Variation of diffracted intensities of significant orders as a function of replay angle $0^\circ < \theta < +20^\circ$. On-Bragg angles $= \pm 10.8^\circ$, otherwise as in figure 7.

$(\xi, \Omega) = (2.97, 0.5)$, both on and off the Bragg angle. These results were obtained by solving equations (6) and varying the parameter P , and model the sort of situation that might be found in a practical application. Several diffraction orders possess significant power when replay is off the $m = \pm 1$ Bragg condition. This off-Bragg, multiwave diffraction, should be compared with the equivalent volume case, shown in figure 8, with $(\xi, \Omega) = (\pi/2, 10)$. Only the two orders (the zero, and in this case, the -1) are present. In situations where multiple higher orders could prove to be deleterious to device performance and significant off-Bragg replay is anticipated then, operation of gratings in the multiwave regime must be treated with caution.

- (iv) In terms of the maxima in the diffraction efficiency, the square wave grating is equal or superior to the sinusoidal case. This is true in the thin, multiwave and volume regimes. Thus square wave modulation should not be regarded as a limitation in a bulk modulated recording medium. As already mentioned, the sawtooth grating is remarkable in its high efficiency in the thin regime. If such modulation profiles could be accurately generated, 100% thin grating diffractive elements could be made. In comparison to the other grating profiles, however, the triangular grating seems to have little to recommend it, having inferior efficiency, in all regimes.

4 CONCLUSIONS

An analysis of bulk, unslanted, lossless, phase-modulated transmission gratings has been carried out. The performance of sinusoidal, square, triangular and sawtooth profiles have been investigated in the thin, multiwave and volume diffraction regimes and the variation of first order diffraction efficiency with modulation and volume parameters presented. The use of these dimensionless parameters should enable the results to be applied easily to most situations and most bulk phase recording media.

The results show that the efficiency achievable with the square wave grating compares favourably with the sinusoidal profile. It was confirmed that the sawtooth grating shows particular merit in the thin regime. In the multiwave regime, several potentially useful operating points exist, some with efficiencies greater than 90%. These may be utilised in certain applications where volume operation is not possible or is undesirable. In the volume regime 100% efficiency is achievable, irrespective of the grating profile.

It is hoped that the results will prove useful in the many areas of optics where gratings are exploited.

REFERENCES

- 1 M. G. Moharam, T. K. Gaylord, R. Magnusson. "Criteria for Bragg regime diffraction by phase gratings" *Opt. Comms.* 32 No. 1, p.14-8 (1980).
- 2 C. V. Raman, N. S. Nagendra Nath "The diffraction of light by high frequency sound waves" Parts I, II, III, IV, V. *Proc. Ind. Acad. Sci.* 2, p. 406-12, 413-20, 3 75-84, 119-25, 459-65 (1935-6).
- 3 R. Magnusson, T. K. Gaylord. "Diffraction efficiencies of thin phase gratings with arbitrary grating shapes". *J. Opt. Soc. Am.* 68 No. 6, p.806-9 (1978).
- 4 H. Kogelnik. "Coupled wave theory for thick hologram gratings". *Bell Syst. Tech. J.* 48 No. 9, p.2909-47 (1969).
- 5 L. Solymar, D. J. Cooke. "Volume holography and volume gratings" Academic Press, London (1981).
- 6 R. Magnusson, T. K. Gaylord. "Diffraction regimes of transmission gratings" *J. Opt. Soc. Am.* 78 No. 6, p.809-14 (1978).
- 7 B. Benlarbi, D. J. Cooke, L. Solymar. "Higher order modes in thick phase gratings" *Optica Acta* 27, p.885-95 (1980).
- 8 E. Guibelalde, M. L. Calvo. "Holographic gratings in the transition regime", *Optics & Laser Tech.* 20 No. 3, p.156-160 (1988).
- 9 P. St. J. Russel. "Optical volume holography" *Phys. Rep.* 71, p.209-312 (1981).
- 10 T. K. Gaylord, M. G. Moharam. "Analysis and applications of optical diffraction by gratings". *Proc. IEEE* 73 No.5, p.894-937 (1985).
- 11 R. Magnusson, T. K. Gaylord. "Analysis of multiwave diffraction of thick gratings". *J. Opt. Soc. Am.* 67, p.1165-1170 (1977).
- 12 M. G. Moharam, T. K. Gaylord. "Rigorous coupled-wave analysis of planar-grating diffraction" *J. Opt. Soc. Am.* 71, p.811-8 (1981).
- 13 R. R. A. Syms, L. Solymar. "Localised one-dimensional theory for volume holograms" *Opt. & Quant. Electron.* 13, p.415-9 (1981).
- 14 M. G. Moharam, T. K. Gaylord. "Diffraction analysis of dielectric surface relief gratings" *J. Opt. Soc. Am.* 72, p.1385-92 (1982).
- 15 J. W. Lewis, L. Solymar. "Spurious waves in thick phase gratings", *Opt. Comm.* 47 No.1, p.23-6 (1984).

- 16 C. W. Slinger, L. Solymar. "Grating interactions in holograms recorded with two object waves" Appl. Opt. 25 p.3283-7 (1986).
- 17 N. S. Nagendra Nath. "The diffraction of light by supersonic waves", Proc. Ind. Acad. Sci. 8 p.499-503 (1938).
- 18 M. G. Moharam, L. Young. "Criterion for Bragg and Raman-Nath diffraction regimes" Appl. Opt. 17 No. 11, p.1757-9 (1978).
- 19 P. Phariseau. "On the diffraction of light by progressive supersonic waves" Proc. Ind. Acad. Sci. 44A, p.165-70 (1956).
- 20 R. R. A. Syms, L. Solymar. "A comparison of the optical path and differential equation method for optically thin phase gratings". Opt. & Quant. Electron. 12, p.383-91 (1980).
- 21 G. Tricoles. "Computer generated holograms: an historical review" Appl. Opt. 26 No. 20, p.4351-60 (1987).
- 22 A. Zakery, C. W. Slinger, P. J. S. Ewen, A. P. Firth, A. E. Owen "Chalcogenide gratings produced by the metal photodissolution effect" J. Physics D:Applied Physics 21 No.108 p.S78-S81 (1988).
- 23 NAG routine D02BAF. The Numerical Algorithms Group Limited, Mayfield House, 256 Banbury Rd., Oxford, UK.
- 24 C. W. Slinger. "Performance predictions for bulk chalcogenide gratings produced by the photodissolution effect". RSRE Memorandum 4121. Royal Signals and Radar Establishment, Great Malvern, Worcestershire, UK.
- 25 C. W. Slinger, A. Zakery, P. J. S. Ewen, A. P. Firth, A. E. Owen. "Photodoped chalcogenides as a far infra-red holographic medium" Paper Tu18, 1988 Annual Meeting of the Optical Society of America, Santa Clara, California USA. To be published.

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Abstract A multiwave coupled wave analysis of phase modulated, transmission gratings is made. The aim is to calculate the first order diffraction efficiency as a function of the modulation and a volume parameter, for arbitrary phase profiles. Sinusoidal, square, triangular and sawtooth gratings are investigated in detail through the thin, multiwave and volume diffraction regimes, using analytic and numerical techniques. The relative merits of these profiles are discussed in terms of efficiency and ease of fabrication. High efficiencies (>90%) are found to be possible in the multiwave regime.				