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Robust Adaptive Control: Stability and Asymptotic Performance

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Robust Adaptive Control: Stability and Asymptotic Performance

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Abstract

Systems containing both compact real parametric uncertainty and frequency-weighted bounded operator uncertainty are addressed. It is shown that any parameter adaptive control system is robustly stable provided only that: (1) the unknown parameters lie in a known compact convex set, (2) the control design rule is lipschitzian, (3) the control design rule would produce a robust controller if given perfect parameter information, and (4) a specified robust parameter estimation algorithm is applied in lieu of perfect parameter information. It is also shown that the asymptotic robust performance level may be made arbitrarily close to that of the nonadaptive design which would result from perfect parameter information.

I. Introduction

Most adaptive control theory begins with a plant description containing uncertain real parameters. However, it is generally recognized that a specific parametric plant description will never exactly describe a physical system's response, regardless of the choice of the parameters. As a consequence, the issue of nonparametric dynamical uncertainty in addition to parametric uncertainty has received a great deal of attention from the adaptive control community. A number of robust adaptive control results have already been obtained (e.g., [13], [15], [14], [4], [6]). The community is currently seeking to expand the definition of robustness and the collection of design techniques.

This paper focuses on a notion of robustness which is common in robust (nonadaptive) control theory and practice. Specifically, we address stability and performance in the presence of two types of uncertainty: parametric uncertainty characterized by an *a priori* known convex membership set, and nonparametric uncertainty as characterized by an *a priori* known frequency domain magnitude bound. Frequency weighted uncertainty bounds have been used to characterize modeling errors since at least the appearance of [2], and remain popular in sensitivity and robustness analyses (e.g., [19], [3], [20], [21], [17]). We adopt the not-uncommon approach of absorbing the weighting function describing the frequency domain magnitude bound into the representation of the known portion of the plant. The residual normalized uncertainty is characterized by the operator norm induced by taking L^2 or fading-memory L^2 norms on the input and output of the uncertain operator. Since the H^∞ norm of an LTI (linear time-invariant) transfer function is its induced L^2 norm, the uncertainty set we treat covers the weighted ball-in-H^∞ with the same weighting function.

There are three themes to our work: perfect parameter information tuned-system robustness, robust parameter estimation, and the interplay of these in the overall system. Of course, the ultimate requirement is that the overall system satisfy robust performance objectives. A reasonable prerequisite is the solution of the subproblem of robust control given perfect parameter information and the subproblem of robust parameter estimation given no control objectives.

In earlier work we provided the solution to these subproblems. The robust control subproblem involves analysis of a set of {plant, controller} pairs, where the set is indexed by the value of the plant parameter vector, and the paired controller is determined through the on-line design rule. In [7] and [10] the robust control subproblem is defined in detail, and a model reference example is analyzed through the use of structured singular value theory. Other approaches to analyzing systems with both parametric degrees of freedom and nonparametric bounded-operator uncertainty are given in [1], [5], and [18].

The robust parameter estimation subproblem involves reduction of parameter uncertainty through the use of measurements obtained on-line. The novelty of the estimation problem, as we have posed it, is that the measurements and physical process are not corrupted by exogenous stochastic noise, but rather by the presence of nonparametric dynamic uncertainty within the system. Others have addressed this issue with time-domain characterizations of uncertainty (which do not match the traditional robust-control characterizations), or by mapping the frequency-domain uncertainty bound to a (generally) conservative pointwise-in-time signal bound. In contrast, our approach is based on a relative deadzone technique using a nonconservative perturbation signal energy (L^2) bound (which is described in various levels of detail in [8], [11] and [12]). This bound is arguably the most natural and tightest bound for our problem formulation since the H^∞ operator norm corresponds directly with the induced-L^2 gain of the operator. Loosely speaking, our parameter adjustment mechanism provides nonincreasing parameter errors, with strictly decreasing parameter errors whenever the parameter error is distinguishable from zero based on available measurements.

The remaining task is the integration of the control and estimation techniques, and analysis of the overall system. In this paper, we state a new stability result: the combination of perfect parameter information robust stability and the use of the robust parameter estimation process together imply bounded-L^2-input - bounded-L^2-output stability. The BIBO property is also obtained with a slightly modified L^2 norm which incorporates exponential de-weighting of older information.

The underlying stability concept is simple: if the system behaves as if the parameter error is zero, then the perfect-information robustness analysis guarantees that the system is behaving in a stable fashion; if the system behaves as if the parameter error is nonzero, then the robust estimation process reduces the euclidean norm of the parameter error vector (which cannot go on forever since the norm is bounded below by zero). A formal version of this argument is given in this paper.

Robust performance results are given as well. Robust performance is often characterized in terms of the worst-case norm of a chosen output signal, given an assumed norm on the input signals, where the worst-case is taken with respect to dynamic uncertainty. In this paper, it is shown that the asymptotic performance of the overall adaptive system is no worse than the guaranteed robust performance of the same system given perfect parameter information. Remarkably, the parametric uncertainty does not degrade the asymptotic system performance guarantees. Unfortunately, the transient performance is not quantified, and can be arbitrarily poor.

II. Preliminaries

A. *Notation*

Consider a function $x : \mathbb{R}^+ \longrightarrow \mathbb{R}^N$. Define the norm

$$\|x\|^{\sigma,t} := \left[\int_0^t e^{-2\sigma(t-\tau)} x^T(\tau)x(\tau)\,d\tau\right]^{1/2} \tag{1}$$

where the superscript σ is omitted when $\sigma = 0$, and the subscript t is omitted when $t = \infty$. When $\|x\|^{\sigma,t}$ exists for all finite t, x is said to be in $\mathbf{L}^{2e,\sigma}$. When $\|x\|^{\sigma,t}$ is uniformly bounded over all $t > 0$, x is said to be in $\mathbf{L}^{2\sigma}$. When $\sigma = 0$, we omit it from the superscript, and use the more common notation $x \in \mathbf{L}^{2e}$ or $x \in \mathbf{L}^2$.

For a vector θ, let $|\theta|$ denote the euclidean norm.

Let H^∞ denote the space of transfer functions $T(s)$ which are analytic and bounded in the open right half plane. Let S^σ be the shift operator defined by

$$S^\sigma T(s) = T(s-\sigma). \tag{2}$$

Let $S^\sigma H^\infty$ be the space of transfer functions $T(s)$ such that $S^\sigma T \in H^\infty$. In this context, σ will be referred to as a "fading memory time constant."

For $T(s) \in H^\infty$, we define $\|T\|_{H^\infty}$ to be the usual H^∞ norm (see [20]). Recall that $\|T\|_{H^\infty}$ is the operator norm induced by the choice of the $L^2[0,\infty)$ norm on the input and output signals of T. We will use $\|T\|_{i2}$ to denote the induced-L^2 norm on T when T is not LTI (Linear Time-Invariant). Similarly, $\|\cdot\|_{i2}^{\sigma}$ will denote the induced-$L^{2,\sigma}$ norm. Likewise, define the shifted H^∞ norm

$$\|T\|_{H^\infty}^{\sigma} := \|S^\sigma T\|_{H^\infty}. \tag{3}$$

Table 1 summarizes the norm notation used in this paper. Note that signal norms take the form "$\|\cdot\|^{\text{norm parameters}}$", while operator norms take the form "$\|\cdot\|_{\text{type of norm}}^{\text{norm parameters}}$."

Table 1
Norm Notation Summary

$\lvert\cdot\rvert$	Euclidean Norm
$\|\cdot\|^{\sigma,t}$	signal norm (equation (1))
$\|\cdot\|^{\sigma}$	signal norm with $t = \infty$
$\|\cdot\|$	signal norm with $t = \infty$ and $\sigma = 0$
$\|\cdot\|_{H^\infty}^{\sigma}$	shifted H^∞ norm (equation (3))
$\|\cdot\|_{H^\infty}$	usual H^∞ norm
$\|\cdot\|_{i2}^{\sigma}$	$L^{2,\sigma}$-induced operator norm
$\|\cdot\|_{i2}$	L^2-induced operator norm

Miscellaneous: For a vector V, "$D(V)$" denotes a diagonal matrix whose row i column i element equals the i^{th} element of the vector V. Superscript T denotes transposition. Throughout the paper, σ is a particular fixed and known nonnegative number.

B. *System Assumptions*

The system is assumed to take the form of Figure 1. In the figure, u_P, y_P, and r denote, respectively, the actuated plant input, measured plant output, and exogenous command input. The signal d_x is a fictitious input which will be added later to account for the effect of nonzero initial conditions.

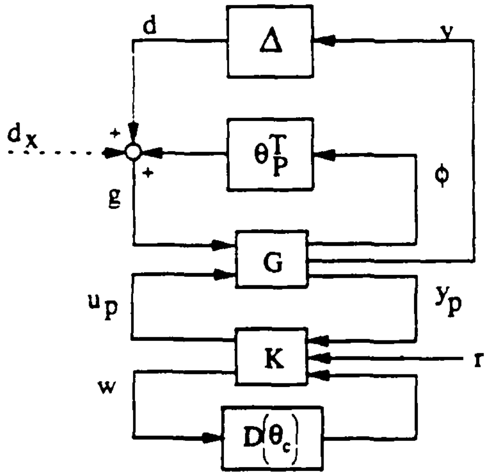

Figure 1. System Structure

The transfer matrix G is the known portion of the plant. The transfer matrix K is the fixed portion of the controller. Both G and K are proper and LTI, and $G \in S^\sigma H^\infty$. The submatrix G_{31}, which maps the input g to the output y, is in $S^\sigma H^\infty$.

The symbol $\Delta(s)$ is an arbitrary (unknown) transfer function which, for some specific known fading memory time constant $\sigma \geq 0$, is analytic on $Re(s) > \sigma$ and satisfies

$$\|\Delta\|_{i2}^{\sigma} \leq 1. \tag{4}$$

Note that for the special case $\sigma = 0$, the set of allowed Δ covers the closed unit ball in H^∞.

Vectors θ_P and θ_C are, respectively, the unknown plant parameter vector and the adjustable controller parameter vector.

Later, ϕ will be called the regression vector. $\hat{\theta}_P(t)$ will denote the estimate of θ_P at time t, and $\tilde{\theta}_P(t)$ will denote the error $\theta_P - \hat{\theta}_P(t)$.

We treat only the case in which the controller's parametric structure is fixed, and only certain parameter values may vary. These controller parameters are defined by a design rule which maps plant parameter values to controller parameter values, that is,

$$\theta_c(t) = f_c(\hat{\theta}_P(t)). \tag{5}$$

We will assume only that f_c is lipschitzian on the orbit $\{\hat{\theta}_P(t) : t \geq 0\}$.

We assume the prior knowledge

$$\theta_P \in \Theta_P \tag{6}$$

where Θ_P is an arbitrary compact convex set.

This prior knowledge may come from an understanding of the physical meaning of the parameters. In this case, it may be useful to think of the physical parameters as having a unique correct value. However, the system performance depends only upon the system input-output response, and, from this input-output perspective, there does not necessarily exist a unique choice for the "correct" plant parameter vector.

A basic principle of mathematical modeling is that no model can be verified through empirical observation; models may only be invalidated. Thus it is reasonable to define the set of "correct" plant parameters (denoted Θ_P^*) for a particular fixed plant input-output mapping P as

$$\Theta_P^* := \{\theta : \text{ the collection of all responses } y_P = P u_P \\ \text{does not contradict } \theta_P = \theta\}, \tag{7}$$

where "all reponses" means all those generated by all u_P. These are correct parameters in the sense that they produce a mathematical model which cannot be invalidated by any experiment.

Of course, in testing for a contradiction of $\theta_P = \theta$ in the above definition, one must evaluate θ_P in the context of the math model in which it is embedded. Specifically, for our plant representation, any choice of θ_P in Θ_P is "correct" if there exists a Δ satisfying (4) such that Figure 1 produces the actual plant's input-output response for all possible inputs.

Throughout this paper, the symbol θ_P represents any element of Θ_P^*, and not some particular unique physical parameter value. Furthermore, no expression depends upon the particular choice of θ_P within Θ_P^*.

Note the distinction between Θ_P and Θ_P^*. The set Θ_P represents the prior knowledge of set of possible plant parameters; its significance is that it limits the set in which we need search for θ_P. The set Θ_P^* is the plant-specific (hence unknown) set of values of θ_P which may be regarded as correct for the particular plant.

Remark: the system of Figure 1 represents a broad class of both direct adaptive control systems (as shown in [9] and [10]), and indirect adaptive control systems (as shown in [11]).

Remark. The feedback representation of embedded uncertainties is based on that of [16]. As is now common with such representations, any frequency-dependent weighting function on the uncertainty Δ is absorbed into the symbol G.

C. *Initial Condition Assumptions*

Assume, for the moment, that d_x of Figure 1 is zero, but that G and Δ may have nonzero initial conditions. In this section, we show that an equivalent system is produced by including a nonzero d_x and assuming that the initial conditions of G and Δ are zero. Moreover, $\|d_x\|^{\sigma,t}$, as a function of time, has a known exponentially decaying bound.

The system will be equivalent in the sense that Figure 1 will continue to describe the input output response of the system, without modification of the assumptions regarding the sets in which Δ and θ_P lie, and without altering the value of G. The time histories of u_P and y_P are precisely the same, although the definitions of the internal signals ϕ, v, d, d_x, and g are modified to reflect the fact that the transient effects are algebraically relocated in the equations.

1. *Initial Condition of G*

If G has nonzero initial conditions, its outputs are the superposition of a natural response and a forced response:

$$\begin{aligned} \phi &= \phi_N + \phi_F \\ v &= v_N + v_F \\ y_P &= y_{PN} + y_{PF}. \end{aligned} \tag{8}$$

—Since $G \in S^{\sigma}H^{\infty}$ and is known, all of the natural response terms decay exponentially, with a known maximum time constant.

The assumed prior knowledge of the initial conditions is an upper bound on the "amplitude" of the exponential response, that is, $|y_{PN}(t)| < ce^{-\sigma t}$, where c and σ are known *a priori*. Note that a decaying exponential bound on a signal implies that the $\|\cdot\|^{\sigma,t}$ norm of the signal also has a bound with an exponential decay rate of at least σ.

2. Initial Condition of Unmodeled Dynamics

Note that when the input to Δ is zero for all time, the assumption $\|\Delta\|_{i2}^{\sigma} \leq 1$ implies that the output of Δ is zero (more precisely, equivalent to zero in the $L^{2,\sigma}$ norm). Consequently, when Δ is thought of as a representation of unmodeled dynamics, the assumption $\|\Delta\|_{i2}^{\sigma} \leq 1$ is generally reasonable only if the unmodeled dynamics are initially at rest. Here we will modify the representation of unmodeled dynamics to remove the at-rest assumption while retaining the operator norm bound on Δ.

Since the unmodeled dynamics need not be linear, we cannot describe their output as the superposition of a forced response and a natural response which is independent of the unmodeled dynamics input. Instead, we decompose the total effect of unmodeled dynamics into the sum of a component d which is smaller than v in norm, and a residual component d_N. The residual d_N is *not* assumed to be independent of v, but is assumed to satisfy $\|d_N\|^{\sigma,t} \leq ce^{-\sigma t}$ for some known c, irrespective of v.

Now the total operator from v to $d + d_N$ need not produce an output of zero even when v is zero.

3. Equivalent At-Rest System with One Added Input

Now define

$$\hat{g} := G_{31}^{-1}(y_P - G_{22}u_P) = G_{31}^{-1}(y_{PF} - G_{22}u_P) + G_{31}^{-1}y_{PN} \tag{9}$$

$$\begin{bmatrix}\hat{\phi} & \hat{v} & \hat{y}_P\end{bmatrix} := G\begin{bmatrix}\hat{g} & u_P\end{bmatrix} \tag{10}$$

(Note that $\hat{y}_P = y_P$.) The system of Figure 1 with non-rest initial conditions on G and Δ is equivalent to the system of Figure 2 with G and Δ initially at rest.

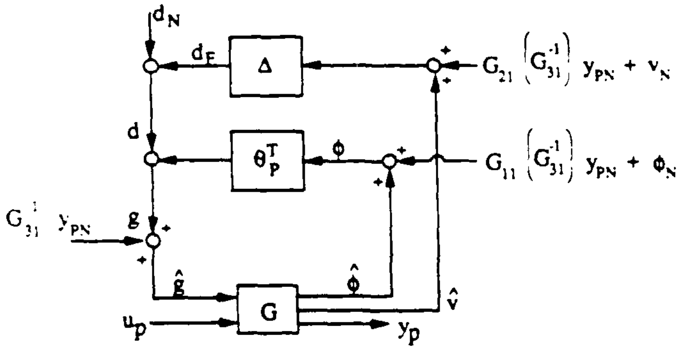

Figure 2. Manipulation of the Effects of Non-Rest Initial Conditions

Although superposition need not apply for Δ, for any signals a and b, one can cover the set $\{c:\ c = \Delta(a+b),\ \|\Delta\|_{i2}^{\sigma} \leq 1\}$ by the set $\{c:\ c = \Delta'a + \Delta''b,\ \|\Delta'\|_{i2}^{\sigma} \leq 1,\ \|\Delta''\|_{i2}^{\sigma} \leq 1\}$. Consequently, the various inputs shown in Figure 2 can be further algebraically moved to the node which defines $\hat{g}$, withgout changing the definitions of $\hat{g}$, $\hat{\phi}$, and $\hat{v}$. The net input at the $\hat{g}$ node is

$$\begin{aligned} d_x := d_N + G_{31}^{-1}y_{PN} + \Delta'(G_{21}G_{31}^{-1}y_{PN} + v_N) \\ + \theta_P^T(G_{11}G_{31}^{-1}y_{PN} + \phi_N). \end{aligned} \tag{11}$$

Furthermore, since v_N, ϕ_N, and y_{PN} have exponential decaying bounds, their $\|\cdot\|^{\sigma,t}$ norms have an exponential decay rate. Since a known induced-$L^{2,\sigma}$ norm bound is known for G, Δ, and θ_P (recall Θ_P is compact), a known exponentially decaying bound exists for $\|d_x\|^{\sigma,t}$:

$$\|d_x\|^{\sigma,t} \leq \bar{d}_x(t) = \bar{d}_o e^{-\sigma t} \quad \forall t \tag{12}$$

for some known function $\bar{d}_x$ and positive constant $\bar{d}_o$.

The result of this last algebraic manipulation is a system structure exactly as shown in Figure 1. The internal signals are different ($\hat{\phi}$ would appear in place of ϕ, et cetera), d_x is now nonzero, and G and Δ are now initially at rest. The signals u_P and y_P are unchanged.

To avoid a proliferation of notation, let ϕ, v, g, and d be re-defined hereafter so that we may use Figure 1 as a representation of the system, with G and Δ initially at rest, and d_x not necessarily zero but satisfying a known exponentially decaying bound on $\|d_x\|^{\sigma,t}$.

III. Perfect Parameter Information Robust Control

The concept of robust stability can be extended to the case of unspecific tuned adaptive systems. As a first step, we represent the system of Figure 1 in the form shown in Figure 3. Note that M of Figure 3 has an input r_x and outputs y and y_x which are not shown in Figure 1. The added output y is simply a signal one wishes to keep small. For example, y could be chosen to be the command tracking error, $r - y_P$. Linear filters can be used to capture the relative importance of different frequencies; these are assumed to be absorbed into M. This is all fairly standard in recent robust multivariable control literature.

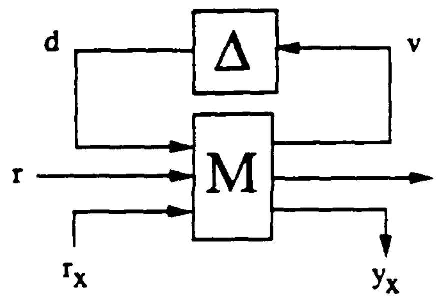

Figure 3. Perfect Parameter Information Tuned System

The signals r_x and y_x are included for a non-standard reason, namely to ensure structural robustness of the robust performance definition to be given later. For now, it suffices to know that r_x and y_x are any added system inputs or outputs which do not involve added dynamics. That is, given a state-space representation of M, y_x is any linear combination of the states (arbitrary output matrix associated with y_x) and r_x enters the state derivative definition in any linear fashion (arbitrary input matrix associated with r_x), but no states have been added to M to allow the inclusion of r_x and y_x.

Note that the system of Figure 1, excluding the Δ block, is known *a priori* except for the parameter vector θ_P and the controller gains θ_C. However, since the desired controller gains are a function of the plant parameters, the tuned system is a function of only the plant parameters. Thus, in the case of the tuned system, Figure 1 can be represented in the form of Figure 3, where M is a linear time invariant transfer function depending only on θ_P. When the functional dependence of M is important, we will write $M(\theta_P, s)$ (s is the Laplace transform variable).

Let M be partitioned such that the submatrix M_{11} is the transfer function from d to v.

Definition 1 : The control law defined by K an f_c is σ*-robustly stabilizing* (given perfect parameter information) if ar . only if

(i) $S^{\sigma}M(\theta_P, s)$ is strictly stable for all $\theta_P \in \Theta_P$

and

(ii) $$\sup_{\theta_P \in \Theta_P} \left\{ \sup_{\substack{\|\Delta\|_{i2}^{\sigma} < 1 \\ \|r\|^{\sigma,t} \neq 0}} \left[\frac{\|y\|^{\sigma,t} + \|v\|^{\sigma,t}}{\|r\|^{\sigma,t}} \right] \right\} < \infty$$

When a control law is robustly stabilizing in this sense, the system will be said to have *tuned system σ-robustness* or *perfect parameter information σ-robustness*.

It is well known that, except for trivial degenerate cases which may be neglected, condition (ii) is equivalent to

(ii) $\sup_{\theta_P \in \Theta_P} \{\|M_{11}(\theta_P, s)\|_{H^{\infty}}^{\sigma}\} < 1.$

This definition is simply the usual definition of strict robust stability, except that now we require it be satisfied for each candidate tuned system. Note that "$\|r\|^{\sigma,t}$ is bounded implies that $\|y\|^{\sigma,t}$ is bounded" is guaranteed for all allowed Δ and θ_P if and only if the tuned system has σ-robustness.

Definition 2 : For a particular σ-robustly stable tuned system M, the *σ-robust performance level*, denoted $\alpha^{\sigma}(M)$, is defined to be the smallest number C such that the following holds:

For any choice of input and output matrices defining r_z and y_z, there exist finite constants C_2, C_3, and C_4 (which may depend on the input and output matrices defining r_z and y_z) such that

$$\|y\|^{\sigma,t} \le C\|r\|^{\sigma,t} + C_2\|r_z\|^{\sigma,t} \tag{13a}$$

$$\|y_z\|^{\sigma,t} \le C_3\|r\|^{\sigma,t} + C_4\|r_z\|^{\sigma,t} \tag{13b}$$

$$\text{for all } r \text{ and for all } d \text{ such that } \|d\|^{\sigma,t} \le \|v\|^{\sigma,t}. \tag{13c}$$

The small gain theorem [19] and σ-robust stability of M guarantees that $\alpha^{\sigma}(M)$ is a well-defined finite number.

Definition 3 : The *tuned-system σ-robust performance guarantee* is defined to be

$$\bar{\alpha}^{\sigma}(M) := \sup_{\theta_P \in \Theta_P} \alpha^{\sigma}(M).$$

The σ-robust performance guarantee resembles the notion of robust performance of multivariable control theory to the extent that it is a bounded-gain definition, as indicated by the following lemma.

Lemma 1 : When $r_z = 0$,

$$\sup_{\theta_P \in \Theta_P} \left\{ \sup_{\substack{\|\Delta\|_{i2}^{\sigma} < 1 \\ \|r\|^{\sigma,t} \ne 0}} \left[\frac{\|y\|^{\sigma,t}}{\|r\|^{\sigma,t}} \right] \right\} \le \bar{\alpha}^{\sigma}(M).$$

The definition of α^{σ} is such that it gives an approximate measure of performance even if very small modeling errors elsewhere in the structure have been neglected, as indicated by the following lemma.

Lemma 2 :

$$\lim_{\varepsilon \to 0} \left\{ \sup_{\substack{r \in L^{2\sigma} \\ \|d\|^{\sigma,t} \le \|v\|^{\sigma,t} \\ \|r_z\|^{\sigma,t} \le \varepsilon \|y_z\|^{\sigma,t} \\ [v\ y\ y_z]^T = M[d\ r\ r_z]^T}} \left[\frac{\|y\|^{\sigma,t}}{\|r\|^{\sigma,t}} \right] \right\} = \alpha^{\sigma}(M).$$

This structural robustness property is crucial to the engineering utility of the value of $\alpha^{\sigma}(M)$. In practice, one prefers not to represent every extremely small error such as roundoff noise in every control computation. Instead, one characterizes the larger modeling uncertainties with some sort of bound which approximates one's intuition about the actual modeling errors, and derives robust performance measures based on the mathematical characterization of the uncertainty. These performance measures are useful if the actual performance is acceptably insensitive to slight deviations in the approximate mathematical characterization of the uncertainty. The structural robustness property above is a fundamental form of insensitivity to slight deviations in uncertainty characterization.

The definition of $\bar{\alpha}^{\sigma}$ is such that it is the tightest structurally-robust *a priori* bound possible on the gain from r to y, for the tuned system.

Remark: the robust peformance definition above ignores the effect of initial conditions on performance. However, when initial condition responses can be represented as exogenous disturbances with an *a priori* known bound, the above framework can incorporate their effects. In this paper, we will examine only asymptotic performance as time approaches infinity for systems with fading memory, and therefore the initial condition effect on performance is null, and need not be included in the robust performance definitions.

Remark: the central problems of robust multivariable control theory are (1) to find analysis techniques to determine the numerical value of the robust performance levels of a system (e.g., $\alpha^{\sigma}(M)$), and (2) to find synthesis techniques to make the robust performance levels as favorable as possible. This paper will not address these questions. Instead, we will show that regardless of the robust control techniques applied, the adaptive system's stability and asymptotic performance guarantee will equal those of the tuned system given perfect parameter information. In effect, the particular robust control design method which produces f_C and K (Figure 1) and the method for calculating $\alpha^{\sigma}(M)$ are irrelevant to the results of this paper. Of course, in practice, the robust control design step is of great importance.

IV. Robust Parameter Estimation

This section summarizes the essential details of [12], [11], with some minor modifications.

A. Estimation Problem Formulation

The estimation algorithm to follow will depend on an error equation which arises in a general class of direct ([10] and [9]) and indirect ([11]) adaptive control systems, which are compatible with the representation of Figure 1.

Note, from Figure 1 that one can construct $g = (G_{31}^{-1})(y_P - G_{32}u_P)$, and from g and u_P one can easily construct ϕ, and v using the known value of G. Then one can construct $e(t) = g(t) - \hat{\theta}_P^T(t)\phi(t)$, which satisfies

$$e(t) = \tilde{\theta}^T(t)\phi(t) + d(t) + d_z(t) \tag{14}$$

$$d = \Delta v \tag{15}$$

$$\|d_z\|^{\sigma,t} \le \bar{d}_z(t). \tag{16}$$

In the above, d_z, e and v are known scalar signals, ϕ is a known vector signal, $\tilde{\theta}$ is the unknown parameter error vector, and Δ is the unknown unstructured plant perturbation.

B. Adjustment Law Definition

Let γ_o, ε_1, and ε_2 be small positive constants, with $\varepsilon_2 \ge \varepsilon_1$. An upper bound on ε_2 will be specified later in the section on stability and asymptotic performance; the parameters are otherwise arbitrary.

We define the parameter adjustment by

$$\eta_t(\tau) := \tilde{\theta}_P^T(t)\phi(\tau) + d(\tau) + d_z(\tau), \quad \tau \in [0,t]. \tag{17}$$

$$I(t) := \|\eta_t\|^{\sigma,t} - \|v\|^{\sigma,t} - \bar{d}_z(t) \tag{18}$$

$$\gamma(t) \text{ is such that } \begin{cases} \gamma(t) \ge 0 \\ \gamma(t) = 0 \quad \text{if } I(t) \le \varepsilon_1\|\phi\|^{\sigma,t} \\ \gamma(t) \ge \gamma_o \quad \text{if } I(t) \ge \varepsilon_2\|\phi\|^{\sigma,t} \end{cases} \tag{19}$$

$$q(t) := \int_0^t e^{-2\sigma(t-\tau)}\phi(\tau)\eta_t(\tau)d\tau \tag{20}$$

$$\frac{d}{dt}\hat{\theta}_P(t) = \pi(-\gamma(t)q(t)) \tag{21}$$

where π denotes the projection into the set Θ_P (i.e., $\hat{\theta}_P(t)$ is not allowed to exit Θ_P).

Remark: A recursive realization of the above may be obtained by differentiating the integral equations for q and $\|\cdot\|^{\sigma,t}$, and may be found in [12] as well.

C. Discussion

The above parameter adjustment has the interpretation of a gradient scheme to minimize $\|\eta_t\|^{\sigma,t}$, with a relative deadzone (γ variation). The deadzone, pictured in Figure 4, has a heuristic explanation.

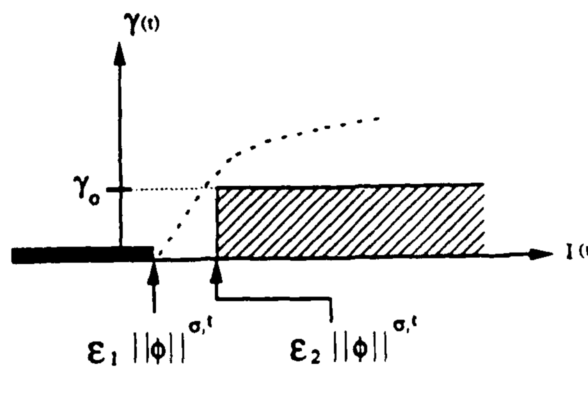

Figure 4. Adaptation Gain Constraints

Note that the known property of d as given by (15) and the $\|\cdot\|_2^{\sigma}$ bound on Δ is $\|d\|^{\sigma,t} \le \|v\|^{\sigma,t}$. Thus when $I(t) \le 0$, it may be that $\tilde{\theta}_P(t) = 0$; the assertion that $\eta_t(\tau) = d(\tau) + d_x(\tau)$ is not contradicted by the norm bound. Thus the definition of γ ensures that the adjustment is disabled when the parameter error $\tilde{\theta}$ is indistinguishable from zero in some sense, based on measured signals. The inclusion of a nonzero ε_1 adds a certain strictness to definition of "distinguishable" to prevent adjustment when the parameter error is arbitrarily close to "indistinguishable from zero."

On the other hand, when $I(t) > 0$, one can deduce from (17) that

$$|\tilde{\theta}_P(t)| \ge \frac{I(t)}{\|\phi\|^{\sigma,t}}. \tag{22}$$

The minimum parameter adjustment gain γ_o is imposed when $|\tilde{\theta}_P(t)|$ is distinguishably "too large," that is, greater than ε_2. Later we will choose ε_2 to correspond to parameter errors which large enough to cause a loss of robust stability or performance. The net effect is this: it is impossible to have unstable behavior (or worse-than-specified performance) without having adaptation turned on ($\gamma(t) \ge \gamma_o > 0$).

D. Parameter Convergence Consequences

Let $\delta(t)$ denote the euclidean distance between the estimate $\hat{\theta}_P(t)$ and the set of valid plant parameters Θ_P^*.

Theorem 1, "Monotone Error Reduction:"
Equations (17) through (21) imply

$$\frac{d}{dt}\,\delta^2(t) \le -2\gamma(t)\left[\|\eta_t\|^{\sigma,t} - \|v\|^{\sigma,t} - \bar{d}_x(t)\right]\|\eta_t\|^{\sigma,t} \le 0 \quad \forall t,$$

with equality if and only if $d/dt\ \hat{\theta}_P(t) = 0$.

Theorem 2, "Asymptotic Time-Invariance:"

$$\lim_{t\to\infty} \hat{\theta}_P(t) =: \hat{\theta}_{P\infty} \ \text{exists.}$$

Taking any fixed (though unknown) choice of $\theta_P \in \Theta_P^*$ and the associated fixed choice of Δ in the open unit ball in H^∞, one can define the asymptotic plant parameter error

$$\tilde{\theta}_{P\infty} := \theta_P - \hat{\theta}_{P\infty} \tag{23}$$

and the asymptotic controller parameter vector

$$\theta_{C\infty} := f_c(\hat{\theta}_{P\infty}). \tag{24}$$

Since f_c is Lipschitzian on the orbit of interest, $\theta_{C\infty}$ is the limit of $\theta_C(t)$, and $\theta_C(t)$ inherits the uniform boundedness of $\hat{\theta}_P$ which is apparent from Theorem 1 and the compactness of Θ_P^*.

V. Stability and Asymptotic Performance

We have not shown convergence of the parameter error to zero. In general, this does not occur, as parameter identifiability is excitation dependent, and we have not made assumptions regarding the excitation. Nonetheless, one can bypass the question of parameter convergence and directly deduce robust stability and performance properties of the overall system using properties of the identification laws alone.

A. Statement of Result

Recall that ε_2 is a free parameter in the identification laws.

Theorem 3, "$L^{2,\sigma}$-BIBO and Asymptotic Performance:" Given

(G1) the control law is σ-robustly stabilizing given perfect parameter information (as defined in section III),

(G2) the design rule f_c is lipschitzian,

(G3) that the robust parameter estimation laws of section IV are used,

(G4) $r \in L^{2,\sigma} \cap L^\infty$,

it follows that

(A) for ε_2 chosen sufficiently small, all signals shown in Figure 1 are in $L^{2,\sigma}$.

If in addition,

(G5) $\sigma > 0$,

it follows that

(B) for any $\alpha_1 > 0$, there exists a sufficiently small choice of ε_2 such that

$$\|y\|^{\sigma,t} \le (\bar{\alpha}^{\sigma}(M) + \alpha_1)\|r\|^{\sigma,t} + \varepsilon(t)\ \text{, where } \lim_{t\to\infty} \varepsilon(t) = 0.$$

Implication (A) of Theorem 3 is a statement of BIBO stability. When $\sigma = 0$, the stability is in the sense of the usual L^2 norm. For $\sigma > 0$, the BIBO property is true with respect to a norm which is similar to the L^2 norm, but with fading memory.

Implication (B) of Theorem 3 is a statement of asymptotic performance. In effect, the guaranteed asymptotic performance of the adaptive system can be made as close as desired to the guaranteed performance of the system given perfect parameter information, as defined in section III.

B. Choice of the Key Identification Coefficient

Theorem 3 involved the choice of the parameter ε_2 which partly defines the adaptive gain. This section shows that the proper choice is governed by a simple rule: the adaptation must be "turned on" when the parameter errors are large enough to produce unacceptable behavior.

Note that $\theta_P = \hat{\theta}_{P\infty} + \tilde{\theta}_{P\infty}$, and that $\theta_c(t) = \theta_{c\infty} + (\theta_c(t) - \theta_{c\infty})$. One can therefore represent the system of Figure 1 in the form shown in Figure 5 (with g being an arbitrary gain). In the figure, M' is the asymptotic system, except for residual errors. It is a tuned system, albeit possibly tuned for the wrong plant.

The value to this representation is this: since M' is a tuned system containing no uncertainty and parameterized by $\hat{\theta}_{P\infty} \in \Theta_P$, the robustness properties of M' can be evaluated *a priori* by taking the worst case over Θ_P, as in section III.

Note that with $\tilde{\theta}_{P\infty} = 0$, the system of Figure 5 is effectively the same as that of Figure 3. Granted, M' has additional outputs $g\phi$ and w which are internal to M, and M' is a function of the asymptotic estimate $\hat{\theta}_{P\infty}$, while M is a function of the true parameters θ_P. Still, the robustness properties involve the image of Θ_P in M-space (or M'-space) and this image is the same for M and M' (modulo added outputs for M'), it follows that M' has σ-robust stability in the presence of Δ if and only if M has σ-robust stability.

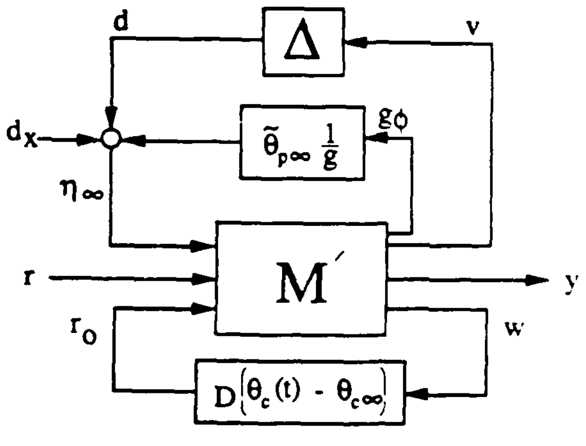

Figure 5. Complete System

Now let us conceptually expand the uncertainty set to allow

$$|\tilde{\theta}_{P\infty}| \le g. \tag{25}$$

Let M' be partitioned so that $v_{new} := \begin{bmatrix} g\phi \\ v \end{bmatrix}$ is regarded as the first output vector, that is, M'_{11} is the transfer function from η_∞ to $\begin{bmatrix} g\phi \\ v \end{bmatrix}$. Let $\bar{\alpha}'^{\sigma}$ be defined in the same manner as that of $\bar{\alpha}^{\sigma}$, except with v_{new} taking the place of v and with $\|v\|^{\sigma,t} + g\|\phi\|^{\sigma,t}$ in place of $\|v_{new}\|^{\sigma,t}$ in (13c).

Lemma 3 : If the original system had perfect parameter information σ-robust stability, then for some $g_s > 0$, $g \le g_s$ implies that the system of Figure 5 has σ-robust stability.

Lemma 4 : For any $\alpha_1 > 0$, there exists a $g_p(\alpha_1) > 0$ such that $g \le g_p$ implies $\bar{\alpha}'^{\sigma}(M') < \bar{\alpha}^{\sigma}(M) + \alpha_1$.

The choice of ε_2 indicated by Theorem 3 is the following: $\varepsilon_2 < g_s$ implies BIBO-$L^{2,\sigma}$ stability, and $\varepsilon_2 < g_p(\alpha_1)$ implies an asymptotic performance level of $\alpha^{\sigma}(M) + \alpha_1$.

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These choices have a heuristic explanation. The lemmas state that small residual parameter errors are not destabilizing, or do not cause violation of a given performance objective. The choice of ε_2 will guarantee that the adaptation gain is bounded away from zero whenever the parameter error norm is distinguishably larger than the level which is tolerated by the robustness of the perfect parameter information tuned system. Because the value of ε_2 depends only on M' and not on $\hat{\theta}_{p\infty}$, it can be determined *a priori*.

VI. Conclusions and Directions

This paper shows that if a lipschitzian control law provides robustness when supplied with the correct parameter vector from a compact convex set, and if the specified robust parameter adjustment laws are applied in lieu of knowledge of the correct parameter vector, then the overall adaptive system provides L^2-BIBO stability. Furthermore, when the norm used contains any degree of exponentially fading memory, the asymptotic performance guarantee is effectively the same as one would obtain with perfect parameter information. No persistency of excitation assumptions were required, and nonzero initial conditions were allowed.

In addition to these theoretical properties, the control laws of this paper have certain practical merits. The parameter adjustment makes engineering sense in that it continually improves the parameter estimate to the greatest degree possible consistent with the nonparametric uncertainty assumptions, unlike stabilizing compensator existence results and impractical dense search constructions. Furthermore, the full magnitude of uncertainty which can be tolerated by the non-adaptive system with parameter knowledge can be tolerated by our adaptive system, unlike earlier "robustness to sufficiently small perturbations" results. Finally, it is a non-mysterious result; the simple engineering heuristic of robust control and plus robust parameter adjustment produce stability in a traceable fashion. These are important strides toward practicality.

Still, these results fall short of a complete practical theory in at least two important respects. First, the transient performance is not quantified, and may be extremely poor for some plants and some command inputs. Second, the uncertain parameters were assumed constant, while adaptive control is often most valuable when the unknown parameters vary in time. Further research is required to overcome these difficulties.

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Proofs: Proofs are available on request from the authors at the address given above. A version of this paper, including proofs, has been submitted for publication elsewhere.

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