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LABORATORY REPLICATION OF SCIENTIFIC DISCOVERY PROCESSES

Technical Report AIP - 94

Yulin Qin & Herbert A. Simon

Carnegie Mellon University
Department of Psychology
Pittsburgh, PA 15213

30 April 1989

The Artificial Intelligence and Psychology Project

Departments of
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Carnegie Mellon University

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Laboratory Replication of Scientific Discovery Processes

Yulin Qin and Herbert A. Simon
Carnegie-Mellon University

Abstract

Fourteen subjects were tape recorded while they undertook to find a law to summarize numerical data they were given. The source of the data was not identified, nor were the variables labeled semantically. Unknown to the subjects, the data were measurements of the distances of the planets from the Sun and the periods of their revolutions about it -- equivalent to the data used by Johannes Kepler to discover his Third Law of planetary motion.

Four of the 14 subjects discovered the same law as Kepler did (the period varies as the $3/2$ power of the distance), and a fifth came very close to the answer. The subjects' protocols provide a detailed picture of the problem-solving search they engaged in, mainly, but not exclusively, in the space of possible functions for fitting the data, and provide explanations as to why some succeeded and the others failed.

The search heuristics used by the subjects are similar to those embodied in the BACON program, a computer simulation of certain scientific discovery processes. The experiment demonstrates the feasibility of examining some of the processes of scientific discovery by recreating in the laboratory discovery situations of substantial historical relevance. It demonstrates also, that under conditions rather similar to those of the original discovery, a law can be rediscovered by persons of ordinary intelligence (i.e. the intelligence needed for academic success in a good university). The data for the successful subjects reveal no "creative" processes in this kind of a discovery situation different from those that are regularly observed in all kinds of problem solving settings.

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In 1618, Johannes Kepler discovered his Third Law of planetary motion: The cube of a planet's distance from the Sun is proportional to the square of its period of revolution, or:

$$D^3/P^2 = C.$$

where D is the distance, P the period, and C a constant. This discovery, along with Kepler's laws of elliptical orbits and equal areas, paved the way for Newton's discovery of the law of universal gravitation, from which Kepler's laws can be deduced logically.

The discovery of the Third Law provides a setting for the study of some of the processes that people (scientists) use to find regularities in data, especially in the frequent circumstances where there exist no bodies of relevant theory to guide the search. In this instance, as in many others in the history of natural science, the discovery requires an induction directly from the data without help from pre-existing theory. Data-driven discovery of this kind has been simulated by the BACON program (Langley, et al., 1987), which, using a few simple heuristics, rediscovered Kepler's Third Law, as well as Ohm's Law of electrical currents, Black's Law of temperature equilibrium, and a substantial number of other important laws of 18th and 19th century chemistry and physics. Langley, et al. (1987) also discuss the significance of data-driven discovery in the overall progress of science.

The purpose of the experiments described in this paper was to compare human data-driven discovery processes with the processes embodied in BACON, and thereby to determine their similarities and differences. Do humans use the same heuristics as BACON when they are confronted with the Kepler data? Unfortunately it is too late to take a protocol from Kepler, and he left behind only a minimal record of how he found the Third Law. As possibly inadequate substitutes for Kepler, we recruited college students for two closely similar experiments. The data we obtained from these

experiments gives us evidence of how human subjects respond to their task and how their methods compare with BACON'S, and perhaps cast some light on the history of scientific discovery in the case of Kepler's Third Law. We will first describe the experiments and their results, next comment upon their significance for the psychology of discovery viewed as problem solving, then ask what light they may cast on Kepler's discovery.

Experiment 1

Method and Material

The data used in this experiment were the average distances from the Sun, and the periods of revolution about the Sun, of Mercury, Venus, Earth, Mars, and Jupiter, obtained from the 1986 *World Almanac*. Kepler, in the 17th Century, used only slightly less precise data (See Harmonies of the World, 1619/1952, chs. 3,4.) The data given to the subjects (Table 1) were not identified by source, and the variables were labeled "s" and "q" (instead of "Distance" and "Period") so as not to reveal their meaning.

Insert Table 1 about here

The experiment generally lasted about 1 hour unless S solved the problem in a shorter time. Subjects were allowed to use pen, scratch paper, and a calculator that had multiplication and division operators as well as exponential and logarithmic functions (The experimenter brought a calculator into the experiment room. However we allowed the subject use his/her own calculator if he/she preferred to.) The subjects were instructed as follows:

We are interested in how a human being discovers a scientific law. This experiment is not designed to test your problem solving ability. It is simply to discover what methods you

would use to build a formula describing the relationship between two groups of given data.

In order to follow your thoughts we ask that you think aloud, explaining each step as thoroughly as you can.

The data will be presented on another sheet of paper, and you should begin by reading the data aloud.

After finishing the experiment, S's were asked if they could identify the law that fitted the data. None identified it as Kepler's law, nor is there any indication from their protocols that they were aware of the meaning of the data or the law that described them. So, while they may have previously encountered Kepler's law in their physics courses, there is no reason to think that memory assisted them in solving the problem.

Subjects

Nine subjects took part in experiment 1. Their academic status is shown in Table 2. Five were undergraduates, all of whom had taken or were taking courses in physics, calculus, and chemistry; one was a graduate student in physics, one an engineer, one a graduate student in art history, and one a graduate student in education.

Insert Table 2 about here

Problem Analysis

The structure of the problem can be illuminated by observing how BACON attacked it. The BACON program found Kepler's third law in about two minutes on a medium-size computer. It did not search "the space of all possible functions," but used the few simple heuristics shown in Table 3 to guide its search. Following these heuristics, it first constructed (Heuristic 4) and tested the function, $P/D = C$, without success. This led it (again by Heuristic 4) to construct and test $P/D^2 = C$.

also without success. Then, using Heuristic 5, it constructed $P^2/D^3 = C$, and concluded that this function fit the data satisfactorily.¹

Insert Table 3 about here

The basic idea underlying BACON's success is to notice when two variables are increasing and decreasing together, and then to test their ratio. (If one increases while the other decreases, test their product instead.) Repeated application of this principle to the original data and to the new functions derived from them quickly produces the desired function. It is interesting to note that if the test of (approximate) equality in BACON is loosened, it will be satisfied with the second function it finds, $P/D^2 = C$, and will stop there. So did Kepler, who was satisfied with the inverse square law for about ten years, until he took up the problem anew to see if he could get a better fit to the data!

Evoking heuristics like those in BACON, and proceeding along the lines sketched above, is only one way to solve the problem. One of the other ways is to take logarithms of the quantities s and q , whereupon the law becomes:

$$\log q = 3/2 \log s + K, \text{ where } K \text{ is } (\log C)/2.$$

BACON's third linear heuristic (Table 3) would find the law immediately from these log-transformed data. Yet another way is to try the square root of s^3 :

$$s^{3/2}/q = K1, \text{ where } K1 \text{ is } C^{1/2}.$$

Behavior of Subjects

Finding the law is not easy for human subjects. A freshman (S3) and the physics graduate student (SY) found the law, and a junior electrical engineering

¹See Langley, et al., p. 85. Various versions of BACON will try slightly different search paths, but none will need more than a half dozen tries to find Kepler's law.

student (S4) came very close; the others failed to find it (See Table 2).

Data-driven scientific discovery is a kind of ill-structured problem solving. Although subjects can, in principle, use means-end analysis etc. to solve the problem, in general, they don't know where the goal is, and don't know the distance between their current solution attempt and the goal. Sometimes, the current solution attempt is very near the goal, but they miss it. For example, in SW's protocol, we find:

How about $s^{3/2}$?
It's too complex.

She didn't check her hypothesis, but turned to $(q_{i+1}-q_i)/(s_{i+1}-s_i)$ instead.

In this experiment, subjects encountered at least three specific difficulties.

1. The relation between q and s is nonlinear. Three subjects, S5, SG and SJ failed because they only tried linear relations. (Note that the latter two subjects were the least sophisticated, mathematically, of the nine.) Other subjects, for example, SW, spent a great deal of their time in unsuccessful efforts to find a linear relation.
2. If we write the law in the form, $q = f(s)$, we get a non-integral power of s , $3/2$. SW failed to solve the problem through not testing non-integral powers, and S1 found no systematic way to arrive at the correct power.
3. The constant coefficient in the law is not unity. This was at the root of the failures of S1, S2, and S4, who neglected to include the coefficient in the functions they were considering.

Let us examine the search strategies that the subjects employed. Their protocols show them generating a sequence of functions and testing these functions against the data. In their fitting of functions, two motives were in evidence: a function might be fitted because it was hypothesized to be the correct one, or it might be fitted simply to gain information about the shape and trend of the data. It is not always easy to determine from the protocols which motive, or combination of them, is operative.

In many cases, a subject considered a particular function, dropped it for another, and at some later time returned to it. Table 4 shows the principal types of

functions that the six subjects who went beyond linear functions generated and examined, and the number of times each function was considered by each subject. Some extreme cases of persistence are S4's examination of linear functions seven times, and S2's examination of sequential functions seven times, but the other subjects are not far behind. All of the six subjects tried linear, sequential and quadratic functions at least once, four tried log functions, three tried cubic functions and two, others. A total of about 32 different functions in these categories were considered by one or more subjects.

 Insert Table 4 about here

The function types recorded in the table are defined as follows:

1. *Linear*. These are relations like q/s , $q - s$, s/C , or q/C , and so on.
2. *Sequential*. These are functions that relate successive values of q , e.g., q_i with q_{i+1} , or s_i with s_{i+1} . Such functions may arise in either of two ways. The subject may be considering first differences of the variables (taking differences between successive values), or may be thinking of possible sequential patterns of the values of each separate variable.² When subjects considered *both* a function of q and a function of s simultaneously, this is counted as one occurrence in Tables 4 and 6; while if they considered only a function of q or a function of s , this is counted as .5.
3. *Quadratic*. These include functions like s^2 , s^2/q , $s^2 + bs + c = q$, and so on.
4. *Logarithmic*. These are functions like $(\log s)/(\log q)$, $\log(s/q)$, and so on.
5. *Cubic*. These are functions like s^3/q , s^3/q^2 .
6. *Other*. Among these, we find functions like s^x , $s^{1/2}$, and $q^{1/2}$.

One other manipulation of the data that should be mentioned is S3's rounding

²Kepler came to the problem of relating period to distance after a long period of search for a pattern of the successive distances of the planets; which included his famous proposal for relating those distances to properties of the regular solids.

of the results of computations to simpler numbers like $5/2$, $10/3$, 4, 5, 9. This abstraction process made it easier for S3 to find trends in the data and ultimately to discover the law.

Most of the subjects made use of some kind of diagram: scatter diagrams of the data, or rough graphs of the functions they were considering. Table 4 shows that S1 used graphs, and all subjects except S1 and S3 used scatter diagrams. (A "0.5" in Table 4 refers to an unfinished scatter diagram.)

In Table 5 are shown the percentages of the function references belonging to the various types, for each subject, and in total. Table 6 provides more detail on percentages of references to the seven functions that subjects considered most frequently, and that account collectively for more than half of all the references.

Insert Table 5 about here

Insert Table 6 about here

From these data we can draw a number of generalizations.

Linear functions were considered most frequently. From Tables 4 and 5, we can see that about 28.6% of function references were to linear functions (excluding the three subjects who considered *only* linear functions). Sequential functions are next most often considered, then quadratic functions, then logarithmic functions.

Simple functions were considered more frequently than complex ones. Although about 32 different functions were considered by one or more subjects, we can see from Table 6 that the seven simple functions listed there account for 52.4% of all references. Of the 38.5 references to these seven functions, 18, or 46.8%, are to the two linear functions (the first and third columns).

There are large individual differences in the functions considered. From Table 4 or Table 5, we can see that S1 gave almost equal consideration to functions in each of five categories. S2 preferred logarithmic and sequential functions. S3 restricted his consideration almost wholly to linear and quadratic functions. S4 thought mostly about linear and "other" functions. SY considered five types of functions, and SW four types. The three subjects excluded from the table considered only linear functions.

Diagrams were used extensively. From Table 4, we see that almost every subject used diagrams: usually scatter diagrams, but in one case, graphs.

These data give some picture of the diversity and similarities of behavior among the subjects, but do not explain immediately why some subjects were successful, and others not. A closer look at the protocols of the two successful subjects will give us a better picture of what they did. Our descriptions of behavior are based on problem behavior graphs (Newell and Simon, 1972) constructed from the protocols of each subject. In constructing the problem behavior graphs, we distinguished nodes at which subjects mentioned functions or facts they had observed about the data from nodes at which subjects commented on their thought processes (meta-nodes). Nodes were numbered in the order in which the subject reached them. To illustrate the method, let us examine the latter portion of the protocol of the successful subject SY.

Protocol of Subject SY

1. You said don't use logarithm? Ok. Try something else.
2. Try a simple function.
3. The simplest one is square. x square.
4. Check if their squares fit or not.
- 5 $88^{1/2} = 9.38$

6. $(36/4)^2 = 88$
7. $(67.25/4)^2 = 282$.
8. The difference (between 282 and 224.7) is big.
9. Again, $(93/4)^2$...
10. No, it's wrong.
11. The difference is too big.
12. The square increases too fast.
13. So, try s^3 and q^2 .
14. $36^3 / 88^2 = \dots$ The easier way (using this calculator) is $88^{3/2} = 19.87$.
15. $88^{2/3} / 36 = 0.54(96)$
16. $224.7^{2/3} / 67.25 = 0.55$
17. $365^{2/3} / 93 = 0.55$
18. $687^{2/3} / 141.75 = 0.55$
19. It looks not bad.
20. $4332^{2/3} / 483.8 = 0.55$.
21. It seems that it is this kind of relationship.
22. E: Write it down.
23. S: That is, s cube is in direct ratio to q square.

Sentences (1) - (6) reflected that SY changed his search direction from trying logarithmic functions to trying quadratic functions and found that $(s_i/4)^2 = q_i$. These sentences form node 22 of his problem behavior graph (See Appendix). To show the change of search direction, we put node 22 in two places on the PBG. The first node, after node 21, shows that the direction is changed after trying node 21. The second one, after node 8, shows that now the search is for quadratic functions again. To connect these two locations of node 22, we label each of them, in parentheses, with the coordinate of the other, and insert an arrow pointing forward or

backward, respectively.

In sentences (7) - (9) SY calculates $(s_j/4)^2$ and $(s_j/4)^2$, and compares them with q_2 and q_3 . Sentences (10)-(12) report the result of the comparison and the trend of the data (Nodes 23 and 24). Sentence (13) proposes a new function. This is a meta-node, indicated by a dotted box instead of a solid box. It is not certain that SY had built a new hypothesis, $s^3 = cq^2$, at this time: he might have been trying to see the trend of s^3/q^2 .

After he had found the rule, he said retrospectively: "When I tried the square, one variable (s) increased very fast, the other, q, increased very slowly. I tried to adjust them, increasing one (q) and decreasing the other (s). At first I tried the direct ratio of s^2 to q. Then I added 1 to the power of s and 1 to the power of q. In this way I made them harmonious." The heuristic he used here is essentially the same as Heuristic 5 in BACON.

Sentences (14)-(18) form node 26, in which SY tried $(q_i)^{2/3}/s_i$, $i=1, \dots, 4$. Sentence (19) forms node 27. Sentence (20) forms node 28. Sentence (21) forms node 29. It seems that SY formed a new hypothesis, $q^{2/3} = ks$, in node 26 and 27. He tested it again in node 28 and confirmed that the rule is $s^3 = cq^2$ in node 29.

To summarize the entire protocol of SY, there are three phases: understanding, initial search, and search in depth.

1. *Understanding (Nodes 1 to 6).* Initially, SY read and characterized the data. He observed that they were not linear, but that both q and s were monotone increasing.

2. *Initial search (Nodes 7 to 15).* During this segment, SY searched in breadth for a suitable function. After examining the scatter diagram, he chose four types of functions: quadratic, exponential, sequential, and logarithmic.

3. *Search in depth (Nodes 16 to 29).* In Nodes 16 to 21, SY sought to estimate

the parameter for the ratio of $\log q$ to $\log s$. Obtaining an estimate, he returned, in Nodes 22-28, to the quadratic, and applied the equivalent of BACON's fifth heuristic to relate powers of q to powers of s . In this way, he found the correct function

Model of SY's Behavior. From the problem behavior graph and the analysis of SY's process, we can describe his rediscovery of Kepler's law in terms of the following models, expressed both at a very general and at a more detailed level. At the most general level, his behavior fits the model proposed by Simon and Lea (1974), see Figure 1. SY searches both in a space of instances (the data) and a space of rules (the hypothesized functions). Information in the instance space (the scatter diagram and the numerical parameters he calculates) suggests functions in the rule space for examination. Manipulation of the functions (fitting them to the data) provides new information in the instance space.

Insert Figure 1 about here

The productions of the more detailed model are shown in Figure 2. The major part of this model consists of productions for searching two levels, function types and parameters, in the space of rules.

Insert Figure 2 about here

Sources of Information. SY and the other subjects obtain new information in four ways: as a result of checking hypotheses, from comparing values of the given data or transformed data, from comparing the trend and shape of data, and from their diagrams. From this information, subjects can make decisions about changing the type of function they are considering, changing a function parameter, performing some operation, or applying a heuristic. Subjects do not always consider the function type and the function parameters separately. Sometimes they choose a

specific function, and if, upon checking it, the result is not satisfactory, they then choose a function of a different type.

Diagram. Subject SY drew a diagram. How did he use it? Upon reading the data, he noted that the relation between s and q was not linear, but was monotone. For more information, he constructed the scatter diagram, and used it to decide that the most likely function types were the quadratic, exponential, sequential, and logarithmic.

Feedback. Most important to SY's success was the way in which he used feedback from the instance space to the space of hypotheses. Beginning at node 22, SY computed s^2 , but found that it increased much faster than q . Since he had noticed that s increases more slowly than q , he multiplied s^2/q by s/q (Heuristic 5 of BACON), obtaining a constant. In other words, from fitting the quadratic, he not only discovered that this was not the correct function, but he also learned in what way it deviated from constancy, enabling him to choose a plausible corrective. From this example, we see that a procedure like Heuristic 5 of BACON is not ad hoc, but is a logical derivative of means-ends analysis.

Inefficiency in Search. SY sometimes fails to use direct methods that are surely within his mathematical repertoire. At node 18, he needed to find the coefficient C and the constant k for the log-linear relation:

$$\log q + C \cdot \log s = k.$$

He could have found these parameters by solving the simultaneous equations obtained from two of the data points. Instead of doing this, he tried to guess the value of C , and failing, he returned to considering the quadratic function.

Best-First Search. From the PBG as a whole, we would conclude that SY conducted a best-first search, although his criteria for choice among different continuations are not always evident. For example, considering linear, quadratic, and

logarithmic functions, he examined the linear (simplest?) functions first, then switched to the logarithmic (most promising?). Only when he had failed to find a fit of the logarithmic functions did he return to the quadratic.

Use of BACON-like Heuristics. We have already noted that SY used the equivalent of BACON's Heuristic 5 in the last step of his solution. Of course he, as well as all the other subjects, used Heuristic 1 (Find a law); that was part of the task instructions. Every subject also knew that finding a law was equivalent to finding a constant function (Heuristic 2). SY also used Heuristic 3 -- fit a linear function -- in his examination of the logarithmic function, although he did not succeed in finding the correct slope and intercept.

Protocol of Subject 3

The same model of the discovery process that fits SY's protocol also fits the protocol of S3. However, there are some details of the process that are different in the two protocols.

Breadth-first Search. S3's protocol is much longer than SY's. A simplified abstract of the PBG is shown in Figure 3. S3 considers four types of functions, linear, $f(q/s)$, quadratic, and sequential patterns, moving from one to another whenever he feels he is not making progress, and revisiting each several times. The PBG gives the appearance of breadth-first search, but the criteria for switching from one branch to another are not evident.

Insert Figure 3 about here

Use of Abstraction. Data abstraction played an important role in S3's finding regularities. In step 15 he re-examined the result, from step 6, of computing q/s , and then simplified these numbers to $5/2$, $10/3$, 4, 5, 9. In step 21, he similarly

abstracted the results of computing s^2/q to 3, 4, 5, 6, and 11. Finding that these two sequences were very close to each other, he evoked Heuristic 4 and solved the problem. This is a clear example in his search of feedback.

Manipulation of Functions and Data. An important difference between SY and S3 is that the former searched mainly in the space of functions, using the data to test his hypotheses, while the latter manipulated the data, and used abstraction (instead of a diagram) to find the regularities in the data.

Unsuccessful Subjects

We discuss next the four other subjects, S1, S2, S4, and SW, who progressed beyond linear functions. From their PBG's it can be seen that their behavior fits the general model of the discovery process that we used for SY and S3. The obvious difference between the unsuccessful and the successful subjects lay in their search strategies and use of heuristics.

Characteristics of Search. The search of the unsuccessful subjects was characterized by shallowness, poor information feedback, and frequent repetition.

1. Some of the unsuccessful subjects, e.g., S1, S2, and S4, searched the function type space quite widely, but they did not pursue the search for parameters of the functions systematically.
2. Some of the unsuccessful subjects, e.g., S1, S2, and SW, obtained little more than a "yes-no" answer from their attempts to fit functions, instead of gaining information about the nature of the discrepancies that might guide the next steps of search. They failed to call productions 5 and 12 of Figure 2 and often called productions 4 and 13. Hence, much of their search could be described as "one-step search."
3. Some of the unsuccessful subjects, for example SW, proposed many hypotheses without examining any but the easiest ones carefully, and often repeated hypotheses that had failed before. This is a further reflection of the lack of informative feedback to guide search.

Use of Heuristics. All the subjects, as we have seen, used BACON's Heuristics 1, 2, and 3. However, the unsuccessful subjects did not use Heuristic 4

and 5 appropriately. For example, at step 2, S1 found s/q to be a decreasing function and wanted to try s^2/q . But he only observed, from s_1^2/q_1 , that the latter functions increased "much faster," and then shifted his attention to the logarithmic functions. The unsuccessful subjects did not use the heuristics systematically. For example, after applying Heuristic 4 to s , q , and getting q/s , SW found q/s increasing. However, this result to her only meant that q/s was *not* a constant. She did not continue to use Heuristic 4, or 5. Instead, she went on to try $s_{i-1}-s_i$, $q_{i-1}-q_i$ and never made one step more along this direction.

Particular Characteristics of S1. While S2, S4, and SW used scatter diagrams to help choose an appropriate function type, S1 drew a graph of the function $y = s^2$ to see if there was a quadratic relation between s and q . Toward the end of his experiment, he recalled the quadratic formula in physics for the acceleration of a falling body, and checked to see if it matched the given data. His style of trying everything in his mental repertory is reminiscent of the phenomena studied by repair theory (VanLehn and Ball, 1987).

Summary of Experiment 1

In experiment 1, nine subjects tried to rediscover Kepler's third law. Two succeeded, while the others failed. Three subjects who failed lacked the mathematical knowledge necessary to find the law. The protocols of the other six subjects, successful and unsuccessful, all fit a basic model, a particularization of that of Simon and Lea (1974).

All the subjects used heuristics like BACON's. Heuristics 1, 2, and 3 were used by everyone. Heuristics 4 and 5 were also used frequently, although not to the same extent by all subjects. The successful subjects proceeded relatively systematically, and obtained relevant information by feedback from the search. The unsuccessful subjects were less systematic, and less able to obtain information from

their tests of hypotheses. As a result, they were not able systematically and successfully to use Heuristics 4 and 5, which depend on feedback.

Experiment 2

Method and General Results

In Experiment 1, the subjects were allowed to use calculators that had operations for computing exponentials and logarithms. Logarithms appeared upon the scientific scene at just about the time that Kepler discovered his Third Law, and in subsequent years Kepler himself played an active role in their further development. Nevertheless, although Kepler learned about logarithms within a year of his discovery of the Third Law, the weight of evidence is that he did not use them in the discovery. We decided, therefore, that we should run a second experiment in which the calculators available to the subjects had no exponential or logarithmic functions. In all other respects, the second experiment was identical with the first.

The change in availability of computing aids had two consequences. One is that the subjects' speeds of calculation decreased slightly. (For example, they had to do $x \cdot x \cdot x$ to compute x^3 .) Second, they now could not calculate square roots. We gave them access to tables of roots to overcome this second difficulty.

Table 7 describes the subjects and their best approximation to the law they were seeking. Two of the five subjects were successful, three unsuccessful.

Insert Table 7 about here

Behavior of Subjects

Corresponding to Tables 4, 5 and 6, respectively, in experiment 1, Table 8 shows the principal types of functions that the five subjects in experiment 2

generated and examined, and the number of times each function type was considered by each subject; Table 9 shows the percentages of the function references belonging to the various types, for each subject, and in total; Table 10 provides more detail on percentages of references to the six functions that subjects in experiment 2 considered most frequently, and that account collectively for more than half of all the references.

Insert Table 8 about here

Insert Table 9 about here

Insert Table 10 about here

Comparing the corresponding Tables of the two experiments, we see that the results of the two experiments are generally consistent: Linear functions were considered most frequently, simple functions were considered more frequently than complex ones, there were large individual differences in the functions considered, and diagrams were used extensively. Of course, there are a few differences. In table 8 the total references per subject (10) is a little less than that in Table 4 (12.2). One reason may be that the calculation took more time in experiment 2 than in experiment 1 because of the change in calculators. In experiment 2, there were a few more references to quadratic and cubic functions than in experiment 1 although the differences are not significant by t-test. Perhaps, the awkwardness of exponential and logarithmic computations caused the subjects to try more quadratic and cubic functions in experiment 2. Nevertheless, None of the differences between Table 4 and Table 8, and between Table 5 and Table 9 are large. The differences between

Table 6 and Table 10 look a little larger. In Table 6, there are seven functions, but in Table 10, there are only six. Four functions (s^x , $\ln q/\ln s$, $q \cdot s$ and $s_{i-1} \cdot s_i$, $q_{i-1} \cdot q_i$) in Table 6 do not appear in Table 10. Instead, there are three new functions in Table 10 ($s^2 + bs + c = q$, s^2/q^2 and s^3 or s^3/q). The disappearance of s^x and $\ln q/\ln s$ can, obviously, be attributed to the changing of the calculators. The others might be caused by the large individual differences among the subjects.

The Successful Subjects, S8 and S9

From the PBGs of S8 and S9, the two subjects who found the law, it appears that their search models closely resembled those of SY and S3 in the first experiment. They searched relatively systematically, obtained feedback from their tests of hypotheses, and used the feedback to guide further search.

Subject S8 manipulated the data, examining only a few functions, and found the law very quickly. His style of search resembled that of S3. Most of his manipulation consisted in computing functions of s , then comparing these with q . He used Heuristic 4 in combination with hill-climbing search (successive approximation), and solved the problem without the help of a diagram.

Subject 9 selected linear and quadratic equations, then constructed a scatter diagram of the data. Next she chose the function $q = as^3$. After observing the behavior of this function, she used Heuristic 5 to find the solution.

Three Unsuccessful Subjects

Subjects 6 and 10 searched over sets of functions rather unsystematically, and without effective feedback of information. For example, S6 examined all of these simple functions:

- (1) s multiplied or divided by a number
(e.g., $s \cdot 2.5$, $3/s$)
- (2) the sum of s and q

(3) differences

(e.g., $q-s$, $(q_2-q_1)/(s_2/s_1)$)(4) the product of q and s (5) the ratio of q to s (e.g., q/s , $(q/s)^2$)

These searches were generally carried to only one step in depth.

S7 proceeded more systematically than S6 or S10, and obtained feedback that he used to guide his search. He searched by selecting successive functions, but failed to solve the problem after having spent an hour and a half. He failed because he did not sufficiently explore simple functions, but tried complex ones such as the hyperbola, and derivatives and integrals.

Summary of Experiment 2

The behavior observed in Experiment 2 is wholly consistent with that in Experiment 1. Even without access to a calculator for logarithmic functions, two subjects succeeded in rediscovering Kepler's third law. The searches covered a somewhat narrower range of functions than were covered by the subjects in Experiment 1.

Perhaps most interesting was the demonstration, in S7's failure to solve the problem, that the effective use of feedback and systematic search are necessary, but not sufficient, conditions for success.

Heuristics

In the experiments, we have seen that the subjects employed numerous heuristics for searching function types and the parameters. We now summarize their heuristics and compare them with BACON's heuristics.

Supervisory heuristics.

1. Try simple functions first.

For example:

- (1) Begin by checking linear functions.
(All subjects except S10, who began by constructing a scatter diagram.)
2. If simple functions don't work, try more complex ones.

For example:
 - (1) If linear functions don't work, try quadratic functions.
(S3, S6, S7, S8, S9).
 - (2) If quadratic functions don't work, try cubic functions.
(SW, S7)
 - (3) If cubic functions don't work, try more complex functions.
(SW)
3. In trying complex functions, try the simplest first.
(S1)
4. If complex functions don't work, try simpler functions.
(S1, S3, S4, SW, S7)
5. If the function looks too complex, don't check it in detail.
(SW, and other subjects)
6. If you find some trends in the data, persist in using them.
(Successful subjects, S7).
7. Use one or two of the pairs of observation to conjecture a formula and test it by other pairs.
(All of the subjects)

That linear functions were considered most frequently and simple functions more frequently than complex ones. can be explained by subjects' employing the heuristics mentioned above.

Operation heuristics.

1. BACON's Heuristic 4.

- (1) If s increases as q increases,
then try s_i/q_i . (S1)
- (1') If s increases as q increases
then try q_i/s_i .
(S2, S3, S4, SY, SW, S6, S7, S9)
- (2) If q/s increases as s^2/q increases,
and the values are very similar,
then try $(s^2/q)/(q/s)$ i.e. s^3/q^2 . (S3)
- (3) If s^2 increases as q increases
then try s^2/q . (S3, S8)

- (4) If s^2/q increases as q increases
then try $(s^2/q)/q$, i.e. s^2/q^2 . (S1)
- (5) If $\ln s$ increases as $\ln q$ increases
then try $\ln q/\ln s$. (S2)
- (6) If $q-s$ increases as s increases
then try $(q-s)/s$. (S4)
- (7) If $q_{i+1}-q_i$ increase as $s_{i+1}-s_i$ increase
then try $(q_{i+1}-q_i)/(s_{i+1}-s_i)$. (SW, S6)
- (8) If s increases as s^2/q increases
then try $s/(s^2/q)$. (S8)
- (9) If $q_{i+1}-q_i/s_{i+1}-s_i$ increases as $s_{i+1}-s_i$ increases,
then try $(q_{i+1}-q_i/s_{i+1}-s_i)/(s_{i+1}-s_i)$. (S10)

Some are more complex:

- (10) If q/s increases as s increases
then try $(q_{i+1}/s_{i+1} - q_i/s_i)/(s_{i+1}-s_i)$. (S9)
- (11) If $q_{i+1}-q_i$ increases as $s_{i+1}-s_i$ increase
then try $\ln((q_{i+1}-q_i)/(s_{i+1}-s_i))$. (S2)

2. BACON's Heuristic 5.

- (1) If s^2 increases much faster than q ,
and S increases more slowly than q
then try s^3/q^2 . (SY)
- (2) If q increases, but not as fast as s^3 (i.e. q/s^3 decreases)
then try q^2/s^3 . (S9)
- (3) If s_i/q_i decreases as s_i increases
then try s_1^2/q_1 . (S1)

3. Hill-climbing combining with BACON's Heuristic 4.

By hill-climbing, we mean repeating a transformation if it produces a more nearly constant function, reversing it if it leads away from constancy.

For example:

- (1) If s^2/q increases then check s^3/q , and if
 s^3/q increases faster than s^2/q
then try $(s^3)^{1/2}/q$. (S8)
- (2) If q/s increases then try $(q/s)^2$, and if
 $(q/s)^2$ increases faster than q/s ,
then try $(q/s)^{1/2}$. (S6)

4. Other heuristics aimed toward constancy.

- (1) Division.

For example:

- (a) If s_i increases as q_i increases
then try s_i/i , q_i/i . (S2)
- (b) If s increases
then try $s/100$, $s/1000$. (S3)
- (c) If $s^2 > q$
then check whether $s^2/q = c$. (S1, SY)
- (d) If s , s^2 and q increase
then try $(s^2 - q)/s$. (S8)

(2) Subtraction.

For example:

- (a) If $q > s$ then try $q - s$. (S3, S4, SW, S6)

(3) Square root.

For example:

- (a) If q/s increases
then try $q^{1/2}/s$. (S6)

(4) Logarithm.

For example:

- (a) If q increases as s increases, and $q > s$
then try $\log q$, $\log s$. (S1, S2, S4)
- (b) If q/s increases
then try $\log(q/s)$. (S2)

5. Sequential laws.

Some subjects tried to find regularities in the sequence of values of s or q , or both. BACON would not attempt this, but as we noted earlier, Kepler tried very hard to find a law for the distances between successive planets.

For example:

- (1) If s increases as q increases
then check $s_{i+1} - s_i$, $q_{i+1} - q_i$. (S1, S2, SW)
- (2) If q/s increases
then try $q_{i+1}/s_{i+1} - q_i/s_i$. (S2)
- (3) If s increases as q increases
then check s_{i+1}/s_i , q_{i+1}/q_i . (S3)
- (4) If s increases
then try $s_{i+1} = x * s_i$. (S4)
- (5) If s and q increase, and $q > s$
then check $(i+1)s_i = q_i$. (S6)

6. Decomposition.

(1) Try $s_1 = 3^2 * 2^2$. (S2, S8)

7. Guessing.

(1) To solve linear equation $\ln q + c * \ln s = \text{constant}$, given that $|c| > 1$, guess a value of c . (S7)

(2) To fit $q = s^x$, guess a value of x . (S1, S4)

(3) To find a function,
seek a law in physics: $x = at^2/2 + c$,
and try the analogue: $q = s^x/y + c$. (S1)

8. Unreasonable or faulty heuristics.

Some heuristics used by S6 are these:

- (1) If s increases as q increases, then try $s+q$.
- (2) If s increases as q increases, then try $s*q$.
- (3) If s increases, then try $3/s$.

One of the faulty heuristics used by S9 and others is:

If q/s is not a constant,
then there is no linear relationship between s and q .

From this survey, we can see that the subjects evoked various strategies and numerous heuristics when they tried to find a law within the given data. BACON's heuristics were used very frequently by the subjects, although some of the objects to which BACON's heuristic 4 or 5 were applied were different from those used by BACON. These heuristics were evoked in somewhat different ways by BACON and the human subjects. BACON uses its heuristics recursively, as explained at the beginning of this paper. The human subjects were not as systematic in their use. Often after the successful subjects evoked one of these heuristics they did not immediately follow up the result. Instead, they first tried some other heuristics before turning back to a new application of the BACON heuristics. Or like S8, they sometimes combined hill-climbing with BACON's Heuristic 4. After using one of BACON's heuristics, unsuccessful subjects generally neither followed up immediately nor return to it later.

Discussion and General Summary

In these experiments, 14 subjects tried to rediscover Kepler's third law using given data. Four succeeded, one came very close, and the other nine failed. What do we learn from the behavior patterns observed in these subjects?

We learn, first of all, that data-driven discovery of a scientific law does not call for unknown or mysterious problem solving processes. Kepler's discovery of his third law was an event of great significance in the history of science. It is regarded as a discovery of first magnitude. From the fact that, with given data, four out of 14 subjects could rediscover this law within one hour, and from the search processes revealed in the subjects' protocols, we can say that some significant discoveries can be made simply by application of the general processes that have been observed in all kinds of problem solving.

Generally, the data driven discovery observed in these experiments is a process of interactive search of a hypothesis space and an instance space, as proposed by Simon and Lea (1974). In these experiments, the hypothesis space has two levels: the level of function type and the parameter level.

There are two stages in the process of discovery, an initial stage of understanding the problem and the data, and a subsequent stage of search. The basic search strategy used by the subjects appears to be best-first search, using a variety of criteria to determine in what direction the search should continue. In terms of the acquisition of new information, the search can incorporate feedback from the results of testing hypotheses or can simply employ "succeed-fail" tests. The effective use of feedback to guide search is a prerequisite for using heuristics successfully.

In the two-level model there must be guidance for both function selection and parameter selection. Scatter diagrams and graphs are important tools for function

selection; while abstraction and other transformations of the data, along with applications of heuristics like those employed in the BACON program are valuable tools for parameter selection, even though sometimes subjects, like SY, employed abstraction to check the function type. Some subjects devote most effort to function selection, others to parameter selection by manipulation of the data, some combine both methods.

Large differences are observed among the strategies of different subjects, and these differences are sufficient to distinguish successful from unsuccessful subjects, as indicated above. The experimental data do not determine sufficient conditions for success in data-driven discovery of scientific laws (although of course the behavior of the successful subjects does exhibit such conditions for this particular law). However, the data do illustrate some necessary conditions: (1) possessing essential knowledge of the domain, (2) applying good search strategies, (3) using heuristics appropriately and systematically, and (4) searching at both function level and parameter level.

We may compare the behavior of the subjects in this experiment with the behavior of the BACON program when it is given the same task. Kepler's third law is only one of the laws rediscovered by BACON, and exercises only a subset of BACON.4's capacities. In trying to rediscover Kepler's third law, most human subjects need to generate and choose among different function types. BACON, because of its structure, need not do so. It carries out its search using only linear functions and ratios.

BACON's Heuristics 1, 2, and 3 are used by almost every subject in our experiments. Successful subjects also used procedures closely resembling Heuristics 4 and/or 5 successfully; heuristic 4 being used more often than 5. Unsuccessful subjects used Heuristics 4 and 5 only very unsystematically, or used them inappropriately. The heuristics used by the subjects are perhaps more general and

flexible than those incorporated in BACON -- for the most part, they can be regarded as forms of means-ends analysis. Sometimes, they are too general to be effective: for example, "If a variable increases, try to decrease it; if it decreases, try to increase it."

Scientific Discovery in History

One motive for this research, the one we have mainly discussed so far, was to characterize human problem solving in a data-driven discovery task, where a priori theory could play no role, and to compare it with the general theoretical model proposed by Simon and Lea (1974) and the more specific theory implemented in BACON (Langley et al., 1987).

Another motive was to see what light such an experiment could cast on an actual historical instance of discovery in a case where there is reasonably good evidence that the discovery was driven by the data and received no substantial guidance from relevant pre-existing theory.

The present experiment was preceded by two others, one informal and the other not yet published, relating to other scientific discoveries. In the informal experiment, five out of eight subjects, given a qualitative description of the data Max Planck had available in October 1900, found Planck's law of blackbody radiation in less than two minutes each. (Planck himself found it, in purely data-driven fashion, in not more than a few hours.) This experiment and the history behind it are recounted by Langley et al. (1987), pp 47-54. The processes used by subjects to find Planck's law (and the processes that, from the historical evidence, Planck used) are processes commonly used by skilled applied mathematicians (which the subjects in that experiment were).

In the other experiment (Kulkarni and Simon, unpublished), a single subject, a

chemical engineering graduate student, employed full-time for the task, succeeded in rediscovering Balmer's formula for the hydrogen spectrum in about 60 hours' working time. Balmer accomplished the same task (in 1883) in some weeks of part-time work. Both the subject and Balmer worked with the same data, and without any relevant pre-existing theory (none existed in Balmer's time). The tape-recorded protocol and notebooks of the subject reveal just the same kind of search as we have described for the subjects in our present experiment, and as are revealed in such documents as Balmer left behind.

Returning to Kepler, we observed earlier that not too much detail is known about how he derived the third law; certainly we can not follow his work on a day-to-day basis. He wrote quite voluminously, however, about his goal and motivations, and, in the manner of his age, was explicit about his philosophical assumptions. We have studied his views with care, especially the Epitome of Copernican Astronomy (1618-21/1952), and Harmonies of the World (1619/1952), as have a number of historians of science, and find that these works provide a consistent view of his procedures.

Kepler's work is characterized by a painstaking attention to data, especially the magnificent data to which he fell heir on the death of his employer, Tycho Brahe. The greatest part of his occupation for a quarter century was working these and earlier data into a parsimonious Copernican description of the heavens. The three laws that bear his name were essential steps along the way. In these respects Kepler was a data-driven discover of laws.

But Kepler was not satisfied with a mere description of the phenomena -- the "geometry" as he regarded it. He wanted to trace the behavior of the Sun, stars and planets to their physical causes. Kepler insistently sought to know not only how things are, but why they have to be that way. Kepler was deeply concerned with

theory.

What roles did data and theory play in Kepler's discovery of his third law? First, he did not invent the problem of the relation between the distances of the planets and their periods. The problem had already been discussed at least as far back as Aristotle (On the Heavens, Bk.ii, ch. 10), and Aristotle had observed that the outer planets moved more slowly than the inner ones. Second, the rather precise data that Kepler used to discover the third law were partly products of his successful investigations of the paths of the planets, in the course of which he found the elliptical shape of their orbits, and from which he could make accurate calculations of the mean diameters of their orbits. (Accurate data on the periods of revolution had already been provided by Brahe and others, and reasonably accurate data on the diameters by Copernicus.)

It is sometimes argued that the real problem of scientific discovery is not to find laws in data but to define the problem and to discover the relevant data. But we have just seen that defining the problem and discovering the data were not Kepler's primary contribution. The problem of describing the heavens parsimoniously he inherited from a long line of predecessors, and the data, as explained above, he mainly inherited from Brahe and Copernicus. His merit was that he converted the data to a form that revealed the geometry of the heavens and laid the foundation for Newton's inertial and gravitational explanation. From a scientific standpoint, his attempts to provide "physical" explanations for his empirically derived laws are now only historical curiosities.

After he had found the third law, Kepler searched for causes, as he had done a decade earlier: when he had erroneously concluded that the periods of the planets varied as the squares of their distances from the Sun. The Sun was the cause, which as it rotated on its axis swept with it the objects (planets) in the space around

it, with a force that became more feeble with distance. How feeble? Just feeble enough to account for the observed law. There were enough hypothetical variables--- the sizes of the planet, their masses (unmeasured), the rate of attenuation of the force---to account for a linear, a square law or a $3/2$ power law, or about any law that the data revealed.

In the philosophical style of his day, the ad hocness of causal explanations was no great concern. The data had revealed a pattern, and causes must exist. Kepler's attitude on this point is quite clear in his treatment of another problem where his "causes" and the data didn't quite agree. One of his great passions was to explain the distance between successive planets in terms of spheres inscribed in, and circumscribed about, the five regular solids (Harmonies of the World, Bk.V, ch.1-3). When the data did not fit the hypothesis, Kepler did not dismiss either hypothesis or data, but openly admitted the discrepancy. Then he sought additional causal forces (celestial musical harmonies in this case) to remedy the defects. The point is that regularities of data came first; causes had to be shaped to fit them.

In words Poincare used to discuss difficulties in the development of the theory of special relativity, "An explanation was necessary, and was forthcoming; they always are. hypotheses are what we lack the least." (quoted in Miller 1984.p.65).

There is every reason to believe, therefore, that Kepler found his third law by examining the data, much as our subjects did. In 1596, as a young man of 25, he asked, as did some of our subjects, whether the ratios of the periods of any two planets might vary as the ratio of their distances. Finding that the ratios of the periods were too large, he tried alternative functions, and arrived at the quadratic law (period varies with the square of the distance). He published this formula thirteen years later, in 1609. Like some of our subjects (e.g., SW, S10), he was then satisfied with the approximate fit of this formula to the data. Moreover, he tried to

support it by the hypothesis that the period was equal to the ratio of the length of the orbital path divided by the strength of the Sun's driving force.

By 1618 Kepler, no longer satisfied with the empirical accuracy of the quadratic law, returned to the problem, and soon found the law we now regard as correct. According to Gingerich (1975), "Kepler says it was conceived on March 8th of this year, 1618, unfelicitously submitted to calculation and rejected as false, and recalled only on May 15 when by a new onset it overcame by storm the darkness of my mind with such full agreements between this idea and my labor of seventeen years on Brahe's observations" It is a pity that he did not leave behind a record of the heuristics he used.

Conclusion

We have already summarized our empirical findings, and have commented on their implications both for the theory of discovery as problem solving, and for historical scientific discoveries. It only remains to put data-driven discovery, like that examined here, in a broader context of scientific activity.

Science is an incremental, cumulative process. No single step in that process is "the real discovery." As Langley et al. (1987) point out, scientists define problems and find new ways of representing them. They generate new phenomena and new data, sometimes with the help of new instruments they or others have invented. With the guidance of data or theories, or both, they find new laws to describe data, and new concepts and mechanisms to explain why the laws hold. They test theories and communicate their findings. All of these and perhaps others, are the incremental steps that make up the cumulative process of scientific discovery.

In this paper, we have examined one class of these incremental steps, data-driven discovery, and have found that it proceeds in the same manner as many other

problem-solving processes that have been studied and described. We believe that this result can be generalized to cover most, perhaps all, of the processes of scientific discovery. But of course, to demonstrate that will require carrying out many, many more incremental steps of the same kind.

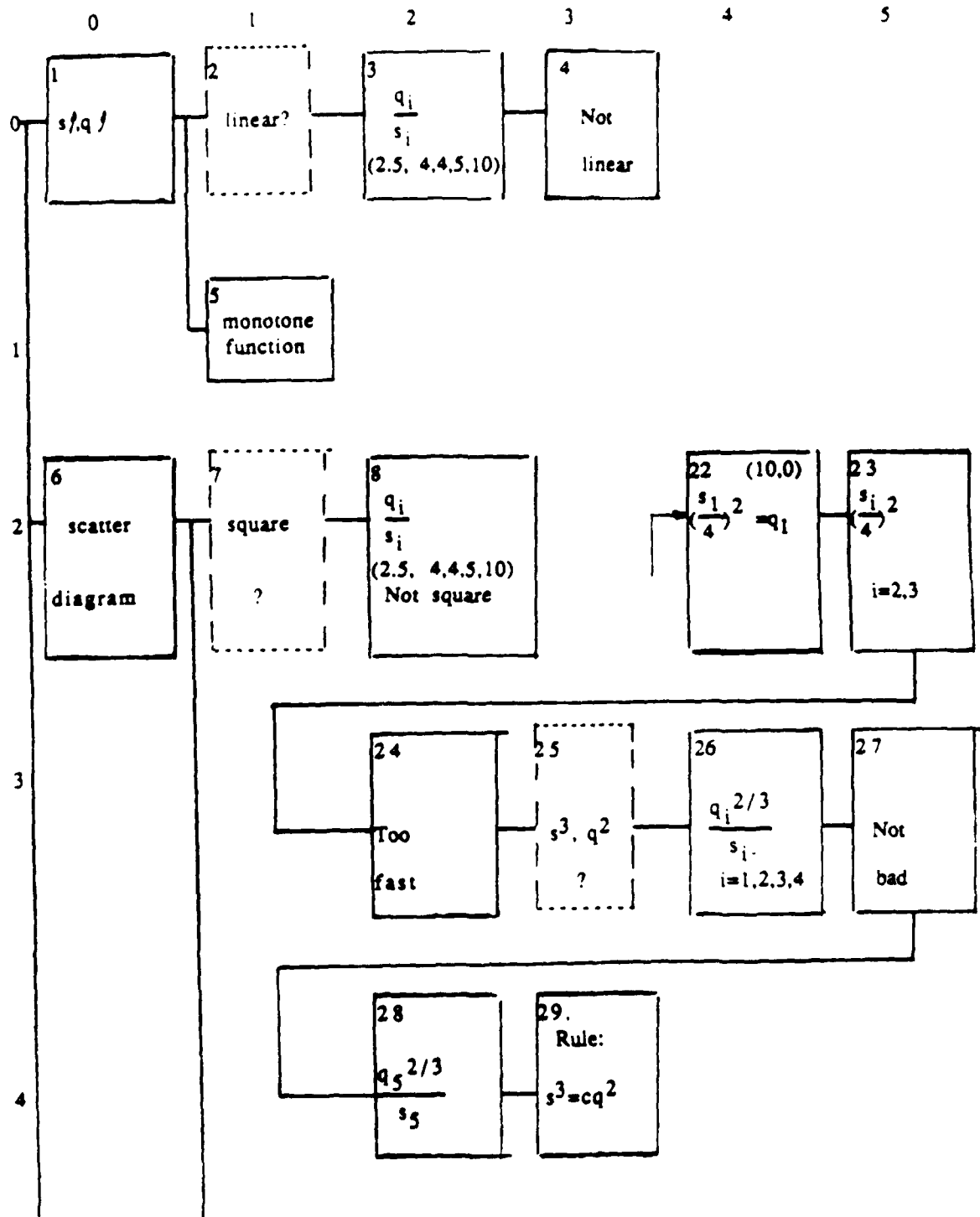
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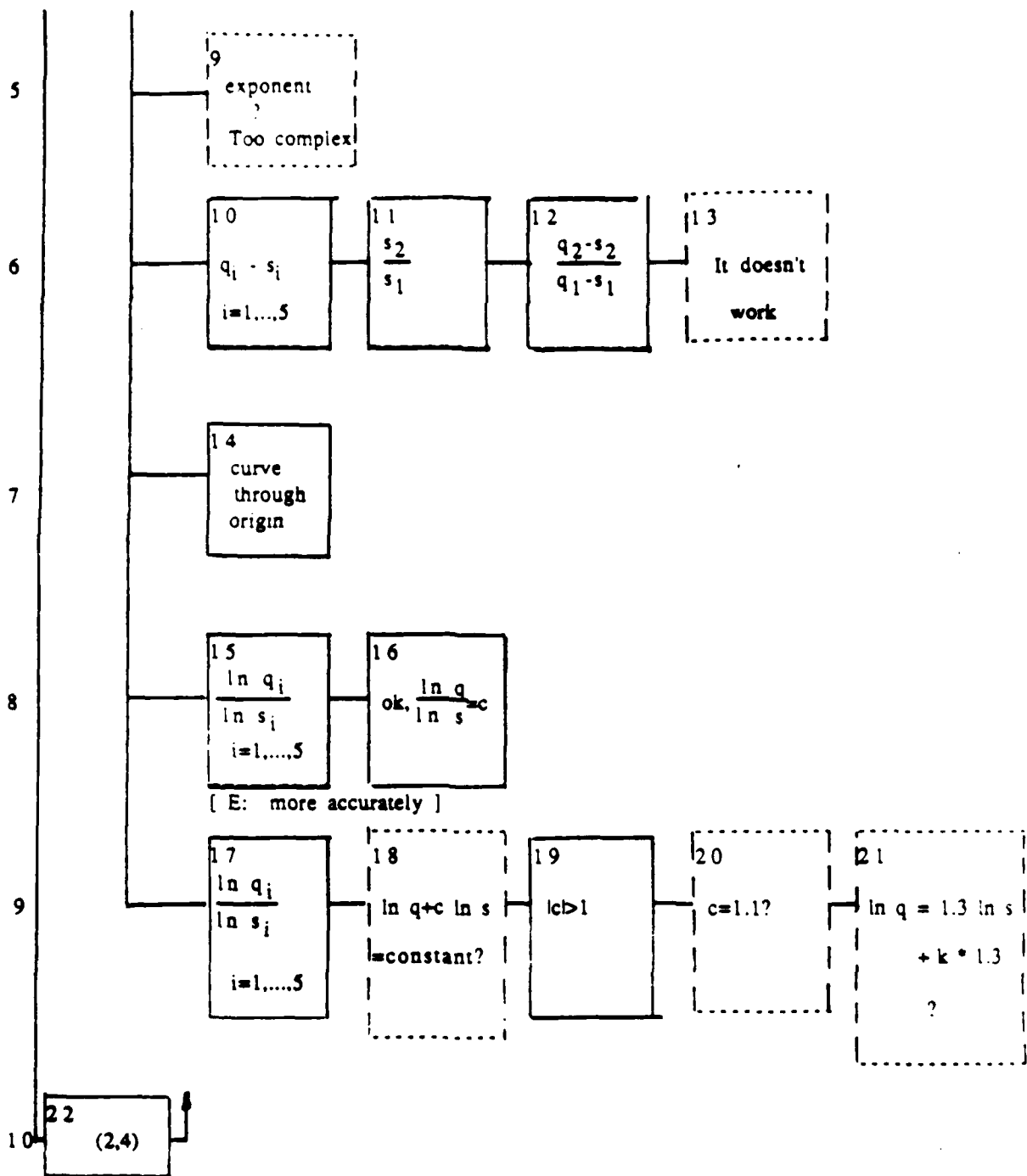
APPENDIX

The problem Behavior Graphs of Subjects SY and S3

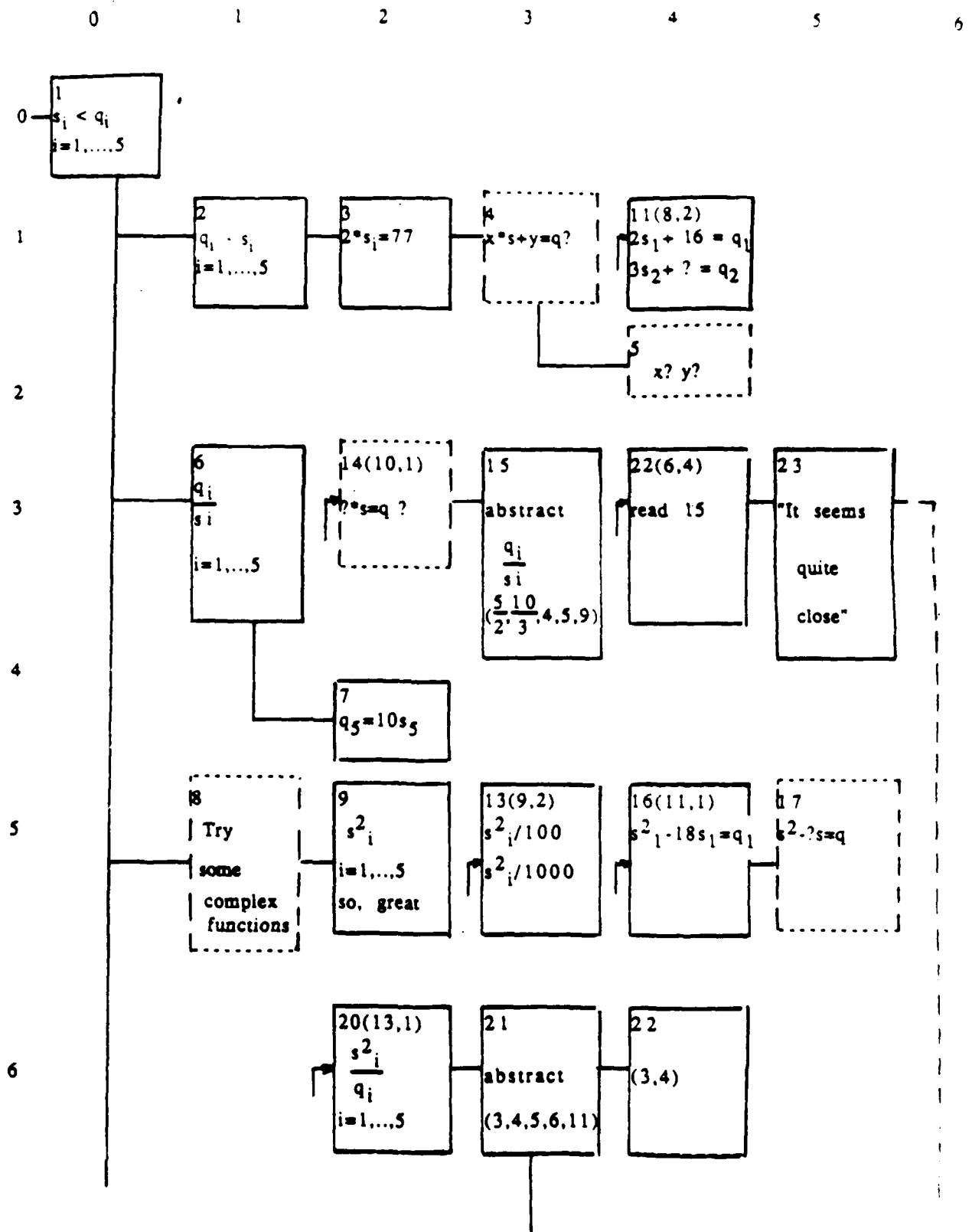
Subject SY

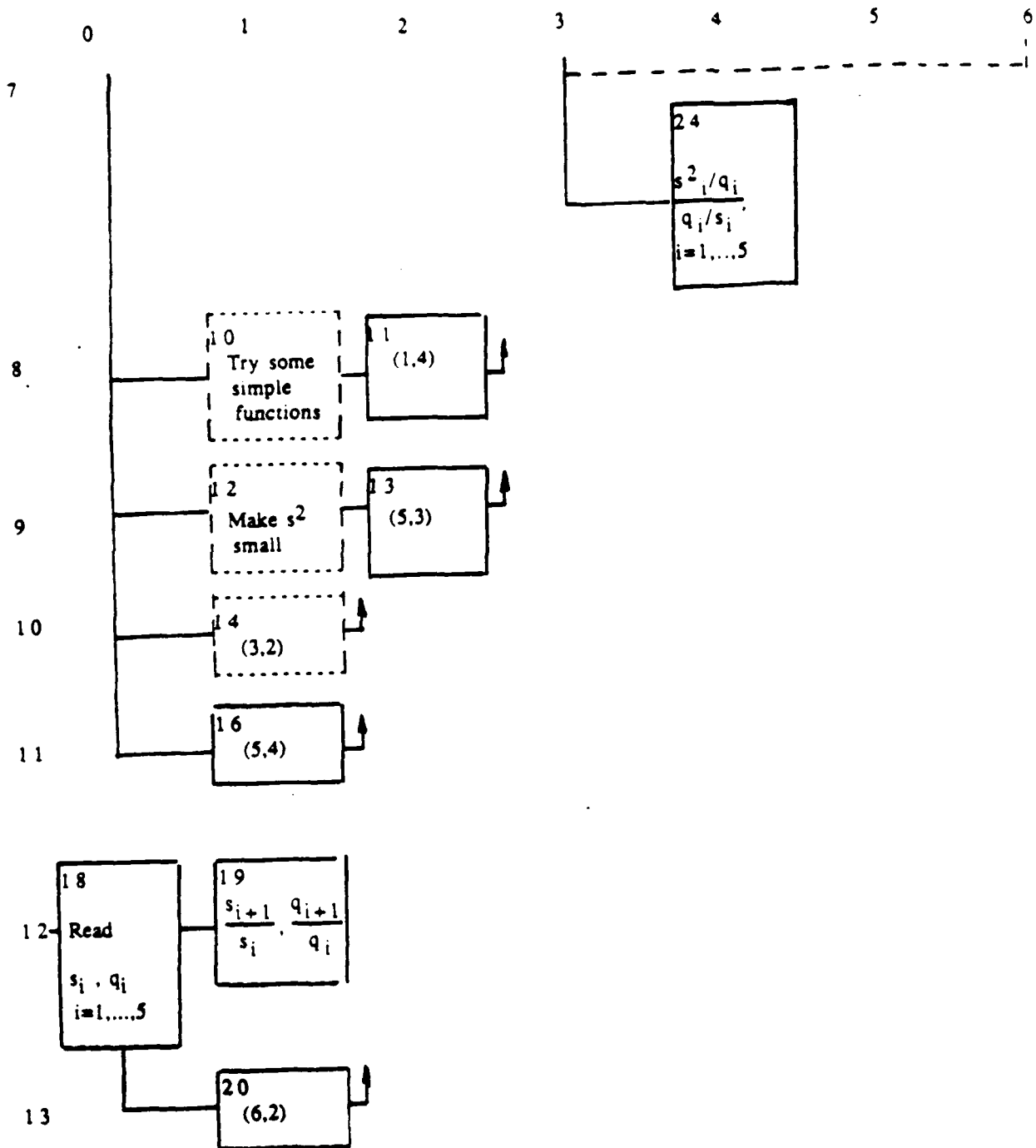


0 1 2 3 4 5



Subject S3





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TABLES

s	q
36	88
67.25	224.7
93	365.3
141.75	687
483.8	4332.1

Table 1: The data given to subjects in Experiment 1
(s = Distance; q = Period of Revolution)

Subject	Situation	The best results
S1	Sophomore.	$s/q=c$, $s^2/q=c$, $s^{1.25}=q$
S2	Senior.	$\ln q/\ln s=c$
S3	Freshman.	$s^3/6.025=q^2$ (correct)
S4	Junior EE.	$s^{1.49}=q$ (nearly correct)
S5	Freshman.	$88-2*36=16$ ($q_i=ks_i+b$, $i=1,2,3,4$)? $22^2=484$
SY	Grad in Phys.	$q^{2/3}=0.55s$ (correct)
SW	Engineer.	$s^2/q=c$, $s^3/q=c$
SG	Grad in Art	$q_1/s_1=x_1*y$
	History.	$q_2/s_2=x_2*y$ $q_3/s_3=x_3*y$
SJ	Grad in Edu.	$q=2^x*s+b$

Table 2: Subjects and their best results in Experiment 1
(s = Distance; q = Period of Revolution)

1. FIND-LAWS

If you want to iterate through the values of independent term I, and you have iterated through all the values of I, then try to find laws for the dependent values you have recorded.

2. CONSTANT

If you want to find laws,
and the term D has value V in all data clusters,
then infer that D always has value V.

3. LINEAR

If you want to find laws,
and you have recorded a set of values for the term X,
and you have recorded a set of values for the term Y,
and the values of X and Y are linearly related
with slope M and intercept B,
then infer that a linear relation exists between X and Y
with slope M and intercept B.

4. INCREASING

If you want to find laws,
and you have recorded a set of values for the term X,
and you have recorded a set of values for the term Y
and the absolute values of X increase,
as the absolute values of Y increase,
and these values are not linearly related,
then consider the ratio of X and Y.

5. DECREASING

If you want to find laws,
and you have recorded a set of values for the term X,
and you have recorded a set of values for the term Y
and the absolute values of X increase,
as the absolute values of Y decrease,
and these values are not linearly related,
then consider the product of X and Y.

Table 3: BACON.1's rules for noting regularities

	FUNCTION						DIAGRAM	
	linear	sequen- tial	quad- ratio	log	cubic	others	total	graph scatter
S1	3	3	2	3	0	2	13	2
S2	1	7	1	4			13	0.5
S3	5	1	6		1		13	
S4	7	2.5	1	1		6	17.5	3
SY	3	1.5	1	2	1		8.5	1
SW	2	3.5	1		2		8.5	1
Total	21	18.5	12	10	4	8	73.5	2 5.5

Table 4: Numbers of references to functions, by type, in Experiment 1

	linear	sequential	quadratic	log	cubic	others	total
S1	23	23	15.5	23	0	15.5	100
S2	7.7	53.8	7.7	30.8	0	0	100
S3	38.5	7.7	46.1	0	7.7	0	100
S4	40	14.3	5.7	5.7	0	34.3	100
SY	35.3	17.6	11.8	23.5	11.8	0	100
SW	23.5	41.2	11.8	0	23.5	0	100
Average	28.6	25.2	16.3	13.6	5.4	10.9	100

Table 5: Function references, percentages of total, in Experiment 1

	q/s*	s ² or s ² /q*	q-s	s _q /s _i q _q /q _i	s [*] q _q -q _i	s _i	lnq/lns	sum	total	%
S1	3	2		1	2	1		9	13	69.2
S2	1			1		1	1	4	13	30.8
S3	3	3	1	1				8	13	61.5
S4	3		2	0.5	2		1	8.5	17.5	48.6
SY	2	1	1	0.5			1	5.5	8.5	64.7
SW	1	0	1	0.5		1		4.5	8.5	52.9
Total	13	6	5	4.5	4	3	3	38.5	73.5	52.4

Table 6: References to seven functions, in Experiment 1

Subject	Situation	The best results
S6	Freshman, Phys.	q^2/s^2
S7	Sophomore, EE/Econ.	$\frac{q^2}{b^2} - \frac{s^2}{a^2} = c$
S8	Sophomore, Chem. Eng.	$s^{3/2}/q = c$ (correct)
S9	Grad, in Civil Eng.	$q^2 = as^3$ (correct)
S10	Freshman, Math.	s_5^2/q_5

Table 7: Subjects and their best results in Experiment 2
(s = Distance; q = Period of Revolution)

	linear	sequen- tial	FUNCTION quad- ratic	cubic	others	total	DIAGRAM graph scatter
S6	8	4	1		2	15	1
S7	2	2	2		1	7	1
S8	2		5	3	2	12	
S9	3	1	1	2	1	8	1
S10		3	3	1	1	8	2
Total	15	10	12	6	7	50	5

Table 8: Numbers of references to functions, by type, in Experiment 2

	linear	sequential	quadratic	cubic	others	total
S6	53.3	26.7	6.7	0	13.3	100
S7	28.6	28.6	28.6	0	14.2	100
S8	16.7	0	41.6	25	16.7	100
S9	37.5	12.5	12.5	25	12.5	100
S10	0	37.5	37.5	12.5	12.5	100
Average	30	20	24	12	14	100

Table 9: Function references, percentages of total, in Experiment 2

	q/s	$s_n - s_i$ $q_n - q_i$	s_i^2 or s_i^2/q	$s_i^2 + bs + c = q$	s_i^2/q^2	s_i^3 or s_i^3/q	sum	total	%
S6	4	3	0	0	1	0	8	15	53.3
S7	2	2	0	1	0	0	5	10	50
S8	1	0	2	0	1	3	7	12	58.3
S9	1	0	0	1	0	1	3	6	50
S10	0	1	1	2	0	0	4	7	57.1
Total	8	6	3	4	2	4	27	50	54

Table 10: References to seven functions, in Experiment 2

FIGURES

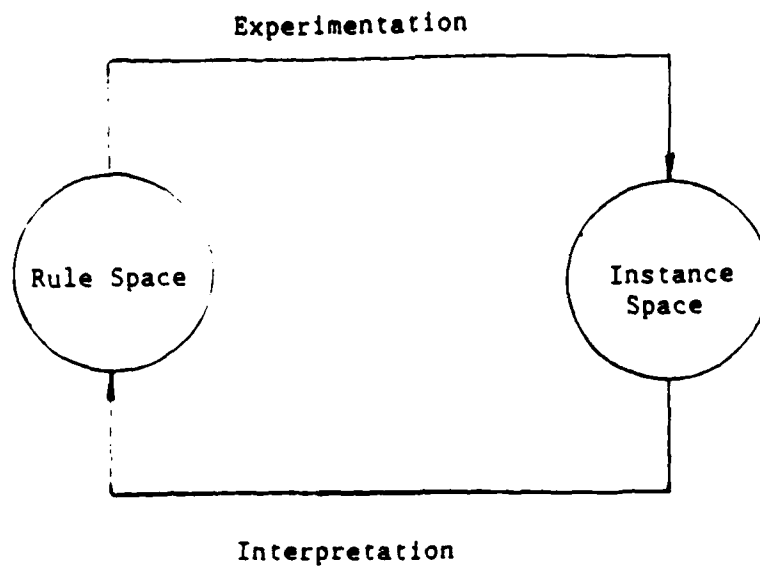


Figure 1. The General Model

Productions for Finding a Law

1. The goal is to find a law;
There is a hypothesis;
-->
Test the hypothesis.
2. The goal is to find a law;
There is a hypothesis;
The result of testing is "Success";
-->
The hypothesis is the law to be found;
Halt.
3. The goal is to find a law;
There is a hypothesis;
The result of testing is "Failure";
-->
Set the hypothesis as a used-hypothesis.
4. The goal is to find a law;
There is no hypothesis;
-->
Set subgoal: build a hypothesis.
5. The goal is to find a law;
There is no hypothesis;
There is a used-hypothesis;
-->
Set subgoal:analyse the used-hypothesis,
find the trend of the data,
build a hypothesis.

Productions for Building a Hypothesis

6. The goal is to build a hypothesis;
-->
Try to find the trend of the data.
7. The goal is to build a hypothesis;
There is a trend;
-->
Form a hypothesis.
8. The goal is to build a hypothesis;
There is a trend;
-->
Select a function type.
9. The goal is to build a hypothesis;
There is a function type;
-->
Check the function type.

10. The goal is to build a hypothesis;
There is a function type;
The result of checking the function type is "Failure";
-->
Delete the function type.
11. The goal is to build a hypothesis;
There is a function type;
There is a trend;
-->
Select a set of parameters, form a hypothesis.
12. The goal is to build a hypothesis;
There is a function type;
There is a used-hypothesis;
There is a set of parameters;
-->
Change the parameters, form a hypothesis.
13. The goal is to build a hypothesis;
There is a trend;
-->
Select a function (including the parameters),
form a hypothesis.

Productions for Finding a Trend

14. Try to find the trend of the data;
-->
Draw a diagram, analyse the diagram,
set the trend of the data.
15. Try to find the trend of the data;
-->
Transform the data, set the trend of the data.

Figure 2. The production system of the detailed model

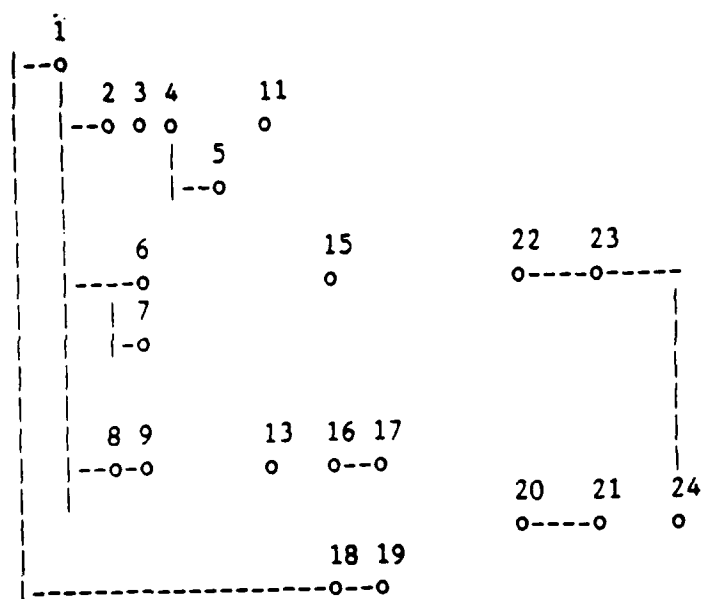


Figure 3. S3's Simplified PBG