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Some battlefield models have a component in them which models the time it takes for an observer to detect a target. Different observers may have different mean detection times due to various factors such as the type of sensor used, environmental conditions, fatigue of the observer, etc. Two parametric models for the distribution of time to target detection are considered which can incorporate these factors. Maximum likelihood estimation procedures for the parameters are described. Results of simulation experiments to study the small sample behavior of the estimators are presented.

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Models for Target Detection Times

by

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ABSTRACT

Some battlefield models have a component in them which models the time it takes for an observer to detect a target. Different observers may have different mean detection times due to various factors such as the type of sensor used, environmental conditions, fatigue of the observer, etc. Two parametric models for the distribution of time to target detection are considered which can incorporate these factors. Maximum likelihood estimation procedures for the parameters are described. Results of simulation experiments to study the small sample behavior of the estimators are presented.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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I. INTRODUCTION

Some battlefield models have a component in them which models the time it takes for an observer to detect a target. A common model is that the time to detect a target is a random variable having a distribution $P_{\infty}(1 - e^{-\lambda t})$, [Ref. 1]. However different observers may have different mean detection times due to various factors such as the type of sensor used, environmental condition, fatigue of the observer, etc. In this thesis we will consider two parametric models for the distribution of the time to detection which can incorporate these factors.

Suppose there are M observers, such as tank crews. Observer i is presented with N_i targets. In Chapter 2, observer i has a parameter A_i which reflects the ability of the observer to detect a target. The parameters of the M observers, $A_1, A_2, A_3, \ldots, A_M$, are assumed independent having a common gamma distribution. Given A_i, the detection times of observer i are conditionally independent, having Weibull distributions. In chapter 2 the parameters of the conditional Weibull distributions are assumed known and interest is in estimating the parameters of the gamma distribution.

In chapter 3 observer i has explanatory variables $x_{ij1}, x_{ij2}, x_{ij3}, \ldots, x_{ijp}$ relating to his j^{th} target representing factors which influence his time to detection. The detection times for the observers are independent random variables having Weibull distributions. The location parameter of the Weibull distribution for the detection time of the j^{th} target by the i^{th} observer is of the form $\mu_{ij} = e^{\underline{x}\,\underline{\beta}}$ where $\underline{x}\,\underline{\beta} = \sum_{k=1}^{p} x_{ijk}\beta_k$; the shape parameter is of the form e^{ξ}. Interest is in estimation of $\{\beta_k\}$ and $\{\xi_i\}$.

In both chapters the data for the observers can be censored. The i^{th} observer has an opportunity time O_{ij} to detect his j^{th} target. If the j^{th} target is not detected in time O_{ij}, the time to detection is censored. Data for the i^{th} observer consist of (possibly censored) times to detect the N_i targets.

Chapter 2 and 3 present iterative procedures based on maximum likelihood to estimate the parameters. Results from simulation studies of small sample size behavior of the estimators are given.

These models and estimation procedures should be of eventual use in the Army MANPRINT program, an objective of which is to better understand the human contribution to battlefield performance [Ref. 2].

II. A HIERARCHICAL MODEL FOR TIMES TO DETECTION

A. MODEL

Suppose there are M observers who attempt to detect targets on a battlefield. Observer i has a parameter A_i , $i = 1,2,3,\ldots,M$, which reflects the ability of the observer to detect a target. Observer i is presented with N_i targets. The targets are presented one at a time. Let U_{ij} be the time it takes for observer i to detect target j. Assume, given $A_i = \theta$, the U_{ij}, $j = 1,2,3,\ldots,N_i$, are conditionally independent random variables with Weibull distributions

$$P\{U_{ij} \le t \mid A_i = \theta\} = 1 - \exp\left\{ - \theta(t/\mu_{ij})^{e^{-\xi_i}} \right\} \qquad t \ge 0 \tag{2.1}$$

independent of other observers.

Further, assume the parameters A_i, $i = 1,2,3,\ldots,M$, are independent identically distributed having a Gamma distribution with density function as follows:

$$g(\theta) = \frac{\alpha(\alpha\theta)^{\gamma-1}}{\Gamma(\gamma)} e^{-\alpha\theta} \tag{2.2}$$

where $\theta > 0$. The A_i variations are introduced to represent individual observer differences. Now let

$$Z_{ij} = \ln U_{ij} \ .$$

It follows that

$$\begin{aligned} P\{Z_{ij} \le z \mid A_i = \theta\} &= P\{U_{ij} \le e^{z} \mid A_i = \theta\} \\ &= 1 - \exp\left\{ - \theta[\exp(z - \ln \mu_{ij})]^{e^{-\xi_i}} \right\} \ . \end{aligned} \tag{2.3}$$

Hence, the conditional density function of Z_{ij}, given $A_i = \theta$, is

$$f_{Z_{ij} \mid A_i}(z \mid \theta) = \theta \exp\left\{(z - \ln \mu_{ij})e^{-\xi_i}\right\} e^{-\xi_i} \exp\left\{ - \theta\left(\exp\left\{(z - \ln \mu_{ij})e^{-\xi_i}\right\}\right)\right\} \ . \tag{2.4}$$

When the i^{th} observer is presented with his j^{th} target, he gets a length of time called opportunity time O_{ij} to detect it. An observer either successfully detects the target within this time or is unsuccessful. Data for the i^{th} observer consist of times of detections for the successes and the lengths of opportunity times for the failures. For each i = 1,2,3,...,M , j = 1,2,3,...,N_i , let

$$Y_{ij} = \min(\ln U_{ij}, \ln O_{ij}) \tag{2.5}$$

and

$$\Delta_{ij} = \begin{cases} 1 & \text{if } \; U_{ij} \le O_{ij} \\ 0 & \text{otherwise} \; . \end{cases} \tag{2.6}$$

The Y_{ij} are the censored ln-detection times and Δ_{ij} is an indicator of whether or not the ln-time to detect the j^{th} target by the i^{th} observer is censored. Let

$$C_i = \sum_{j=1}^{N_i} \Delta_{ij} \tag{2.7}$$

be the number of targets detected by observer i.

In this Chapter we will assume $\{\mu_{ij}\}$ and $\{\xi_i\}$ are known constants. We are interested in estimating the parameters η and β_0 with $\alpha \equiv e^{\eta}$ and $\frac{\gamma}{\alpha} \equiv e^{\beta_0}$ using maximum likelihood. These parameter estimates can be used to predict future times to detection for an observer given his past performance. In the next section we describe a Newton - Raphson procedure for solving the likelihood equations, [Ref. 3], [Ref. 4]. In the final section of this chapter we describe a simulation experiment to study the small sample properties of the estimators. Results of the simulation are also presented.

B. ESTIMATION

1. The Likelihood Equation and Maximum Likelihood Estimates

Given $A_i = \theta$, it follows from equation (2.3) and (2.4) that the conditional likelihood function for observer i using the censored ln-times y_{ij} is

$$L_i(\underline{\mu}, \underline{\xi}; \theta) = \prod_{j=1}^{N_i} \Big[\theta e^{(y_{ij} - \ln \mu_{ij})e^{-\xi_i}} e^{-\xi_i} \Big]^{\Delta_{ij}} \exp\Big[- \theta e^{(y_{ij} - \ln \mu_{ij})e^{-\xi_i}} \Big] \; . \tag{2.8}$$

Let

$$S_i = \sum_{j=1}^{N_i} \exp\left[(y_{ij} - \ln \mu_{ij}) e^{-\xi_i}\right] . \tag{2.9}$$

Rewriting equation (2.8)

$$\begin{aligned} L_i(\underline{\mu}, \underline{\xi}; \theta) &= \theta^{C_i} \exp\left\{ \sum_{j=1}^{N_i} \Delta_{ij}\left[(y_{ij} - \ln \mu_{ij}) e^{-\xi_i} - \xi_i\right] \right\} \exp(-\theta S_i) \\ &= \theta^{C_i} K_i \exp(-\theta S_i) \end{aligned} \tag{2.10}$$

where

$$K_i = \exp\left\{ \sum_{j=1}^{N_i} \Delta_{ij}[(y_{ij} - \ln \mu_{ij}) e^{-\xi_i} - \xi_i] \right\}. \tag{2.11}$$

The unconditional likelihood for observer i is

$$\begin{aligned} L_i(\underline{\mu}, \underline{\xi}, \alpha, \gamma) &= \int_0^\infty L_i(\underline{\mu}, \underline{\xi}; \theta)\ g(\theta) d\theta \\ &= K_i \int_0^\infty \theta^{C_i} e^{-\theta S_i} \frac{\alpha(\alpha\theta)^{\gamma-1}}{\Gamma(\gamma)} e^{-\alpha\theta} d\theta \\ &= K_i \alpha^\gamma \left(\frac{1}{S_i + \alpha} \right)^{C_i + \gamma} \prod_{k=0}^{C_i - 1} (\gamma + k) . \end{aligned} \tag{2.12}$$

If $C_i = 0$, then $\prod_{k=0}^{C_i-1} (\gamma + k) = 1$. Recall the parametrization $\alpha = e^\eta$ and $\frac{\gamma}{\alpha} = e^{\beta_0}$. The unconditional ln-likelihood function for observer i can be rewritten as

$$\begin{aligned} \ln L_i(\underline{\mu}, \underline{\xi}, \eta, \beta_0) &= \ln K_i + e^{\eta + \beta_0}\eta - (C_i + e^{\eta + \beta_0}) \ln(S_i + e^\eta) \\ &\quad + \sum_{k=0}^{C_i - 1} \ln(e^{\eta + \beta_0} + k) \end{aligned} \tag{2.13}$$

where if $C_i = 0$, then $\sum_{k=0}^{C_i-1} \ln(e^{\eta - \beta_0} + k) = 0$. Since the observers are independent, the unconditional ln-likelihood for all M observers is

$$\ln L = \sum_{i=1}^{M} \left\{ \ln K_i + e^{\eta + \beta_0}\eta - (C_i + e^{\eta + \beta_0}) \ln(S_i + e^\eta) + \sum_{k=0}^{C_i - 1} \ln(e^{\eta + \beta_0} + k) \right\} .$$

(2.14)

The derivative of the ln-likelihood with respect to η is

$$\frac{\partial \ln L}{\partial \eta} = \sum_{i=1}^{M}\left\{e^{\eta+\beta_0}\eta + e^{\eta+\beta_0} - e^{\eta+\beta_0}\ln(S_i + e^{\eta}) - \left(\frac{C_i + e^{\eta+\beta_0}}{S_i + e^{\eta}}\right)e^{\eta}\right\} + \sum_{i=1}^{M}\sum_{k=0}^{C_i-1}\left(\frac{e^{\eta+\beta_0}}{e^{\eta+\beta_0}+k}\right). \tag{2.15}$$

The derivative with respect to β_0 is

$$\frac{\partial \ln L}{\partial \beta_0} = \sum_{i=1}^{M}\left\{e^{\eta+\beta_0}\eta - e^{\eta+\beta_0}\ln(S_i + e^{\eta}) + \sum_{k=0}^{C_i-1}\frac{e^{\eta+\beta_0}}{e^{\eta+\beta_0}+k}\right\}. \tag{2.16}$$

We are assuming $\{\mu_{i_j}\}$ and $\{\xi_i\}$ are known. The problem is to find the maximum likelihood estimates of η and β_0; that is, find η and β_0 such that $\frac{\partial \ln L}{\partial \eta} = 0$ and $\frac{\partial \ln L}{\partial \beta_0} = 0$. Note that

$$\frac{\partial \ln L}{\partial \eta} = \frac{\partial \ln L}{\partial \beta_0} + \sum_{i=1}^{M}\left[e^{\eta+\beta_0} - \left(\frac{C_i + e^{\eta+\beta_0}}{S_i + e^{\eta}}\right)e^{\eta}\right]. \tag{2.17}$$

Thus, if $\frac{\partial \ln L}{\partial \beta_0} = 0$, then to solve the equation $\frac{\partial \ln L}{\partial \eta} = 0$, we need to solve the equation

$$
\begin{aligned}
0 = f(\eta, \beta_0) &= \sum_{i=1}^{M} \left[e^{\eta + \beta_0} - \left(\frac{C_i + e^{\eta + \beta_0}}{S_i + e^{\eta}} \right) e^{\eta} \right] \\
&= \sum_{i=1}^{M} \frac{e^{\eta + \beta_0}(S_i + e^{\eta}) - C_i e^{\eta} - e^{2\eta + \beta_0}}{S_i + e^{\eta}} \\
&= \sum_{i=1}^{M} e^{\eta} \frac{(e^{\beta_0} S_i - C_i)}{S_i + e^{\eta}} \quad .
\end{aligned}
\tag{2.18}
$$

The derivatives of $f(\eta, \beta_0)$ with respect to η and β_0 are as follows :

$$
\begin{aligned}
\frac{\partial f}{\partial \eta} &= - \sum_{i=1}^{M} \frac{(e^{\beta_0} S_i - C_i) e^{\eta}}{(S_i + e^{\eta})} \\
&= e^{\eta} f(\eta, \beta_0) \ ;
\end{aligned}
\tag{2.19}
$$

$$
\frac{\partial f}{\partial \beta_0} = \sum_{i=1}^{M} \frac{e^{\beta_0}}{S_i + e^{\eta}} \quad .
\tag{2.20}
$$

Solving the equation $\frac{\partial \ln L}{\partial \beta_0} = 0$ is equivalent to solving the equation

$$
0 = g(\eta, \beta_0) \equiv \sum_{i=1}^{M} \left[\eta - \ln(S_i + e^{\eta}) + \sum_{k=0}^{C_i - 1} \frac{1}{e^{\eta + \beta_0} + k} \right] .
\tag{2.21}
$$

The derivatives of $g(\eta, \beta_0)$ with respect to η and β_0 are

$$
\frac{\partial g}{\partial \eta} = \sum_{i=1}^{M} \left[1 - \frac{e^{\eta}}{S_i + e^{\eta}} - \sum_{k=0}^{C_i - 1} \frac{e^{\eta + \beta_0}}{(e^{\eta + \beta_0} + k)^2} \right] ;
\tag{2.22}
$$

$$\frac{\partial g}{\partial \beta_0} = -\sum_{i=1}^{M} \sum_{k=0}^{C_i - 1} \frac{e^{\eta + \beta_0}}{(e^{\eta + \beta_0} + k)^2} \tag{2.23}$$

where if $C_i = 0$, then the sum involving C_i - 1 is zero. A Newton procedure to solve the equations $\frac{\partial L}{\partial \eta} = 0$ and $\frac{\partial L}{\partial \beta_0} = 0$ would use the following equations

$$0 = f(\eta^0, \beta_0^0) + \frac{\partial f(\eta^0, \beta_0^0)}{\partial \eta}(\eta - \eta^0) + \frac{\partial f(\eta^0, \beta_0^0)}{\partial \beta_0}(\beta_0 - \beta_0^0) \tag{2.24}$$

$$0 = g(\eta^0, \beta_0^0) + \frac{\partial g(\eta^0, \beta_0^0)}{\partial \eta}(\eta - \eta^0) + \frac{\partial g(\eta^0, \beta_0^0)}{\partial \beta_0}(\beta_0 - \beta_0^0) \tag{2.25}$$

where η^0 and β_0^0 are current values for η and β_0. However note that if $f(\eta, \beta_0) = 0$, then $\frac{\partial f}{\partial \eta} = 0$. Hence, a Newton procedure that is more stable numerically would use the equations

$$0 = f(\eta^0, \beta_0^0) + \frac{\partial f(\eta^0, \beta_0^0)}{\partial \beta_0}(\beta_0 - \beta_0^0) \tag{2.26}$$

$$0 = g(\eta^0, \beta_0^0) + \frac{\partial g(\eta^0, \beta_0^0)}{\partial \eta}(\eta - \eta^0) \tag{2.27}$$

which results in

$$\beta_0 - \beta_0^0 = \frac{-f(\eta^0, \beta_0^0)}{\frac{\partial f}{\partial \beta_0}(\eta^0, \beta_0^0)} \tag{2.28}$$

$$\eta_0 - \eta_0^0 = \frac{-g(\eta^0, \beta_0)}{\frac{\partial g}{\partial \eta}(\eta^0, \beta_0^0)} \tag{2.29}$$

2. Initial Condition

In this subsection, we describe a rough way to provide initial estimates of η and

β_0 to start the iterative Newton procedure of the previous section.

The uncensored random variable U_{ij} has the same distribution as

$$U_{ij} = \mu_{ij}\left(\frac{W_{ij}}{A_i}\right)^{e^{\xi_i}} \tag{2.30}$$

where W_{ij} is unit exponential random variable. Let

$$\begin{aligned} Z_{ij} &= \ln U_{ij} \\ &= \ln \mu_{ij} + e^{\xi_i} \ln W_{ij} - e^{\xi_i} \ln A_i \ ; \end{aligned} \tag{2.31}$$

then

$$E[\ \ln W_{ij}] = -\ 0.5772 \tag{2.32}$$

$$VAR[\ \ln W_{ij}] = \frac{\pi^2}{6} \ . \tag{2.33}$$

[Ref. 5: p.943]. An approximation to the moments of $\ln A_i$ is given by the first two terms of a Taylor expansion of $\ln(A_i)$ about the mean $E[A_i] = \frac{\gamma}{\alpha}$

$$\ln(A_i) \cong \ln\left(\frac{\gamma}{\alpha}\right) + \frac{1}{\left(\frac{\gamma}{\alpha}\right)}\left(A_i - \frac{\gamma}{\alpha}\right); \tag{2.34}$$

thus

$$E[\ \ln A_i] \cong \ln \gamma - \ln \alpha \ ; \tag{2.35}$$

$$\begin{aligned} VAR[\ \ln A_i] &\cong \left(\frac{\alpha}{\gamma}\right)^2 VAR[A_i] \\ &= \left(\frac{\alpha}{\gamma}\right)^2 \frac{\gamma}{\alpha^2} \\ &= \frac{1}{\gamma} \ . \end{aligned} \tag{2.36}$$

Let

$$\begin{aligned} V_{ij} &= (Z_{ij} - \ln \mu_{ij})e^{-\xi_i} \\ &= \ln W_{ij} - \ln A_i \ . \end{aligned} \tag{2.37}$$

Hence,

$$E[V_{ij}] = -0.5772 - (\ln \gamma - \ln \alpha) \tag{2.38}$$

$$\begin{aligned} VAR[V_{ij}] &= VAR[\ln W_{ij}] + VAR[\ln A_{ij}] - 2cov(\ln W_{ij}, \ln A_i) \\ &= \frac{\pi^2}{6} + \frac{1}{\gamma} \quad . \end{aligned} \tag{2.39}$$

Rough estimates of the first and second moments of V_{ij} are given by

$$\hat{M}_1 = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N_i} \Delta_{ij}(Z_{ij} - \ln \mu_{ij}) e^{-\xi_i}}{\sum_{i=1}^{M} \sum_{j=1}^{N_i} \Delta_{ij}} \tag{2.40}$$

and

$$\hat{M}_2 = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N_i} \Delta_{ij} \left[(Z_{ij} - \ln \mu_{ij}) e^{-\xi_i} \right]^2}{\sum_{i=1}^{M} \sum_{j=1}^{N_i} \Delta_{ij}} \quad . \tag{2.41}$$

Recall the parametrization

$$\alpha = e^{\eta} \; ; \tag{2.42}$$

$$\frac{\gamma}{\alpha} = e^{\beta_0} \quad . \tag{2.43}$$

Thus

$$\gamma = e^{\eta} e^{\beta_0} \tag{2.44}$$

and

$$\begin{aligned} E[V_{ij}] &= -0.5772 - \ln \gamma + \ln \alpha \\ &= -0.5772 - [\eta + \beta_0] + \eta \\ &= -0.5772 - \beta_0 \; ; \end{aligned} \tag{2.45}$$

$$VAR[V_{ij}] = \frac{\pi^2}{6} + \frac{1}{\gamma}$$
$$= \frac{\pi^2}{6} + e^{-(\eta+\beta_0)} \quad . \tag{2.46}$$

Equating the rough estimates of the moments of V_{ij} with the mean and variance of V_{ij} we obtain initial estimates

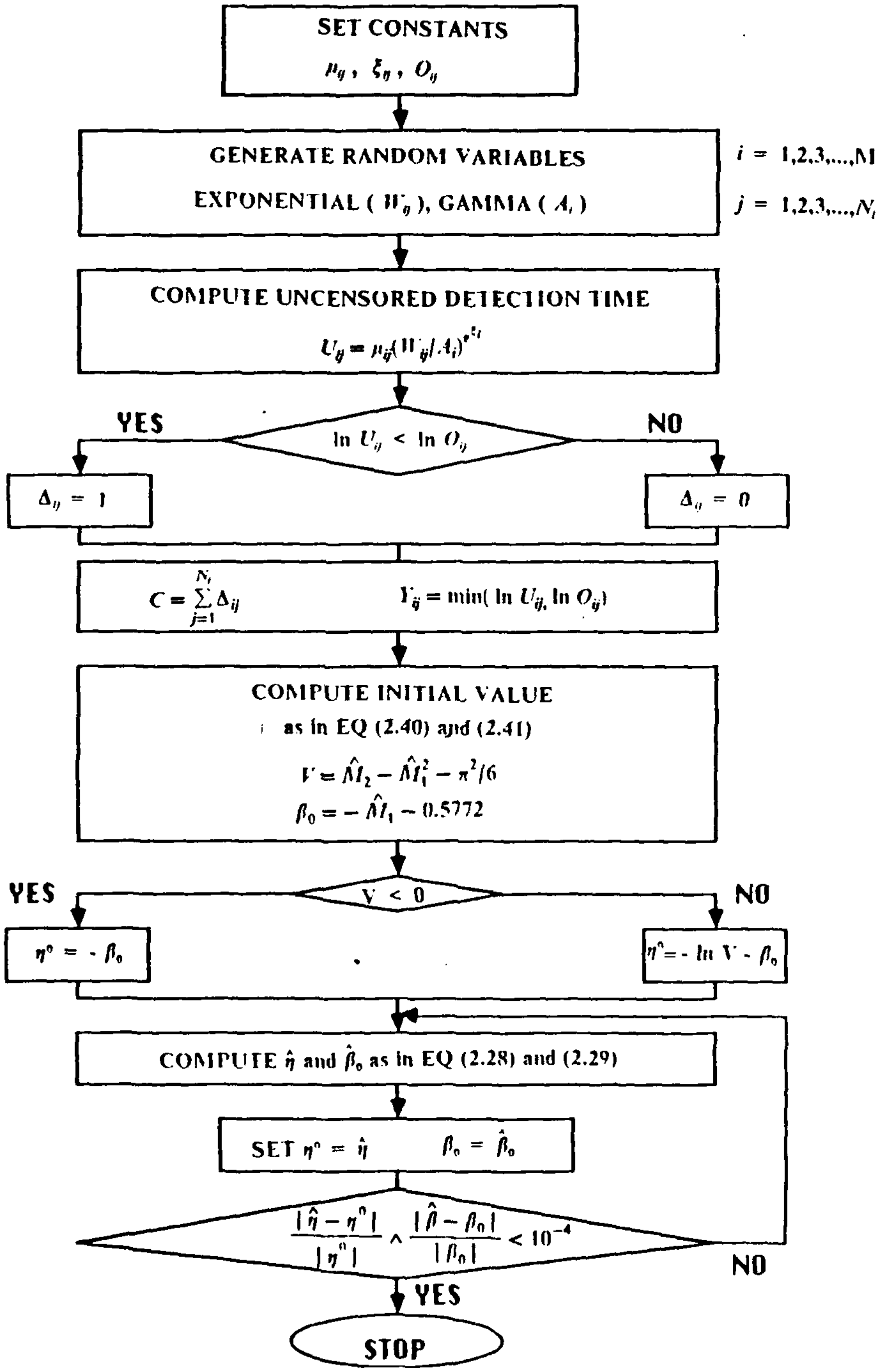
$$\hat{\beta}_0 = -\hat{M}_1 - 0.5772 \ ; \tag{2.47}$$

$$\hat{\eta} = -\ln[\hat{M}_2 - (\hat{M}_1)^2 - \frac{\pi^2}{6}] - \hat{\beta}_0 \ . \tag{2.48}$$

If $\hat{M}_2 - (\hat{M}_1)^2 - \frac{\pi^2}{6} < 0$, we set $\hat{\eta} = -\hat{\beta}_0$. The latter condition usually occurs in the simulations indicating that a better initial condition might be found.

3. Implementation in Simulation

The following flow diagram describes one replication of a simulation experiment.



C. SIMULATION PROCEDURES AND RESULTS

1. Simulation

All simulations were carried out on an IBM 3179 G computer at the Naval Postgraduate School using the APL GRAFSTAT random number package [Ref. 6]. Histograms of simulated estimates were produced by an experimental APL package GRAFSTAT which the Naval Postgraduate School is using under a test agreement with IBM Watson Research center, Yorktown. Height, NY. The simulation is replicated for R = 100 replications. Each simulation experiment of 100 replications starts with the same random number seed. The mean bias (M.B) and its standard error (S.E(M.B)), mean square error (M.S.E) and its standard error (S.E(M.S.E)) from R = 100 replications are computed as follows :

$$M.B = \frac{1}{R} \sum_{i=1}^{R} (\hat{\beta}_i - \beta) \tag{2.49a}$$

$$S.E(M.B) = \sqrt{\frac{1}{R(R-2)} \sum_{i=1}^{R} \left((\hat{\beta}_i - \beta) - M.B\right)^2} \tag{2.49b}$$

$$M.S.E = \frac{1}{R} \sum_{i=1}^{R} (\hat{\beta}_i - \beta)^2 \tag{2.50a}$$

$$S.E(M.S.E) = \sqrt{\frac{1}{R(R-2)} \sum_{i=1}^{R} \left((\hat{\beta}_i - \beta)^2 - M.S.E\right)^2} \tag{2.50b}$$

where $\hat{\beta}_i$ is the point estimate of the true value β for the ith replication and R is the number of replications.

The simulation experiment to study the sampling properties of the estimators of η and β_0 is as follows.

1. Give arbitrary constants for μ_o and ξ_i (μ_o = 4.2 , ξ_i = 0)
2. Set the same opportunity time , O_{ij}, for M observers, i = 1,2,3,....,M, j = 1,2,3,...., N. The constant opportunity time is changed to give different censoring levels. The different values for O_{ij} are 10, 25, 40.

3. Generate gamma random variables for M observers ($A_1, A_2, A_3, ..., A_M$ with shape parameter γ = 1.5, scale parameter α = 6.5).
4. Generate independent exponential random variables with mean 1 (W_{ij}, i = 1,2,3,...,M, j = 1,2,3,...,N_i) .
5. Compute the detection time U_{ij} for observer i to detect target j

$$U_{ij} = \mu_{ij}\left(\frac{W_{ij}}{A_i} \right)^{e^{\xi_i}}$$

6. Compare ln U_{ij} with ln O_{ij}, then choose the smaller one for the data

$$Y_{ij} = \min(\ln U_{ij}, \ln O_{ij})$$

and compute the censoring indicator Δ_{ij} as in equation (2.6)

7. Compute the moments M_1 and M_2 for the observations that are not censored as in equation (2.40) and equation (2.41).
8. Compute the initial value for η and β_0 as in equation (2.47) and equation (2.48)
9. Use equation (2.28) and (2.29) to find new values for η and β_0
10. Iterate the procedure until the differences between successive values of η and β_0 are small. (less than 10^{-4}).

2. Results

In this section results from the simulation experiments will be reported. Simulation experiments were done for various numbers of observers and targets and values of the opportunity time. The numbers of observers considered are M = 5, 15, and 30. The numbers of targets considered are N = 5, 15, 30, and 50 for each observer. For all simulation experiments, the μ_{ij} = 4.2, ξ_i = 0, γ = 1.5 and α = 6.5. From equations (2.42) and (2.44) the true values of η and β_0 are found to be 1.8718 and -1.4663. Histograms of the difference between the estimates and the true values, $\hat{\eta}_i$ - η and $\hat{\beta}_0$ - β_0 are shown in Figures 1 through 18 of Appendix A. For Figures 1 through 6, O_{ij} = 10; for Figures 7 through 12, O_{ij} = 25; for Figures 13 through 18, O_{ij} = 40. The fraction of detected level or uncensoring level (UC) for R replications of each simulation is determined by following equation

$$UC = \frac{\sum_{r=1}^{R} \sum_{i=1}^{M} C_i(r)}{\sum_{r=1}^{R} \sum_{i=1}^{M} N_i(r)} \tag{2.51}$$

where $C_i(r)$ is numbers of targets detected by observer i in the r^{th} replication, $N_i(r)$ is the number of targets presented to observer i in the r^{th} replication. The UC's for the simulations using the same opportunity time are then averaged to obtain the mean UC for that opportunity time. For each Figure of Appendix A and B, the mean uncensoring level for the opportunity time is given in parenthesis. Mean bias and mean square error are recorded in tables 1 through 9 for each of the estimators; their standard errors appear in parenthesis below. Tables 1, 4 and 7 present all of the means and standard errors for different opportunity times. The other tables present the same results in a more convenient fashion; the mean biases are displayed with the mean square error in parenthesis. For each Table, the opportunity time(O) and average uncensoring level(UC) for that opportunity time are given at the top of table.

- Results for $\hat{\eta}$

  The histograms of $\hat{\eta}$ - η are centered about 0 with a slight amount of skewness to the right. Increasing the opportunity time, which results in less censoring, has very little effect on mean bias and mean square error. Increasing the number of targets for a fixed number of observers has some tendency to decrease the mean square error and bias. Increasing the number of observers for a fixed number of targets has the greatest effect on decreasing the mean bias and mean square error.

- Results for $\hat{\beta}_0$

  The histograms of $\hat{\beta}_0$ - β_0 are centered around 0 with some skewness to the left. Once again changing the opportunity time has little effect on mean bias and mean square error. Changing the number of targets for a fixed number of observers also has little effect. Increasing the number of observers for a fixed number of targets has the greatest effect on decreasing the mean bias and mean square error.

Table 1. **MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 10 AND UC = 37%**

Number of Observers	Number of Targets	$\hat{\eta}$		$\hat{\beta}_0$	
		M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)
5	5	0.79 (0.12)	2.04 (0.17)	-0.07 (0.06)	0.36 (0.09)
	15	0.58 (0.11)	1.33 (0.15)	-0.08 (0.05)	0.25 (0.05)
	30	0.42 (0.09)	0.97 (0.12)	-0.12 (0.04)	0.19 (0.03)
	50	0.43 (0.07)	0.68 (0.08)	-0.05 (0.04)	0.14 (0.03)
15	5	0.19 (0.09)	0.81 (0.10)	-0.02 (0.03)	0.12 (0.02)
	15	0.19 (0.08)	0.72 (0.01)	-0.01 (0.03)	0.07 (0.01)
	30	0.09 (0.04)	0.20 (0.03)	-0.07 (0.03)	0.07 (0.01)
	50	0.14 (0.05)	0.23 (0.04)	-0.02 (0.03)	0.06 (0.01)
30	5	0.29 (0.07)	0.54 (0.10)	-0.03 (0.02)	0.05 (0.01)
	15	0.13 (0.05)	0.21 (0.04)	-0.01 (0.02)	0.03 (0.01)
	30	0.08 (0.03)	0.10 (0.01)	-0.04 (0.02)	0.03 (0.01)
	50	0.01 (0.03)	0.09 (0.01)	+ 0.004 (0.02)	0.02 (0.003)

Table 2. MEAN BIAS AND MEAN SQUARE ERROR AT O = 10 AND UC = 37%

	M.B(M.S.E) for $\hat{\eta}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	0.79(2.04)	0.58(1.3)	0.42(0.97)	0.43(1.68)
15 OBS	0.19(0.81)	0.19(0.72)	0.09(0.20)	0.14(0.23)
30 OBS	0.29(0.54)	0.13(0.21)	0.08(0.09)	0.01(0.09)

Table 3. MEAN BIAS AND MEAN SQUARE ERROR AT O = 10 AND UC = 37%

	M.B(M.S.E) for $\dot{\beta}_0$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.07(0.36)	-0.08(0.25)	-0.12(0.19)	-0.05(0.14)
15 OBS	-0.02(0.12)	-0.009(0.07)	-0.07(0.07)	-0.02(0.06)
30 OBS	-0.03(0.05)	-0.01(0.03)	-0.04(0.03)	-0.004(0.02)

Table 4. MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 25 AND UC = 62%

Number of Observers	Number of Targets	$\hat{\eta}$		$\hat{\beta}_0$	
		M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)
5	5	0.83 (0.13)	2.23 (0.23)	-0.06 (0.05)	0.25 (0.07)
	15	0.52 (0.08)	0.97 (0.14)	-0.08 (0.05)	0.20 (0.04)
	30	0.47 (0.01)	1.00 (0.14)	-0.12 (0.04)	0.18 (0.03)
	50	0.47 (0.07)	0.76 (0.10)	-0.05 (0.04)	0.13 (0.02)
15	5	0.20 (0.07)	0.55 (0.08)	-0.01 (0.03)	0.09 (0.01)
	15	0.15 (0.05)	0.29 (0.05)	-0.01 (0.03)	0.07 (0.01)
	30	0.10 (0.04)	0.20 (0.03)	-0.06 (0.03)	0.06 (0.01)
	50	0.16 (0.05)	0.23 (0.04)	-0.12 (0.03)	0.06 (0.01)
30	5	0.17 (0.06)	0.37 (0.12)	-0.03 (0.02)	0.04 (0.01)
	15	0.13 (0.04)	0.18 (0.03)	-0.005 (0.02)	0.03 (0.01)
	30	0.09 (0.03)	0.09 (0.01)	-0.03 (0.02)	0.03 (0.01)
	50	0.01 (0.03)	0.10 (0.01)	0.02 (0.02)	0.02 (0.01)

Table 5. MEAN BIAS AND MEAN SQUARE ERROR AT O=25 AND UC=62%

	M.B(M.S.E) for $\hat{\eta}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	0.83(2.20)	0.52(0.97)	0.47(1.00)	0.47(0.76)
15 OBS	0.20(0.55)	0.15(0.29)	0.10(0.20)	0.16(0.22)
30 OBS	0.17(0.37)	0.13(0.18)	0.09(0.08)	0.008(0.10)

Table 6. MEAN BIAS AND MEAN SQUARE ERROR AT O=25 AND UC=62%

	M.B(M.S.E) for $\hat{\beta}_0$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.06(0.25)	-0.08(0.20)	-0.12(0.18)	-0.05(0.13)
15 OBS	-0.01(0.09)	-0.007(0.07)	-0.06(0.06)	-0.02(0.01)
30 OBS	-0.03(0.04)	-0.005(0.03)	-0.03(0.03)	0.02(0.02)

Table 7. MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 40 AND UC = 75%

Number of Observers	Number of Targets	$\hat{\eta}$ M.B (S.E)	$\hat{\eta}$ M.S.E (S.E)	$\hat{\beta}_0$ M.B (S.E)	$\hat{\beta}_0$ M.S.E (S.E)
5	5	0.89 (0.12)	2.22 (0.28)	-0.06 (0.05)	0.20 (0.04)
	15	0.55 (0.09)	1.08 (0.16)	-0.06 (0.04)	0.18 (0.03)
	30	0.49 (0.09)	1.03 (0.16)	-0.12 (0.04)	0.18 (0.03)
	50	0.61 (0.12)	1.71 (0.17)	-0.05 (0.04)	0.13 (0.02)
15	5	0.24 (0.08)	0.70 (0.18)	-0.01 (0.03)	0.09 (0.01)
	15	0.17 (0.05)	0.29 (0.05)	-0.01 (0.03)	0.07 (0.01)
	30	0.12 (0.04)	0.20 (0.03)	-0.06 (0.03)	0.06 (0.01)
	50	0.35 (0.04)	0.19 (0.03)	-0.03 (0.03)	0.06 (0.01)
30	5	0.13 (0.05)	0.24 (0.03)	-0.02 (0.02)	0.04 (0.01)
	15	0.20 (0.07)	0.55 (0.03)	-0.01 (0.02)	0.04 (0.01)
	30	0.16 (0.08)	0.68 (0.03)	-0.04 (0.02)	0.03 (0.01)
	50	0.02 (0.03)	0.09 (0.01)	+0.01 (0.02)	0.02 (0.01)

Table 8. MEAN BIAS AND MEAN SQUARE ERROR AT O=40 AND UC=75%

	$M.B(M.S.E)$ for $\hat{\eta}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	0.89(2.20)	0.55(1.1)	0.49(1.00)	0.61(1.70)
15 OBS	0.24(0.70)	0.17(0.29)	0.12(0.20)	0.35(2.00)
30 OBS	0.13(0.24)	0.20(0.55)	0.16(0.68)	0.02(0.09)

Table 9. MEAN BIAS AND MEAN SQUARE ERROR AT O=40 AND UC=75%

	$M.B(M.S.E)$ for $\hat{\beta}_0$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.06(0.20)	-0.06(0.18)	-0.12(0.18)	-0.05(0.13)
15 OBS	-0.007(0.09)	-0.009(0.07)	-0.06(0.06)	-0.03(0.06)
30 OBS	-0.02(0.04)	-0.01(0.04)	-0.04(0.03)	-0.01(0.02)

III. A WEIBULL REGRESSION MODEL

A. MODEL

Suppose there are M observers. Each observer is presented with N_i targets. Let U_{ij} be the time it takes for observer i to detect target j. Let x_{ij1} , x_{ij2} , x_{ij3} ,..., x_{ijp} be the values of explanatory variables which may effect U_{ij} (e.g., terrain, atmospheric conditions, fatigue etc.)

Assume that U_{ij} are independent random variables with Weibull distributions

$$\begin{aligned} P\{U_{ij} \le t\} &= 1 - \exp\Big[- \big(te^{-\underline{x}_{ij}\underline{\beta}}\big)^{e^{-\xi_i}}\Big] \\ &= 1 - \exp\Big[- (t/\mu_{ij})^{e^{-\xi_i}}\Big] \end{aligned} \tag{3.1}$$

where $\mu_{ij} = e^{\underline{x}_{ij}\underline{\beta}}$ with

$$\underline{x}_{ij}\underline{\beta} = \sum_{k=1}^{p} x_{ijk}\beta_k \ . \tag{3.2}$$

It follows from equation (3.1) that the distribution of the ln-detection time is

$$\begin{aligned} P\{ \ln U_{ij} \le t\} &= P\{U_{ij} \le e^t\} \\ &= 1 - \exp\Big\{ - \big(e^t e^{-\underline{x}_{ij}\underline{\beta}}\big)^{e^{-\xi_i}}\Big\} \\ &= 1 - \exp\Big\{ - \exp\big[(t - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\big]\Big\} \ . \end{aligned} \tag{3.3}$$

The derivative of equation (3.3) with respect to t is following

$$\begin{aligned} P\{ \ln U_{ij} \in dt\} &= \exp\Big\{ - \exp\big[(t - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\big]\Big\} \exp\big\{(t - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\big\} e^{-\xi_i} \\ &= \exp\Big\{ - \exp\big[(t - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\big]\Big\} \exp\big\{(t - \underline{x}_{ij}\underline{\beta})e^{-\xi_i} - \xi_i\big\} \ . \end{aligned} \tag{3.4}$$

When the i^{th} observer is presented with his j^{th} target, he gets a length of time called an opportunity time O_{ij} to detect it. The observer either successfully detects the target within the opportunity time or is unsuccessful. Data for the i^{th} observer consist of times of detections for the successes and the lengths of opportunity times for the failures. For each i = 1,2,3,....,M , j = 1,2,3,....,N_i , let

$$Y_{ij} = \min(\ln U_{ij}, \ln O_{ij}) \tag{3.5}$$

and

$$\Delta_{ij} = \begin{cases} 1 & if \;\; U_{ij} \le O_{ij} \\ 0 & otherwise \; . \end{cases} \tag{3.6}$$

The Y_{ij} are the censored ln-detection times and Δ_{ij} is an indicator of whether or not the ln-time to detect the j^{th} target by the i^{th} observer is censored. Let

$$C_i = \sum_{j=1}^{N_i} \Delta_{ij} \tag{3.7}$$

be the number of targets detected by observer i. The next section of this chapter will discuss estimation of the parameters $\{\beta_k\}$ [*Ref.* 4]. These parameter estimates might be used to predict future times to detection for an observer given his past performance. In the last section of the chapter results of a simulation study of the estimation precedure will be given.

B. ESTIMATION

1. The likelihood Equation and Maximum Likelihood Estimators.

The likelihood for the i^{th} observer is

$$L_i(\underline{\beta}, \underline{\xi}) = \prod_{j=1}^{N_i} \exp\{\Delta_{ij}[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i} - \xi_i]\} \exp\{-\exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]\} \; . \tag{3.8}$$

The ln likelihood for the i^{th} observer is

$$\ln L_i(\underline{\beta}, \underline{\xi}) = \sum_{j=1}^{N_i} \{\Delta_{ij}[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i} - \xi_i] - \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]\} \; . \tag{3.9}$$

Since the observers are assumed to be independent, the ln likelihood for all observers is

$$\ln L(\underline{\beta}, \underline{\xi}) = \sum_{i=1}^{M} \ln L_i(\underline{\beta}, \underline{\xi})$$
$$= \sum_{i=1}^{M} \sum_{j=1}^{N_i} \{\Delta_{ij}[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i} - \xi_i] - \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]\} \ . \tag{3.10}$$

a. Newton's Procedure to solve for β_k

The partial derivative of equation (3.10) with respect to β_k is

$$\frac{\partial}{\partial \beta_k} \ln L_i(\underline{\beta}, \underline{\xi}) = \sum_{i=1}^{M} \sum_{j=1}^{N_i} \{\Delta_{ij}[(-x_{ijk})e^{-\xi_i}] - \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}](-x_{ijk}e^{-\xi_i})\}$$
$$= \sum_{i=1}^{M} \sum_{j=1}^{N_i} \{-\Delta_{ij} + \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]\} x_{ijk} e^{-\xi_i} \ . \tag{3.11}$$

A Newton procedure to solve equation (3.11) for $\underline{\beta}$ assuming $\{\xi_i\}$ known uses the second derivatives

$$\frac{\partial^2 L}{\partial \beta_k \partial \beta_h} = \sum_{i=1}^{M} \sum_{j=1}^{N_i} - \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}] x_{ijh} e^{-\xi_i} x_{ijk} e^{-\xi_i} \ . \tag{3.12}$$

Let

$$w_{ij} = \sqrt{\exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]} \ ; \tag{3.13}$$

$$u_{ijh} = x_{ijh} e^{-\xi_i} w_{ij} \ . \tag{3.14}$$

The Newton procedure to solve equations (3.11) for $\underline{\beta}$ can be written as

$$
\begin{aligned}
0 = \frac{\partial}{\partial \beta_k} \ln L_i(\underline{\beta}, \underline{\xi}) &= \sum_{i=1}^{M} \sum_{j=1}^{N_i} \frac{\left[-\Delta_{ij} + w_{ij}^2 \right]}{w_{ij}} u_{ijk} \\
&- \sum_{i=1}^{M} \sum_{j=1}^{N_i} \sum_{h=1}^{p} w_{ij}^2 x_{ijh} e^{-\xi_i} x_{ijk} e^{-\xi_i} \left[\beta_k - \beta_k^0 \right] \\
&= \sum_{i=1}^{M} \sum_{j=1}^{N_i} \left\{ \frac{\left[-\Delta_{ij} + w_{ij}^2 \right]}{w_{ij}} + \sum_{h=1}^{p} u_{ijh} \beta_h^0 - \sum_{h=1}^{p} u_{ijh} \beta_h \right\}
\end{aligned}
\tag{3.15}
$$

which are of the form of the normal equations for Least Squares regression with dependent variables

$$
z_{ij} = \frac{\left[-\Delta_{ij} + w_{ij}^2 \right]}{w_{ij}} + \sum_{h=1}^{p} u_{ijh} \beta_h^0 \tag{3.16}
$$

and independent variables

$$
u_{ijk} = w_{ij} x_{ijk} e^{-\xi_i} \; . \tag{3.17}
$$

b. Newton Procedure to solve for ξ_i

The partial derivative of equation (3.10) with respect to ξ_i is

$$
\begin{aligned}
\frac{\partial}{\partial \xi_i} \ln L_i(\underline{\beta}, \underline{\xi}_i) &= \sum_{j=1}^{N_i} \Big[\left\{ \Delta_{ij} \left[(y_{ij} - \underline{x}_{ij} \underline{\beta}) e^{-\xi_i} (-1) - 1 \right] \right\} \\
&- \left\{ \exp\left[(y_{ij} - \underline{x}_{ij} \underline{\beta}) e^{-\xi_i} \right] (y_{ij} - \underline{x}_{ij} \underline{\beta}) e^{-\xi_i} (-1) \right\} \Big] \\
&= -C_i + \sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij} \underline{\beta}) e^{-\xi_i} \left\{ -\Delta_{ij} + \exp\left[(y_{ij} - \underline{x}_{ij} \underline{\beta}) e^{-\xi_i} \right] \right\}
\end{aligned}
\tag{3.18}
$$

A Newton procedure to solve equation (3.15) for ξ_i asuming $\underline{\beta}$ known uses the second derivative

$$\frac{\partial^2}{\partial \xi_i^{\,2}} \ln L_i(\underline{\beta}, \underline{\xi}_i) = -\sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i} \{ -\Delta_{ij} + \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]\}$$

$$+\sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i} \{ \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}](y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}(-1)\}$$

$$= -C_i - \sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij}\underline{\beta})^2 e^{-2\xi_i} \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}] \tag{3.19}$$

if $\frac{\partial}{\partial \xi_i} \ln L_i(\underline{\beta}, \underline{\xi}_i) = 0$. Therefore the Newton equation for ξ_i is

$$0 = \frac{\partial}{\partial \xi_i} \ln L_i(\underline{\beta}, \underline{\xi}) = -C_i + \sum_{j=1}^{N_i} \Delta_{ij}(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i} \{ -\Delta_{ij} + \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]\}$$

$$+\left(-C_i - \sum_{j=1}^{N_i} \{[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]^2 \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]\} \right)[\xi_i - \xi_i^0] \tag{3.20}$$

where ξ_i^0 is the current value of ξ_i. Solving results in the equation

$$\xi_i - \xi_i^0 = \frac{\left\{ +C_i - \sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}[-\Delta_{ij} + \exp\{(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}\}] \right\}}{-C_i - \sum_{j=1}^{N_i} \{[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]^2 \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta}) e^{-\xi_i}]\}} \tag{3.21}$$

2. Initial Condition

In this subsection, we describe a rough way to provide initial estimates of $\underline{x}_{ij}\underline{\beta}$ to start the iterative Newton procedure of the previous section. It follows from equation (3.1) that

$$P\left\{(U_{ij})^{e^{-\xi_i}} \le t\right\} = P\left\{U_{ij} \le t^{e^{\xi_i}}\right\}$$
$$= 1 - \exp\left[- \left(t^{e^{\xi_i}} e^{-\underline{x}_{ij}\underline{\beta}}\right)^{e^{-\xi_i}}\right] \quad (3.22)$$
$$= 1 - \exp\left[- e^{-\underline{x}_{ij}\underline{\beta} e^{-\xi_i}} t\right]$$

Hence,

$$E\left[(U_{ij})^{e^{-\xi_i}}\right] = \left(e^{-\underline{x}_{ij}\underline{\beta} e^{-\xi_i}}\right)^{-1}$$
$$= e^{\underline{x}_{ij}\underline{\beta} e^{-\xi_i}} \quad (3.23)$$

and

$$\ln E\left[(U_{ij})^{e^{-\xi_i}}\right] = \underline{x}_{ij}\underline{\beta} e^{-\xi_i} \quad (3.24)$$

As a result, for all the observed U_{ij} censored or uncensored, we will put

$$e^{\xi_i} \ln U_{ij} = \underline{x}_{ij}\underline{\beta}^0 \quad (3.25)$$

We will take $\xi_i^0 = 0$ for convenience. Thus the initial value of $\underline{x}_{ij}\underline{\beta}^0$ for each observation u_{ij} is

$$\underline{x}_{ij}\underline{\beta}^0 = \ln u_{ij} \quad (3.26)$$

3. Recursive Procedure to find $\{\beta_k\}$ and ξ_i

1. Put $\xi_i = 0$ for $i = 1,2,3,\ldots,M$
2. Compute the initial value

$$\underline{x}_{ij}\underline{\beta}^0 = e^{\xi_i} \ln U_{ij} \quad (3.27)$$

3. Iteration

   a. Compute dependent and independent variables for regression

$$W_{ij} = \sqrt{\exp\left\{(y_{ij} - \underline{x}_{ij}\underline{\beta}^0)e^{-\xi_i}\right\}} \quad (3.28)$$

$$u_{ijk} = W_{ij}\, x_{ijk}\, e^{-\xi_i} \qquad (\textit{dependent variable})$$

$$z_{ij} = \frac{-\Delta_{ij} + W_{ij}^2}{W_{ij}} + W_{ij}\underline{x}_{ij}\underline{\beta}^0 \qquad (\textit{independent variable})$$

Let

$$
\underline{Z} = \begin{bmatrix} z_{11} \\ \vdots \\ z_{1N_1} \\ z_{21} \\ \vdots \\ z_{2N_2} \\ \vdots \\ z_{M1} \\ \vdots \\ z_{MN_M} \end{bmatrix}
\qquad
\underline{U} = \begin{bmatrix}
u_{111} & u_{112} & \cdots & u_{11p} \\
u_{121} & u_{122} & \cdots & u_{12p} \\
\vdots & \vdots & \vdots & \vdots \\
u_{1N_11} & u_{1N_12} & \cdots & u_{1N_1p} \\
u_{211} & u_{212} & \cdots & u_{21p} \\
\vdots & \vdots & \vdots & \vdots \\
u_{2N_21} & u_{2N_22} & \cdots & u_{2N_2p} \\
\vdots & \vdots & \vdots & \vdots \\
u_{M11} & u_{M12} & \cdots & u_{M1p} \\
\vdots & \vdots & \vdots & \vdots \\
u_{MN_M1} & u_{MN_M2} & \cdots & u_{MN_Mp}
\end{bmatrix}
$$

b. Compute regression estimates

$$\underline{\beta} = (\underline{U}^T\underline{U})^{-1}\underline{U}^T\underline{Z} \tag{3.29}$$

c. Recompute the initial value

$$\underline{x}_{ij}\underline{\beta} = \sum_{k=1}^{p} x_{ijk}\beta_k \tag{3.30}$$

d. Update ξ_i as in equation (3.21)

$$\xi_i - \xi_i^0 = \frac{\left\{ + C_i - \sum_{j=1}^{N_i} (y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\left[- \Delta_{ij} + \exp\{(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}\}\right] \right\}}{- C_i - \sum_{j=1}^{N_i} \{[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]^2 \exp[(y_{ij} - \underline{x}_{ij}\underline{\beta})e^{-\xi_i}]\}}$$

e. Put

$$\xi_i^0 = \xi_i$$

$$\beta_k^0 = \beta_k$$

f. Return to step 3

4. Iterate until the following conditions are satisfied

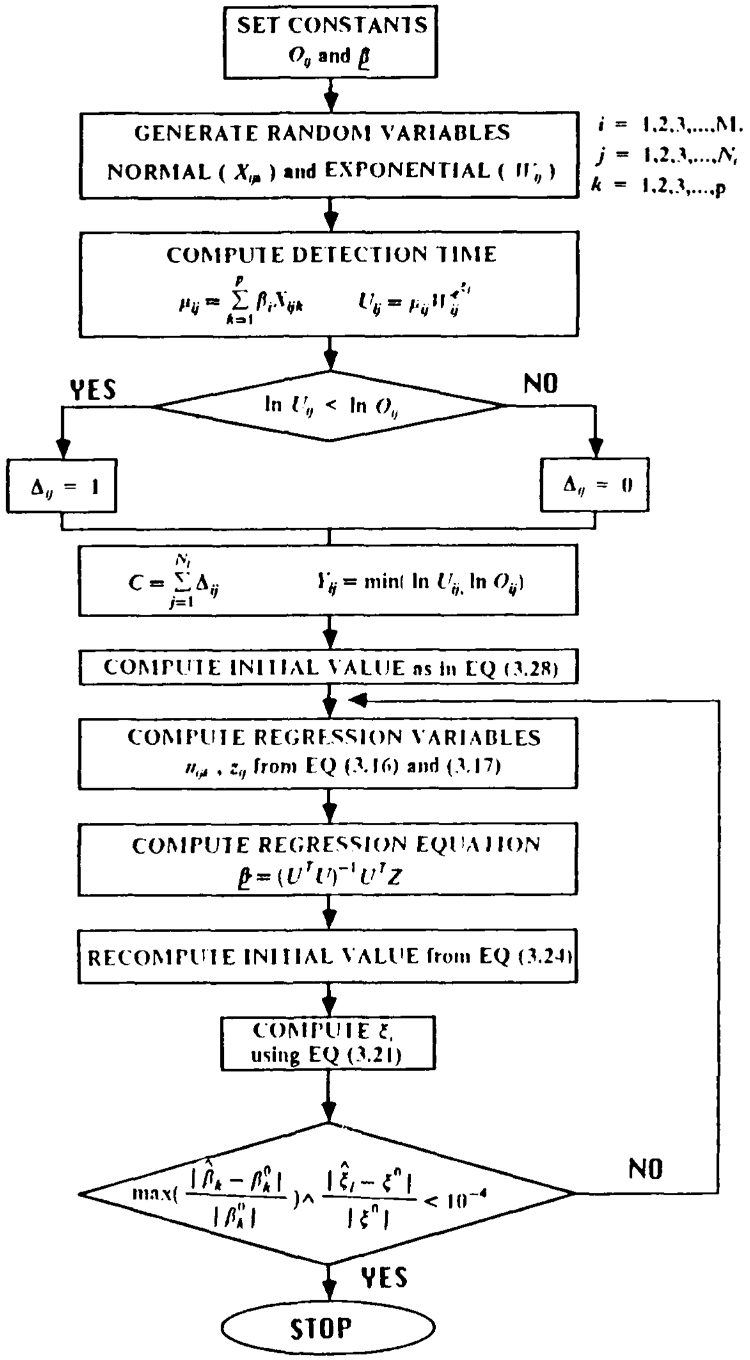
$$\max\left(\frac{|\beta_k - \beta_k^0|}{|\beta_k^0|} \right) \le 1E-4, \quad k = 1,2,3,\ldots,p$$

and

$$\max\left(\frac{|\xi_i - \xi_i^0|}{|\xi_i^0|}\right) \le 1E-4, \quad i = 1,2,3,\ldots,M.$$

4. Implementation in Simulation

The following flow diagram describes one replication of a simulation experiment.



C. SIMULATION PROCEDURE AND RESULTS

1. Simulation

The number of replications for each simulation experiment is 100. Each simulation experiment of 100 replications starts with the same random number seed. The mean bias,its standard error and mean square error, standard error of mean square error from R = 100 replications are computed as in equations (2.49a) through (2.50b). In each of the experiments the number of covariates for each observer is 2.

The simulation experiment to study the sampling properties of the estimators of $\{\beta_k\}$ and ξ_i is as follows.

1. Give arbitrary constant values for the true values of β_k and ξ_i. In all the simulations $\beta_1 = 0.2$, $\beta_2 = 0.3$ and $\xi_i = 0$.
2. The same opportunity time, O_{ij}, $i = 1,2,3,....,M$, and $j = 1,2,3,....,N_i$, is used for M observers. The values of the constant opportunity time are 1.3, 2.5, 4.0.
3. Generate random numbers
   a. Generate independent normal random numbers with mean 1 and variance 0.5, x_{ij1}, $i = 1,2,3,....,M$, $j = 1,2,3,....,N_i$.
   b. Generate independent normal random numbers with mean 2 and variance 1 , x_{ij2}, $i = 1,2,3,....,M$, $j = 1,2,3,....,N_i$.
   c. Generate exponential random numbers with mean 1, W_{ij}, $i = 1,2,3,....,M$, $j = 1,2,3,....,N_i$.
4. Compute μ_{ij} as follows :
$$\mu_{ij} = \exp(\beta_1 x_{ij1} + \beta_2 x_{ij2}) . \tag{3.31}$$
5. Compute the detection time U_{ij} that it takes for observer i to detect target j as
$$U_{ij} = \mu_{ij}(W_{ij})^{e^{\xi_i}} . \tag{3.32}$$
6. Compare ln U_{ij} with ln O_{ij} , then choose smaller one for the data
$$Y_{ij} = \min(\ln U_{ij}, \ln O_{ij})$$
   and compute Δ_{ij} as equation (3.7)
7. Compute the initial value as in equation (3.27)
8. Compute the values of the regression variables , z_{ij} and u_{ijk} as in equation (3.16) and (3.17)
9. Compute regression estimates as in equation (3.29)
10. Recompute the initial value with new $\underline{\beta}$ value as in equation (3.30)
11. Compute ξ_i as in equation (3.21)
12. Iterate the procedure until the differences between successive values of β_1, β_2 and ξ_i are small. (less than 10^{-4}).

2. Results

In this section results from the simulation experiments will be reported. For R replications, the uncensoring level (UC) for an experiment is computed as follows :

$$UC = \frac{\sum_{r=1}^{R}\sum_{i=1}^{M} C_i(r)}{\sum_{r=1}^{R}\sum_{i=1}^{M} N_i(r)}$$

where $C_i(r)$ is the number of the targets detected by observer i in replication r and $N_i(r)$ is the number of targets presented to observer i in replication r. The UC's for the simulations using the same opportunity time are then averaged to obtain the mean UC for that opportunity time. Figures 19 through 45 show histograms of ($\hat{\beta}_k - \beta_k$), $k = 1,2$ and ($\hat{\xi}_i - \xi$). Tables 10 through 21 show mean square errors and mean biases for each of the estimates, their standard errors appear in parenthesis below. Tables 10, 14 and 18 present all the simulation results for different values of opportunity times. The other tables present the same results in a more convenient fashion. The mean biases are displayed with the mean square error in parenthesis; for each table, the opportunuty time(O) and average uncensoring(UC) for that are given at the top of table.

- Results for $\hat{\beta}_1$ and $\hat{\beta}_2$

  The histograms for $\hat{\beta}_1 - \beta_1$ and $\hat{\beta}_2 - \beta_2$ tend to be somewhat centered around 0. For small numbers of targets and observers the histograms tend to be slightly skewed to the left. Increasing the observation time has little effect on the mean bias and mean square error. Increasing the number of observers for a fixed number of targets tends to decrease the mean square error but has less effect on the mean bias. Increasing the number of targets for a fixed number of observers decreases the mean bias and mean square error. The standard errors of the mean biases mean square errors are large.

- Results for $\hat{\xi}_i$

  The histograms of $\hat{\xi}_i - \xi_i$ tend to be centered about 0. There is a tendency for slight skewness to the right for small numbers of observers and targets. Changing the opportunity time has little effect on the mean bias and mean square error. The more targets there are for observer i, the smaller the mean bias and mean square error for $\hat{\xi}_i$. The standard errors of the mean biases and mean square errors are large.

Table 10. MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 1.3 AND UC = 45%

Number of Observers	Number of Targets	$\hat{\beta}_1$		$\hat{\beta}_2$		$\hat{\xi}_i$	
		M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)
5	5	-0.06 (0.03)	0.07 (0.01)	-0.02 (0.02)	0.03 (0.01)	-0.05 (0.01)	0.02 (0.01)
	15	-0.02 (0.02)	0.02 (0.01)	-0.02 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)
	30	-0.003 (0.01)	0.02 (0.01)	-0.02 (0.01)	0.01 (0.002)	-0.03 (0.01)	0.01 (0.001)
	50	-0.01 (0.001)	0.01 (0.001)	-0.01 (0.01)	0.003 (0.001)	-0.01 (0.01)	0.004 (0.001)
15	5	-0.05 (0.01)	0.02 (0.003)	-0.04 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)
	15	-0.03 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.003 (0.001)	-0.01 (0.01)	0.002 (0.0003)
	30	-0.01 (0.01)	0.01 (0.001)	-0.01 (0.004)	0.002 (0.001)	-0.01 (0.004)	0.002 (0.0002)
	50	-0.01 (0.001)	0.003 (0.0004)	-0.01 (0.003)	0.001 (0.0004)	-0.01 (0.004)	0.001 (0.0002)
30	5	-0.05 (0.01)	0.01 (0.002)	-0.04 (0.01)	0.005 (0.001)	-0.02 (0.005)	0.003 (0.001)
	15	-0.02 (0.01)	0.003 (0.0004)	-0.02 (0.003)	0.001 (0.0002)	-0.01 (0.003)	0.001 (0.0002)
	30	-0.01 (0.01)	0.003 (0.0004)	-0.01 (0.003)	0.001 (0.0001)	-0.004 (0.003)	0.001 (0.0001)
	50	-0.01 (0.004)	0.002 (0.0002)	-0.01 (0.003)	0.001 (0.0001)	-0.01 (0.002)	0.001 (0.0001)

Table 11. MEAN BIAS AND MEAN SQUARE ERROR AT O = 1.3 AND UC = 45%

	$M.B(M.S.E)$ for $\hat{\beta}_1$			
	5 TGT	15 TGT	30 TGT	50 TGT
5OBS	-0.04(0.06)	-0.02(0.02)	-0.004(0.02)	-0.003(0.01)
15OBS	-0.05(0.02)	-0.02(0.007)	-0.01(0.005)	-0.01(0.003)
30OBS	-0.008(0.01)	-0.05(0.003)	-0.006(0.003)	-0.01(0.002)

Table 12. MEAN BIAS AND MEAN SQUARE ERROR AT O = 1.3 AND UC = 45%

	$M.B(M.S.E)$ for $\hat{\beta}_2$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.03(0.02)	-0.02(0.007)	-0.02(0.005)	-0.01(0.002)
15 OBS	-0.04(0.007)	-0.009(0.003	-0.01(0.002)	-0.01(0.001)
30 OBS	-0.04(0.004)	-0.01(0.001)	-0.01(0.001)	-0.009(0.0006)

Table 13. MEAN BIAS AND MEAN SQUARE ERROR AT O = 1.3 AND UC = 45%

	$M.B(M.S.E)$ for $\hat{\xi}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.05(0.14)	-0.02(0.007)	-0.02(0.005)	-0.006(0.004)
15 OBS	-0.01(0.005)	-0.009(0.002)	-0.005(0.002)	-0.008(0.001)
30 OBS	-0.02(0.002)	-0.01(0.001)	-0.004(0.0007)	0.008(0.0006)

Table 14. MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 2.5 AND UC = 75%

Number of Observers	Number of Targets	$\hat{\beta}_1$		$\hat{\beta}_2$		$\hat{\xi}_i$	
		M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)
5	5	-0.04 (0.02)	0.06 (0.01)	-0.03 (0.01)	0.02 (0.003)	-0.05 (0.01)	0.14 (0.01)
	15	-0.02 (0.02)	0.02 (0.003)	-0.02 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)
	30	-0.004 (0.01)	0.02 (0.003)	-0.02 (0.01)	0.005 (0.001)	-0.02 (0.01)	0.01 (0.001)
	50	-0.003 (0.01)	0.01 (0.001)	-0.01 (0.01)	0.002 (0.0003)	-0.01 (0.01)	0.004 (0.001)
15	5	-0.05 (0.01)	0.02 (0.003)	-0.04 (0.01)	0.01 (0.001)	-0.01 (0.01)	0.005 (0.001)
	15	-0.02 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.003 (0.0004)	-0.01 (0.01)	0.002 (0.0003)
	30	-0.01 (0.01)	0.01 (0.001)	-0.01 (0.004)	0.002 (0.0002)	-0.01 (0.004)	0.002 (0.0002)
	50	-0.01 (0.01)	0.003 (0.0004)	-0.01 (0.003)	0.001 (0.0002)	-0.01 (0.003)	0.001 (0.0002)
30	5	-0.01 (0.01)	0.01 (0.001)	-0.04 (0.01)	0.004 (0.001)	-0.02 (0.01)	0.003 (0.0004)
	15	-0.05 (0.01)	0.003 (0.0004)	-0.02 (0.003)	0.001 (0.0002)	-0.01 (0.003)	0.001 (0.0002)
	30	-0.01 (0.01)	0.003 (0.0003)	-0.01 (0.003)	0.001 (0.0001)	-0.004 (0.003)	0.001 (0.0001)
	50	-0.01 (0.004)	0.0012 (0.0002)	-0.01 (0.002)	0.001 (0.0001)	-0.01 (0.002)	0.001 (0.0001)

Table 15. MEAN BIAS AND MEAN SQUARE ERROR AT O=2.5 AND UC=75%

	$M.B(M.S.E)$ *for* $\hat{\beta}_1$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.06(0.07)	-0.02(0.20)	-0.003(0.02)	-0.005(0.01)
15 OBS	-0.05(0.02)	-0.03(0.007)	-0.01(0.006)	-0.008(0.003)
30 OBS	-0.05(0.01)	-0.02(0.003)	-0.006(0.003)	-0.01(0.002)

Table 16. MEAN BIAS AND MEAN SQUARE ERROR AT O=2.5 AND UC=75%

	$M.B(M.S.E)$ *for* $\hat{\beta}_2$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.02(0.03)	-0.02(0.007)	-0.02(0.005)	-0.009(0.003)
15 OBS	-0.04(0.007)	-0.02(0.003)	-0.01(0.002)	-0.01(0.001)
30 OBS	-0.04(0.004)	-0.02(0.001)	-0.01(0.001)	-0.009(0.001)

Table 17. MEAN BIAS AND MEAN SQUARE ERROR AT O=2.5 AND UC=75%

	$M.B(M.S.E)$ *for* $\hat{\xi}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.046(0.02)	-0.03(0.008)	-0.03(0.004)	-0.006(0.004)
15 OBS	-0.02(0.01)	-0.01(0.002)	-0.006(0.002)	-0.009(0.001)
30 OBS	-0.02(0.003)	-0.01(0.001)	-0.004(0.0008)	-0.009(0.0006)

Table 18. MEAN BIAS, MEAN SQUARE ERROR AND STANDARD ERROR AT O = 4.0 AND UC = 89%

Number of Observers	Number of Targets	$\hat{\beta}_1$		$\hat{\beta}_2$		$\hat{\xi}_i$	
		M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)	M.B (S.E)	M.S.E (S.E)
5	5	-0.03 (0.01)	0.02 (0.003)	-0.03 (0.02)	0.03 (0.01)	-0.07 (0.01)	0.02 (0.003)
	15	-0.01 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)
	30	-0.01 (0.01)	0.01 (0.001)	-0.02 (0.01)	0.01 (0.001)	-0.03 (0.01)	0.004 (0.001)
	50	-0.002 (0.01)	0.01 (0.001)	-0.01 (0.01)	0.004 (0.001)	-0.01 (0.01)	0.003 (0.0004)
15	5	-0.03 (0.01)	0.01 (0.001)	-0.04 (0.01)	0.01 (0.001)	-0.04 (0.01)	0.01 (0.001)
	15	-0.02 (0.01)	0.003 (0.0004)	-0.02 (0.001)	0.004 (0.0001)	-0.02 (0.0004)	0.002 (0.0003)
	30	-0.01 (0.001)	0.002 (0.0003)	-0.01 (0.001)	0.002 (0.0003)	-0.01 (0.004)	0.001 (0.0002)
	50	-0.01 (0.004)	0.002 (0.0002)	-0.01 (0.04)	0.002 (0.0002)	-0.01 (0.003)	0.001 (0.0002)
30	5	-0.03 (0.01)	0.004 (0.001)	-0.04 (0.01)	0.006 (0.001)	-0.04 (0.01)	0.004 (0.001)
	15	-0.01 (0.004)	0.002 (0.0002)	-0.02 (0.004)	0.002 (0.0003)	-0.02 (0.003)	0.001 (0.0002)
	30	-0.03 (0.0003)	0.001 (0.00)	-0.01 (0.0004)	0.001 (0.00)	-0.01 (0.0003)	0.001 (0.00)
	50	-0.01 (0.0003)	0.001 (0.00)	-0.01 (0.003)	0.001 (0.0001)	-0.01 (0.002)	0.001 (0.0001)

Table 19. MEAN BIAS AND MEAN SQUARE ERROR AT O=4.0 AND UC=89%

	$M.B(M.S.E)$ for $\hat{\beta}_1$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.03(0.02)	-0.009(0.008)	-0.01(0.008)	-0.002(0.005)
15 OBS	-0.03(0.01)	-0.02(0.003)	-0.008(0.002)	-0.006(0.002)
30 OBS	-0.03(0.004)	-0.01(0.002)	-0.003(0.001)	-0.008(0.0008)

Table 20. MEAN BIAS AND MEAN SQUARE ERROR AT O=4.0 AND UC=89%

	$M.B(M.S.E)$ for $\hat{\beta}_2$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.03(0.03)	-0.02(0.01)	-0.02(0.008)	-0.008(0.004)
15 OBS	-0.04(0.009)	-0.02(0.004)	-0.01(0.002)	-0.0074(0.002)
30 OBS	-0.04(0.004)	-0.02(0.002)	-0.0097(0.001)	-0.008(0.0007)

Table 21. MEAN BIAS AND MEAN SQUARE ERROR AT O=4.0 AND UC=89%

	$M.B(M.S.E)$ for $\hat{\xi}$			
	5 TGT	15 TGT	30 TGT	50 TGT
5 OBS	-0.07(0.02)	-0.02(0.008)	-0.03(0.004)	-0.0005(0.003)
15 OBS	-0.04(0.004)	-0.02(0.002)	-0.01(0.001)	-0.011(0.001)
30 OBS	-0.04(0.004)	-0.02(0.001)	-0.008(0.0006)	-0.011(0.0006)

IV. CONCLUSION

This thesis considers two models for the times until detection of targets. Each model has M observers. The i^{th} observer is presented with N_i targets. In the model of Chapter 2, observer i has a random variable A_i which reflects his ability to detect a target. The random variables $\{A_i\}$ are independent identically distributed having a gamma distribution. Given A_i, the times to target detections for observer i are conditionally independent Weibull random variables with known parameters. Simulation experiments indicate that increasing the number of observers for a fixed number of targets provides the greatest decrease in the mean bias and mean square error of the estimates of the parameters of the gamma distribution that describes the variation between individuals. This is not surprising, since observing more individuals sampled from a fixed population should better estimate properties of that population. The model of Chapter 3 is a Weibull regression model. In this case the simulation experiments indicate that increasing the number of targets for a fixed number of observers provides the greatest decrease in the mean bias and mean square error of the estimators.

A topic for future investigation is to combine the two models and estimation procedures to provide estimates for a hierarchical gamma Weibull regression model. Another topic is to investigate using the fitted hierarchical model to predict future performance of the observers.

V. LIST OF REFERENCES

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APPENDIX A. HISTOGRAMS FOR THE ESTIMATED GAMMA PARAMETERS

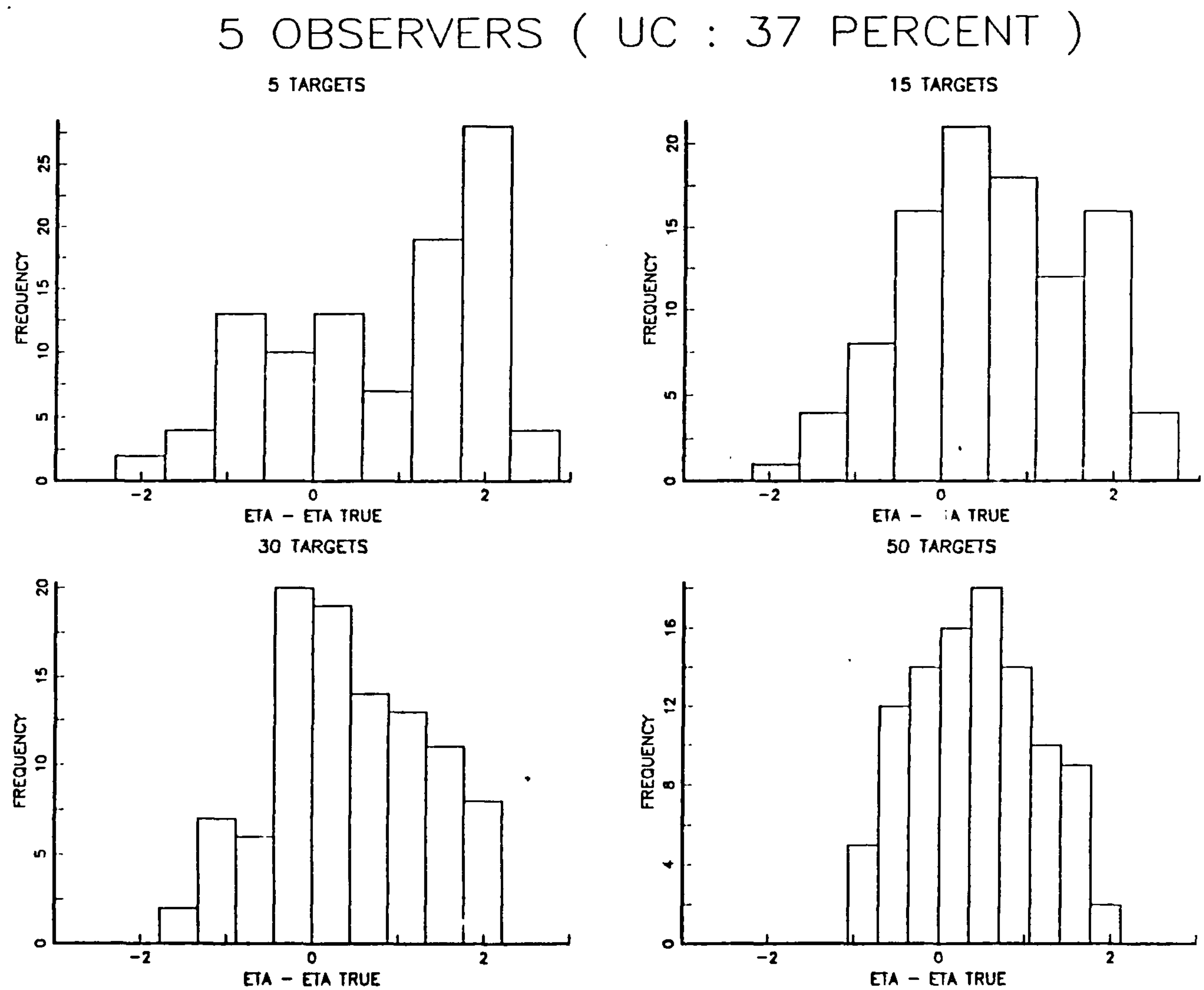

Figure 1. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 5 observers ($\hat{\eta} - \eta$)

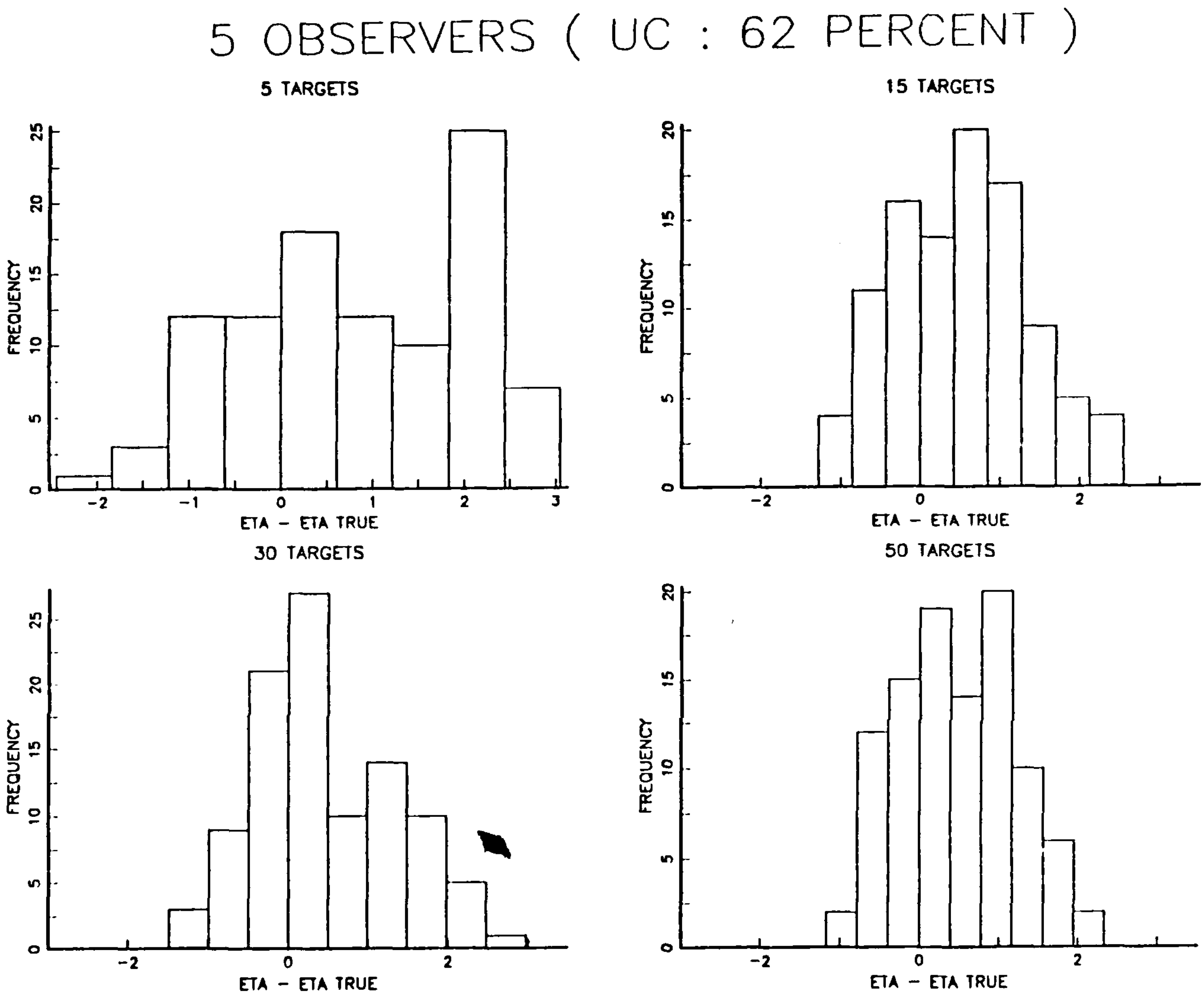

Figure 2. **COMPARISON BETWEEN DIFFERENT ESTIMATES (O=25, UC=62%):** 5,15,30,50 targets for 5 observers ($\hat{\eta} - \eta$)

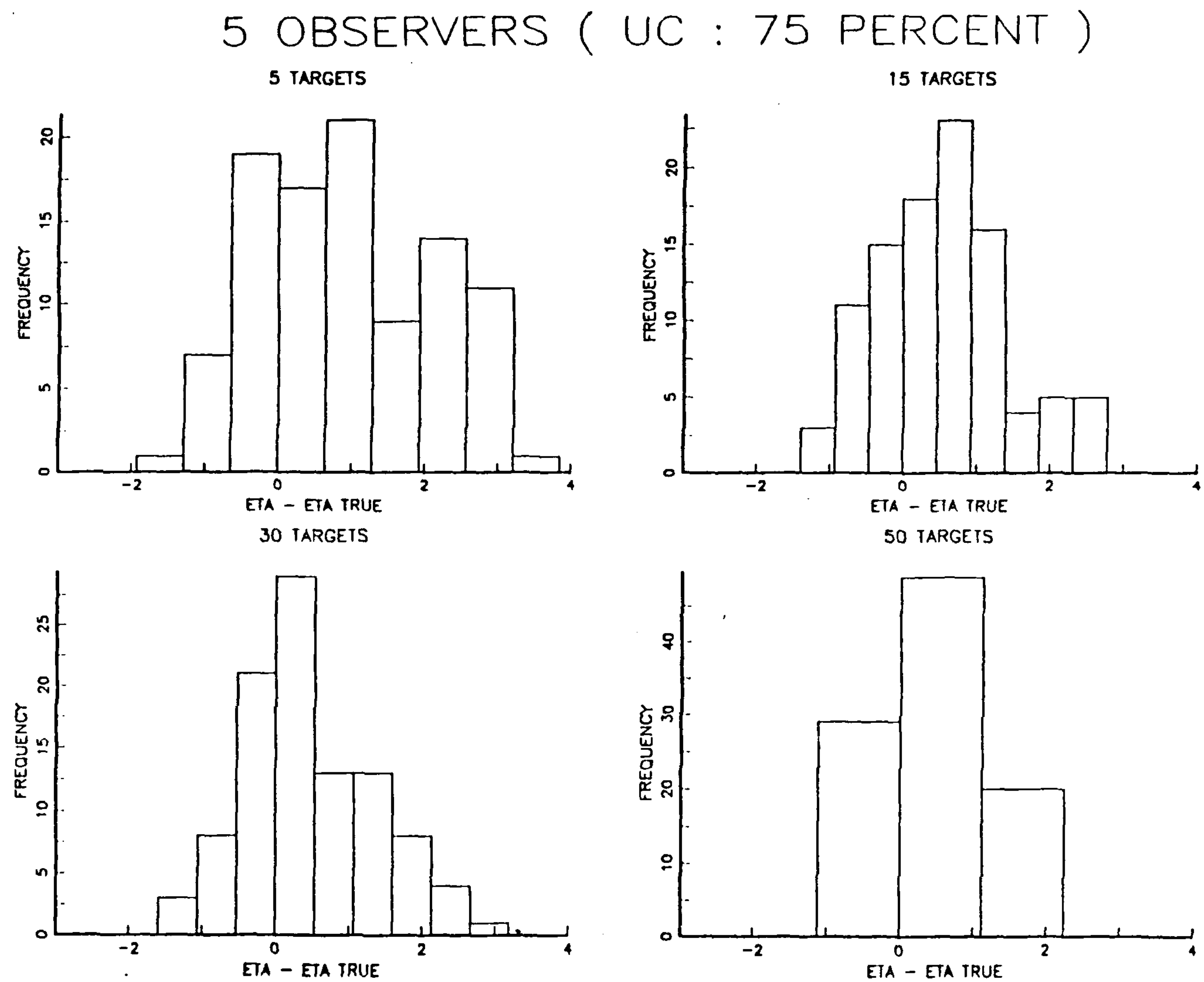

Figure 3. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 40, UC = 75%): 5,15,30,50 targets for 5 observers ($\hat{\eta} - \eta$)

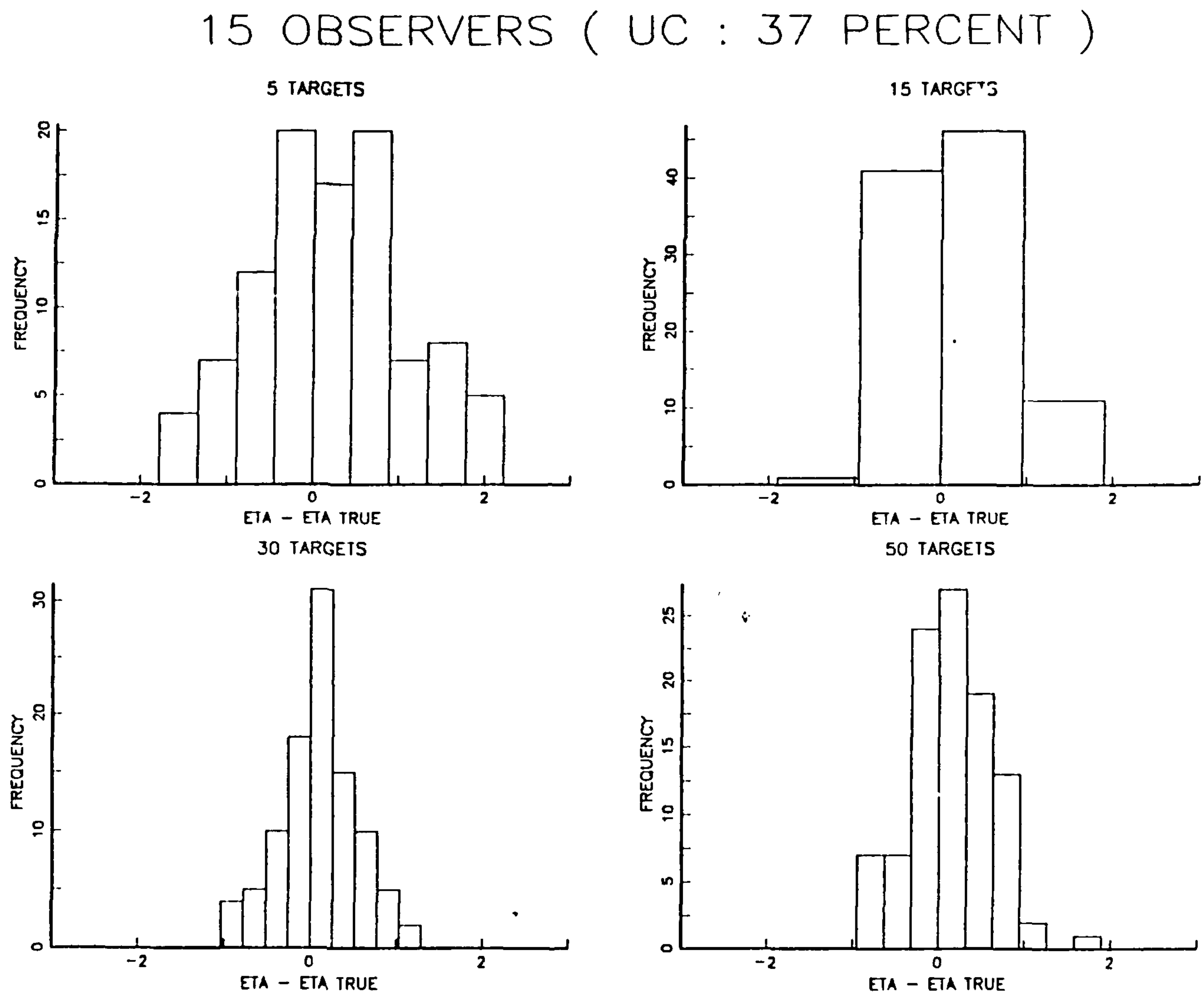

Figure 4. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 15 observers ($\hat{\eta} - \eta$)

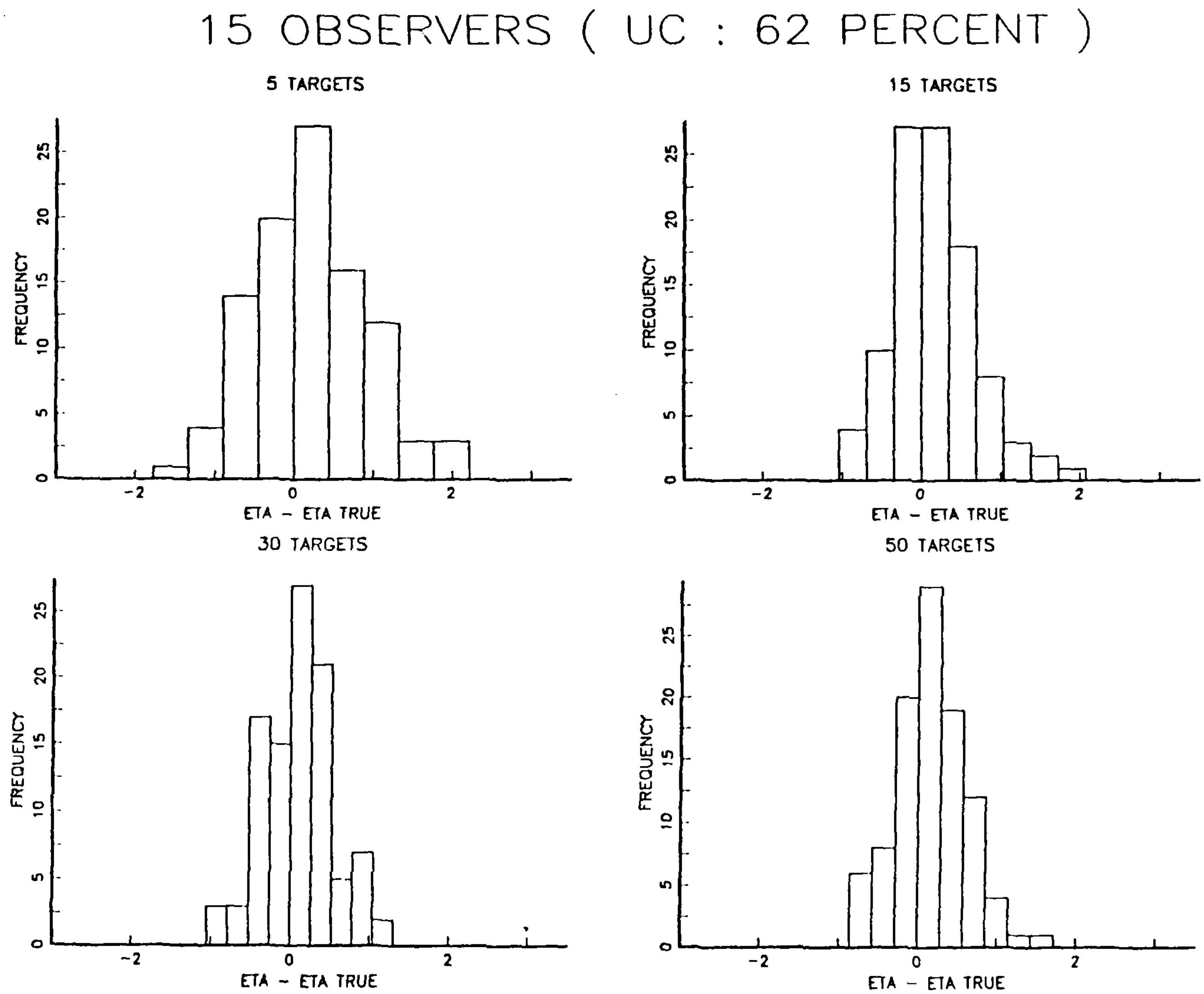

Figure 5. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=25, UC=62%): 5,15,30,50 targets for 15 observers ($\hat{\eta} - \eta$)

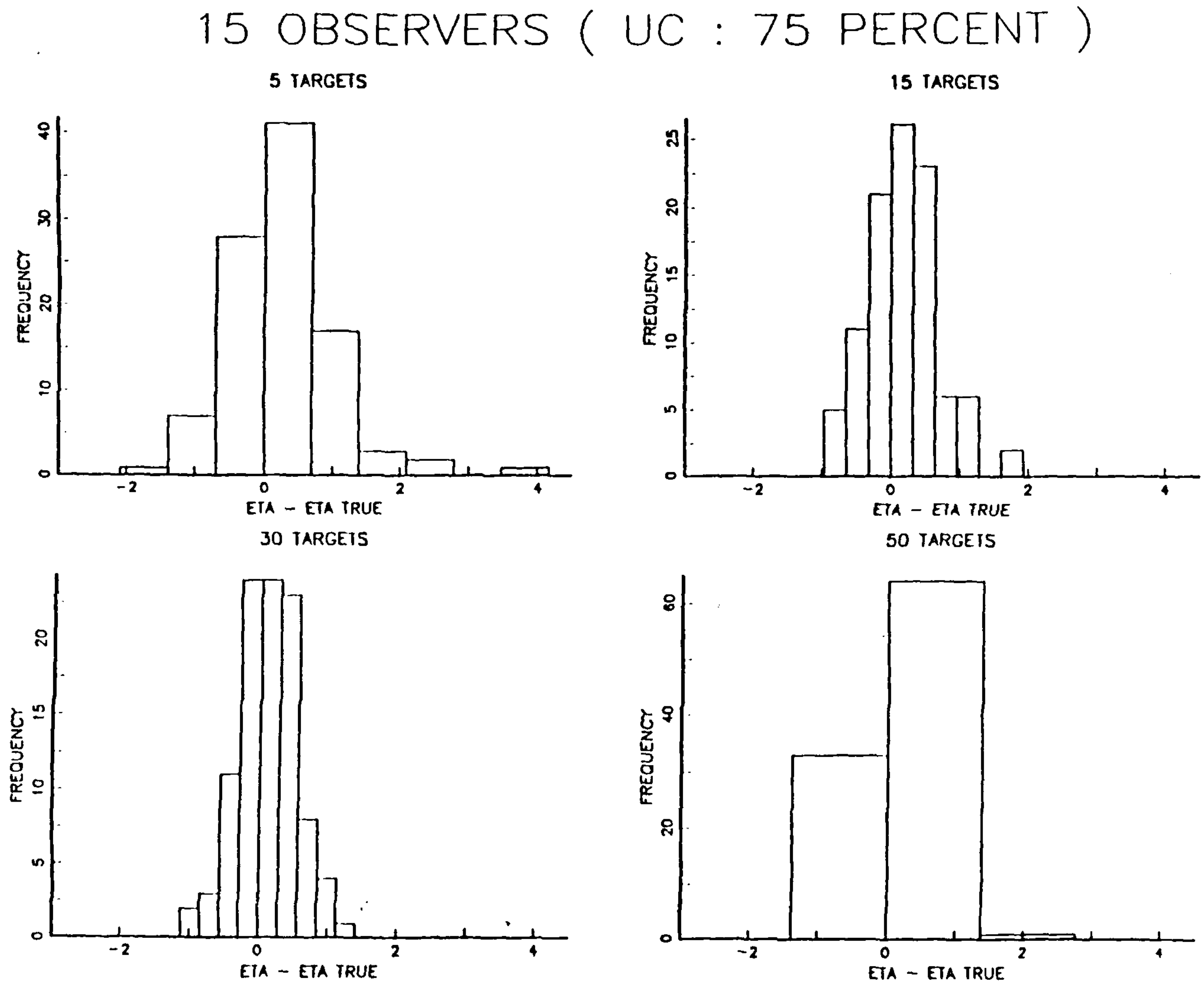

Figure 6. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 40, UC = 75%): 5,15,30,50 targets for 15 observers ($\hat{\eta} - \eta$)

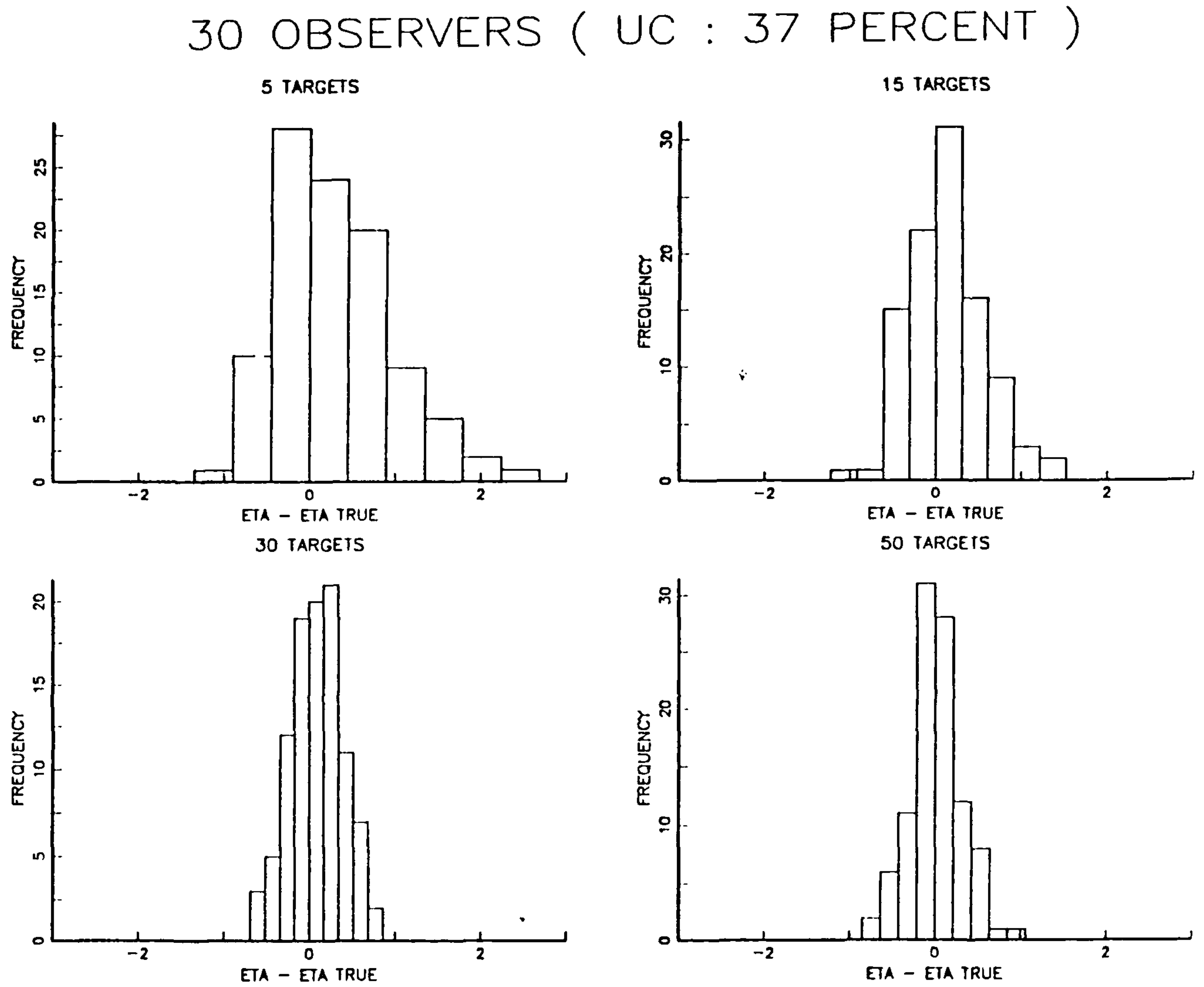

Figure 7. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 30 observers ($\hat{\eta} - \eta$)

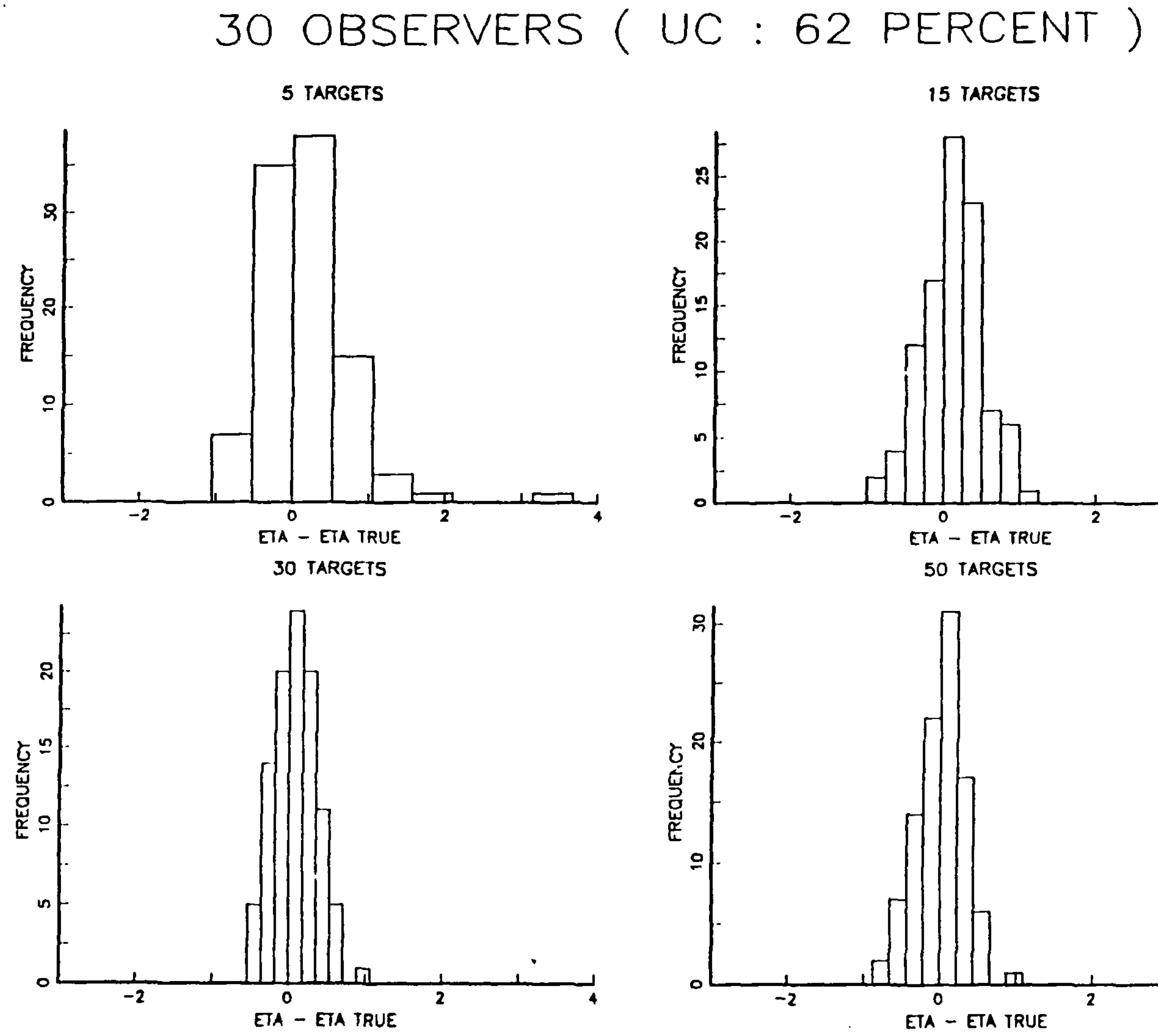

Figure 8. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=25, UC=62%): 5,15,30,50 targets for 30 observers ($\hat{\eta} - \eta$)

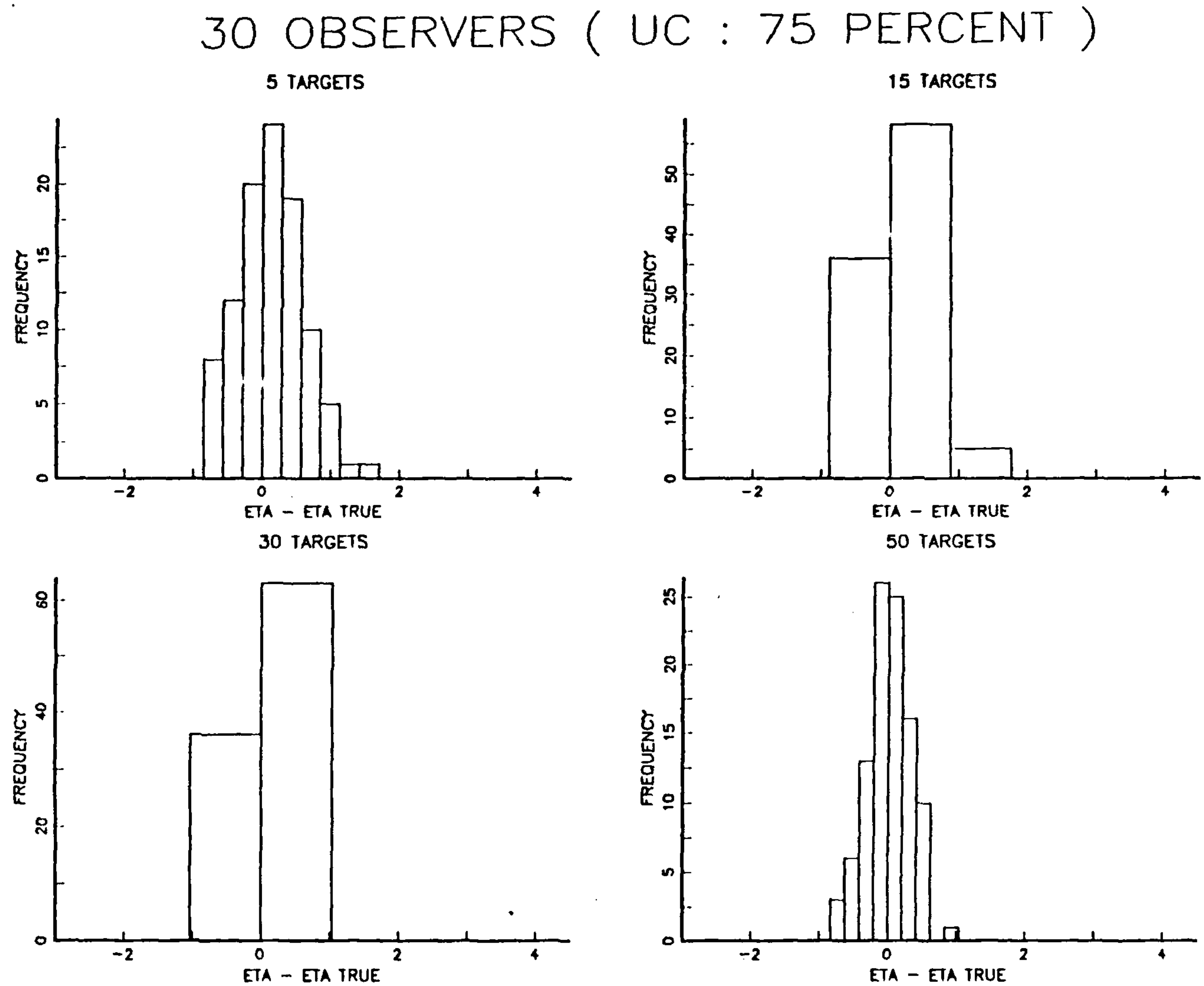

Figure 9. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=40, UC=75%): 5,15,30,50 targets for 30 observers ($\hat{\eta} - \eta$)

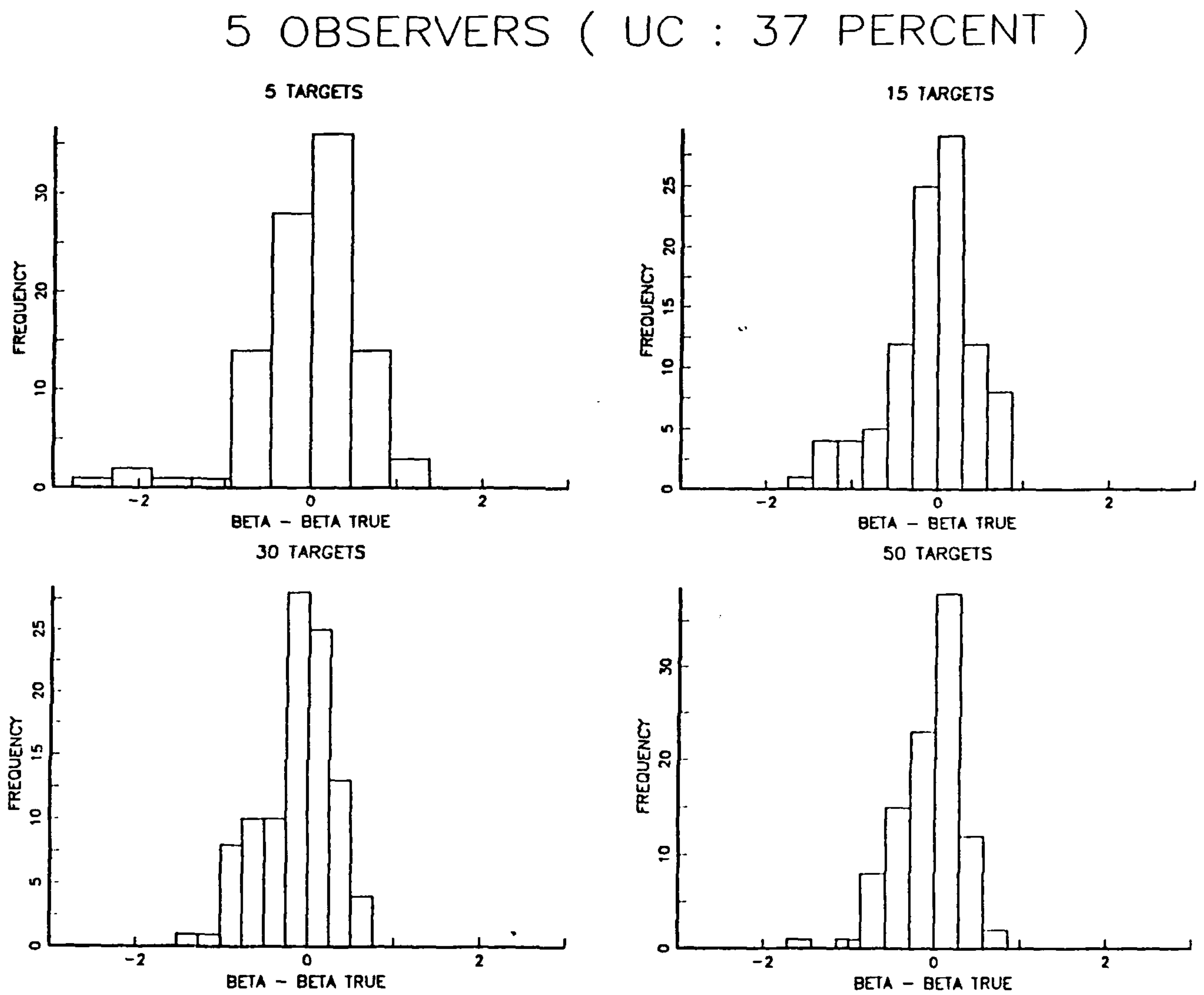

Figure 10. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 5 observers ($\hat{\beta}_0 - \beta_0$)

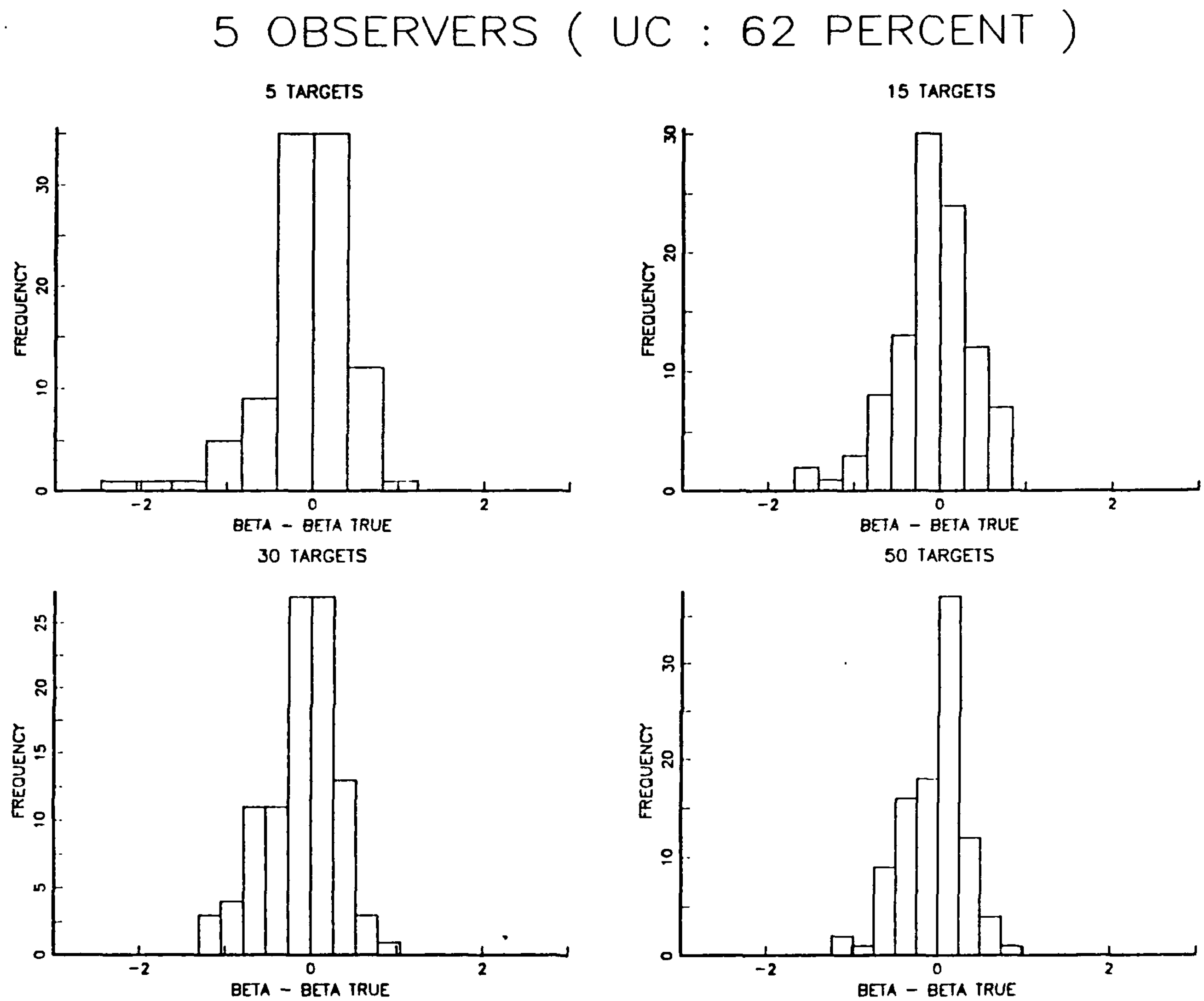

Figure 11. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 25, UC = 62%): 5,15,30,50 targets for 5 observers ($\hat{\beta}_0 - \beta_0$)

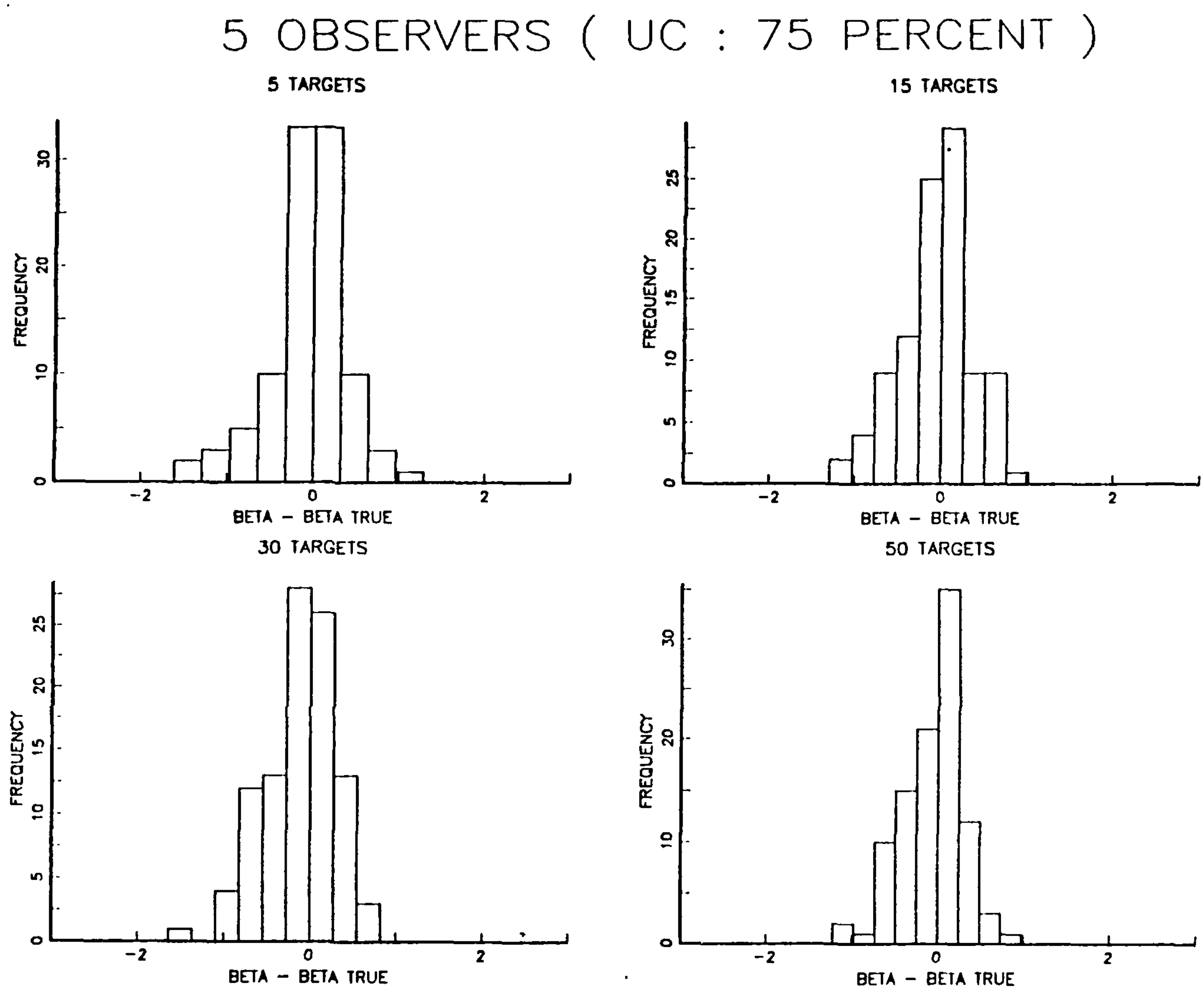

Figure 12. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=40, UC=75%): 5,15,30,50 targets for 5 observers ($\hat{\beta}_0 - \beta_0$)

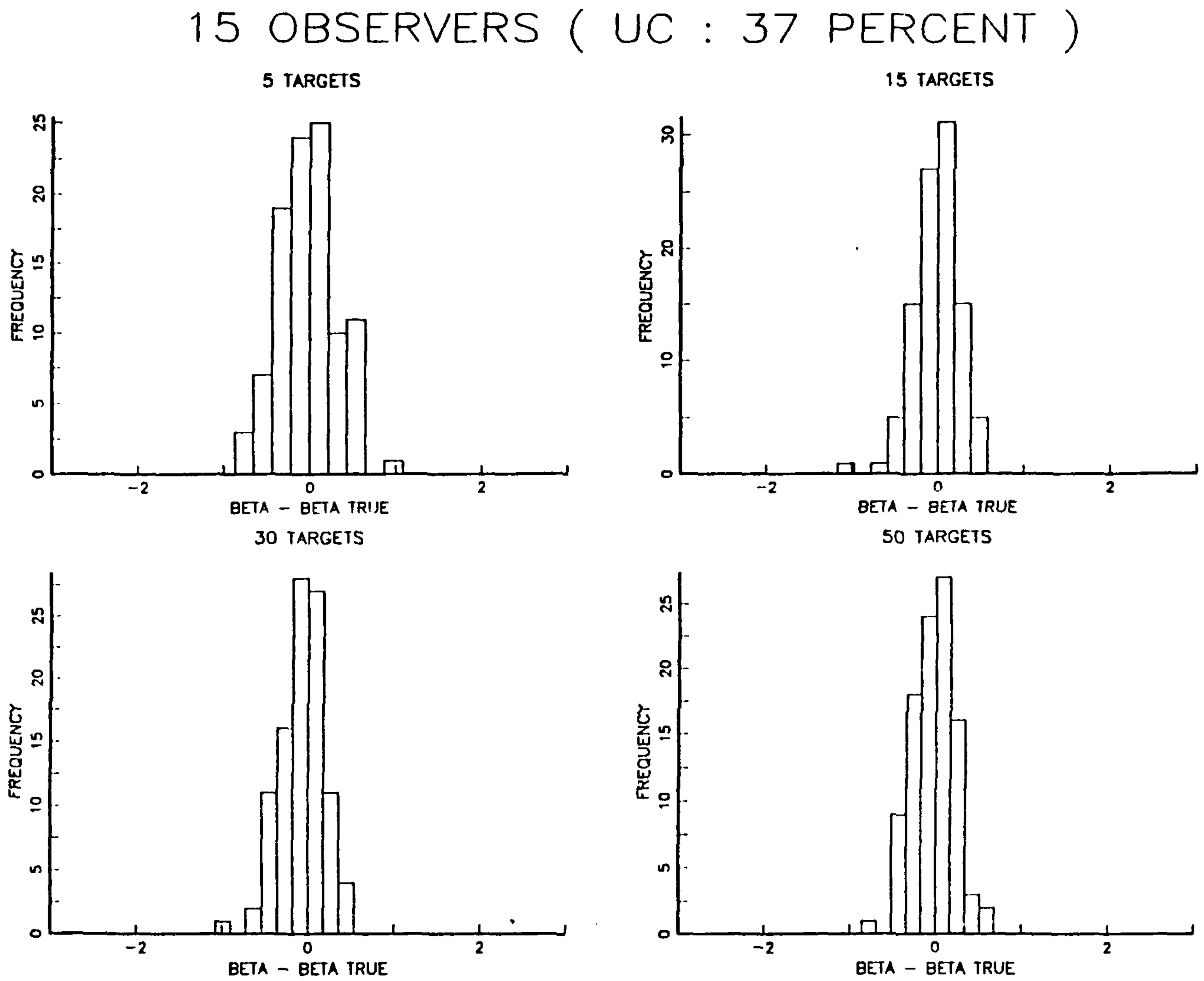

Figure 13. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 15 observers ($\hat{\beta}_0 - \beta_0$)

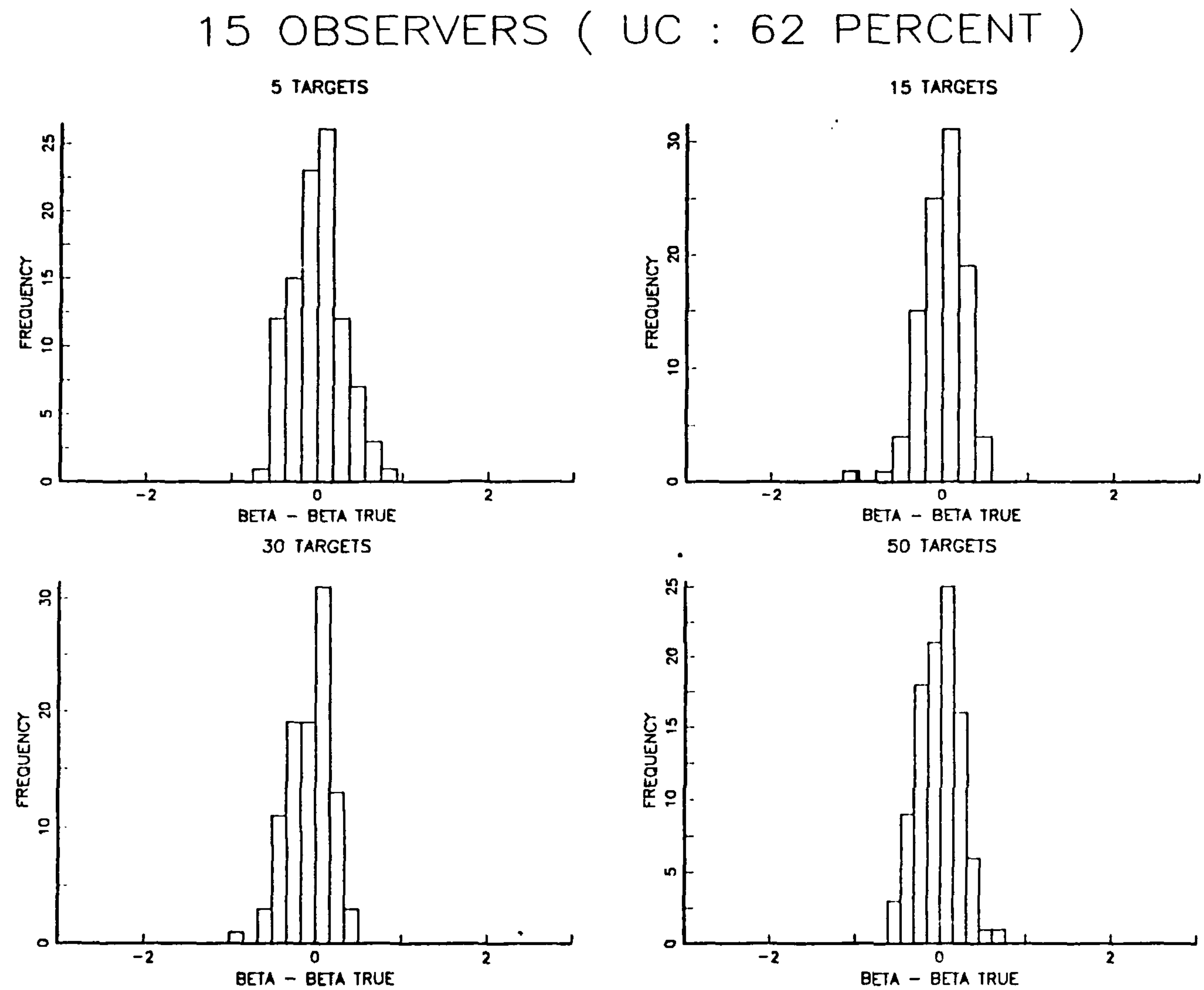

Figure 14. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=25, UC=62%): 5,15,30,50 targets for 15 observers ($\hat{\beta}_0 - \beta_0$)

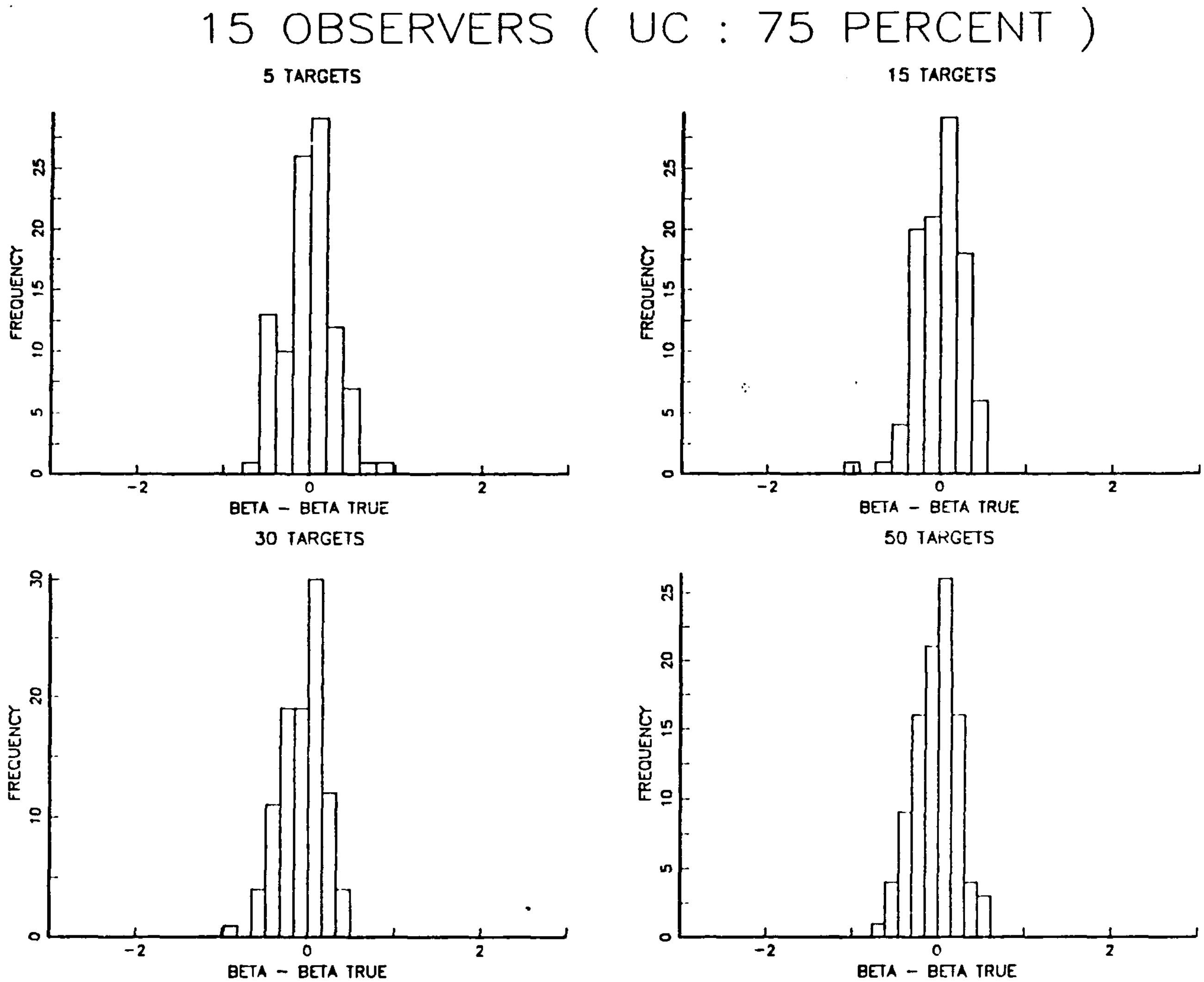

Figure 15. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=40, UC=75%): 5,15,30,50 targets for 15 observers ($\hat{\beta}_0 - \beta_0$)

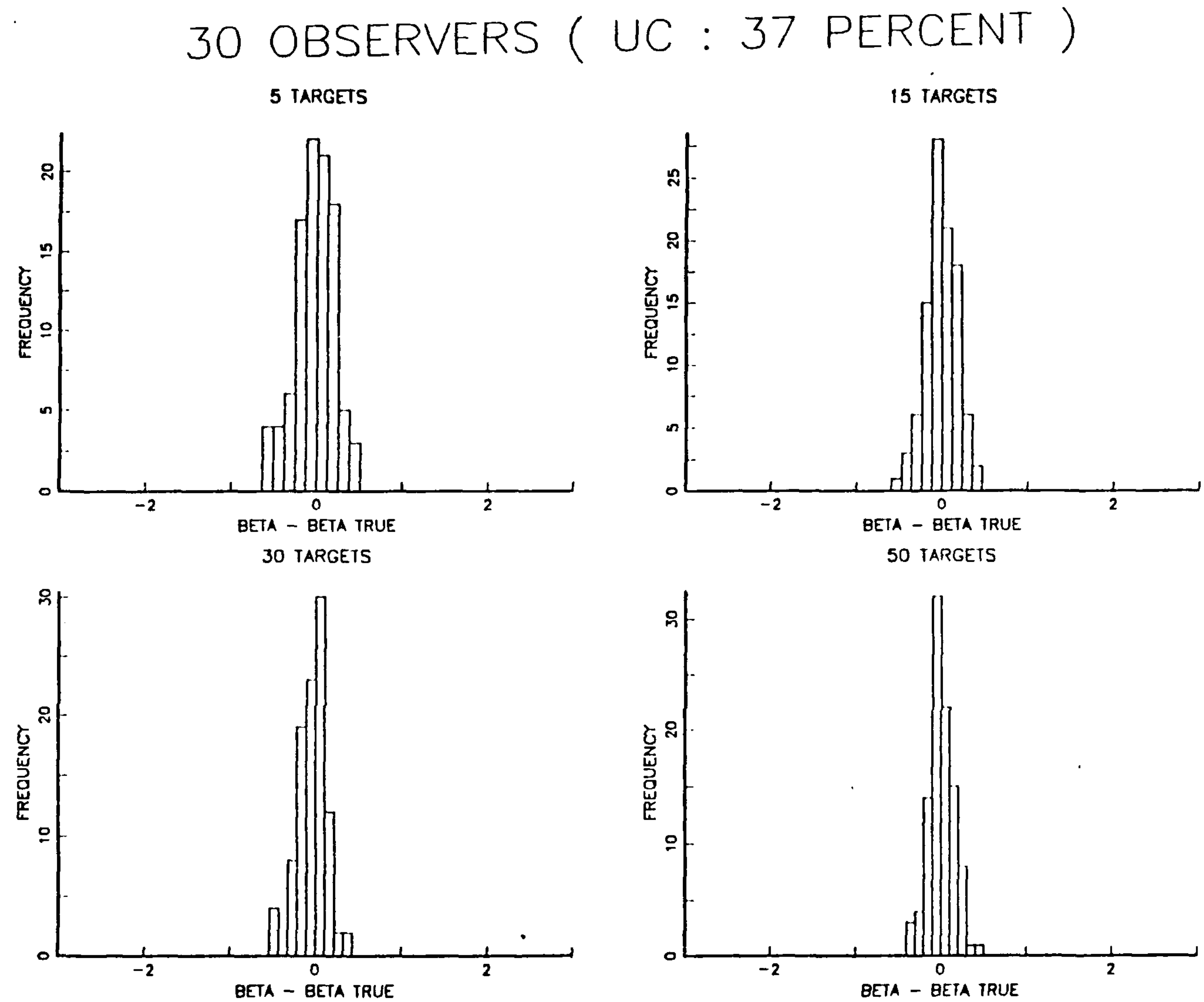

Figure 16. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 10, UC = 37%): 5,15,30,50 targets for 30 observers ($\hat{\beta}_0 - \beta_0$)

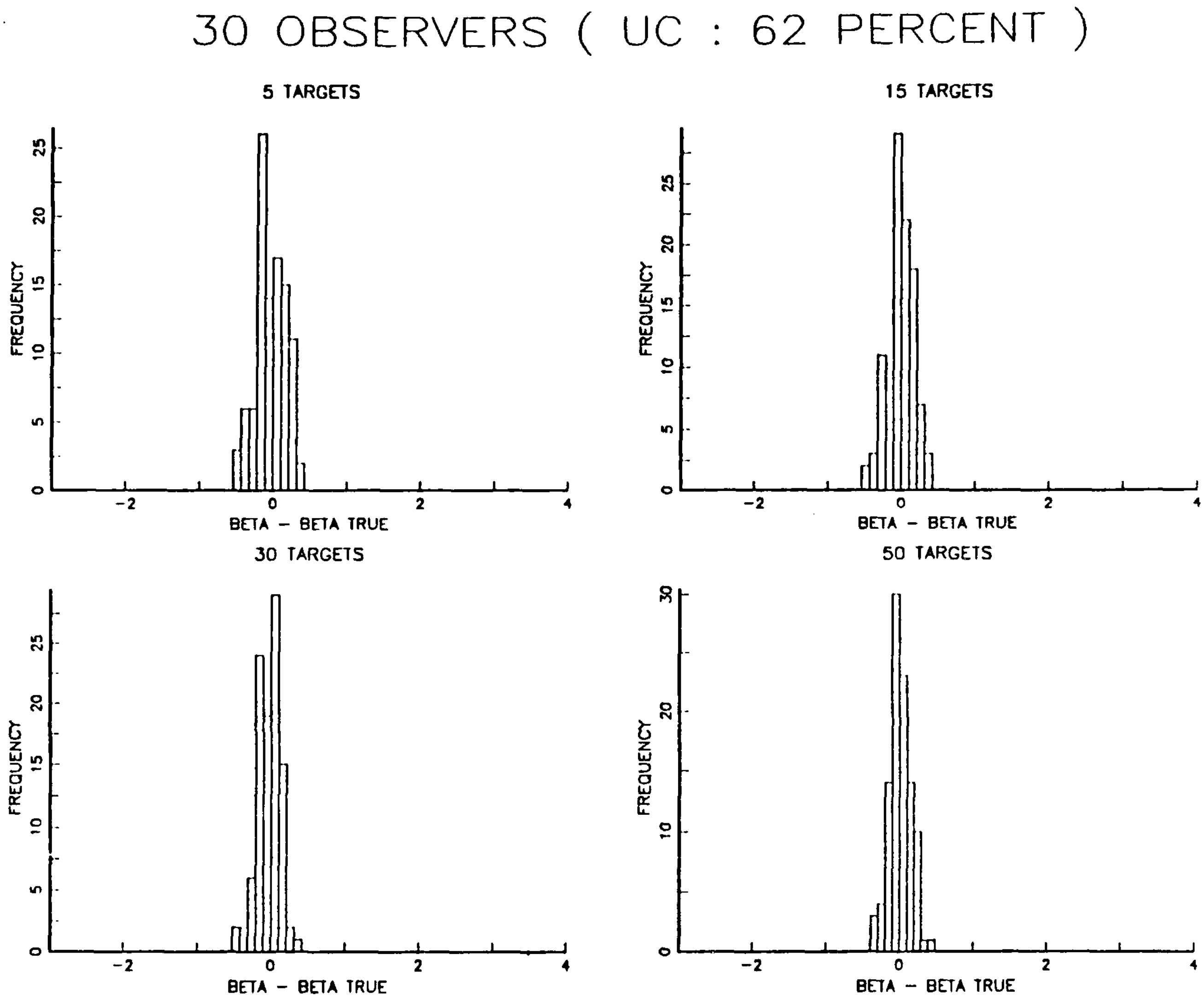

Figure 17. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 25, UC = 62%): 5,15,30,50 targets for 30 observers ($\hat{\beta}_0 - \beta_0$)

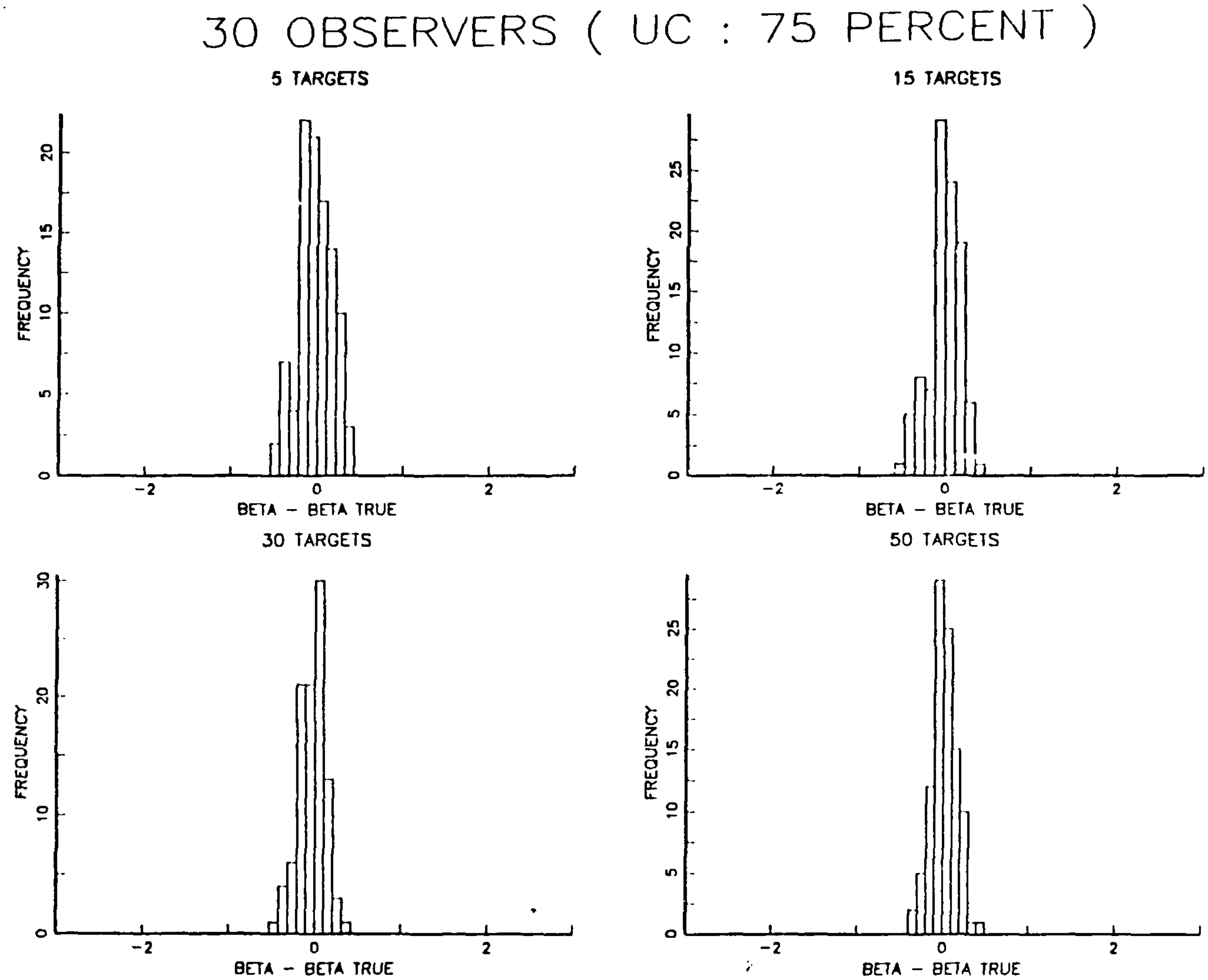

Figure 18. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=40, UC=75%): 5,15,30,50 targets for 30 observers ($\hat{\beta}_0 - \beta_0$)

APPENDIX B. HISTOGRAM FOR THE WEIBULL REGRESSION PARAMETERS

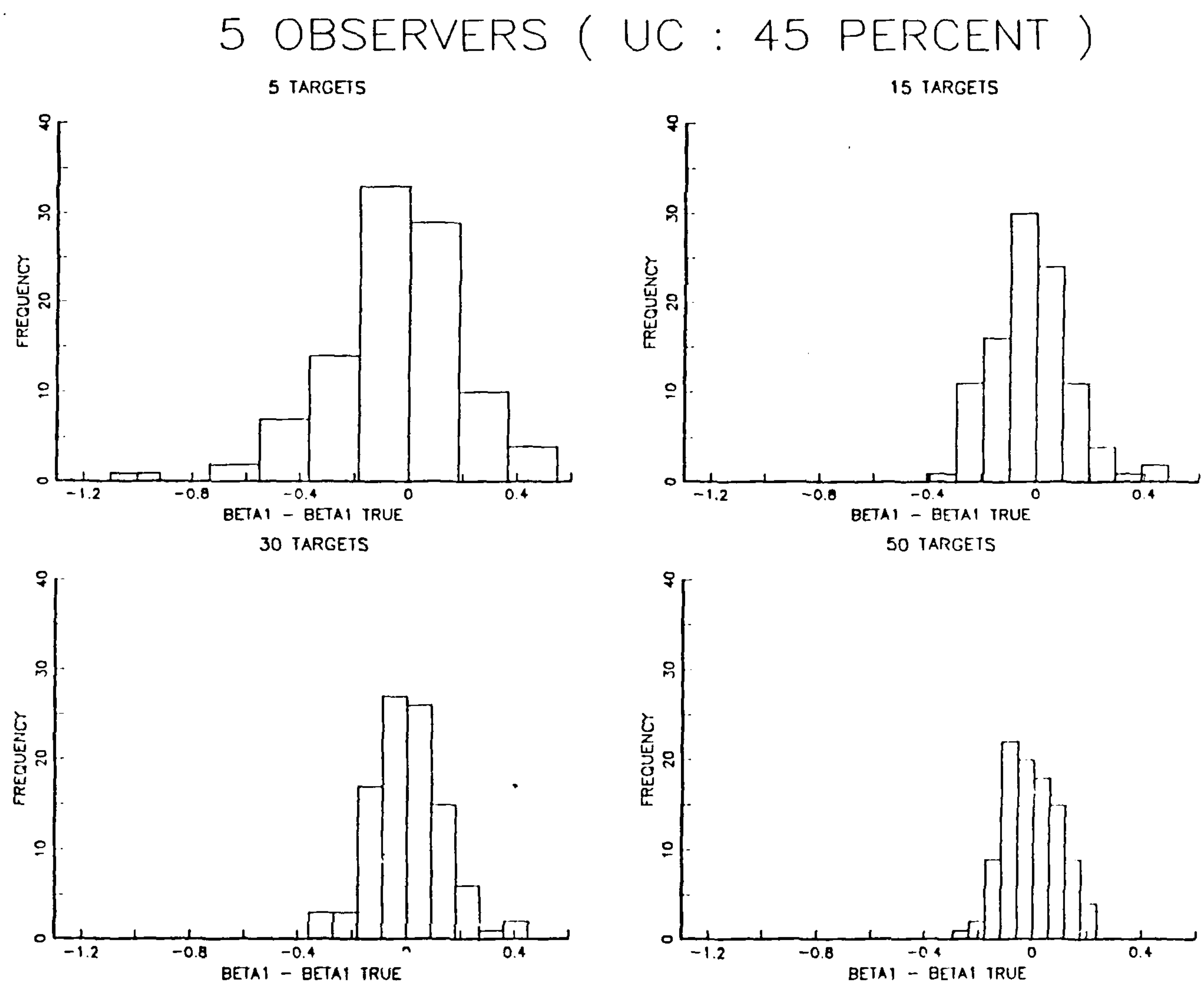

Figure 19. COMPARISON BETWEEN DIFFERENT ESTIMATES (O=1.3, UC=45%): 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_1 - \beta_1$)

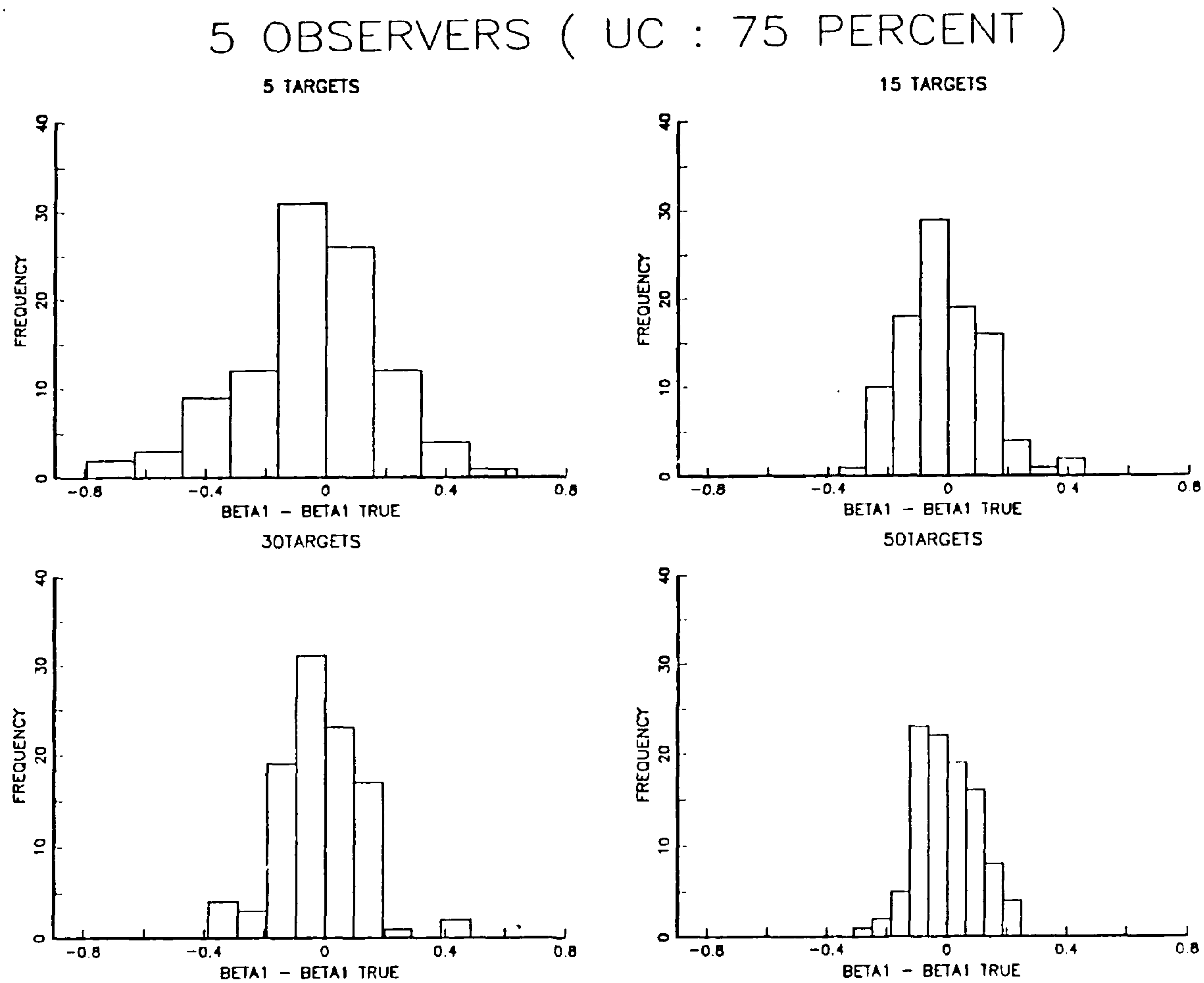

Figure 20. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_1 - \beta_1$)

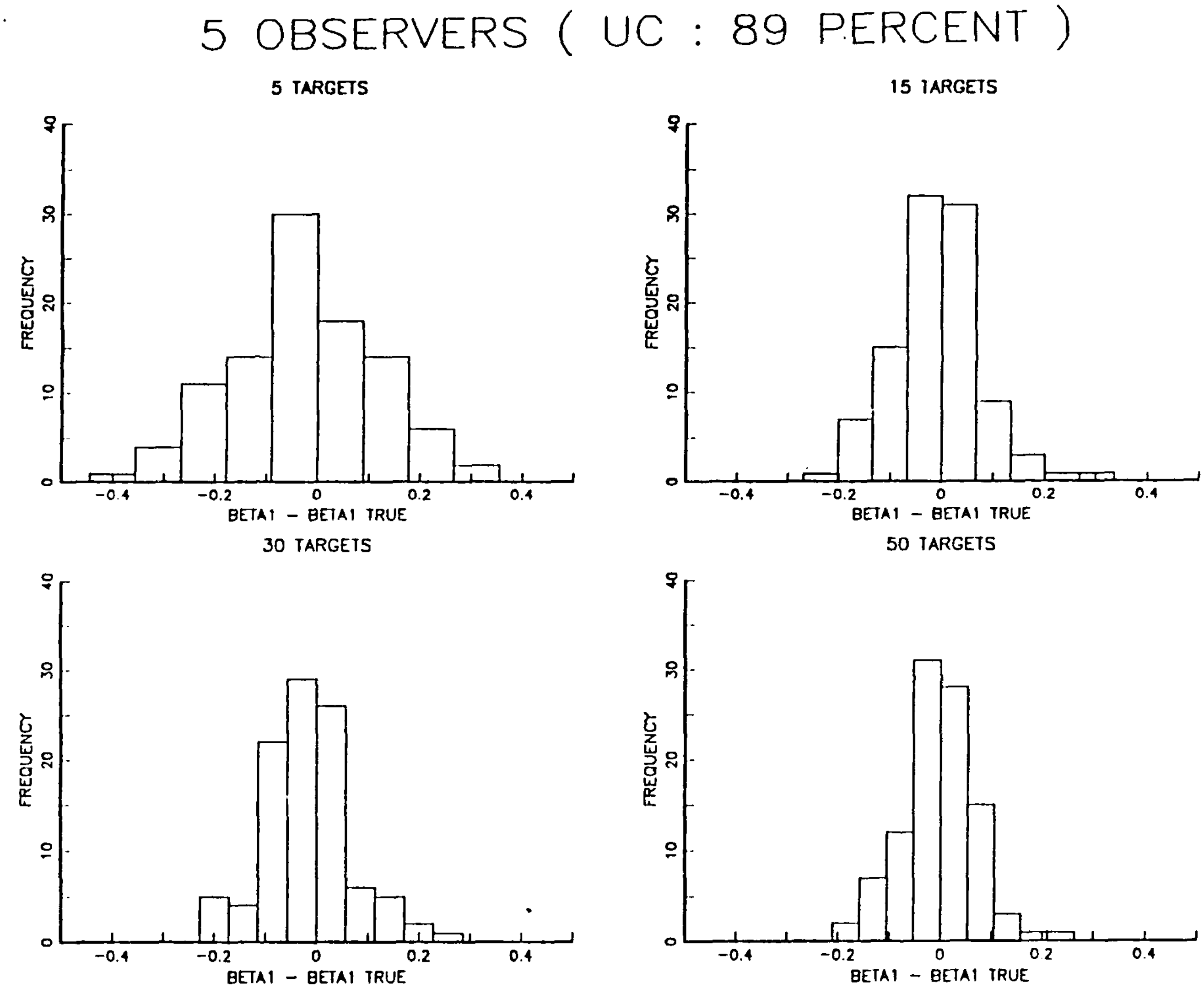

Figure 21. **COMPARISON BETWEEN DIFFERENT ESTIMATES (O=4.0, UC=89%):** 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_1 - \beta_1$)

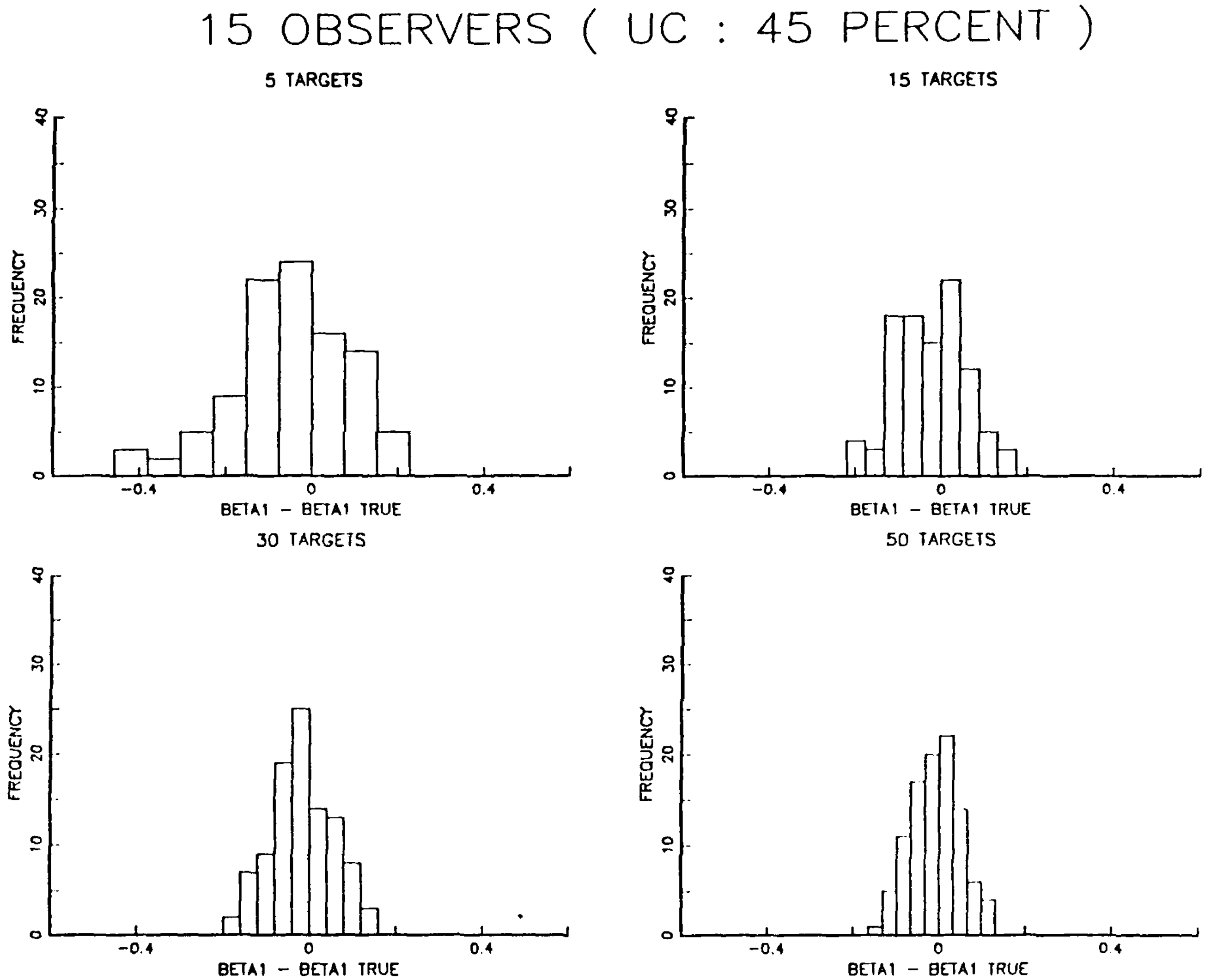

Figure 22. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_1 - \beta_1$)

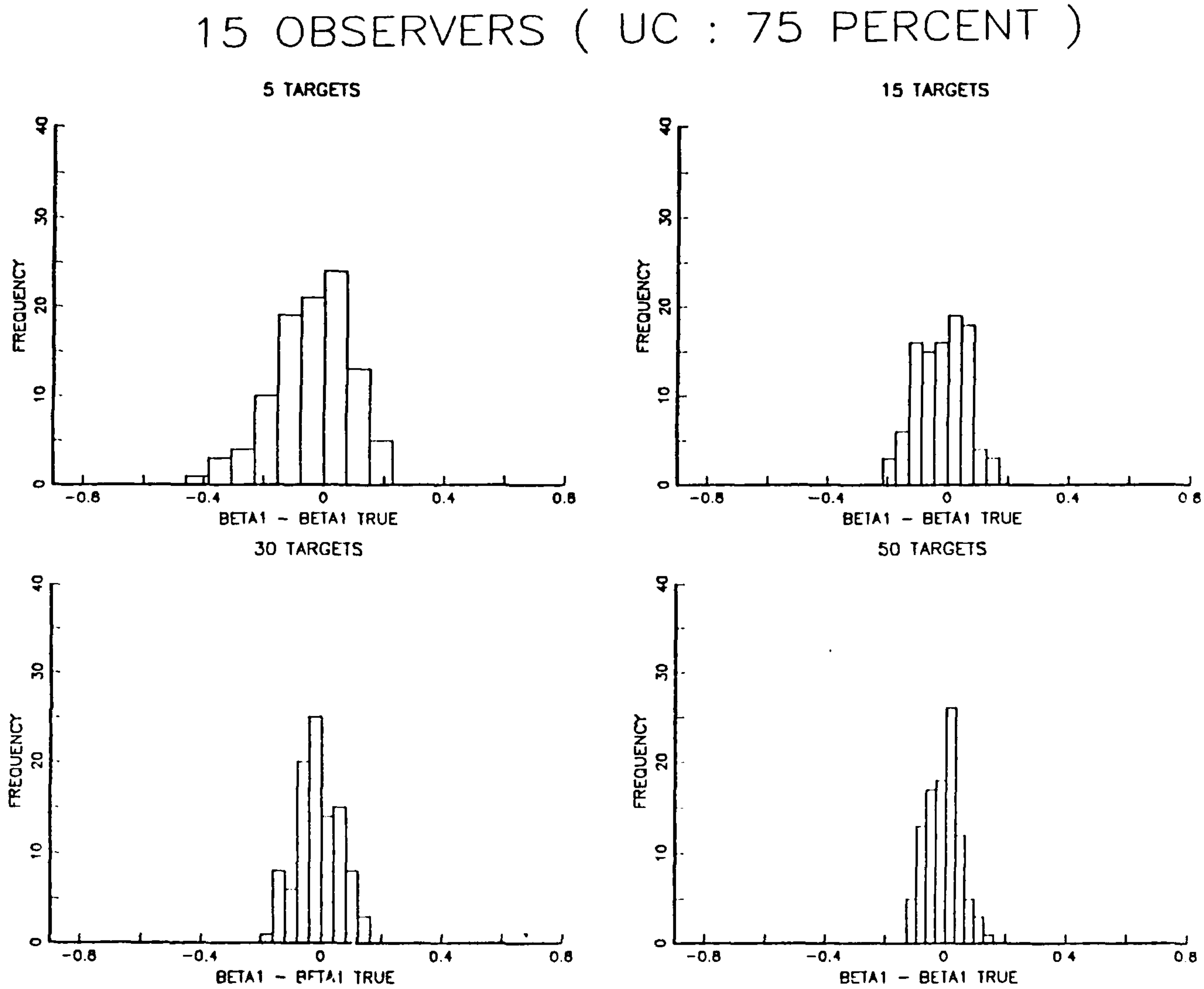

Figure 23. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_1 - \beta_1$)

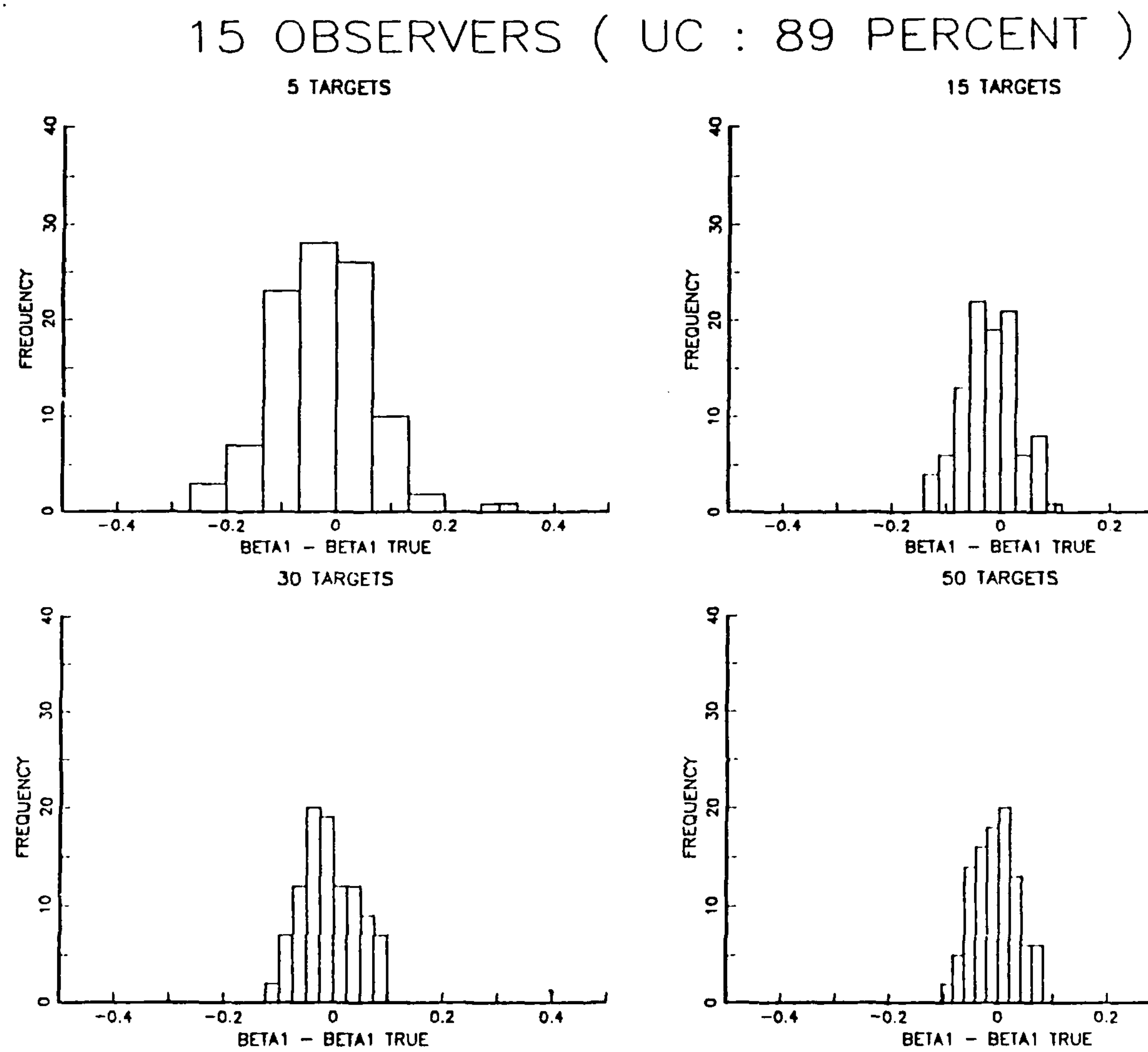

Figure 24. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_1 - \beta_1$)

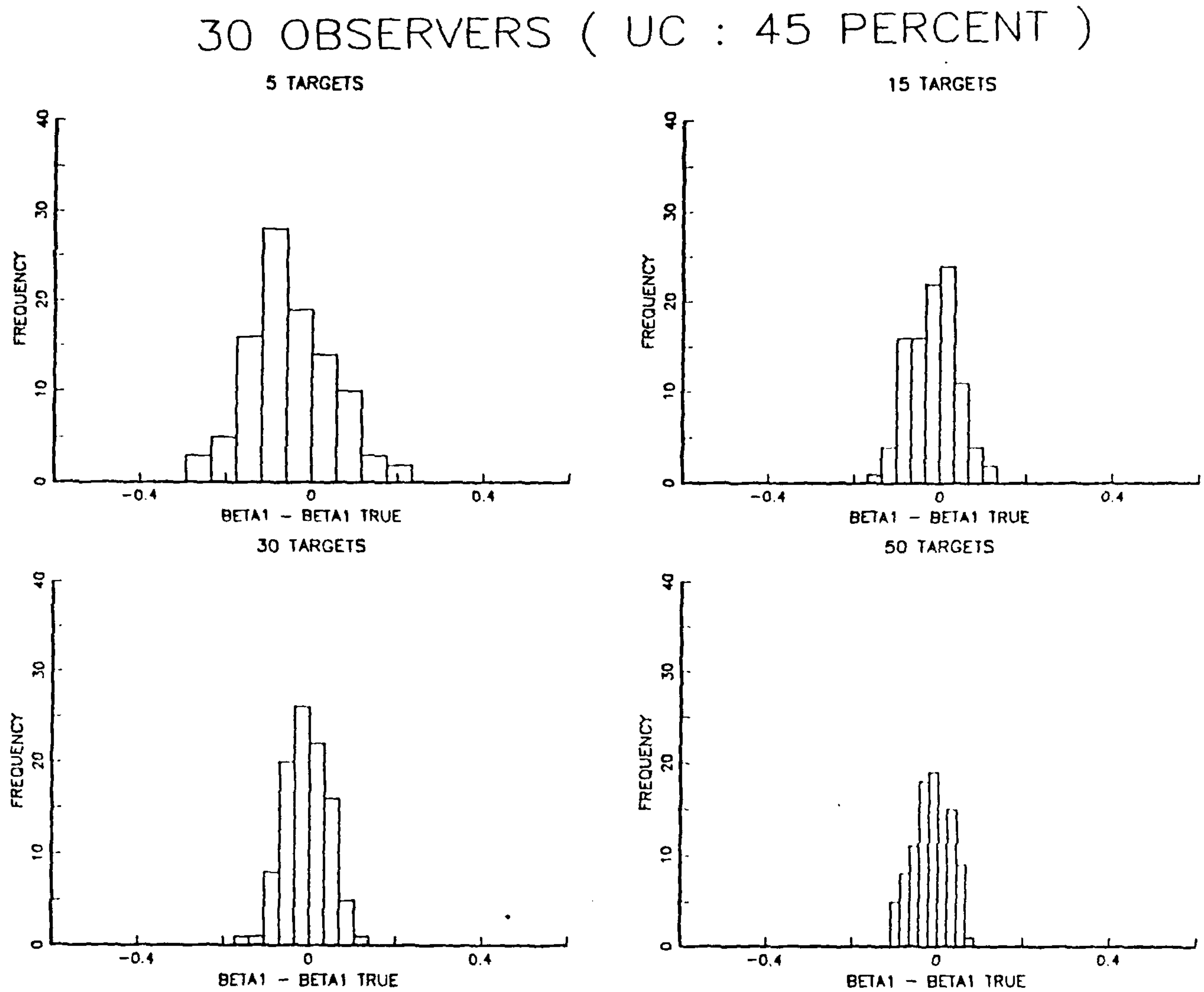

Figure 25. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 30 observers ($\hat{\beta}_1 - \beta_1$)

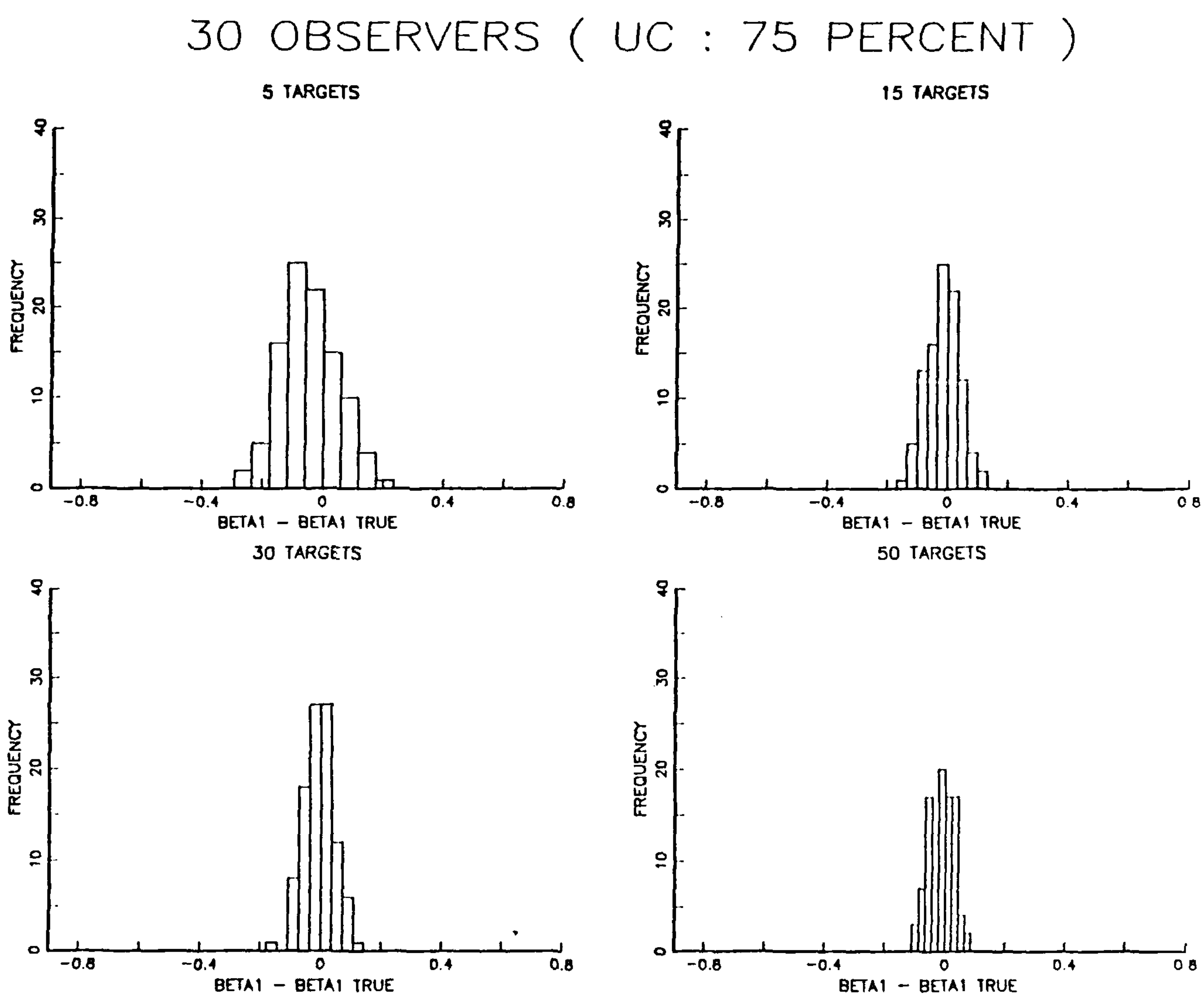

Figure 26. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.3, UC = 75%): 5, 15, 30, 50 targets for 30 observers ($\hat{\beta}_1 - \beta_1$)

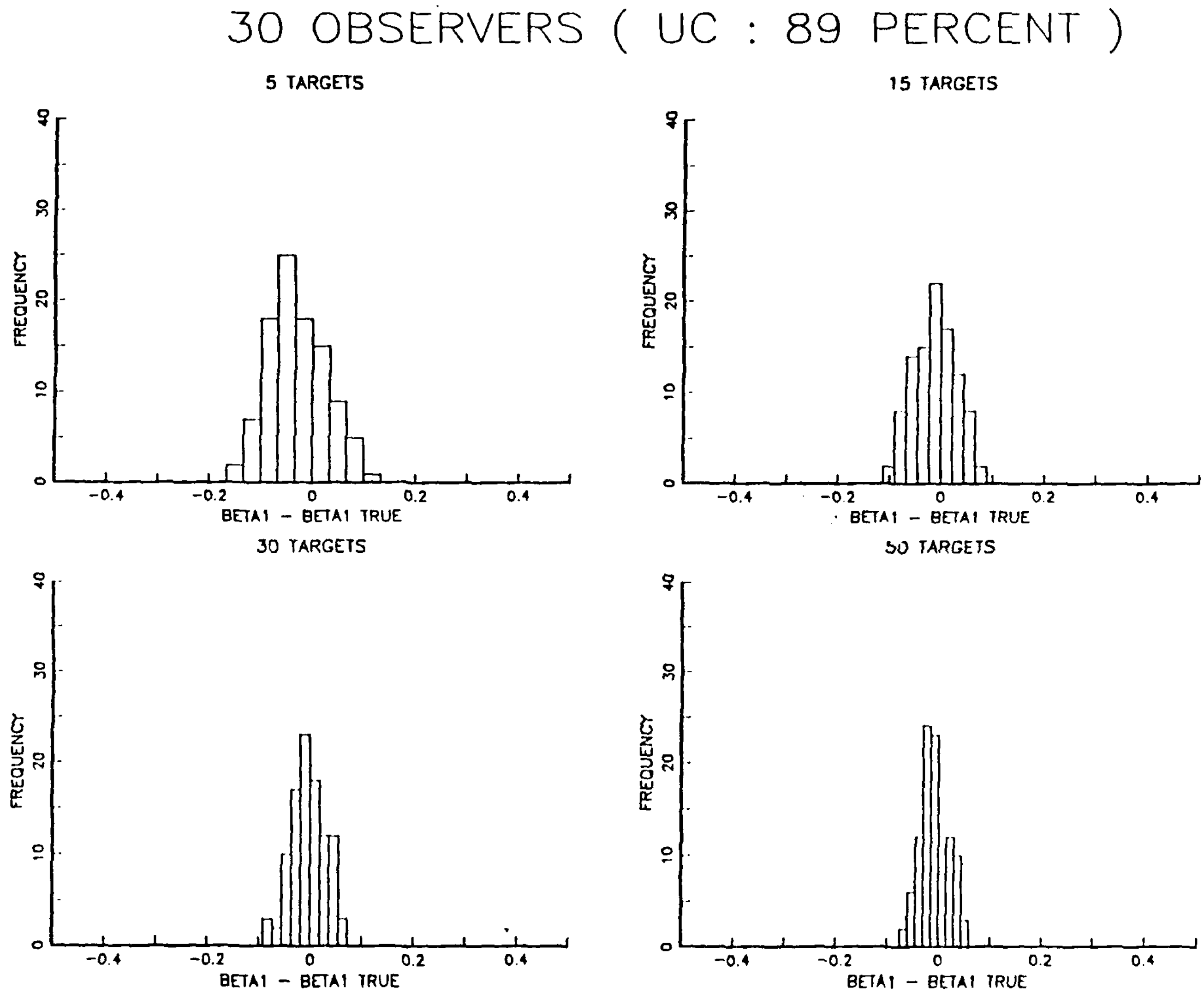

Figure 27. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 30 observers ($\hat{\beta}_1 - \beta_1$)

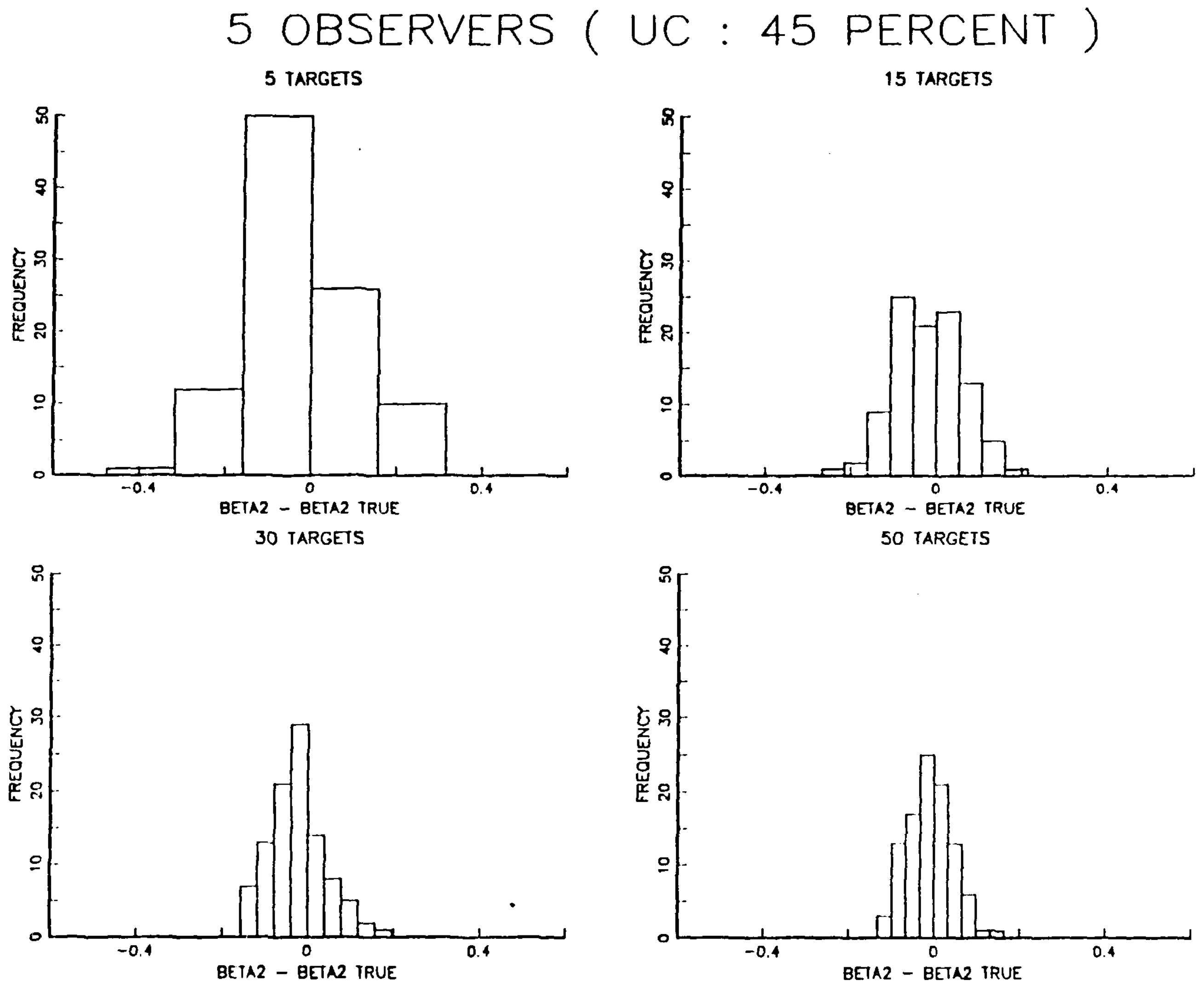

Figure 28. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_2 - \beta_2$)

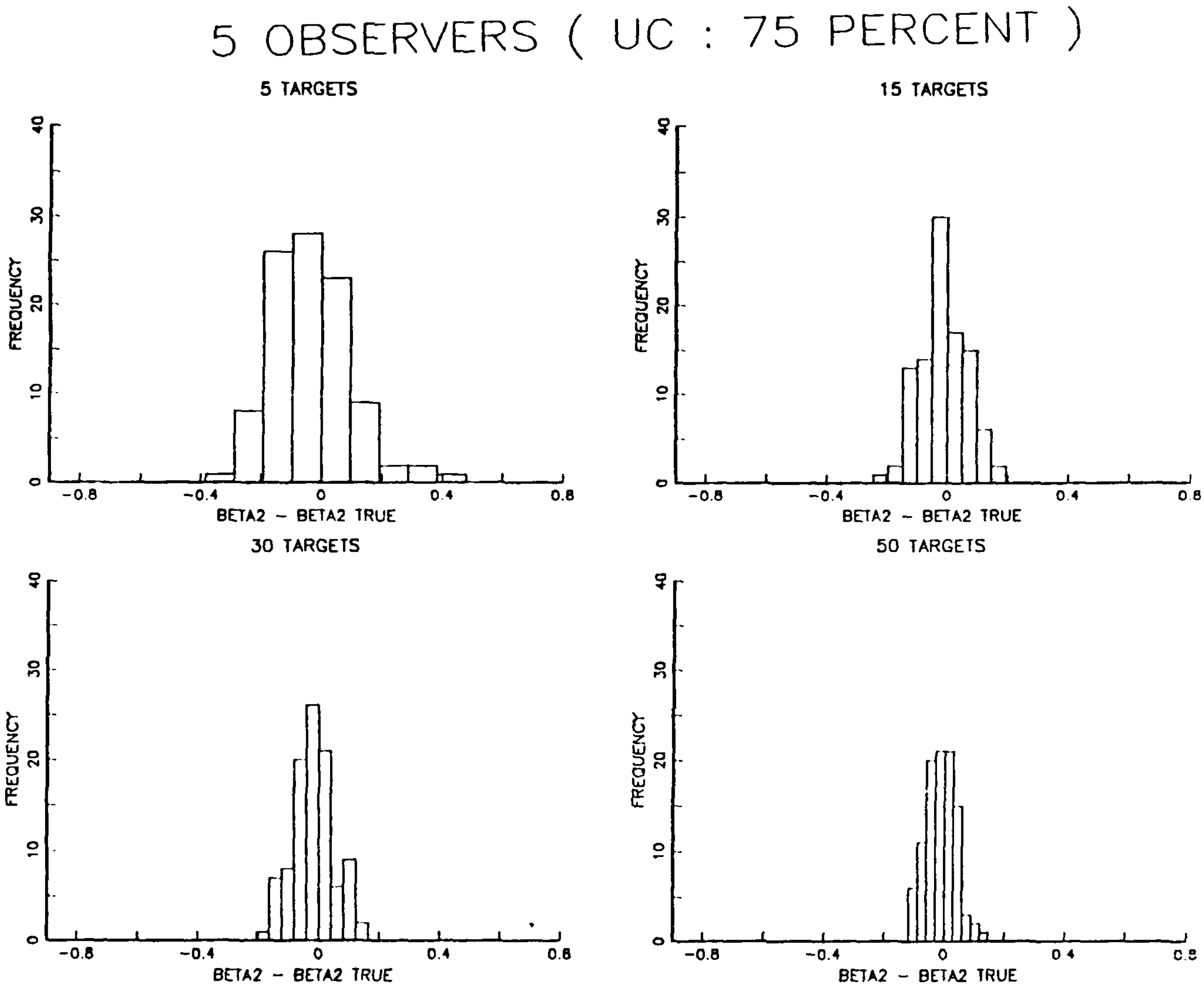

Figure 29. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_2 - \beta_2$)

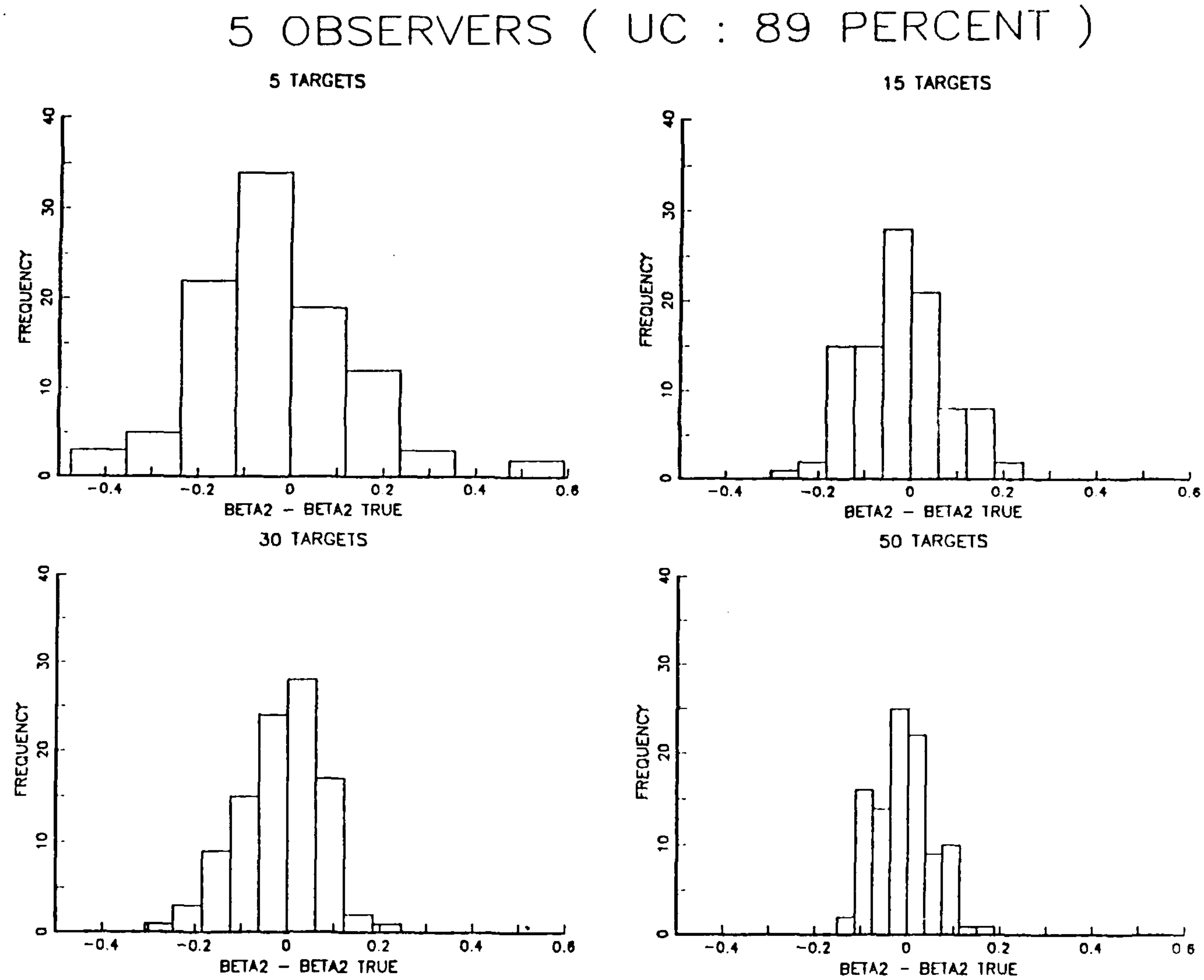

Figure 30. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 5 observers ($\hat{\beta}_2 - \beta_2$)

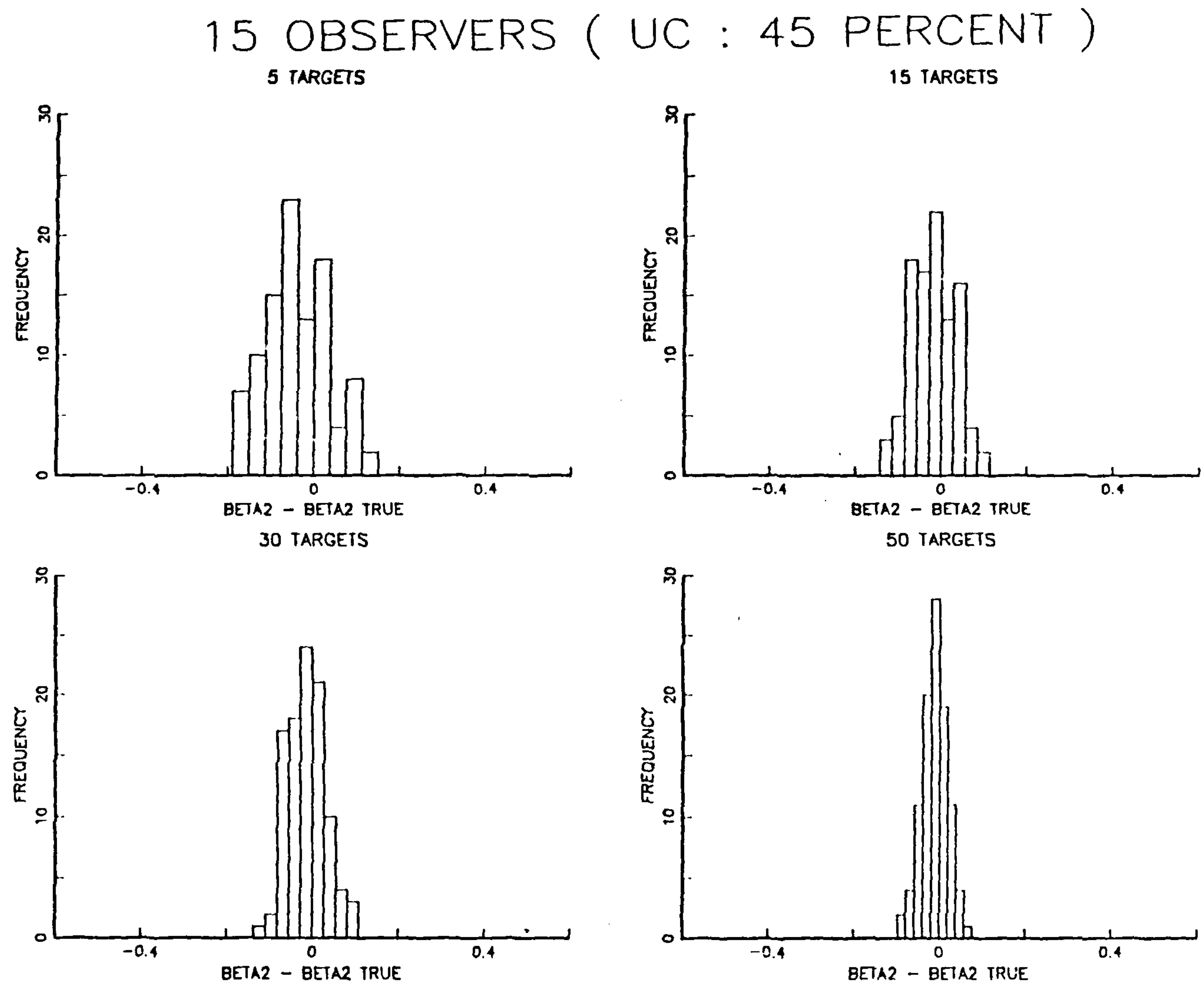

Figure 31. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_2 - \beta_2$)

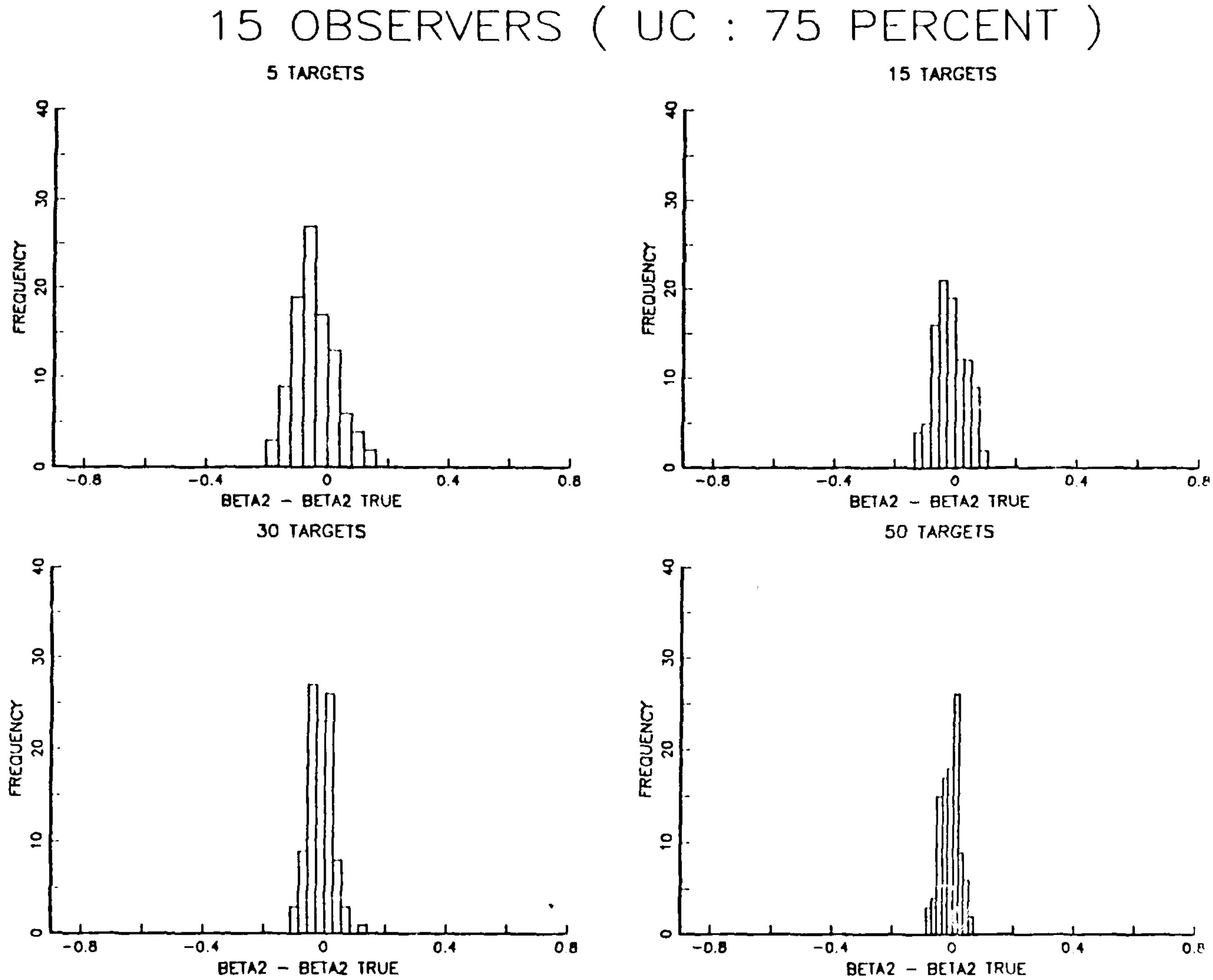

Figure 32. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_2 - \beta_2$)

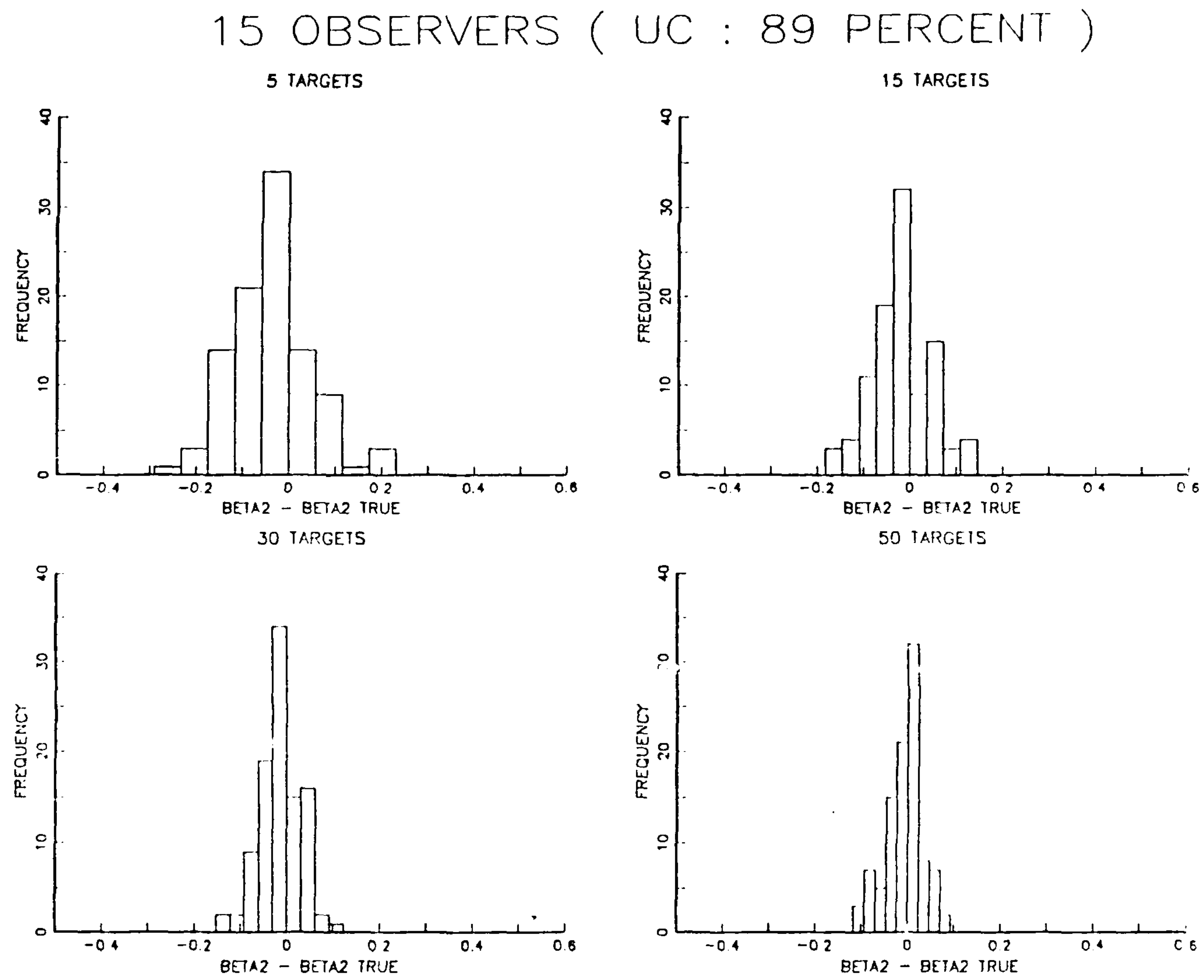

Figure 33. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 15 observers ($\hat{\beta}_2 - \beta_2$)

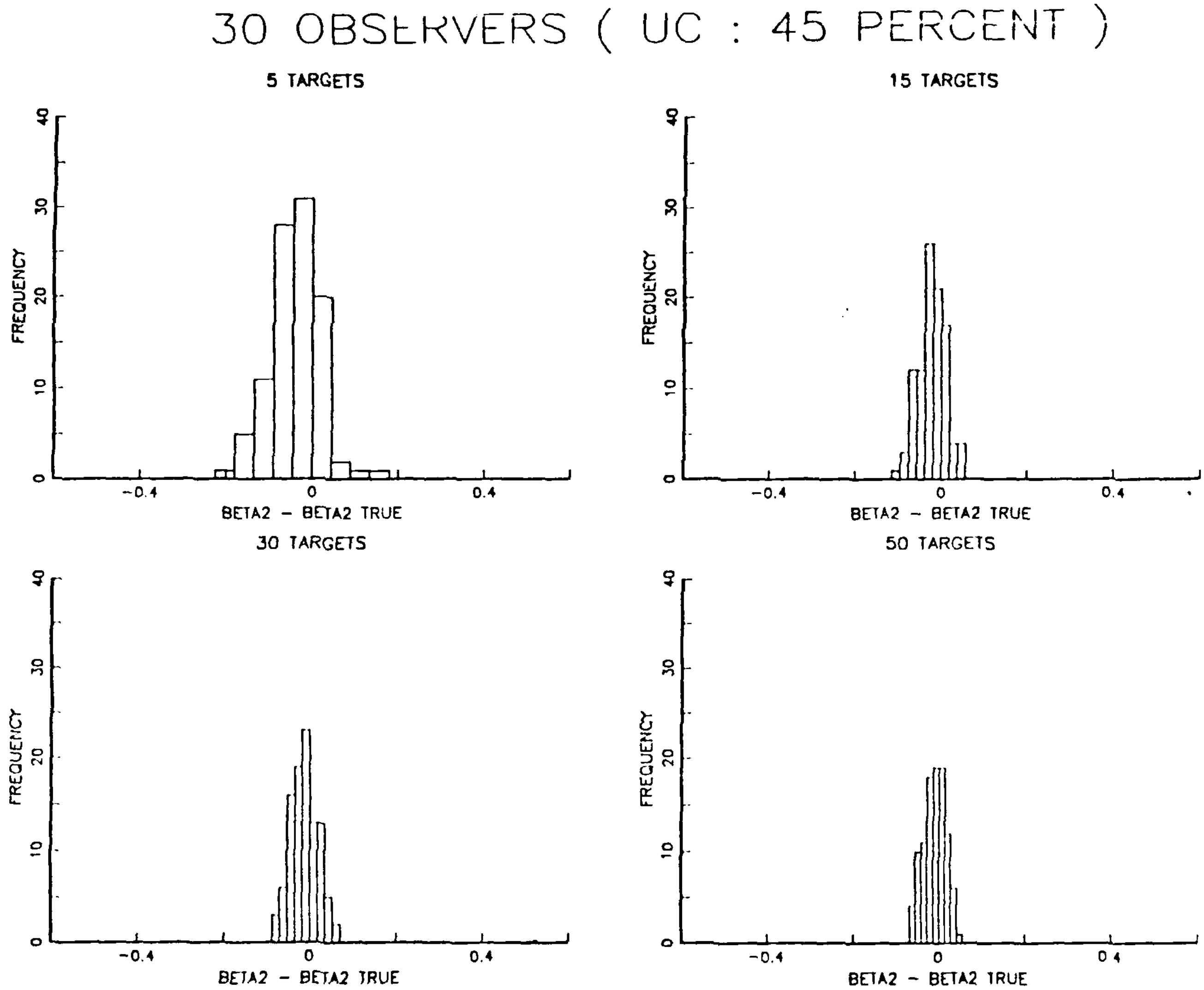

Figure 34. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 30 observers ($\hat{\beta}_2 - \beta_2$)

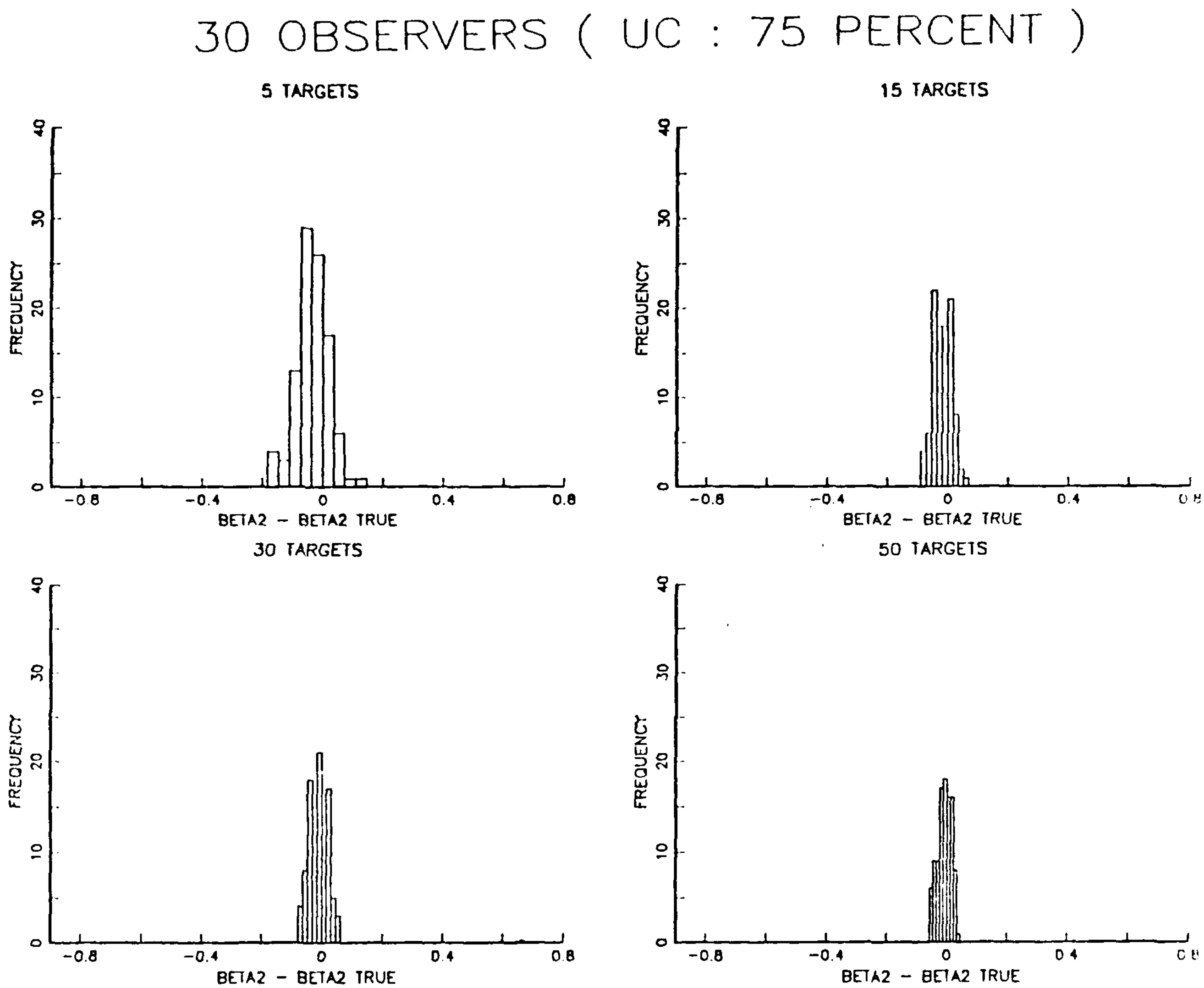

Figure 35. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 30 observers ($\hat{\beta}_2 - \beta_2$)

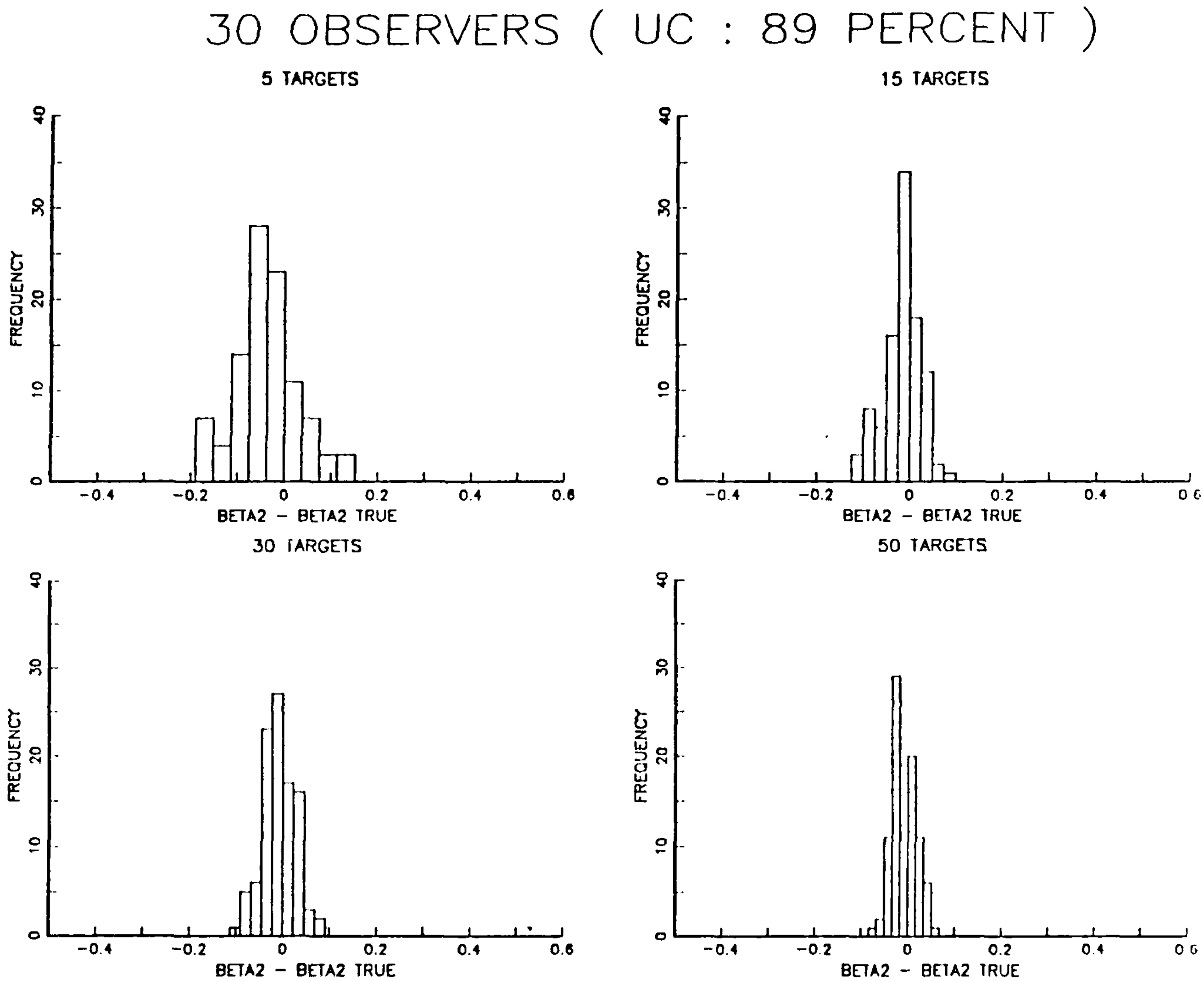

Figure 36. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 30 observers $(\hat{\beta}_2 - \beta_2)$

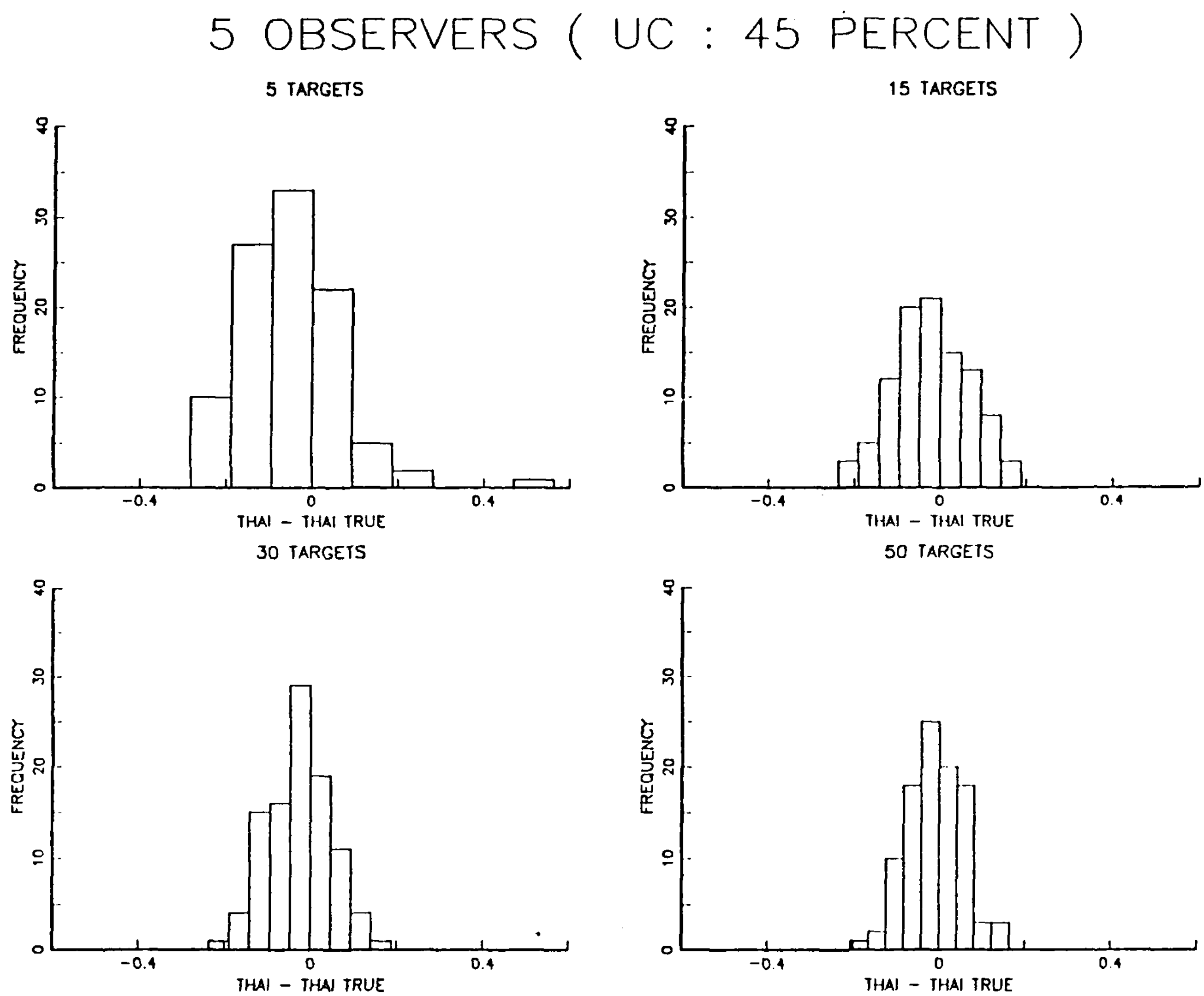

Figure 37. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 5 observers ($\hat{\xi}_i - \xi_i$)

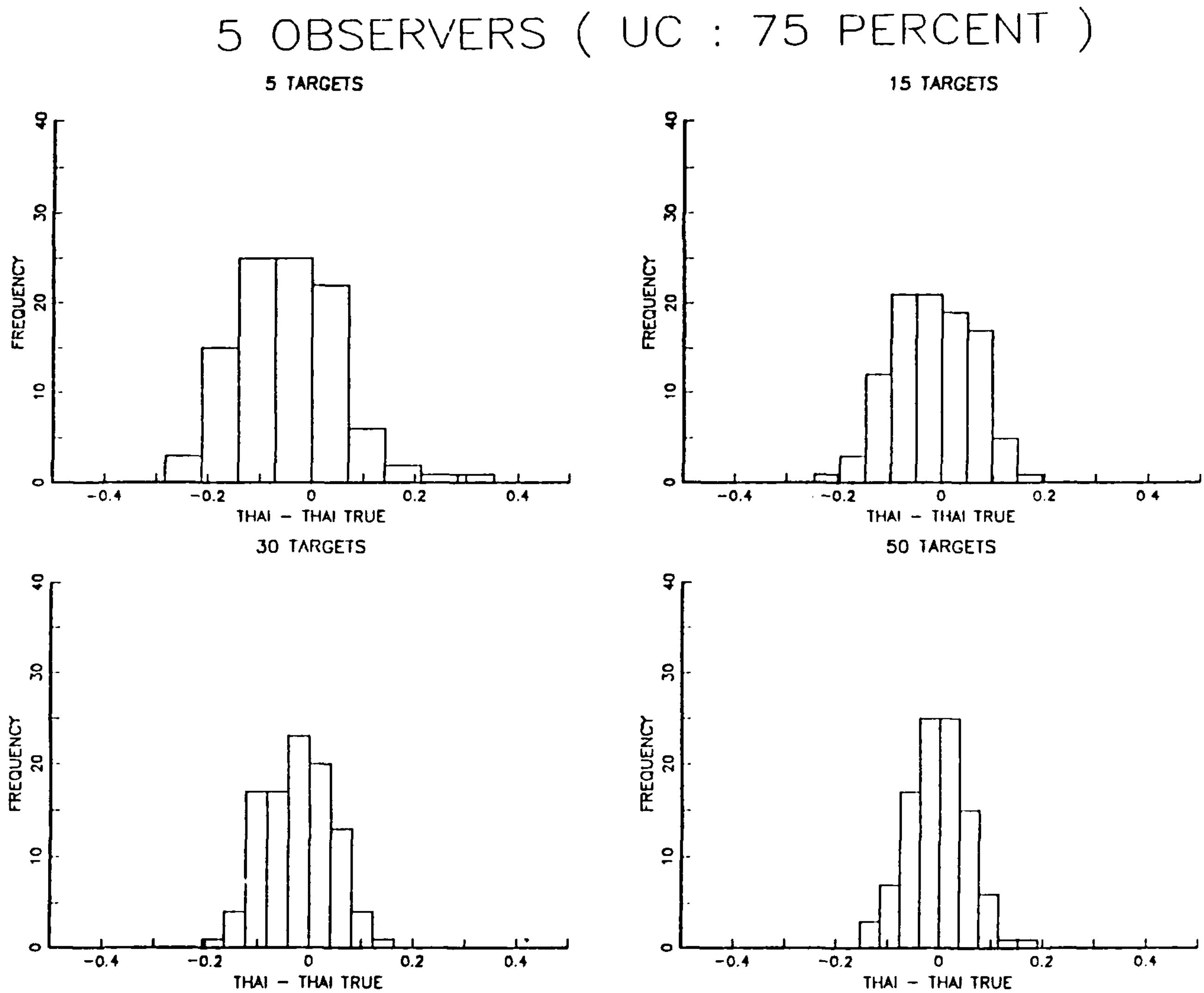

Figure 38. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 5 observers ($\hat{\xi}_i - \xi_i$)

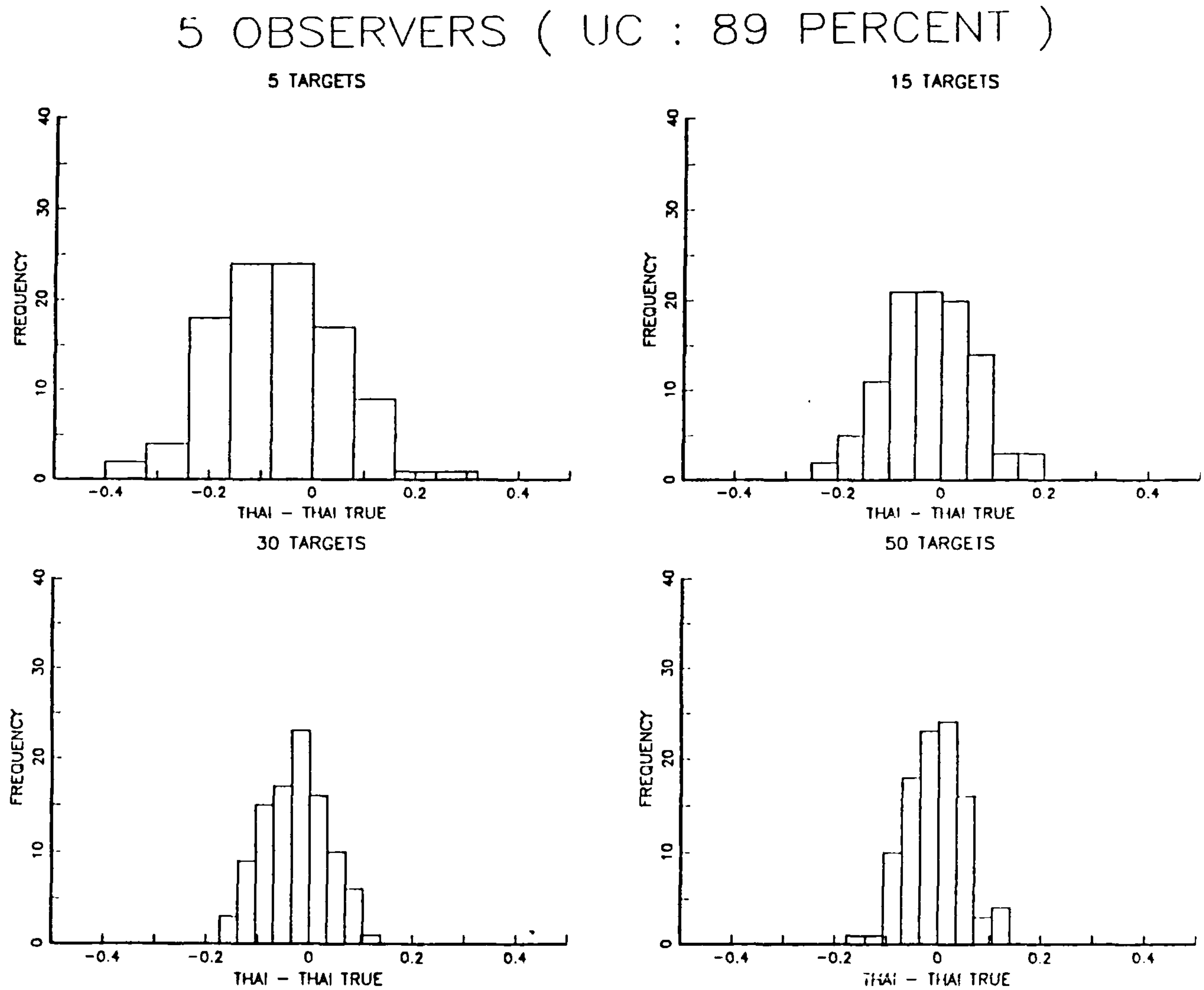

Figure 39. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 5 observers ($\hat{\xi}_i - \xi_i$)

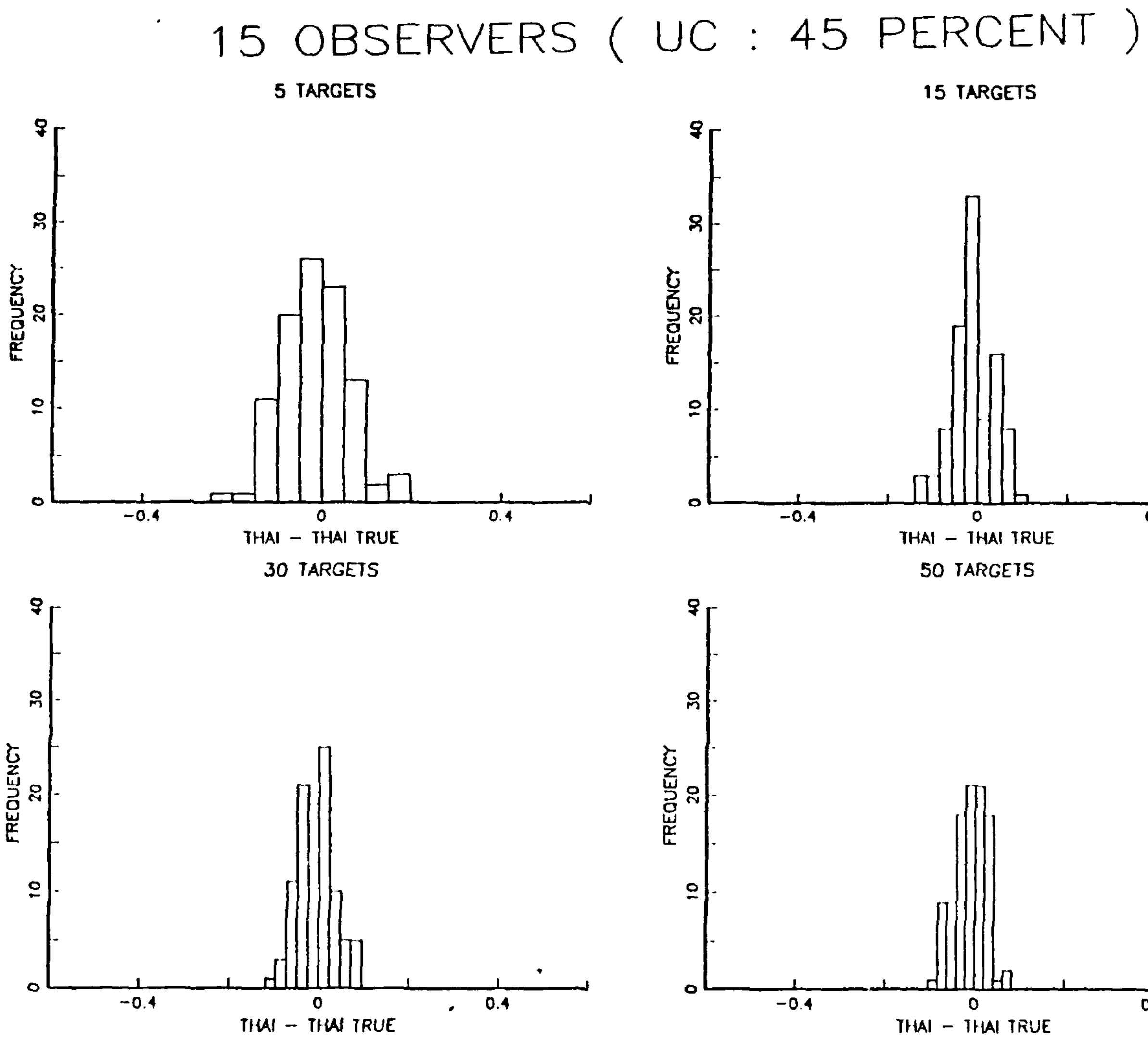

Figure 40. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 15 observers ($\hat{\xi}_i - \xi_i$)

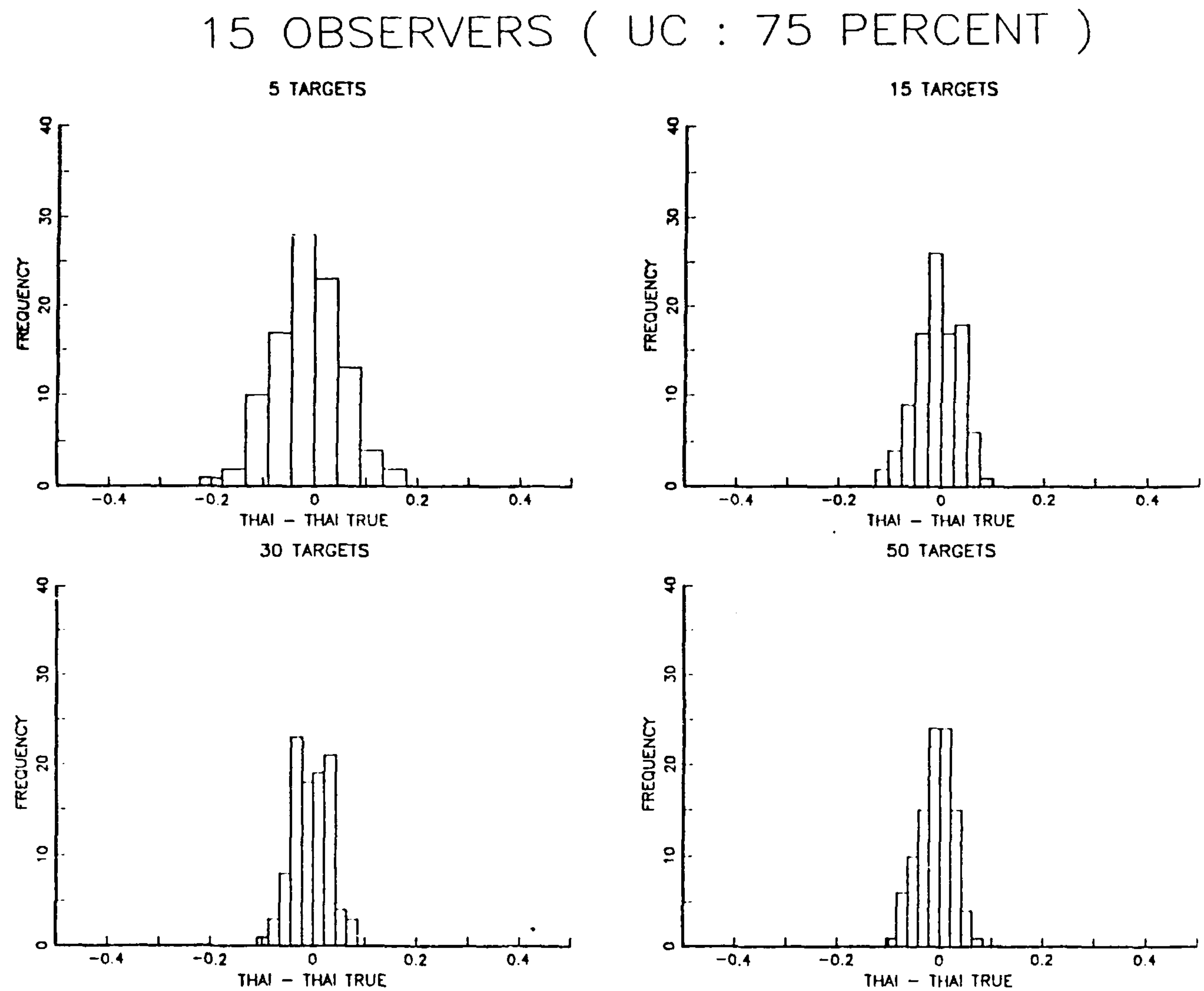

Figure 41. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 2.5, UC = 75%): 5, 15, 30, 50 targets for 15 observers ($\hat{\xi}_i - \xi_i$)

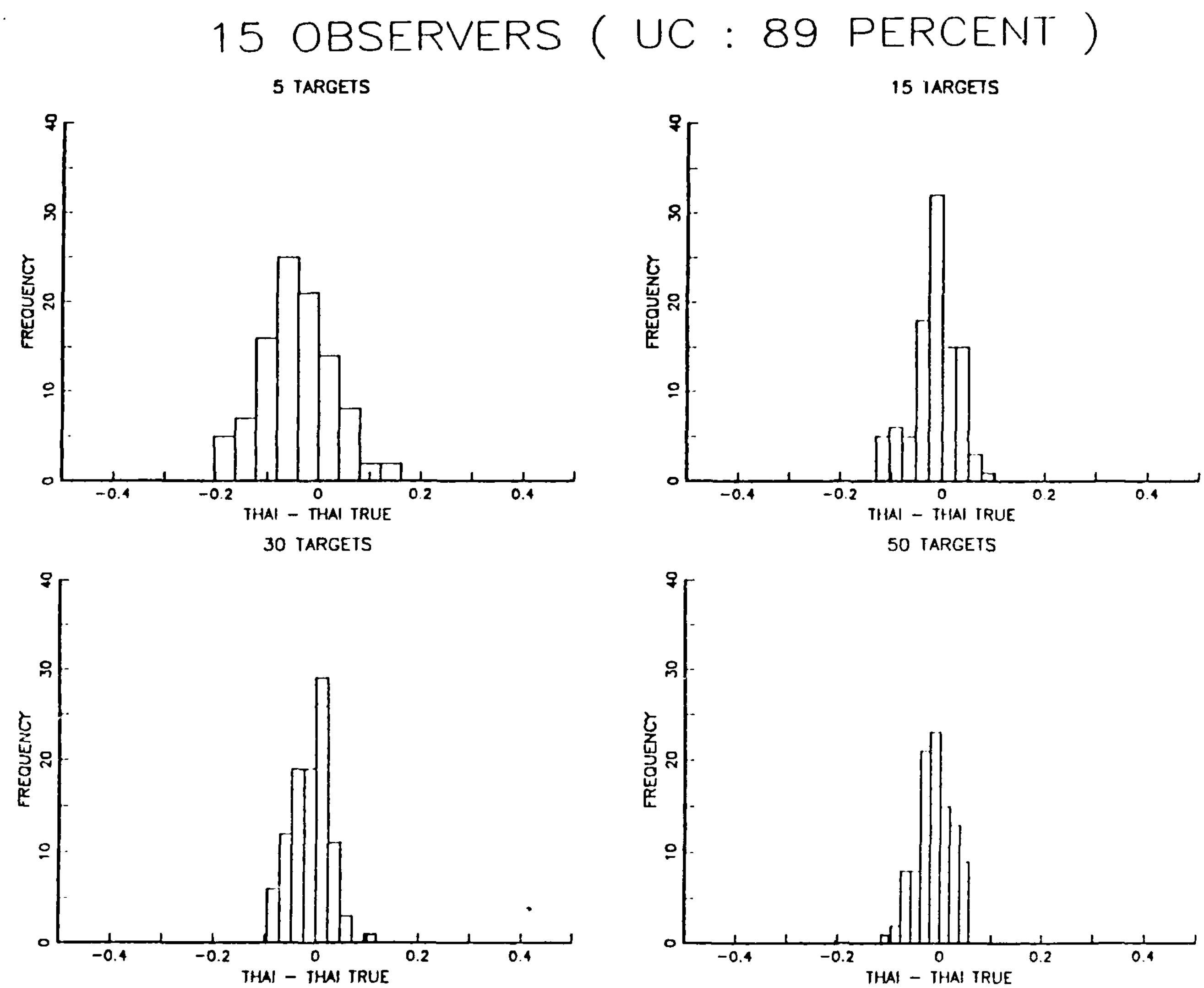

Figure 42. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 15 observers ($\hat{\xi}_i - \xi_i$)

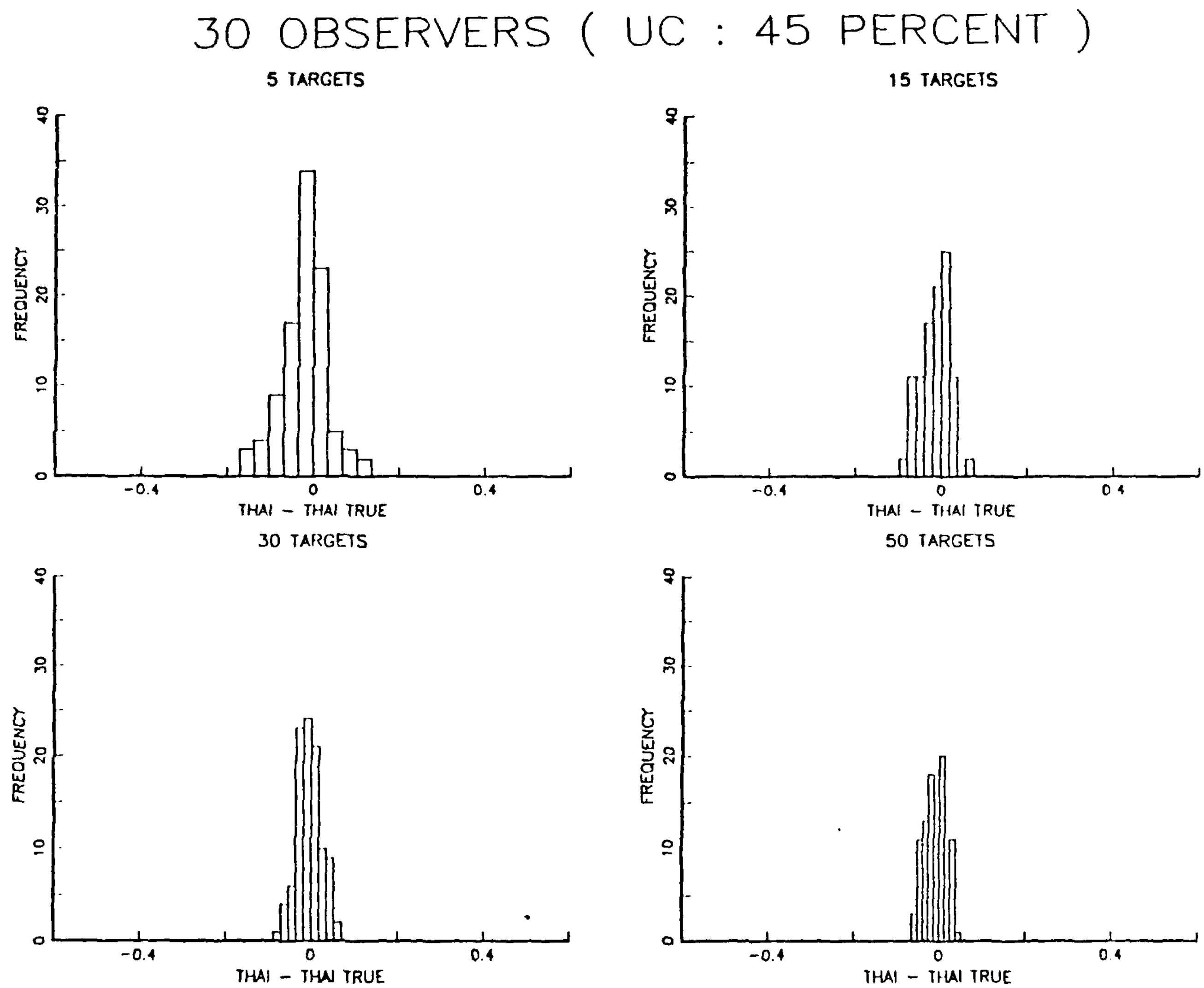

Figure 43. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 1.3, UC = 45%): 5, 15, 30, 50 targets for 30 observers ($\hat{\xi}_i - \xi_i$)

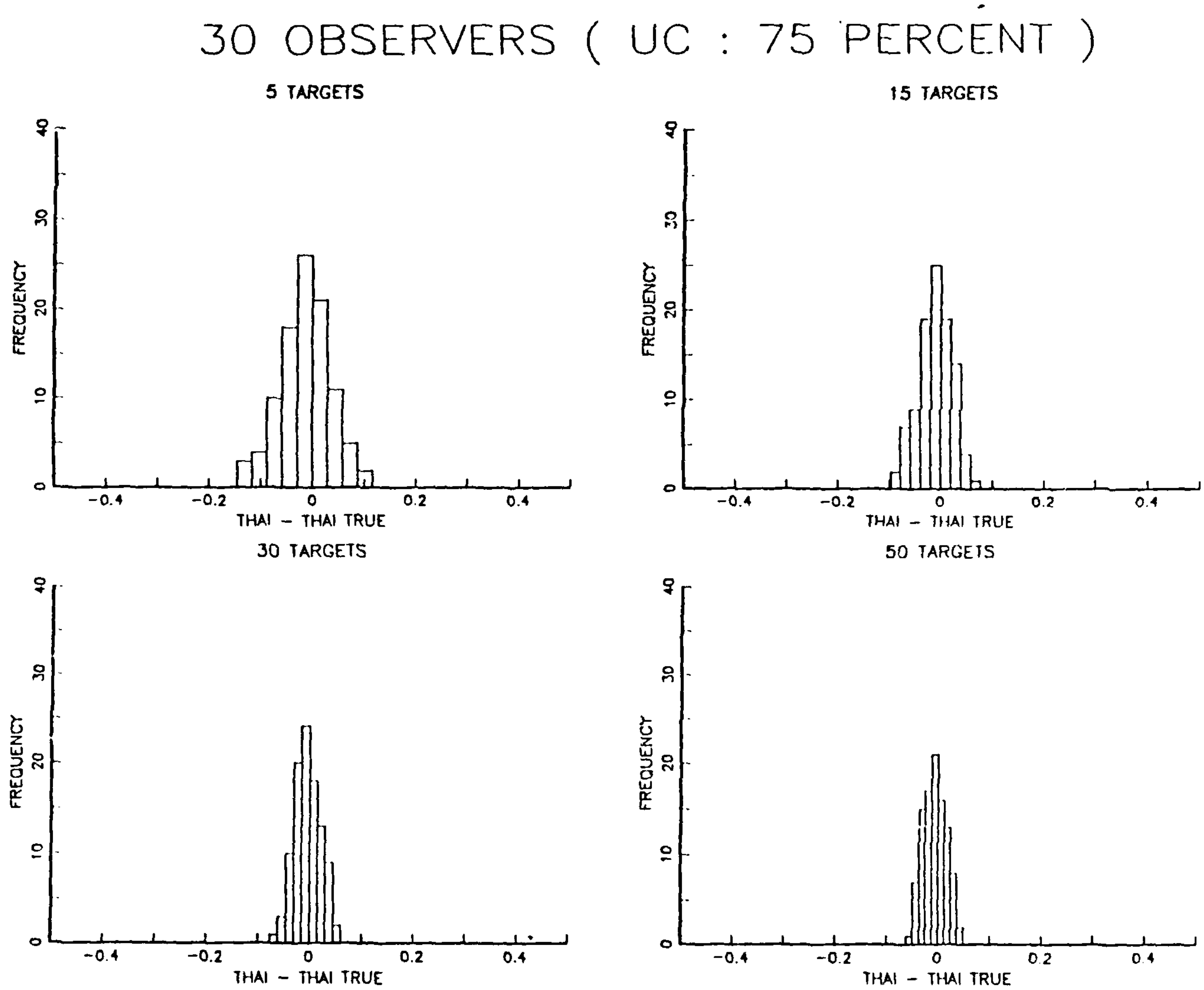

Figure 44. COMPARISON BETWEEN DIFFERENT FSTIMATES (O = 2.3, UC = 75%): 5, 15, 30, 50 targets for 30 observers ($\hat{\xi}_i - \xi_i$)

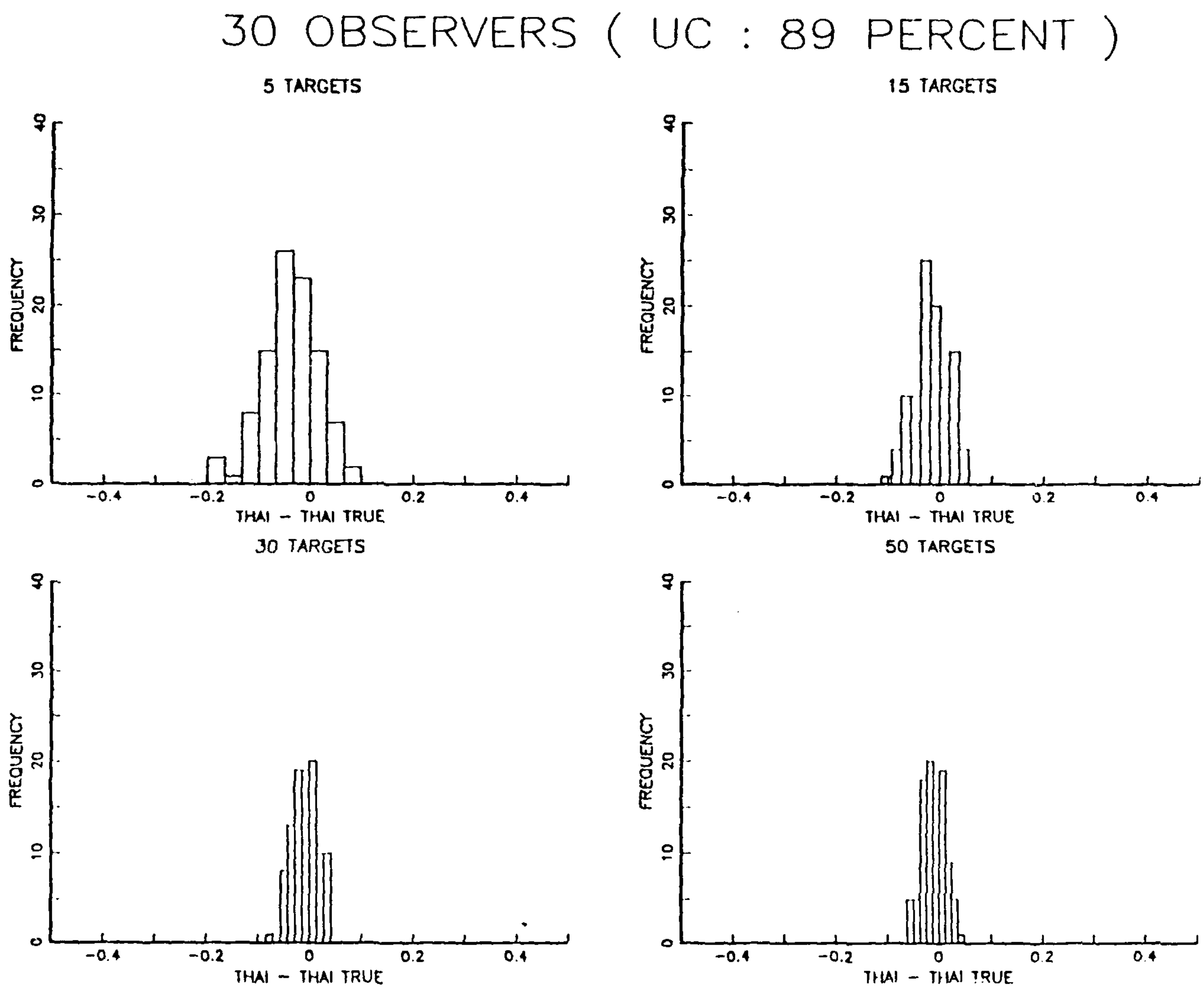

Figure 45. COMPARISON BETWEEN DIFFERENT ESTIMATES (O = 4.0, UC = 89%): 5, 15, 30, 50 targets for 30 observers ($\hat{\xi}_i - \xi_i$)

APPENDIX C. SIMULATION PROGRAM FOR THE ESTIMATION OF GAMMA PARAMETERS

```
        ∇BAE[⎕]∇
        ∇ R←M BAE N;MU;TA;H;E;L;OBS;W;G1;G;Y;C;DEL;S;
        M1;M2;NIN;L;V;F;A;B;IN;JN;C1
[1]     MU←4.2
[2]     TA←0
[3]     IN←NIN←JN←2⍴0
[4]     OBS←40
[5]     W←(M,N)⍴(M×N) EXPRAND 1
[6]     G1←⍉(N,M)⍴G←M GAMRAND 1.5 0.154
[7]     Y←⍟MU×(W÷G1)*(*TA)
[8]     C1←+/C←+/DEL←Y≤⍟OBS
[9]     Y←Y⌊⍟OBS
[10]    S←+/S1←*H←(Y-⍟MU)×*¯1×TA
[11]    M1←(+/+/DEL×H)÷(L←+/C)
[12]    M2←(+/+/DEL×(H*2))÷L
[13]    V←(M2-(M1*2))-((○1)*2)÷6
[14]    IN[2]←(¯1×M1)-0.5772
[15]    →A2×⍳(V>0)
[16]    V←1
[17]   A2:IN[1]←(¯1×⍟V)-IN[2]
[18]    →L7
[19]   L6:IN←NIN
[20]   L7:NIN[2]←IN[2]+(¯1×+/(A-C)÷B)÷(+/(A←(*IN[2])×S)÷
(B←S+*IN[1]))
[21]    JN←IN[1],NIN[2]
[22]    F←C SUM JN
[23]    NIN[1]←IN[1]+(¯1×+/(IN[1]-(⍟B))+F[;1])÷
(+/(1-*IN[1]÷B)-F[;2])
[24]    →L6×⍳((|(IN[1]-NIN[1])÷IN[1])>0.0001)∧
((|(IN[2]-NIN[2])÷IN[2])>0.0001)
[25]    ⎕←NIN
[26]    R←(NIN+(¯1.872 1.4665)),(C1÷M×N)
        ∇
        ∇SUM[⎕]∇
        ∇ R←C SUM IN;DD;BB;AA;D1;D2;I
[1]     DD←⍳0
[2]     I←1
[3]    L3:→L4×⍳(C[I]=0)
[4]     D1←+/÷(AA←*IN[1]+IN[2])+BB←(⍳C[I])-1
[5]     D2←+/AA÷(AA+BB)*2
[6]     →L5
[7]    L4:D1←D2←0
[8]    L5:DD←DD,D1,D2
[9]     →L3×⍳(⍴C)≥I←I+1
[10]    R←DD←((⍴C),2)⍴DD
```

```
        ∇
        ∇THESIS[⎕]
        ∇ R←K THESIS J;I;RES;M;N
[1]     ⎕RL←466801747
[2]     M←1↑J
[3]     N←¯1↑J
[4]     I←1
[5]
[6]     RES←(K,3)⍴0
[7]    L0:RES[I;]←M BAE N
[8]     →L0×⍳K≥I←I+1
[9]     R←RES
        ∇
```

APPENDIX D. SIMULATION PROGRAM FOR THE ESTIMATION OF REGRESSION PARAMETERS

```
      ∇BAE2[⎕]∇
      ∇ R←M BAE2 N;OBS;BO;BN;W;U1;M1;M2;MU;ZZ;RR;T0;T1;S;
      Y;C;IV;W1;X1;X2;DEL;T;A;T2;C1
[1]   OBS←4.0
[2]   J←1
[3]   T1←T2←⍉(N,M)⍴T←T0←M⍴0
[4]   BO←BN←2⍴0
[5]   X1←(M,N)⍴(M×N) NORRAND 1 0.5
[6]   X2←(M,N)⍴(M×N) NORRAND 2 1
[7]   W←(M,N)⍴(M×N) EXPRAND 1
[8]   U1←*((0.2×X1)+0.3×X2)
[9]   Y←⍟U1×W*(*T2)
[10]  DEL←Y≤⍟OBS
[11]  C1←+/C←+/DEL
[12]  Y←Y⌊⍟OBS
[13]  IV←(*T2)×Y
[14] LL:BO←BN
[15]  T0←T
[16]  T2←T1
[17]  W1←(*((Y-IV)×*(¯1×T2)))*0.5
[18]  M1←W1×X1×*(¯1×T2)
[19]  M2←W1×X2×*(¯1×T2)
[20]  MU←⍉(2,(M×N))⍴(,M1),(,M2)
[21]  Z←⍉,Z←(W1×IV)+((¯1×DEL)+W1*2)÷W1
[22]  BN←(⌹(⍉MU)+.×MU)+.×((⍉MU)+.×Z)
[23]  IV←(X1×BN[1])+(X2×BN[2])
[24]  RR←(Y-IV)×*(¯1×T2)
[25]  T1←⍉(N,M)⍴T1←T←,T←T+(C-+/RR×((¯1×DEL)+*RR))÷
((¯1×C)-+/(RR*2)××RR)
[26]  A←(⌈/|(BN-BO)÷BN)>0.0001
[27]  S←|(T0-T)÷T
[28]  →LE×⍳300≤J←J+1
[29]  →LL×⍳(A∨(⌈/S)>0.0001)
[30] LE:R←(BN←BN- 0.2 0.3),(C1÷(M×N)),((+/T)÷M)
      ∇
      ∇THESIS[⎕]
      ∇ R←K THESIS J;I;RES;M;N
[1]   ⎕RL←466801747
[2]   M←1↑J
[3]   N←¯1↑J
[4]   I←1
[5]   RES←(K,4)⍴0
[6]  L0:RES[I;]←M BAE2 N
[7]   →L0×⍳K≥I←I+1
[8]   R←RES
```

```
      ∇
      ∇STAT[⎕]
      ∇ R←STAT K;DF;ANS;II;JJ;KK;TR;MS
[1]     II←JJ←KK←1
[2]
[3]     DF← 5 10 20 30 40 50
[4]     TR← 10 20 50
[5]     ANS←(((ρDF)×(ρTR)),K,4)ρ0
[6]     MSE←(((ρDF)×(ρTR)),7)ρ0
[7]   LO:ANS[II;;]←MS←K THESIS(DF[JJ],TR[KK])
[8]     MSE[II;]←((+/MS)÷K),((+/MS[; 1 2 4]*2)÷K)
[9]     II←II+1
[10]    →LO×ι(ρTR)≥KK←KK+1
[11]    →LO×ι(ρDF)≥JJ←JJ+KK←1
[12]    R←ANS
      ∇
```

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