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13. ABSTRACT (Maximum 200 words)

Generalized Korteweg - de Vries equation (GKdV). This equation is written as $u_t+u\ u_x+u_{xxx}=0$ where p is a positive integer. (p = 1 gives the KdV equation). The convergence estimates obtained for the KdV equation have been generalized, under the assumption that the solution u is sufficiently regular. For p 4, it is not known whether a global smooth solution exists corresponding to smooth initial data. It is in fact conjectured that for these cases, the solution may develop a singularity in finite time. A code that uses a spatially and temporally adaptive strategy has been implemented. We are currently investigating the stability of solitary type solutions. As conjectured, these solutions are highly unstable for initial amplitudes larger than one. With the code, we are able to track the solution to the point where it develops a pulse-like shape with an amplitude of 2.25×10^5 supported over an interval of width 10^{-15} (See attached graphs). These experiments will be reported in [3].

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1. **The Stokes problem.** Nonconforming finite element approximations to solutions of the Stokes equations are constructed in [1]. Optimal rates of convergence are proved for the velocity and pressure approximations, in the usual Sobolev norms of positive as well as negative index. For the pressure, C^0 piecewise polynomial functions are used. The class of vector fields used to approximate the velocity field have piecewise polynomial components, discontinuous across interelement boundaries. On each triangle, these vector fields satisfy the incompressibility condition pointwise. It is shown that these piecewise solenoidal vector fields possess optimal approximation properties to smooth solenoidal vector fields on domains with curved boundaries. Bases for these approximating spaces can be constructed very easily, and bases for higher order spaces can be constructed by adding suitable terms to those for spaces of lower order.

2. **Korteweg – de Vries equation.** (KdV). In [2], we show that the well-known order reduction phenomenon affecting Implicit Runge-Kutta methods does not occur when approximating periodic solutions of the KdV equation, i.e. that the rates of convergence are those that one expects in the case of nonstiff systems of ODE's. For example, if a q-stage IRK method of Gauss-Legendre type is used, then the rate of convergence is 2q. Interestingly, our estimates may be viewed as superconvergent if the IRK method is viewed as a special case of a continuous finite element in time formulation.

3. **Generalized Korteweg – de Vries equation** (GKdV). This equation is written as

$$u_t + u^p u_x + u_{xxx} = 0$$

where p is a positive integer. ($p = 1$ gives the KdV equation). The convergence estimates obtained for the KdV equation have been generalized, under the assumption that the solution u is sufficiently regular. For $p \geq 4$, it is not known whether a global smooth solution exists corresponding to smooth initial data. It is in fact conjectured that for these cases, the solution may develop a singularity in finite time. A code that uses a spatially and temporally adaptive strategy has been implemented. We are currently investigating the stability of solitary type solutions. As conjectured, these solutions are highly unstable for initial amplitudes larger than one. With the code, we are able to track the solution to the point where it develops a pulse-like shape with an amplitude of 2.25×10^{5} supported over an interval of width 10^{-15} (See attached graphs). These experiments will be reported in [3].

4. **Nonlinear (Cubic) Schrodinger equation (NLS).** We consider the initial-boundary value problem

$$u_t = i\Delta u + i\lambda|u|^2 u \qquad x \in \Omega, t > 0$$

$$u = 0 \qquad on \quad \partial\Omega$$

$$u(x,0) = u^0(x) \quad .$$

here Ω is a bounded domain in R^n and λ is a real parameter. In [4], The solution u is approximated by the Galerkin method in space and two second-order accurate temporal discretizations. In [5], high order temporal discretizations generated by IRK methods are analyzed. For general domains, a rate of convergence of $q + 3$ is shown to hold, while for the case of rectangular domains with sides perpendicular to the coordinate axes, (including the important one dimensional case) the full rate of $2q$ comes into force. The computational work consists in solving, at each time level, a set of q linear systems. These matrices, albeit different from one another, are independent from time, resulting in a very efficient scheme. In addition, these q systems may be solved independently on a "parallel" computer with at least q processors.

5. **Parallel methods.** The inherent parallel features of IRK methods are exploited, resulting in schemes that give high accuracy while requiring execution times similar to those of the lowest order methods. Such schemes for the KdV and NLS equations have already been implemented and tested on the CRAY and the Hypercube, two machines that typify popular but different philosophies in parallel computing. Speedup measurements are under way, as well as the task of gathering data regarding the relative efficiency of high-order methods versus those of low order. The conclusions of these experiments will be reported in the near future.

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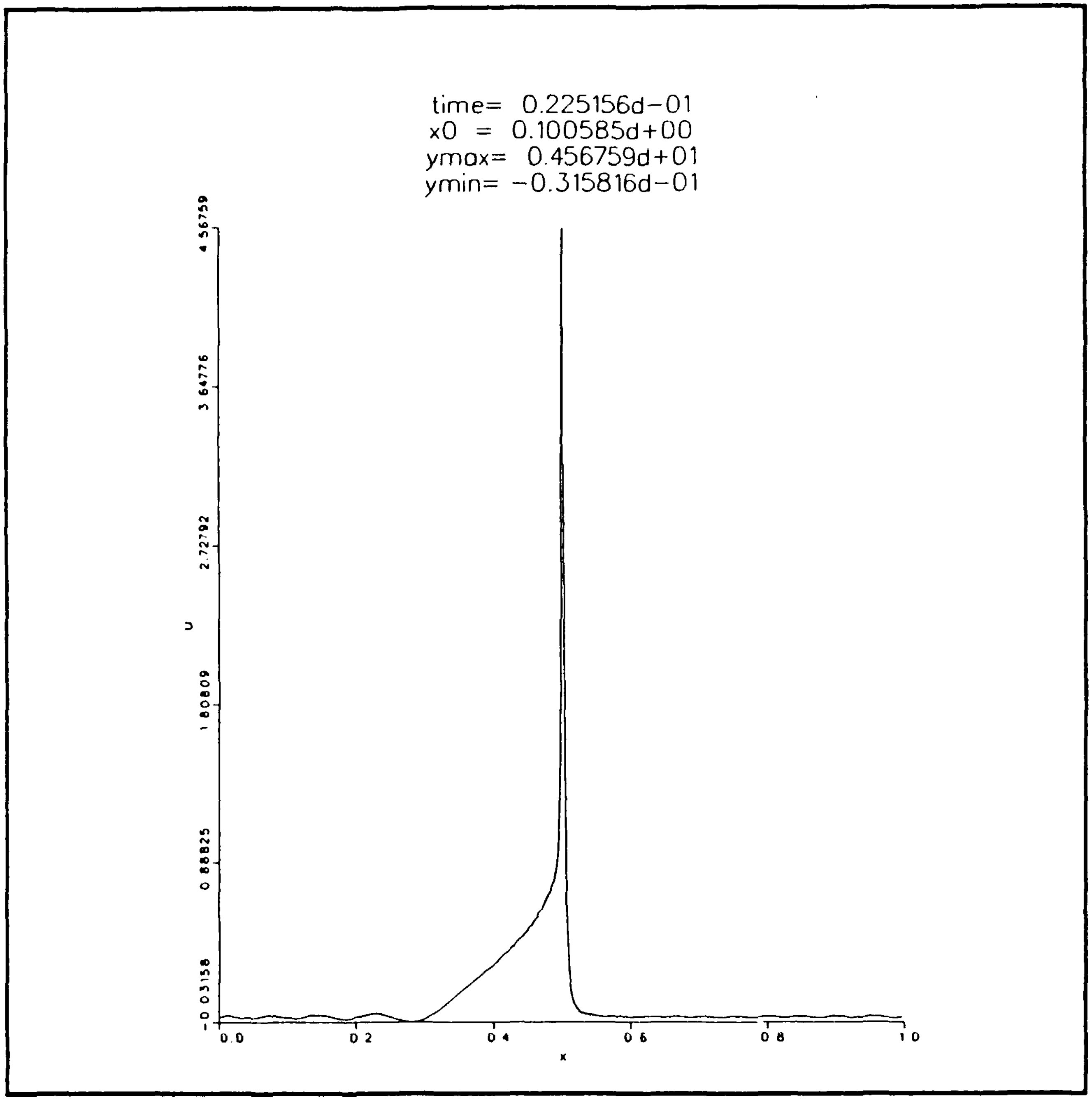
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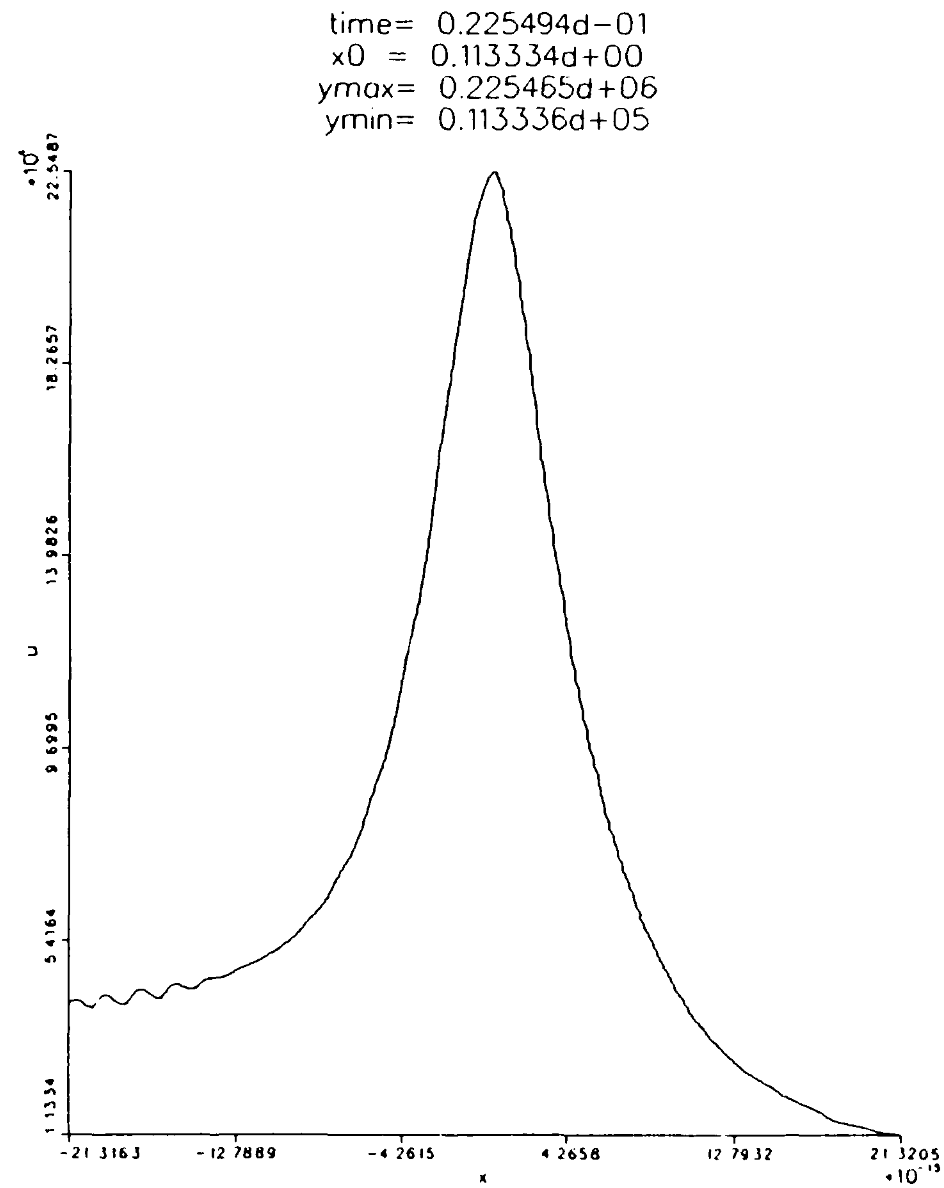
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