REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	February 1990	Final Report, 15 Dec 87 to 14 Dec 89

4. TITLE AND SUBTITLE
A THEORY OF CONTROL FOR INFINITE DIMENSIONAL SYSTEMS WITH APPLICATION TO LARGE SPACE STRUCTURES

5. FUNDING NUMBERS
AFOSR-88-0078
61102F 2304/A1

6. AUTHOR(S)
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AD-A219 937

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8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR- 90-0332

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg. 410
Bolling AFB, DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-88-0078

11. SUPPLEMENTARY NOTES

DTIC
ELECTE
MAR 28 1990
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12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

An extended and unifying system identification technique for a class of systems that include all main signal models that arise in the Harmonic Decomposition Problem has been obtained. This technique unifies and extends the previously developed system identification techniques which are improvements on the PISARENKO (MUSIC, dually) Harmonic Decomposition as they arise in arrays of sensors. The advantages of the technique and some of its specializations given here include having no assumptions of stationarity on the stochastic processes involved. Another contribution on this technique is to show that it can also be used without any resort to probability theoretic concepts, thus bypassing the approximation of autocorrelations via time averages, yielding the system parameters exactly. This technique can be utilized to determine the dominant modes of vibrations of flexible structures as well. An analogy is established between arrays of sensors for target signal returns and those that can be used for vibrations in flexible structures.

14. SUBJECT TERMS	15. NUMBER OF PAGES
	33
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

Texas Tech University

Department of Electrical Engineering

Report to Air Force Office of Scientific Research on

Grant No: AFOSR-88-0078

"A Theory of Control for Infinite Dimensional Systems with Application to Large Space Structures"

by

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Professor,

Principal Investigator

February 1990

Accession For
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Availability Codes
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1. INTRODUCTION

Our research has been progressing in two directions:

1) A traveling wave approach directed to furthering the results as in e.g., [10], [11], [12]. In particular, we have concentrated on the problem of identification of dominant modes of vibrations (dominant traveling wave frequencies) [8]. In [8], we have established an analogy between vibrations traveling along a flexible structure and signal returns from targets to an array of sensors.

2) H^2/H^∞ theory of infinite dimensional systems along the lines of the results we had established in [1-3].

Our work in both of these is still in progress, and will continue. These are described in more detail in Sections 2 and 3, respectively.

2. CONTROL OF FLEXIBLE LARGE SPACE STRUCTURES

As it is evidenced in many recent publications on the subject (e.g., many papers in several sessions of recent conferences), the main theme of research is on Identification and Control of finitely many dominant modes of vibrations, using finite dimensional compensators. Whether this be in State-Space form or in Frequency Domain (e.g., considering those structures whose transfer matrices fall into the **Callier-Desoer Algebra**), it requires the modeling of a finite number of the (infinitely many) modes of vibrations, and thus involves an approximation of **Hyperbolic Partial Differential equations** (see, e.g., [10]). The modes to be modeled for control must satisfy the differential equation describing the structure. A common procedure for this is to consider hyperbolic differential equations of the form

$$(1) \qquad \frac{\partial^2 u(t,x)}{\partial t^2} + A_1 \frac{\partial u(t,x)}{\partial t} + A_2(x)u(t,x) = f(x,t)$$

where $A_2(x)$ is a partial differential operator in x (usually the second or the fourth order). Using an orthonormal set of functions $\phi(x)$ which are the eigenfunctions of $A_2(x)$, (1) can be decoupled into an infinite set of ordinary differential equations for the (temporal oscillation) modes of (1), each associated with one of $\phi(x)$ (that describes the spatial amplitude distribution of the mode) (See e.g., [4], [9]). Among these infinitely many modes, using a finite dimensional compensator, we can only control a finite number of ones.

These can be then described in the usual state-space form as

(2) $\dot{x} = Fx + Gu$

(3) $y = Hx + v_1 + v.$

In (2), F describes the dominant modes of vibrations we wish to control, G describes the distribution of control inputs, and u is the control input. It is important to note here that in our approach to identification of the dominant modes, we do not need to assume any a priori knowledge of the frequencies of vibration.

In (3) y is the total of the measurements from the structure; v is the measurement noise; H describes the distribution of the sensors, and v_1 describes the effect of the modes that are not modeled in F.

The problems that are being considered are

i) identification of the dominant modes (i.e., the unknown F),

ii) choosing the right sensor structure and then identification of H,

iii) choosing the right distribution of actuators (determination of G)

iv) finding the right controller structure to provide enough damping to the dominant vibrations despite

a) the measurement noise v,

b) the unmodeled modes v_1.

Particularly important is the consideration of v_1, as it has been shown that it can cause instabilities ([4]). This is referred to as **Spillover.** The noise v can be handled considerably by usual stochastic control techniques.

It appears that certain system identification/signal processing techniques have already been studied for applicability to this problem. For example, ARMA models and their identification [5,7], frequency filtering for spectral decomposition via Discrete Fourier Transform [5], and for isolation of the control input from v_1 (to prevent spillover) [4]. It is recognized that [5], the dominant modes must be identified on-line. Certain such techniques have already been in use in approximation of partial differential equations for a long time [10].

We have recently concentrated on System Identification, Frequency Filtering and finding the right location of the sensors for this purpose. We have established (see [8], Appendix A) an analogy with arrays of sensors used for target signal returns (especially those for Radars). As a consequence, we have developed several new results in ([8], Appendix A). It turns out that suitable location of sensors can be helpful simultaneously for identification of the dominant modes as well as in isolation (filtering) of the unmodeled modes from the output to prevent the Spillover. These are described in some detail in [8] (Appendix A, Section 5).

i) The system identification problem we have considered in [8] (Appendix A) is for systems with zero inputs. We have been working on extending these techniques to systems with possibly nonzero inputs, and to time-varying systems.

ii) We have been investigating the Toeplitz (and related) matrices that arise in the covariances in our technique. This

would enable us to approximate the dominant modes among infinitely many modes of vibrations in a more effective way.

iii) We have been investigating the dualization of the results on sensor location to actuator locations for use in disturbance cancelling, and extension of our system identification techniques to control of flexible structures.

Our approach here has been traveling wave modeling of the vibrations, and the analogy we have established with arrays of sensors arising in target signal returns, taking into consideration in full the structure of the partial differential equations that arise. This, in fact, appears to be a very general approach to systems described via partial differential equations (see, e.g., [12]).

3. H^2/H^∞ THEORY OF INFINITE DIMENSIONAL SYSTEMS

The problem we have been working on is the following.

Let G be a semi-infinite matrix whose columns are in ℓ_2. Let F be linear operator on ℓ_2. Consider the sequence

$$\{G,\ FG,\ F^2G,\ \ldots.\} = \{F^kG\}_{k=0}^{\infty}$$

Now consider a generalized Gram-Schmidt process whereby we consider the orthogonal direct sums

$$X_k = \text{lm}G + \text{lm}FG + \ldots + \text{lm}F^{k-1}G$$

$$= Y_0 \oplus Y_1 \oplus \ldots. \oplus Y_{k-1}$$

where the subspaces Y_j are mutually orthogonal. This is similar to the idea of a Lancoz Sequence where for a finite vector x and a finite matrix A, the sequence

$$\{x,\ Ax,\ A^2x,\ \ldots.\}$$

is orthogonalized via Gram-Schmidt. In this case, the orthogonal projection of A_x^k to the space spanned by $\{x,\ Ax,\ldots..,\ A_x^{k-1}\}$ can be recursively obtained. This is related to orthogonal polynomials (Lancoz polynomials).

A similar technique has been obtained in [7] in the ℓ_2 case when F is the shift operator.

We are working on extension of this to operators (F,G) where (F,G,H) is the realization [1] of a transfer matrix

$$H = Q^{-1}R = P_1 Q_1^{-1}$$

where Q, R, P_1 Q_1 are H^{∞} matrices Q, Q_1 being admissible [1].

Such an extension applied to the reachability map [G,FG,....] yields a finite dimensional approximation to H. One can recursively obtain higher order approximations. This would also have application to Least squares identification of a linear system, and is also related to Lattice filters. This research is presently continuing.

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APPENDIX A

(Proceedings American Control Conference, 1989)

To appear in

IEEE Transactions on Aerospace and Electronic Systems

A NEW TECHNIQUE FOR SYSTEM IDENTIFICATION FOR HARMONIC DECOMPOSITION WITH APPLICATION TO ARRAYS OF SENSORS AND FLEXIBLE STRUCTURES

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ABSTRACT

In this paper, we give a new extended and unifying system identification technique for a class of systems that include all main signal models that arise in the **Harmonic Decomposition Problem.** This technique unifies and extends the previously developed system identification techniques which are improvements on the PISARENKO (MUSIC, dually) Harmonic Decomposition as they arise in arrays of sensors. The advantages of the technique and some of its specializations given here include having no assumptions of stationarity on the stochastic processes involved. Another contribution of this technique is to show that it can also be used without any resort to probability theoretic concepts, thus bypassing the approximation of autocorrelations via time averages, yielding the system parameters exactly. This technique can be utilized to determine the dominant modes of vibrations of flexible structures as well. An analogy is established between arrays of sensors for target signal returns and those that can be used for vibrations in flexible structures.

1. INTRODUCTION

In this paper, we establish a new extended and unifying system identification technique for a class of systems that include all basic signal models that arise in the **Harmonic Decomposition Problem.**

The problem considered is the following:

Given the system model

$$\begin{aligned} x_{t+1} &= Fx_t, \quad ; \quad x(0) \in R^n, \\ y_t &= Hx_t + v_t, \quad t = 0, 1, 2, \ldots \end{aligned} \tag{1}$$

and the output sequence $y_0, \ldots, y_M$; determine H and F. Here $x_t \in R^n$, and $y_t \in R^p$. v_t is the measurement noise, with $E\{v_t v_t^T\} = R_t$. The only assumption on $\{v_t\}$ is that it should decorrelate after a finite time $\sigma \geq 1$ (this assumption is also used in [3]). This signal model includes most of the signals arising in the Harmonic Decomposition Problem (e.g., determining the frequency spectrum of a given signal, as in arrays of sensors, or as in vibrations arising in certain flexible structures possibly with some damping).

In Sections 2 and 3, we develop our general system identification technique. In Sections 4 and 5, we describe the applications to two important situations: i) Arrays of Sensors, ii) Vibrations arising in Flexible Space Structures. Also, we compare our technique to previously developed techniques such as MUSIC, PISARENKO HARMONIC DECOMPOSITION, and their previous

extensions that can be found in the literature (See e.g., [2,3] and the references there). The advantages of the technique and some of its specializations given here include having no assumptions of stationarity on the stochastic processes involved. Another contribution of this technique is to show that it can also be used without any resort to probability theoretic concepts, bypassing the approximation of autocorrelations via time averages. Thus it yields the system parameters exactly.

2. IDENTIFICATION FROM OUTPUT COVARIANCES (PROBABILISTIC APPROACH)

Consider

$$\begin{bmatrix} y_t \\ \cdot \\ \cdot \\ \cdot \\ y_{t+N} \end{bmatrix} , \tag{2}$$

and

$$\begin{bmatrix} y_{t-N-\sigma} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-\sigma} \end{bmatrix} \quad ; \quad t \geq N + \sigma \ ,$$

corresponding to the signal model in (1).

Using (1), (2) can be written as

$$(3) \qquad Y_t := \begin{bmatrix} y_t \\ \cdot \\ \cdot \\ \cdot \\ y_{t+N} \end{bmatrix} = \begin{bmatrix} H \\ HF \\ \cdot \\ \cdot \\ \cdot \\ HF^N \end{bmatrix} x_t + \underbrace{\begin{bmatrix} v_t \\ v_{t+1} \\ \cdot \\ \cdot \\ \cdot \\ v_{t+N} \end{bmatrix}}_{V_t},$$

and

$$(4) \qquad Y_{t-N-\sigma} := \begin{bmatrix} y_{t-N-\sigma} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-\sigma} \end{bmatrix}$$

$$= \begin{bmatrix} H \\ HF \\ \cdot \\ \cdot \\ \cdot \\ HF^N \end{bmatrix} x_{t-N-\sigma} + \underbrace{\begin{bmatrix} v_{t-N-\sigma} \\ \cdot \\ \cdot \\ \cdot \\ v_{t-\sigma} \end{bmatrix}}_{V_{t-N-\sigma}}$$

Assuming $\sigma \geq 1$ is chosen large enough so that V_t and $V_{t-N-\sigma}$ are uncorrelated, and assuming that $\{v_t\}$ is a zero-mean process, we obtain

$$(5) \qquad E\{Y_t Y_{t-N-\sigma}^T\} = \underbrace{\begin{bmatrix} H \\ HF \\ \cdot \\ \cdot \\ \cdot \\ HF^N \end{bmatrix}}_{\Omega_N} E\{x_t x_{t-N-\sigma}^T\} \Omega_N^T .$$

If

$$P_t = E\{x_t x_t^T\} ; \qquad t = 0, 1, \ldots,$$

then,

$$E\{x_t x_{t-N-\sigma}^T\} = F^{N+\sigma} P_{t-N-\sigma} ; \; t-N-\sigma \geq 0.$$

Thus,

$$E\{Y_t Y_{t-N-\sigma}^T\} = \Omega_N F^t P_0 (F^T)^{t-N-\sigma} \Omega_N^T = K_N ; \quad t-N-\sigma \geq 0.$$

It is straightforward to see that the outputs produced by a system of the form

$$x_{t+1} = Fx_t ,$$

$$y_t = Hx_t ; \quad x_0 \in R^n, \quad t \geq 0,$$

satisfies a difference equation (i.e., has an Autoregressive (AR) model)

$$A_r y_t + \ldots + A_0 y_{t-r} = 0 \; ; \; t \geq r,$$

for every $x_0 \in R^n$, if and only if

$$[A_0, \ldots, A_r]\Omega_r = 0,$$

where A_i's are pxp matrices. Thus we obtain

7. **THEOREM.**

If F is nonsingular (no zero eigenvalues), and P_0 is nonsingular, then the AR models for (1) are characterized as solutions of

(7a) $$AK_N = 0,$$

or, equivalently,

$$K_N A^T = 0,$$

where

$$A = [A_0, \ldots, A_N].$$

It is clear that usually in system identification, we are not interested in finding just any AR model, neither all of them. All the AR models other then minimal ones will contain redundant modes (eigenvalues of F in (6)), that do not, in fact, exist in the true system that produces the data $y_0, \ldots, y_M$. The same is true for the models (6), or (1). Assuming an irredundant state-space model in (1), (F,H) is an observable pair. Then, it is well-known that the minimal (irredundant) AR models are characterized via the observability indices of (F,H).

Contrary to the usual assumption in most literature, for multi-output systems, in the minimal AR models, A_r is not I_p. From the results obtained above, and from the elementary realization theory, it follows that a minimal AR model can be obtained by finding the linear dependence relationships among the rows or the columns of the matrix K_N. r is the largest observability index, and N must be chosen to be greater than or equal to the largest observability index. This can be done exactly the same way that canonical forms for a pair (F,G) or (H,F) are obtained (see, e.g., [1], for a detailed exposition of this). This leads to minimal AR representations of the form

$$Q(D)y_t = 0; \qquad t \geq 0 \ ,$$

where Q(D) is a pxp polynomial matrix

$$Q(D) = A_r D^r + A_{r-1} D^{r-1} + \ldots + A_0, \qquad r \geq N,$$

D being the advance operator. In particular, a minimal Q(D) can be computed which is row proper with row degrees equal to the observability indices, from which a minimal state-space realization of the system producing the data $y_0, \ldots, y_M$ can be easily obtained (see, e.g., [1]) by inspection of A_i's. Clearly, N must be chosen to be greater than or equal to the largest observability index. Once a minimal (F,H) is obtained as described above (which is unique up to a nonsingular state-space transformation), it is straightforward to compute P_0 uniquely up to the particular state-space transformation, since Ω_N is left invertible, and (by assumption) F has no zero eigenvalues.

Once F, H, P_0 are determined, the autocorrelation function of v_t can be easily obtained from

$$(7b) \qquad E\left\{ \begin{bmatrix} y_t \\ y_{t+1} \\ \cdot \\ \cdot \\ \cdot \\ y_{t+\sigma} \end{bmatrix} [Y_t^T, y_{t+1}^T, \ldots y_{t+\sigma}^T] \right\}$$

$$= \Omega_\sigma F^t P_0 (F^T)^t \Omega_\sigma^T + \hat{R}_t ,$$

where

$$(8) \qquad \hat{R}_t = E\left\{ \begin{bmatrix} v_t \\ \cdot \\ \cdot \\ \cdot \\ v_{t+\sigma} \end{bmatrix} [v_t^T, \ldots, v_{t+\sigma}^T] \right\}$$

yields the autocorrelation function of $\{v_t\}$. (Note that the first term in (7b) is known).

9. **REMARK.**

Note that contrary to most system identification techniques, our technique, in particular, has **no stationarity assumption** on the processes $\{y_t\}$ or $\{v_t\}$. In fact, in many situations, the eigenvalues of F are exactly on the unit circle. This violates the conditions for stationarity of $\{y_t\}$, even if $\{v_t\}$ were assumed to be a stationary process. The technique given above has no such assumptions.

10. **REMARK.**

Here we did not assume any special structure on

$$R_t^1 := E\{v_t v_t^T\}.$$

If we assume that i) R_t^1 **has the special diagonal structure with equal diagonal entries**

$$R_t^1 := \text{diag}(r_1); \quad r_1 \geq 0,$$

and, ii) $\{v_t\}$ is a **stationary white-noise process**, then we can directly consider

$$\text{(10a)} \qquad Z := E\left\{ \begin{bmatrix} y_t \\ y_{t+1} \\ \vdots \\ y_{t+N} \end{bmatrix} [y_t^T, \ldots, y_{t+N}^T] \right\}$$

$$= \Omega_N F^t P_0 (F^T)^t \Omega_N^T + R^2.$$

Then R^2 will be a diagonal matrix whose diagonal entries will be the smallest eigenvalue of $Z(= r_1)$. From an eigenvalue-eigenvector decomposition of Z, we can then compute

$$K_N^1 := \Omega_N F^t P_0 (F^T)^t \Omega_N^T.$$

In this case, we can obtain P_0, F and H from K_N^1 by the same procedure that was applied to K_N, explained in this section.

Then, this special case becomes a generalization of the well-known PISARENKO Harmonic Decomposition technique to the case where the output of (1) is not necessarily assumed to be scalar.

11. REMARK.

It is clear that the above technique also applies to the case where (F,H) may have complex numbers as their entries, and $\{v_t\}$, $\{y_t\}$ are complex valued. In this case, we only need to replace "T" (transpose) with "*" (complex conjugate transpose) throughout, and change the condition (7a) to

$$AK_N = 0,$$

or, equivalently,

$$K_N A^* = 0 .$$

12. REMARK.

There is a special case where the technique developed here can be simplified considerably. The reason why y_t and $Y_{t-N-\sigma}$ are both considered is that for our technique, Ω_N must have linearly independent columns. Suppose that the columns of H are

linearly independent. Then consider

$$(12a) \qquad E\{Y_t y_{t-\sigma}^T\} = \Omega_N F^t P_0 (F^*)^{t-\sigma} H^* =: C_N .$$

Then, as F, P_0 are nonsingular, and since the rows of H are linearly independent, one can obtain (H,F) and the minimal AR representations of (1) via the left kernel of C_N, as explained in this section. Next, P_0 can be obtained from C_N. In certain cases, one is interested in finding a nonzero row vector a such that

$$ay_t = 0 \; ; \quad t = 0, 1, \ldots,$$

for all initial conditions. (See section 4). Clearly, this corresponds to the condition

$$aH = 0.$$

Obviously, such a vector a can be obtained by computing a nonzero vector a^* which is in the right kernel of C_N, i.e.,

$$C_N a^* = 0.$$

If we are only interested in finding such a nulling vector, it is enough to consider the right or left kernel of

$$C_N^1 := E\{y_t y_{t-\sigma}^T\} = HF^t P_0 (F^*)^{t-\sigma} H^* .$$

i.e.,

$$aH = 0 ,$$

if and only if

$$aC_N^1 = 0,$$

if and only if

$$H^* a^* = 0,$$

if and only if

$$C_N^1 a^* = 0.$$

3. NONPROBABILISTIC APPROACH

There are standard techniques of estimating P_0 such as

$$P_0 = \frac{1}{\alpha} \sum_{t=0}^{M} \beta_t F^t x_0 x_0^T (F^T)^t \tag{13}$$

where β_t is a weighting factor, and α is some constant. It is clear that nonsingularity of P_0 is equivalent to the reachability of the pair $(F\gamma, x_0)$ provided $\beta_t = \gamma^{2t}$ for all t, where γ is a nonzero number, and M is larger than the order of F.

It is important to note that, as obvious from the results in Section 2, provided that P_0 in (13) is nonsingular, (F,H) and P_0 can be obtained without having to use any probability theory concepts. In this latter case, the autocorrelations and covariances of all the quantities are their time-averages as in (13). Thus, then the assumption on $\{v_t\}$ becomes that its time-average autocorrelation function becomes exactly zero after some finite time. Thus, everything described so far can be carried out (with no approximation, as opposed to the Probabilistic Approach) in terms of time-averages. Thus, the technique developed here is perfectly applicable to many realistic situations where there is usually no data available other than $y_0, y_1, \ldots, y_M$.

4. HARMONIC DECOMPOSITION FOR ARRAYS OF SENSORS

There is a substantial amount of literature on Harmonic Decomposition oriented system identification techniques (see, e.g., [2-3] and the references there). Among these, [2-3] appear to us as among the representative generalizations of the PISARENKO Harmonic Decomposition technique.

In this case, usually target returns are narrowband signals with center frequencies $\omega_1, \ldots, \omega_n$. Then F has the form

$$F = \text{diag}\,(e^{j\omega_i \Delta t}),$$

where Δt is the sampling interval. The matrix H in (1) now corresponds to the measurements by an array of sensors. It has the structure

$$H = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w_1 & w_2 & \cdots & w_n \\ \vdots & \vdots & & \vdots \\ w_1^{p-1} & w_2^{p-1} & \cdots & w_n^{p-1} \end{bmatrix}, \tag{14}$$

where

$$w_i = e^{j\omega_i \tau_i}.$$

τ_i is the delay due to spacing of the sensors. We have assumed in (14) that the sensors are equally spaced. Clearly, our techniques in Section 3 are applicable even if the sensors were not placed with equal distances. The problem is to determine both $\{\omega_1, \ldots, \omega_n\}$, and $\{\tau_1, \ldots, \tau_n\}$. The first set

yields the Doppler frequencies. r_i's yield the angles of arrivals. In this case the technique given in Remark 10 coincides with the technique developed in [2] for the special case that $\{v_t\}$ is assumed to be a **stationary white-noise** process. Clearly, the primary technique developed in Sections 2 and 3 does not have this assumption on $\{v_t\}$.

A main difference between the technique in Remark 10, and the one given in [2] is that, in [2] the target return signals are assumed to be **stationary zero-mean stochastic processes** with a positive definite covariance matrix. However, in many cases, the return signals are, in fact, **deterministic signals (sinusoidal signals** in the case of **Radars**, for example), with the only uncertainty being at the initial state x_0. We exploit this fact in Remark 10. Thus, the technique in Remark 10 is a directly applicable exact technique that can be used purely in terms of exact time averages as opposed to the technique in [2] which has to approximate probabilistic covariances via time averages, introducing an additional approximation error. Also, we can incorporate the uncertainty in signal magnitudes into x_0 since F is diagonal. Thus the H matrix in (14) has a simpler structure.

Another difference between the techniques in [2] and in Remark 10 is that, in our case, the determination of the number of targets is a problem of determining linear dependencies among the rows (or columns) of a positive semidefinite matrix, whereas this is approached in [2] as a hypothesis testing problem via a likelihood-ratio.

In the technique developed in Section 2, the observability of the pair (H,F) (linear independence of the columns of Ω_N) is possible via different r_i's for sources with the same center frequency, and these cannot exceed p. But the technique in Section 2 (as well as the one in [2]) are applicable to the situation of an arbitrary number of targets that can be larger than p (= number of sensors). For example, then, two sensors would be enough for any number of targets.

Next, suppose that each target return signal has a different frequency, and that p (= number of sensors) is greater than n = number of return signals. In this case the columns of H in (14) will be linearly independent. Then, the simplified technique in Remark (12) will yield $\{\omega_1, \ldots, \omega_n\}$ and $\{r_1, \ldots, r_n\}$ in terms of the left and the right kernels of C_N as in Remark 12.

If all the incoming signals to the array are interference, and if we want to find a row vector a such that

$$ay_t = 0; \quad t = 0, 1, \ldots,$$

then any nonzero column vector in the right kernel of C_N will do as a^*, as explained in Remark 12. Thus we can both null and estimate interference sources using the left and right kernels of C_N. If we do not want to estimate ω_i's and r_i's it is enough to consider only the right kernel of C_N. In fact, if we require only output nulling, clearly we can only consider C_N^1 as explained in Remark 12. These latter cases constitute generalizations of the techniques given in [3] even in this case as follows. Suppose (H,F) is observable, but the columns of H are not linearly independent. In this case, the row vectors a such that

$$aH = 0 \ ,$$

still coincide with the row vectors a such that

$$C_N a^* = 0.$$

Thus, our technique still works even if the number of interference signals are greater than or equal to the number of sensors as far as (H,F) remains observable, contrary to the situation assumed in [3]. We should also note that the technique in [3] is probabilistic, and as such it introduces an approximation error in estimating the probabilistic covariances via time averages, whereas our techniques can be used directly via time-averages as well if we define P_0 as in (14), and they do not require this extra approximation, (see Section 3). Also, note that via our development, the results obtained are still applicable even if sensors are not equally spaced. In the case of equally spaced sensors, the output (interference) nulling vectors a's have also the following interpretation that has been missed in [3]. If

$$\alpha = [\alpha_0, \ldots, \alpha_{p-1}] \tag{15}$$

is a vector such that

$$C_N \alpha^* = 0, \text{ or, } C_N^1 \alpha^* = 0, \tag{16}$$

then, clearly

$$\alpha(w_i) = \alpha_0 + \alpha_1 w_i + \ldots + \alpha_{p-1} w_i^{p-1} = 0. \tag{17}$$

Thus, if $\alpha(w)$ denotes the polynomial obtained from a vector α as in (15-17), the roots of a greatest common divisor of all such polynomials are $w_1, \ldots, w_{p-1}$.

17a. **REMARK**

A technique for finding this polynomial whose roots are $w_1, \ldots, w_{p-1}$ is as follows:

Let $\{\gamma_1^*, \gamma_2^*, \ldots, \gamma_r^*\}$ be a basis for the kernel of C_N or (equivalently, of C_N^1). Then all vectors α in this subspace are given by

$$\sum_{k=1}^{r} \alpha_k \gamma_k^*,$$

for some arbitrary $\alpha_1, \ldots, \alpha_r \in \mathbf{R}$ (or $\mathbf{C}$). Thus a greatest common divisor of the polynomials $\gamma_1(w), \ldots, \gamma_r(w)$ obtained as described above from $\gamma_1, \ldots, \gamma_r$ is the desired polynomial.

18. **REMARK.**

It is well-known that MUSIC is the spatial dual of the PISARENKO technique. Thus our results can be similarly dualized to obtain spatial generalizations of MUSIC.

5. VIBRATIONS IN FLEXIBLE STRUCTURES

In what follows, first we establish an analogy between an array of sensors receiving target return signals after delays, and the vibrations occurring in a flexible structure. For example, consider two types of flexible structures;

$$\frac{\partial^2 u(t,x)}{\partial t^2} = c^2 \frac{\partial^2 u(t,x)}{\partial x^2} \quad \text{(elastic string)}, \tag{19}$$

or,

$$\frac{\partial^2 v(t,x)}{\partial t^2} = c^2 \frac{\partial^4 v(t,x)}{\partial x^4} \quad \text{(rod)}. \tag{20}$$

We can approach the rod problem (20) first considering

$$u(t,x) := \frac{\partial^2 v(t,x)}{\partial x^2} .$$

This transforms the situation (20) to the wave equation situation (19). Thus we will concentrate on the wave equation (19).

It is well known that the general solution of (19) can be written as

$$u(t,x) = f(t - \frac{x}{c}) + g(t + \frac{x}{c})$$

which represents two waves f(t) and g(t) traveling in opposite directions along the string with speed c. The functions f and g are determined via the boundary and the initial conditions. For simplicity, let us consider a semi-infinite string. In this case

$$u(t,x) = f(t - \frac{x}{c})$$

which is a wave f(t) traveling in one direction. The case where g also exists can also be approached similarly. But for simplicity of exposition of the ideas here, we assume g = 0. A more general traveling wave approach for more complicated structures can be found, e.g., in [5].

The function f(t) which represents the oscillations at x = 0 is a sum of sinusoids (infinitely many)

$$f(t) = \sum_{k=1}^{\infty} A_k e^{j\omega_k t}.$$

Now we have a situation quite analogous to the case of an array of sensors receiving f(t) as a signal emitted by targets after a certain delay each. This analogy can now be exploited to place the sensors on a vibrating structure in certain ways that makes the problem of estimating the vibration modes much easier.

To explain this, first consider a single sinusoid

$$s(t) = Ae^{j\omega t} \quad ; \; t \in \mathbf{R}.$$

Suppose that we place sensors at equally spaced points along the string, where d > 0 is the distance between the sensors (although our techniques in Sections 2 and 3 are still applicable for arbitrary number of sensors spaced arbitrarily).

Then each sensor measures

$$y_r(t) = Ae^{j\omega(t-r\frac{d}{c})} + v_r(t); \quad t \in R; \quad 0 \le r \le p,$$

where $v_r(t)$ is the measurement noise.

Thus, we have

$$(21) \qquad y_k := \begin{bmatrix} y_0(k\Delta t) \\ \cdot \\ \cdot \\ \cdot \\ y_p(k\Delta t) \end{bmatrix}$$

$$= A \begin{bmatrix} 1 \\ e^{j\tau\omega} \\ e^{j2\tau\omega} \\ \cdot \\ \cdot \\ \cdot \\ e^{j(p-1)\tau\omega} \end{bmatrix} e^{j\omega k\Delta t} + v(k\Delta t)$$

where

$$\tau = \frac{d}{c}; \quad k = 0, 1, 2, \ldots,$$

and Δt is the sampling interval. Suppose that there are n dominant modes ω_i's that we want to identify, and the unmodeled modes contribute negligibly. Then we have

$$x_{t+1} = Fx_t,$$

where F describes the dominant modes of vibrations, and

$$y_t = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ w_1 & w_2 & & w_n \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ w_1^{p-1} & w_2^{p-1} & & w_n^{p-1} \end{bmatrix} x_t + \underset{\text{noise}}{v_t}$$

$$= Hx_t + v_t; \quad t = 0, 1, 2, \ldots,$$

where

$$w_i = e^{j\omega_i \tau},$$

$$v_k = v(k\Delta t); \quad k = 0, 1, 2, \ldots .$$

Thus, now we have a simpler system identification problem where H has a nice structure as in Section 4. Clearly, F, H can be found via the system identification techniques developed here. In fact, in this case $\tau = \frac{d}{c}$ is known. Thus if we place more sensors than the number of dominant modes, the much simpler technique given in Remark 12 (specialized in Section 4) can be utilized to estimate ω_i's, via kernel of C_N^1, or C_N.

The case where there is damping of the modes of vibrations can be very similarly solved via these techniques. One can pursue the analogy we have established further. There are alternatives to placing p different sensors as described above. One can consider placing a moving sensor along the x-axis, and effectively achieve the same result. This is similar to the idea of a Synthetic Aperture Radar (SAR). Note that in the case of flexible structures, the problem is simpler because no angle of arrival estimation is needed.

22. REMARK

The analogy with array of sensors we described above leads to some other results for flexible structures. For example, suppose we weigh the output of each sensor via a_k and add them. Then we obtain the response to an incoming wave of frequency ω as

$$y(t) = A \cdot \sum_{k=0}^{N-1} a_k y_k(t)$$

$$= A\cdot\left(\sum_{k=0}^{N-1} a_k e^{-jk\frac{d}{c}\omega}\right) e^{j\omega t} + \bar{v}(t)$$

$$= A\cdot H(\omega) e^{j\omega t} + H(\omega)v(t).$$

Suppose, for example that, to prevent spillover (see e.g., [4]), we choose a_k's in such a way that $H(\omega)$ is a low pass finite impulse response filter whose passband includes the highest dominant frequency of vibration.

Thus, the spillover problem [4] due to high frequency unmodeled modes would be eliminated.

ACKNOWLEDGEMENTS

The preliminary part of the results of this paper on system identification were developed while the author was visiting Texas Instruments, Defense Systems and Electronics Group (DSEG) during early 1988. Discussions with Joe Mynk and Sidney Theis of DSEG have been instrumental for this. The author would like to thank A. A. Beex of Virginia Polytech for providing the references [2-3], during early 1989.

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