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THE HLLSRF HULL REPRESENTATION SYSTEM

David Hally

Defence Research Establishment Atlantic

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THE HLLSRF HULL REPRESENTATION SYSTEM

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m_3, m_4 Numbers of the offset points which delimit the region of discontinuity on the second station of a full discontinuity (Section 6.2.2)

N Number of B-splines in any of the subsidiary splines (Section 3.1)

N_s Number of offset stations (Section 6.1)

N_x Number of hull B-splines in the x direction (Section 3)

N_y Number of hull B-splines in the y direction (Section 3)

$\mathbf{n}$ Normal to the transom (Section 3.2)

n_d Number of a fully discontinuous offset station (Section 6.2.2)

P Vector space of piecewise polynomials spanned by a knot sequence $\{t_i\}$ (Section 6.2.3)

P^* Vector space of piecewise polynomials spanned by a knot sequence $\{t_i^*\}$ (Section 6.2.3)

$\mathcal{R}$ Rectangle which limits the range of the parameters x and y (Section 2.2)

r Radial coordinate in the propeller coordinate system.

R_p Radius of the propeller disk (Section 3.4.3)

$\mathcal{S}$ Surface in Cartesian space generated by the hull splines (Section 2.2)

s Non-cartesian coordinate of the hull defined below the waterline.

s_i Value of y for the i^{th} knuckle (Section 6.2.3)

t_i Sequence of knots for any of the subsidiary splines (Section 3.1)

$t_i^{(x)}$ Sequence of knots for the hull splines in the x direction (Section 3)

$t_i^{(y)}$ Sequence of knots for the hull splines in the y direction (Section 3)

t_i^* Knot sequence with a multiple knot removed (Section 6.2.3)

X Cartesian coordinate of the hull (Section 2.1)

$\mathbf{X}$ Cartesian coordinate vector: (X, Y, Z) (Section 2.1)

$\mathbf{X}_p$ Centre of the propeller disk (Section 3.4.3)

$\mathbf{X}_t$ A Cartesian point used to define the transom (Section 3.2)

x Parameter of the hull representation defined in HLLSRF to be the station number (Section 2.1)

x_{AP} Station number of the aft perpendicular (Section 3)

x_{APm} Station number of the aft perpendicular on a modified hulls (Section 4)

α_{jn} Hull spline coefficients (Equation (3.3))

β_n Spline coefficients for any of the subsidiary splines (Section 3.1)

β_{jn} Hull spline coefficients (Equation (3.4))

γ_n Normalization factor defined in equation (6.5)

$\mu_j(x)$ Function defined in equation (6.23)

ν Multiplicity of a knot (Section 3.3)

$\nu_j(x)$ Function defined in equation (6.24)

θ Angular coordinate in the propeller coordinate system.

Bold face symbols refer to vectors.

1 Introduction

Computer programs are now used routinely to support a wide variety of ship-related research. At DREA, computer programs have been developed to aid in ship design as well as to determine ship motions, ship structural vibrations, and flows around the hull and into the propeller plane. Each of these programs requires, as input, a description of the geometry of the ship hull, but each, at present, requires a different format for this description. Thus, when performing a series of calculations on the same hull, much time is wasted preparing several different data files each of which describes the hull geometry. To circumvent this problem, a system for describing the hull geometry has been developed which is capable of describing all relevant characteristics of the hull for the above programs. The system is called HLLSRF.

The HLLSRF hull representation system has grown out of earlier hull modelling programs associated with the HLLFLO programs for calculating the flow around hulls[1,2,3,4]. HLLSRF has extended these programs to allow the representation of many common hull features which were not allowed previously: notably, bulbous bows, complex stern profiles, slanted transoms, and stepped decks.

A useful feature of the HLLSRF representation scheme is that variations of existing hulls can be represented by specifying how they differ from their parent. For example, the length of the hull could be changed simply by specifying the new length in a modification file; all other aspects of the hull would remain unchanged. Similarly, the position of the propeller disk could be altered or its draft lowered. Using a method similar to that of Lackenby[5], the form of the hull can also be modified to change its prismatic coefficients or its longitudinal centre of buoyancy.

The main function of a HLLSRF representation is to provide a regular coordinate system covering the entire hull. The coordinate system provides the means for performing a wide variety of hydrodynamic calculations ranging in complexity from simple evaluation of sectional properties for strip theory, to the solution of complicated differential equations to determine the viscous flow around the hull. HLLSRF has been designed to maintain the regularity of this coordinate system so that numerical difficulties can be avoided. To facilitate flow calculations, an additional regular coordinate system is provided which covers only the portion of the hull below the waterline. Using the method of hull modification described above, the waterline itself may be changed without changing the rest of the hull representation. This allows a simple and efficient means for calculating flows at different drafts, at different trim angles, or at different Froude numbers.

HLLSRF hull representations are also extensible: extra data may be included in the data file describing a hull. To make use of the extra data, the user must supply subroutines which read and access the data in a manner appropriate to his needs; otherwise they will be ignored. The extensibility of HLLSRF is very convenient, since it allows for the inclusion of data which may be required for one program but not for others. It also allows a convenient means for extending the system to include hull features not considered important now but which may be

2 Hull Representation Strategy

In this section an overview of the HLLSRF representation is given. A more precise definition is given in Section 3.

2.1 Coordinate Systems

The hull is defined in terms of a Cartesian coordinate system whose axes are aligned with the waterlines and centreplane. The three Cartesian coordinates are denoted (X, Y, Z) and are defined as follows.

- The coordinate X increases from bow to stern. The forward perpendicular is $X = 0$ and the aft perpendicular is $X = L$, where L is the length between perpendiculars. The planes of constant X are hull sections.
- The coordinate Y increases from the centreplane to the point of maximum breadth. The centreplane is $Y = 0$. Owing to the symmetry of the hull, it is usually not necessary to decide whether Y increases to port or to starboard, but, to maintain a right-handed coordinate system, Y should increase to starboard. Surfaces of constant Y are parallel to the centreplane.
- The coordinate Z increases from the baseline to the point of maximum height above the baseline. The surfaces of constant Z are waterplanes. The waterplane which contains the baseline is $Z = 0$.

The position vector (X, Y, Z) will sometimes be denoted by the single bold face symbol $\mathbf{X}$.

A hull representation is simply a parameterization of the hull surface so that the Cartesian coordinate vector, $\mathbf{X}$, of a point on the hull can be obtained as functions of the two parameters (x, y). That is,

$$\mathbf{X} = \mathbf{f}(x, y) \tag{2.1}$$

for some vector function $\mathbf{f} = (f_X, f_Y, f_Z)$. The parameters x and y define a non-Cartesian coordinate system on the hull surface. In general x increases along the length of the hull (lines of constant y run from bow to stern) while y increases around its girth (lines of constant x run from the keel to the deck edge). Both coordinate systems are summarized in Figure 1.

For computer applications, the vector function $\mathbf{f}$ is usually expressed as a linear combination of a series of basis functions, $b_n(x)$ and $b_j(y)$.

$$\mathbf{f}(x, y) = \sum_{n=1}^{N_x} \sum_{j=1}^{N_y} \boldsymbol{\alpha}_{jn} b_j(y) b_n(x) \tag{2.2}$$

Each hull is then described by the coefficients $\boldsymbol{\alpha}_n$ which may be stored in a data file.

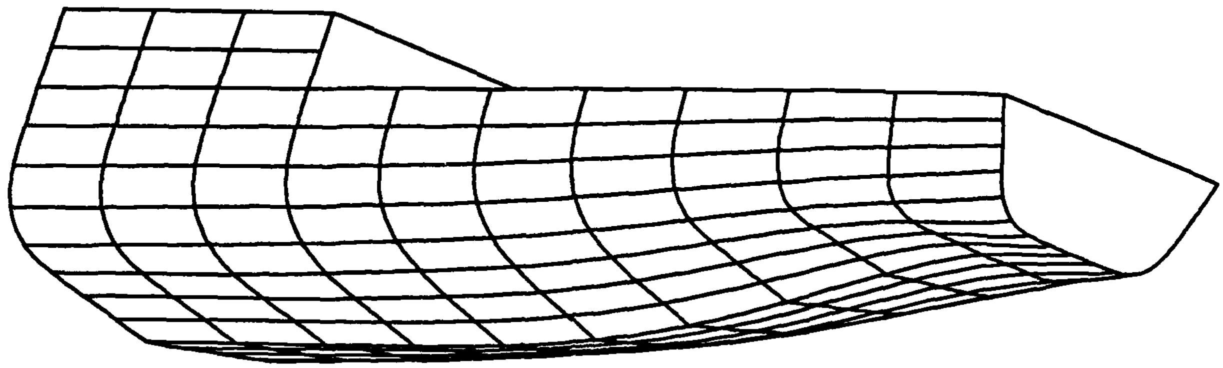

Figure 2: A ship hull with a stepped deck

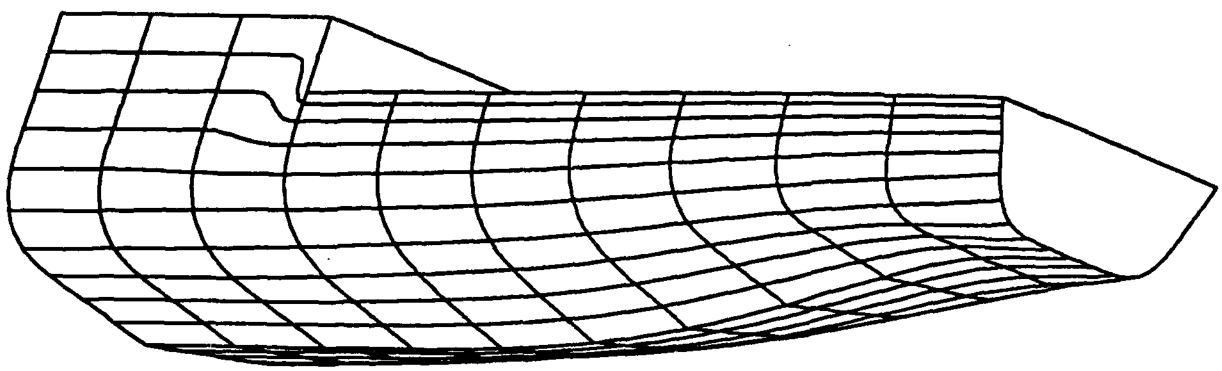

Figure 3: xy-coordinate grid which preserves a rectangular domain

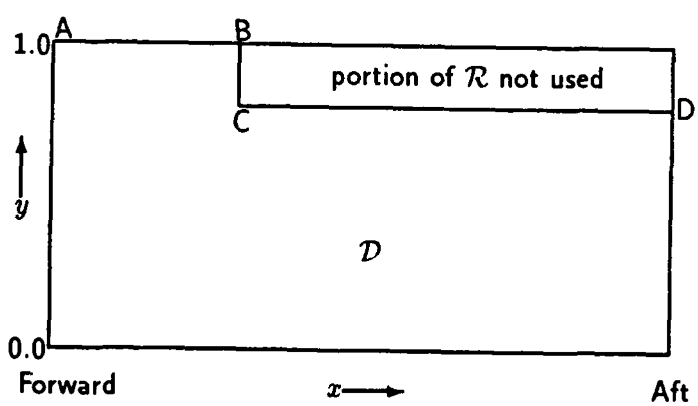

Figure 4: Non-rectangular xy-domain used to generate Figure 2

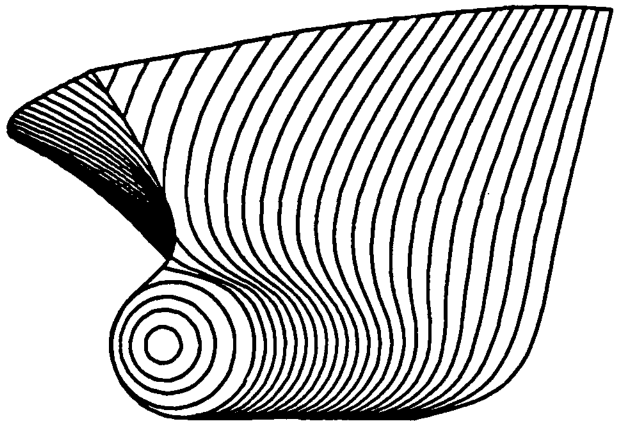

Figure 5: A bulbous bow

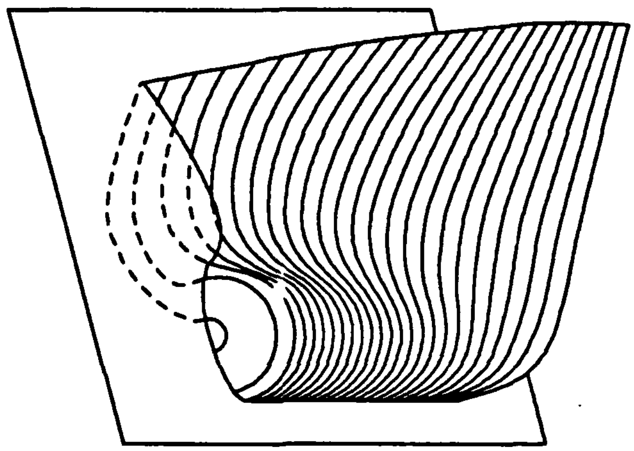

Figure 6: Surface $\mathcal{S}$ for Figure 5

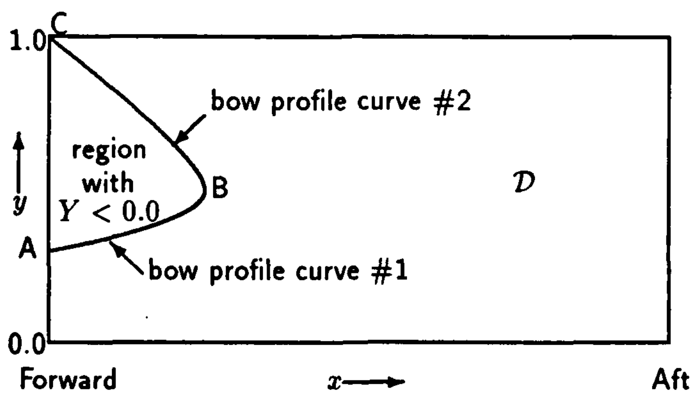

Figure 7: (x, y) domain for Figure 5

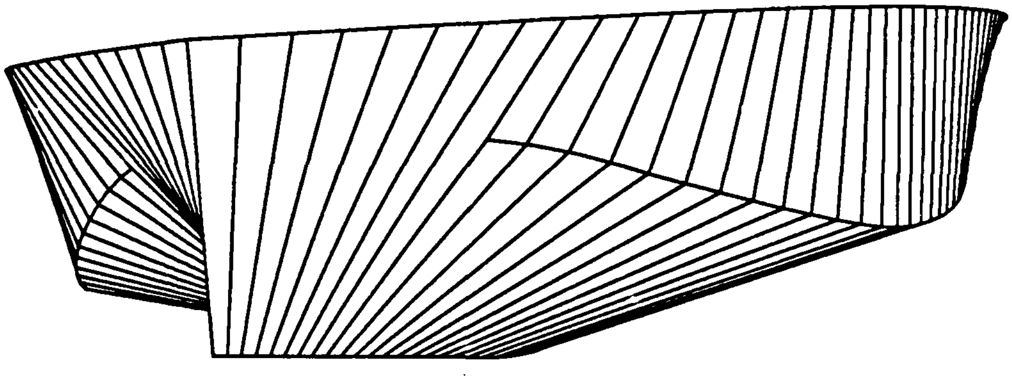

Figure 8: Hull with a chine

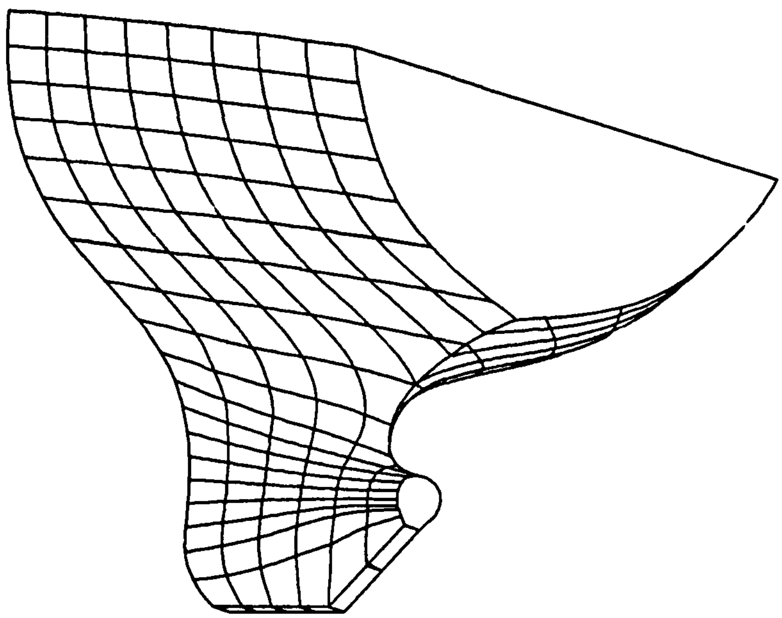

Figure 9: Stern with a propeller bossing

which, in the way it is represented, is essentially the same as the chine line of Figure 8; the function $\partial \mathbf{f}/\partial y$ is discontinuous. The second is the corner where the stern frame meets the flat of keel; in this case the function $\partial \mathbf{f}/\partial x$ is discontinuous. The discontinuity is localized to the region of the keel.

otherwise unconstrained. When it is important to be able to model the hull on both sides of the centreplane, y is allowed to vary between -1.0 and 1.0.

$$\left.\begin{array}{l} X(x,y)= \ f_X(x,-y) \\ Y(x,y)=-f_Y(x,-y) \\ Z(x,y)= \ f_Z(x,-y) \end{array}\right\} \quad \text{if } -1 \leq y < 0 \tag{3.2}$$

Before the functional forms of f_Y and f_Z can be specified, it is necessary to choose a set of basis functions $b_n(x,y)$. HLLSRF uses B-splines of arbitrary order (for further information on B-splines see de Boor[6]). B-splines are a popular choice as the basis functions for hull representations[2,7-16] as they can be calculated both accurately and efficiently and are capable of describing a wide variety of complex shapes[7,8,9,10]. By manipulation of the B-spline knots, it is possible to generate the 'cuts' in the hull and the discontinuities in derivative described in Section 2.4; details are given in Section 3.3.

HLLSRF uses the following functional forms to define the Cartesian coordinates Y and Z in terms of B-splines.

$$Y = f_Y(x,y) \quad = \quad B\sum_{j=1}^{N_y}\sum_{n=1}^{N_x} \alpha_{jn} B_{n,k_x}(x) B_{j,k_y}(y) \tag{3.3}$$

$$Z = f_Z(x,y) \quad = \quad D\sum_{j=1}^{N_y}\sum_{n=1}^{N_x} \beta_{jn} B_{n,k_x}(x) B_{j,k_y}(y) \tag{3.4}$$

where

B is the half-breadth of the ship,

D is its depth from the baseline to the highest point above the baseline,

$B_{n,k_x}(x)$ is a B-spline of order k_x corresponding to a knot sequence $t_i^{(x)}, i = 1,\ldots,N_x + k_x$, for each $n = 1,\ldots,N_x$,

$B_{n,k_y}(y)$ is a B-spline of order k_y corresponding to a knot sequence $t_i^{(y)}, i = 1,\ldots,N_y + k_y$, for each $n = 1,\ldots,N_y$, and

α_{jn}, β_{jn} are coefficients,

The B-splines are normalized to sum to one:

$$\sum_{i=1}^{N} B_{i,k}(x) = 1 \quad \text{if } t_k \leq x \leq t_{N-k+1} \tag{3.5}$$

The sums of the B-splines return non-zero values for Y and Z if $x \in (t_1^{(x)}, t_{N_x+k_x}^{(x)})$ and $y \in (t_1^{(y)}, t_{N_y+k_y}^{(y)})$. Hence it is necessary that $t_1^{(x)} \leq x_{min}$, $t_{N_x+k_x}^{(x)} \geq x_{max}$, $t_1^{(y)} \leq 0$ and $t_{N_y+k_y}^{(y)} \geq 1$.

derivative of $y_d(x)$; it may be generated by including a knot of multiplicity $k-1$. The number of deck edge discontinuities, the station numbers at which they occur, and their types (i.e. whether they are steps or corners) are all stored in additional HLLSRF data structures.

3.1.2 Bow and Stern Profile Curves

The functions $y_b^m(x)$ used to approximate the bow profile curve are also specified in terms of B-splines. Thus, for given x, the point (x,y) lies (approximately) on the bow profile curve if $a_m \le x \le b_m$ (see equation (2.3)) and $y = y_b^m(x)$ for some $m \in [1, M_b]$ where

$$y_b^m(x) = \sum_{n=1}^{N_m} \beta_n^m B_{n,k_m}^m(x) \tag{3.8}$$

and

$B_{n,k_m}^m(x)$ is a B-spline of order k_m corresponding to a knot sequence $t_i^m, i = 1, \ldots, N_m + k_m$, for each $n = 1, \ldots, N_m$, and

β_n^m are coefficients,

At the stern a completely analogous set of splines is used to represent the stern profile curve.

3.2 Slanted Transoms

HLLSRF represents a transom as a plane which intersects the hull near the stern (see Figure 12). It does not allow the representation of curved transoms or more complex transoms consisting of several different planes as in Figure 13; however, for most purposes, the hull of Figure 13 could be adequately represented by including the line A–B in the deck edge rather than the transom.

The transom plane is defined by specifying a point through which it passes, and an aft-facing normal vector. Let the Cartesian coordinate vector of the point through which the transom passes be $\mathbf{X}_t$ and its normal $\mathbf{n}$. A point $\mathbf{X}$ lies on the transom if

$$(\mathbf{X} - \mathbf{X}_t) \cdot \mathbf{n} = 0 \tag{3.9}$$

By symmetry, n_Y is zero, and one can choose $\mathbf{X}_t$ to lie on the centreplane so that $Y_t = 0$. Hence, to define the transom, only X_t, Z_t, n_X and n_Z need be given.

When a transom is defined, the hull domain $\mathcal{D}$ must be restricted to discard points which lie beyond the transom: $\mathcal{D} = \{(x,y) : (\mathbf{X}(x,y) - \mathbf{X}_t) \cdot \mathbf{n} \le 0\}$.

3.3 Hull Discontinuities

Discontinuities may be accommodated in B-spline hull representations in three different ways. Rogers and Satterfield[11], Kukner and Ergur[12] and Rosović[13] generate knuckles by a simple manipulation of the hull B-spline coefficients α_{jn} and β_{jn} (see equations (3.3) and (3.4)). If $\alpha_{j-k+1,n} = \alpha_{j-k+2,n} = \ldots = \alpha_{j-1,n}$ and $\beta_{j-k+1,n} = \beta_{j-k+2,n} = \ldots = \beta_{j-1,n}$ for all $n \in [n_{min}, n_{max}]$, then the line $y = t_j$ is a knuckle for $x \in [t_{n_{min}+k_x}, t_{n_{max}}]$. However, Hally[14] has pointed out that the xy-coordinate generated by this method is singular at the knuckle; the singularity will often cause difficulties when used in numerical calculations. Moreover, in HLLSRF, because the Cartesian coordinate X is not described using B-splines, this method cannot be used to generate hull discontinuities along a hull station (as in Figure 10, for example). Furthermore, this method cannot be used to generate complete discontinuities in the surface $\mathcal{S}$ such as the 'cut' used to define the propeller bossing in Figure 10. Hence, a better method for representing hull discontinuities is required.

The other two methods for generating knuckles use the fact that multiple knots can be used to decrease the degree of continuity of a spline (see de Boor[6], chapter IX); in particular, if a knot sequence of a B-spline of order k has a knot of multiplicity ν, then the spline curve can have discontinuous $(k-\nu)^{\text{th}}$ derivative at that knot. Knuckles can be defined by having knots of multiplicity $k_y - 1$ in the knot sequence $t_i^{(y)}$.

Hayes[10], Stroobant and Mars[9], and Hally[3] have generated knuckles by allowing the knot sequence $\{t_i^{(y)}\}$ to vary with x. Suppose that $\{t_i^{(y)}\}$ is such that all knots are distinct for $x < x_{lo}$ and $x > x_{hi}$, but that $k_y - 1$ knots coalesce when $x_{lo} \leq x \leq x_{hi}$. Then the hull will generally exhibit a discontinuity in derivatives with respect to y for $x_{lo} \leq x \leq x_{hi}$ but will be smooth outside this range; that is, there will be a knuckle beginning at x_{lo} and extending to x_{hi}. Similarly, discontinuities along hull stations can be defined by letting the knot sequence $t_i^{(x)}$ vary with y.

While this method seems to work well when defining the hull representation, it has distinct shortcomings when calculations are performed using the representation. In particular, evaluation of points on the hull given (x, y) is less efficient since the knot sequence must be calculated before the B-splines can be evaluated and summed. Techniques for speeding up the calculation are described by Hally[3]. The inefficiency is much worse when y-derivatives of the functions $Y(x,y)$ and $Z(x,y)$ are to be calculated (for example, when calculating a normal to the hull at a given point) since now the derivatives of the B-splines with respect to the knots must also be calculated. This overhead is too costly for hull representations which are to be used in hydrodynamic computations; it is not used by HLLSRF.

The third method also uses multiple knots to generate the knuckle but does so while keeping the knot sequences fixed. A knot of multiplicity ν in the knot sequence does not necessarily imply that the spline will have a discontinuous $(k-\nu)^{\text{th}}$ derivative at the knot; it merely *allows* the spline to a have a discontinuous $(k-\nu)^{\text{th}}$ derivative. In fact, by proper adjustment of the spline coefficients, the spline can be kept smooth at the multiple knot. The third method of knuckle representation generates a discontinuity in the spline where the knuckle is intended, but adjusts the spline coefficients so that the splines are smooth elsewhere.

For example, suppose $t_i^{(y)} = \ldots = t_{i+k-2}^{(y)} = t$, so that there is a knot of multiplicity $k_y - 1$ at $y = t$. The hull will then normally have discontinuous first derivative with respect to y at $y = t$: i.e. there will be a knuckle. By adjusting the spline coefficients α_{jn} and β_{jn} for $n = r, \ldots, s$, one can remove the discontinuity over the range $x \in [t_{r+k_x-1}^{(x)}, t_{s-k_x+1}^{(x)}]$. Hence, by subsidiary spline curve, $y = y_w(x)$.

$$y_w(x) = \begin{cases} \sum_{n=1}^{N} \beta_n B_{n,k}(x) & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \tag{3.10}$$

If the waterline curve is not present in the hull representation, it is assumed that the waterline curve is $y = 1$. Use of the waterline curve avoids regeneration of the hull whenever the waterline is changed: for example, if there is a change of draft or Froude number. However, the waterline will not usually be a line of constant y, a property which is desirable for performing calculations below the waterline. Therefore a new coordinate, s, is defined by

$$s = \frac{y}{y_w(x)} \tag{3.11}$$

It is 0.0 at the keel and 1.0 at the waterline. The coordinate system defined by (x, s) is preferred for most hydrodynamic calculations. The Cartesian coordinates corresponding to a point (x, s) are easily obtained using the inverse relation $y = sy_w(x)$.

$$\mathbf{X}(x, s) = \mathbf{f}(x, sy_w(x)) \tag{3.12}$$

Derivatives may be obtained using the chain rule. Use of (x, s) coordinates implies some loss of efficiency in comparison with (x, y), but this is easily regained by the requirement that only one hull need be generated to accommodate all possible waterlines. The time taken to calculate the spline representation of the waterline curve $y_w(x)$ is not significant.

3.4.2 The Curve of Sectional Areas

Many strip theory programs use sectional properties of the hull that can be determined once the curve of sectional areas is known. To avoid repeated calculation of the sectional areas, the HLLSRF representation allows the representation of the curve of sectional areas by a subsidiary spline.

$$A(x) = \begin{cases} \sum_{n=1}^{N} \beta_n B_{n,k}(x) & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

where

$A(x)$ is the area of the half-section at x divided by BZ_{dw},

B is the half-breadth of the ship, and

Z_{dw} is the design waterline of the ship.

4 Hull Modifications

One of the most useful features of the HLLSRF representation scheme is that variations of existing hulls can be generated using modification files: that is, a new hull can be defined by specifying how it differs from an existing hull. The changes are specified by data in the modification file. For example, the waterline for a hull may be changed by specifying a new spline representation for the waterline curve in a modification file. All other aspects of the hull remain unchanged. Similarly, the position of the propeller disk could be altered or the deck edge changed.

Changes to the hull itself are also possible. The simplest of these involve changes of the hull dimensions, L, B, and D. The form of equations (3.1), (3.3) and (3.4) has been chosen so that these quantities can be changed without the need of altering the spline coefficients.

Lackenby[5] first introduced the idea of modifying hulls by shifting their stations between the perpendiculars. This idea has been developed for use in computer hull representations by Hally[16] and Hally and Gilbert[4]. The shift of stations is specified by defining a transformation between the station numbers on the modified hull and the station numbers on the parent. The modified hull is defined by

$$X = \frac{(x - x_{FPm})L}{x_{APm} - x_{FPm}} \qquad (4.1)$$

$$Y = f_Y(x_p(x)) \qquad (4.2)$$

$$Z = f_Z(x_p(x)) \qquad (4.3)$$

where

x_{FPm} is the station number of the forward perpendicular of the modified hull,

x_{APm} is the station number of the aft perpendicular of the modified hull, and

$x_p(x)$ is a function which returns the station number on the parent hull which corresponds to the station x on the modified hull.

When the hull is modified in this way, there is no need to modify any of the subsidiary splines in the hull representation. For example, the waterline of a hull with modified station numbers is generated from that of the parent hull by

$$y = y_w(x_p(x)) \qquad (4.4)$$

The function $x_p(x)$ is represented by B-splines as follows.

$$x_p(x) = x_P(x) + \sum_{n=1}^{N} \beta_n B_{n,k}(x_P(x)) \qquad (4.5)$$

5 Using the HLLSRF Library

The HLLSRF library consists of many sub-programs which perform calculations using the HLLSRF hull representation of a hull. The most commonly used calculations for which sub-programs are provided are described in this section.

5.1 Loading the Representation of a Hull

Data defining a HLLSRF hull representation is stored in a computer file organized into a series of labelled records using the format described by Hally[17]; details of the records recognized by HLLSRF are described in the HLLSRF User's Guide[18].

Before any calculations can be performed, a hull representation must be loaded by reading a data file. Although there are two two different types of hull data files, one for a parent hull and one for hull modifications (see Section 4), a single sub-program is used to read all files: the existence of modifications is transparent to the user. This is also true for all of the HLLSRF sub-programs described here: they all work equally well whether the hull has been modified or not.

5.2 Coordinate Transformations

As was pointed out in the introduction, the main purpose of the hull representation is to provide coordinate transformations between the Cartesian coordinates (X,Y,Z) and the non-Cartesian hull coordinates (x,y). Accordingly, HLLSRF provides sub-programs for performing such coordinate transformations.

A sub-program is provided which, given a point (x,y), will calculate any of the Cartesian components X, Y, and Z, or any of their derivatives with respect to x and y. This involves a simple evaluation of the splines according to equations (3.1) (3.3) and (3.4). When a modification of hull form has been specified (see Section 4), then equations (4.1), (4.2), (4.3), (4.6), and (4.5) are used.

For hydrodynamic calculations, it is more convenient to use the coordinates (x,s) which are defined only on the submerged portion of the hull (see Section 3.4.1). A sub-program is provided which, given a point (x,y), will calculate any of the Cartesian components X, Y, and Z, or any of their derivatives (up to second order) with respect to x and s. This involves simple evaluation of the hull splines, the waterline spline, $y_w(x)$, and their derivatives.

The HLLSRF library also provides sub-programs which will determine points (x,y) or (x,s) given values of the Cartesian components. Given a value for x and a value for Y, say Y_0, there is a sub-program which will determine the value of y (or s) such that $f_Y(x,y) = Y_0$. Similarly, a value of Z could be specified. Likewise, if y (or s) is specified, there is a sub-program which a vector to which it is tangent, **t**. The line can then be parameterized in terms of the variable u by: $\{\mathbf{X} : \mathbf{X} = \mathbf{X}_0 + u\mathbf{t}\}$. A sub-program is provided which will determine a point (x, s) at which the line intersects the hull: that is, it solves the three simultaneous equations

$$\mathbf{X}_0 + u\mathbf{t} = \mathbf{X}(x, y) \tag{5.1}$$

for the unknowns x, y, and u. For example, by setting X_0 to the centre of the propeller disk and **t** to its normal, this sub-program could be used to determine the point at which a propeller shaft meets the hull.

A plane may be defined by specifying a Cartesian point through which it passes, $\mathbf{X}_0$, and its normal, **n**. A point (x, y) lies on the intersection of this plane and the hull if

$$(\mathbf{X}(x, y) - X_0) \cdot \mathbf{n} = 0 \tag{5.2}$$

Sub-programs are provided which will solve this equation for x given a value for y, or which will solve it for y given a value for x. In addition, a sub-program is provided which will calculate a series of points on the line of intersection of a plane with the hull. This could be used, for example, to determine the intersection of the waterplane with the hull for different trim or roll angles.

1. The values of y''_{mn} are set to the fractional arclength around the station, where the arclength is approximated by straight line segments joining the offset points.

$$y''_{1n} = 0 \tag{6.1}$$

$$y''_{mn} = \frac{\sum_{i=2}^{m} \sqrt{(Y_{in} - Y_{i-1,n})^2 + (Z_{in} - Z_{i-1,n})^2}}{\sum_{i=2}^{M_n} \sqrt{(Y_{in} - Y_{i-1,n})^2 + (Z_{in} - Z_{i-1,n})^2}} \tag{6.2}$$

This method was originally proposed by Sachdeva and Preston[20] and was used as the basis of the hull representation system in the DREA hydrodynamic package HLLFLO[1,2]. The fractional arclength method has the advantage that y''_{mn} is always well-defined for any set of non-coincident offset points; however, it has the disadvantage that the value of y''_{mn} depends on all the offset points on the station. Adverse effects from this are illustrated in Figure 14 which shows the y''-coordinate lines on the afterbody of a hull with a skeg (and no knuckles). The corner in the skeg at the keel causes a 'kink' in all the lines of constant y'' all the way to the deck edge, although with severity which decreases as the deck edge is approached. Thus local variations in geometry can have non-local effects in the definition of y''.

2. The y''_{mn} are set to the fractional angle subtended by the point at the intersection of the centreplane and the deck. This method requires that $Y_{1n} = 0$.

$$y''_{mn} = \frac{2}{\pi}\arctan\left(\frac{Y_{mn}}{Z_{M_n,n} - Z_{mn}}\right) \tag{6.3}$$

The fractional angle method has been used by Muraoka and Shirose in calculations of turbulent boundary layer flow around ship hulls[21].

If $Z_{M_n,n}$ is the same on all stations, as is often the case, the fractional angle method has the advantage that y''_{mn} depends only on the offset point (Y_{mn}, Z_{mn}); thus, the non-local effects found with the fractional arclength method can be avoided. Figure 15 shows the y''-lines generated for the same hull as in Figure 14.

On the other hand, this method does not work well on stations which are long and thin, as often occur near the bow. Figure 16 shows the distribution of y''-values on such a station using this method. The clustering of the y''-lines at the upper part of the station is not desirable. Moreover, in the limit that the width of the station vanishes, this method of calculation fails. It will also fail when there are offset points with negative Y-values, for then it would be possible to have two different offset points with the same value of y''.

3. The cartesian components Y and Z are first normalized by $Y_{M_n,n}$ and $(Z_{m_n,n} - Z_{1n})$ and then the y''_{mn} are set to the fractional angle subtended by the point at the intersection of the centreplane and the deck.

$$y''_{mn} = \frac{2}{\pi}\arctan\left(\gamma_n \frac{Y_{mn}}{(Z_{M_n,n} - Z_{mn})}\right) \tag{6.4}$$

where γ_n is the normalization factor for the n^{th} offset station, given by

$$\gamma_n = \frac{Z_{M_n,n} - Z_{1n}}{Y_{M_n,n}} \tag{6.5}$$

By normalizing the Y and Z values before calculating the angles, this method avoids the clustering of the y''-values obtained using the method 2. Figure 17 shows the thin section of Figure 16 with y''-values as calculated using the method 3; the clustering of y''-values has been avoided. However, the normalization can bring back non-local effects; use of this method on the hull of Figure 14 generates similar kinks in the y''-lines.

4. A composite of methods 2 and 3 can be defined as follows. The y''_{mn} are calculated using method 3 up to midships. Beyond midships, the normalization factor retains a constant value: whatever its value was at the midship station. Thus, if x_j is the midship station, then

$$\gamma_n = \begin{cases} \dfrac{Z_{M_n,n} - Z_{1n}}{Y_{M_n,n}} & \text{for } n \leq j \\ \dfrac{Z_{M_j,j} - Z_{1j}}{Y_{M_j,j}} & \text{for } n > j \end{cases} \tag{6.6}$$

This method works well for most destroyer and frigate hulls in which the problems in the method 2 tend to occur near the bow, while the problems in method 3 tend to occur near the stern. As with methods 2 and 3, it will not work with offsets with negative Y-values.

5. The y''_{mn} are calculated in a user-supplied sub-program. For complex hulls, it is possible that none of the above methods will work satisfactorily. In this case the user may determine the y''-values by some other method of his choosing, but must supply a sub-program to perform the calculation.

6. The y''_{mn} are read from a user-supplied file.

As has been illustrated above, the first four methods for determining y'' offered by HLLSRF all have disadvantages for certain hull forms. Other methods for defining y'' have been proposed, notably, conformal mapping onto a unit circle[22,23] and requiring that the coordinate system be orthogonal[24]. The conformal mapping method suffers from the same non-local behaviour as methods 1 and 3. The method of orthogonal coordinates is not always well-defined if there are hull discontinuities and will not always meet the requirement that the boundaries of the spline surface $\mathcal{S}$ be curves of constant y''. Thus, to date, no method for defining y''_{mn} has been discovered that is fully satisfactory for all typical hull forms. Therefore, HLLSRF offers the user the two additional options, 5 and 6, whereby the method for determining y''_{mn} can be tailored to the particular hull for which a representation is desired. As the complexity of the given hull increases, so also increases the likelihood that the user will have to resort to one of these options. Development of more satisfactory methods for generating the y''_{mn} is an area deserving of further research.

6.2.2 Adjustment of the y''-values at Fully Discontinuous Stations

At a fully discontinuous station, the offset file must contain two sets of offset points: one set which lies on the surface $\mathcal{S}$ just in front of the discontinuity, and another set which lies on the surface just beyond the discontinuity. In the offset file this is implemented by having two offset stations, x_{n_d} and x_{n_d+1}, at the fully discontinuous station: $x_{n_d} = x_{n_d+1}$. The y''-values of these offset points are y''_{mn_d} and y''_{m,n_d+1}.

The discontinuity will extend only over a portion of the station. On each of the two offset stations, the two offset points lying at each end of the discontinuous portion of the station must be marked: on the station n_d, these are the points m_1 and m_2; on the station $n_d + 1$ they are the points m_3 and m_4. Hence on station n_d, the discontinuous portion is $y'' \in (y''_{m_1 n_d}, y''_{m_2 n_d})$, while on the station $n_d + 1$ the discontinuous portion is $y'' \in (y''_{m_3 n_d}, y''_{m_4 n_d})$. Outside the region of discontinuity, the two stations are identical: hence, the offset points are also required to be identical outside the region of discontinuity. Thus,

$$m_1 = m_3 \tag{6.7}$$

$$(Y_{mn_d}, Z_{mn_d}) = (Y_{m,n_d+1}, Z_{m,n_d+1}) \quad \text{for } m \leq m_1 \tag{6.8}$$

$$(Y_{mn_d}, Z_{mn_d}) = (Y_{m+m_4-m_2,n_d+1}, Z_{m+m_4-m_2,n_d+1}) \quad \text{for } m \geq m_2 \tag{6.9}$$

Figure 18 shows the offset points on the two stations used to define the end of the propeller bossing in Figure 9. The points on station x_n are marked with dots, while those on station x_{n+1} are marked with crosses. The centreplane is also shown. In this case the region of discontinuity extends all the way to the keel so that $m_1 = m_3 = 0$.

So that the hull splines are continuous outside the range of discontinuity, it is also necessary that the y-values of the offset points on the two stations are identical outside the region of discontinuity. However, when the y''-values are calculated using methods 2 or 3, this will usually not be the case. For method 1, the mismatch is caused by the the normalization by the total arclength, since the two stations will usually have different total arclength. For method 3, it is caused by the fact that the Y-values at the deck edge may differ for the two stations, as may the Z-values at the keel; thus the normalization of the angles causes differing y''-values for the two stations.

Hence, it is necessary to adjust these values to y'_{mn} for which

$$y'_{mn_d} = y''_{m,n_d+1} \quad \text{for } m \leq m_1 \tag{6.10}$$

$$y'_{mn_d} = y''_{m+m_4-m_2,n_d+1} \quad \text{for } m \geq m_2 \tag{6.11}$$

This is done by defining a transformation $g(y'')$ which is applied to the second station of offsets and all subsequent stations.

$$y'_{mn} = y''_{mn} \quad \text{for } n \leq n_d \tag{6.12}$$

$$y'_{mn} = g(y''_{mn}) \quad \text{for } n > n_d \tag{6.13}$$

with the condition that

$$g(y''_{m,n_d+1}) = y''_{m,n_d} \quad \text{for } m \leq m_1 \tag{6.14}$$

$$g(y''_{m,n_d+1}) = y''_{m-m_4+m_2,n_d} \quad \text{for } m \geq m_2 \tag{6.15}$$

$\{t_i^{(x)}\}$ and $\{t_i^{(y)}\}$ are constructed with multiple knots reflecting the locations of the knuckles and discontinuous stations. For example, there is a knot of multiplicity $k_y - 1$ in the y-knot sequence at each of the knuckle y-values, s_i.

In order to determine the hull spline coefficients α_{jn} and β_{jn}, equations (3.3) and (3.4) are first rewritten as follows.

$$Y = B\sum_{j=1}^{N_y} \mu_j(x)B_{j,k_y}(y) \tag{6.21}$$

$$Z = D\sum_{j=1}^{N_y} \nu_j(x)B_{j,k_y}(y) \tag{6.22}$$

$$\mu_j(x) = \sum_{n=1}^{N_x} \alpha_{jn}B_{n,k_x}(x) \tag{6.23}$$

$$\nu_j(x) = \sum_{n=1}^{N_x} \beta_{jn}B_{n,k_x}(x) \tag{6.24}$$

The specification of the two-dimensional spline coefficients, α_{jn} and β_{jn}, may then proceed by fitting a series of data sets with one-dimensional splines.

First, at each offset station, the data set $\{(y_{mn}, Y_{mn}) : m = 1,\ldots,M_n\}$ is fitted according to equation (6.21) to yield $\mu_j(x_n)$. Similarly, the data set $\{(y_{mn}, Z_{mn}) : m = 1,\ldots,M_n\}$ is fitted according to equation (6.22) to yield $\nu_j(x_n)$. Then, for each $j = 1,\ldots,N_y$, the data set $\{(x_n, \mu_j(x_n)) : n = 1,\ldots,N_s\}$ is fitted according to equation (6.23) to yield α_{jn}, and $\{(x_n, \nu_j(x_n)) : n = 1,\ldots,N_s\}$ is fitted according to equation (6.24) to yield β_{jn}.

Each of these splines is determined using a single spline fitting sub-program based on the subroutine BSMTH described by Hally[26]. The algorithm used has the following properties.

1. The spline may be smoothed by an amount specified by the user. 'Errors' may also be assigned to the data points to allow the splines to pass more closely to or further from a given point. The errors may be changed interactively by the user.

2. The user may assign 'stiffness weights' to portions of the spline to allow or inhibit curvature of the spline in a given region; thus a high stiffness weight is assigned to the flat bottom of a hull but a low stiffness weight near the bilge. The stiffness weights are calculated automatically but may be altered interactively by the user.

3. The spline knots are independent of the offset points; moreover, unlike most spline algorithms, the number of B-splines comprising the spline can exceed the number of data points. This allows flexibility in the choice of knots and great freedom when digitizing offset diagrams to create the offset data file.

4. After each spline has been calculated, it may be plotted and altered interactively by the user by varying the required amount of smoothing, the data point errors, and the stiffness weights.

6.4 Calculating the Subsidiary Splines

Once the hull splines have been calculated, the subsidiary splines for the deck edge, the waterline, the bow profile, the stern profile, and the curve of sectional areas are determined. Each of these is optional. The subsidiary spline curves are all calculated using the same interactive spline sub-program (based on BSMTH) that is used for the hull splines.

6.4.1 Calculating the Deck Edge Splines

The OFFSRF representation of offset data[17] allows the user to mark those offset points which lie on the deck edge. On the station x_n, denote the number of the point on the deck edge by m_d. Then the data set $\{(x_n, y_{m_d,n}) : n = 1, \ldots, N_s\}$ can be splined to yield th. 'eck edge curve $y_d(x)$. The knot sequence for this spline is constructed so that the spline allows discontinuity of first derivative at all deck edge corners, and discontinuity of the spline itself at steps.

6.4.2 Calculating the Waterline Spline

The user may set the design waterline, Z_{dw}, to any desired height above the baseline provided $Z_{dw} < D$. At a series of stations $\{x_i\}$, the equation

$$f_Z(x_i, y_i) = Z_{dw} \tag{6.25}$$

is solved for y_i using the Newton-Raphson method. The resulting data set $\{(x_i, y_i)\}$ is then splined to yield the waterline curve $y_w(x)$.

6.4.3 Calculating the Bow and Stern Profile Splines

The bow profile spline is calculated by starting on the keel and searching for a series of points for which

$$f_Y(x_i, y_i) = 0 \tag{6.26}$$

The search is started by choosing a series of values for x_i and solving for y_i. If the solution procedure fails (as will happen, for example, at the forward station of a bulbous bow) then a series of values y_i is chosen and equation (6.26) is solved for x_i. This procedure of alternating between solving for x and solving for y is continued until the deck edge is reached. The resulting series of points $\{(x_i, y_i)\}$ is then split into sub-series over which x_i varies monotonically: for example, $\{(1.0,0.1),(2.0,0.2),(3.0,0.2),(1.0,0.2)\}$ would be split into the two sub-series $\{(1.0,0.1),(2.0,0.2),(3.0,0.2)\}$ and $\{(3.0,0.2),(1.0,0.2)\}$. The sub-series are then data sets which can be splined (perhaps after re-ordering to ensure that x_i is increasing) to yield the different segments of the bow profile curve $y_b^m(x)$. The procedure to calculate the stern profile is similar.

7 Concluding Remarks

The designer of any useful hull representation system is beset by two main obstacles: the complexity and variety of hull forms, and the efficiency and ease of use demanded by the user. HLLSRF has been designed to surmount both obstacles in a manner which will find favour with hydrodynamicists.

Efficiency and ease of use has been addressed by using the station number as the parameter x: ease of use is provided as this is the parameter of choice for nearly all hydrodynamic computations; efficiency is provided by avoiding costly spline inversions in favour of efficient spline evaluations. These objectives have also been addressed in the implementation of the computer code itself:

- data files are structured so that they are easy to read and easy to edit,
- sub-programs are provided in the HLLSRF library which perform many commonly used calculations, and
- the code is modular, efficient, well-documented, and provides clear error-reporting.

The ability to describe complex hull shapes has been addressed by allowing the parametric domain, $\mathcal{D}$, to be non-rectangular. Figures 2, 6, 9, 8, and 15 are an indication of the variety of hull shapes that HLLSRF can represent.

The program HLLSPL has been provided for generating HLLSRF representations from offset data. It must be recognized that, while every effort has been made to make the program HLLSPL easy to use, the level of effort required to generate a hull representation will always rise as its complexity increases. Easing the generation of complex hulls is the area in which future research effort can be most profitably spent.

HLLSRF differs from most hull representation schemes in that it was developed to provide a method of representing existing hulls accurately so that their hydrodynamic properties can be calculated. Similar representation schemes have been developed for the purposes of designing new hulls[8–16] but Hally[14] has pointed out that the requirements for these two objectives can be quite different. To date, no representation scheme has been designed that is fully satisfactory for both purposes. When a user wishes to represent a hull for which inadequate offset data have been provided, a problem arises: the hull generation task is one partly of approximation to the offsets and partly of design of those portions of the hull which are not described adequately by the offsets. All current systems, including HLLSRF, require some user 'artistry' when applied to such hulls. However, it is also important to note that, while HLLSRF is not yet the perfect hull representation system, it is currently at a state of development which surpasses the programs in which it is to be applied. That is, if a hull is sufficiently complex that it is difficult to represent properly, then it is probably too complex to be analyzed satisfactorily

References

1. D. Hally, "HLLFLO User's Guide," DREA Tech. Comm. 86/306, 1986.

2. D. Hally, "Numerical Approximation of Ship Hulls," DREA Tech. Memo. 85/207, 1985.

3. D. Hally, "Numerical Representation of Hulls with Knuckles," DREA Tech. Memo. 86/209, 1986.

4. D. Hally and N. Gilbert, "MODHLL: A Program for Modifying Computer Representations of Ship Hulls," DREA Tech. Memo. 86/224, 1986.

5. H. Lackenby, "On the Systematic Geometrical Variation of Ship Forms," *Trans. INA* **92**, 289 (1950).

6. C. de Boor, *A Practical Guide to Splines* (Springer Verlag, New York, 1978).

7. R. E. Barnhill and R. F. Reisenfeld, "Surface Representation," in *International Symposium on Computer Aided Hull Surface Definition*[28], pp. 97–104.

8. D. F. Rogers, "B-Spline Curves and Surfaces for Ship Hull Definition," in *International Symposium on Computer Aided Hull Surface Definition*[28], pp. 79–96.

9. G. Stroobant and D. Mars, "Ship Hull Form Fairing," in Rogers et al.[29], pp. 177–181.

10. J. G. Hayes, "New Shapes from Bi-Cubic Splines," Report 58, NPL, 1974.

11. D. F. Rogers and S. G. Satterfield, "Dynamic B-Spline Surfaces," in Rogers et al.[29], pp. 189–196.

12. A. Kukner and H. Ergur, "Modeling of a Ship using B-Surface Spline," in Murthy et al.[27], pp. 173–188.

13. B. Rosović, "SURFIT – Interactive Program for Designing Free Forms," in *Computer Applications in the Automation of Shipyard Operation and Ship Design V*, edited by P. Banda, C. Kuo, and G. di Filippo (North-Holland, Amsteram, 1985), pp. 413–420.

14. D. Hally, "A Computer Hull Representation for Use in Hydrodynamic Programs," in Murthy et al.[27], pp. 131–142.

15. D. Hally, "Representation of Knuckles in Tensor Product B-spline Hull Representations," DREA Tech. Comm., in preparation.

16. D. Hally, "On the Systematic Variation of Hull Representations for Computers," *Trans. R.I.N.A.* **130**, 77–83 (1988).

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