

AD-A221 735

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) 1

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER No. 38	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER ①
4. TITLE (and Subtitle) Photon Statistical Properties of an Optical Wiggler Free Electron Laser		5. TYPE OF REPORT & PERIOD COVERED Technical Report 10/01/84 - 09/30/85
AUTHOR(s) G. Dattoli and J. Gallardo		6. PERFORMING ORG. REPORT NUMBER
PERFORMING ORGANIZATION NAME AND ADDRESS University of California Quantum Institute Santa Barbara, CA 93106		8. CONTRACT OR GRANT NUMBER(s) N00014-80-C-0308
CONTROLLING OFFICE NAME AND ADDRESS ONR 1030 E. Green St. Pasadena, CA 91106		10. PROGRAM ELEMENT, PROJECT, TASK, AREA & WORK UNIT NUMBERS 61153N; RR011-07-06; NR603-001
MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 1985
DTIC ELECTE MAY 21 1990 DS		13. NUMBER OF PAGES
		15. SECURITY CLASS. (of this report) Unclassified/Unlimited
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) ADVISORY GROUP DECEMBER FEB 20 1988 ON ELECTRON DEVICES		
18. SUPPLEMENTARY NOTES Journal of the Optical Science B, 1985.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Photon, Properties, Wiggler, FEL		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We use the Free Electron Laser Hamiltonian to study the statistical properties of the photons emitted in a scattering process between a coherent electromagnetic wave (optical wiggler) and a counter-propagating relativistic electron beam. It is shown that the probability distribution of exchanged photons is binomial and that coherent states are preserved by the Free Electron Laser interaction only in the absence of gain.		

1. INTRODUCTION.

In a Free Electron Laser (FEL) device an electron beam interacts with a permanent magnet wiggler. The magnetic field can be treated as a real electromagnetic wave according to the prescription of Weizsäcker-Williams. In the quantized version of the FEL interaction it is assumed that the wiggler is a classical wave with a fixed but infinitely large number of photons. In this short note we will assume the wiggler to be a real electromagnetic field described by a coherent state. This problem was suggested by the fact that coherence in the Glauber sense is not preserved by the FEL interaction even at zero electron recoil if the wiggler is a classical wave and the input laser field is a coherent wave¹; furthermore, the photon distribution was shown to be sub-Poissonian. We now ask the equivalent questions: does the laser field remain in a coherent state during the interaction when the wiggler is in a coherent state and there are not laser photons present at $t = 0$? what does the photon distribution look like? These questions have relevance in fully understanding coherence in an FEL. Our treatment can be extended to understand other problems in Quantum Optics, i.e. the evolution of coupled harmonic oscillators and consequently that of coherent Bloch states².

2. THEORY.

The single mode, single particle non-relativistic Hamiltonian describing the FEL process reads

$$H = \frac{P_z^2}{2m} + \hbar\omega \left(a_L^\dagger a_L + a_w^\dagger a_w + 1 \right) + \hbar\Omega \left[a_L^\dagger a_w e^{-2ikz} + a_w^\dagger a_L e^{2ikz} \right] \quad (1)$$

where P_z , z are the electron linear momentum and longitudinal position, respectively; $a_L^\dagger$ ($a_w^\dagger$) is the laser (undulator) creation operators and $\Omega = \frac{2\pi c^2}{\lambda} \frac{e^2}{m\omega}$ the coupling constant with V the interaction volume.

From Eq. (1) it can be readily seen that the total linear momentum and the number of photons are conserved quantities¹. In the assumption of both laser and undulator initial fields are in coherent states with mean number of photons $|\alpha_0|^2$, $|\beta_0|^2$ respectively, the state of the system at time t is given by

$$|\Psi(t)\rangle = e^{-i\frac{P_z^2}{2m}t} e^{-i\omega t} \sum_{n=0}^{\infty} \frac{e^{-\frac{1}{2}|\alpha_0|^2}}{\sqrt{n!}} \alpha_0^n \sum_{m=0}^{\infty} \frac{e^{-\frac{1}{2}|\beta_0|^2}}{\sqrt{m!}} \beta_0^m \times \sum_{l=-\infty}^m C_l^{m,n}(t) |P_0 - 2l\hbar, n+l, m-l\rangle \quad (2)$$

where P_0 is the initial electron momentum, l is an integer enumerating the exchanged photons and $C_l^{m,n}(t)$ is the probability amplitude for interchanging l -photons in the presence of m and n undulator and laser photons, respectively. Replacing Eq. (2) in the Schrödinger equation we obtain a differential-difference equation of the spherical Raman-Nath type³

$$i \frac{d}{dt} C_l^{m,n}(t) = (-W_0 + \epsilon l) C_l^{m,n}(t) + \Omega_R \left[\sqrt{(n+l+1)(m-l)} C_{l+1}^{m,n}(t) \right.$$

Photon statistical properties of an optical wiggler Free Electron Laser

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ABSTRACT: We use the Free Electron Laser Hamiltonian to study the statistical properties of the photons emitted in a scattering process between a coherent electromagnetic wave (optical wiggler) and a counter-propagating relativistic electron beam. It is shown that the probability distribution of exchanged photons is binomial and that coherent states are preserved by the Free Electron Laser interaction only in the absence of gain.

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$$+ \sqrt{(n+l)(m-l+1)} C_{l-1}^{m,n}(\tau) \quad (3)$$

with the initial condition $C_l^{m,n}(0) = \delta_{l,0}$, where τ is the dimensionless time ($\tau = \gamma_L^2$), $W_0 = \omega^2 P_0 L / mc^2$ is the resonance parameter, $\Omega_R = \Omega L / c$ is the coupling constant and $\epsilon = 2\hbar k^2 / mc$ is the electron recoil; ϵ is of the order of 10^{-7} for most cases of interest.

Choosing a vacuum as input field ($|\alpha_0|^2 = 0$), the solution of the Raman-Nath Equation³ in first order in ϵ reads

$$C_l^m(\tau) = e^{i\Delta(\tau) - im\Phi(\tau)} [A_l^m(\tau) + iD_l^m(\tau)] \quad (4)$$

where $\Delta(\tau)$ and $\Phi(\tau)$, which we need not to specify here, have been given in Ref. 3. The functions appearing in Eq. (4) are given by

$$\begin{aligned} A_l^m(\tau) &= B_l^m(\tau) + \epsilon \Omega_R \frac{\partial}{\partial W_0} \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right) \left\{ (2l+1) \sqrt{(l+1)(m-l)} B_{l+1}^m(\tau) \right. \\ &\quad \left. - (2l-1) \sqrt{l(m-l+1)} B_{l-1}^m(\tau) \right\} - \frac{1}{2} \epsilon \Omega_R^2 \frac{\partial}{\partial W_0} \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right)^2 \\ &\quad \times \left\{ \sqrt{(l+1)(l+2)(m-l-1)} B_{l+2}^m(\tau) - \sqrt{l(l-1)(m-l+1)(m-l+2)} B_{l-2}^m(\tau) \right\} + \dots \end{aligned} \quad (5a)$$

and

$$\begin{aligned} D_l^m(\tau) &= \frac{1}{2} \epsilon \Omega_R \tau \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right) \left\{ (2l+1) \sqrt{(l+1)(m-l)} B_{l+1}^m(\tau) \right. \\ &\quad \left. + (2l-1) \sqrt{l(m-l+1)} B_{l-1}^m(\tau) \right\} - \frac{1}{2} \epsilon \Omega_R^2 \tau \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right)^2 \\ &\quad \times \left\{ \sqrt{(l+1)(l+2)(m-l-1)} B_{l+2}^m(\tau) + \sqrt{l(l-1)(m-l+1)(m-l+2)} B_{l-2}^m(\tau) \right\} + \dots \end{aligned} \quad (5b)$$

where we have denoted the binomial distribution as

$$B_l^m(\tau) = \sqrt{\binom{m}{l}} p^{l/2} (1-p)^{(m-l)/2} \quad (6a)$$

with

$$p = \frac{4\Omega_R^2}{(4\Omega_R^2 + W_0^2)} \sin^2 \frac{\tau}{2} \sqrt{4\Omega_R^2 + W_0^2} \quad (6b)$$

An interesting question is the definition of coherent state in the context of this model and within the $\epsilon = 0$ approximation. The most obvious difficulty we are faced with is how to deal with the electron variables. Now, since the Hamiltonian contains two coupled (through e^{2ikz}) harmonic oscillators, we can follow the definition suggested in Ref. 4 for the coherent Bloch states, namely a simultaneous eigenstate of the laser and undulator destruction operator. This suggests to define the *BOSON* operators

$$A = e^{2ikz} \frac{\partial L}{\partial \alpha_0} \frac{\partial \alpha_0}{\partial \beta_0} \quad (7a)$$

with the commutation relation

$$(\Psi | [A, A^\dagger] | \Psi) = 1 \quad (7b)$$

It is not difficult to prove that $|\Psi(\tau)\rangle$ is an eigenstate of A and the average value is given by

$$(\Psi(\tau) | A | \Psi(\tau))_{\epsilon=0} = \sqrt{p(1-p)} |\beta_0|^2$$

Hence, the state $|\Psi\rangle$ of Eq. (2) for zero electron recoil is a coherent Bloch state in the sense of Ref. 4. Clearly, for $\epsilon \neq 0$ the FEL interaction does not preserve coherence. Next, we evaluate the average value of the exchanged photons

$$\langle l \rangle_{\epsilon \neq 0} = |\beta_0|^2 \left\{ p - 2\epsilon \Omega_R \sqrt{p(1-p)} \frac{\partial}{\partial W_0} \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right) \right\} + \dots \quad (8)$$

where p is given in Eq. (6b); the additional terms are higher order in Ω_R . This result is obtained using recursive properties of the binomial distribution. In the limit of $|\beta_0|^2 \rightarrow \infty$ and $\Omega_R \rightarrow 0$, with $|\beta_0|^2 \Omega_R \equiv \Omega = \text{constant}$, Eq. (8) becomes

$$\langle l \rangle_{\epsilon \neq 0} \rightarrow \Omega^2 \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right)^2 - \epsilon \Omega^2 \frac{\partial}{\partial W_0} \left(\frac{\sin W_0 \tau / 2}{W_0 / 2} \right) + \dots$$

which is the usual expression for the FEL gain¹ with permanent magnet undulator. We also evaluate

$$(\Delta l^2)_{\epsilon \neq 0} - \langle l \rangle_{\epsilon \neq 0}^2 \approx O(\Omega_R^4)$$

quantity which measures the departure from Poisson statistics⁶, finding agreement with previous results^{1,5}.

In conclusion, we have shown that in a parametric process involving the FEL interaction of a single electron with a coherent wave (optical wiggler) the spontaneously emitted photons distribution function is binomial (Eq. (6)). We have also shown that in first order in the electron recoil the average number of exchanged photon contains, besides the spontaneous emission term, a stimulated emission factor similar to the one appearing in the undulator FEL process.

ACKNOWLEDGEMENT

The authors acknowledge discussion with L. Elias. They also thank P. Burris for her assistance in the preparation of the manuscript. (G.D.) is grateful to the FEL group of the Quantum Institute, University of California at Santa Barbara for partial financial support. (J.G.) has been supported by the Office of Naval Research, ONR Contract N00014-80-C-0308.

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