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ROCK STRENGTH UNDER CONFINED SHOCK CONDITIONS

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This report addresses the laboratory measurements of the static strength of rock needed to simulate the response of rock to an underground explosion. The approach is to identify the variables that affect the strength of rock and to discuss each effect in terms of the underlying processes that cause it. Most of the report is the result of a literature review, although some new analyses and concepts are presented.

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I. INTRODUCTION

1.1 SCOPE OF THE REPORT

The problem to be addressed may be stated as follows: given laboratory measurements of the static strength of rock, what is the appropriate value to be used in simulating the response of rock to confined shock loading, as in an underground explosion?

This is a complex problem which cannot be answered unequivocally at the present time. It is, rather, the scope of this report to attempt to properly state the problem by separating it into its relevant parts, discussing each in appropriate detail, and pointing out the unresolved problem areas.

1.2 MODE OF PRESENTATION

The approach that will be used in this report is to separate out various variables that affect the strength of rock and to discuss each effect in terms of the underlying processes that cause it. It is only by understanding these processes that a coherent and predictive understanding of rock failure can be achieved. Since it is intended that this report be readable by nonspecialists as well as specialists in the field of rock mechanics, the discussion begins at a fairly elementary level. Most of the report is the result of a literature review, although some new analyses and concepts are pesented.

Attention is directed at three basic rock types: low porosity brittle rock such as granodiorite, high porosity brittle rock such as volcanic tuff, and a rock that may be ductile under the relevant conditions, salt. These three rock types are sufficiently different that somewhat different constitutive laws may have to be used to model their behavior. We consequently discuss them separately.

Following these discussions of the basic physics of the problem, we turn to some actual observations of the response of rock to underground nuclear explosions in an attempt to reconcile the expected and observed behavior.

II. LOW POROSITY BRITTLE ROCK

2.1 NATURE OF THE PORE STRUCTURE

These rocks differ from high porosity rock not only in the degree but also in the nature of their porosity. The porosity of those rocks, which includes crystalline igneous and metamorphic rocks and well consolidated sedimentary rocks, is usually less than one percent and is almost entirely in the form of high aspect ratio cavities, i.e., cracks (Brace, 1965; Tapponier and Brace, 1976; Kranz, 1979). Therefore, the porosity of these rocks is typically readily closed, reversibly, by confining pressure (Walsh and Brace, 1966; Brace, 1965). Furthermore, studies of the effect of pressure on rock properties such as permeability (Brace, et al., 1968) and elastic wave velocity (Hadley, 1975; Scholz, 1978) have shown that the crack porosity in such rocks constitutes an almost completely connective network. Therefore in this type of rock all cracks will be in communication with any pore fluid present. This latter feature has important consequences on the mechanical behavior of the rock.

2.2 THE BASIC FRACTURE PROCESS

The microscopic phenomena that lead to the macroscopic brittle fracture of rock in compression can be understood through examination of the stress-strain behavior, as shown schematically in Figure 2.1 (after Brace, et al., 1966). It is particularly instructive to study the plot of volumetric strain versus stress. The curve is divided into four parts. In Stage I, the volume of the rock decreases faster than would be expected from elasticity and is nonlinear: cracks are closing under the influence of the applied stress. In Stage II, the rock behaves nearly linearly elastically. In Stage III, the rock exhibits dilatancy, i.e., it increases in

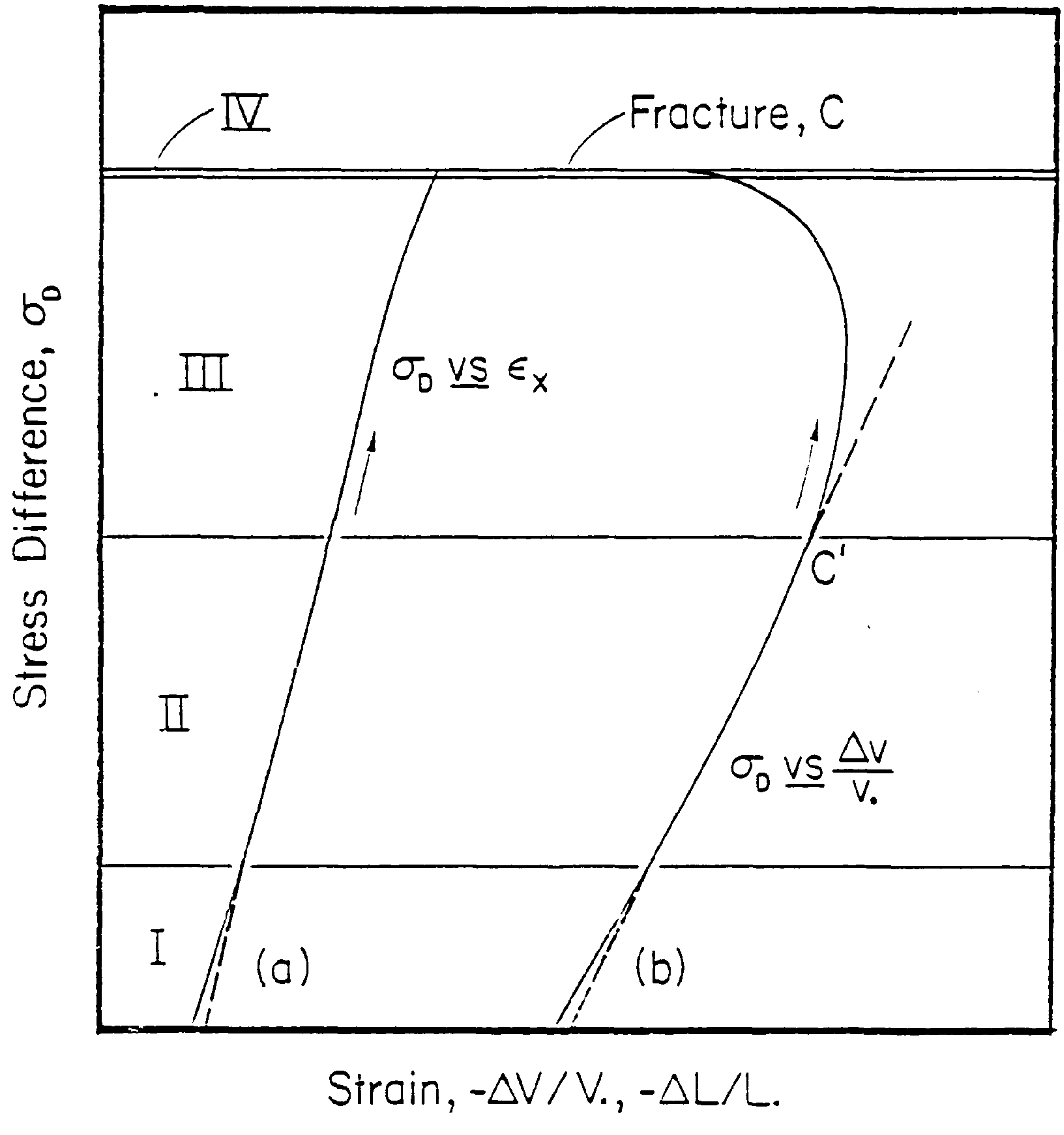

Figure 2.1 Idealized stress-strain curves. (a) Stress versus linear (axial) strain; (b) stress versus volumetric strain. Behavior in each of the four regions, I, II, III, and IV, is discussed in the text. (Brace, et al., 1966)

volume greater than would be expected from elasticity. The stress at which dilatancy begins is called C', or the dilation stress. In Stage IV, there is a rapid acceleration of dilatancy, which leads to ultimate failure of the rock.

Dilatancy is accompanied by acoustic emissions that occur at a rate proportional to the rate of dilatant strain, indicating that the dilatancy is caused by the dynamic propagation of microcracks within the rock (Scholz, 1968a). During stage IV the microcracks coalesce to form a macroscopic fault which leads to failure of the specimen (Scholz, 1968b; Mogi, 1969). The mechanism of crack coalescence during this stage is poorly understood, but what is known is that it leads to a loss of strength of the rock which results in a dynamic instability (for a discussion of the post-yield behavior of rock see Wawersik and Brace, 1971; and for a discussion of this type of instability see Rice, 1979).

In early literature on this subject, it was thought that a modified version of the Griffith theory (McClintock and Walsh, 1962), for a general state of stress and which took into account the friction acting across crack walls, could be used to predict the fracture strength of brittle rock (Brace, 1960). However, this theory is a weakest link theory, i.e., it assumes that when the single "most critical" crack exceeds the Griffith energy balance, the crack propagates through the specimen, causing failure. The observation, however, is that a great many cracks propagate within the specimen prior to macroscopic failure.

The principal explanation of the failure of the theory is that it assumes that the rock is a homogeneous elastic solid, whereas the rock is highly heterogeneous. As a result the stress which is locally applied to a crack may be significantly different from the stress applied to the sample. Although the crack will propagate because it locally exceeds the Griffith energy balance, it does not propagate through the sample because it does not globally meet the energy balance (Scholz, 1968a). As a consequence, macroscopic failure does not occur until pervasive fracturing results in a loss of strength such that the criterion for dynamic instability within the rock-loading machine system occurs (the rock loses strength faster than the loading system can unload).

As a result of the above mentioned developments, the modified Griffith theory was abandoned and has been neglected in the more recent literature. However, for reasons that will become clear later in the text, its qualitative successes should not be forgotten. It predicts, for example, that the compressive strength of rock is about ten times the tensile strength, and that the strength of rock increases nearly linearly in compression with confining pressure, both of which are borne out by the observations.

These successes led Brace and Byerlee (1966) and Brace and Riley (1972) to suggest that, while the modified Griffith theory does not predict the failure strength, it may predict, say, the dilation stress. While this is probably not correct either, it is possible to understand the qualitative success of the theory. Consider two cases, one of a rock loaded triaxially under a certain confining pressure and another loaded similarly but subject to a much higher confining pressure. In the second case, because of heterogeneity, we can expect that the normal stress across each crack in the rock to be not equal to the confining pressure, but to, on average, be proportionally greater than in the first case. Hence each crack in the second case can be expected to propagate at a proportionally higher deviatoric stress than in the first case. Therefore, the microstructural state at which failure occurs in the second case is achieved at a proportionally higher deviatoric stress than in the first state. Thus, the gross process can be understood in terms of the theory, although no quantitative link can be achieved since we cannot quantitatively describe the state of stress within the rock.

It should be emphasized, as was pointed out by Nur (1975), that the dilatancy of the large scale rock mass may not be the same as the microcrack dilatancy, described above, of laboratory specimens. The large scale rock mass will certainly dilate before failure, but the relationship between stress and dilatant strain may not be the same as in laboratory samples. Nur discussed various simple possibilities.

2.3 THE EFFECT OF SPECIMEN SIZE ON ROCK STRENGTH

When scaling laboratory strength data to large scale applications, it is very important to take into account the dependence of rock strength on specimen size. Although early work on this subject produced conflicting results, the thorough study of Pratt, et al., (1972) on granodiorite appears reliable, is consistent with the independent work of Brown and Swanson (1970), and will be taken as essentially correct.

Their results are shown in Figure 2.2. They found that rock strength decreased by about one order of magnitude as specimen size was increased about two orders of magnitude. For blocks above 1 m in size, a plateau was observed in which strength was found to no longer diminish with size.

The unusual explanation for such a size effect on strength of brittle materials is that of Weibull (1939). In its essence Weibull's analysis holds that it is simply a consequence of containing larger cracks in the larger samples. The Weibull theory, like the Griffith theory, is a weakest link theory, and Pratt, et al. (1972) argue that it is not applicable to their results because

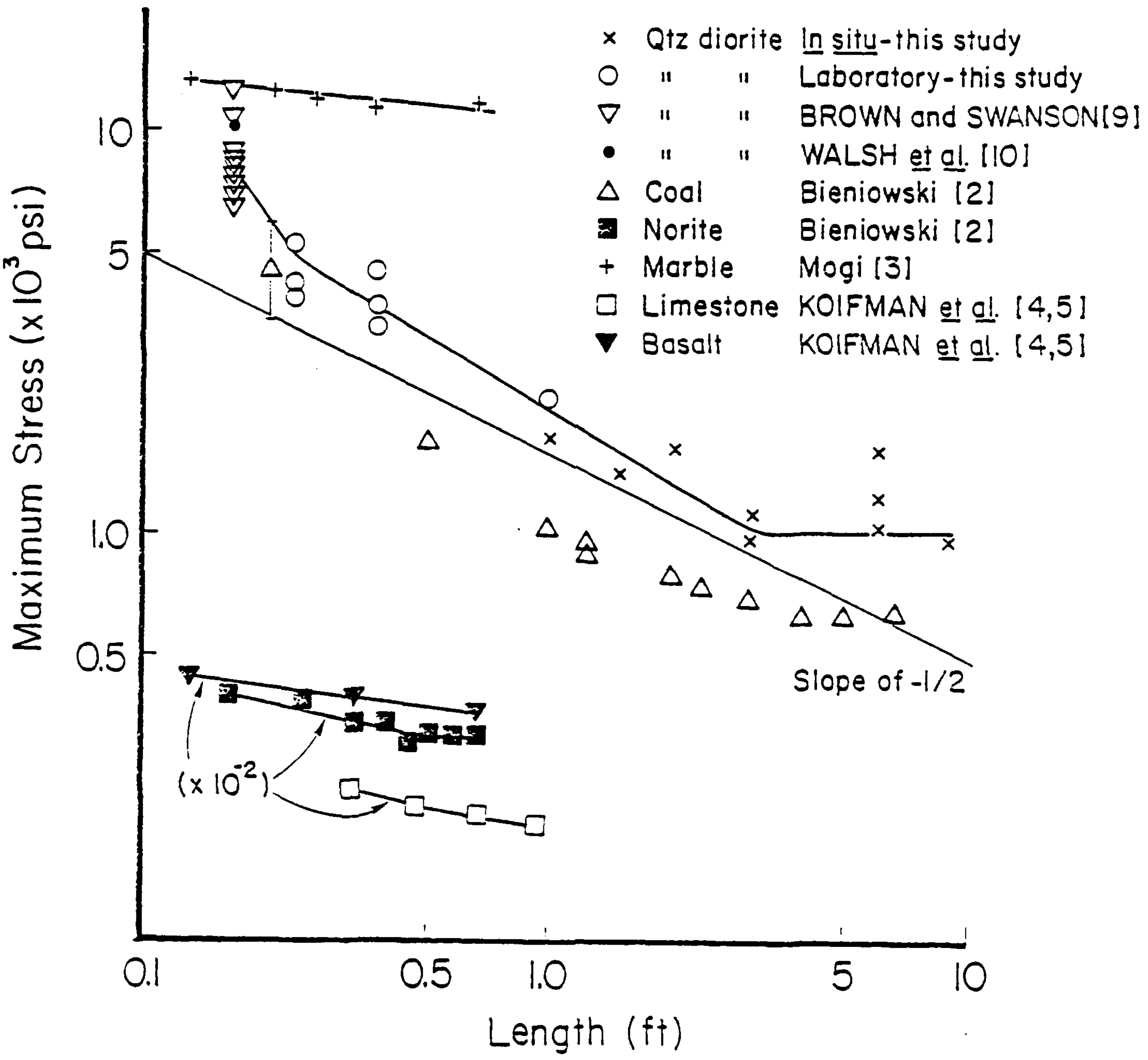

Figure 2.2 Maximum stress versus specimen size. (Pratt, et al., 1972).

rock strength does not show the scatter implicit in the theory. Their objection is justified; virtually an identical argument as that made above concerning the inapplicability of the Griffith theory can be used to show why the Weibull theory should not be strictly applicable as well. After all, the Weibull theory is simply a corollary of the Griffith theory. The reason little scatter is observed in rock strength data is because, as we discussed above, many cracks propagate before failure occurs, so the effect of individual cracks is averaged out.

Nonetheless, the basic assumption underlying it, that the size of the largest cracks increases with sample size, is probably still valid. By analogy with our use of the Griffith theory to understand qualitatively the strength behavior of the rock, we can use the Weibull model to understand qualitatively the size effect.

Suppose we assume that strength depends primarily on the largest flaws, and that the mean length x of the set of largest flaws is proportional to sample size D,

$$x = \alpha D \tag{2.1}$$

The mean stress intensity factor at the tip of this set of cracks is,

$$K = \sigma\sqrt{x} \tag{2.2}$$

where σ is the mean stress, which from equilibrium is equal to the applied stress. A statement equivalent to the Griffith energy balance is that a crack will propagate instably when the stress intensity factor reaches a critical value, K_c, the fracture toughness of the material (see, e.g., Lawn and Wilshaw, 1975). Therefore, the mean critical state for the set of largest cracks is, combining (2.1) and (2.2),

$$\frac{K_c}{(\alpha D)^{1/2}} = \sigma_f \qquad (2.3)$$

which we identify qualitatively with the strength, σ_f.

Equation 2.3 indicates that strength should be proportional to $D^{-1/2}$. A line of slope -1/2 has been drawn in Figure 2.2, and fits the data as well as could be expected. Notice that we are assuming that the strength is determined by the net effect of a set of cracks, so that we do not expect to see wide scatter in the measurements. This model emphasizes the fact that K_c is a fundamental material property, while the macroscopic strength is not.

How, then, do we explain the plateau in strength for large sample sizes since our model would not predict it This probably is because Pratt, et al. were studying the strength of unjointed rock, so that for the largest samples they were specifically searching for sites in which there were no macroscopic fractures. This selection procedure probably had the effect of limiting the size of the largest flaws in the largest specimens.

There is, however, a minimum strength for the rock mass. This is the limit when the rock is completely broken into blocks. In that case the strength is determined by the frictional strength, which is,

$$\tau = 0.85\ \sigma_n \qquad (2.4)$$

where τ is the shear stress and σ_n the normal stress applied to the frictional surface. Equation 2.4 is applicable at low normal stresses (< 1-2 Kb). At higher normal stress the applicable law is,

$$\tau = 0.5 + 0.6\ \sigma_n \qquad (2.5)$$

expressed in kilobars (Byerlee, 1979). For reasons discussed by Scholz (1977) Equations 2.4 and 2.5 are independent of lithology, temperature (< 500°C) and scale. There is a weak negative time or rate dependence, in which the frictional strength decreases a few percent per decade of increase in sliding velocity (see e.g., Scholz and Engelder, 1976; Dieterich, 1978). At low normal stress there is a scale effect (Bandis, et al., 1981) which is probably a masked roughness effect (Byerlee, 1967).

In Figure 2.3 we compare the strength of intact rock with the frictional strength. If we consider the frictional strength to be the lower limit of the size effect, it can be seen from the figure that it is much lower than the strength of an intact sample at low normal stress but at high normal stress (> 1-2 Kb) the frictional strength becomes comparable to the intact strength. Therefore it appears that the size effect cannot be significant at high confining pressures. This is probably because Equation 2.3 is appropriate for the results of Pratt, et al., which were uniaxial compression tests, but is not valid at high confining pressures where large frictional tractions act across the crack faces. Equation 2.3 is a fracture criterion that considers only the stresses at the crack tip. It can be tied to the Griffith energy balance through a path independent integral around the crack tip (Rice, 1968) but to calculate the complete energy balance that includes frictional work on the crack faces requires a global integration around the entire crack, which has not proven to be mathematically tractable (Kostrov, 1974).

One can quantitatively guess the result, however. The crack tip stresses, and hence the work done by them, depend critically on

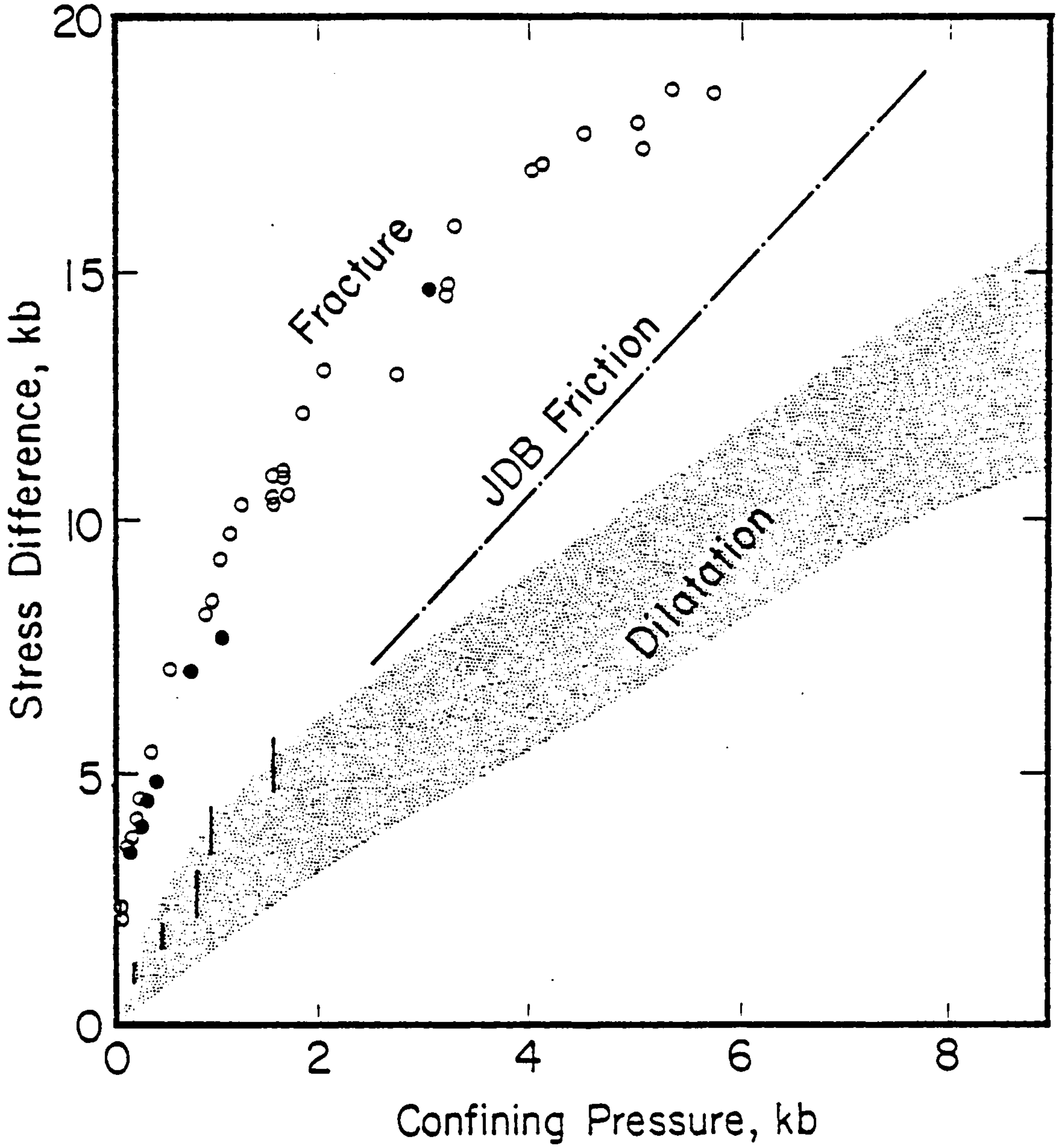

Figure 2.3 Pressure dependence of fracture, sliding and dilatation stress (C') of Westerly granite. Filled circles are from Hadley, (1973); open circles from Brace et al., (1966), Byerlee (1967), and Mogi (1966). Vertical bars are stress for sliding from Hadley, (1973); dashed-dot line is from Byerlee (1967). (Hadley, 1973)

the length of the crack, whereas the frictional work on the crack faces does not (expect insofar as the area of the crack increases). Hence, at high confining pressure the frictional term in the energy balance becomes large with respect to the crack tip term, and the length of the crack becomes much less critical in the energy balance. As a result, the size effect becomes small.

In summary, for rock mechanics applications involving low normal stresses, the size effect on strength is important and must be taken into account. For applications at high confining pressures it can be neglected.

2.4 DYNAMIC VERSUS STATIC STRENGTH

In attempting to predict rock strength under shock conditions from static strength data, it is clearly important to consider the effect of strain rate on strength. There are three different mechanisms that lead to a strain rate dependence of strength. The most well studied of these is subcritical crack growth due to stress corrosion (e.g., Scholz, 1972; Atkinson, 1979). It is a small effect and is only important in very low strain rate applications. The other two effects are strong and only occur under dynamic loading conditions. We will therefore restrict our discussion to the latter.

In the early literature on the dynamic strength of rock it appears that there was an implicit assumption that the rock failed in compression under conditions of uniaxial strain, i.e., the conditions imposed by a plane compressive shock wave. Quasistatic uniaxial strain experiments showed an equation of state that agreed very well with that deduced from shock wave studies, but the so-called "Hugoniot elastic limit" was not found to correspond to the locus of dilation stresses (Brace and Jones, 1971; Brace and Riley, 1972; Schock, et al., 1973). Thus, the inertial confining pressure that results in uniaxial strain conditions in a plane shock wave is just sufficient to prevent any dilatancy from occurring. Since other work (e.g., Kumar, 1968) has shown that the processes that lead up to rock failure, as outlined in Section 2.2 above, are essentially identical under dynamic as under static conditions, it seems evident that it is not possible for a rock of this type to fail under uniaxial strain conditions. We are left then with two possibilities: either the rock fails during the release of the shock wave or under the actual conditions the shock wave is sufficiently nonplanar that the conditions of uniaxial strain are not imposed. The latter will be particularly important at small radii, the rock may fail in extension, rather than in compression.

In Figure 2.4 we show a typical example of the effect of strain rate on the compressive strength of rock (see Kumar, 1968; Green and Perkins, 1969; Perkins, et al., 1970). At strain rates below about 10^2 sec^{-1} (for this rock) the effect of strain rate is rather small and is due to stress-corrosion cracking as alluded to above. At higher strain rates, however, a very steep rise in strength is observed. This rapid strengthening at high strain rate has been interpreted as being due to a transition to a new more highly rate-dependent mechanism of crack growth (Perkins, et al., 1970), or to a change in the stress state from uniaxial stress to uniaxial strain with increasing strain rate (Green and Perkins, 1969). Brace and Jones (1971) favored the latter. The argument is that at sufficiently high strain rates where uniaxial strain conditions are approached, an inertial confining pressure is imposed on the rock that leads to an increase in strength. Later work has shown, however, that both mechanisms operate. Blanton (1981) recently suggested that the observed effect was an artifact of incorrectly accounting for inertial effects in the loading apparatus. Whereas that is a problem with the type of apparatus Blanton used, it is not a valid criticism of the results we are discussing, which were obtained using the Hopkinson bar technique.

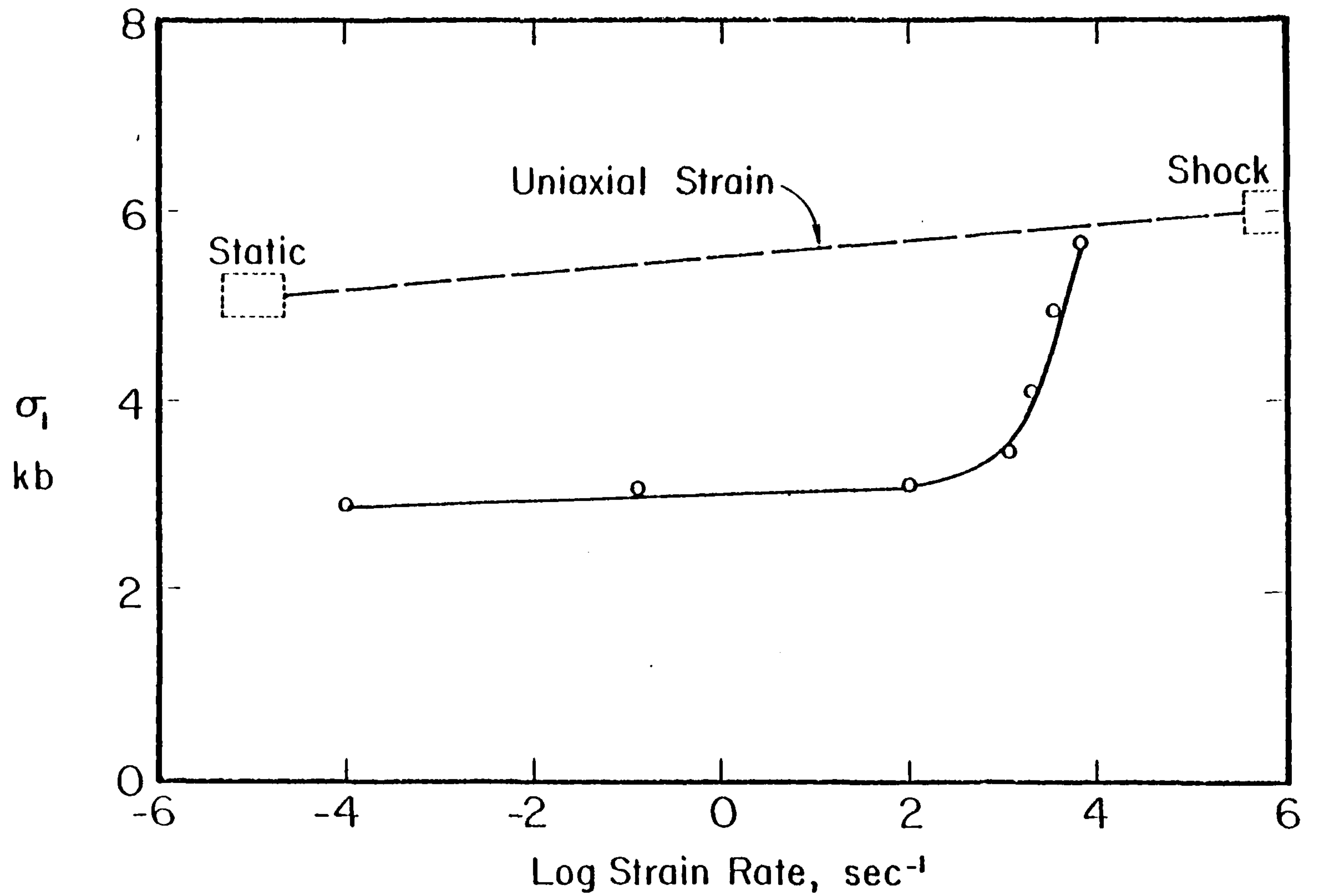

Figure 2.4 Maximum stress σ_1 as a function of strain rate for Solenhofen limestone (Green and Perkins, 1969) is given by the circles and continuous curve. For comparison, static and shock values of σ_1 are given assuming an axial strain ϵ_1 of about 0.7 percent. (Brace and Jones, 1971.)

Blanton's experiments were not conducted at a high enough strain rate to observe the behavior we are discussing.

The work Lundberg (1976) and Janach (1976) confirmed the mechanism implicit in the discussion of Brace and Jones, which was described above. They both showed that dilatancy occurs prior to failure of rock in dynamic compression. Janach called it "bulking". Janach essentially solved the problem posed earlier: how rock fails in compression under shock conditions. The mechanism he proposed is shown in Figure 2.5, which shows the split Hopkinson bar apparatus used in his experiments. Immediately after the loading shock front the rock is under a state of uniaxial strain, and immediately behind it an unloading wave propagates from the free surface into the sample at a much slower velocity than that of the shock wave. Just inside this "bulking wave" the material is subject to radial inertial stress imposed by the material outside it, which is literally exploding outwards (a process that may be considered to be "dynamic dilatancy"). The radial stress in the interior of the sample will not drop to zero until the failure process is fully terminated. Thus Janach reached the conclusion discussed above: failure in shock compression does not occur during loading, but during unloading, and the rapid increase of strength at high strain rates is caused by a transition from uniaxial stress to uniaxial strain. Unfortunately his results also point out the importance of the free surface in a Hopkinson bar test and that leads to another kind of size effect. In dynamic tests, the "strength" of the material is actually the stress level it can sustain for some relevant amount of time. The implication of Janach's work is that this depends on the distance to the nearest free surface from which an unloading wave can propagate. This means that "strength" is not a fundamental material property but depends on the size and geometry of the specimen. In this case strength would be expected to increase with sample size. In the case of an underground nuclear explosion, the only relevant surface from which an unloading wave may propagate

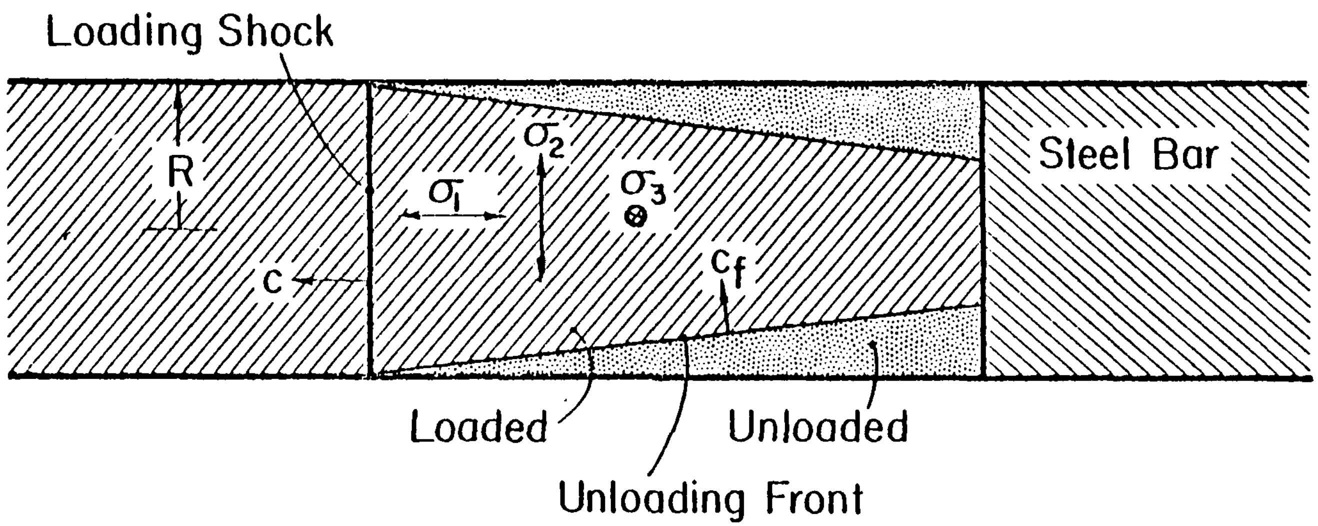

Figure 2.5 Unloading process in cylindrical specimen. (Janach, 1976)

appears to be that of the cavity. The geometry of this problem is sufficiently different from that of a Hopkinson bar experiment that it is not clear how relevant the experimental strength data are to the problem.

In conventional blasting applications it is usually thought that transient waves or the interaction of transient waves with local free surfaces carry regions of the rock into tension, and that rock fails by tensile failure. Many studies, beginning with the work of Reinhart (1965) have shown that the dynamic tensile strength of rock may be as much as ten times the static value. Goldsmith, et al., (1976), for example, studied both the dynamic tensile and compressive strength of a granite, using a Hopkinson bar arrangement. They found that at a strain rate of $10^3\ s^{-1}$ that both tensile and compressive strength were about twice their static values. The latter result agrees with that of Lundberg (1976) and Janach (1976), also for granite.

Shockey, et al., (1974) and Grady and Kipp (1979) studied the tensile strength of novaculite (a very fine grained quartzite) at higher strain rates, using the impact method. They found that although cracking initiated at tensile stresses of about 50 MPa, the rock had a tensile strength of 70-100 MPa at strain rates of about $10^4\ s^{-1}$. Grady and Kipp found that at these rates the strength increased with the cube root of the strain rate. They showed that this occurred because the stress intensity factor at a crack tip cannot instantly attain its static value because of inertia of material near the crack tip (Achenbach and Brock, 1973; Freund, 1973). This then is the second strong strain rate effect that occurs in dynamic loading: the one that had been hinted at by Perkins, et al., (1970). This effect, of course, will also occur during compressive failure too, so it must be included with the effect discussed by Janach.

There is yet another effect that has not been discussed by those authors. It results from the fact that the rupture velocity of a crack is limited by an elastic wave speed of the medium. This will be either the P, Rayleigh, or S wave velocity, depending on the mode of propagation (see e.g., Lawn and Wilshaw, 1975). This will limit the distance a crack can grow within a given time interval, and will result in enhanced strength at very high strain rates.

In chemical explosions a gas pressure is produced in the cavity, which, while being of considerably smaller amplitude than the detonation pulse, is of longer duration. The gases vent into cracks, enhancing the fragmentation (see Durney, et al., 1975). If a similar phenomenon occurs during a nuclear explosion it may lead to enhanced seismic coupling over a limited range of frequencies.

2.5 PORE PRESSURE EFFECTS

When the rock contains a fluid within its pore structure at an internal pore pressure, p, it is known that the properties of the rock do not depend on the externally applied stress, but on the effective stresses, defined as:

$$\sigma_{ij} = \sigma_{ij} - \lambda\delta_{ij}p \qquad (2.6)$$

where σ_{ij} are the applied stresses and λ is a coefficient less than 1 (Garg and Nur, 1973; Nur and Byerlee, 1972). For both fracture strength and frictional strength $\lambda = 1$ so Equations 2.4 and 2.5 should be written more completely as:

$$\tau = 0.85\ (\sigma_n - p) \qquad (2.7)$$

and,

$$\tau = 0.5 + 0.6\ (\sigma_n - p) \qquad (2.8)$$

and these properties are hence said to obey a "law of effective stress."

In general, there will be an interaction between the applied stresses and the pore pressure. In Figure 2.6 are illustrated two extreme cases in which stress (in this case hydrostatic pressure) is applied to a rock containing a pore fluid. In Case A it is assumed that the confining pressure is applied very slowly as compared to the hydraulic time constant of the sample. In this case the fluid is squeezed out of the rock as the cracks close, as symbolized by the standpipe, and the internal pore pressure is not changed. In Case B, the pressure is applied very rapidly with respect to the hydraulic time constant, there is not time for the fluid to flow and the pore pressure increases to a value $p < P_c$, where P_c is the applied confining pressure. In soil mechanics these are referred to as drained and undrained conditions, respectively. As a result of this the rock takes on a viscoelastic rheology similar to a standard linear solid, in which Case A is the relaxed state and Case B is the unrelaxed state. The elastic moduli will be higher in Case B than Case A.

For the present application we can safely say that the conditions will, at all relevant times, be undrained, so that if the rock is saturated, compressive shock loading will induce a pore pressure. We could assume some geometry for the cracks and calculate the pore pressure induced, using the analysis of Eshelby (1957). However, for simplicity we will assume the cracks are very long and thin: in this case the pore pressure induced will be very nearly equal the applied pressure. Hence the effective confining pressure during shock loading will be zero.

If the rock is partially saturated, slightly different behavior will occur. We show in Figure 2.7 a volumetric compression curve for a typical low porosity rock (see Brace, 1965). At low pressure, cracks are closing which produces the pronounced "toe" in the curve. At a sufficiently high pressure, P_1 (usually 1-2 Kb)

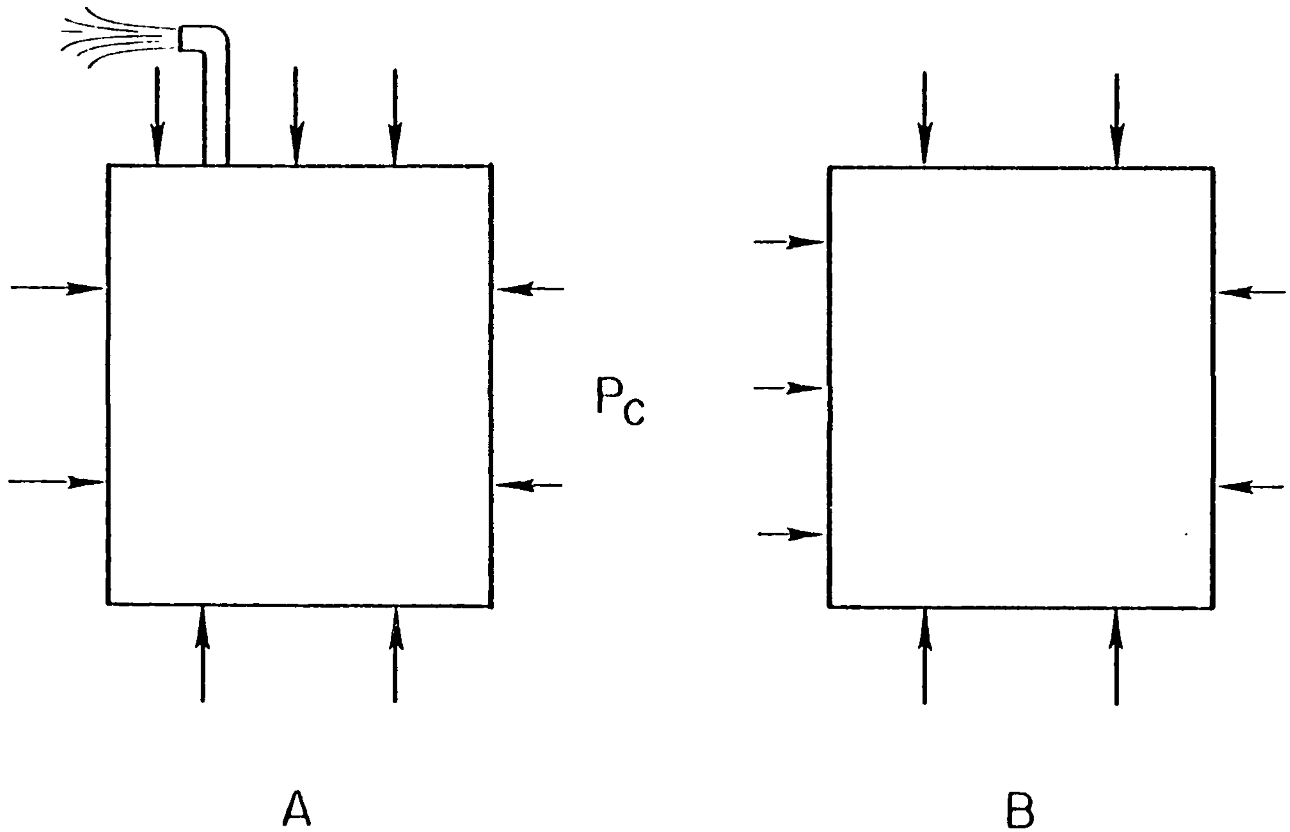

Figure 2.6 Schematics of drained (A) and undrained (B) test conditions.

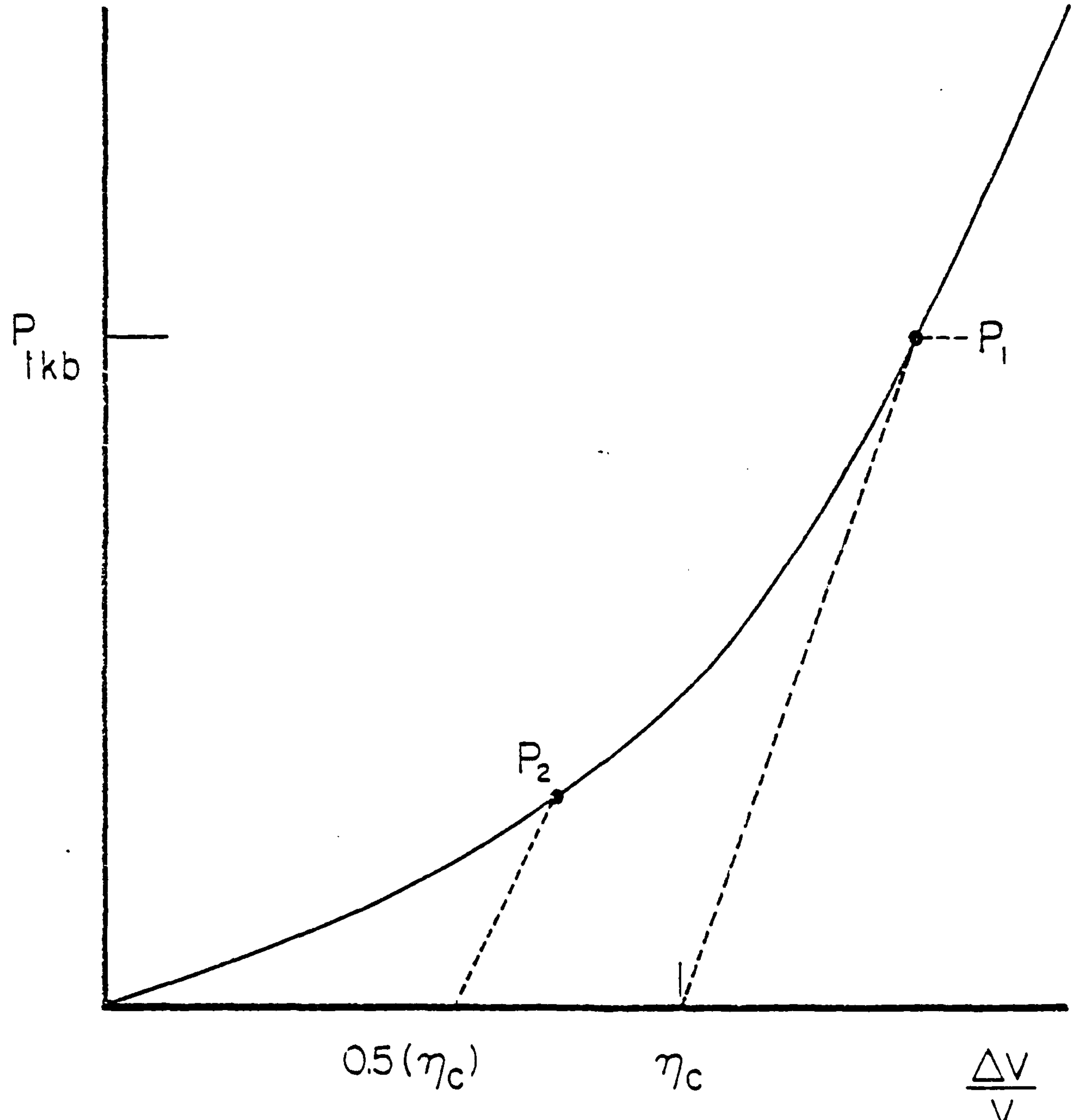

Figure 2.7 Volumetric compression for a typical low porosity rock.

all cracks are closed and the behavior becomes linearly elastic. An extrapolation of the elastic part of the curve intersects the volumetric strain axis at n_c, the crack porosity. Now suppose the saturation is 50 percent. Then upon loading a pore pressure will not be induced until the pressure p_2 is reached (obtained by extrapolating from the 50 percent n_c point on the volume strain axis along the elastic slope). Above p_2 pore pressure will be induced and to a first approximation will be $p_c - p_2$.

From these considerations one might at first conclude that under shock loading the pore pressure will equal the confining pressure for a fully saturated rock, hence the correct value of strength to be used is the uniaxial compressive strength. It turns out, however, not to be that simple. If dilatancy occurs there will be an increase in pore volume which will lead to an increase in the effective stress and a consequent increase in strength, a phenomenon known as dilatancy hardening (Frank, 1966). This is illustrated, for low porosity rocks, in Figure 2.8 (from Brace and Martin, 1968). The solid line in the figure shows the strength of dry or saturated unconfined rock as a function of strain rate. The dashed lines indicate the strength of a rock in which the pore pressure initially equals confining pressure and the pore pressure in the rock is connected to a large reservoir by means of a tube. At low strain rates the strengths correspond: dilatancy occurs slow enough such that the conditions are completely drained. There is sufficient time for fluid to flow into the rock from the reservoir to maintain the pore pressure at its initial value. A critical strain rate is observed, the value of which depends on the hydraulic diffusivity of the rock, above which the pore pressure begins to fall and the strength correspondingly rises. At higher rates there is a second break in the dashed line, above which pore pressure falls to zero, dilatancy hardening is complete, and the rock takes on a strength appropriate to the confining pressure. Above this point, the condition of the sample is completely undrained. Under shock conditions, we can expect that this will always be the case.

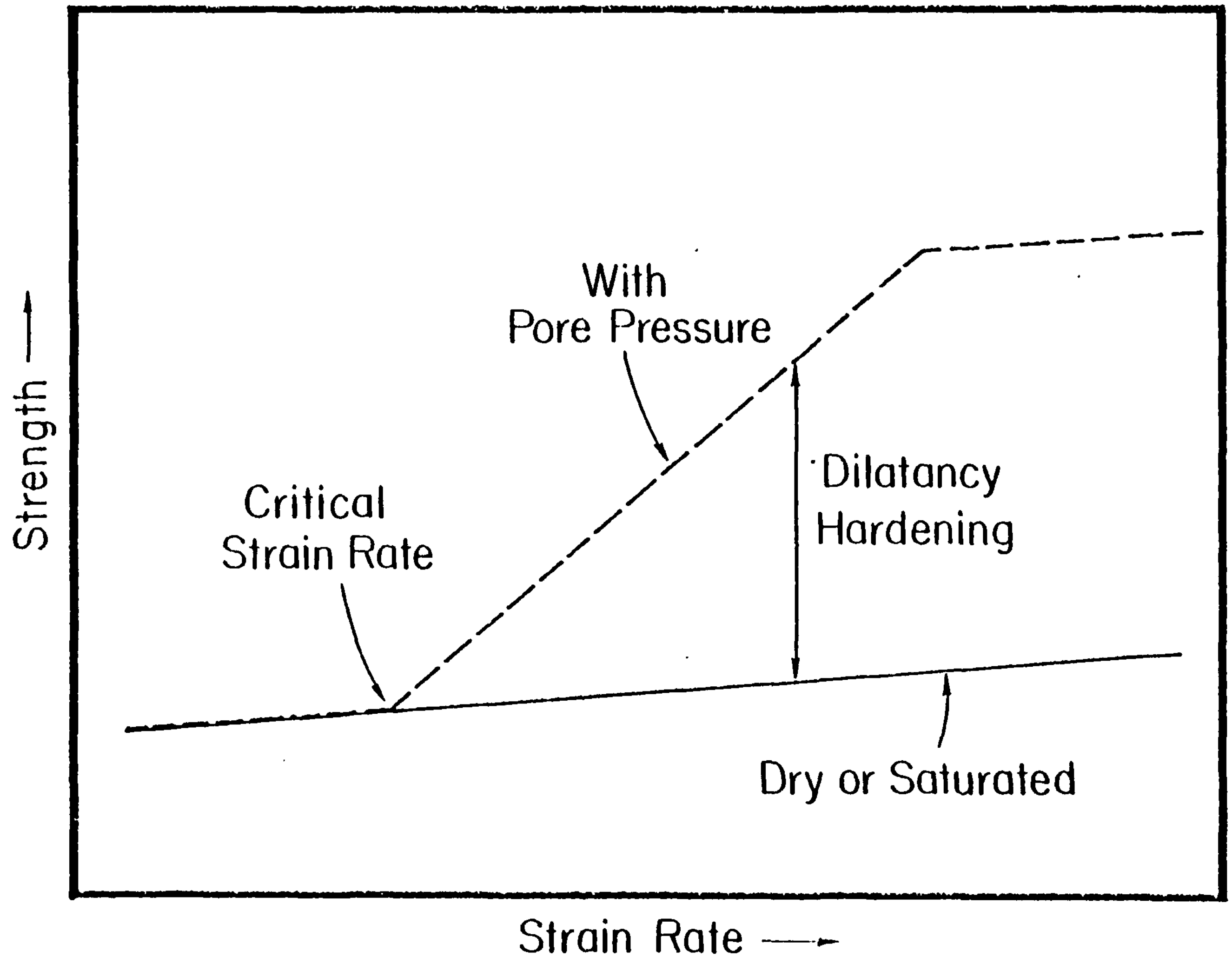

Figure 2.8 Generalized dependence of strength on strain rate for a given pore fluid. (Brace and Martin, 1968)

In conclusion, while the shock front can be expected to produce high pore pressures on loading, dilatancy hardening would completely nullify this effect. If dilatancy occurs, then the pore pressure effects would not significantly influence rock strength.

III. HIGH POROSITY BRITTLE ROCK

These rocks, which include poorly consolidated sedimentary rocks and vesicular igneous rocks, principally differ from the rocks discussed in Section II in their pore structure. This difference leads to very strong differences in the strength behavior.

3.1 PORE STRUCTURE

In contrast to the low prosity rocks, most of the void volume in these rocks is in the form of nearly equant void spaces which we will call pores to contrast them with cracks. Under the action of compressive stress, their behavior is unlike that of cracks, which close reversibly; i.e., pores may crush, producing permanent strains and a net decrease in volume.

The other important difference occurs in the vesicular rocks, such as volcanic tuff. In these rocks, unlike the low porosity rocks, the pore space is not generally connective. The clearest demonstration of this is to note that pumice will float indefinitely: it does not eventually become waterlogged and sink. As a result, such rocks may retain a considerable amount of air-filled porosity even when totally saturated, the amount depending not only on the degree of vesicularity, but also on how much the rock has been fractured (since cracks in general increase the connectivity of pores).

3.2 DIFFERENCE FROM LOW POROSITY ROCKS

There are two important differences that arise because of the difference in pore structure. The first, is unlike the low porosity rocks, these rocks will yield under uniaxial strain conditions by the mechanism of pore crushing. In Figure 3.1 are shown the stress strain behavior of a variety of rocks under uniaxial strain (after Brace and Riley, 1972). On the left of the figure are the results for some low porosity rocks, such as granites. They behave nearly linearly elastic, with little or no irreversible strain. Contrast this with the behavior of the rhyolite tuff, a rock of about 40 percent porosity. That rock yields under these conditions, and large permanent strains occur. Volume compaction will occur in these rocks, the amount proportional to their porosity (Figure 3.2). Thus, even if significant cracking occurs, the dilatancy it causes will be small compared to the volume reduction due to pore collapse, so that the volumetric constraint discussed in reference to low porosity rocks will not be valid for these rocks. The effect this has on the dynamic behavior of these rocks has been discussed in detail by Crowley (1973). They basically exhibit very low strength and highly attenuate the stress wave.

The results of Crowley (1973) show that the behavior of these rocks is highly dependent on the amount of air-filled porosity. This is because water filled pores will tend to crush at much higher pressures than air filled pores, since the compressibility of water is only about three times that of rock while the compressibility of air is very large compared to that of rock. Because of the noncon-nectivity of pores in vesicular rock such as tuff, the percentage of void space that is air filled is likely to be high in such rock, even when it is beneath the water table. Hence such rock should exhibit low strength under most conditions.

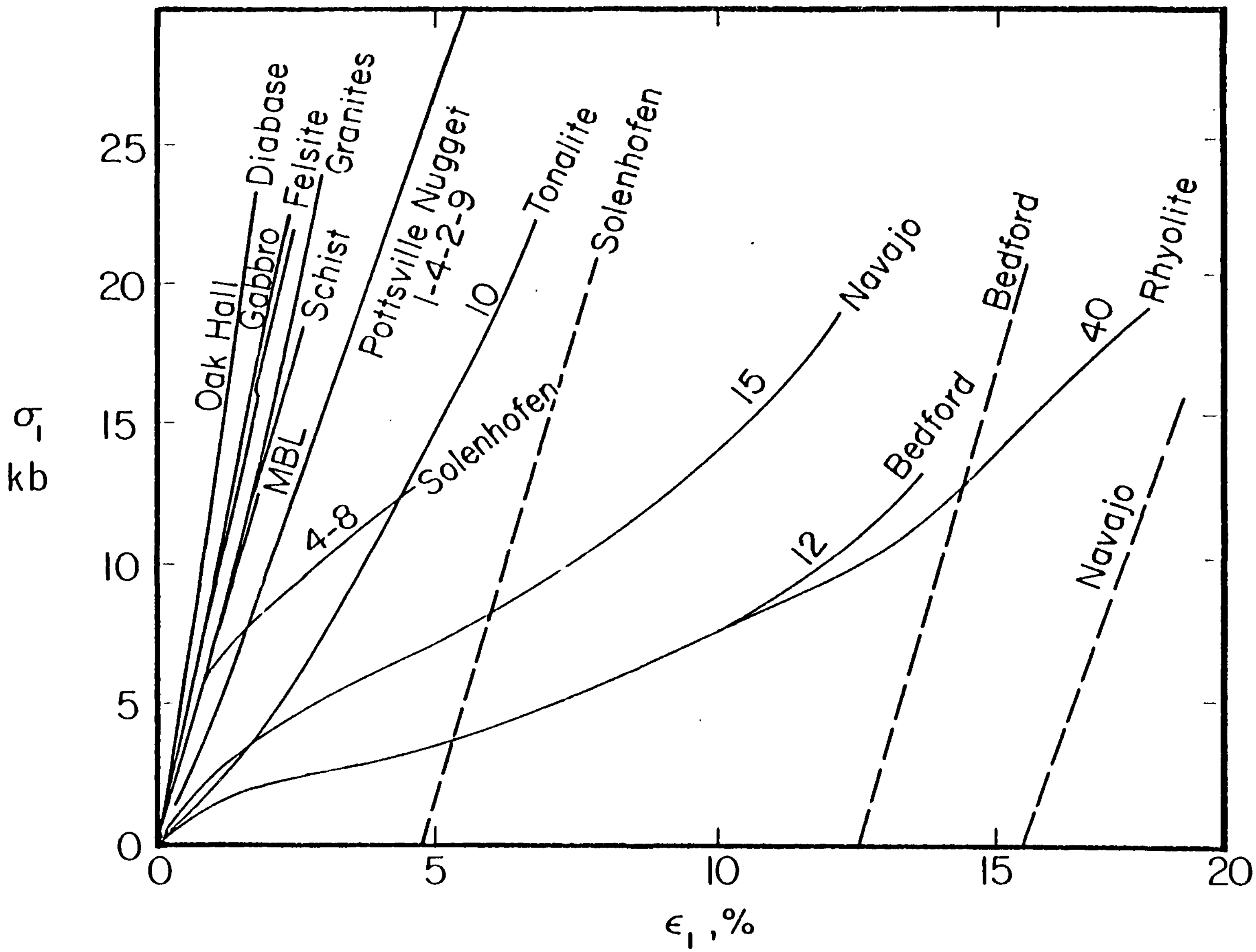

Figure 3.1 Stress in the axial direction as a function of axial strain. The small number on some of the curves is porosity in percent. The dotted lines are stress-strain curves which would be followed if porosity were zero. (Brace and Riley, 1972)

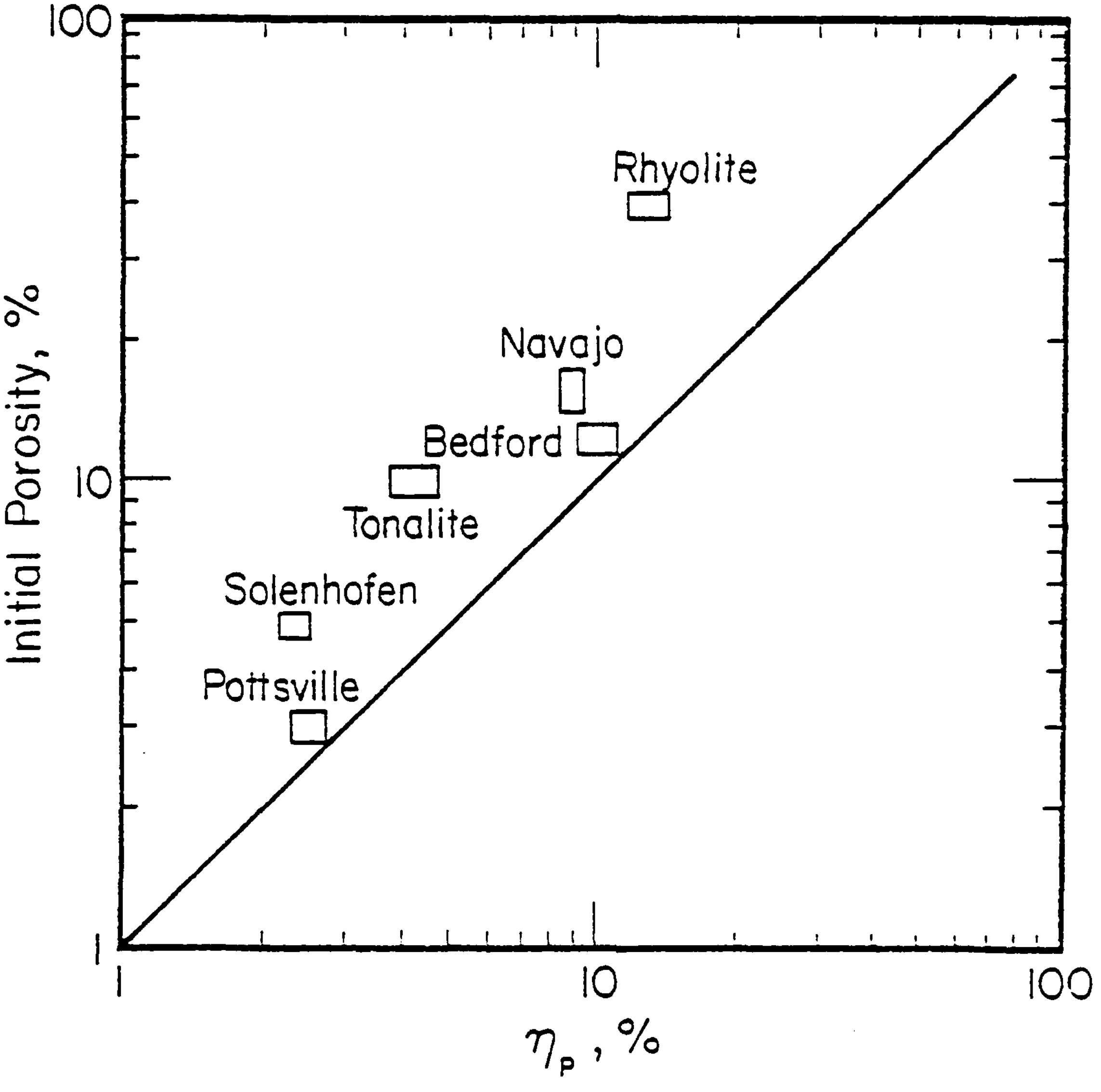

Figure 3.2 Comparison of permanent volumetric compaction with initial high-porosity rock. (Brace and Riley, 1972)

IV. DUCTILE ROCKS

4.1 INTRODUCTION

Salt and marble are rocks that behave cataclastically at standard temperature and pressure. That is to say, pervasive microfracturing occurs within the rock, but the typical features of brittle failure, formation of a macroscopic shear fault and dynamic instability, do not occur. Rather, the rheology of the rock, in general aspect, resembles that of a ductile material.

Cataclastic behavior is typical of materials that are very close to the brittle-ductile transition. Although salt, for example, is brittle under ordinary experience, it is ductile at low strain rates under room pressure and temperature conditions and it is also ductile at slightly elevated temperatures and pressures. This is because cataclastic behavior results from the internal deformation being a mixture of brittle processes involving crack growth and intracrystalline plasticity involving dislocation motion.

4.2 THE BRITTLE TO DUCTILE TRANSITION

In order to demonstrate the brittle to ductile transition for such rocks we consider some results for marble in Figure 4.1 (from Scholz, 1968a, see also Edmund and Paterson, 1972). This rock is somewhat more brittle than salt, which will exhibit lower strength and undergo the transition at lower pressure.

In the upper part of the figure is shown the stress-axial strain curves for this material at various confining pressures. Notice that the stress-strain curves at all pressures appear macroscopically like that of a ductile material, but that the yield point and degree of strain hardening (slope of the post-yield stress-strain

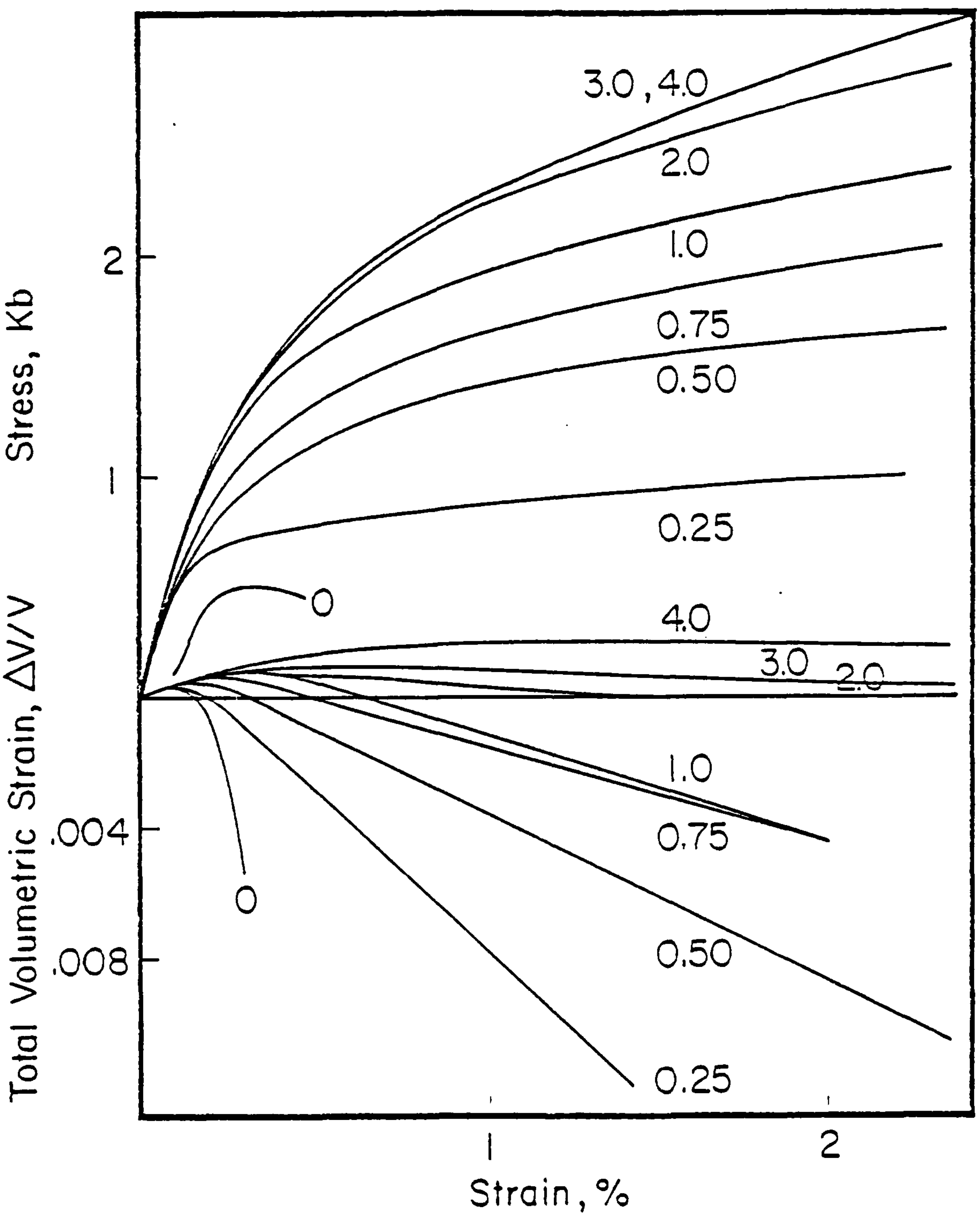

Figure 4.1 Stress versus axial strain and volumetric strain versus axial strain for marble deformed under several confining pressures. (Scholz, 1968a).

curve), both increase with pressure, featues not expected for a purely plastic material. True plasticity is insensitive to pressure, to first and second order.

In order to understand this behavior one needs to study the lower part of the figure, which shows volumetric strain plotted against axial strain. Since plastic deformation produces no volume change, any dilation must be due to cracking. At the lowest pressure, volume strain increases rapidly in the post-yield region, indicating a substantial degree of cracking is occurring within the material. At high pressures, this behavior is suppressed (recall that the volume increase associated with cracking must do work on the confining pressure), until finally, at 3 Kb, no volume change occurs. Thus there is a gradual change in internal deformation from a brittle mode to a plastic mode, during which the macroscopic behavior can be said to be ductile. However, it is only above 3 Kb can the internal deformation be considered to be fully plastic and produced entirely by the motion of dislocations. Thus at 3 Kb and above, deformation occurs without volume change, and there is no further increase of yield point or the strain hardening index.

The discussion above illustrates how the brittle-ductile transition occurs with increasing pressure. Pressure inhibits cracking, since the cracks must do work on the pressure, but has no influence on dislocation motion, since no volume change is involved. Similarly, increased temperature will induce a similar transition, since temperature has little influence on cracks but aids dislocation motion. Strain rate has a more complex effect on the transition, and, since it is important to our problem, needs to be discussed in some detail.

4.3 THE EFFECT OF STRAIN RATE

As discussed in some detail above, rock can sustain higher loads at higher strain rates before undergoing brittle fracture and this effect can be tied to several mechanisms involving crack propagation. Similarly, the strength of plastic materials increases with strain rate because of constraints on dislocation mobility. Unlike crack propagations, dislocations do not (usually) move instably. Therefore, the velocity of dislocation motion depends on the applied stress. Dislocation velocity is usually a power law function of stress, and, like cracks, is limited to an elastic wave speed of the medium (Gilman, 1960). The plastic strain rate is given by:

$$\dot{\varepsilon} = \Gamma v b \tag{4.1}$$

where Γ is the dislocation density, b is the burgers vector, and v is the dislocation velocity. It is clear that increased strain rate will requie increased dislocation density or dislocation velocity, or both, and both of these will increase the strength of the rock. An increase in velocity requires an increase in stress for reasons given above, and an increase in dislocation density will produce an increase in strain hardening because of pileups and tangles, leading to increased strength.

Usually the strain rate effect is stronger for plastic flow than for brittle fracture so that an increase in strain rate will casue a shift in the brittle-ductile transition in P-T space: brittle behavior is favored at high strain rates. This effect was noticed by Schock, et al., (1973) for a sandstone, though to be exact, from their description of the ductile behavior of the rock it was deforming cataclastically and not plastically.

V. COMPARISON WITH OBSERVATIONS

5.1 SUMMARY OF OBSERVATIONS

In this section we will discuss the results of attempts to explain near field ground motion data with numerical simulation of nuclear explosions. Our concern will be about the constitutive bahavior that was assumed in the models as compared with that implied by the data and how the two may be reconciled. What is given below is abstracted from J. T. Cherry (1981). Both here, and later we will discuss the rocks in increasing order of complexity.

5.1.1 Tuff

For this rock the importance of air-filled cavities in reducing the strength by the mechanism of pore collapse was recognized early. A nonlinear constitutive model was developed to describe this behavior and simulation using this model fit the data well.

Considerable scatter was observed for shots in tuff beneath the water table. This was ascribed to variations in air-filled porosity, which it was assumed could exist even under totally saturated conditions.

5.1.2 Salt

The response of salt was well modeled by using the static compressive strength. The data show a small precursor, which was modeled by assuming that the salt was saturated. However, following the precursor the data indicate that the strength greatly increased. This was explained by invoking dilatancy hardening.

5.1.3 Granite

This rock presents the greatest difficulty. Initially the dry compressive strength of the rock was used in simulations, but the fit was unsatisfactory. According to the data, the strength values used were much too high. Then the strength of fractured granite was used, which did not help much. Finally, it was assumed that the rock was partially saturated, and that the pore pressure equilibrated with the confining pressure at a pressure of approximately one kilobar.

This worked well, but the French HOGGAR test in granite exhibited apparently much higher strength than PILEDRIVER. HOGGAR could be modeled using the dry compressive strength. The explanation for the difference was that the HOGGAR granite was dry, whereas the PILEDRIVER granite was partially saturated, so that high pore pressures were induced in the latter from shock compression, weakening it.

5.2 DISCUSSION

5.2.1 Tuff

The assumptions that went into the constitutive model for the tuff that was successful are essentially identical to those used for PILEDRIVER granite except that the rock is very compressible because of collapse of air filed pores. Air-filled pores will generally constitute a high portion of the rock's porosity even under saturated conditions because of the nonconnective nature of the pore structure of vesicular rocks. It seems quite reasonable that variations in air-filled porosity causes the scatter in the data. The grout used in the experiments described by Cherry and Rimer (1982) was also a material of this type. The constitutive law which they used to model the experiments was one in which fluid filled pores were pressurized so that the uniaxial strength could be used at high confining pressures. For this material the dilatancy hardeing mechanism will be unimportant, for reasons discussed in Section 3.2 and failure will occur by pervasive pore failure. Hence, the constitutive law used in fitting the experimental data was quite appropriate in this case.

5.2.2 Salt

For this material, it was assumed that the rock was initially saturated and work hardened during loading. However, if one invokes dilatancy hardening in the case of salt, one would also have to invoke it for the granite, too.

5.2.3 Granite

As discussed in Section II, if the rock fails in compression under conditions of high confining pressure, then neither the size effect or dynamic effects can explain strength less than that given by the "dry" static value, and would, in fact, predict higher values. We also concluded that loading induced pore pressure could not be the explanation either, because dilatancy hardening would negate it.

These remarks are also relevant to the constitutive law employed by Cherry and Rimer (1982) to model PILEDRIVER. Even though the shock loading will increase the pore pressure in the cracks, dilatancy during failure will reduce the pore pressure to zero, since the conditions are undrained, resulting in dilatancy hardening. Hence, although the constitutive law that was assumed fits the data well, some other explanation must exist to explain why the strength was so low.

5.3 ALTERNATIVES

As noted above the apparent low strength of granite deduced by fitting data from several nuclear explosions with a constitutive model cannot be explained by invoking the internal generation of high pressure and hence low effective stress. Other possible effects, explored in this report, also do not offer any reasonable explanation for this problem. The basic conclusion is that if the rock fails in compression, there is no way for the strength to be much lower than would be calculated as the "dry" compressive strength. Even if the rock is completely shattered, the frictional strength should be used, which is not much lower.

One possibility is that the dilatancy hardening mechanism does not operate under shock conditions because of some as yet unkonwn reason. This possibility should be explored. Another is that the basic assumptions concerning the mode of failure under the conditions imposed is incorrect. That is, maybe the rock does not fail in compression, or even more fundamental, maybe a stress fracture criterion is not appropriate for this problem.

It was pointed out in Section 2.4 that an early assumption that the rock failed under uniaxial strain conditions turned out to be incorrect: rock cannot fail under those conditions. In any case, uniaxial strain conditions only occur at large distances from the shotpoint, when the radius of curvature of the shock wave is large and hence the wave can be considered planar.

At small radii from the shotpoint, where to first approximation the shock wave can be considered spherical, the state upon shock loading will not be uniaxial strain. In this case, since the velocities of adjacent particles diverge, extensional strains are set up tangent to the shock front. Because of the very nature of the atomic bond, no real brittle material like rock can sustain more than a few percent of extensional strain relative to its equilibrium condition. If subject to such a strain the material will fail, in extension, even if the overall stress field is compressive. This extensile failure will have the same general characteristics of what is generally known as tensile fracture, but the new term is used to avoid the connotation that an applied tensile stress is required for failure.

Since the above statement seems contrary to conventional wisdom about fracture, an illustrative example is useful to jog the intuition. Bridgman (1931) reported an experiment in which, for apparently inadvertent reasons, he slipped a rubber ring snugly over a steel rod, placed both in a pressure vessel and subjected them to high hydrostatic compression. On removal, he found that the ring had split on a parting parallel to the axis of the rod. The behavior was repeatable. The appearance of the ring was that it had failed "in tension" but by any account both the rod and the ring were subject to high compressive stress throughout the experiments, and furthermore, the strains in both ring and rod were also compressive.

Bridgman's solution to this puzzle is very illuminating for the present problem. He reasoned that the ring, being made of a material much more compressible than that of the rod, when subject to high pressure would have shrunk to a much smaller diameter than the rod, had it not been constrained by the rod. Thus, relative to its equilibrium condition at high pressure, the ring was in extension, and it failed, even though relative to its initial condition, room temperature and pressure, the strain in the ring was compressive (equal to that of the rod).

A thought experiment is useful for fully understanding Bridgman's solution. Imagine placing the ring and rod separately in the pressure vessel and subjecting them to high pressure. The inner diameter of the ring will now be much smaller than the outer diameter of the rod. Now imagine, by some means, stretching the ring in order to slip it over the rod, and having it fail in the process.

The key to Bridgman's solution is realizing that for some problems in fracture a stress fracture criterion is completely inapplicable. In those cases a strain fracture criterion is appropriate, but in using it one must be careful to reference the strains to the correct equilibrium state. The present problem appears to be such a case.

Consider again the problem of the spherical compressive shock wave. Since adjacent particles are forced to diverge, tangential extensional strains (relative to the initial state) are developed on loading. Since there is insufficient time for elastic tangential expansion to occur, the correct equilibrium state from which to measure those strains is the initial state, and those strains are thus extensional. If the tangential extensional strains exceed a critical level, extensonal failure will occur by the propagation of Mode I (opening) cracks radial to the shotpoint, and this may be so even if all stresses are calculated to be compressive.

To the author's knowledge, all numerical simulations of underground nuclear explosions have assumed that failure is in compression and have employed a stress failure criterion. The above remarks suggest that failure occurs in extension and that a strain failure criterion should be employed. Since the dynamic tensile strength of compact, crystalline rock is of the order of several hundred bars, much lower than the compressive strength, it is quite reasonable that in order to fit the data, seemingly artificially low values for the strength of rock had to be used in the simulations. In other words, the simulations were right but for the wrong reasons.

There is nothing in the above discussion that would explain the apparently great difference between the HOGGAR and PILEDRIVER tests. If the critical strain criterion approach is the correct one, then there may be significant differences in the critical strain for the two rocks, due to differences in weathering, fracturing, etc. The overall results may be quite sensitive to this critical strain. Further simulations, using a strain criterion, can check this. Otherwise, there appears to be nothing in the fracture behavior of brittle rock that will readily explain the difference. Other possibilities, of course, may be differences in the devices themselves, or differences in the size or shape of the initial cavity.

VI. RECOMMENDATIONS

As stated in the introduction, it was not possible to unequivocally "solve" the problem posed. What was possible was to narrow down the possibilities to several, which can be tested by further work. These have been discussed in Section 5.3.

The suggestion that the rock failure may be extension rather than compression, and that a critical strain, rather than a critical stress, failure criterion should be employed can be checked with numerical simulations. Such simulations should be directed first of all at determining whether or not such an approach works, and secondly at a parameter study to see how sensitive it is to varitions in parameters such as the critical strain level.

The second possibility, that some mechanism inhibits dilatancy hardening under shock conditions, can only be tested by experiment. It is suggested that controlled experiments such as those described by Cherry and Rimer (1982) on grout be carried out, but on granite or a granite-like material under both saturated and dry conditions.

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