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The objective of our research was to design a computer code capable of simulating interactions between complicated flows and shock waves.
We ~~have shown that~~ it is possible to completely overcome the Gibbs phenomenon from the approximation (or signal processing) point of view. The main theoretical breakthrough has been the observation that the Gibbs phenomenon does not apply to moments of the approximated function. These moments are obtained with high accuracy. The only remaining difficulty is the accurate reconstruction of a given function from its moments. We have suggested and tested several different methods. Computationally the above ideas are carried out as low-pass filters. We have proved that when a discontinuous signal is being evolved in time by a linear system of hyperbolic equations, the moments are obtained with spectral accuracy.
~~We have developed~~ an essentially non-oscillatory spectral method is developed for the numerical

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simulations of non-linear equations. we developed
~~We have also developed~~ Cell Averages techniques for Fourier and Chebyshev Spectral methods. This techniques is crucial for modern shock capturing techniques.

COMPUTATIONAL METHODS FOR PROBLEMS IN AERODYNAMICS USING PARALLEL AND VECTOR ARCHITECTURE

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Computational Methods for Problems in Aerodynamics Using Parallel and Vector Architecture

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- High order accurate methods for Shock-Turbulence interactions.
- The Bluff-Body Wake
- Numerical Boundary Conditions for Spectral and Finite Difference Methods.
- Parallel Spectral Methods for Incompressible and Compressible Flows.
- Efficient Implementation of Spectral Methods.
- Various Topics

High Order Methods for Shock-Turbulence Interactions

OBJECTIVE

- To design a computer code capable of simulating interactions between complicated flows and shock waves.

METHODOLOGY

- To adapt **Spectral Methods** to discontinuous problems.

Background

The crucial role of high order accuracy computational methods for the numerical simulation of complicated flows has been widely recognized by the scientific community. In fact most of the computational turbulence simulations employ spectral methods as the building block of the numerical technique. Spectral methods, being highly accurate, offer substantial savings in computer resources (memory and cpu time), since even delicate features of the simulated flow can be captured with minimal number of grid points.

Spectral methods are based on the global expansions of the unknowns in term of Fourier series or Chebyshev polynomials. These expansions converge exponentially fast for smooth functions, but the rate of convergence deteriorates in the presence of discontinuities (e.g. shock waves). This phenomenon (known as the Gibbs phenomenon) was found in the beginning of this century and seems to rule out the possibility of applying spectral methods to shocked flows.

Under the above grant we had explored ideas to overcome this fundamental difficulty, both from the theoretical point of view and from the practical one. The goal was and still is to design a spectral code that handles shocks well without loss of the spectral accuracy.

Achievements

1. We have shown that it is possible to completely overcome the Gibbs phenomenon from the approximation (or signal processing) point of view. Mathematically, the result can stated as follows : "Given the first N Fourier coefficients of a piecewise smooth function (signal) then it is possible to reconstruct the function (signal) such that

a) The approximation is non-oscillatory.

b) The rate of convergence in the smooth regions of the function is spectral."

The main theoretical breakthrough has been the observation that the Gibbs phenomenon does not apply to **moments** of the approximated function. These moments are obtained with high accuracy. The only remaining difficulty is the accurate reconstruction of a given function from its moments. We have suggested and tested several different methods. Also some very promising new ideas in this direction are being developed. Computationally the above ideas are carried out as low-pass filters.

2. We have proved that when a discontinuous signal is being evolved in time by a linear system of hyperbolic equations the moments are obtained with spectral accuracy. We have demonstrated theoretically as well as computationally that for **linear problems** one can achieve spectral accuracy even for discontinuous problems.

3. We have developed an essentially non-oscillatory spectral method for the numerical simulations of non-linear equations (such as the Euler equations of gas-dynamics).

The method is based on the theoretical achievements reported in 1. and 2., the discontinuity is automatically detected and subtracted and a low-pass filter retrieves the spectrally accurate solutions away from the shock. The method has been successfully applied to shock-density wave interactions and is currently being applied to shock-vortex interactions.

A slightly different version of this code is being applied now to the studies of compressible effects in homogeneous turbulence.

4. We have developed Cell Averages techniques for Fourier and Chebyshev Spectral methods. This technique is crucial for modern shock capturing techniques.

The research is summarized in two papers (enclosed) and a Ph.D. thesis

[1] "Essentially Nonoscillatory Spectral Fourier Methods for Shock wave Calculations" by W. Cai, D. Gottlieb and C.W. Shu, Math. Comp. 1989, pp. 389-410.

[2] "Cell Averaging Chebyshev Methods for Hyperbolic Problems" by W. Cai, D. Gottlieb and A. Harten, 1989 (to appear).

[3] "Spectral Methods for Shock Wave Calculations" W. Cai, Ph.D. thesis, Brown University 1989.

Other papers related to this projects will be mentioned in other sections.

The Bluff Body Wake

OBJECTIVE

- To design a code that will be able to simulate *compressible* flows past circular cylinder for a wide range of Reynolds and Mach numbers.

Background

One of the ambitions of fluid dynamicists has been to start with a steady laminar flow and to observe transition to turbulence by increasing the flow Reynolds number gradually.

We feel that such an experimental program should have a computational counterpart. Experiments and computations are complementary, since some features of the flow can be obtained easier in experiments and others are more readily available in computations. Also the wind tunnel and the computer code can verify each other's results.

Based on this belief we have developed a two dimensional computer code with the following characteristics

- The full time dependent compressible viscous Navier-Stokes equations are discretized.

- The code is fully spectral in all space dimensions, using Fourier collocation in the azimuthal direction and Chebyshev collocation in the radial direction.

- The code is fully vectorized.

- The code has an adaptive grid capability.

- The boundary conditions are applied in a way that insure not only stability but also the elimination of spurious frequencies.

Achievements

- The numerical results settled a controversy between experimentalists by demonstrating that no secondary frequency besides the primary vortex shedding frequency can be created in open systems for Reynolds number less than 160. The controversy stemmed from the fact that some experiments had observed a secondary frequency while other experiments did not detect such a frequency. Our numerical results indicated that the source of the secondary frequency is the wind tunnel wall.

- The boundary procedure developed for outer computational boundary has been applied to a variety of finite difference codes and helped in eliminating spurious frequencies.

- Vortex shedding frequency predicted by the spectral code agrees very well with experimental data for Reynolds number up to 500 using a fixed number of collocation points.

- The code serves as a tool of checking issues that cannot be answered easily by experiment, such as the critical Reynolds number for vortex shedding or the importance of the actual body to the transition to turbulence.

The results of the above research are summarized in the following two papers and a Ph.D. thesis.

[4] "Spectral Simulations of an Unsteady Compressible Flow Past a Circular Cylinder" by W.S. Don and D. Gottlieb, Computer Methods in Applied Mechanics and Engineering **78**, 1990 (to appear).

[5] "Secondary Frequencies in The Wake of a Circular Cylinder with Vortex Shedding" by S.S. Abarbanel, W.S. Don, D. Gottlieb, and D. Rudy, J. Fluid Mech. (to appear).

[6] "Theory and Application of Spectral Methods for the Unsteady Viscous Compressible Wake Flow Past a Two-Dimensional Circular Cylinder" by W.S. Don, Ph.D. thesis, Brown University 1989.

Numerical Boundary Conditions for Spectral and Finite Difference Methods

Background

The importance of the numerical treatment of the boundary conditions (BC) in the simulation of hyperbolic partial differential equations has been well recognized by practitioners as well as theoretical computational scientists. An extensive literature exists for finite difference methods, and a complete stability theory had been developed in the early seventies.

Achievements

The need for such a theory for spectral methods had been felt in the last decade. This theory was provided under this grant. Spectral methods are very sensitive to boundary treatments even more than finite difference schemes, and there are only very limited number of ways to implement correctly the boundary conditions.

The stability theory developed for finite difference schemes has been often ignored in the application. We have shown that incorrect implementation of BC can lead to stable but spurious results and gave a procedure such that those spurious frequencies disappear.

We also showed the importance of the *intermediate* boundary conditions, in particular we showed that an incorrect implementation of the intermediate BC led to restrictions on the allowable time step.

The research on spectral methods is summarized in the following papers.

[7] "Stability Analysis of Intermediate Boundary Conditions in Approximate Factorization Schemes" by J.C. South, M.M. Hafez and D. Gottlieb, Appl. Numer. Math. **2** 1986, pp. 181-192.

[8] "Convergence of Spectral Methods for Hyperbolic Initial-Boundary Value Systems" by D. Gottlieb, L. Lustman and E. Tadmor, SIAM J. Numer. Anal. **24** no. **3** 1987.

[9] "A new Method of Imposing Boundary Conditions in Pseudospectral Approximations of Hyperbolic Equations" by D. Funaro and D. Gottlieb, Math. Comput. 1990.

[10] "Convergence Results for Pseudospectral Approximations of Hyperbolic Systems by a Penalty Boundary Treatment" by D. Funaro and D. Gottlieb, Math. Comput. (to appear).

Parallel Spectral Methods

OBJECTIVE

- To apply Spectral Methods to Incompressible and Compressible Flows Using Parallel Computers.

Background

Spectral methods are standard methods in large scale computing. The main attractive feature of these methods is their high accuracy, stemming from the fact that the methods use global expansions.

Domain decomposition methods, in which the computational domain is divided into simple sub-domains, seem to be the ideal way to go about parallelizing the spectral methods. In each of these elements the classical spectral method is applied. The way that the interface boundary conditions are applied is of course crucial.

The structure of the domain decomposition spectral methods makes it very attractive in parallel environments since each subdomain can be assigned to a different processor.

Achievements

- Bernardi and Maday showed how to treat several subdomains when different degree of the polynomials are used in the different subdomains.

- Munoz and Maday analyzed multigrid algorithms for the spectral element methods which are domain decomposition methods for incompressible flows.

- Funaro analyzed the multidomain approach for second-order equations and classified the interface boundary conditions.

- Gottlieb and Hirsh discussed ways to minimize the communication time between processors in using domain decomposition spectral methods with parallel computers. They showed that judicious choice of interface boundary conditions can completely decouple a given problem to several subproblems.

The following publications reports on the results presented above.

[11] "Approximation Results for Spectral Methods with Domain Decomposition" by C. Bernardi and I. Maday (to appear).

[12] "Spectral Element Multigrid" by I. Maday and R. Munoz, J. Sci. Comput. **3** no. **4** 1988, pp 323-353.

[13] "Domain Decomposition Methods fo Pseudospectral Approximations" by D. Funaro (to appear).

[14] "Parallel Pseudospectral Domain Decomposition Techniques" by D. Gottlieb and R. Hirsh, J. Sci. Comput. **4** no. **4** 1989.

Efficient Implementation of Spectral Methods

OBJECTIVE

- To overcome difficulties resulting from the grid distribution of Chebyshev methods.

Background

Chebyshev methods are based on collocating the equation at the extrema of the Nth order Chebyshev polynomial. Those points are not evenly distributed, they are crowded in the neighborhood of the boundaries.

This fact may create difficulties when solving time dependent problems involving changes that occur in the *middle* of the domain. In particular the distribution of the grid points causes a severe restriction on the stable time step. Also many problems require denser mesh in the interior of the domain rather than at the boundaries.

A substantial progress has been made in overcoming these difficulties.

Achievements

- Gottlieb and Bayliss have developed an *adaptive* procedure, based on minimizing new error functionals. They applied it successfully on various combustion problems in one and two dimensions. Adaptive methods that did not use the above functionals failed.

- Tal-Ezer introduced a new set of interpolating points and greatly improved the allowable time steps. Since publication of this work the method has been tried successfully on many real life problems.

- Tal-Ezer found the optimal way to evaluate $f(A)$ where A is a matrix resulting from some spectral approximation of an operator (like the derivative operator).

The following papers summarize the research in this topic

[15] "An adaptive Pseudospectral Method for Reaction Diffusion Problems" by Bayliss, Gottlieb, Matkowski and Minkoff, J. Comput. Phys. **81** no. **2** 1989, pp 421-443.

[16] "Polynomial Approximation of Functions of Matrices and Applications" by H. Tal-Ezer, J. Sci. Comput. **4** no. **1** 1989.

[17] "Modified Chebyshev Pseudospectral Method with $O(N^{-1})$ Time Step Restriction" by D. Kosloff and H. Tal-Ezer, J. Comput. Phys. (to appear).

Multiple Steady States

Achievements

We have analyzed the steady isentropic flow in a dual-throat nozzle with equal areas. It is known that the flow between the throats can be either completely subsonic or supersonic depending on the initial state of the flow and the path taken to reach the steady state. We have provided a complete analysis explaining mathematically the above phenomenon. We showed that the non-uniqueness aspect of the steady state is a by-product of the fact that the boundary conditions for the evolution equation are prescribed along characteristic curves.

The following publication summarizes the above results

[18] "Multiple Steady States for Characteristic Initial Value Problems" by S.S. Abarbanel and D. Gottlieb, Appl. Numer. Math. **2** 1986, pp. 193-210.

Various Topics

The following research topics had been investigated and reports were published in the literature.

[19] "Splitting Methods for Low Mach Number Euler and Navier-Stokes Equations" by Abarbanel, Dutt and Gottlieb, Computer and Fluids.

In this paper we pointed out shortcomings of proposed splitting techniques and suggested an explanation for their inadequacy. A symmetric splitting is developed and shown to be stable.

[20] "Spectral Methods for the Approximation of Fourth Order Problems" by Bernardi and Maday (to appear).

Discusses spectral methods for the bi-harmonic equation.

[21] "Computing with Spectral Matrices" by Funaro (to appear in Calcolo).

The paper presents various numerical techniques to invert the pseudospectral derivative matrix.

[22] "Microphone Array Optimization by Stochastic Region Contraction" by Berger and Silverman, (to appear).

This work is concerned with optimal microphone placement and gain for a linear one dimensional array. A Power Spectral Dispersion function is used as a core element for a min-max objective function. A new method (SRC) is proposed that achieves a computational speed up of 30-50 compares to the simulated annealing method. SRC is shown to be ideally suited for coarse grain parallel processing.

[23] "On Non-Smooth Beam Shapes for Maximal Transverse Natural Frequencies" by Berger and Porat, J. Sound and Vibration **132 (3)** 1989, pp. 423-432.

The optimal shape of an Euler Beam subject to space constraints is studied. Three previously unknown solutions are found and verified numerically.