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NEW PRESSURE GRADIENT EQUATIONS FOR
LUMPED-PARAMETER INTERIOR BALLISTIC CODES

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MAY 1990

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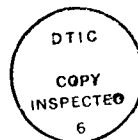
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13. ABSTRACT (Maximum 200 words) Lumped-parameter models of interior ballistics require a relation between the space mean pressure and the pressure at the base of the projectile. Typically, this is furnished using the closed form analyses of Lagrange or of Pidduck and Kent. These relations do not take into account the non-uniform cross-sectional area of the chamber of the gun and the two-phase structure of the flow propellant grains and their products of combustion. <i>THIS</i> In the present report we describe analytical relations which reflect first the variation in cross-sectional area, second the two-phase structure of the flow and third, the combination of both variable area and two-phase flow. These three sets of relations are programmed into a lumped-parameter code the results of which are compared with those from a quasi-one-dimensional two-phase flow code. Significant improvements in predictive capacity are observed relative to the traditional Lagrange formula when high performance charges are considered. <i>Keywords:</i>				
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TABLE OF CONTENTS

	<u>Page</u>
LIST OF FIGURES	v
LIST OF TABLES	vii
ACKNOWLEDGMENTS	ix
1. INTRODUCTION	1
1.1 Models	1
1.2 Influence of Chambrage	1
1.3 Influence of Propellant Velocity Lag	3
1.4 Combined Influence of Chambrage and Propellant Velocity Lag	9
2. CALCULATIONS	14
3. RESULTS	15
3.1 Chambrage	15
3.2 Multiphase Flow	17
3.3 Combined	21
4. CONCLUSIONS	21
5. REFERENCES	25
6. LIST OF SYMBOLS	27
APPENDIX 1: INFLUENCE OF CHAMBRAGE	29
APPENDIX 2: INFLUENCE OF PROPELLANT VELOCITY LAG	39
APPENDIX 3: COMBINED INFLUENCE OF CHAMBRAGE AND PROPELLANT VELOCITY LAG	61
APPENDIX 4: INPUT DESCRIPTION FOR IBRGA	87
APPENDIX 5: FORTRAN LISTING OF IBRGA	93
APPENDIX 6: INPUT DATA FOR IBRGA TEST	121
APPENDIX 7: OUTPUT FROM IBRGA	125
APPENDIX 8: INPUT DATA FOR XKTC	133
DISTRIBUTION LIST	137



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Distribution/	
Availability Codes	
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LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1 XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage	18
2 XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage	18
3 XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage	18
4 IBRGA Calculation With Chambrage Gradient With Figure 1 Values	18
5 IBRGA Calculation With Chambrage Gradient With Figure 2 Values	18
6 IBRGA Calculation With Chambrage Gradient With Figure 3 Values	18
7 XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and No Chambrage	20
8 XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and No Chambrage	20
9 XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and No Chambrage	20
10 IBRGA Calculation With Two-Phase Gradient With Figure 7 Values	20
11 IBRGA Calculation With Two-Phase Gradient With Figure 8 Values	20
12 IBRGA Calculation With Two-Phase Gradient With Figure 9 Values	20
13 XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage	23
14 XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage	23
15 XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage	23
16 IBRGA Calculation With RGA Gradient With Figure 13 Values	23
17 IBRGA Calculation With RGA Gradient With Figure 14 Values	23
18 IBRGA Calculation With RGA Gradient With Figure 15 Values	23

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LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	Chambrage Calculations	16
2	Two-Phase Calculations	19
3	RGA Calculations	22

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1. INTRODUCTION

During the analysis of 120-mm gun firings, designed to look at the interior ballistic characteristics of combustible cartridge cases (Robbins, Koszoru, and Minor 1986), XNOVAKTC (XKTC) (Gough 1980) was noted to be in agreement with measured pressure-time curves as well as with pressure-difference curves, while IBHVG2 (Anderson and Fickie, 1987) gave a calculated maximum breech pressure which was 42 MPa higher than measured. Parametric studies were performed using XKTC to attempt to attribute this disparity to the various processes omitted from IBHVG2. The boattail intrusion was calculated to account for 14 MPa, with effects of flamespreading and intergranular stress accounting for 3 MPa each. Subsequent calculations (Robbins 1986) indicated chambrage, propellant packaging, wave dynamics, and multiphase effects (the solid propellant velocity lag and concomitant formation of an ullage region between the projectile base and the propellant bed) as contributors to the differences between the lumped-parameter and two-phase interior ballistic codes. In this report, we show that the influence of chambrage and propellant velocity lag on the pressure gradient may be represented in analytical form. We compare the analytical pressure gradient with that predicted by XKTC and assess the extent to which the effects of chambrage and propellant velocity lag account for the differences in ballistic predictions.

The NOVA codes, of which XKTC is the latest version, have been used with uncompromised data bases to model gun systems with much success (Robbins, Koszoru, and Minor 1986; Robbins 1983; Robbins and Horst 1984). Since XKTC calculates the pressure gradient from first principles and agrees with gun firings, XKTC is assumed correct. Accordingly, all the lumped-parameter computer runs, with different gradient equations, are compared with equivalent XKTC computer runs.

1.1 Models. Gradient equations have been developed by Gough (Gough, no date) to look at (1) the effects of chambrage, (2) the effects of propellant velocity lag, and (3) both effects at the same time. The governing partial differential equations, assumptions, and definitions are given for each effect followed by the resultant gradient equations and an equation for the total kinetic energy of the propellant and gas. The details of the solution technique and mathematical procedure are documented in Appendices 1-3.

1.2 Influence of Chambrage. For the chambrage gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and the variation in area is assumed to be confined to the chamber. The basic results presented in this section have been previously derived (Vinti 1942) and, more recently, in a somewhat different context, (Morrison and Wren 1988).

The continuity and momentum equations for unsteady flow of a homogeneous inviscid substance are

$$\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial \rho A u}{\partial z} = 0 \quad (C.01)$$

and

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial z} + g_o \frac{\partial p}{\partial z} = 0. \quad (C.02)$$

Making the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0 \quad (C.03)$$

and using

$$\rho = \frac{C}{V(z_p)} \quad (C.04)$$

results in the pressure distribution

$$p(z) = p(0) + a(t)J_1(z) + b(t)J_2(z), \quad (C.05)$$

where

$$a(t) = \frac{CA_B}{g_o V^2(z_p)} \left[\frac{A_B V_p^2}{V(z_p)} - \dot{V}_p \right], \quad (C.06)$$

$$b(t) = - \frac{CV_p^2 A_B^2}{2g_o V^3(z_p)}, \quad (C.07)$$

$$J_1(z) = \int_0^z \frac{V(z) dz}{A(z)}, \quad (C.08)$$

and

$$J_2(z) = \frac{V^2(z)}{A^2(z)}. \quad (C.09)$$

Substituting the projectile acceleration $\dot{V}_p$ given by

$$\dot{V}_p = \frac{g_o A_B}{M_p} [P_B - P_{ra}] \quad (C.10)$$

into (C.05) results in a pressure distribution in terms of the pressure at the base of the projectile P_B

$$p(z) = p(0) + a_1(t)J_1(z) + a_2(t)J_1(z)P_B + b(t)J_2(z), \quad (C.11)$$

where

$$a_1(t) = \frac{CA_b}{g_o V^2(z_p)} \left[\frac{A_B V_p^2}{V(z_p)} + \frac{g_o A_B}{M_p} P_{ra} \right] \quad (C.12)$$

and

$$a_2(t) = - \frac{CA_b^2}{M_p V^2(z_p)}. \quad (C.13)$$

With the mean pressure P_m defined as

$$P_m = \frac{\int_0^{z_p} P(z)A(z)dz}{\int_0^{z_p} A(z)dz} \quad (C.14)$$

and using (C.11) and identifying $P(0)$ with P_{Br} , then the base and breech pressures are given in terms of the mean pressure by

$$P_m + a_1(t)J_1(z_p) + b(t)J_2(z_p) - \frac{a_1(t)J_3(z_p)}{V(z_p)} - \frac{b(t)J_4(z_p)}{V(z_p)} \\ P_B = \frac{1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)}}{\quad} \quad (C.15)$$

and

$$P_{Br} = (1 - a_2(t)J_1(z_p)) P_B - a_1(t)J_1(z_p) - b(t)J_2(z_p), \quad (C.16)$$

where

$$J_3(z_p) = \int_0^{z_p} A(z)J_1(z)dz \quad (C.17)$$

and

$$J_4(z_p) = \int_0^{z_p} A(z)J_2(z)dz. \quad (C.18)$$

The kinetic energy of the gas/solid mixture is

$$KE = \frac{A_B^2 V_p^2 C J_4(z_p)}{2g_o V^3(z_p)}. \quad (C.19)$$

1.3 Influence of Propellant Velocity Lag. For the two-phase gradient equation developed in this section, the chamber and tube are assumed to have the same, uniform diameter. The coupling of the two-phase effects with chambrage is addressed in the next section. The propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed with the leading edge of the bed being initially at the base of the projectile. However, once the projectile begins to move, the leading edge of the propellant bed is not assumed to remain in contact with the base of the projectile. Instead, the motion of the leading edge of the bed is explicitly modeled by reference to the equation of motion of the solid phase which includes the influence of both pressure gradient (buoyancy force) and interphase drag. The existence of a region of

ullage between the propellant bed and the base of the projectile is also recognized explicitly. The requirement that the gas-phase density be everywhere uniform causes the gas velocity at the leading edge of the propellant bed (U_g) to have a much more complex time dependence than that in the simple Lagrange analysis. Instead of being a fixed fraction of the projectile velocity at all times, the value of U_g may be significantly greater than the projectile velocity during the pressurization phase since the ullage requires a net compression to maintain equilibrium with the conditions in the mixture region where combustion is occurring. The complex time dependence of U_g will be seen to have important repercussions in respect to the behavior of the pressure gradient. It is believed that this is the first time that these effects have been modeled analytically.

The continuity equations for the mixture region are

$$\frac{\partial \epsilon \rho}{\partial t} + \frac{\partial \epsilon \rho u}{\partial z} = \dot{m}(t) \quad (L.1)$$

$$\rho_p \frac{\partial \epsilon}{\partial t} - \frac{\partial [(1-\epsilon)\rho_p u_p]}{\partial z} = \dot{m}(t). \quad (L.2)$$

Making assumptions analogous to the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0 \quad (\text{gas density constant throughout the tube}) \quad (L.3)$$

$$\frac{\partial \epsilon}{\partial z} = 0 \quad (\text{porosity constant throughout the mixture region}) \quad (L.4)$$

and noting the assumption $\dot{m}(t)$ is only a function of time, then

$$u_p = U_p \left[\frac{z}{z_b} \right] \quad (L.5)$$

and

$$u = U_g \left[\frac{z}{z_b} \right]. \quad (L.6)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\epsilon (U_g - U_p) = U_{g+} - U_p.$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.

Consider the momentum equation for the mixture region,

$$\frac{\partial \epsilon \rho u}{\partial t} + \frac{\partial \epsilon \rho u^2}{\partial z} + \epsilon g_o \frac{\partial P}{\partial z} = -f_s + \dot{m} u_p \quad (L.7)$$

and

$$\frac{\partial (1-\epsilon) \rho_p u_p}{\partial t} + \frac{\partial (1-\epsilon) \rho_p u_p^2}{\partial z} + (1-\epsilon) g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p, \quad (L.8)$$

where

$$f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc}, \quad (L.9)$$

and for

$$REN = \frac{\rho D_p |U_g - U_p|}{\mu}$$

$$\lambda = \left[\frac{0.5 + \frac{\alpha}{GD}}{\left(1.5 \frac{\alpha}{GD}\right)^{2/3}} \right]^{2.17}$$

$$\epsilon_o = 1 - \frac{C}{\rho_p V(z_p)}$$

Then

$$f_{sc} = \begin{cases} \frac{2.5\lambda}{REN^{.081}} f_{so} & \epsilon < \epsilon_o \\ \frac{2.5\lambda}{REN^{.081}} \left(\frac{(1-\epsilon)}{(1-\epsilon_o)} \frac{\epsilon_o}{\epsilon} \right)^{0.45} f_{so}, & \epsilon \geq \epsilon_o. \end{cases} \quad (L.10)$$

Defining $\phi_s = \phi - \rho A_B L / C$,

(L.11)

and

$$\frac{d^2 \ln \rho}{dt^2} = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{V_F M_p},$$

where

$$c_1(t) = \ddot{m} \left(\frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}$$

and

$$V_F = V_o + A_B(z_p - z_{po}) - \frac{c}{\rho_p} + \frac{m}{\rho_p},$$

$$\dot{m} = \rho_p S \frac{dx}{dt}, \quad (L.12)$$

$$\begin{aligned} \ddot{m} &= \rho_p \dot{S} \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2} \\ &= \rho_p \frac{dS}{dx} \left(\frac{dx}{dt} \right)^2 + \rho_p S \frac{d^2x}{dt^2}, \end{aligned} \quad (L.13)$$

with

$$\frac{dx}{dt} = aP_m^n \quad (L.14)$$

and

$$\frac{d^2x}{dt^2} = anP_m^{n-1} \frac{dP_m}{dt}, \quad (L.15)$$

where dP_m/dt is determined numerically, or from

$$\frac{dP_m}{dt} = \frac{\frac{nRT}{m_w} - P_m \dot{V} + \frac{TRm}{m_w}}{V}$$

and

$$\dot{V}_p = \frac{g_o A_B}{M_p} (P_B - P_{res}). \quad (L.16)$$

Then

$$P(z) = P_{Br} - \frac{(k_{11}P_B + k_{12})}{2} z^2, \quad (L.17)$$

where

$$k_{11} = \frac{C \frac{\phi_c}{z} \left(1 - \frac{LA_B}{V_F} \right) \frac{A_B}{M_p} k_2}{z_b V(z_b)} \quad (L.18)$$

and

$$k_2 = \frac{1}{1 - \frac{\phi_c C}{\rho_p V(z_b)}} \quad (L.19)$$

and

$$k_{12} = \frac{\phi_1' C k_2}{g_o z_b V(z_b)} - k_{11} P_{res}, \quad (L.20)$$

$$\begin{aligned} \phi_1' &= \phi_s U_s - \phi U_p - \frac{\phi_s \dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \\ &+ \frac{\phi_s L}{\epsilon} \frac{d \ln \rho}{dt} + \frac{2 \phi_s U_s}{z_b} (U_s - U_p) \\ &+ \phi_2 \frac{\rho}{D_p \rho_p} (U_s - U_p)^2 f_{sc} + \frac{L \phi_s C_1(t)}{\epsilon}, \end{aligned} \quad (L.21)$$

and

$$\phi_2 = 1 - \phi - \phi_s \frac{(1 - \epsilon)}{\epsilon}. \quad (L.22)$$

For the ullage region, the momentum equation for the gas is

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right), \quad (L.23)$$

and, therefore, the pressure in the ullage region is

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta)(z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right], \quad (L.24)$$

where

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt} \right)^2.$$

Defining

$$P_m = \frac{\int_{z_o}^{z_p} P(z) A(z) dz}{\int_{z_o}^{z_p} A(z) dz}, \quad (L.25)$$

then

$$P_B = \frac{P_m - D}{G} \quad (L.26)$$

and

$$P_{Br} = AP_B + B, \quad (L.27)$$

where

$$\begin{aligned} D = & \frac{k_{12}z_b^2}{2} - \frac{\rho LA_B P_{res}}{M_p} - \frac{k_{12}z_b^2}{2z_p} \left(\frac{z_b}{3} + L \right) + \frac{\rho L^2 A_B P_{res}}{2z_p M_p} \\ & + \frac{\rho L^2}{2g_o} \left(1 - \frac{2L}{3z_p} \right) \left(c_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 \right) \\ & + \frac{A_B^2 \rho L^2}{2M_p V_F} \left(1 - \frac{2L}{3z_p} \right) P_{res}, \end{aligned} \quad (L.28)$$

$$\begin{aligned} G = & 1 + \frac{k_{11}z_b^2}{2} + \frac{\rho LA_B}{M_p} - \frac{A_B^2 \rho L^2}{2V_F M_p} \left(1 - \frac{2L}{3z_p} \right) - \frac{k_{11}z_b^3}{6z_p} \\ & - \frac{k_{11}Lz_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p}, \end{aligned} \quad (L.29)$$

$$A = 1 + \frac{k_{11}z_b^2}{2} + \frac{\rho LA_B}{M_p} - \frac{A_B^2 \rho L^2}{2V_F M_p}, \quad (L.30)$$

and

$$\begin{aligned} B = & \frac{k_{12}z_b^2}{2} - \frac{\rho LA_B}{M_p} P_{res} \\ & + \frac{\rho L^2}{2g_o} \left(c_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 \right) + \frac{A_B^2 \rho L^2}{2V_F M_p} P_{res}. \end{aligned} \quad (L.31)$$

The kinetic energy of the propellant and gases is

$$\begin{aligned} KE = & \frac{A_B z_b}{6g_o} [\epsilon \rho U_g^2 + (1-\epsilon) \rho_p U_p^2] \\ & + \frac{\rho A_B L}{6g_o} \left(3V_p^2 + 3V_p L \frac{d \ln \rho}{dt} + L^2 \left(\frac{d \ln \rho}{dt} \right)^2 \right). \end{aligned}$$

The mean pressure over the mixture region is

$$P_{mix} = P_{Br} - \left(\frac{k_{11}P_B + k_{12}}{6} \right) z_b^2.$$

1.4 Combined Influence of Chambrage and Propellant Velocity Lag. The Robbins-Gough-Anderson (RGA) gradient equation, which combines the influences of chambrage and multiphase flow, assumes that the variation in area is confined to the propellant chamber and that the propellant initially fills the chamber. The cross-sectional area within the ullage is accordingly uniform and equal to A_B at all points. Nomenclature is as in the previous gradient equations.

The continuity equations for the mixture region are

$$\frac{\partial \epsilon \rho}{\partial t} + \frac{1}{A} \frac{\partial \epsilon \rho u A}{\partial z} = \dot{m}(t) \quad (R.1)$$

$$\rho_p \frac{\partial \epsilon}{\partial t} - \frac{1}{A} \frac{\partial (1 - \epsilon) \rho_p u_p A}{\partial z} = \dot{m}(t). \quad (R.2)$$

Making assumptions analogous to the Lagrange assumption, namely

$$\frac{\partial \rho}{\partial z} = \frac{\partial \epsilon}{\partial z} = 0 \quad \text{in the mixture region} \quad (R.3)$$

and

$$\frac{\partial \rho}{\partial z} = 0 \quad \text{in the ullage region} \quad (R.4)$$

and noting the assumption $\dot{m}(t)$ is only a function of time, then

$$u_p = \frac{U_p A_B V(z)}{V(z_b) A(z)} \quad (R.5)$$

and

$$u = \frac{U_g A_B V(z)}{V(z_b) A(z)}. \quad (R.6)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\epsilon (U_g - U_p) = U_{g+} - U_p$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.

Consider the momentum equations for the mixture region,

$$\frac{1}{A} \left[\frac{\partial A \epsilon \rho u}{\partial t} + \frac{\partial A \epsilon \rho u^2}{\partial z} \right] + \epsilon g_o \frac{\partial P}{\partial z} = -f_s + \dot{m} u_p \quad (R.7)$$

and

$$\frac{1}{A} \left[\frac{\partial A (1-\epsilon) \rho_p u_p}{\partial t} + \frac{\partial A (1-\epsilon) \rho_p u_p^2}{\partial z} \right] + (1-\epsilon) g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p, \quad (R.8)$$

where

$$f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc} \quad (R.9)$$

and for

$$REN = \frac{\rho D_p [U_s - U_p]}{\mu}$$

and for

$$\lambda = \left[\frac{0.5 + \frac{\alpha}{GD}}{\left(1.5 \frac{\alpha}{GD}\right)^{2/3}} \right]^{2.17}$$

and with ϵ_o the initial porosity

$$\epsilon_o = 1 - \frac{C}{\rho_p V(z_{p_o})}$$

Then

$$f_{sc} = \begin{cases} \frac{2.5\lambda}{REN^{.081}} f_{so} & \epsilon < \epsilon_o \\ \frac{2.5\lambda}{REN^{.081}} \left(\frac{(1-\epsilon)}{(1-\epsilon_o)} \frac{\epsilon_o}{\epsilon} \right)^{0.45} f_{so}, & \epsilon \geq \epsilon_o. \end{cases}$$

Defining

$$\phi_s = \phi - \rho A_B L / C \quad (R.10)$$

and

$$\frac{d^2 \ln \rho}{dt^2} = C_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{M_p V_F}, \quad (R.11)$$

where

$$C_1(t) = \bar{m} \left(\frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2} \quad (R.12)$$

and

$$\begin{aligned} V_F &= V_o + A_B (z_p - z_{po}) - \frac{C}{\rho_p} + \frac{m}{\rho_p}, \\ \dot{m} &= \rho_p S \frac{dx}{dt}, \\ \ddot{m} &= \rho_p S \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2}, \end{aligned} \quad (R.13)$$

or

$$\ddot{m} = \rho_p \frac{dS}{dx} \left(\frac{dx}{dt} \right)^2 + \rho_p S \frac{d^2x}{dt^2}$$

with

$$\frac{dx}{dt} = a P_m^n \quad (R.14)$$

and

$$\frac{d^2x}{dt^2} = a n P_m^{n-1} \frac{dP_m}{dt} \quad (R.15)$$

where dP_m/dt is determined numerically, or from

$$\frac{dP_m}{dt} = \frac{\frac{mRT}{mV} - P_m \dot{V} + \frac{mRT}{mV}}{V}$$

and

$$\dot{V}_p = \frac{g_o A_B}{M_p} (P_B - P_{res}). \quad (R.16)$$

Then

$$P(z) = P_{Br} + (a_1(t) + a_2(t)P_B) J_1(z) + b(t)J_2(z) \quad (R.17)$$

in the mixture region, where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_b)} \left(\frac{\phi_2 A}{V(z_b)} - \frac{\phi'_1 + D - \frac{E \phi_2 A_B}{e M_p} P_{res}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \quad (R.18)$$

and

$$a_2(t) = \frac{-\frac{CE\phi_s A_B^2 P_B}{V^2(z_b) \epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \quad (R.19)$$

and

$$b(t) = -\frac{C\phi_s A_B^2}{2g_o V^3(z_b)} \quad (R.20)$$

$$J_1(z) = \int_0^z \frac{V(z)}{A(z)} dz, \quad (R.21)$$

$$J_2(z) = \frac{V^2(z)}{A^2(z)},$$

$$\begin{aligned} \phi'_1 = & \phi_s U_s - \phi U_p - \frac{\phi_s \dot{\epsilon}}{\epsilon^2} \left[V_p + L \frac{d \ln \rho}{dt} - U_p \right] + \frac{L \phi_s}{\epsilon} \frac{d \ln \rho}{dt} \\ & + \frac{2A_B \phi_s U_s}{V(z_b)} [U_s - U_p] + \phi_2 \frac{\rho}{\rho_p D_p} (U_s - U_p)^2 f_{sc}, \end{aligned} \quad (R.21a)$$

and

$$\begin{aligned} \phi_2 &= 1 - \phi - \phi_s \frac{(1-\epsilon)}{\epsilon}, \\ \phi_3 &= \phi_s U_s^2 + (1-\phi) U_p^2, \\ D &= \frac{L \phi_s}{\epsilon} C_1(t), \\ E &= 1 - \frac{L A_B}{V_p}. \end{aligned} \quad (R.22)$$

Also in the ullage region

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right] \quad (R.23)$$

and

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right], \quad (R.24)$$

where

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt} \right)^2.$$

Defining the mean pressure as

$$P_m = \frac{\int_0^{z_p} P(z) A(z) dz}{\int_0^{z_p} A(z) dz} \quad (R.25)$$

and substituting for $\dot{V}_p$ and $d^2 \ln \rho / dt^2$,

then

$$\begin{aligned} P_{Br} = P_B & \left(1 - a_2(t) J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2 V_F M_p} \right) \\ & + \frac{\rho L^2}{2 g_o} \left(C_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F} \right) \\ & - a_1(t) J_1(z_b) - b(t) J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p} \end{aligned} \quad (R.26)$$

and

$$\begin{aligned} P_B = & \left[P_m - \frac{\rho L^2}{2 g_o} \left(C_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F} \right) \right. \\ & - \frac{\rho A_B L^3}{3 g_o V(z_p)} \left(C_1(t) + \frac{g_o A_B^2 P_{res}}{M_p V_F} - \left(\frac{d \ln \rho}{dt} \right)^2 \right) + \frac{a_1(t) J_3(z_b)}{V(z_p)} \\ & + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2 V(z_p) M_p} \\ & \left. - a_1(t) J_1(z_b) - b(t) J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p} \right] / \\ & \left(1 + \frac{A_B L a_2(t) J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2 V(z_p) M_p} + \frac{a_2(t) J_3(z_b)}{V(z_p)} - a_2(t) J_1(z_b) \right. \\ & \left. + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2 V_F M_p} + \frac{\rho A_B L^3 A_B^2}{3 V(z_p) V_F M_p} \right), \end{aligned} \quad (R.27)$$

where

$$J_3(z_b) = \int_0^{z_b} A(z) J_1(z) dz \quad (R.28)$$

and

$$J_4(z_b) = \int_0^{z_b} \frac{V^2(z)}{A^2(z)} dz. \quad (R.29)$$

The kinetic energy is

$$\begin{aligned} KE = & \left(\frac{1-\epsilon}{2g_o} \right) \frac{U_p^2 A_B^2 \rho_p J_4(z_b)}{V(z_b)^2} + \frac{\epsilon}{2g_o} \frac{U_g^2 A_B^2 \rho J_4(z_b)}{V(z_b)^2} \\ & + \frac{\rho A_B L}{6g_o} \left(3V_p^2 + 3V_p \rho L \frac{d \ln \rho}{dt} + L^2 \left(\frac{d \ln \rho}{dt} \right)^2 \right). \end{aligned} \quad (R.30)$$

The mean pressure over the mixture region is

$$P_{mix} = P_{Br} + \frac{(a_1(t) + a_2(t)P_B)}{V(z_b)} J_3(z_b) + \frac{b(t)J_4(z_b)}{V(z_b)}. \quad (R.31)$$

2. CALCULATIONS

A number of propellant charges were simulated with XKTC to probe the influence of chambrage and velocity lag. The same charges were simulated with IBRGA (input description and listing given in Appendices 4 and 5) using data bases as consistent with those of XKTC as the physical scope of the lumped-parameter model would permit, and using each of the various pressure gradient models. The resultant maximum breech pressures, velocities, and histories of pressure-time and mean-pressure-to-base-pressure curves are compared.

The calculations performed with XKTC involved data bases with evenly distributed seven-perforated propellant having an initial porosity of 0.4, zero barrel resistance, and with all the propellant ignited at the initial instant. All calculations were performed for a flat-based projectile and nominal heat loss.

The parameters used in the computer codes were:

Large chamber simulations:

Bore diameter	127 mm
Volume	9,832.2 cm ³
Travel	4.572 m
Propellant mass	9.8 kg
Projectile mass	2.45, 9.8, 39.2 kg

Small chamber simulations:

Bore diameter	28.65 mm
Volume	98.322 cm ³
Travel	1.880 m
Propellant mass	.098 kg
Projectile mass	.0245, .098, .392 kg

Propellant characteristics:

Impetus	1,136 J/g
Covolume	.976 cm ³ /g
Gamma	1.23
Flame temperature	3,141 K
Molecular weight	23.0 g/gmole
Density	1.66 g/cm ³
Burning rate	1.10519p ^{1.0} mm/s (p is in MPa)

Three maximum breech pressures were studied, nominally 517 MPa, 345 MPa, and 172 MPa. The maximum pressures were achieved by allowing the web to vary, with the grain length kept between two to three times the outer diameter. Calculations were performed for both bore diameter chambers and right circular cylindrical chambers with a larger diameter than the bore. Chambrage (defined as the distance over which the chamber tapers down to the bore area) was 76.2 mm for the large volume chamber, with the chamber length being changed from 776.22 mm (straight chamber) to 541.02 mm. For the smaller volume chamber, we had 25.4 mm of chambrage with the chamber length being changed from 152.4 mm (straight chamber) to 101.6 mm.

3. RESULTS

3.1 Chambrage. The effect of chambrage was assessed at a nominal pressure of 345 MPa using XKTC. Calculations were performed for a straight chamber to determine grain dimensions corresponding to a maximum breech pressure of 345 MPa. These grain dimensions were then used with a chamber with the same volume but having chambrage. The presence of chambrage was found to reduce the maximum breech pressure substantially. Finally, XKTC was run with chambrage and the dimensions of the grains changed to restore the maximum breech pressure to 345 MPa. These sets of grain dimensions were then used with a version of IBRGA into which the chambrage gradient equation had been encoded. Equivalent computer runs on IBRGA with the Lagrange gradient are also given in Table 1.

Adding chambrage to the gradient equation in the lumped-parameter code captures the large drop in maximum breech pressure and the concomitant drop in muzzle velocity seen when chambrage is added to XKTC. The gradient equation with chambrage also gives the proper amount of increase in maximum breech pressure and muzzle velocity, as seen in XKTC calculations, when the grain dimensions are changed to

TABLE 1. Chambrage Calculations.

Chamber volume, cm ³	c/m	Chambrage, mm	XKTC		Chambrage gradient		Lagrange gradient	
			Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s
9,832.2	0.25	none	345	769	349	775	350	775
9,832.2	0.25	76.2	336	764	337	766		
9,832.2	0.25	76.2	345	774	346	776	358	784
9,832.2	1.0	none	345	1,352	347	1,363	347	1,363
9,832.2	1.0	76.2	310	1,303	307	1,308		
9,832.2	1.0	76.2	345	1,376	339	1,375	384	1,429
9,832.2	4.0	none	345	1,964	360	1,962	360	1,962
9,832.2	4.0	76.2	268	1,764	273	1,771		
9,832.2	4.0	76.2	345	1,964	339	1,990	452	2,184
98.322	0.25	none	345	895	348	880	348	880
98.322	0.25	25.4	337	895	334	872		
98.322	0.25	25.4	345	901	341	879	355	886
98.322	1.0	none	345	1,560	347	1,517	347	1,517
98.322	1.0	25.4	308	1,526	305	1,469		
98.322	1.0	25.4	345	1,595	340	1,536	387	1,582
98.322	4.0	none	345	2,298	371	2,198	371	2,198
98.322	4.0	25.4	275	2,149	280	2,028		
98.322	4.0	25.4	345	2,354	342	2,230	457	2,389

restore the maximum breech pressure to 345 MPa. If the Lagrange gradient equation were to be used, a difference from XKTC of approximately 10% at a charge mass to projectile mass ratio (c/m) of one and approximately 30% at a c/m of four would occur in the predicted values of maximum breech pressure.

The chambrage gradient equation gives calculated maximum breech pressures, which are as much as 7% high and muzzle velocities low by 3 to 4% when compared to XKTC calculations. The resultant gradient structure is illustrated, in Figures 1-6, with representative plots of the ratio of the mean pressure to the base pressure for both XKTC and the analytic chambrage gradient equation. The figures also include histories of breech and base pressure.

3.2 Multiphase Flow. The effects of multiphase flow were examined using XKTC. Calculations were based on a straight chamber; nominal maximum breech pressures of 172 MPa, 345 MPa, and 517 MPa; large and small chamber volumes; and values of c/m equal to .25, 1, and 4. The grain dimensions from the XKTC calculations were used to define equivalent data bases for IBRGA into which the two-phase gradient equation had been encoded.

The pressure at which the propellant burns is taken as the mean pressure over the mixture region instead of the mean pressure over the entire volume behind the projectile, as is conventionally assumed. Comparable Lagrange gradient calculations are also supplied in Table 2.

The calculated maximum breech pressures and velocities for both the two-phase and Lagrange gradient for c/m of .25 are in very close agreement with XKTC at all pressures. For calculations at c/m 's of 1 and 4, the Lagrange and two phase gradients tend to give close maximum breech pressures, at worst about 6% different from XKTC.

The history of the ratio of space mean to base pressures for representative two-phase gradient calculations and for corresponding XKTC calculations is plotted in Figures 7-12, together with associated breech-pressure and base-pressure histories. The Lagrange gradient would be a straight line with a ratio value of 1.083 for a c/m of 0.25; 1.333 for a c/m of 1; and 2.333 for a c/m of 4. The two-phase gradient equation has the same shape and magnitude of the first peak in the mean pressure to base pressure ratio as XKTC, with the maximum occurring earlier in the analytic two-phase gradient calculations and the recovery to the Lagrange ratio value not being as fast as XKTC. The comparison of shape and magnitude of the pressure ratio histories strongly suggests that the dominant physical processes are captured by the two-phase gradient equation. In particular, the undulatory character of the curve is seen to be attributable, at least in part, to the two-phase aspects of the flow, rather than being due to transient or wave propagation phenomena as might at first be thought.

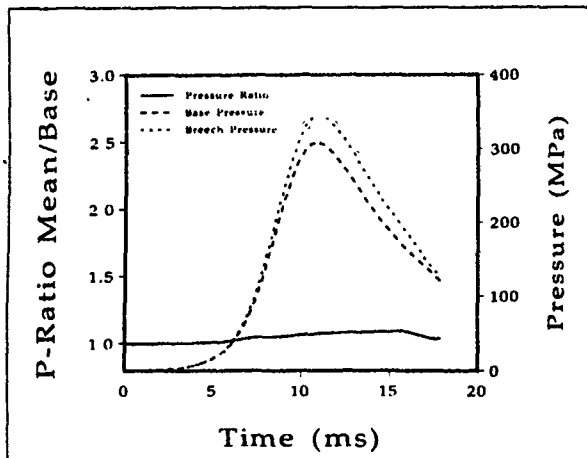


Figure 1. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

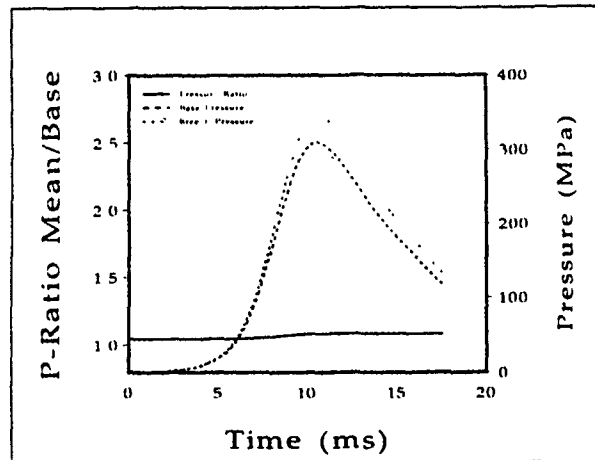


Figure 4. IBRGA Calculation With Chambrage Gradient With Figure 1 Values.

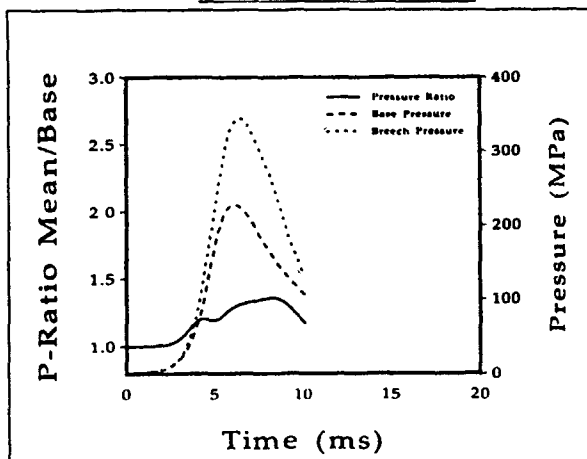


Figure 2. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

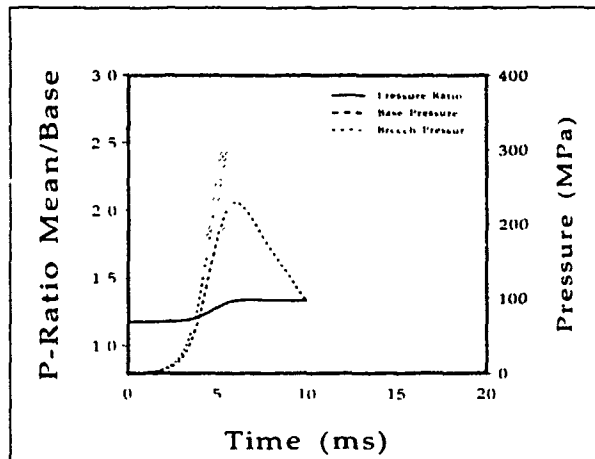


Figure 5. IBRGA Calculation With Chambrage Gradient With Figure 2 Values.

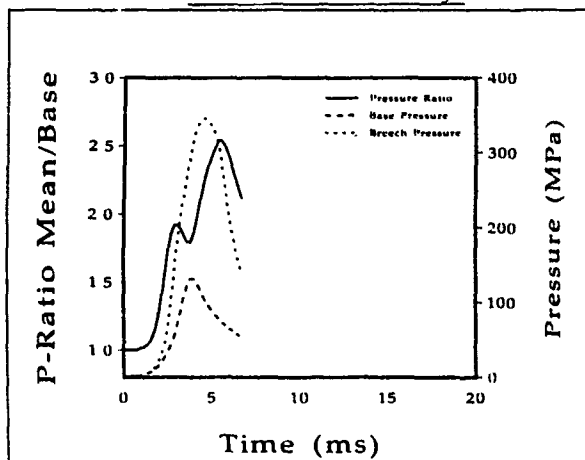


Figure 3. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

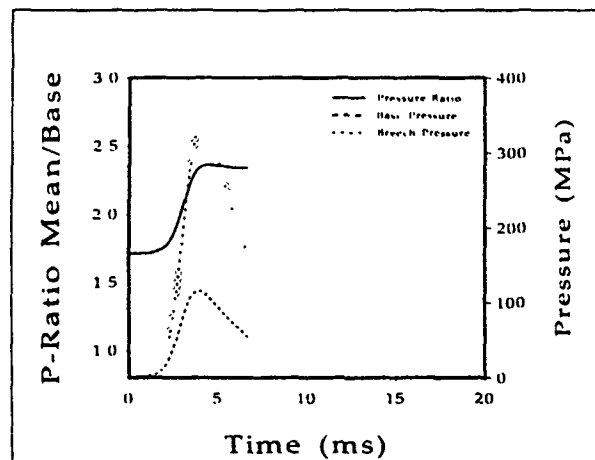


Figure 6. IBRGA Calculation With Chambrage Gradient With Figure 3 Values.

TABLE 2. Two-Phase Calculations.

Chamber volume, cm ³	c/m	XKTC		Two-Phase gradient		Lagrange gradient	
		Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s
9,832.2	0.25	174	549	177	557	179	557
9,832.2	0.25	345	769	348	773	350	775
9,832.2	0.25	518	887	521	891	519	891
9,832.2	1.0	173	957	177	970	184	966
9,832.2	1.0	345	1,352	343	1,355	347	1,363
9,832.2	1.0	516	1,593	498	1,556	497	1,560
9,832.2	4.0	172	1,389	172	1,380	184	1,354
9,832.2	4.0	345	1,964	351	1,930	360	1,962
9,823.2	4.0	518	2,391	524	2,290	528	2,306
98.322	0.25	172	641	177	632	179	630
98.322	0.25	345	895	346	880	348	880
98.322	0.25	518	987	514	974	513	974
98.322	1.0	172	1,130	180	1,107	186	1,094
98.322	1.0	345	1,560	343	1,516	347	1,517
98.322	1.0	516	1,756	510	1,703	512	1,705
98.322	4.0	172	1,682	174	1,581	185	1,524
98.322	4.0	345	2,298	360	2,188	371	2,198
98.322	4.0	518	2,685	545	2,516	552	2,527

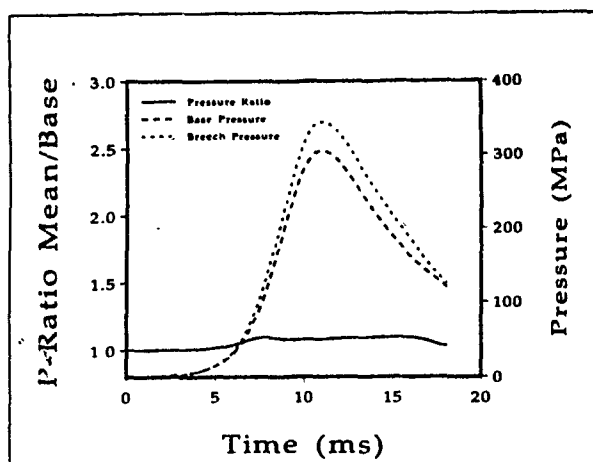


Figure 7. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and No Chambrage.

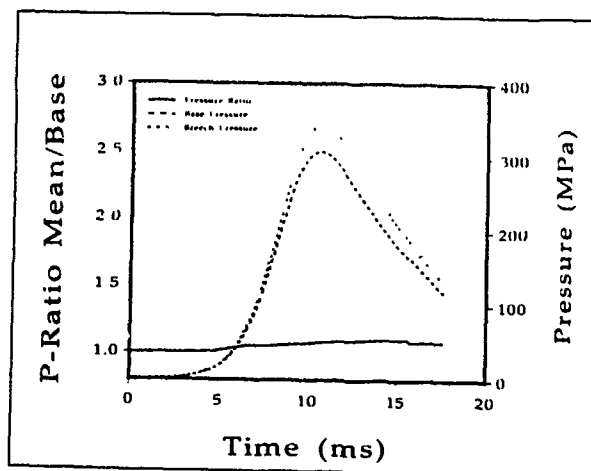


Figure 10. IBRGA Calculation With Two-Phase Gradient With Figure 7 Values.

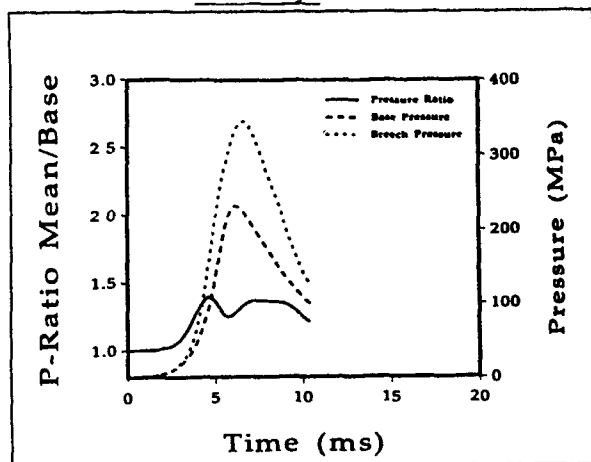


Figure 8. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and No Chambrage.

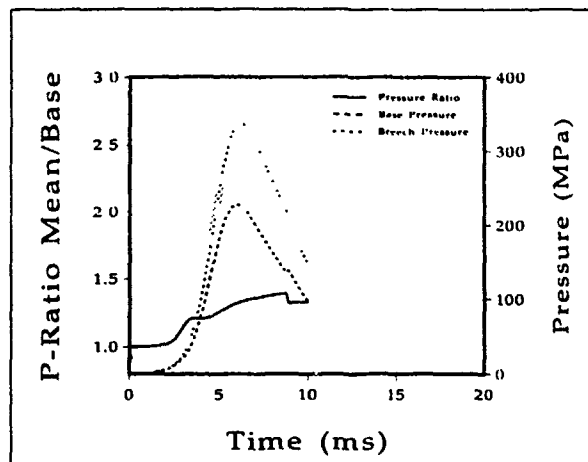


Figure 11. IBRGA Calculation With Two-Phase Gradient With Figure 8 Values.

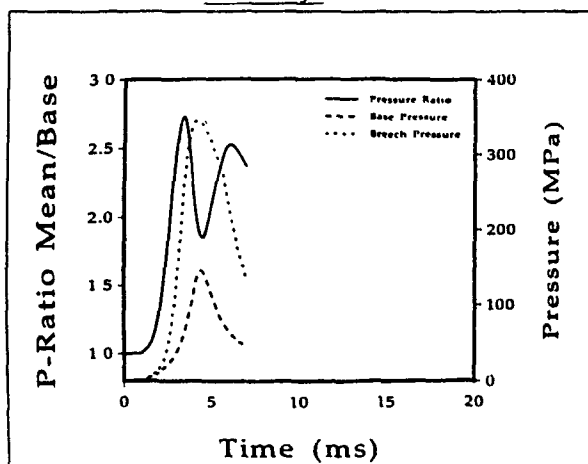


Figure 9. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and No Chambrage.

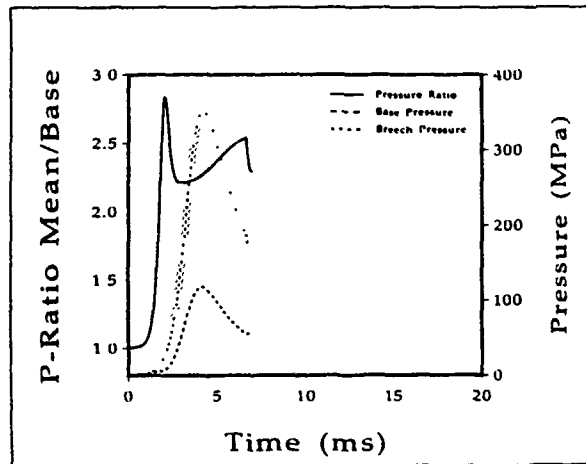


Figure 12. IBRGA Calculation With Two-Phase Gradient With Figure 9 Values.

The drop off of the pressure ratio at the slivering event seen in IBRGA calculations may cause numerical problems. (This event is smoothed over in XKTC since all the grains do not sliver at the same time because of the different pressure fields seen by the grains, as well as the finite time required for the transfer of information.) This problem is not felt to affect the calculated velocity significantly, but a smoothing routine has been incorporated into IBRGA to circumvent any difficulty.

The two-phase gradient equation also has, as input, a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

3.3 Combined. Table 1 is reproduced in Table 3 with the chambrage gradient equation replaced by the RGA gradient equation with the appropriate kinetic energy term. As with the two-phase gradient equation, the burning rate is calculated from mean pressure over the mixture region.

The shapes of the ratio and pressure-time plots, Figures 13-18, are qualitatively similar to those produced by XKTC. This reinforces the idea that both two phase and chambrage effects are important factors in respect to the time-dependent behavior of the pressure gradient. The result of the RGA gradient equation calculations are in general slightly better, when compared to XKTC, than the corresponding calculations with the chambrage gradient equation.

One of the RGA gradient equation inputs is a multiplier to the friction factor. By making this value small, an approximation to stick propellant behavior can be captured.

4. CONCLUSIONS

It is concluded that incorporating chambrage into a gradient equation is important and can make as much as a 20 to 30% difference in predicted maximum breech pressure at c/m of 4. The inclusion of two-phase effects into the gradient equation seems to be well motivated in that a large portion of the structure of the gradient history is captured.

At low c/m 's (0.25), any of the gradient equations will give good results; but as the c/m gets larger (>1), the correction for chambrage is required.

The use of the RGA gradient equation offers a natural way to account for stick propellant phenomena having to do with stick propellant motion and the pressure history in which it burns.

The use of the proper kinetic energy term for the gradient equation, as well as the calculation of the mean pressure over the mixture region, should be included in any gradient model when applicable.

TABLE 3. RGA Calculations.

Chamber volume, cm ³	c/m	Chambrage, mm	XKTC		RGA gradient		Lagrange gradient	
			Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s	Maximum breach pressure, MPa	Muzzle velocity, m/s
9,832.2	0.25	none	345	769	348	773	350	775
9,832.2	0.25	76.2	336	764	336	766		
9,832.2	0.25	76.2	345	774	345	775	358	784
9,832.2	1.0	none	345	1,352	343	1,355	347	1,363
9,832.2	1.0	76.2	310	1,303	304	1,300		
9,832.2	1.0	76.2	345	1,376	337	1,369	384	1,429
9,832.2	4.0	none	345	1,964	351	1,930	360	1,961
9,832.2	4.0	76.2	268	1,764	266	1,747		
9,832.2	4.0	76.2	345	1,964	334	1,964	452	2,183
98.322	0.25	none	345	895	346	879	348	880
98.322	0.25	25.4	337	895	332	872		
98.322	0.25	25.4	345	901	340	878	355	886
98.322	1.0	none	345	1,560	342	1,515	347	1,517
98.322	1.0	25.4	308	1,526	302	1,468		
98.322	1.0	25.4	345	1,595	337	1,535	387	1,582
98.322	4.0	none	345	2,298	361	2,190	371	2,198
98.322	4.0	25.4	275	2,149	272	2,023		
98.322	4.0	25.4	345	2,354	336	2,222	457	2,389

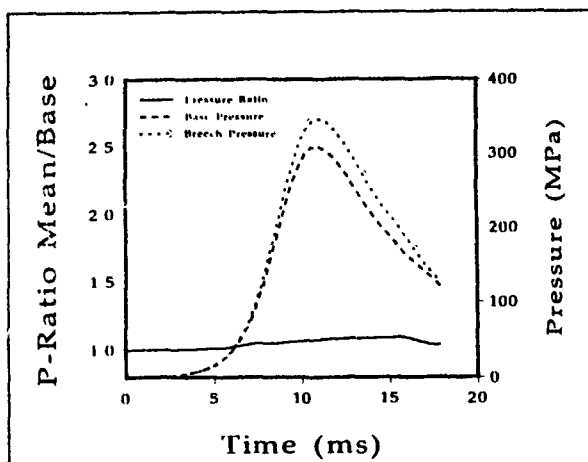


Figure 13. XKTC Calculation With Large Volume, c/m of 0.25, 345 MPa and 76.2 mm of Chambrage.

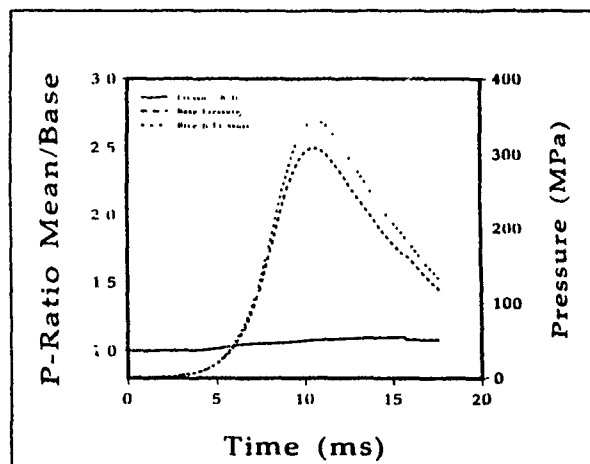


Figure 16. IBRGA Calculation With RGA Gradient With Figure 13 Values.

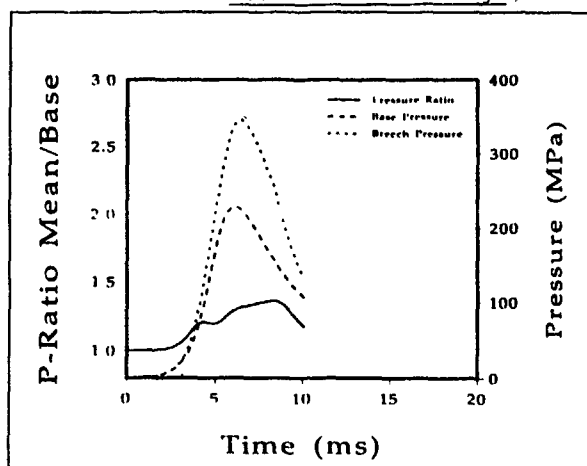


Figure 14. XKTC Calculation With Large Volume, c/m of 1.0, 345 MPa and 76.2 mm of Chambrage.

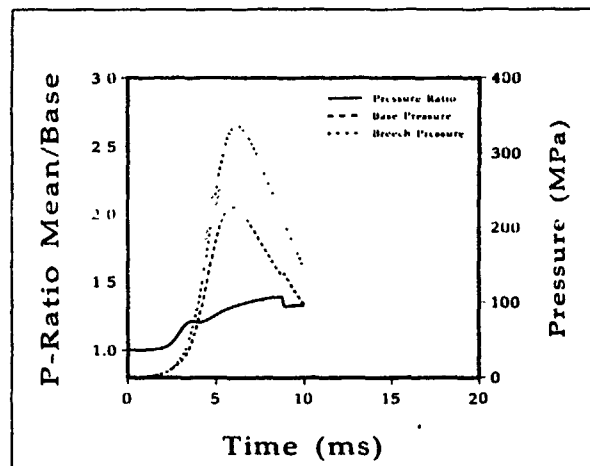


Figure 17. IBRGA Calculation With RGA Gradient With Figure 14 Values.

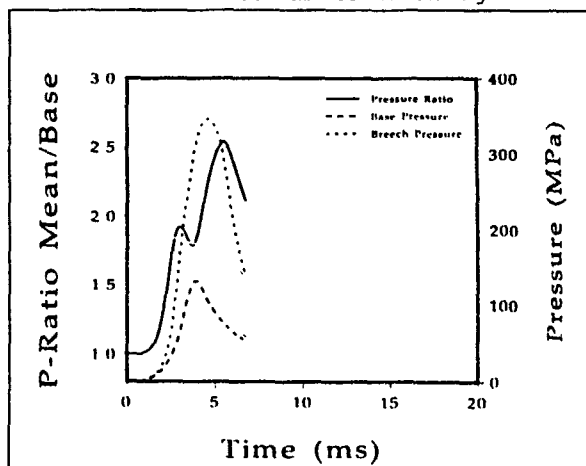


Figure 15. XKTC Calculation With Large Volume, c/m of 4.0, 345 MPa and 76.2 mm of Chambrage.

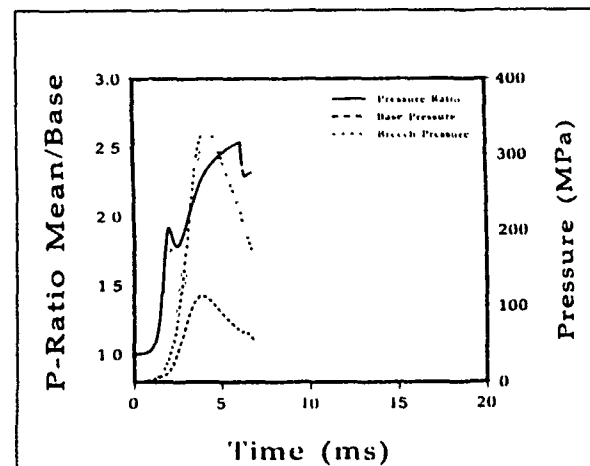


Figure 18. IBRGA Calculation With RGA Gradient With Figure 15 Values.

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6. LIST OF SYMBOLS

ε ,	porosity defined as the volume of gas/total volume
λ ,	particle shape factor for gas flow resistance
μ ,	viscosity of gas
ρ ,	density
ρ_p ,	propellant density
ϕ ,	mass fraction burned
$A(z)$,	area at position z
A_B ,	tube area
C ,	total charge mass
D_p ,	effective diameter of the propellant grain, i.e., $6 \times$ volume of propellant grain/surface area of a propellant grain
f_s ,	interphase drag
f_{so} ,	baseline friction factor
f_{sc} ,	dimensionless friction factor, a function of Reynolds number and particle shape
GD ,	grain diameter
GL ,	grain length
g_0 ,	a constant to reconcile dimensions
L ,	the distance between the leading edge of the propellant bed and the base of the projectile, i.e., $L = (z_p - z_b)$
$\dot{m}(t)$,	rate of combustion of propellant
m ,	mass of propellant burned
M_p ,	mass of projectile
m_w ,	molecular weight of propellant gas
P ,	pressure
P_B ,	projectile base pressure
P_{Br} ,	breech pressure
P_m ,	mean pressure
P_{res} ,	resistive pressure
R ,	universal gas constant
REN ,	particle reynolds number
S ,	surface area of the propellant
T ,	gas temperature
t ,	time
u ,	velocity

U_g , gas velocity on the mixture side at the leading edge of the propellant bed, i.e., $u(z_b)$
 U_{g*} , gas velocity on the ullage side of the leading edge of the propellant bed
 u_p , propellant velocity
 U_p , velocity of the leading edge of the propellant bed, i.e., $u_p(z_b)$
 $V(z)$, volume between the breech face and position z
 V_F , free volume for gases to expand into
 V_o , initial empty volume of chamber
 V_p , velocity of the projectile
 x , distance burned into the propellant
 z , axial position measured from the breech face
 z_b , position of the leading edge of the propellant bed
 z_p , position of the base of the projectile
 z_{po} , initial position of the base of the projectile
 $\dot{}$, derivative with respect to time
 $\ddot{}$, second derivative with respect to time

APPENDIX 1:
INFLUENCE OF CHAMBRAGE

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The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

Influence of Chambrage. For the effects of chambrage on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech and the base of the projectile, and all variations in tube area are confined to the chamber.

The continuity and momentum equations for unsteady flow of a homogeneous inviscid substance through a tube with variable area are

$$\frac{\partial \rho}{\partial t} + \frac{1}{A} \left(\frac{\partial \rho A u}{\partial z} \right) = 0 \quad (C.1)$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho \frac{\partial P}{\partial z} = 0. \quad (C.2)$$

Making the Lagrange assumption

$$\frac{\partial \rho}{\partial z} = 0, \quad (C.3)$$

then C.1 becomes

$$\frac{\partial \rho}{\partial t} = - \frac{\rho}{A} \left(\frac{\partial A u}{\partial z} \right). \quad (C.4)$$

The density may be written as

$$\rho = \frac{C}{V(z_p)}. \quad (C.5)$$

Differentiating (C.5), we get

$$\frac{d\rho}{dt} = - \left(\frac{C}{V^2(z_p)} \right) \left(\frac{dV(z_p)}{dt} \right). \quad (C.6)$$

But,

$$V(z_p) = V(z_{po}) + (z_p - z_{po}) A_B, \quad (C.7)$$

and

$$\frac{dV(z_p)}{dt} = A_B \dot{z}_p = A_B V_p, \quad (C.8)$$

therefore,

$$\frac{d\rho}{dt} = - \frac{CA_B V_p}{V^2(z_p)}. \quad (C.9)$$

But,

$$\rho = \frac{C}{V(z_p)},$$

therefore,

$$\frac{d\rho}{dt} = - \frac{A_B \rho V_p}{V(z_p)}. \quad (C.10)$$

Substituting (C.10) into (C.4), we get

$$- \frac{A_B \rho V_p}{V(z_p)} = - \frac{\rho}{A} \left(\frac{\partial A u}{\partial z} \right) \quad (C.11)$$

or

$$\frac{A_B A V_p}{V(z_p)} = \frac{\partial A u}{\partial z}. \quad (C.12)$$

Integrating (C.12) with $u(0) = 0$ and noting that V_p and $V(z_p)$ are functions of time only,

$$A(z)u(z) = Au = \frac{A_B V_p}{V(z_p)} \int_0^z A(z) dz = \frac{A_B V_p V(z)}{V(z_p)}. \quad (C.13)$$

Therefore, the velocity u at a given time is given by:

$$u = \left(\frac{A_B V_p}{V(z_p)} \right) \left(\frac{V(z)}{A(z)} \right). \quad (C.14)$$

Note that if $A = A_B$, then $V(z) = A_B z$, then $u = (z/z_p) V_p$ as for the Lagrange gradient. (C.15)

The pressure distribution follows from the momentum equation (C.2)

$$g_0 \frac{\partial P}{\partial z} = - \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right). \quad (C.16)$$

Differentiating (C.14), we get $\partial u / \partial t$ and $\partial u / \partial z$ for substitution into (C.16).

To get $\partial u/\partial t$ we differentiate (C.14), noting that only V_p and $V(z_p)$ are functions of time and

$$\frac{\partial V(z_p)}{\partial t} = \frac{\partial(V(z_{p0}) + (z_p - z_{p0}) A_B)}{\partial t} = A_B \dot{z}_p = A_B V_p, \quad (C.17)$$

then

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{A_B \dot{V}_p V(z)}{A(z) V(z_p)} - \left(\frac{A_B V_p V(z)}{A(z) V^2(z_p)} \right) \left(\frac{\partial V(z_p)}{\partial t} \right) \\ &= \frac{A_B \dot{V}_p V(z)}{A(z) V(z_p)} - \frac{A_B^2 V_p^2 V(z)}{A(z) V^2(z_p)}. \end{aligned} \quad (C.18)$$

To get $\partial u/\partial z$, we differentiate (C.14), noting that only $V(z)$ and $A(z)$ are functions of z and

$$\frac{\partial V(z)}{\partial z} = \frac{\partial \int_0^z A(z) dz}{\partial z} = A(z), \quad (C.19)$$

then

$$\begin{aligned} \frac{\partial u}{\partial z} &= \frac{A_B V_p A(z)}{V(z_p) A(z)} - \left(\frac{A_B V_p V(z)}{A^2(z) V(z_p)} \right) \left(\frac{dA}{dz} \right) \\ &= \frac{A_B V_p}{V(z_p)} - \left(\frac{A_B V_p V(z)}{A^2(z) V(z_p)} \right) \left(\frac{dA}{dz} \right). \end{aligned} \quad (C.20)$$

Substituting (C.5), (C.14), (C.18), and (C.20) into (C.16) yields

$$g_o \frac{\partial P}{\partial z} = - \frac{CA_B V(z) \dot{V}_p}{A(z) V^2(z_p)} + \left(\frac{CV^2(z) V_p^2 A_B^2}{A^3(z) V^3(z_p)} \right) \left(\frac{\partial A}{\partial z} \right). \quad (C.21)$$

Integrating (C.21), and noting $\dot{V}_p$ and $V(z_p)$ are functions only of time, then

$$\begin{aligned} g_o \int_0^z \partial P &= g_o (P(z) - P(0)) \\ &= - \frac{CA_B \dot{V}_p}{V^2(z_p)} \int_0^z \frac{V(z) dz}{A(z)} \\ &\quad + \frac{CV_p^2 A_B^2}{V^3(z_p)} \int_0^z \left(\frac{V^2(z)}{A^3(z)} \right) \left(\frac{\partial A}{\partial z} \right) dz. \end{aligned} \quad (C.22)$$

Noting

$$\int_0^z \left(\frac{V^2(z)}{A^3(z)} \right) \left(\frac{\partial A}{\partial z} \right) dz = \int_0^z \frac{V(z)}{A(z)} dz - \frac{V^2(z)}{2A^2(z)}$$

(from integration by parts), then (C.22), the pressure distribution behind the projectile is

$$\begin{aligned} g_o (P(z) - P(0)) &= \left(\frac{CA_B^2 V_p^2}{V^3(z_p)} - \frac{CA_B \dot{V}_p}{V^2(z_p)} \right) J_1(z) \\ &\quad - \frac{CV_p^2 A_B^2}{2V^3(z_p)} J_2(z), \end{aligned} \quad (C.23)$$

where

$$J_1(z) = \int_0^z \frac{V(z)}{A(z)} dz, \quad (C.24)$$

which can be evaluated algebraically or numerically if $V(z)$ and $A(z)$ are known, and

$$J_2(z) = \frac{V^2(z)}{A^2(z)}. \quad (C.25)$$

Rewriting (C.23) in the form

$$P(z) = P(0) + a(t)J_1(z) + b(t)J_2(z) \quad (C.26)$$

gives

$$a(t) = \frac{1}{g_o} \left(\frac{CA_B^2 V_p^2}{V^3(z_p)} - \frac{CA_B \dot{V}_p}{V^2(z_p)} \right) \quad (C.27)$$

and

$$b(t) = - \frac{CV_p^2 A_B^2}{2g_o V^3(z_p)}. \quad (C.28)$$

Defining the mean pressure in the chamber to be

$$P_m = \frac{\int_0^{z_p} P(z) A(z) dz}{\int_0^{z_p} A(z) dz}, \quad (C.29)$$

then using (C.26),

$$P_m = \frac{\int_0^{z_p} [P(0) + a(t)J_1(z) + b(t)J_2(z)] A(z)dz}{\int_0^{z_p} A(z)dz} \quad (C.30)$$

$$\begin{aligned} P_m &= \frac{P(0) \int_0^{z_p} A(z)dz}{V(z_p)} \\ &+ \frac{a(t) \int_0^{z_p} J_1(z)A(z)dz}{V(z_p)} \\ &+ \frac{b(t) \int_0^{z_p} J_2(z)A(z)dz}{V(z_p)} \end{aligned} \quad (C.31)$$

or

$$P_m = P(0) + \frac{a(t)}{V(z_p)} J_3(z_p) + \frac{b(t)}{V(z_p)} J_4(z_p) \quad (C.32)$$

where

$$\begin{aligned} J_3(z_p) &= \int_0^{z_p} A(z)J_1(z)dz \\ &= \int_0^{z_p} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A(z)dz \end{aligned} \quad (C.33)$$

and

$$\begin{aligned} J_4(z_p) &= \int_0^{z_p} A(z)J_2(z)dz \\ &= \int_0^{z_p} \frac{V^2(z)}{A(z)} dz, \end{aligned} \quad (C.34)$$

and where, if $A(z)$ and $V(z)$ are known, $J_3(z_p)$ and $J_4(z_p)$ may be evaluated algebraically or numerically. The acceleration of the projectile is given by

$$\dot{V}_p = \frac{g_0 A_B}{M_p} [P_B - P_{ra}]. \quad (C.35)$$

Substitution into (C.27) gives

$$a(t) = a_1(t) + a_2(t)P_B \quad (C.36)$$

where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_p)} \left(\frac{A_B V_p^2}{V(z_p)} + \frac{g_o A_B P_{ra}}{M_p} \right) \quad (C.37)$$

and

$$a_2(t) = - \frac{CA_B^2}{M_p V^2(z_p)} \quad (C.38)$$

Substituting (C.36) into (C.26) and (C.32) and noting $P_{Br} = P(0)$,

then

$$P_{Br} = [1 - a_2(t)J_1(z_p)]P_B - a_1(t)J_1(z_p) - b(t)J_2(z_p) \quad (C.39)$$

and

$$P_m = P_{Br} + \frac{[a_1(t) + a_2(t)P_B]}{V(z_p)} J_3(z_p) + \frac{b(t)}{V(z_p)} J_4(z_p). \quad (C.40)$$

Substituting (C.39) into (C.40) and rearranging terms,

$$\begin{aligned} P_m = & \left(1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)} \right) P_B - a_1(t)J_1(z_p) \\ & - b(t)J_2(z_p) + \frac{a_1(t)J_3(z_p)}{V(z_p)} + \frac{b(t)J_4(z_p)}{V(z_p)}. \end{aligned} \quad (C.41)$$

Solving (C.41), we get P_B in terms of P_m

$$\begin{aligned} P_m + a_1(t)J_1(z_p) + b(t)J_2(z_p) - \frac{a_1(t)J_3(z_p)}{V(z_p)} - \frac{b(t)J_4(z_p)}{V(z_p)} \\ P_B = \frac{\quad}{1 - a_2(t)J_1(z_p) + \frac{a_2(t)J_3(z_p)}{V(z_p)}} \end{aligned} \quad (C.42)$$

Therefore, assuming the mean pressure, P_m , can be calculated from an equation of state, then (C.42) will give the projectile base pressure, P_B . The breech pressure, P_{Br} , is calculated from (C.39) knowing P_B . The pressure distribution is given by (C.26).

The evaluation of $J_1(z_p)$, $J_2(z_p)$, $J_3(z_p)$, and $J_4(z_p)$ can be simplified by noting that the variation in area is confined to the chamber and, therefore,

$$V(z) = V(z_{po}) + A_B(z - z_{po}), \text{ for } z \geq z_{po}, \quad (C.43)$$

and

$$\begin{aligned} J_1(z_p) &= \int_0^{z_p} \frac{V(z)}{A(z)} dz = \int_0^{z_{po}} \frac{V(z)}{A(z)} dz + \int_{z_{po}}^{z_p} \frac{V(z)}{A(z)} dz \\ &= J_1(z_{po}) + \int_{z_{po}}^{z_p} \frac{V(z_{po}) + A_B(z - z_{po})}{A_B} dz \\ &= J_1(z_{po}) + \frac{1}{A_B} \left(V(z_{po}) (z_p - z_{po}) + \frac{A_B(z_p - z_{po})^2}{2} \right) \end{aligned} \quad (C.44)$$

$$J_2(z_p) = \frac{V^2(z)}{A^2(z)} \Big|_{z_p} = \frac{[V(z_{po}) + A_B(z_p - z_{po})]^2}{A_B^2}, \quad (C.45)$$

$$\begin{aligned} J_3(z_p) &= \int_0^{z_p} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A(z) dz \\ &= \int_0^{z_{po}} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A(z) dz + \int_{z_{po}}^{z_p} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A_B dz \\ &= J_3(z_{po}) + \int_{z_{po}}^{z_p} \left(J_1(z_{po}) + \frac{1}{A_B} \left(V(z_{po})(z - z_{po}) + \frac{A_B}{2} (z - z_{po})^2 \right) \right) A_B dz \\ &= J_3(z_{po}) + A_B J_1(z_{po}) (z_p - z_{po}) + \frac{V(z_{po}) (z_p - z_{po})^2}{2} \\ &\quad + \frac{A_B}{6} (z_p - z_{po})^3, \end{aligned} \quad (C.46)$$

$$\begin{aligned} J_4(z_p) &= \int_0^{z_p} \frac{V^2(z)}{A(z)} dz = \int_0^{z_{po}} \frac{V^2(z)}{A(z)} dz + \int_{z_{po}}^{z_p} \frac{[V(z_{po}) + A_B(z - z_{po})]^2}{A_B} dz \\ &= J_4(z_{po}) + \frac{[V(z_{po}) + A_B(z_p - z_{po})]^3 - V^3(z_{po})}{3A_B^2}. \end{aligned} \quad (C.47)$$

Equations (C.44) - (C.47) require the evaluation of the integrals $J_1(z_{po})$ - $J_4(z_{po})$ only once.

The kinetic energy (KE) of the gas/solid mixture will be required, and since $\rho dV = dm$ and $dV = A(z) dz$, then

$$g_o KE = \frac{1}{2} \int_0^{z_p} u^2 dm = \frac{1}{2} \int_0^{z_p} u^2 \rho dV = \frac{1}{2} \int_0^{z_p} u^2 \rho A(z) dz \quad (C.48)$$

and from (C.5),

$$\rho = \frac{C}{V(z_p)},$$

and from (C.14)

$$u = \frac{A_B V_p V(z)}{V(z_p) A(z)},$$

therefore,

$$\begin{aligned} g_o KE &= \frac{1}{2} \int_0^{z_p} \frac{C A_B^2 V_p^2 V^2(z)}{A(z) V^3(z_p)} dz \\ &= \frac{C A_B^2 V_p^2}{2 V^3(z_p)} \left(\int_0^{z_p} \frac{V^2(z)}{A(z)} dz \right), \end{aligned}$$

or

$$KE = \frac{C A_B^2 V_p^2}{2 g_o V^3(z_p)} J_4(z_p). \quad (C.49)$$

APPENDIX 2:
INFLUENCE OF PROPELLANT VELOCITY LAG

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The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

Influence of Propellant Velocity Lag. For the effects of the propellant velocity lag on the form of the gradient equation, the propellant is assumed to be uniformly distributed between the breech face and the leading edge of the bed, with the leading edge of the bed being initially at the base of the projectile. The tube diameter is the same as the bore diameter.

The continuity equations for each of the phases in the mixture region are

$$\frac{\partial \epsilon \rho}{\partial t} + \frac{\partial \epsilon \rho u}{\partial z} = \dot{m}(t). \quad (L.1)$$

$$\rho_p \frac{\partial \epsilon}{\partial t} - \frac{\partial [(1-\epsilon) \rho_p u_p]}{\partial z} = \dot{m}(t). \quad (L.2)$$

Making assumptions analogous to the Lagrange assumption,

$$\frac{\partial \rho}{\partial z} = 0 \quad (\text{gas density constant throughout the tube}) \quad (L.3)$$

$$\frac{\partial \epsilon}{\partial z} = 0 \quad (\text{porosity constant throughout the mixture region}) \quad (L.4)$$

and noting the assumption that $\dot{m}(t)$ is only a function of time, then from (L.2),

$$\rho_p \frac{\partial \epsilon}{\partial t} - u_p \frac{\partial (1-\epsilon) \rho_p}{\partial z} - (1-\epsilon) \rho_p \frac{\partial u_p}{\partial z} = \dot{m}(t). \quad (L.5)$$

Noting (L.4), then

$$u_p \frac{\partial (1-\epsilon) \rho_p}{\partial z} = 0. \quad (L.6)$$

Therefore,

$$(1-\epsilon) \rho_p \frac{\partial u_p}{\partial z} = \rho_p \frac{\partial \epsilon}{\partial t} - \dot{m}(t) = f(t) \quad (L.7)$$

and

$$u_p(z) = \int \partial u_p = \frac{\rho_p \frac{\partial \epsilon}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p} z + f_1(t). \quad (L.8)$$

Since $u_p(0) = 0$, then $f_1(t) = 0$, and

$$u_p(z_b) = U_p = \frac{\rho_p \frac{\partial \epsilon}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p} z_b,$$

or

$$\frac{U_p}{z_b} = \frac{\rho_p \frac{\partial \epsilon}{\partial z} - \dot{m}(t)}{(1-\epsilon) \rho_p}. \quad (L.9)$$

Substituting (L.9) into (L.8) gives

$$u_p(z) = \frac{U_p}{z_b} z. \quad (L.10)$$

From (L.1),

$$\frac{\partial \epsilon \rho}{\partial t} + u \left(\rho \frac{\partial \epsilon}{\partial z} + \epsilon \frac{\partial \rho}{\partial z} \right) + \epsilon \rho \frac{\partial u}{\partial z} = \dot{m}(t),$$

and using (L.3) and (L.4), then

$$\frac{\partial u}{\partial z} = \frac{-\frac{\partial \epsilon \rho}{\partial t} + \dot{m}(t)}{\epsilon \rho} \quad (\text{a function of time only}). \quad (L.11)$$

Integrating (L.11) gives

$$u(z) = \int \partial u = \frac{-\frac{\partial \epsilon \rho}{\partial t} + \dot{m}(t)}{\epsilon \rho} z + f_2(t). \quad (L.12)$$

Since $u(0) = 0$, then $f_2(t) = 0$, and

$$u(z_b) = U_s = \frac{-\frac{\partial \epsilon \rho}{\partial t} + \dot{m}(t)}{\epsilon \rho} z_b$$

or

$$\frac{U_s}{z_b} = \frac{-\frac{\partial \epsilon \rho}{\partial t} + \dot{m}(t)}{\epsilon \rho}. \quad (L.13)$$

Substituting (L.13) into (L.12) gives

$$u(z) = \frac{U_g}{z_b} z. \quad (L.14)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\epsilon (U_g - U_p) = U_{g+} - U_p. \quad (L.15)$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption L.3.

Also, in the ullage region, the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0. \quad (L.16)$$

Using (L.3), we get

$$\frac{\partial u}{\partial z} = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right) \quad (\text{a function of time only}). \quad (L.17)$$

Integrating (L.17) we get

$$u(z) = - \frac{1}{\rho} \left(\frac{d\rho}{dt} \right) z + f_3(t). \quad (L.18)$$

Using the boundary condition

$$u(z_p) = V_p \quad (L.19)$$

gives

$$u(z) = V_p + (z_p - z) \frac{d \ln \rho}{dt}. \quad (L.20)$$

Evaluating (L.20) at z_b gives

$$U_{g+} = V_p + \frac{L}{\rho} \left(\frac{d\rho}{dt} \right) = V_p + L \frac{d \ln \rho}{dt}. \quad (L.21)$$

Eliminating U_{g+} from (L.15) with (L.21) gives

$$\epsilon (U_g - U_p) = V_p + L \frac{d \ln \rho}{dt} - U_p \quad (L.22)$$

or

$$U_s = U_p + \frac{1}{\epsilon} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right). \quad (L.23)$$

Differentiating (L.23) yields

$$\begin{aligned} \dot{U}_s = & - \dot{U}_p \left(\frac{1-\epsilon}{\epsilon} \right) - \frac{\dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \\ & + \frac{\dot{V}_p}{\epsilon} + \frac{L}{\epsilon} \left(\frac{d^2 \ln \rho}{dt^2} \right) + \frac{L}{\epsilon} \left(\frac{d \ln \rho}{dt} \right). \end{aligned} \quad (L.24)$$

We anticipate the need to factor out the base pressure P_B dependence of both $\dot{V}_p$ and $d^2 \ln \rho / dt^2$.

The projectile acceleration is given by

$$\dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{rco}}{M_p}. \quad (L.25)$$

To get the base pressure P_B dependence associated with $d^2 \ln \rho / dt^2$, we start with the definition of the gas density

$$\rho = \frac{m}{V_F} \quad (L.26)$$

where m = mass of gas burned and V_F = free volume. Therefore,

$$V_F = V_o + A_B (z_p - z_{po}) - \frac{C}{\rho_p} + \frac{m}{\rho_p} \quad (L.27)$$

and

$$\dot{V}_F = A_B \dot{z}_p + \frac{\dot{m}}{\rho_p} = A_B \dot{V}_p + \frac{\dot{m}}{\rho_p} \quad (L.28)$$

and

$$\ddot{V}_F = A_B \ddot{V}_p + \frac{\ddot{m}}{\rho_p} \quad (L.29)$$

where it is understood that m/ρ_p , $\dot{m}/\rho_p$, and $\ddot{m}/\rho_p$ may be in the form $\sum \frac{m_i}{\rho_{p_i}}$, $\sum \frac{\dot{m}_i}{\rho_{p_i}}$, $\sum \frac{\ddot{m}_i}{\rho_{p_i}}$, where the i

index refers to different propellants.

From (L.26) we get

$$\ln \rho = \ln m - \ln V_F, \quad (L.30)$$

and

$$\frac{d \ln \rho}{dt} = \frac{\dot{m}}{m} - \frac{\dot{V}_F}{V_F} \quad (L.31)$$

and

$$\frac{d^2 \ln \rho}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{\ddot{V}_F}{V_F} + \frac{\dot{V}_F^2}{V_F^2}. \quad (L.32)$$

Substituting (L.29) and (L.25), we get

$$\frac{d^2 \ln \rho}{dt^2} = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{V_F M_p}, \quad (L.33)$$

where

$$c_1(t) = \ddot{m} \left(\frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}.$$

We still require $\ddot{m}$, which we get from

$$\dot{m} = \rho_p S \frac{dx}{dt} \quad (L.34)$$

where S is the surface area of the propellant and dx/dt is the burning rate of the propellant, where dx/dt is of the form

$$\frac{dx}{dt} = a P_m^n. \quad (L.35)$$

Therefore, from (L.34),

$$\begin{aligned} \ddot{m} &= \rho_p S \frac{d^2 x}{dt^2} + \rho_p S \frac{dx}{dt} \frac{dS}{dx} \\ &= \rho_p S \frac{d^2 x}{dt^2} + \rho_p S \left(\frac{dx}{dt} \right)^2 \frac{dS}{dx} \end{aligned} \quad (L.36)$$

and from (L.35),

$$\frac{d^2 x}{dt^2} = a n P_m^{n-1} \frac{dP_m}{dt} \quad (L.37)$$

where dP_m/dt is determined numerically or from

$$\frac{dP_m}{dt} = \frac{\frac{mRT}{m_w} - P_m \dot{V} + \frac{mRT}{m_w}}{V}.$$

Also defining ϕ as the mass fraction of propellant burned, then

$$\rho = \frac{\phi C - \rho A_B L}{\epsilon V(z_b)} = \frac{\phi_* C}{\epsilon V(z_b)} \quad (L.38)$$

where

$$\phi_* = \phi - \frac{\rho A_B L}{C} \quad (L.39)$$

and

$$\rho_p = \frac{(1-\phi)C}{(1-\epsilon)V(z_b)}. \quad (L.40)$$

Differentiating (L.39), we get

$$\dot{\phi}_* = \dot{\phi} - \frac{\rho A_B}{C} \left(L + L \frac{d \ln \rho}{dt} \right). \quad (L.41)$$

Solving (L.40) for ϵ , we get

$$\epsilon = 1 - \frac{(1-\phi)C}{\rho_p V(z_b)}. \quad (L.42)$$

Differentiating (L.42) and noting that

$$V(z_b) = V(z_{po}) + (z_b - z_{po}) A_B \quad (L.43)$$

and, therefore,

$$\dot{V}(z_b) = \dot{z}_b A_B = U_p A_B, \quad (L.44)$$

then

$$\dot{\epsilon} = \frac{\dot{\phi} C}{\rho_p V(z_b)} + \frac{(1-\phi) C \dot{V}(z_b)}{\rho_p V^2(z_b)} = \frac{\dot{\phi} C}{\rho_p V(z_b)} + \frac{(1-\phi) C U_p A_B}{\rho_p V^2(z_b)}. \quad (L.45)$$

We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distribution in the mixture region.

Consider the momentum equations for each phase

$$\frac{\partial \epsilon \rho u}{\partial t} + \frac{\partial \epsilon \rho u^2}{\partial z} + \epsilon g_o \frac{\partial P}{\partial z} = -f_s + \dot{m} u_p \quad (L.46)$$

$$\frac{\partial (1-\epsilon) \rho_p u_p}{\partial t} + \frac{\partial (1-\epsilon) \rho_p u_p^2}{\partial z} + (1-\epsilon) g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p \quad (L.47)$$

where

$$f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc} \quad (L.48)$$

and

$$f_{sc} = \begin{cases} \frac{2.5\lambda}{REN^{.081}} f_{so} & \epsilon < \epsilon_o \\ \frac{2.5\lambda}{REN^{.081}} \left(\frac{(1-\epsilon)}{(1-\epsilon_o)} \frac{\epsilon_o}{\epsilon} \right)^{0.45} f_{so}, & \epsilon \geq \epsilon_o \end{cases} \quad (L.49)$$

where

$$\lambda = \left(\frac{0.5 + \frac{GL}{GD}}{\left(1.5 \frac{GL}{GD} \right)^{2/3}} \right)^{2.17} \quad (L.50)$$

and

$$REN = \rho D_p |U_s - U_p|/\mu.$$

Adding (L.46) and (L.47), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{\partial \epsilon \rho u}{\partial t} - \frac{\partial (1-\epsilon) \rho_p u_p}{\partial t} - \frac{\partial \epsilon \rho u^2}{\partial z} - \frac{\partial (1-\epsilon) \rho_p u_p^2}{\partial z} \quad (L.51)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{\partial}{\partial t} [\epsilon \rho u + (1-\epsilon) \rho_p u_p] - \frac{\partial}{\partial z} [\epsilon \rho u^2 + (1-\epsilon) \rho_p u_p^2]. \quad (L.52)$$

From (L.10) we get $u_p(z) = U_p[z/z_b]$, from (L.14) we get $u(z) = U_s[z/z_b]$, from (L.38) we get

$$\varepsilon\rho = \frac{\phi C - \rho A_B L}{V(z_b)} = \frac{\phi C}{V(z_b)},$$

and from (L.40) we get

$$(1-\varepsilon)\rho_p = \frac{(1-\phi)C}{V(z_b)},$$

which, when substituted into (L.52), yields

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{\partial}{\partial t} \left(\frac{\phi C U_s z}{V(z_b) z_b} + \frac{(1-\phi) C U_p z}{V(z_b) z_b} \right) \\ & - \frac{\partial}{\partial z} \left(\frac{\phi C U_s^2 z^2}{V(z_b) z_b^2} + \frac{(1-\phi) C U_p^2 z^2}{V(z_b) z_b^2} \right). \end{aligned} \quad (L.53)$$

Differentiating (L.53) and noting z_b , $V(z_b)$, ϕ , ϕ , U_s , and U_p are functions of time only and that $\dot{z}_b = U_p$, $\dot{V}(z_b) = A_B U_p$, and $V(z_b) = A_B z_b$, we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - zC \left(\frac{\phi \dot{U}_s + \dot{\phi} U_s + (1-\phi) \dot{U}_p - \dot{\phi} U_p}{z_b V(z_b)} \right) \\ & + \frac{zC [\phi U_s + (1-\phi) U_p] \dot{z}_b}{z_b^2 V(z_b)} \\ & + \frac{zC [\phi U_s + (1-\phi) U_p] \dot{V}(z_b)}{z_b V^2(z_b)} \\ & - \frac{2zC [\phi U_s^2 + (1-\phi) U_p^2]}{z_b^2 V(z_b)} \end{aligned} \quad (L.54)$$

or

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{zC}{z_b V(z_b)} \left(\phi \dot{U}_s + \dot{\phi} U_s + (1-\phi) \dot{U}_p - \dot{\phi} U_p \right. \\ & - \frac{[\phi U_s + (1-\phi) U_p] U_p}{z_b} - \frac{[\phi U_s + (1-\phi) U_p] U_p}{z_b} \\ & \left. + 2 \frac{[\phi U_s^2 + (1-\phi) U_p^2]}{z_b} \right) \end{aligned} \quad (L.55)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_s \dot{U}_s + \phi_s U_s + (1-\phi) \dot{U}_p - \phi U_p + \frac{2\phi_s U_s (U_s - U_p)}{z_b} \right). \quad (L.56)$$

To get $\dot{U}_p$ for substitution into (L.56), we use another form of the solid phase momentum equation

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} = \frac{f_s}{(1-\epsilon)\rho_p}, \quad (L.57)$$

and noting that at $z = z_b$, $\partial u_p(z_b)/\partial t = \dot{U}_p$, and that $\partial U_p/\partial z = 0$ (since U_p is a function of time only), then

$$\dot{U}_p + \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \bigg|_{z_b} = \frac{f_s}{(1-\epsilon)\rho_p}. \quad (L.58)$$

Substituting (L.48) for f_s in (L.58), we get

$$\dot{U}_p = \frac{\rho}{D_p \rho_p} \frac{(U_s - U_p)^2}{f_{sc}} - \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \bigg|_{z_b}. \quad (L.59)$$

Substituting (L.24) into (L.56), we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{zC}{z_b V(z_b)} \left(\phi_s \frac{\dot{V}_p}{\epsilon} - \frac{(1-\epsilon)}{\epsilon} \dot{U}_p \phi_s \right) \\ & - \frac{\dot{\epsilon}}{\epsilon^2} \phi_s \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L \phi_s}{\epsilon} \frac{d^2 \ln \rho}{dt^2} \\ & + \frac{\phi_s L}{\epsilon} \frac{d \ln \rho}{dt} + \phi_s U_s + (1-\phi) \dot{U}_p - \phi U_p \\ & + \frac{2\phi_s U_s}{z_b} (U_s - U_p) \bigg). \end{aligned} \quad (L.60)$$

Substituting (L.58) into (L.60), we get

$$\begin{aligned}
 g_o \frac{\partial P}{\partial z} = & - \frac{zC}{z_b V(z_b)} \left(\phi_* \frac{\dot{V}_p}{\epsilon} + \dot{\phi}_* U_s - \dot{\phi} U_p \right. \\
 & - \frac{\dot{\epsilon} \phi_*}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{\phi_* L}{\epsilon} \frac{d^2 \ln \rho}{dt^2} \\
 & + \frac{\phi_* L}{\epsilon} \frac{d \ln \rho}{dt} + \frac{2 \phi_* U_s}{z_b} (U_s - U_p) + \frac{\phi_2 \rho}{D_p \rho_p} (U_s - U_p)^2 f_{sc} \\
 & \left. - \frac{\phi_2 g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \right) \quad (L.61)
 \end{aligned}$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_1 - \frac{\phi_2 g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \right) \quad (L.62)$$

where

$$\phi_1 = \phi_1' + \frac{\phi_* \dot{V}_p}{\epsilon} + \frac{L \phi_*}{\epsilon} \frac{d^2 \ln \rho}{dt^2} \quad (L.63)$$

and

$$\begin{aligned}
 \phi_1' = & \dot{\phi}_* U_s - \dot{\phi} U_p - \frac{\phi_* \dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \\
 & + \frac{\phi_* L}{\epsilon} \frac{d \ln \rho}{dt} + \frac{2 \phi_* U_s}{z_b} (U_s - U_p) + \frac{\phi_2 \rho}{D_p \rho_p} (U_s - U_p)^2 f_{sc} \quad (L.64a)
 \end{aligned}$$

and

$$\phi_2 = 1 - \phi - \phi_* \frac{(1 - \epsilon)}{\epsilon}. \quad (L.64b)$$

To get $\frac{\partial P}{\partial z} \Big|_{z_b}$ for elimination from (L.62), we evaluate (L.62) at z_b and solve for $\frac{\partial P}{\partial z} \Big|_{z_b}$. (L.62) evaluated at z_b is

$$g_o \frac{\partial P}{\partial z} \Big|_{z_b} = - \frac{z_b C}{z_b V(z_b)} \left(\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \right) \quad (L.65)$$

or

$$\frac{\partial P}{\partial z} \Big|_{z_b} = - \frac{C \phi_1}{g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)}. \quad (L.66)$$

Substituting (L.66) back into (L.62), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{zC}{z_b V(z_b)} \left(\phi_1 + \frac{\phi_2 g_o C \phi_1}{\rho_p g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)} \right) \quad (L.67)$$

or

$$\frac{\partial P}{\partial z} = - \frac{zC}{g_o z_b V(z_b)} \left(\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \quad (L.68)$$

or

$$\frac{\partial P}{\partial z} = - k_1(t) z \quad (L.69)$$

where

$$k_1(t) = k_1 = \frac{C}{g_o z_b V(z_b)} \left(\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \quad (L.70)$$

k_1 depends upon the base pressure $P_B = P(z_p)$ (one of the values we wish to solve for) through $\dot{V}_p$ (the acceleration of the projectile) and also through $(d^2 \ln \rho)/(dt^2)$.

Substituting (L.25),

$$\dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{res}}{M_p}$$

and (L.33) into (L.70), we get

$$k_1 = \frac{C}{g_o z_b V(z_b)} \frac{\phi_1' + D - EP_{res} + EP_B}{\left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)} \quad (L.71)$$

where

$$D = \frac{L \phi_o c_1(t)}{\epsilon} \quad (L.72)$$

and

$$E = \frac{\phi_o}{\epsilon} \left(1 - \frac{LA_B}{V_F} \right) \frac{g_o A_B}{M_p} \quad (L.73)$$

or

$$k_1 = k_{11}P_B + k_{12} \quad (L.74)$$

where

$$k_{11} = \frac{CEk_2}{g_0 z_b V(z_b)} \quad (L.75)$$

$$k_2 = \frac{1}{1 - \frac{\rho_2 C}{\rho_1 V(z_b)}} \quad (L.76)$$

and

$$k_{12} = \frac{(\phi_1' + D) C k_2}{g_0 z_b V(z_b)} - k_{11} P_{res}. \quad (L.77)$$

Therefore, (L.69) becomes

$$\frac{\partial P}{\partial z} = - (k_{11}P_B + k_{12}) z. \quad (L.78)$$

Integrating (L.78) gives us the pressure distribution in the mixture $0 < z < z_b$

$$P(z) = P(0) - \frac{k_{11}P_B + k_{12}}{2} z^2. \quad (L.79)$$

We now need the pressure distribution in the ullage region. For the ullage region, the momentum equation for the gas is

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_0} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right). \quad (L.80)$$

The equation of continuity in the ullage region is

$$\frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{d\rho}{dt} = - \frac{d \ln \rho}{dt}. \quad (L.81)$$

Integrating (L.81) and noting $(d \ln \rho)/dt$ is a function of time only, we get

$$u(z) = - \frac{d \ln \rho}{dt} z + f(t) \quad (L.82)$$

with the boundary condition,

$$u(z_p) = V_p = -z_p \frac{d \ln \rho}{dt} + f(t) \quad (L.83)$$

Therefore

$$f(t) = V_p + z_p \frac{d \ln \rho}{dt} \quad (L.84)$$

and

$$u(z) = -\frac{d \ln \rho}{dt} z + V_p + z_p \frac{d \ln \rho}{dt} = V_p + (z_p - z) \frac{d \ln \rho}{dt}. \quad (L.85)$$

Differentiating (L.85) gives

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} + z_p \frac{d \ln \rho}{dt} \quad (L.86)$$

or

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} + V_p \frac{d \ln \rho}{dt}. \quad (L.87)$$

Substituting (L.81), (L.85), and (L.87) into (L.80), we get

$$\begin{aligned} \frac{\partial P}{\partial z} = & -\frac{\rho}{g_o} \left(\dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} + V_p \frac{d \ln \rho}{dt} \right. \\ & \left. - \left(V_p + (z_p - z) \frac{d \ln \rho}{dt} \right) \left(\frac{d \ln \rho}{dt} \right) \right) \end{aligned} \quad (L.88)$$

or

$$\frac{\partial P}{\partial z} = -\frac{\rho}{g_o} \left(\dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} - (z_p - z) \left(\frac{d \ln \rho}{dt} \right)^2 \right). \quad (L.89)$$

Integrating (L.89) from z_b to z and noting z_p , $\dot{V}_p$, and $\ln \rho$ are functions of time only, we get

$$\begin{aligned} P(z) = P(z_b) & - \frac{\rho}{g_o} \left(\dot{V}_p z - \frac{(z_p - z)^2}{2} \left(\frac{d^2 \ln \rho}{dt^2} \right) \right. \\ & \left. + \frac{(z_p - z)^2}{2} \left(\frac{d \ln \rho}{dt} \right)^2 \right) \Big|_{z_b}^z \end{aligned} \quad (L.90)$$

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left(\dot{V}_p z - \frac{(z_p - z)^2}{2} \Delta \right) \Big|_{z_b}^{z_p} \quad (L.91)$$

where

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt} \right)^2 \quad (L.92)$$

Therefore, we get the pressure distribution in the ullage region as

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left(\dot{V}_p (z - z_b) + [(z_p - z_b)^2 - (z_p - z)^2] \frac{\Delta}{2} \right) \quad (L.93)$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left((\dot{V}_p + z_p \Delta) (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right) \quad (L.94)$$

Defining the mean pressure P_m to be

$$P_m = \frac{\int_0^{z_p} P(z) A(z) dz}{\int_0^{z_p} A(z) dz} \quad (L.95)$$

and, since $A(z) = A_B$ is a constant, then

$$P_m = \frac{\int_0^{z_b} P(z) dz + \int_{z_b}^{z_p} P(z) dz}{z_p} \quad (L.96)$$

Substituting the pressure distribution in the mixture region (L.79) and the pressure distribution in the ullage region (L.94) into (L.96) to get the mean pressure, and noting that $P(0) = P_{Br}$ (the pressure at the breech) we get

$$P_m = \frac{1}{z_p} \left(\int_0^{z_b} \left(P_{Br} - \frac{(k_{11} P_B + K_{12})}{2} z^2 \right) dz + \int_{z_b}^{z_p} \left(P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right] \right) dz \right) \quad (L.97)$$

or

$$\begin{aligned}
 P_m = & \left[P_{Br} z_b - \frac{(k_{11} P_B + k_{12})}{6} z_b^3 + P(z_b) (z_p - z_b) \right. \\
 & - \frac{\rho}{2g_o} \left((\dot{V}_p + z_p \Delta) (z_p - z_b)^2 \right. \\
 & \left. \left. - \frac{\Delta}{3} (z_p^3 - z_b^3) + z_b^2 \Delta (z_p - z_b) \right) \right] / z_p
 \end{aligned} \quad (L.98)$$

or, since $L = z_p - z_b$,

$$\begin{aligned}
 P_m = & P_{Br} \frac{z_b}{z_p} - \frac{k_{11} P_B z_b^3}{6z_p} - \frac{k_{12} z_b^3}{6z_p} + \frac{P(z_b)L}{z_p} \\
 & - \frac{\rho L^2}{2g_o z_p} \left(\dot{V}_p + \frac{2}{3} \Delta L \right).
 \end{aligned} \quad (L.99)$$

We need to substitute for $P(z_b)$. To get $P(z_b)$, we evaluate (L.79) at z_b and get

$$P(z_b) = P_{Br} - \frac{k_{11} z_b^2 P_B}{2} - \frac{k_{12} z_b^2}{2}. \quad (L.100)$$

Substituting (L.100) into (L.99), we get

$$\begin{aligned}
 P_m = & P_{Br} \frac{z_b}{z_p} - \frac{k_{11} z_b^3 P_B}{6z_p} - \frac{k_{12} z_b^3}{6z_p} \\
 & + \frac{L}{z_p} \left(P_{Br} - \frac{k_{11} z_b^2 P_B}{2} - \frac{k_{12} z_b^2}{2} \right) - \frac{\rho L^2 \dot{V}_p}{2g_o z_p} - \frac{\rho L^3 \Delta}{3g_o z_p}.
 \end{aligned} \quad (L.101)$$

Substituting (L.25) for $\dot{V}_p$ and combining terms, we get

$$\begin{aligned}
 P_m = & P_{Br} + P_B \left(-\frac{k_{11} z_b^3}{6z_p} - \frac{L k_{11} z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p} \right) \\
 & + \frac{\rho L^2 A_B P_{ra}}{2z_p M_p} - \frac{k_{12} z_b^3}{6z_p} - \frac{L k_{12} z_b^2}{2z_p} - \frac{\rho L^3 \Delta}{3g_o z_p}.
 \end{aligned} \quad (L.102)$$

Evaluating (L.94) at z_p , we get

$$P(z_p) = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) (z_p - z_b) - (z_p^2 - z_b^2) \frac{\Delta}{2} \right], \quad (L.103)$$

and substituting (L.100) into (L.103) and noting $L = z_p - z_b$, we get

$$P(z_p) = P_B = P_{Br} - \frac{k_{11}z_b^2 P_b}{2} - \frac{k_{12}z_b^2}{2} - \frac{\rho L \dot{V}_p}{g_o} - \frac{\rho L z_p \Delta}{g_o} + \frac{\rho}{g_o} (z_p^2 - z_b^2) \frac{\Delta}{2}. \quad (L.104)$$

Substituting (L.25) for $\dot{V}_p$, and collecting P_B terms for (L.104), we get

$$0 = P_{Br} + P_B \left(-1 - \frac{k_{11}z_b^2}{2} - \frac{\rho L A_B}{M_p} \right) - \frac{k_{12}z_b^2}{2} + \frac{\rho L A_B P_{res}}{M_p} - \frac{\rho L z_p \Delta}{g_o} + \frac{\rho \Delta}{2g_o} (z_p^2 - z_b^2) \quad (L.105)$$

or

$$0 = P_{Br} + P_B \left(-1 - \frac{k_{11}z_b^2}{2} - \frac{\rho L A_B}{M_p} \right) - \frac{k_{12}z_b^2}{2} + \frac{\rho L A_B P_{res}}{M_p} - \frac{\rho L^2 \Delta}{2g_o}. \quad (L.106)$$

Subtracting (L.106) from (L.102) to eliminate P_{Br} and get P_B in terms of P_m , we get

$$P_m = P_B \left(-\frac{k_{11}z_b^3}{6z_p} - \frac{Lk_{11}z_b^2}{2z_p} - \frac{\rho L^2 A_B}{2z_p M_p} + 1 + \frac{k_{11}z_b^2}{2} + \frac{\rho L A_B}{M_p} \right) + \frac{\rho L^2 A_B P_{res}}{2z_p M_p} - \frac{k_{12}z_b^3}{6z_p} - \frac{Lk_{12}z_b^2}{2z_p} + \frac{k_{12}z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} + \frac{\rho L^2 \Delta}{2g_o} \left(1 - \frac{2L}{3z_p} \right). \quad (L.107)$$

Noting the dependence of P_B in Δ , we use (L.92)

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt} \right)^2$$

or, substituting for

$$\frac{d^2 \ln \rho}{dt^2}$$

from (L.33) and eliminating $\dot{V}_p$ with (L.25), we get

$$\Delta = c_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B P_{res}}{M_p V_F} - \left(\frac{d \ln \rho}{dt} \right)^2 \quad (L.108)$$

Substituting (L.108) into (L.107) and collecting P_B terms, we get

$$\begin{aligned} P_m = P_B & \left[-\frac{k_{11} z_b^3}{6 z_p} - \frac{L k_{11} z_b^2}{2 z_p} - \frac{\rho L^2 A_B}{2 z_p M_p} + 1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} \right. \\ & - \frac{A_B^2 \rho L^2}{2 V_F M_p} \left(1 - \frac{2L}{3 z_p} \right) \left. \right] + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} - \frac{k_{12} z_b^2}{2 z_p} \left(\frac{z_b}{3} + L \right) \\ & + \frac{\rho L^2 A_B P_{res}}{2 z_p M_p} + \frac{\rho L^2}{2 g_o} \left(1 - \frac{2L}{3 z_p} \right) \left(c_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 \right) \\ & + \frac{A_B^2 \rho L^2}{2 M_p V_F} \left(1 - \frac{2L}{3 z_p} \right) P_{res}. \end{aligned} \quad (L.109)$$

Substituting (L.108) into (L.106) and collecting P_B terms, we get

$$\begin{aligned} P_{Br} = P_B & \left(1 + \frac{k_{11} z_b^2}{2} + \frac{\rho L A_B}{M_p} - \frac{A_B^2 \rho L^2}{2 V_F M_p} \right) + \frac{k_{12} z_b^2}{2} - \frac{\rho L A_B P_{res}}{M_p} \\ & + \frac{\rho L^2}{2 g_o} \left(c_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 \right) + \frac{A_B^2 \rho L^2}{2 V_F M_p} P_{res}. \end{aligned} \quad (L.110)$$

Equation (L.109) gives P_B in terms of P_m (which is determined using an equation of state) and then (L.110) gives P_{Br} in terms of P_B .

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

$$g_o \quad KE = \frac{1}{2} \int_0^r u^2 dm \quad (L.111)$$

where u is the velocity and dm is the differential of mass; and since $\rho dV = dm$ and $dV = A(z) dz = A_B dz$, then

$$g_o KE = \frac{1}{2} \int_0^{z_b} u^2 dm + \frac{1}{2} \int_{z_b}^{z_p} u^2 dm$$

or

$$\begin{aligned} g_o KE &= \frac{1}{2} (1-\epsilon) \int_0^{z_b} u_p^2 \rho_p A_B dz + \frac{1}{2} \epsilon \int_0^{z_b} u^2 \rho A_B dz \\ &+ \frac{1}{2} \int_{z_b}^{z_p} u^2 \rho A_B dz. \end{aligned} \quad (L.112)$$

Substituting (L.10) for u_p , and (L.14) for u in the mixture region, and (L.20) for u in the ullage region, we get

$$\begin{aligned} g_o KE &= \frac{1}{2} (1-\epsilon) \rho_p A_B \frac{U_p^2}{z_b^2} \int_0^{z_b} z^2 dz + \frac{\epsilon}{2} \frac{\rho A_B U_g^2}{z_b^2} \int_0^{z_b} z^2 dz \\ &+ \frac{\rho A_B}{2} \int_{z_b}^{z_p} \left(V_p + (z_p - z) \frac{d \ln \rho}{dt} \right)^2 dz, \end{aligned} \quad (L.113)$$

or

$$\begin{aligned} g_o KE &= \frac{A_B z_b}{6} [\epsilon \rho U_g^2 + (1-\epsilon) \rho_p U_p^2] \\ &- \frac{\rho A_B}{6 \frac{d \ln \rho}{dt}} \left(V_p + (z_p - z) \frac{d \ln \rho}{dt} \right)^3 \Big|_{z_b}^{z_p} \end{aligned} \quad (L.114)$$

or

$$\begin{aligned} KE &= \frac{A_B z_b}{6 g_o} [\epsilon \rho U_g^2 + (1-\epsilon) \rho_p U_p^2] \\ &+ \frac{\rho A_B L}{6 g_o} \left[3 V_p^2 + 3 V_p L \frac{d \ln \rho}{dt} + L^2 \left(\frac{d \ln \rho}{dt} \right)^2 \right]. \end{aligned} \quad (L.115)$$

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.

The mean pressure over the mixture region is

$$P_{mix} = \frac{\int_0^{z_b} P(z)A(z)dz}{\int_0^{z_b} A(z)dz}, \quad (L.116)$$

and since $A(z) = A_B$ and using the pressure distribution in the mixture region given by (L.79), then

$$P_{mix} = \frac{A_B \int_0^{z_b} \left(P_{Br} - \left(\frac{k_{11}P_B + k_{12}}{2} \right) z^2 \right) dz}{A_B \int_0^{z_b} dz} \quad (L.117)$$

or

$$P_{mix} = P_{Br} - \frac{(k_{11}P_B + k_{12})}{6} z_b^2. \quad (L.118)$$

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APPENDIX 3:
COMBINED INFLUENCE OF CHAMBRAGE AND
PROPELLANT VELOCITY LAG

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The following, is work by Gough (Gough, in preparation) with modifications and elaborations by Robbins.

For the effects of the propellant velocity lag and chambrage on the form of the gradient equation, the variation in area is assumed confined to the chamber of the gun and the propellant initially uniformly fills the chamber.

The continuity equations for each of the phases in the mixture region are

$$\frac{\partial \epsilon \rho}{\partial t} + \frac{1}{A} \frac{\partial \epsilon \rho u A}{\partial z} = \dot{m}(t) \quad (R.1)$$

$$\rho_p \frac{\partial \epsilon}{\partial t} - \frac{1}{A} \frac{\partial (1 - \epsilon) \rho_p u_p A}{\partial z} = \dot{m}(t). \quad (R.2)$$

Making assumptions analogous to the Lagrange assumption,

$$\frac{\partial \rho}{\partial z} = 0 \quad (\text{gas density constant throughout the tube}) \quad (R.3)$$

$$\frac{\partial \epsilon}{\partial z} = 0 \quad (\text{porosity constant throughout the mixture region}) \quad (R.4)$$

and noting the assumption that $\dot{m}(t)$ is a function of time, then (R.1) becomes,

$$\frac{\partial \epsilon \rho}{\partial t} + \frac{\epsilon \rho}{A} \frac{\partial u A}{\partial z} = \dot{m}(t), \quad (R.4a)$$

or

$$\frac{\partial u A}{\partial z} = \frac{\left(\dot{m}(t) - \frac{\partial \epsilon \rho}{\partial t} \right) A}{\epsilon \rho}. \quad (R.5)$$

Integrating R.5 and noting that the right side is a function of time only except for A which is a function of z only, we get

$$u A = f(t) \int A dz + k(t) = f(t) V(z) + k(t). \quad (R.6)$$

Applying the boundary condition $u(0) = 0$ implies that $k(t) = 0$ since $V(0) = 0$.

Also $u(z_b) = U_g$ implies that

$$f(t) = \frac{U_g A(z_b)}{V(z_b)}, \quad (R.7)$$

and, since $A(z_b) = A_B$,

$$u(z) = \frac{U_g A_B V(z)}{A(z) V(z_b)}. \quad (R.8)$$

Also (R.2) becomes after using (R.3) and (R.4)

$$\rho_p \frac{\partial \epsilon}{\partial t} - \frac{(1-\epsilon) \rho_p}{A} \frac{\partial u_p A}{\partial z} = \dot{m}(t) \quad (R.9)$$

or

$$\frac{\partial u_p A}{\partial z} = \frac{\dot{m}(t) - \rho_p \frac{\partial \epsilon}{\partial t}}{(1-\epsilon) \rho_p} A. \quad (R.10)$$

Since the right side of (R.10) is a function of time only except for A , which is a function of z only, on integrating, we get

$$u_p A = f_1(t) \int A dz + k_1(t) = f_1(t) V(z) + k_1(t). \quad (R.11)$$

Applying the boundary condition, $u_p(0) = 0$ implies that $k_1(t) = 0$ since $V(0) = 0$.

Also $u_p(z_b) = U_p$ implies that

$$f_1(t) = \frac{U_p A(z_b)}{V(z_b)}, \quad (R.12)$$

and since $A(z_b) = A_B$,

$$u_p(z) = \frac{U_p A_B V(z)}{A(z) V(z_b)}. \quad (R.13)$$

At the internal boundary defined by the leading edge of the propellant bed we may write the macroscopic balance of mass in the following form

$$\epsilon (U_g - U_p) = U_{g+} - U_p \quad (R.14)$$

This result includes the assumption that the density jump across the boundary is negligible, as will always be the case if the Mach number is small compared with unity, and which is in any case consistent with the present assumption R.3.

As with the velocity lag derivation, the ullage region is the same.

That is the continuity equation given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0, \quad (R.15)$$

and using (R.3),

$$\frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{d\rho}{dt} \quad (R.16)$$

where the right side of (R.16) is a function of time only.

Therefore,

$$u = - \frac{1}{\rho} \frac{d\rho}{dt} z + f_3(t). \quad (R.17)$$

Since $u(z_b) = U_{s+}$, then

$$U_{s+} = - \frac{1}{\rho} \frac{d\rho}{dt} z_b + f_3(t), \quad (R.18)$$

and since $u(z_p) = V_p$, then

$$V_p = - \frac{1}{\rho} \frac{d\rho}{dt} z_p + f_3(t). \quad (R.19)$$

Subtracting (R.18) from (R.19) and noting

$$\frac{1}{\rho} \frac{d\rho}{dt} = \frac{d \ln \rho}{dt}$$

we get

$$U_{s+} = \frac{d \ln \rho}{dt} (z_p - z_b) + V_p. \quad (R.20)$$

Also solving (R.19) for $f_3(t)$ and substituting into (R.17), we get

$$u(z) = V_p + (z_p - z) \frac{d \ln \rho}{dt}. \quad (R.21)$$

Eliminating U_g from (R.14) with (R.20) gives

$$\epsilon (U_g - U_p) = V_p + L \frac{d \ln \rho}{dt} - U_p \quad (R.22)$$

or

$$U_g = U_p + \frac{1}{\epsilon} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \quad (R.23)$$

Differentiating (R.23) yields

$$\begin{aligned} \dot{U}_g = & - \dot{U}_p \left(\frac{1-\epsilon}{\epsilon} \right) - \frac{\dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \\ & + \frac{\dot{V}_p}{\epsilon} + \frac{L}{\epsilon} \frac{d^2 \ln \rho}{dt^2} + \frac{L}{\epsilon} \frac{d \ln \rho}{dt} \end{aligned} \quad (R.24)$$

We anticipate the need to factor out the base pressure P_B dependence of both $\dot{V}_p$ and $d^2 \ln \rho / dt^2$.

The projectile acceleration is given by

$$\dot{V}_p = \frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{res}}{M_p} \quad (R.25)$$

To get the base pressure P_B dependence associated with $d^2 \ln \rho / dt^2$ we start with the definition of the gas density

$$\rho = \frac{m}{V_F} \quad (R.26)$$

where m = mass of gas burnt and V_F = free volume.

Therefore

$$V_F = V_o + A_B (z_p - z_{po}) - \frac{C}{\rho_p} + \frac{m}{\rho_p} \quad (R.27)$$

and

$$\dot{V}_F = A_B \dot{z}_p + \frac{\dot{m}}{\rho_p} = A_B V_p + \frac{\dot{m}}{\rho_p} \quad (R.28)$$

and

$$\dot{V}_F = A_B \dot{V}_p + \frac{\dot{m}}{\rho_p} \quad (R.29)$$

where it is understood that

$$\frac{m}{\rho_p}, \quad \frac{\dot{m}}{\rho_p}, \quad \text{and} \quad \frac{\ddot{m}}{\rho_p}$$

may be in the form

$$\sum \frac{m_i}{\rho_{pi}}, \quad \sum \frac{\dot{m}_i}{\rho_{pi}}, \quad \text{and} \quad \sum \frac{\ddot{m}_i}{\rho_{pi}}$$

where the i index refers to different propellants.

From (R.26) we get

$$\ln \rho = \ln m - \ln V_F \quad (R.30)$$

and

$$\frac{d \ln \rho}{dt} = \frac{\dot{m}}{m} - \frac{\dot{V}_F}{V_F} \quad (R.31)$$

and

$$\frac{d^2 \ln \rho}{dt^2} = \frac{\ddot{m}}{m} - \frac{\dot{m}^2}{m^2} - \frac{\ddot{V}_F}{V_F} + \frac{\dot{V}_F^2}{V_F^2} \quad (R.32)$$

Substituting (R.29) and then (R.25), we get

$$\frac{d^2 \ln \rho}{dt^2} = C_1(t) - \frac{g_o A_B^2 P_B}{V_F M_p} + \frac{g_o A_B^2 P_{res}}{M_p V_F} \quad (R.33)$$

where

$$C_1(t) = \ddot{m} \left(\frac{1}{m} - \frac{1}{\rho_p V_F} \right) + \frac{\dot{V}_F^2}{V_F^2} - \frac{\dot{m}^2}{m^2}$$

We still require $\ddot{m}$, which we get from

$$\dot{m} = \rho_p S \frac{dx}{dt} \quad (R.34)$$

where S is the surface area of the propellant and dx/dt is the burning rate of the propellant where dx/dt is of the form

$$\frac{dx}{dt} = aP_m^n \quad (R.35)$$

Therefore, from (R.34) we get

$$\begin{aligned} \ddot{m} &= \rho_p S \frac{dx}{dt} + \rho_p S \frac{d^2x}{dt^2} \\ &= \rho_p \frac{dS}{dx} \left(\frac{dx}{dt} \right)^2 + \rho_p S \frac{d^2x}{dt^2} \end{aligned} \quad (R.36)$$

and from (R.35) we get

$$\frac{d^2x}{dt^2} = an P_m^{n-1} \frac{dP_m}{dt} \quad (R.37)$$

where dP_m/dt is determined numerically or from

$$\frac{dP_m}{dt} = \frac{\frac{nRT}{m_w} + \frac{TR_m}{m_w} - P_m \dot{V}}{V}$$

Also, defining ϕ as the mass fraction of propellant burned, then

$$\epsilon \rho = \frac{\phi c - \rho A_B L}{V(z_b)} = \frac{\phi_* C}{V(z_b)} \quad (R.38)$$

where

$$\phi_* = \phi - \frac{\rho A_B L}{C} \quad (R.39)$$

and

$$(1 - \epsilon) \rho_p = \frac{(1 - \phi) C}{V(z_b)} \quad (R.40)$$

Differentiating (R.39), we get

$$\dot{\phi}_* = \dot{\phi} - \frac{\rho A_B}{C} \left[L + L \frac{d \ln \rho}{dt} \right] \quad (R.41)$$

Solving (R.40) for ϵ , we get

$$\epsilon = 1 - \frac{(1-\phi)C}{\rho_p V(z_b)} \quad (R.42)$$

Differentiating (R.42) and noting that

$$V(z_b) = V(z_{po}) + (z_b - z_{po}) A_B \quad (R.43)$$

and therefore

$$\dot{V}(z_b) = \dot{z}_b A_B = U_p A_B, \quad (R.44)$$

then

$$\dot{\epsilon} = \frac{\dot{\phi}C}{\rho_p V(z_b)} + \frac{(1-\phi)C\dot{V}(z_b)}{\rho_p V^2(z_b)} = \frac{\dot{\phi}C}{\rho_p V(z_b)} + \frac{(1-\phi)CU_p A_B}{\rho_p V^2(z_b)} \quad (R.45)$$

We now have the quantities which are required for the solution of the momentum equation from which we get the pressure distribution in the mixture region.

Consider the momentum equations for each phase

$$\frac{1}{A} \left[\frac{\partial A \epsilon \rho u}{\partial t} + \frac{\partial A \epsilon \rho u^2}{\partial z} \right] + \epsilon g_o \frac{\partial P}{\partial z} = -f_s + \dot{m} u_p \quad (R.46)$$

$$\frac{1}{A} \left[\frac{\partial A (1-\epsilon) \rho_p u_p}{\partial t} + \frac{\partial A (1-\epsilon) \rho_p u_p^2}{\partial z} \right] + (1-\epsilon) g_o \frac{\partial P}{\partial z} = f_s - \dot{m} u_p \quad (R.47)$$

where

$$f_s = \frac{(1-\epsilon)}{D_p} \rho (u - u_p)^2 f_{sc} \quad (R.48)$$

and

$$f_{sc} = \begin{cases} \frac{2.5\lambda}{REN^{.081}} f_{so}, & \epsilon < \epsilon_o \\ \frac{2.5\lambda}{REN^{.081}} \left(\frac{(1-\epsilon)}{(1-\epsilon_o)} \frac{\epsilon_o}{\epsilon} \right)^{.45} f_{so}, & \epsilon \geq \epsilon_o \end{cases} \quad (R.49)$$

where

$$\lambda = \left(\frac{0.5 + \frac{GL}{GD}}{\left(1.5 \frac{GL}{GD}\right)^{2/3}} \right)^{2.17} \quad (R.50)$$

and

$$REN = \rho D_p |U_s - U_p| / \mu$$

Adding (R.46) and (R.47) we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{1}{A} \frac{\partial}{\partial t} [A \epsilon \rho u + A(1-\epsilon) \rho_p u_p] \\ & - \frac{1}{A} \frac{\partial}{\partial z} [A \epsilon \rho u^2 + A(1-\epsilon) \rho_p u_p^2]. \end{aligned} \quad (R.51)$$

From (R.8) we get

$$u(z) = \frac{U_s A_B V(z)}{A(z) V(z_b)},$$

from (R.13) we get

$$u_p(z) = \frac{U_p A_B V(z)}{A(z) V(z_b)},$$

from (R.38) we get

$$\epsilon \rho = \frac{\phi \cdot C}{V(z_b)},$$

and from (R.40) we get

$$(1-\epsilon) \rho_p = \frac{(1-\phi)C}{V(z_b)}$$

which, when substituted into (R.51), yields

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{1}{A} \frac{\partial}{\partial t} \left[\frac{\phi \cdot C U_s A_B V(z)}{V^2(z_b)} + \frac{(1-\phi) C U_p A_B V(z)}{V^2(z_b)} \right] \\ & - \frac{1}{A} \frac{\partial}{\partial z} \left[\frac{\phi \cdot C U_s^2 A_B^2 V^2(z)}{V^3(z_b) A(z)} + \frac{(1-\phi) C U_p^2 A_B^2 V^2(z)}{V^3(z_b) A(z)} \right], \end{aligned} \quad (R.52)$$

or

$$g_o \frac{\partial P}{\partial t} = - \frac{CA_B V(z)}{A(z)} \frac{\partial}{\partial t} \left[\frac{\phi_s U_s}{V^2(z_b)} + \frac{(1-\phi)U_p}{V^2(z_b)} \right] - \frac{CA_B^2}{A(z)V^3(z_b)} \frac{\partial}{\partial z} \left[\frac{\phi_s U_s^2 V^2(z)}{A(z)} + \frac{(1-\phi)U_p^2 V^2(z)}{A(z)} \right]. \quad (R.53)$$

Performing the indicated differentiation in (R.53), we get

$$g_o \frac{\partial P}{\partial z} = - \frac{CA_B V(z)}{A(z)} \left[\frac{\dot{\phi}_s U_s}{V^2(z_b)} + \frac{\phi_s \dot{U}_s}{V^2(z_b)} - 2 \frac{\phi_s U_s \dot{V}(z_b)}{V^3(z_b)} - \frac{\dot{\phi} U_p}{V^2(z_b)} + \frac{(1-\phi)\dot{U}_p}{V^2(z_b)} - 2 \frac{(1-\phi)U_p \dot{V}(z_b)}{V^3(z_b)} \right] - \frac{CA_B^2}{A(z)V^3(z_b)} [\phi_s U_s^2 + (1-\phi)U_p^2] \frac{\partial}{\partial z} \left(\frac{V^2(z)}{A(z)} \right). \quad (R.54)$$

Noting that, since

$$V(z_b) = V(z_{bo}) + A_B (z_b - z_{bo}), \quad (R.55)$$

and therefore

$$\dot{V}(z_b) = A_B \dot{z}_b = A_B U_p, \quad (R.56)$$

and since $\partial V(z)/\partial z = A(z)$, and $A(z)$ and $V(z)$ are function of z only,

$$\frac{\partial}{\partial z} \left(\frac{V^2(z)}{A(z)} \right) = 2V(z) - \frac{V^2(z)}{A^2(z)} \frac{dA(z)}{dz}, \quad (R.57)$$

then

$$g_o \frac{\partial P}{\partial z} = - \frac{CA_B V(z)}{A(z)V^2(z_b)} \left[\phi_s U_s + \phi_s \dot{U}_s - 2 \frac{\phi_s U_s A_B U_p}{V(z_b)} - \dot{\phi} U_p + (1-\phi)\dot{U}_p - 2 \frac{(1-\phi)U_p^2 A_B}{V(z_b)} + \frac{2A_B}{V(z_b)} (\phi_s U_s^2 + (1-\phi)U_p^2) - \frac{A_B V(z)}{A^2(z)V(z_b)} (\phi_s U_s^2 + (1-\phi)U_p^2) \frac{\partial A(z)}{\partial z} \right] \quad (R.58)$$

or

$$\begin{aligned}
 g_o \frac{\partial P}{\partial z} = & - \frac{CA_B V(z)}{A(z)V^2(z_b)} \left[\phi \dot{U}_g + (1-\phi)\dot{U}_p + \phi U_g - \phi U_p \right. \\
 & + 2 \frac{A_B \phi U_g}{V(z_b)} (U_g - U_p) \\
 & \left. - \frac{A_B V(z)}{A^2(z)V(z_b)} (\phi U_g^2 + (1-\phi)U_p^2) \frac{\partial A(z)}{\partial z} \right]. \quad (R.59)
 \end{aligned}$$

Eliminating $\dot{U}_g$ from (R.59) by using (R.24), we get

$$\begin{aligned}
 g_o \frac{\partial P}{\partial z} = & - \frac{CA_B V(z)}{V^2(z_b)A(z)} \left[\frac{\phi \dot{V}_p}{\epsilon} + \frac{L\phi}{\epsilon} \frac{d^2 \ln \rho}{dt^2} \right] \\
 & + \dot{U}_p \left(1 - \phi - \frac{\phi(1-\epsilon)}{\epsilon} \right) - \phi \frac{\dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) \\
 & + \phi \frac{L}{\epsilon} \frac{d \ln \rho}{dt} + \phi U_g - \phi U_p + \frac{2A_B \phi U_g}{V(z_b)} (U_g - U_p) \\
 & - \frac{A_B V(z)}{V(z_b)A^2(z)} (\phi U_g^2 + (1-\phi)U_p^2) \frac{\partial A(z)}{\partial z}. \quad (R.60)
 \end{aligned}$$

We need to eliminate $\dot{U}_p$.

From (R.2) we get, by multiplying by u_p ,

$$u_p \rho_p \frac{\partial \epsilon}{\partial t} - \frac{u_p}{A} \frac{\partial (1-\epsilon) \rho_p u_p A}{\partial z} = \dot{m}(t) u_p \quad (R.61)$$

which, when substituted into (R.47), gives

$$\begin{aligned}
 \frac{1}{A} \frac{\partial A(1-\epsilon) \rho_p u_p}{\partial t} + \frac{1}{A} \frac{\partial A(1-\epsilon) \rho_p u_p^2}{\partial z} + (1-\epsilon) g_o \frac{\partial P}{\partial z} \\
 = f_s - u_p \rho_p \frac{\partial \epsilon}{\partial t} + \frac{u_p}{A} \frac{\partial (1-\epsilon) \rho_p u_p A}{\partial z} \quad (R.62)
 \end{aligned}$$

or

$$\begin{aligned}
 (1-\epsilon)\rho_p \frac{\partial u_p}{\partial t} + \frac{(1-\epsilon)\rho_p u_p}{A} \frac{\partial A}{\partial t} + 2(1-\epsilon)\rho_p \frac{\partial u_p}{\partial z} \\
 + \frac{(1-\epsilon)\rho_p u_p^2}{A} \frac{\partial A}{\partial z} + (1-\epsilon)g_o \frac{\partial P}{\partial z} \\
 = f_s + u_p (1-\epsilon)\rho_p \frac{\partial u_p}{\partial z} + \frac{u_p(1-\epsilon)\rho_p u_p}{A} \frac{\partial A}{\partial z}. \quad (R.63)
 \end{aligned}$$

Evaluating R.63 at z_b and noting $\partial A/\partial t = 0$,

$$\frac{\partial U_p}{\partial z}|_{z_b} = 0$$

and

$$\frac{\partial A}{\partial z}|_{z_b} = 0 \quad \text{for } z_b \geq z_{po},$$

then

$$(1-\epsilon) g_o \frac{\partial P}{\partial z} + (1-\epsilon)\rho_p \dot{U}_p = f_s. \quad (R.63a)$$

Using (R.48) with (R.63a), we get

$$\dot{U}_p = \frac{\rho(U_s - U_p)^2 f_{sc}}{\rho_p D_p} - \frac{g_o}{\rho_p} \frac{\partial P}{\partial z}|_{z_b}. \quad (R.64)$$

Substituting (R.64) into (R.60) gives

$$\begin{aligned}
 g_o \frac{\partial P}{\partial z} = & - \frac{CA_B V(z)}{V^2(z_b) A(z)} \left[\frac{\phi \dot{V}_p}{\epsilon} + L \frac{\phi}{\epsilon} \frac{d^2 \ln \rho}{dt^2} \right. \\
 & + \left(1 - \phi - \frac{\phi(1-\epsilon)}{\epsilon} \right) \frac{\rho(U_g - U_p)^2}{\rho_p D_p} f_{sc} \\
 & - \left(1 - \phi - \frac{\phi(1-\epsilon)}{\epsilon} \right) \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \\
 & - \frac{\dot{\epsilon} \phi}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L \phi}{\epsilon} \frac{d \ln \rho}{dt} \\
 & + \phi U_g - \phi U_p + \frac{2A_B \phi U_g}{V(z_b)} (U_g - U_p) \\
 & \left. - \frac{A_B V(z)}{V(z_b) A^2(z)} \left(\frac{\partial A(z)}{\partial z} \right) (\phi U_g^2 + (1-\phi) U_p^2) \right] \quad (R.65)
 \end{aligned}$$

or

$$\begin{aligned}
 g_o \frac{\partial P}{\partial z} = & - \frac{V(z)}{A(z)} \frac{CA_B}{V^2(z_b)} \left[\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \right. \\
 & \left. - \frac{\phi_3 A_B V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z} \right] \quad (R.66)
 \end{aligned}$$

and

$$\phi_1 = \phi_1' + \frac{L \phi}{\epsilon} \frac{d^2 \ln \rho}{dt^2} + \frac{\phi \dot{V}_p}{\epsilon} \quad (R.67)$$

and

$$\begin{aligned}
 \phi_1' = & \phi U_g - \phi U_p - \frac{\phi \dot{\epsilon}}{\epsilon^2} \left(V_p + L \frac{d \ln \rho}{dt} - U_p \right) + \frac{L \phi}{\epsilon} \frac{d \ln \rho}{dt} \\
 & + \frac{2A_B \phi U_g (U_g - U_p)}{V(z_b)} + \phi_2 \frac{\rho(U_g - U_p)^2 f_{sc}}{\rho_p D_p} \quad (R.68)
 \end{aligned}$$

and

$$\phi_2 = 1 - \phi - \frac{\phi(1-\epsilon)}{\epsilon} \quad (R.69)$$

and

$$\phi_3 = \phi_* U_t^2 + (1-\phi) U_p^2. \quad (R.70)$$

To get

$$\frac{\partial P}{\partial z} \Big|_{z_b}$$

for elimination from (R.66), we evaluate (R.66) at z_b and solve for

$$\frac{\partial P}{\partial z} \Big|_{z_b}$$

With (R.66) evaluated at z_b and noting

$$\frac{\partial A}{\partial z} \Big|_{z_b} = 0$$

and $A(z_b) = A_B$, we get

$$g_o \frac{\partial P}{\partial z} \Big|_{z_b} = - \frac{C}{V(z_b)} \left[\phi_1 - \phi_2 \frac{g_o}{\rho_p} \frac{\partial P}{\partial z} \Big|_{z_b} \right] \quad (R.71)$$

or

$$\frac{\partial P}{\partial z} \Big|_{z_b} = - \frac{C \phi_1}{g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)} \quad (R.72)$$

Substituting (R.72) back into (R.66), we get

$$\begin{aligned} g_o \frac{\partial P}{\partial z} = & - \frac{V(z)}{A(z)} \frac{C A_B}{V^2(z_b)} \left[\phi_1 + \frac{\phi_2 g_o C \phi_1}{\rho_p g_o V(z_b) \left(1 - \frac{\phi_2 C}{\rho_p V(z_b)} \right)} \right. \\ & \left. - \frac{\phi_3 A_B V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z} \right] \end{aligned} \quad (R.73)$$

or

$$g_o \frac{\partial P}{\partial z} = - \frac{V(z)}{A(z)} \frac{C A_B}{V^2(z_b)} \left[\frac{\phi_1}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} - \frac{\phi_3 A_B V(z)}{V(z_b) A^2(z)} \frac{\partial A(z)}{\partial z} \right] \quad (R.74)$$

ϕ_1 depends upon the base pressure $P_B = P(z_p)$ (one of the values we wish to solve for) through $\dot{V}_p$ (the acceleration of the projectile) and also through $d^2 \ln p / dt^2$.

Substituting (R.33) into (R.67), we get

$$\phi_1 = \phi_1' + \frac{L\phi_* C_1(t)}{\epsilon} - \frac{L\phi_* A_B}{\epsilon V_F} \dot{V}_p + \frac{\phi_*}{\epsilon} \dot{V}_p \quad (R.75)$$

or

$$\phi_1 = \phi_1' + D + \frac{E\phi_*}{\epsilon} \dot{V}_p \quad (R.76)$$

where

$$D = \frac{L\phi_* C_1(t)}{\epsilon} \quad (R.77)$$

and

$$E = \left(1 - \frac{LA_B}{V_F} \right). \quad (R.78)$$

Substituting (R.25) into (R.76), we get

$$\phi_1 = \phi_1' + D + \frac{E\phi_*}{\epsilon} \left(\frac{g_o A_B P_B}{M_p} - \frac{g_o A_B P_{ra}}{M_p} \right) \quad (R.79)$$

or

$$\phi_1 = \phi_1' + D - \frac{E\phi_* g_o A_B P_{ra}}{\epsilon M_p} + \frac{E\phi_* g_o A_B P_B}{\epsilon M_p}. \quad (R.80)$$

Substituting (R.80) into (R.74), we get

$$\begin{aligned} \frac{\partial P}{\partial z} = & - \frac{V(z)CA_B}{A(z)g_o V^2(z_b)} \left[\frac{\phi_1' + D - \frac{E\phi_* g_o A_B P_{ra}}{\epsilon M_p} + \frac{E\phi_* g_o A_B P_B}{\epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right. \\ & \left. - \frac{\phi_3 A_B V(z)}{V(z_b)A^2(z)} \frac{\partial A}{\partial z} \right] \end{aligned} \quad (R.81)$$

or

$$\begin{aligned} \frac{\partial P}{\partial z} = \frac{CA_B}{g_o V^2(z_b)} \left[- \frac{\phi_1' + D - \frac{E\phi_2 g_o A_B P_{1a}}{cM_p} + \frac{E\phi_2 g_o A_B P_B}{cM_p}}{1 - \frac{\phi_2 C}{p_p V(z_b)}} \frac{V(z)}{A(z)} \right. \\ \left. + \frac{\phi_3 A_B V^2(z)}{V(z_b) A^3(z)} \frac{\partial A(z)}{\partial z} \right]. \end{aligned} \quad (R.82)$$

Integrating (R.82) and noting that A(z) and V(z) are functions of z and only of z and that

$$\int \frac{V^2(z)}{A^3(z)} \frac{\partial A}{\partial z} dz$$

(by parts) can be written

$$- \frac{V^2(z)}{2A^2(z)} + \int \frac{V(z)}{A(z)} dz$$

we get

$$\begin{aligned} P(z) - P(0) = \frac{CA_B}{g_o V^2(z_b)} \left[- \frac{\phi_1' + D - \frac{E\phi_2 g_o A_B P_{1a}}{cM_p} + \frac{E\phi_2 g_o A_B P_B}{cM_p}}{1 - \frac{\phi_2 C}{p_p V(z_b)}} \int \frac{V(z)}{A(z)} dz \right. \\ \left. - \frac{\phi_3 A_B V^2(z)}{2V(z_b) A^2(z)} + \frac{\phi_3 A_B}{V(z_b)} \int \frac{V(z)}{A(z)} dz \right]. \end{aligned} \quad (R.83)$$

Noting $P(0) = P_{Br}$ and collecting terms, we get

$$P(z) = P_{Br} + \left[\frac{CA_B}{g_o V^2(z_b)} \left(\frac{\phi_3 A_B}{V(z_b)} - \frac{\phi_1' + D - \frac{E\phi_3 g_o A_B P_{res}}{\epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) - \frac{\frac{CE\phi_3 A_B^2 P_{res}}{V^2(z_b) \epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right] \int \frac{V(z)}{A(z)} dz - \frac{\phi_3 C A_B^2 V^2(z)}{2g_o V^3(z_b) A^2(z)} \quad (R.83a)$$

or

$$P(z) = P_{Br} + (a_1(t) + a_2(t) P_B) J_1(z) + b(t) J_2(z) \quad (R.84)$$

where

$$a_1(t) = \frac{CA_B}{g_o V^2(z_b)} \left(\frac{\phi_3 A_B}{V(z_b)} - \frac{\phi_1' + D - \frac{E\phi_3 g_o A_B P_{res}}{\epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \right) \quad (R.85)$$

and

$$a_2(t) = - \frac{\frac{CE\phi_3 A_B^2}{V^2(z_b) \epsilon M_p}}{1 - \frac{\phi_2 C}{\rho_p V(z_b)}} \quad (R.86)$$

and

$$b(t) = - \frac{C\phi_3 A_B^2}{2g_o V^3(z_b)} \quad (R.87)$$

and

$$J_1(z) = \int_0^z \frac{V(z)}{A(z)} dz \quad (R.88)$$

and

$$J_2(z) = \frac{V^2(z)}{A^2(z)} \quad (R.89)$$

(R.84) is the equation for the pressure distribution in the mixture region. To complete the pressure distribution description, we require the distribution in the ullage region which should be identical to the description in the velocity lag derivation that is in the ullage region. The momentum equation for the gas is

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} \right) \quad (R.90)$$

The equation of continuity in the ullage region is

$$\frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{d\rho}{dt} = - \frac{d \ln \rho}{dt} \quad (R.91)$$

Integrating (R.91) and noting $d \ln \rho / dt$ is a function of time only we get

$$u(z) = - \frac{d \ln \rho}{dt} z + f(t). \quad (R.92)$$

With the boundary condition $u(z_p) = V_p$ we get

$$V_p = - z_p \frac{d \ln \rho}{dt} + f(t) \quad (R.93)$$

and therefore,

$$f(t) = V_p + z_p \frac{d \ln \rho}{dt} \quad (R.94)$$

and

$$u(z) = V_p + (z_p - z) \frac{d \ln \rho}{dt}. \quad (R.95)$$

Differentiating (R.95) and noting $\dot{z}_p = V_p$ gives

$$\frac{\partial u}{\partial t} = \dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} + V_p \frac{d \ln \rho}{dt}. \quad (R.96)$$

Substituting (R.91), (R.95), and (R.96) into (R.90), we get

$$\begin{aligned} \frac{\partial P}{\partial z} = & - \frac{\rho}{g_o} \left[\dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} + V_p \frac{d \ln \rho}{dt} \right. \\ & \left. + \left(V_p + (z_p - z) \frac{d \ln \rho}{dt} \right) \left(- \frac{d \ln \rho}{dt} \right) \right] \end{aligned} \quad (R.97)$$

or

$$\frac{\partial P}{\partial z} = - \frac{\rho}{g_o} \left[\dot{V}_p + (z_p - z) \frac{d^2 \ln \rho}{dt^2} - (z_p - z) \left(\frac{d \ln \rho}{dt} \right)^2 \right]. \quad (R.98)$$

Integrating (R.98) from z_b to z and noting z_p , $\dot{V}_p$, and $\ln \rho$ are functions of time only, we get

$$\begin{aligned} P(z) = & P(z_b) - \frac{\rho}{g_o} \left[\dot{V}_p z - \frac{(z_p - z)^2}{2} \frac{d^2 \ln \rho}{dt^2} \right. \\ & \left. + \frac{(z_p - z)^2}{2} \left(\frac{d \ln \rho}{dt} \right)^2 \right]_{z_b} \end{aligned} \quad (R.99)$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[\dot{V}_p z - \frac{(z_p - z)^2}{2} \Delta \right] \Big|_{z_b}^z \quad (R.100)$$

where

$$\Delta = \frac{d^2 \ln \rho}{dt^2} - \left(\frac{d \ln \rho}{dt} \right)^2 \quad (R.101)$$

Therefore, we get the pressure distribution in the ullage region as

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[\dot{V}_p (z - z_b) + ((z_p - z_b)^2 - (z_p - z)^2) \frac{\Delta}{2} \right] \quad (R.102)$$

or

$$P(z) = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right] \quad (R.103)$$

Defining the mean pressure P_m to be

$$P_m = \frac{\int_0^{z_p} P(z) A(z) dz}{\int_0^{z_p} A(z) dz} \quad (R.104)$$

and since $\int_0^{z_p} A(z) dz = V(z_p)$ and using (R.84) and (R.103)

$$\begin{aligned} P_m &= \frac{\int_0^{z_b} (P_{Br} + a_1(t) J_1(z) + a_2(t) P_B J_1(z) + b(t) J_2(z)) A(z) dz}{V(z_p)} \\ &+ \frac{\int_{z_b}^{z_p} \left(P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) (z - z_b) - (z^2 - z_b^2) \frac{\Delta}{2} \right] \right) A(z) dz}{V(z_p)} \end{aligned} \quad (R.105)$$

or

$$\begin{aligned} P_m &= \frac{P_{Br} V(z_b) + a_1(t) J_3(z_b) + a_2(t) P_B J_3(z_b) + b(t) J_4(z_b)}{V(z_b)} \\ &+ \frac{A_B L P(z_b)}{V(z_p)} - \frac{A_B \rho}{g_o} \left[\frac{\frac{(\dot{V}_p + z_p \Delta) (z_p - z_b)^2}{2} - \frac{\Delta}{6} (z_p^3 - z_b^3)}{V(z_p)} \right. \\ &\left. + \frac{\frac{z_b^2 \Delta (z_p - z_b)}{2}}{V(z_p)} \right] \end{aligned} \quad (R.106)$$

or

$$P_m = P_{Br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) P_B J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L P(z_b)}{V(z_p)} - \frac{\rho A_B}{g_o} \left[\dot{V}_p + \frac{2L\Delta}{3} \right] \frac{L^2}{2V(z_p)} \quad (R.107)$$

where

$$J_3(z_b) = \int_0^{z_b} A(z) J_1(z) dz \quad (R.108)$$

and

$$J_4(z_b) = \int_0^{z_b} \frac{V^2(z)}{A(z)} dz. \quad (R.109)$$

Substituting in for $\dot{V}_p$ from (R.25) into (R.107), we get

$$P_m = P_{Br} \frac{V(z_b)}{V(z_p)} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L P(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_o V(z_p)}. \quad (R.110)$$

To eliminate $P(z_b)$, we evaluate (R.84) at z_b and get

$$P(z_b) = P_{Br} + (a_1(t) + a_2(t) P_B) J_1(z_b) + b(t) J_2(z_b) \quad (R.111)$$

and substitute into (R.110) to get

$$P_m = P_{Br} + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{a_2(t) J_3(z_b) P_B}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L a_2(t) J_1(z_b) P_B}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2 P_B}{2V(z_p) M_p} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_o V(z_p)} \quad (R.112)$$

or

$$P_m = P_{Br} + P_B \left(\frac{A_B L a_2(t) J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p) M_p} + \frac{a_2(t) J_3(z_b)}{V(z_p)} \right) + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3g_o V(z_p)}. \quad (R.113)$$

We now need P_{Br} in terms of P_B to eliminate P_{Br} from (R.113) and give us an equation for P_B .

Evaluating (R.103) at z_p , we get

$$P(z_p) = P_B = P(z_b) - \frac{\rho}{g_o} \left[(\dot{V}_p + z_p \Delta) L - (z_p^2 - z_b^2) \frac{\Delta}{2} \right] \quad (R.114)$$

or, since $z_p^2 - z_b^2 = L(z_p + z_b)$,

$$P_B = P(z_b) - \frac{\rho L}{g_o} \left(\dot{V}_p + \frac{L\Delta}{2} \right), \quad (R.115)$$

and substituting (R.25) for $\dot{V}_p$, we get

$$P_B = P(z_b) - \frac{\rho L A_B P_B}{M_p} + \frac{A_B \rho L P_{res}}{M_p} - \frac{\rho L^2 \Delta}{2 g_o}. \quad (R.116)$$

Substituting (R.111) for $P(z_b)$ into (R.116) and collecting terms, we get

$$\begin{aligned} 0 = P_{Br} + P_B & \left(-1 + a_2(t) J_1(z_b) - \frac{\rho L A_B}{M_p} \right) + a_1(t) J_1(z_b) \\ & + b(t) J_2(z_b) + \frac{A_B \rho L P_{res}}{M_p} - \frac{\rho L^2 \Delta}{2 g_o}. \end{aligned} \quad (R.117)$$

Subtracting (R.117) from (R.113) eliminates P_{Br} and gives P_B in terms of P_m or

$$\begin{aligned} P_m = P_B & \left[1 - a_2(t) J_1(z_b) + \frac{\rho L A_B}{M_p} + \frac{A_B L a_2(t) J_1(z_b)}{V(z_p)} \right. \\ & - \frac{\rho A_B^2 L^2}{2 V(z_p) M_p} + \left. \frac{a_2(t) J_3(z_b)}{V(z_p)} \right] + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} \\ & + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2 V(z_p) M_p} - \frac{\rho A_B L^3 \Delta}{3 g_o V(z_p)} \\ & - a_1(t) J_1(z_b) - b(t) J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p} + \frac{\rho L^2 \Delta}{2 g_o}. \end{aligned} \quad (R.118)$$

Since Δ has a dependence on P_B through $d^2 \ln \rho / dt^2$ using (R.33) with (R.117), we get

$$\begin{aligned} P_{Br} = P_B & \left(1 - a_2(t)J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p} \right) \\ & + \frac{\rho L^2}{2g_o} \left(C_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F} \right) - a_1(t)J_1(z_b) \\ & - b(t)J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p}, \end{aligned} \quad (R.119)$$

and using (R.33) with (R.118), we get

$$\begin{aligned} P_m = P_B & \left[1 + \frac{A_B L a_2(t) J_1(z_b)}{V(z_p)} - \frac{\rho A_B^2 L^2}{2V(z_p) M_p} + \frac{a_2(t) J_3(z_b)}{V(z_p)} \right. \\ & - a_2(t) J_1(z_b) + \frac{\rho L A_B}{M_p} - \frac{\rho L^2 A_B^2}{2V_F M_p} + \left. \frac{\rho A_B^3 L^3}{3V(z_p) V_F M_p} \right] \\ & + \frac{\rho L^3}{2g_o} \left(C_1(t) - \left(\frac{d \ln \rho}{dt} \right)^2 + \frac{g_o A_B^2 P_{res}}{M_p V_F} \right) \left(1 - \frac{2A_B L}{3V(z_p)} \right) \\ & + \frac{a_1(t) J_3(z_b)}{V(z_p)} + \frac{b(t) J_4(z_b)}{V(z_p)} + \frac{A_B L a_1(t) J_1(z_b)}{V(z_p)} \\ & + \frac{A_B L b(t) J_2(z_b)}{V(z_p)} + \frac{\rho A_B^2 L^2 P_{res}}{2V(z_p) M_p} \\ & - a_1(t) J_1(z_b) - b(t) J_2(z_b) - \frac{A_B \rho L P_{res}}{M_p}. \end{aligned} \quad (R.120)$$

Equation (R.120) gives P_B in terms of P_m (which is determined using an equation of state) and then (R.119) gives P_{Br} from P_B .

The evaluation of $J_1(z_b)$, $J_2(z_b)$, $J_3(z_b)$, and $J_4(z_b)$ can be simplified by noting that the variation in area is confined to the chamber and, therefore,

$$V(z) = V(z_{bo}) + A_B(z - z_{bo}), \text{ for } z \geq z_{bo}, \quad (R.121)$$

and

$$\begin{aligned} J_1(z_b) &= \int_0^{z_b} \frac{V(z)}{A(z)} dz = \int_0^{z_{bo}} \frac{V(z)}{A(z)} dz + \int_{z_{bo}}^{z_b} \frac{V(z)}{A(z)} dz \\ &= J_1(z_{bo}) + \int_{z_{bo}}^{z_b} \frac{V(z_{bo}) + A_B(z - z_{bo})}{A_B} dz \\ &= J_1(z_{bo}) + \frac{1}{A_B} \left(V(z_{bo}) (z_b - z_{bo}) + \frac{A_B (z_b - z_{bo})^2}{2} \right) \end{aligned} \quad (R.122)$$

$$J_2(z_b) = \frac{V^2(z)}{A^2(z)} \Big|_{z_b} = \frac{[V(z_{bo}) + A_B (z_b - z_{bo})]^2}{A_B^2}, \quad (R.123)$$

$$\begin{aligned} J_3(z_b) &= \int_0^{z_b} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A(z) dz \\ &= \int_0^{z_{bo}} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A(z) dz + \int_{z_{bo}}^{z_b} \left(\int_0^z \frac{V(x)}{A(x)} dx \right) A_B dz \\ &= J_3(z_{bo}) + \int_{z_{bo}}^{z_b} \left[J_1(z_{bo}) + \frac{1}{A_B} \left[V(z_{bo}) (z - z_{bo}) + \frac{A_B}{2} (z - z_{bo})^2 \right] \right] A_B dz \\ &= J_3(z_{bo}) + A_B J_1(z_{bo}) (z_b - z_{bo}) + \frac{V(z_{bo}) (z_b - z_{bo})^2}{2} \\ &\quad + \frac{A_B}{6} (z_b - z_{bo})^3, \end{aligned} \quad (R.124)$$

$$\begin{aligned} J_4(z_b) &= \int_0^{z_b} \frac{V^2(z)}{A(z)} dz = \int_0^{z_{bo}} \frac{V^2(z)}{A(z)} dz + \int_{z_{bo}}^{z_b} \frac{[V(z_{bo}) + A_B (z - z_{bo})]^2}{A_B} dz \\ &= J_4(z_{bo}) + \frac{[V(z_{bo}) + A_B (z_b - z_{bo})]^3 - V^3(z_{bo})}{3A_B^2}. \end{aligned} \quad (R.125)$$

Equations (R.122) - (R.125) require the evaluation of the integrals $J_1(z_{bo})$ - $J_4(z_{bo})$ only once.

The kinetic energy (KE) of the gas and solid will be required. The kinetic energy is given by

$$g_o \quad KE = \frac{1}{2} \int_0^{z_b} u^2 \, dm$$

where u is the velocity and dm is the differential of mass, and since $\rho dV = dm$ and $dV = A(z)dz$, then

$$\begin{aligned} g_o \quad KE &= \frac{1}{2} (1 - \epsilon) \int_0^{z_b} u^2 \rho_p A(z) dz \\ &\quad + \frac{1}{2} \epsilon \int_0^{z_b} u^2 \rho A(z) dz + \frac{1}{2} \int_{z_b}^{z_p} u^2 \rho A_B dz. \end{aligned} \quad (R.126)$$

We get u in the mixture region from (R.8) and u_p from (R.13) and u in the ullage region from (R.21), and therefore

$$\begin{aligned} g_o KE &= \frac{1}{2} (1-\epsilon) \int_0^{z_b} \frac{U_p^2 A_B^2 \rho_p V^2(z)}{V^2(z_b) A(z)} dz \\ &+ \frac{1}{2} \epsilon \int_0^{z_b} \frac{U_s^2 A_B^2 \rho_p V^2(z)}{V^2(z_b) A(z)} dz \\ &+ \frac{\rho A_B}{2} \int_{z_b}^{z_p} \left(V_p + \frac{d \ln \rho}{dt} (z_p - z) \right)^2 dz \end{aligned} \quad (R.127)$$

or

$$\begin{aligned} KE &= \frac{(1-\epsilon) U_p^2 A_B^2 \rho_p J_4(z_b)}{2 g_o V^2(z_b)} + \frac{\epsilon U_s^2 A_B^2 \rho J_4(z_b)}{2 g_o V^2(z_b)} \\ &+ \frac{\rho A_B L}{6 g_o} \left[3 V_p^2 + 3 V_p L \frac{d \ln \rho}{dt} + L^2 \left(\frac{d \ln \rho}{dt} \right)^2 \right]. \end{aligned} \quad (R.128)$$

It would appear that the pressure used in evaluating the burning rate should be the mean pressure over the region occupied by the propellant.

The mean pressure over the mixture region is

$$P_{mix} = \frac{\int_0^{z_b} P(z) A(z) dz}{\int_0^{z_b} A(z) dz} \quad (R.129)$$

and using (R.84), we get

$$P_{mix} = \frac{\int_0^{z_b} [P_{Br} + (a_1(t) + a_2(t)P_B) J_1(z) + b(t)J_2(z)] A(z) dz}{V(z_b)} \quad (R.130)$$

or

$$P_{mix} = P_{Br} + \frac{(a_1(t) + a_2(t)P_B)}{V(z_b)} J_3(z_b) + \frac{b(t)}{V(z_b)} J_4(z_b). \quad (R.131)$$

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APPENDIX 4:
INPUT DESCRIPTION FOR IBRGA

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USER'S MANUAL FOR IBRGA

IBRGA relies on an input data base consisting of all numerical parameters essential for running the code. All values are in metric units. Below is a compilation of a typical data base showing the name and location of each parameter. The names for the numerical values are prefixed with an alphabetical designator corresponding to the position at which the data are to appear, that is, from left to right. The data may be separated by blanks or commas. The units are shown to the right of each input.

A B C D E F G H I J K

record 1

- A. - chamber volume (cm³)
- B. - groove diameter (cm)
- C. - land diameter (cm)
- D. - groove/land ratio (none)
(land, groove, and groove/land ratio used to calculate the projectile bore area)
- E. - twist (turns/caliber)
- F. - projectile travel (cm)
- G. - gradient switch (none)
(integer value designating the gradient equation to use
1 = Lagrange, 2 = Chambrage, 3 = Two-phase, 4 = RGA)
- H. - friction factor (none)
(normally 1 for granular, 0.01 for stick, and 0.1 for partially cut propellant, only used with the two-phase and RGA gradient)

record 1a (Read if and only if gradient switch = 2 or 4)

- A. - number of pairs of points to describe chamber geometry, integer I ≤ 5 (none)
- B. - initial distance from breech (cm)
(must be 0.0)
- C. - diameter at initial distance (cm)
-
- . - Ith distance from breech (cm)
(initial position of the base of the projectile)
- . - Ith diameter at Ith distance (cm)
(used to calculate bore area overriding record 1 groove and land diameter specifications)
(note chamber geometry is used to calculate the chamber volume overriding record 1 chamber volume description)

record 2

- A. - projectile mass (kg)
- B. - switch to calculate energy lost to air resistance,
integer (none) (0 = no loss, 1 = loss)
- C. - fraction of bore resistance work used to heat the tube (none) (≥ 0.0, ≤ 1.0)
- D. - gas pressure in front of the projectile (Pa)

record 3

- A. - number of pairs of barrel resistance points (none)
(integer $J \leq 10$)
- B. - bore resistance (MPa)
- C. - travel (cm)
- .
- .
- .
- .
- Jth bore resistance (MPa)
- Jth travel (cm)

record 4

- A. - mass of recoiling parts (kg)
- B. - number of recoil point pairs (none) (must be an integer = 2)
- C. - recoil force (N) (force to overcome before start of recoil - rod preload)
- D. - time of rod preload (s) (must be 0.0)
- E. - recoil force (N) (constant resistive force after rise time)
- F. - rise time (s) (time to go from start of recoil to constant resistive recoil force)

record 5

- A. - free convective heat transfer coefficient ($W/cm^2 \cdot K$)
- B. - chamber wall thickness (cm) (wall depth which is heated uniformly)
- C. - heat capacity of chamber wall ($J/g \cdot K$)
- D. - initial temperature of chamber wall (K)
- E. - heat loss coefficient (none) (usually 1, but may be set to 0.0 to eliminate heat loss)
- F. - density of chamber wall (g/cm^3)

record 6

- A. - impetus of igniter (J/g)
- B. - covolume of igniter (cm^3/g)
- C. - adiabatic flame temperature of igniter (K)
- D. - mass of igniter (kg)
- E. - ratio of specific heats of igniter (none)

record 7

- A. - number of propellants (none) (integer ≤ 10)

record 8

- A. - impetus of propellant (J/g)
- B. - adiabatic flame temperature of propellant (K)
- C. - covolume of propellant (cm^3/g)
- D. - mass of propellant (kg)
- E. - density of propellant (g/cm^3)
- F. - ratio of specific heats of propellant (none)
- G. - propellant form function indicator (none)
(integer, may be
0, zero perforated cylindrical grain
1, one perforated cylindrical grain
7, seven perforated cylindrical grain
15, nineteen perforated hexagonal grain
19, nineteen perforated cylindrical grain)
- H. - length of propellant grain (cm)
- I. - diameter of perforations in the propellant grains (cm) (value ignored if not required but must be present)

- J. - outside diameter of propellant grain (cm)
(for the hexagonal grain, it is the distance between rounded corners)

record 8 repeated for each propellant

record 9

- A. - number of burning rate triplet points (none)
(integer $J \leq 10$)
- B. - exponent (none)
- C. - coefficient (cm/s-MPaⁿ)
- D. - pressure (MPa) (upper pressure limit for which the previous exponent and coefficient are usable)

.

.

.

. - Jth exponent (none)

. - Jth coefficient (cm/s-MPaⁿ)

. - Jth pressure (MPa) (if pressure exceeds this limit, then this burning rate equation is used)

record 9 repeated for each propellant

record 10

- A. - integration time increment (ms)
- B. - print increment (ms)
- C. - upper limit on integration time to stop calculation (ms)

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APPENDIX 5:
FORTRAN LISTING OF IBRGA

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```

program ibrga
common nsl1,kpr,fracsl(10),dsdxsl(10),surfsl(10),
& nslp(10),tsl(10),pbrch,pbase,pmean,bbr(10),abr(10),
& deltat,y(20),igrad
character outfil*10
character bdfil*10
dimension br(10),trav(10),rp(10),tr(10),forcp(10),tempp(10),covp(
&10)
dimension chwp(10),rhop(10),gamap(10),nperfs(10),glenp(10),pdp(10
&),gdiap(10),alpha(10,10),beta(10,10),pres(10,10)
dimension a(4),b(4),ak(4),d(20),p(20),z(20),frac(10),surf(10
&),volp(10),dsdx(10),nbr(10),ibo(10),tbo(10),d2xdt2(10),tng(10)
real lambda,j1zp,j2zp,j3zp,j4zp,j1zb,j2zb,j3zb,j4zb
dimension chdist(5),chdiam(5),bint(4)
dimension nsl(10),surfo(10),dsdxn(10)
c call gettim(ihr,imin,isech,ihuns)
pi=3.14159
write(*,15)
15 format(' input name of data file to be used as input ')
read(*,10)bdfil
10 format(a10)
open(unit=2,err=999,file=bdfil,status='old',iostat=ios)
rewind 2
write(*,25)
25 format(' input name of output file ')
read(*,10)outfil
open(unit=6,err=998,file=outfil)
write(6,16)bdfil
16 format(' the input file is ',a10)
do 9 i=1,20
p(i)=0.
y(i)=0.
z(i)=0.
d(i)=0.
9 continue
read(2,*,end=20,err=30)cham,grvc,aland,glr,twst,travp,igrad
&,fs0
if(igrad.gt.1)go to 51
write(6,55)
55 format(1x,'using Lagrange pressure gradient')
igrad=1
go to 52
c define chambrage assumes nchpts=number of points to define
c chamber > or = 2 < or = 5 (?),chdiam(I) defines chamber diameter
c at chdist (I) chamber distance. chdiam(nchpts) is assumed to be
c the bore diameter and chdist(i) is assumed to be 0, i.e. at the
c breech. Assumes truncated cones.
51 if(igrad.eq.3)go to 401
if(igrad.eq.4)go to 434
write(6,47,err=30)
47 format(1x,'Using chambrage pressure gradient')
go to 436

```

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434 write(6,437)
437 format(1x,'using rga gradient')
go to 436

436 read(2,*,end=20,err=30)nchpts,(chdist(I),chdiam(I),I=1,nchpts)
write(6,53,err=30)(chdist(I),chdiam(I),I=1,nchpts)
53 format(///,' chamber distance cm chamber diameter cm',/(5x,e14
&.6,5x,e14.6))
do 54 I=1,nchpts
chdist(I)=0.01*chdist(I)
54 chdiam(I)=0.01*chdiam(I)
c calculate chamber integrals and volume
if(nchpts.gt.5) write(6,44,err=30)
44 format(1x,'use first 5 points')
if(nchpts.gt.5)nchpts=5
bore=chdiam(nchpts)
if(chdist(1).ne.0.0)write(6,45,err=30)
45 format(1x,' # points ? ')
chdist(1)=0.0
pi3=pi/3.0
b1=0.0
b2=0.0
b3=0.0
b4=0.0
points=25.0
56 points=points+points
step=chdist(nchpts)/points
zz=0.0
bint(1)=0.0
bint(3)=0.0
bint(4)=0.0
bvol=0.0
r2=0.5*chdiam(1)
k=1
j=int(points+0.5)
do 57 I=1,j
zz=zz+step
if(k.eq.nchpts-1)go to 46
do 58 I1=k,nchpts-1
if(zz.gt.chdist(I1).and. zz.lt.chdist(I1+1))go to 59
58 continue
I1=nchpts-1
59 k=I1
46 diam=(zz-chdist(k))/(chdist(k+1)-chdist(k))
diam=chdiam(k)+diam*(chdiam(k+1)-chdiam(k))
r1=0.5*diam
area=pi*(r1+r2)*(r1+r2)/4.
bvol=bvol+step*pi3*(r1*r1+r1*r2+r2*r2)
bint(1)=bint(1)+step*bvol/area
bint(3)=bint(3)+step*area*bint(1)
bint(4)=bint(4)+step*bvol*bvol/area
57 r2=r1
temp=abs(1.0-b1/bint(1))
if(abs(1.0-b3/bint(3)).gt.temp)temp=abs(1.0-b3/bint(3))

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```

      if(abs(1.0-b4/bint(4)).gt.temp)temp=abs(1.0-b4/bint(4))
      if(temp.le.0.001)go to 41
      b1=bint(1)
      b3=bint(3)
      b4=bint(4)
      go to 56
41    cham=bvol*1.e6
c    write(6,47,err=30)bint(1),bint(3),bint(4)
c    format(1x,'bint 1 = ',e14.6,' bint 3 = ',e14.6,' bint 4 = ',e14.
c    &6)
      chmlen=chdist(nchpts)
      go to 52
401  write(6,402)
402  format(1x,'using 2 phase gradient equation')
      goto 52
52    write(6,40,err=30)cham,grve,aland,glr,twst,travp,igrad,fs0
40    format(1x,'chamber volume cm**3',e14.6,' groove diam cm',e14.6/
&' land diam cm',e14.6,' groove/land ratio',e14.6,' twist turns
&/caliber ',e14.6,' projectile travel cm',e14.6
&,' gradient # ',i3,' friction factor ',e14.6//)
      cham=cham*1.e-6
      grve=grve*1.e-2
      aland=aland*1.e-2
      travp=travp*1.e-2
      read(2,*,end=20,err=30)prv,t,iair,htfr,pgas
      write(6,50,err=30)prv,t,iair,htfr,pgas
50    format(1x,'projectile mass kg',e14.6,' switch to calculate energ
&y lost to air resistance J',i2,' fraction of work against bore u
&sed to heat the tube',e14.6/1x,' gas pressure Pa' ,e14.6)
      read(2,*,end=20,err=30)npts,(br(i),trav(i),i=1,npts)
      write(6,60,err=30)npts,(br(i),trav(i),i=1,npts)
60    format(1x,'number barrel resistance points',i2,' bore resistance
& MPa - travel cm'/(1x,e14.6,e14.6))
      write(6,65)
      do 62 i=1,npts
      br(i)=br(i)*1.e6
      trav(i)=trav(i)*1.e-2
62    continue
65    format(1x)
      read(2,*,end=20,err=30)rcwt,nrp,(rp(i),tr(i),i=1,nrp)
      write(6,70,err=30)rcwt,nrp,(rp(i),tr(i),i=1,nrp)
70    format(1x,' mass of recoiling parts kg',e14.6,' number of recoi
&l point pairs',i2,' recoil force N',' recoil time sec'/(1x,e14
&.6,3x,e14.6))
      write(6,65)
      read(2,*,end=20,err=30)ho,tshl,csht,twal,hl,rhocs
      write(6,75,err=30)ho,tshl,csht,twal,hl,rhocs
75    format(1x,' free convective heat transfer coefficient w/cm**2 K',
&e14.6,' chamber wall thickness cm',e14.6,' heat capacity of st
&eel of chamber wall J/g K',e14.6,' initial temperature of chambe
&r wall K',e14.6,' heat loss coefficient',e14.6,' density of ch
&amber wall steel g/cm**3',e14.6//)
      ho=ho/1.e-4
      tshl=tshl*1.e-2

```

```

cshl=cshl*1.e+3
rhocs=rhocs*1.e-3/1.e-6
read(2,*,end=20,err=30)forcig,covi,tempi,chw, gamai
write(6,85,err=30)forcig,covi,tempi,chw, gamai
85  format(1x,' impetus of igniter propellant J/g',e14.6,' covolume
& of igniter cm**3/g',e14.6,' adiabatic flame temperature of igni
& ter propellant K',e14.6,' initial mass of igniter kg',e14.6,' r
&atio of specific heats for igniter',e14.6//)
forcig=forcig*1.e+3
covi=covi*1.e-6/1.e-3
read(2,*)nprop
write(6,98)nprop
98  format(' there are ',i2,' propellants')
read(2,*,end=20,err=30)(forcp(i),tempp(i),covp(i),chwp(i),
& rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),i=1,nprop)
write(6,95,err=30)(i,forcp(i),tempp(i),covp(i),chwp(i)
& ,rhop(i),gamap(i),nperfs(i),glenp(i),pdp(i),gdiap(i),i=1,nprop)
95  format((' for propellant number',i2,' impetus of propellant J/g
& ',e14.6,' adiabatic temperature of propellant K',e14.6,' covol
& ume of propellant cm**3/g',e14.6,' initial mass of propellant kg'
& ,e14.6,' density of propellant g/cm**3',e14.6,' ratio of specifi
& c heats for propellant',e14.6,' number of perforations of propell
& ant',i2,' length of propellant grain cm',e14.6,' diameter of per
& foration in propellant grains cm',e14.6,' outside diameter of pro
& pellant grain cm',e14.6//)
tmpi=0.0
do 96 i=1,nprop
forcp(i)=forcp(i)*1.e+3
covp(i)=covp(i)*1.e-6/1.e-3
rhop(i)=rhop(i)*1.e-3/1.e-6
glenp(i)=glenp(i)*0.01
pdp(i)=pdp(i)*0.01
gdiap(i)=gdiap(i)*0.01
tmpi=tmpi+chwp(i)
kpr=i
call prf710(pdp(i),gdiap(i),glenp(i),nperfs(i),0.,
& frac(i),volp(i),surf(i),dsdx(i))
tng(i)=chwp(i)/rhop(i)/volp(i)
surfo(i)=surf(i)
write(6,408)i,tng(i)
408  format(' for propellant ',i2,' the total number of grains'
& ', is ',e14.6)
96  continue
tmpi=tmpi+chw
do 97 j=1,nprop
read(2,*,end=20,err=30)nbr(j),(alpha(j,i),beta(j,i),pres(j,i),
& i=1,nbr(j))
write(6,110,err=30)nbr(j),(alpha(j,i),beta(j,i),pres(j,i),
& i=1,nbr(j))
110  format(1x,'number of burning rate points',i2/3x,' exponent',8x,'
& coefficient',10x,' pressure',5x,'-',15x,'cm/sec-MPa**ai',10x,'MP
& a',/(1x,e14.6,5x,e14.6,15x,e14.6))
do 112 i=1,nbr(j)
beta(j,i)=beta(j,i)*1.e-2

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```

pres(j,i)=pres(j,i)*1.e6
112 continue
97 continue
write(6,65)
read(2,*,end=20,err=30)deltat,deltap,tstop
write(6,120,err=30)deltat,deltap,tstop
120 format(1x,'time increment msec',e14.6,' print increment msec',e14
&.6/1x,'time to stop calculation msec ',e14.6)
write(*,130)
deltat=deltat*0.001
deltap=deltap*0.001
tstop=tstop*.001
130 format(1x,'end input data -- I.B. calculation start')
if(igrad.eq.2.or.igrad.eq.4)go to 131
bore=(glr*grve*grve+aland*aland)/(glr+1.)
bore=sqrt(bore)
131 areab=pi*bore*bore/4.
lambda=1./((13.2+4.*log10(100.*bore))**2)
iplot=0
pltdt=deltat
pltt=0.
pmaxm=0.0
pmaxbr=0.0
pmaxba=0.0
tpmaxm=0.0
tpmxb=0.0
tpmxb=0.0
tpmax=0.0
a(1)=0.5
a(2)=1.-sqrt(2.)/2.
a(3)=1.+sqrt(2.)/2.
a(4)=1./6.
b(1)=2.
b(2)=1.
b(3)=1.
b(4)=2.
ak(1)=0.5
ak(2)=a(2)
ak(3)=a(3)
ak(4)=0.5
vp0=0.0
tr0=0.0
tcw=0.0
if(igrad.eq.3)chmlen=cham/areab
zb=chmlen
zp=chmlen
grlen=0.
grdiam=0.
egama=0.
do 5 i=1,nprop
grlen=grlen+chwp(i)*glenp(i)
grdiam=grdiam+chwp(i)*gdiap(i)
ibo(i)=0
egama=egama+chwp(i)*gamap(i)

```

```

    nsl(i)=0
5   vp0=chwp(i)/rho(i)+vp0
    volgi=cham-vp0-chwi*covi
    grlen=grlen/(tmpi-chwi)
    grdiam=grdiam/(tmpi-chwi)
    egama=(egama+chwi*gamai)/tmpi
    ism=0
    odlnr=0.
    vf0=cham-vp0
    eps0=1.-vp0/cham
    eps=eps0
    gasden=chwi/vf0
    prden=tmpi/vp0
    ug=0.
    up=0.
    pmean=forcig*chwi/volgi
    pbase=pmean
    pbrch=pmean
    opbase=pmean
    volg=volgi
    volgi=volgi+vp0
    wallt=twal
    tgas=tempi
    told=0.
    tgaso=tgas
    dtgaso=0.
    cov1=covi
    t=0.
    ptime=0.0
    ibrp=10
    .z(3)=1.
    nde=ibrp+nprop
    write(6,132)areab,pmean,vp0,volgi
132  format(1x,'area bore m^2 ',e16.6,/' pressure from ign pa',e16.6,/
    &,' volume of unburnt prop m^3 ',e16.6,/
    &' init cham vol-cov ign m^3 ',e16.6)
    write(6,6)
6   format(1x,'   time       acc       vel       dis       mpress
    & pbase       pbrch   ')
    isw1=0
19  continue
    do 11 J=1,4
c   FIND BARREL RESISTANCE
    do 201 k=2,npts
    if(y(2)+y(7).ge.trav(k))go to 201
    go to 203
201  continue
    k=npts
203  resp=(trav(k)-y(2)-y(7))/(trav(k)-trav(k-1))
    resp=br(k)-resp*(br(k)-br(k-1))
c   FIND MASS FRACTION BURNED
    do 211 k=1,nprop
    kpr=k
    if(ibo(k).eq.1)goto211

```

```

      nsl1=0
      call prf710(pdp(k),gdiap(k),glenp(k),nperfs(k),y(ibrp+k)
&,frac(k),volp(k),surf(k),dsdx(k))
      nsl(k)=nsl1
      if(nsl(k).eq.0)goto 212
      if(nslp(k).eq.1)go to 212
      write(6,2213)k
2213  format(' propellant',i2,' has slivered')
      nslp(k)=1
      tsl(k)=y(3)
      ism=1
212  continue
      if(frac(k).lt..9999) go to 211
      frac(k)=1.
      tbo(k)=y(3)
      ibo(k)=1
      ism=1
      write(6,456)k
456  format(' propellant',i2,' has burned out')
211  continue
c    ENERGY LOSS TO PROJECTILE TRANSLATION
      elpt=prwt*y(1)*y(1)/2.
      eptdot=prwt*y(1)*z(1)
c    ENERGY LOSS DUE TO PROJECTILE ROTATION
      elpr=pi*pi*prwt*y(1)*y(1)/4.*twst*twst
      eprdot=pi*pi*prwt*y(1)*z(1)/2.*twst*twst
c    ENERGY LOSS DUE TO GAS AND PROPELLANT MOTION
      if(igrad.eq.1)go to 214
      if(igrad.eq.3)goto 217
      if(igrad.eq.4)go to 438
      pt=y(2)+y(7)
      vzb=bvol+areab*pt
      j4zp=bint(4)+((bvol+areab*pt)**3-bvol**3)/3./areab/areab
      elgpm=tmpi*y(1)*y(1)*areab*areab*j4zp/2./vzp/vzp/vzp
      go to 216
438  pb=y(7)+y(10)
      vzb=bvol+areab*pb
      j4zb=bint(4)+(vzb**3-bvol**3)/3./areab/areab
      elgpm=(1.-eps)*up*up*areab**2*prden*j4zb+
& eps*ug*ug*areab**2*gasden*j4zb
      elgpm=elgpm/2./vzb/vzb+gasden*areab*ullen/6.*
& (3.*y(1)*y(1)+3.*y(1)*ullen*dlnrho+ullen**2*dlnrho**2)
c    approximate epdot
      epdot=tmpi*y(1)*z(1)/3.
      go to 216
214  elgpm=tmpi*y(1)*y(1)/6.
      go to 216
217  elgpm=areab*z(3)/6.*(eps*gasden*ug*ug+(1.-eps)*prden*up*up)
      elgpm=elgpm+gasden*areab*ullen/6.*(3.*y(1)*y(1)+
& 3.*y(1)*ullen*dlnrho+ullen**2*dlnrho**2)
c    approximate epdot
      epdot=tmpi*y(1)*z(1)/3.
c    ENERGY LOSS FROM BORE RESISTANCE
216  elbr=y(4)

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z(4)=areab*resp*y(1)
ebrdot=z(4)
c ENERGY LOSS DUE TO RECOIL
elrc=rcwt*y(6)*y(6)/2.
erdot=rcwt*y(6)*z(6)
c ENERGY LOSS DUE TO HEAT LOSS
areaw=cham/areab*pi*bore+2.*areab*pi*bore*(y(2)+y(7))
avden=0.0
avc=0.0
avcp=0.0
z18=0
z19=0
do 213 k=1,nprop
z18=forc(k)*gamap(k)*chwp(k)*frac(k)/(gamap(k)-1.)/tempp(k)+z18
z19=chwp(k)*frac(k)+z19
avden=avden+chwp(k)*frac(k)
213 continue
avcp=(z18+forcig*gamai*chwi/(gamai-1.)/tempi)/(z19+chwi)
avden=(avden+chwi)/(volg+cov1)
avvel=.5*y(1)
htns=lambda*avcp*avden*avvel+ho
z(5)=areaw*htns*(tgas-wallt)*hl
elht=y(5)
ehdot=z(5)
wallt=(elht+htfr*elbr)/cshl/rhocs/areaw/tshl+twal
c write(6,*)lambda,avcp,avden,avvel,ho,areaw,htns,tgas,wallt,hl,z(5)
c &,elht
c ENERGY LOSS DUE TO AIR RESISTANCE
air=iair
z(8)=y(1)*pgas*air
elar=areab*y(8)
eddot=z(8)*areab
c RECOIL
z(6)=0.0
if(pbrch.le.rp(1)/areab)go to 221
rfor=rp(2)
if(y(3)-tr0.ge.tr(2))go to 222
rfor=(tr(2)-(y(3)-tr0))/(tr(2)-tr(1))
rfor=rp(2)-rfor*(rp(2)-rp(1))
222 z(6)=areab/rcwt*(pbrch-rfor/areab-resp)
if(y(6).lt.0.0)y(6)=0.0
z(7)=y(6)
goto 223
221 tr0=y(3)
223 continue
c CALCULATE GAS TEMPERATURE
eprop=0.0
rprop=0.0
dmfugt=0.0
dmfugt=0.0
do 231 k=1,nprop
eprop=eprop+forc(k)*chwp(k)*frac(k)/(gamap(k)-1.)
rprop=rprop+forc(k)*chwp(k)*frac(k)/(gamap(k)-1.)/tempp(k)
dmfugt=dmfugt+forc(k)*rhop(k)*tng(k)*surf(k)*z(ibrp+k)/

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& ((gamap(k)-1.)*tempp(k))
  dmfog=dmfog+forcp(k)*rhop(k)*tng(k)*surf(k)*z(ibrp+k)/
& (gamap(k)-1.)
231  continue
    tenerg=elpt+elpr+elgpm+elbr+elrc+elht+elar
    tgas=(cprop+forcig*chwi/(gamai-1.))-elpt-elpr-elgpm-elbr-elrc-elht
& -elar)/(rprop+forcig*chwi/(gamai-1.)/tempi)
    tedot=epdot+eprdot+eddot+ebrdot+erdot+ehdot+eptdot
    dtgas=(dmfog-tedot-tgas*dmfog)/(rprop+forcig*chwi/
& (gamai-1.)/tempi)
c    FIND FREE VOLUME
    v1=0.0
    cov1=0.0
    do 241 k=1,nprop
      v1=chwp(k)*(1.-frac(k))/rhop(k)+v1
      cov1=cov1+chwp(k)*covp(k)*frac(k)
241  continue
    volg=volgi+areab*(y(2)+y(7))-v1-cov1
c    CALCULATE MEAN PRESSURE
    r1=0.0
    do 251 k=1,nprop
      r1=r1+forcp(k)*chwp(k)*frac(k)/tempp(k)
251  continue
    pmean=tgas/volg*(r1+forcig*chwi/tempi)
259  resp=resp+pgas*air
    if(igrad.eq.1)go to 252
    if(igrad.eq.2)goto 403
    if(igrad.eq.3)go to 404
    if(igrad.eq.4)go to 441
403  if(isw1.ne.0)go to 253
    pbase=pmean
    pbrch=pmean
    if(pbase.gt.resp+1.)isw1=1
    go to 257
c    USE CHAMBRAGE PRESSURE GRADIENT EQUATION
253  j1zp=bint(1)+(bvol*pt+areab/2.*pt*pt)/areab
    j2zp=(bvol+areab*pt)**2/areab/areab
    j3zp=bint(3)+areab*bint(1)*pt+bvol*pt*pt/2.+areab*pt*pt*pt/6.
    a2t=-tmpi*areab*areao/prwt/vzp/vzp
    alf=1.-a2t*j1zp
    a1t=tmpi*areab*(areab*y(1)*y(1)/vzp+areab*resp/prwt)/vzp/vzp
    bt=-tmpi*y(1)*y(1)*areab*areab/2./vzp/vzp/vzp
    bata=-a1t*j1zp-bt*j2zp
    gamma=alf+a2t*j3zp/vzp
    delta=bata+a1t*j3zp/vzp+bt*j4zp/vzp
c    calculate base pressure
    pbase=(pmean-delta)/gamma
c    calculate breech pressure
    pbrch=alf*pbase+bata
    go to 254
c    USE 2 PHASE GRADIENT EQUATION
404  IF(ISW1.NE.0)GOTO 407
    pbase=pmean
    pbrch=pmean

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```

if(pbase.gt.resp+1)isw1=1
go to 257
407 if(isw1.eq.2)go to 411
vzp=cham+areab*(y(2)+y(7))
vzb=cham+areab*(y(10)+y(7))
phi=0.
phidot=0.
dmorho=0.
dmcov=0.
dmromw=0.
rromw=0.
vfree=vzp-v1
do 405 k=1,nprop
rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/tempp(k)
phi=chwp(k)*frac(k)+phi
if(ibo(k).eq.1)go to 405
dmorho=dmorho+tng(k)*surf(k)*z(ibrp+k)
phidot=rhop(k)*tng(k)*surf(k)*z(ibrp+k)+phidot
dmcov=rhop(k)*tng(k)*surf(k)*z(ibrp+k)*covp(k)+dmcov
dmromw=dmromw+rhop(k)*tng(k)*surf(k)*z(ibrp+k)*
& forcp(k)/tempp(k)
405 continue
rmomw=rmomw+chwi*forcig/tempi
gasmass=phi+chwi
gasden=gasmass/vfree
phi=(phi+chwi)/tempi
if (phi.gt.0.999) then
isw1=2
rbm=pbase/pmean
rbm=pbrch/pmean
if(phi.ge.1.)go to 411
endif
dmdt=phidot
phidot=phidot/tempi
vdotov=(dmorho+areab*y(1))/vfree
dlnrho=dmdt/gasmass-vdotov
dvoldt=dmorho+areab*y(1)-dmcov
c GET TIME DERIVATIVE OF MEAN PRESSURE
dpmtdt=(dmromw*tgas-pmean*dvoldt+dtgas*rmomw)/volg
volprp=0.
effdia=0.
dmdmdt=0.
dmdmor=0.
avelen=0.
avedia=0.
do 406 k=1,nprop
if(ibo(k).eq.1)go to 406
volprp=volprp+(1.-frac(k))*chwp(k)/rhop(k)
dmdmdt=dmdmdt+rhop(k)*tng(k)*dsdx(k)*z(ibrp+k)*z(ibrp+k)
dmdmdt=dmdmdt+rhop(k)*tng(k)*surf(k)*d2xdt2(k)
dmdmor=dmdmor+(dsdx(k)*z(ibrp+k)**2+surf(k)*d2xdt2(k))*tng(k)
effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
406 continue
c1t=dmdmdt/gasmass-dmdmor/vfree+vdotov**2-(dmdt/gasmass)**2

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```

d2lnr=c1t-areab**2*pbase/vfree/prwt
d2lnr=d2lnr+areab*areab*resp/vfree/prwt
zp=chmlen+y(2)+y(7)
zb=chmlen+y(10)+y(7)
ullen=zp-zb
cnow=tmpi-gasmas
vp=y(1)
effdia=effdia/cnow
prden=cnow/volprp
up=y(9)
phistr=phi-gasden*areab*ullen/tmpi
ulldot=vp-up
dphist=phidot-gasden*areab/tmpi*(ulldot+ullen*dlnrho)
eps=1.-(1.-phi)*tmpi/prden/vzb
epsdot=phidot*tmpi/prden/vzb+(1.-phi)*tmpi*up*areab/
& prden/vzb/vzb
ug=up+(vp+ullen*dlnrho-up)/eps
alam=(1.5*grlen/grdiam)**.666666667
alam=(0.5+grlen/grdiam)/alam
alam=alam**2.17
c VIS kg/s/m
vis=.00007
ren=gasden/vis*effdia*abs(ug-up)
if(ren.lt.1.)ren=1.
fsrg=2.5*alam/ren**.081*((1.-eps)/(1.-eps0)*eps0/eps)**.45
fsc=fsrg*fs0
phi2=1.-phi-phistr*(1.-eps)/eps
phi1p=dphist*ug-phidot*up-phistr*epsdot/eps/eps
& *(vp+ullen*dlnrho-up)+phistr*ulldot*dlnrho/eps
& +2.*phistr*ug/zb*(ug-up)
phi1p=phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc
ak2=1./(1.-phi2*tmpi/prden/vzb)
phi1=phi1p+phistr*z(1)/eps+ullen*phistr*d2lnr/eps
c ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED
z(9)=gasden*(ug-up)**2*fsc/prden/effdia+tmpi*phi1*ak2
&/vzb/prden
z(10)=y(9)
e=phistr/eps*(1.-ullen*areab/vfree)*areab/prwt
dd=ullen*phistr*c1t/eps
ak11=tmpi*e*ak2/zb/vzb
ak12=tmpi*ak2*(phi1p+dd)/zb/vzb-ak11*resp
pbase=pmean-ak12*zb*zb/2.+gasden*ullen*resp*areab/prwt
pbase=pbase+ak12*zb*zb*(zb/3.+ullen)/2./zp
pbase=pbase-gasden*ullen**2*areab*resp/2./zp/prwt
pbase=pbase-gasden*ullen**2/2.*(1.-2.*ullen/3./zp)*
& (c1t-dlnrho**2)
pbase=pbase-areab**2*gasden*ullen**2*
&(1.-2.*ullen/3./zp)*resp/prwt/vfree/2.
deno=-ak11*zb**3/6./zp-ullen*ak11*zb*zb/2./zp
deno=deno+gasden*ullen*areab/prwt-areab**2*gasden*ullen**2
&(1.-2.*ullen/3./zp)/2./vfree/prwt
deno=deno-gasden*ullen**2*areab/2./zp/prwt+1.+ak11*zb*zb/2.
pbase=pbase/deno
if(ism.eq.0)goto453

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```

    if(ism.eq.1)goto451
    goto452
451  ism=2
    tss=sqrt(egama*rmomw/gasmas*tgas)
    write(6,*)tss
    tss=ullen/(ullen*odlnr+tss)
    tso=y(3)
    write(6,*)tss,tso
452  coefbp=(tss+tso-y(3)-deltat)/tss
    if(coefbp.gt.1.)coefbp=1.
    if(coefbp.le.0.)then
        coefbp=0.
        ism=0
    endif
    pbase=coefbp*opbase+(1.-coefbp)*pbase
    write(6,*)coefbp,opbase,pbase,ism
453  odlnr=dlnrho
    opbase=pbase
    pbrch=pbase*(1.+ak11*zb*zb/2.+gasden*ullen*areab/prwt
& -areab**2*gasden*ullen**2/2./vfree/prwt)
    pbrch=pbrch+ak12*zb*zb/2.-gasden*ullen*areab*resp/prwt
    pbrch=pbrch+gasden*ullen**2/2.*(c1t-dlnrho**2)
    pbrch=pbrch+areab**2*gasden*ullen**2*resp/2./vfree/prwt
    go to 254
C    USING RGA GRADIENT
441  if(isw1.ne.0)go to 444
    pbase=pmean
    pbrch=pmean
    if(pbase.gt.resp+1.)isw1=1
    go to 257
444  if(isw1.eq.2)go to 411
    vzp=cham+areab*(y(2)+y(7))
    vzb=cham+areab*(y(10)+y(7))
    j1zb=bint(1)+(bvol*pb+areab/2.*pb*pb)/areab
    j2zb=(bvol+areab*pb)**2/areab/areab
    j3zb=bint(3)+areab*bint(1)*pb+bvol*pb*pb/2.+areab/6.*pb**3
    phi=0.
    phidot=0.
    dmorho=0.
    dmcov=0.
    dmromw=0.
    rmomw=0.
    vfree=vzp-v1
    do 442 k=1,nprop
        rmomw=rmomw+chwp(k)*frac(k)*forcp(k)/tempp(k)
        phi=chwp(k)*frac(k)+phi
        if(ito(k).eq.1)go to 442
        dmorho=dmorho+tng(k)*surf(k)*z(ibrp+k)
        phidot=rhop(k)*tng(k)*surf(k)*z(ibrp+k)+phidot
        dmcov=rhop(k)*tng(k)*surf(k)*z(ibrp+k)*covp(k)+dmcov
        dmromw=dmromw+rhop(k)*tng(k)*surf(k)*z(ibrp+k)*
& forcp(k)/tempp(k)
442  continue
    rmomw=rmomw+chwi*forcig/tempi

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```

gasmas=phi+chwi
gasden=gasmas/vfree
phi=(phi+chwi)/tmpi
if (phi.gt.0.99) then
isw1=2
rbm=pbase/pmean
rbm=pbrch/pmean
if(phi.ge.1.)go 411
endif
dmdt=phidot
phidot=phidot/tmpi
vdotov=(dmorho+areab*y(1))/vfree
dlnrho=dmdt/gasmas-vdotov
dvoldt=dmorho+areab*y(1)-dmcov
c get time derivative of mean pressure
dpmtd=(dmromw*tgas-pmean*dvoldt+dtgas*rmomw)/volg
volprp=0.
effdia=0.
dmdmdt=0.
dmdmor=0.
avelen=0.
avedia=0.
do 443 k=1,nprop
if(ibo(k).eq.1)go to 443
volprp=volprp+(1.-frac(k))*chwp(k)/rho(k)
dmdmdt=dmdmdt+rho(k)*tng(k)*dsdx(k)*z(ibrp+k)*z(ibrp+k)
dmdmdt=dmdmdt+rho(k)*tng(k)*surf(k)*d2xdt2(k)
dmdmor=dmdmor+(dsdx(k)*z(ibrp+k)**2+surf(k)*d2xdt2(k))*tng(k)
effdia=effdia+6.*volp(k)/surf(k)*(1.-frac(k))*chwp(k)
443 continue
c1t=dmdmdt/gasmas-dmdmor/vfree+vdotov**2-(dmdt/gasmas)**2
d2lnr=c1t-areab**2*pbase/vfree/prwt
d2lnr=d2lnr+areab*areab*resp/vfree/prwt
zp=chmlen+y(2)+y(7)
zb=chmlen+y(10)+y(7)
ullen=zp-zb
cnow=tmpi-gasmas
vp=y(1)
effdia=effdia/cnow
prden=cnow/volprp
up=y(9)
phistr=phi-gasden*areab*ullen/tmpi
ulldot=vp-up
dphist=phidot-gasden*areab/tmpi*(ulldot+ullen*dlnrho)
eps=1.-(1.-phi)*tmpi/prden/vzb
epsdot=phidot*tmpi/prden/vzb+(1.-phi)*tmpi*up*areab/
& prden/vzb/vzb
ug=up+(vp+ullen*dlnrho-up)/eps
alam=(1.5*grlen/grdiam)**.66666667
alam=(0.5+grlen/grdiam)/alam
alam=alam**2.17
c VIS kg/s/m
vis=.00007
ren=gasden/vis*effdia*abs(ug-up)

```

```

if(ren.lt.1.)ren=1.
fsrg=2.5*alam/ren**.081*((1.-eps)/(1.-eps0)*eps0/eps)**.45
fsc=fsrg*fs0
phi2=1.-phi-phistr*(1.-eps)/eps
phi1p=dphistr*ug-phidot*up-phistr*epsdot/eps/eps
& *(vp+ullen*dlnrho-up)+phistr*ullen*dlnrho/eps
& +2.*areab*phistr*ug/vzb*(ug-up)
phi1p=phi1p+phi2*gasden/effdia/prden*(ug-up)**2*fsc
ak2=1./(1.-phi2*tmpi/prden/vzb)
phi1=phi1p+phistr*z(1)/eps+ullen*phistr*d2lnr/eps
c ACCELERATION OF FORWARD BOUNDARY OF PROPELLANT BED
z(9)=gasden*(ug-up)**2*fsc/prden/effdia+tmpi*phi1*ak2
&/vzb/prden
z(10)=y(9)
phi3=phistr*ug*ug+(1.-phi)*up*up
e=1.-ullen*areab/vfree
dd=ullen*phistr*c1t/eps
a1t=tmpi*areab/vzb/vzb*(phi3*areab/vzb-(phi1p+dd-e
& *phistr*areab*resp/eps/prwt)*ak2)
a2t=(-tmpi*e*phistr*areab**2/vzb/vzb/eps/prwt)*ak2
bt=-tmpi*phi3*areab**2/2./vzb/vzb/vzb
pbase=pmean-gasden*ullen**2/2.*(c1t-dlnrho**2
$ +areab**2*resp/prwt/vfree)*(1.-2.*areab*ullen/3./vzp)
pbase=pbase-a1t*j3zb/vzp-bt*j4zb/vzp-areab*ullen*a1t*j1zb/vzp
pbase=pbase-areab*bt*ullen*j2zb/vzp-gasden*areab**2*ullen**2
&*resp/2./vzp/prwt+a1t*j1zb+bt*j2zb+areab*gasden*ullen*resp/prwt
deno=1.+areab*ullen*a2t*j1zb/vzp-gasden*areab**2*ullen**2
& /2./vzp/prwt+a2t*j3zb/vzp-a2t*j1zb+gasden*ullen*areab/prwt
deno=deno-gasden*ullen**2*areab**2/2./vfree/prwt
& +gasden*areab**3*ullen**3/3./vzp/vfree/prwt
pbase=pbase/deno
pbrch=pbase*(1.-a2t*j1zb+gasden*ullen*areab/prwt
& -gasden*ullen**2*areab**2/2./vfree/prwt)
&+gasden*ullen**2/2.*(c1t-dlnrho**2+areab**2*resp/prwt/vfree)
& -a1t*j1zb-bt*j2zb-areab*gasden*ullen*resp/prwt
go to 254
411 pbase=rbrm*pmean
pbrch=rbrm*pmean
go to 254
c USE LAGRANGE PRESSURE GRADIENT EQUATION
252 if(isw1.ne.0)go to 256
if(pmean.lt.resp)resp=pmean
c CALCULATE BASE PRESSURE
256 pbase=(pmean+tmpi*resp/3./prwt)/(1.+tmpi/3./prwt)
if(pbase.gt.resp+1.)isw1=1
c CALCULATE BREECH PRESSURE
pbrch=pbase+tmpi/2./prwt*(pbase-resp)
c CALCULATE PROJECTILE ACCELERATION
254 z(1)=areab*(pbase-resp)/prwt
if(z(1).lt.0.0)go to 257
go to 258
257 if(isw1.eq.0)z(1)=0.0
258 if(y(1).lt.0.0)y(1)=0.0
z(2)=y(1)

```

```

c   GET BURNING RATE
do 264 m=1,nprop
  z(ibrp+m)=0.0
  d2xdt2(m)=0.0
  if(ibo(m).eq.1) goto 264
  do 262 k=1,nbr(m)
    if(pmean.gt.pres(m,k))go to 262
    go to 263
262  continue
    k=nbr(m)
263  pmix=pmean
    if(igrad.eq.3)pmix=pbrch-(ak11*pbase+ak12)/6.*zb*zb
    if(igrad.eq.4)pmix=pbrch+(a1t+a2t*pbase)*j3zb/vzb+bt*j4zb/vzb
    if(pmix.lt..99*pmean)pmix=pmean
    z(ibrp+m)=beta(m,k)*(pmix*1.e-6)**alpha(m,k)
    abr(m)=alpha(m,k)
    bbr(m)=beta(m,k)
    d2xdt2(m)=beta(m,k)*alpha(m,k)*(pmix*1.e-6)
    & *(alpha(m,k)-1.)*dpmdt*1.e-6
264  continue
    do 21 i=1,nde
      d(i)=(z(i)-b(j)*p(i))*a(j)
      y(i)=deltat*d(i)+y(i)
      p(i)=3.*d(i)-ak(j)*z(i)+p(i)
21    continue
11  continue
    t=t+deltat
    told=y(3)
    if(pmaxm.gt.pmean)go to 281
    pmaxm=pmean
    tpmaxm=y(3)
281  if(pmaxba.gt.pbase)go to 282
    pmaxba=pbase
    tpmxba=y(3)
282  if(pmaxbr.gt.pbrch)go to 283
    pmaxbr=pbrch
    tpmxbr=y(3)
283  continue
    if(y(3).lt.ptime)go to 272
    ptime=ptime+deltap
    pjt=y(2)+y(7)
    write(6,7)y(3),z(1),y(1),pjt,pmean,pbase,pbrch
7    format(1x,7e11.4)
    if(igrad.gt.2)then
      pjt=y(2)+y(7)
      prt=y(10)+y(7)
      write(6,427)prt,pjt
427  format(1x,'prop travel',e11.4,'proj travel',e11.4)
    endif
272  continue
    if(t.gt.tstop)goto 200
    if(y(2)+y(7).gt.travp)go to 200
    rmvelo=y(1)
    tmvelo=y(3)

```

```

    disto=y(2)+y(7)
    go to 19
200  write(6,311)t,y(3)
311  format(1x,'deltat t',e14.6,'intg t',e14.6)
    write(6,312)pmaxm,tpmaxm
312  format(1x,'PMAXMEAN Pa ',e14.6,'time at PMAXMEAN sec ',e14.6)
    write(6,313)pmaxba,tpmxba
313  format(1x,'PMAXBASE Pa ',e14.6,'time at PMAXBASE sec ',e14.6)
    write(6,314)pmaxbr,tpmxbbr
314  format(1x,'PMAXBREECH Pa ',e14.6,'time at PMAXBREECH sec ',e14.6)
    if(y(2)+y(7).le.travp)go to 303
    dfrc=(travp-disto)/(y(2)+y(7)-disto)
    rmvel=(y(1)-rmvelo)*dfrc+rmvelo
    tmvel=(y(3)-tmvelo)*dfrc+tmvelo
    write(6,318)rmvel,tmvel
318  format(1x,'muzzle VELOCITY m/s ',e14.6,'time of muzzle velocity s
    &ec ',e14.6)
    goto 319
303  write(6,327)y(1),y(3)
327  format(1x,'velocity of projectile m/s ',e14.6,'at this time msec
    &',e14.6)
319  efi=chwi*forcig/(gamai-1.)
    efp=0.0
    do 315 i=1,nprop
    efp=efp+chwp(i)*forcp(i)/(gamap(i)-1.0)
315  continue
    tenerg=efi+efp
    write(6,317)tenerg
317  format(1x,'total initial energy available J = ',e14.6)
    tengas=chwi*forcig*tgas/(gamai-1.)/tempi
    do 135 i=1,nprop
    tengas=(frac(i)*chwp(i)*forcp(i)*tgas/tempp(i)/(gamap(i)-1.))+teng
    &as
    write(6,328)i,frac(i),tbo(i)
328  format('FOR PROPELLANT ',I2,'MASSFRACT BURNT IS ',e14.6
    &,'at time in sec ',e14.6)
135  continue
    write(6,136)tengas
136  format(1x,'total energy remaining in gas J= ',e14.6)
    write(6,320)elpt
320  format(1x,'energy loss from projectile translation J= ',e14.6)
    write(6,321)elpr
321  format(1x,'energy loss from projectile rotation J= ',e14.6)
    write(6,322)elgpm
322  format(1x,'energy lost to gas and propellant motion J= ',e14.6)
    write(6,323)elbr
323  format(1x,'energy lost to bore resistance J= ',e14.6)
    write(6,324)elrc
324  format(1x,'energy lost to recoil J= ',e14.6)
    write(6,325)elht
325  format(1x,'energy loss from heat transfer J= ',e14.6)
    write(6,326)elar
326  format(1x,'energy lost to air resistance J= ',e14.6)
    stop

```

```

20  write(*,140)
140 format(1x,'end of file encounter')
    stop
30  write(*,150)
999  continue
998  continue
150  format(1x,'read or write error')
    stop
    end
    subroutine prf710(pd,gd,gl,np,x,frac,vol,surf,dsdx)
    common nsl,kpr,fracsl(10),dsdxsl(10),surfsl(10),
& nslp(10),tsl(10),pbrch,pbase,pmean,bbr(10),abr(10),
& deltat,yar(20) igrad
    dimension ts(10),coef(10)
    pi=3.141593
    nsl=0
C
C   pd=perforation diameter
C   gd=OUTER DIA
C   gl=GRAIN LENGTH
C   NP=NUMBER OF PERFS
C
C   SURF=OUTPUT SURFACE AREA
C   frac=OUTPUT MASS FRACTION OF PROPELLANT BURNED
C
C   w = web = distance between perforation edges
C       = distance between outside perf edge and edge of grain
C
C   p = distance between perforation centers
C
C   x1 = distance to inner sliver burnout
C
C   x2 = distance to outer sliver burnout (frac=1)
C
    if(np.eq.0) go to 2000
    IF(NP.EQ.1)GO TO 3000
    IF(NF.eq.7)GO TO 61
    if(np.eq.19)go to 4000
    if(np.eq.15)go to 5000
60  WRITE(6,90)
90  FORMAT(1X,'UNACCEPTABLE GRANULATION')
    STOP
61  w=(gd-3.*pd)/4.
    d=w+pd
    sqr3=sqrt(3.)
    x1=d/sqr3-pd/2.
    x2=(14.-3.*sqr3)*d/13.-pd/2.
    v0=pi/4.*gl*(gd*gd-7.*pd*pd)
    s0=2.*v0/gl+pi*gl*(gd+7.*pd)
    if (x.gt.w/2.+0.000001) goto 20
    vol=pi/4.*(gl-2.*x)*((gd-2.*x)**2-7.*(pd+2.*x)**2)
    surf=2.*vol/(gl-2.*x)+pi*(gl-2.*x)*((gd-2.*x)+
& 7.*(pd+2.*x))
    frac=1.-vol/v0

```

```

dsdx=-4*pi*(gd+7.*pd-3.*gl+18.*x)
dsdxsl(kpr)=dsdx
fracsl(kpr)=frac
surfsl(kpr)=surf
return
20  nsl=1
    coef(kpr)=0.
    if(igrad.eq.1.or.igrad.eq.2)go to 726
    if(nslp(kpr).eq.1)goto 26
    tsl(kpr)=yar(3)
    ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
    & (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
26  continue
    coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
    if(coef(kpr).gt.1.)coef(kpr)=1.
    if(coef(kpr).lt.0.)coef(kpr)=0.
726  if(x.ge.x2)goto 30
    s1=0.
    s2=0.
    v1=0.
    v2=0.
    ds1dx=0.
    ds2dx=0.
    y=sqrt((pd+2.*x)**2-d*d)
    theta=atan(y/d)
    a1=theta/4.*(pd+2.*x)**2-d/4.*y
    if(x.ge.x1)goto 25
    v1=3./4.*(gl-2.*x)
    v1=v1*(2.*sqr3*d*d-pi*(pd+2.*x)**2+24.*a1)
    s1=2.*v1/(gl-2.*x)
    s1=s1+3.*(gl-2.*x)*(pi-6.*theta)*(pd+2.*x)
25  y1=sqrt((gd-2.*x)**2-(5.*d-2.*(pd+2.*x))**2)
    chi=atan(y1/(5.*d-2.*(pd+2.*x)))
    y2=sqrt((pd+2.*x)**2-(3.*d-2.*(pd+2.*x))**2)
    phi=atan(y2/(3.*d-2.*(pd+2.*x)))
    a2=phi*(pd+2.*x)**2-chi*(gd-2.*x)**2
    a2=(a2+2.*sqr3*d*d*sqrt((3.*d-pd-2.*x)*(3.*d-gd+2.*x)))/8.
    v2=pi*(gd-2.*x)**2-6.*sqr3*d*d-4.*pi*(pd+2.*x)**2
    v2=(v2+24.*(a1+2.*a2))*(gl-2.*x)/4.
    s2=2.*v2/(gl-2.*x)
    s2=s2+(gl-2.*x)*((pi-6.*chi)*(gd-2.*x)+2.*(2.*pi-3.*phi-3.*theta
    & )*(pd+2.*x))
    vol=v1+v2
    surf=s1+s2
    frac=1.-vol/v0
    dsdx=-surf/(x2-x)
    dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
    dsdxsl(kpr)=dsdx
    frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
    fracsl(kpr)=frac
    surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
    surfsl(kpr)=surf
    return
30  vol=0.

```

```

    surf=0.
    frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
    fracsl(kpr)=frac
    if(frac.gt..9999) frac=1.
    if(frac.gt..9999)return
    dsdx=0.
    dsdx=dsdxsl(kpr)*coef(kpr)
    dsdxsl(kpr)=dsdx
    if(abs(dsdx).lt.1.)dsdx=0.
    surf=surfsl(kpr)*coef(kpr)
    surfsl(kpr)=surf
    return
C
C ZERO PERF CALCULATIONS START HERE.
C
2000  if(gd-2.*x.le.0.0) go to 2001
      v0=pi*gd*gd/4.*gl
      vol=pi*(gd-2.*x)**2/4.*(gl-2.*x)
      frac=1.-vol/v0
      surf=pi/2.*(gd-2.*x)**2+pi*(gd-2.*x)*(gl-2.*x)
      dsdx=-2.*pi*(gd+gl-6.*x)
      return
2001  surf=0.
      frac=1.0
      vol=0.
      dsdx=0.
      nsl=1
      return
c
c   one perf calculation starts here
c
3000  if(gd-pd-4.*x.le.0.0) goto 3001
      v0=pi/4.*(gd*gd-pd*pd)*gl
      vol=pi/4.*((gd-2.*x)**2-(pd+2.*x)**2)*(gl-2.*x)
      frac=1.-vol/v0
      surf=pi/2.*((gd-2.*x)**2-(pd+2.*x)**2)
      surf=surf+pi*(gd-2.*x)*(gl-2.*x)
      surf=surf+pi*(pd+2.*x)*(gl-2.*x)
      dsdx=-4.*pi*(gd+pd)
      return
3001  surf=0.
      frac=1.0
      vol=0.
      dsdx=0.
      nsl=1
      return
C
C Below is the calculation for the cylindrical 19 perf grain.
C
C INPUT
C
C   P = PERF DIAMETER
C   D = GRAIN DIAMETER
C   GL = GRAIN LENGTH

```

```

C   X = DISTANCE BURNT
C
C   OUTPUT
C
C   VOL = THE VOLUME OF ONE GRAIN AT X.
C   SURF = THE SURFACE AREA OF ONE GRAIN AT X.
C   FRAC = THE FRACTION OF GRAIN BURNT AT X.
C
C   W=WEB
C
C   4000 p=pd
C       d=gd
C       W=(D-5.*P)/6.
C       PI=3.141592654
C       SQRT3=SQRT(3.)
C       SQRT5=SQRT(5.)
C       SQRT6=SQRT(6.)
C       SQRT10=SQRT(10.)
C
C   INITIAL VOLUME AND SURFACE AREA
C
C       V0=PI/4.*GL*(D*D-19.*P*P)
C       S0=2.*V0/GL+PI*GL*(D+19.*P)
C
C   X1 = DISTANCE TO INNER SLIVERR BURNOUT
C   X2 = DISTANCE TO OUTER SLIVER BURNOUT
C   DBC = DISTANCE BETWEEN PERFORATION CENTERS
C   ASSUMES BURNOUT DOES NOT OCCUR IN LONGITUDINAL DIRECTION
C   W1 = SECONDARY WEB
C
C       DBC=W+P
C       W1=0.5*(D-P-2.*SQRT3*DBC)
C       X1=DBC/SQRT3-P/2.
C       X2=0.25*(DBC*(6.-SQRT10)-2.*P)
C       IF(X.GT.W/2.)GO TO 110
C
C   NOT SLIVERED YET
C
C       VOL=PI/4.*(GL-2.*X)*((D-2.*X)**2-19.*(P+2.*X)**2)
C       SURF=2.*VOL/(GL-2.*X)+PI*(GL-2.*X)*(D-2.*X+19.*(P+2.*X))
C       dsdx=pi*(-4*D+36*GL-76*P-216*x)
C       FRAC=1.-VOL/V0
C       dsdxsl(kpr)=dsdx
C       fracsl(kpr)=frac
C       surfsl(kpr)=surf
C       RETURN
C
C   V1=TOTAL VOLUME OF INNER SLIVER, V2=TOTAL VOLUME OF OUTER SLIVER
C   S1=TOTAL SURFACE AREA OF INNER SLIVERS, S2=TOTAL SURFACE AREA OF
C       OUTER SLIVERS
C
C   110  V1=0.
C       V2=0.
C       S1=0.

```

```

S2=0.
DELTA=0.
CHI=0.
NSL=1
coef(kpr)=0.
if(igrad.eq.1.or.igrad.eq.2)go to 727
if(nslp(kpr).eq.1)goto 728
tsl(kpr)=yar(3)
ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
& (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
728 continue
coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
if(coef(kpr).gt.1.)coef(kpr)=1.
if(coef(kpr).lt.0.)coef(kpr)=0.
727 A3=0.
IF(X.GE.X2)GO TO 130
THETA=ACOS(DBC/(P+2.*X))
A1=THETA/4.*(P+2.*X)**2-DBC/4.*SQRT((P+2.*X)**2-DBC*DBC)
IF(X.GT.X1)GO TO 120
V1=3.*(GL-2.*X)*(2.*SQRT3*DBC*DBC-PI*(P+2.*X)**2+24*A1)
S1=2.*V1/(GL-2.*X)+12.*(GL-2.*X)*(PI-6.*THETA)*(P+2.*X)
120 PHI=ACOS((5.*D-13.*P-36.*X)/(12.*(P+2.*X)))
XI=ACOS((13.*D-5.*P-36.*X)/(12.*(D-2.*X)))
IF(X.LE.W1/2.)GO TO 125
DELTA=ACOS((2.*D-P-6.*X)/(SQRT3*(D-2.*X)))
CHI=ACOS((D-2.*P-6.*X)/(SQRT3*(P+2.*X)))
A3=.125*(CHI*(P+2.*X)**2-DELTA*(D-2.*X)**2
& +2.*SQRT6*DBC*SQRT(6.*DBC*(P+2.*X-DBC)-(P+2.*X)**2))
125 A2=.125*(PHI*(P+2.*X)**2-XI*(D-2.*X)**2
& +2.*SQRT5*DBC*SQRT((5.*DBC-P-2.*X)*(5.*DBC-D+2.*X)))
V2=.25*(GL-2.*X)*(PI*(D-2.*X)**2-7.*PI*(P+2.*X)**2
& -24.*SQRT3*DBC*DBC+48.*(A1+A2+A3))
S2=2.*V2/(GL-2.*X)+(GL-2.*X)*((D-2.*X)*(PI-6.*(XI+DELTA))
& +(P+2.*X)*(7.*PI-6.*(2.*THETA+CHI+PHI)))
VOL=V1+V2
SURF=S1+S2
DSDX=-SURF/(X2-X)
FRAC=1.-VOL/V0
dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
dsdxsl(kpr)=dsdx
frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
fracsl(kpr)=frac
surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
surfsl(kpr)=surf
RETURN
130 VOL=0.
SURF=0.
frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
fracsl(kpr)=frac
if(frac.gt..9999) frac=1.
if(frac.gt..9999)return
dsdx=0.
dsdx=dsdxsl(kpr)*coef(kpr)
dsdxsl(kpr)=dsdx

```

```

    if(abs(dsdx).lt.1.)dsdx=0.
    surf=surfsl(kpr)*coef(kpr)
    surfsl(kpr)=surf
    RETURN

```

```

C
C Below is the calculation for the 19 perf hex grain.
C
C
C Translation of the input values.
C   p= perf diameter
C   d= grain diameter
C   gl= grain length
C   x= distance burnt
C
C Translation of the output values.
C   vol= volume of one grain at x.
C   surf= surface area of one grain at x.
C   frac= mass fraction of the grain burnt at x.
C
C Assignment statement for pi.
5000 pi=3.141592654
      sqrt3=sqrt(3.)
      p=pd
      d=gd
C
C   d=6w + 5p is the statement for the grain diameter which will be
C used to calculate the web.
C
C To calculate the web.
      w= (d-5.*p)/6.
C
C Below is the equation to calculate the distance between the perf cen-
C ters.
      dpc= p + w
C To calculate the grain diameter between the flats.
      f= 2.*(sqrt3*dpc + p/2. + w)
C
C To calculate the distance burnt.
      X1=dpc/sqrt3-p/2.
      X2= 0.125*(5.*dpc-4.*p)
C
C To calculate the area.
      A=sqrt3/3.*((w+p/2.)**2)-pi/6.*((w+p/2.)**2)
C To calculate the initial volume of the sharp corner grain.
      Vs=gl/4.*(2.*sqrt3*f**2-19.*pi*p**2)
C
C To calculate the volume that will be removed from the grain.
      Vr=6.*A*gl
C
C To calculate the initial volume for the 19hex grain with rounded
C corners.
      Vo= Vs - Vr
C
C To calculate the initial surface area of the sharp corner grain.

```

```

      Ss=2.*Vs/gl+gl*(2.*sqrt3*f+19.*pi*p)
C
C To calculate the surface area that will be removed from the grain.
      Sr=12.*A+gl*(w+p/2.)*(4.*sqrt3-2.*pi)
C
C To calculate the initial surface area for the 19hex grain with rounded
C comers.
      So= Ss-Sr
C
C To calculate the unknowns of the grain under the condition  $x \leq 0.5w$ .
      if(0.le.x.and.x.le.w/2.) then
        A=sqrt3/3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
        &(w-2.*x+(p+2.*x)/2.)**2
C To calculate the volume that will be removed from the sharp corner grain.
      Vr=6.*A*(gl-2.*x)
C To calculate the volume for the sharp corner grain at some distance burnt.
      Vn=.25*(gl-2.*x)*(2.*sqrt3*(f-2.*x)**2.
      & -19.*pi*(p+2.*x)**2.)
C
C To calculate the volume for the 19hex grain with rounded corners.
      V= Vn-Vr
C
C To calculate the surface area that will be removed from the sharp
C corner grain.
      Sr=12.*A+(gl-2.*x)*(w-2.*x+(p+2.*x)/2.)*(4.*sqrt3-2.*pi)
C To calculate the surface area for the sharp corner grain.
      Sn=2.*V/(gl-2.*x)+(gl-2.*x)*(2.*sqrt3*(f-2.*x)+
      & 19.*pi*(p+2.*x))
C
C To calculate the surface area for 19hex grain with rounded comers.
      S= Sn-Sr
C To calculate the mass fraction.
      frac= 1-V/Vo
      dsdx=-8.*sqrt3*(f-2.*x)-76.*pi*(p+2.*x)+(gl-2.*x)*(-4.*sqrt3+38.
      & *pi)+16*sqrt3*(w+p/2.-x)-8.*pi*(w+p/2.-x)+(gl-2.*x)*
      & (4.*sqrt3-2.*pi)
      surf=S
      vol=V
      dsdxsl(kpr)=dsdx
      fracsl(kpr)=frac
      surfsl(kpr)=surf
      return
      endif
C
C Due to the cross section at the sliver point  $x=.5w$  there will be 24
C identical inner slivers,12 identical side slivers. After slivering the
C surface area and the volume function become more complex. Each type of
C sliver will be treated seperately and later the volumes will be combined
C to complete the function.
C
C To calculate the 12 identical side slivers for the grain  $x=.5/w$ .
      nsl=1
      coef(kpr)=0.
      if(igrad.eq.1.or.igrad.eq.2)go to 729

```

```

    if(nslp(kpr).eq.1)goto 730
    tsl(kpr)=yar(3)
    ts(kpr)=w/2.*(-1.+(pbrch/pmean)**abr(kpr))/
    & (bbr(kpr)*(pbase*1.e-6)**abr(kpr))
730  continue
    coef(kpr)=(ts(kpr)+tsl(kpr)-(deltat+yar(3)))/ts(kpr)
    if(coef(kpr).gt.1.)coef(kpr)=1.
    if(coef(kpr).lt.0.)coef(kpr)=0.
729  if(w/2.lt.x.and.x.lt.X1.and.x.lt.X2) then
C
C To calculate the areas of the grain.
    A=sqrt(3./3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
    &(w-2.*x+(p+2.*x)/2.)**2
    theta=acos(dpc/(p+2.*x))
    A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
    omega=acos(2.*dpc/(p+2.*x)-1.)
    A2=0.125*(p+2.*x)*((p+2.*x)*(omega+sin(omega))-2.*dpc*sin(omega))
C To calculate the volumes of the grain.
    V1=3.*(gl-2.*x)*(2.*sqrt(3)*dpc**2-pi*(p+2.*x)**2+24.*A1)
    V2=6.*(gl-2.*x)*(2.*dpc**2-dpc*(p+2.*x)-pi/4.*(p+2.*x)**2
    &+2.*A1+4.*A2)
C To calculate the surface areas of the grain.
    S1=2.*V1/(gl-2.*x)+12.*(gl-2.*x)*(pi-6.*theta)*(p+2.*x)
    S2=2.*V2/(gl-2.*x)+12.*(gl-2.*x)*(dpc+(p+2.*x)*(pi/2.-omega
    &-theta-sin(omega)))
C To calculate the total volume and total surface area.
    Vf=V1+V2
    Sf=S1+S2
C To calculate the mass fraction.
    frac=1.-Vf/Vo
    surf=Sf
    dsdx=-surf/(x2-x)
    vol=Vf
    dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
    dsdxsl(kpr)=dsdx
    frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
    fracsl(kpr)=frac
    surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
    surfsl(kpr)=surf
    return
    endif
    if(x.gt.X1.and.x.lt.X2)then
C To calculate the area of the grain.
    A=sqrt(3./3.*(w-2.*x+(p+2.*x)/2.)**2-pi/6.*
    &(w-2.*x+(p+2.*x)/2.)**2
    theta=acos(dpc/(p+2.*x))
    A1=theta/4.*(p+2.*x)**2-dpc/4.*sqrt((p+2.*x)**2-dpc**2)
    omega=acos(2.*dpc/(p+2.*x)-1.)
    A2=0.125*(p+2.*x)*((p+2.*x)*(omega+sin(omega))-2.*dpc*sin(omega))
C To calculate the volume of the grain.
    V2=6.*(gl-2.*x)*(2.*dpc**2-dpc*(p+2.*x)-pi/4.*(p+2.*x)**2
    &+2.*A1+4.*A2)
C To calculate the surface area of the grain.
    S2=2.*V2/(gl-2.*x)+12.*(gl-2.*x)*(dpc+(p+2.*x)*(pi/2.-omega

```

```

    &-theta-sin(omega)))
C To calculate the volume and the surface area.
    Vf=V2
    Sf=S2
C To calculate the the mass fraction.
    frac=1-Vf/Vo
    surf=Sf
    dsdx=-surf/(x2-x)
    vol=Vf
    dsdx=coef(kpr)*dsdxsl(kpr)+(1.-coef(kpr))*dsdx
    dsdxsl(kpr)=dsdx
    frac=coef(kpr)*fracsl(kpr)+(1.-coef(kpr))*frac
    fracsl(kpr)=frac
    surf=coef(kpr)*surfsl(kpr)+(1.-coef(kpr))*surf
    surfsl(kpr)=surf
    return
endif
if(x.gt.X2)then
    dsdx=0.
    surf=0.
    vol=0.
    frac=fracsl(kpr)*coef(kpr)+1.-coef(kpr)
    fracsl(kpr)=frac
    if(frac.gt..9999) frac=1.
    if(frac.gt..9999)return
    dsdx=0.
    dsdx=dsdxsl(kpr)*coef(kpr)
    dsdxsl(kpr)=dsdx
    if(abs(dsdx).lt.1.)dsdx=0.
    surf=surfsl(kpr)*coef(kpr)
    surfsl(kpr)=surf
    return
endif
end

```

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APPENDIX 6:
INPUT DATA FOR IBRGA TEST

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9832.2384 12.7 12.7 1.0 0.0 457.2 4 1.00
 3 0. 15.381732 46.482 15.381732 54.102 12.7
 9.796 0 0.0 0.0
 5 0.0 0.0 0.0 .6 0.0 1.3 0.0 300. 0. 457.
 1.e20 2 3.0e+4 0.0 8.0e+5 0.2
 .001135 .01143 .46028 273. 1. 7.8612
 106.34 .9755 294. .004712 1.4
 1
 1135.84 3142.75 .97544 9.7959 1.6605 1.23 7 3.175 .0508 1.23063
 1 1.0 .1105187 689.476
 .01 .1 30.

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APPENDIX 7:
OUTPUT FROM IBRGA

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the input file is i6315a
using rga gradient

chamber distance cm chamber diameter cm
0.000000E+00 0.153817E+02
0.464820E+02 0.153817E+02
0.541020E+02 0.127000E+02
chamber volume cm**3 0.982089E+04
groove diam cm 0.127000E+02
land diam cm 0.127000E+02
groove/land ratio 0.100000E+01
twist turns/caliber 0.000000E+00
projectile travel cm 0.457200E+03
gradient # 4
friction factor 0.100000E+01

projectile mass kg 0.979600E+01
switch to calculate energy lost to air resistance J 0
fraction of work against bore used to heat the tube 0.000000E+00
gas pressure Pa 0.000000E+00
number barrel resistance points 5
bore resistance MPa - travel cm
0.000000E+00 0.000000E+00
0.000000E+00 0.600000E+00
0.000000E+00 0.130000E+01
0.000000E+00 0.300000E+03
0.000000E+00 0.457000E+03

mass of recoiling parts kg 0.100000E+21
number of recoil point pairs 2
recoil force N recoil time sec
0.300000E+05 0.000000E+00
0.800000E+06 0.200000E+00

free convective heat transfer coefficient w/cm**2 K 0.113500E-02
chamber wall thickness cm 0.114300E-01
heat capacity of steel of chamber wall J/g K 0.460280E+00
initial temperature of chamber wall K 0.273000E+03
heat loss coefficient 0.100000E+01
density of chamber wall steel g/cm**3 0.786120E+01

impetus of igniter propellant J/g 0.106340E+03
covolume of igniter cm**3/g 0.975500E+00
adiabatic flame temperature of igniter propellant K 0.294000E+03
initial mass of igniter kg 0.471200E-02
ratio of specific heats for igniter 0.140000E+01
there are 1 propellants
for propellant number 1
impetus of propellant J/g 0.113584E+04

adiabatic temperature of propellant K 0.314275E+04
 covolume of propellant cm**3/g 0.975440E+00
 initial mass of propellant kg 0.979590E+01
 density of propellant g/cm**3 0.166050E+01
 ratio of specific heats for propellant 0.123000E+01
 number of perforations of propellant 7
 length of propellant grain cm 0.317500E+01
 diameter of perforation in propellant grains cm 0.508000E-01
 outside diameter of propellant grain cm 0.123063E+01

for propellant 1 the total number of grains is 0.158099E+04
 number of burning rate points 1

exponent	coefficient	pressure
-	cm/sec-MPa**ai	MPa
0.100000E+01	0.110519E+00	0.689476E+03

time increment msec 0.100000E-01 print increment msec 0.100000E+00

time to stop calculation msec 0.300000E+02

area bore m^2 0.126677E-01

pressure from ign pa 0.127925E+06

volume of unburnt prop m^3 0.589937E-02

init cham vol-cov ign m^3 0.981629E-02

time	acc	vel	dis	mpress	pbase	pbrch
0.1000E-04	0.1696E+03	0.1399E-02	0.5582E-08	0.1312E+06	0.1312E+06	0.1313E+06
prop travel	0.8638E-11	proj travel	0.5582E-08			
0.1100E-03	0.2182E+03	0.2070E-01	0.1070E-05	0.1688E+06	0.1688E+06	0.1689E+06
prop travel	0.1936E-08	proj travel	0.1070E-05			
0.2100E-03	0.2790E+03	0.4545E-01	0.4327E-05	0.2159E+06	0.2158E+06	0.2160E+06
prop travel	0.8074E-08	proj travel	0.4327E-05			
0.3100E-03	0.3543E+03	0.7699E-01	0.1039E-04	0.2742E+06	0.2740E+06	0.2743E+06
prop travel	0.2001E-07	proj travel	0.1039E-04			
0.4100E-03	0.4464E+03	0.1169E+00	0.2000E-04	0.3454E+06	0.3452E+06	0.3455E+06
prop travel	0.4001E-07	proj travel	0.2000E-04			
0.5100E-03	0.5576E+03	0.1669E+00	0.3410E-04	0.4315E+06	0.4312E+06	0.4317E+06
prop travel	0.7140E-07	proj travel	0.3410E-04			
0.6100E-03	0.6904E+03	0.2291E+00	0.5379E-04	0.5344E+06	0.5339E+06	0.5346E+06
prop travel	0.1190E-06	proj travel	0.5379E-04			
0.7100E-03	0.8474E+03	0.3058E+00	0.8040E-04	0.6560E+06	0.6553E+06	0.6563E+06
prop travel	0.1898E-06	proj travel	0.8040E-04			
0.8100E-03	0.1031E+04	0.3995E+00	0.1155E-03	0.7986E+06	0.7975E+06	0.7990E+06
prop travel	0.2943E-06	proj travel	0.1155E-03			
0.9100E-03	0.1245E+04	0.5130E+00	0.1610E-03	0.9646E+06	0.9631E+06	0.9653E+06
prop travel	0.4479E-06	proj travel	0.1610E-03			
0.1010E-02	0.1493E+04	0.6497E+00	0.2189E-03	0.1157E+07	0.1155E+07	0.1158E+07
prop travel	0.6734E-06	proj travel	0.2189E-03			
0.1110E-02	0.1780E+04	0.8130E+00	0.2918E-03	0.1380E+07	0.1376E+07	0.1381E+07
prop travel	0.1005E-05	proj travel	0.2918E-03			
0.1210E-02	0.2110E+04	0.1007E+01	0.3825E-03	0.1637E+07	0.1632E+07	0.1639E+07
prop travel	0.1494E-05	proj travel	0.3825E-03			
0.1310E-02	0.2492E+04	0.1237E+01	0.4944E-03	0.1934E+07	0.1927E+07	0.1936E+07
prop travel	0.2217E-05	proj travel	0.4944E-03			
0.1410E-02	0.2931E+04	0.1507E+01	0.6312E-03	0.2276E+07	0.2266E+07	0.2281E+07

prop travel 0.3292E-05proj travel 0.6312E-03
 0.1510E-02 0.3437E+04 0.1825E+01 0.7974E-03 0.2673E+07 0.2658E+07 0.2679E+07
 prop travel 0.4893E-05proj travel 0.7974E-03
 0.1610E-02 0.4021E+04 0.2197E+01 0.9981E-03 0.3131E+07 0.3110E+07 0.3141E+07
 prop travel 0.7289E-05proj travel 0.9981E-03
 0.1710E-02 0.4694E+04 0.2632E+01 0.1239E-02 0.3662E+07 0.3630E+07 0.3676E+07
 prop travel 0.1089E-04proj travel 0.1239E-02
 0.1810E-02 0.5470E+04 0.3140E+01 0.1527E-02 0.4277E+07 0.4230E+07 0.4298E+07
 prop travel 0.1630E-04proj travel 0.1527E-02
 0.1910E-02 0.6361E+04 0.3730E+01 0.1870E-02 0.4989E+07 0.4919E+07 0.5020E+07
 prop travel 0.2447E-04proj travel 0.1870E-02
 0.2010E-02 0.7384E+04 0.4416E+01 0.2276E-02 0.5814E+07 0.5710E+07 0.5859E+07
 prop travel 0.3681E-04proj travel 0.2276E-02
 0.2110E-02 0.8556E+04 0.5212E+01 0.2757E-02 0.6769E+07 0.6616E+07 0.6836E+07
 prop travel 0.5544E-04proj travel 0.2757E-02
 0.2210E-02 0.9892E+04 0.6133E+01 0.3323E-02 0.7874E+07 0.7650E+07 0.7974E+07
 prop travel 0.8353E-04proj travel 0.3323E-02
 0.2310E-02 0.1141E+05 0.7196E+01 0.3988E-02 0.9153E+07 0.8825E+07 0.9300E+07
 prop travel 0.1258E-03proj travel 0.3988E-02
 0.2410E-02 0.1313E+05 0.8421E+01 0.4767E-02 0.1063E+08 0.1015E+08 0.1085E+08
 prop travel 0.1891E-03proj travel 0.4767E-02
 0.2510E-02 0.1507E+05 0.9829E+01 0.5678E-02 0.1234E+08 0.1165E+08 0.1265E+08
 prop travel 0.2835E-03proj travel 0.5678E-02
 0.2610E-02 0.1724E+05 0.1144E+02 0.6740E-02 0.1431E+08 0.1333E+08 0.1475E+08
 prop travel 0.4232E-03proj travel 0.6740E-02
 0.2710E-02 0.1967E+05 0.1328E+02 0.7974E-02 0.1658E+08 0.1521E+08 0.1719E+08
 prop travel 0.6278E-03proj travel 0.7974E-02
 0.2810E-02 0.2238E+05 0.1538E+02 0.9405E-02 0.1918E+08 0.1731E+08 0.2003E+08
 prop travel 0.9242E-03proj travel 0.9405E-02
 0.2910E-02 0.2541E+05 0.1777E+02 0.1106E-01 0.2217E+08 0.1965E+08 0.2332E+08
 prop travel 0.1348E-02proj travel 0.1106E-01
 0.3010E-02 0.2882E+05 0.2047E+02 0.1297E-01 0.2560E+08 0.2229E+08 0.2710E+08
 prop travel 0.1943E-02proj travel 0.1297E-01
 0.3110E-02 0.3268E+05 0.2354E+02 0.1517E-01 0.2951E+08 0.2527E+08 0.3144E+08
 prop travel 0.2765E-02proj travel 0.1517E-01
 0.3210E-02 0.3709E+05 0.2702E+02 0.1769E-01 0.3397E+08 0.2869E+08 0.3637E+08
 prop travel 0.3877E-02proj travel 0.1769E-01
 0.3310E-02 0.4217E+05 0.3098E+02 0.2059E-01 0.3903E+08 0.3261E+08 0.4194E+08
 prop travel 0.5351E-02proj travel 0.2059E-01
 0.3410E-02 0.4802E+05 0.3548E+02 0.2391E-01 0.4475E+08 0.3714E+08 0.4820E+08
 prop travel 0.7263E-02proj travel 0.2391E-01
 0.3510E-02 0.5475E+05 0.4060E+02 0.2770E-01 0.5119E+08 0.4234E+08 0.5518E+08
 prop travel 0.9691E-02proj travel 0.2770E-01
 0.3610E-02 0.6243E+05 0.4644E+02 0.3205E-01 0.5841E+08 0.4828E+08 0.6294E+08
 prop travel 0.1271E-01proj travel 0.3205E-01
 0.3710E-02 0.7109E+05 0.5310E+02 0.3702E-01 0.6645E+08 0.5497E+08 0.7153E+08
 prop travel 0.1641E-01proj travel 0.3702E-01
 0.3810E-02 0.8072E+05 0.6068E+02 0.4270E-01 0.7536E+08 0.6242E+08 0.8100E+08
 prop travel 0.2085E-01proj travel 0.4270E-01
 0.3910E-02 0.9131E+05 0.6926E+02 0.4919E-01 0.8514E+08 0.7061E+08 0.9139E+08
 prop travel 0.2611E-01proj travel 0.4919E-01
 0.4010E-02 0.1028E+06 0.7895E+02 0.5659E-01 0.9580E+08 0.7948E+08 0.1027E+09
 prop travel 0.3228E-01proj travel 0.5659E-01
 0.4110E-02 0.1151E+06 0.8983E+02 0.6502E-01 0.1073E+09 0.8898E+08 0.1150E+09

prop travel 0.3944E-01proj travel 0.6502E-01
 0.4210E-02 0.1280E+06 0.1020E+03 0.7460E-01 0.1197E+09 0.9901E+08 0.1282E+09
 prop travel 0.4767E-01proj travel 0.7460E-01
 0.4310E-02 0.1416E+06 0.1154E+03 0.8546E-01 0.1327E+09 0.1095E+09 0.1421E+09
 prop travel 0.5707E-01proj travel 0.8546E-01
 0.4410E-02 0.1555E+06 0.1303E+03 0.9773E-01 0.1464E+09 0.1203E+09 0.1568E+09
 prop travel 0.6774E-01proj travel 0.9773E-01
 0.4510E-02 0.1697E+06 0.1465E+03 0.1116E+00 0.1605E+09 0.1313E+09 0.1721E+09
 prop travel 0.7980E-01proj travel 0.1116E+00
 0.4610E-02 0.1840E+06 0.1642E+03 0.1271E+00 0.1748E+09 0.1423E+09 0.1877E+09
 prop travel 0.9336E-01proj travel 0.1271E+00
 0.4710E-02 0.1981E+06 0.1833E+03 0.1444E+00 0.1893E+09 0.1532E+09 0.2035E+09
 prop travel 0.1085E+00proj travel 0.1444E+00
 0.4810E-02 0.2118E+06 0.2038E+03 0.1638E+00 0.2036E+09 0.1638E+09 0.2192E+09
 prop travel 0.1254E+00proj travel 0.1638E+00
 0.4910E-02 0.2249E+06 0.2256E+03 0.1852E+00 0.2175E+09 0.1739E+09 0.2347E+09
 prop travel 0.1442E+00proj travel 0.1852E+00
 0.5010E-02 0.2372E+06 0.2487E+03 0.2089E+00 0.2308E+09 0.1834E+09 0.2496E+09
 prop travel 0.1648E+00proj travel 0.2089E+00
 0.5110E-02 0.2485E+06 0.2730E+03 0.2350E+00 0.2433E+09 0.1922E+09 0.2637E+09
 prop travel 0.1876E+00proj travel 0.2350E+00
 0.5210E-02 0.2587E+06 0.2983E+03 0.2636E+00 0.2547E+09 0.2001E+09 0.2769E+09
 prop travel 0.2124E+00proj travel 0.2636E+00
 0.5310E-02 0.2677E+06 0.3247E+03 0.2947E+00 0.2651E+09 0.2070E+09 0.2889E+09
 prop travel 0.2395E+00proj travel 0.2947E+00
 0.5410E-02 0.2754E+06 0.3518E+03 0.3285E+00 0.2741E+09 0.2129E+09 0.2998E+09
 prop travel 0.2689E+00proj travel 0.3285E+00
 0.5510E-02 0.2817E+06 0.3797E+03 0.3651E+00 0.2819E+09 0.2178E+09 0.3092E+09
 prop travel 0.3006E+00proj travel 0.3651E+00
 0.5610E-02 0.2867E+06 0.4081E+03 0.4045E+00 0.2883E+09 0.2217E+09 0.3172E+09
 prop travel 0.3348E+00proj travel 0.4045E+00
 0.5710E-02 0.2903E+06 0.4370E+03 0.4467E+00 0.2933E+09 0.2245E+09 0.3239E+09
 prop travel 0.3714E+00proj travel 0.4467E+00
 0.5810E-02 0.2928E+06 0.4661E+03 0.4919E+00 0.2970E+09 0.2264E+09 0.3291E+09
 prop travel 0.4105E+00proj travel 0.4919E+00
 0.5910E-02 0.2940E+06 0.4955E+03 0.5400E+00 0.2995E+09 0.2274E+09 0.3329E+09
 prop travel 0.4520E+00proj travel 0.5400E+00
 0.6010E-02 0.2942E+06 0.5249E+03 0.5910E+00 0.3008E+09 0.2275E+09 0.3355E+09
 prop travel 0.4961E+00proj travel 0.5910E+00
 0.6110E-02 0.2934E+06 0.5543E+03 0.6449E+00 0.3010E+09 0.2269E+09 0.3369E+09
 prop travel 0.5428E+00proj travel 0.6449E+00
 0.6210E-02 0.2918E+06 0.5835E+03 0.7018E+00 0.3003E+09 0.2257E+09 0.3371E+09
 prop travel 0.5919E+00proj travel 0.7018E+00
 0.6310E-02 0.2894E+06 0.6126E+03 0.7616E+00 0.2988E+09 0.2238E+09 0.3364E+09
 prop travel 0.6435E+00proj travel 0.7616E+00
 0.6410E-02 0.2864E+06 0.6414E+03 0.8243E+00 0.2965E+09 0.2215E+09 0.3348E+09
 prop travel 0.6976E+00proj travel 0.8243E+00
 0.6510E-02 0.2828E+06 0.6699E+03 0.8899E+00 0.2936E+09 0.2187E+09 0.3323E+09
 prop travel 0.7542E+00proj travel 0.8899E+00
 0.6610E-02 0.2788E+06 0.6979E+03 0.9583E+00 0.2901E+09 0.2156E+09 0.3293E+09
 prop travel 0.8132E+00proj travel 0.9583E+00
 0.6710E-02 0.2744E+06 0.7256E+03 0.1029E+01 0.2862E+09 0.2122E+09 0.3256E+09
 prop travel 0.8746E+00proj travel 0.1029E+01
 0.6810E-02 0.2697E+06 0.7528E+03 0.1103E+01 0.2820E+09 0.2085E+09 0.3214E+09

prop travel 0.9384E+00proj travel 0.1103E+01
 0.6910E-02 0.2647E+06 0.7795E+03 0.1180E+01 0.2774E+09 0.2047E+09 0.3169E+09
 prop travel 0.1005E+01proj travel 0.1180E+01
 0.7010E-02 0.2596E+06 0.8058E+03 0.1259E+01 0.2727E+09 0.2008E+09 0.3120E+09
 prop travel 0.1073E+01proj travel 0.1259E+01
 0.7110E-02 0.2544E+06 0.8315E+03 0.1341E+01 0.2677E+09 0.1968E+09 0.3069E+09
 prop travel 0.1144E+01proj travel 0.1341E+01
 0.7210E-02 0.2492E+06 0.8566E+03 0.1426E+01 0.2627E+09 0.1927E+09 0.3016E+09
 prop travel 0.1217E+01proj travel 0.1426E+01
 0.7310E-02 0.2439E+06 0.8813E+03 0.1512E+01 0.2576E+09 0.1886E+09 0.2961E+09
 prop travel 0.1292E+01proj travel 0.1512E+01
 0.7410E-02 0.2385E+06 0.9054E+03 0.1602E+01 0.2525E+09 0.1845E+09 0.2905E+09
 prop travel 0.1369E+01proj travel 0.1602E+01
 0.7510E-02 0.2333E+06 0.9290E+03 0.1694E+01 0.2473E+09 0.1804E+09 0.2849E+09
 prop travel 0.1449E+01proj travel 0.1694E+01
 0.7610E-02 0.2281E+06 0.9521E+03 0.1788E+01 0.2422E+09 0.1764E+09 0.2793E+09
 prop travel 0.1530E+01proj travel 0.1788E+01
 0.7710E-02 0.2229E+06 0.9746E+03 0.1884E+01 0.2372E+09 0.1724E+09 0.2737E+09
 prop travel 0.1614E+01proj travel 0.1884E+01
 0.7810E-02 0.2179E+06 0.9967E+03 0.1983E+01 0.2322E+09 0.1685E+09 0.2681E+09
 prop travel 0.1700E+01proj travel 0.1983E+01
 0.7910E-02 0.2129E+06 0.1018E+04 0.2083E+01 0.2272E+09 0.1646E+09 0.2625E+09
 prop travel 0.1787E+01proj travel 0.2083E+01
 0.8010E-02 0.2081E+06 0.1039E+04 0.2186E+01 0.2224E+09 0.1609E+09 0.2570E+09
 prop travel 0.1877E+01proj travel 0.2186E+01
 0.8110E-02 0.2033E+06 0.1060E+04 0.2291E+01 0.2176E+09 0.1572E+09 0.2516E+09
 prop travel 0.1969E+01proj travel 0.2291E+01
 0.8210E-02 0.1987E+06 0.1080E+04 0.2398E+01 0.2130E+09 0.1537E+09 0.2463E+09
 prop travel 0.2062E+01proj travel 0.2398E+01
 0.8310E-02 0.1943E+06 0.1100E+04 0.2507E+01 0.2084E+09 0.1502E+09 0.2411E+09
 prop travel 0.2158E+01proj travel 0.2507E+01
 0.8410E-02 0.1899E+06 0.1119E+04 0.2618E+01 0.2040E+09 0.1469E+09 0.2360E+09
 prop travel 0.2255E+01proj travel 0.2618E+01
 0.8510E-02 0.1857E+06 0.1138E+04 0.2731E+01 0.1997E+09 0.1436E+09 0.2310E+09
 prop travel 0.2354E+01proj travel 0.2731E+01
 0.8610E-02 0.1817E+06 0.1156E+04 0.2845E+01 0.1954E+09 0.1405E+09 0.2261E+09
 prop travel 0.2455E+01proj travel 0.2845E+01
 0.8710E-02 0.1777E+06 0.1174E+04 0.2962E+01 0.1913E+09 0.1374E+09 0.2213E+09
 prop travel 0.2558E+01proj travel 0.2962E+01
 propellant 1 has slivered
 0.8810E-02 0.1772E+06 0.1192E+04 0.3080E+01 0.1852E+09 0.1370E+09 0.2116E+09
 prop travel 0.2662E+01proj travel 0.3080E+01
 0.8910E-02 0.1773E+06 0.1209E+04 0.3200E+01 0.1814E+09 0.1371E+09 0.2052E+09
 prop travel 0.2768E+01proj travel 0.3200E+01
 0.9010E-02 0.1726E+06 0.1227E+04 0.3322E+01 0.1765E+09 0.1335E+09 0.1997E+09
 prop travel 0.2875E+01proj travel 0.3322E+01
 0.9110E-02 0.1674E+06 0.1244E+04 0.3446E+01 0.1714E+09 0.1294E+09 0.1938E+09
 prop travel 0.2982E+01proj travel 0.3446E+01
 0.9210E-02 0.1621E+06 0.1260E+04 0.3571E+01 0.1662E+09 0.1254E+09 0.1879E+09
 prop travel 0.3091E+01proj travel 0.3571E+01
 0.9310E-02 0.1570E+06 0.1276E+04 0.3698E+01 0.1610E+09 0.1214E+09 0.1821E+09
 prop travel 0.3200E+01proj travel 0.3698E+01
 0.9410E-02 0.1519E+06 0.1292E+04 0.3826E+01 0.1559E+09 0.1175E+09 0.1763E+09
 prop travel 0.3310E+01proj travel 0.3826E+01

0.9510E-02 0.1470E+06 0.1307E+04 0.3956E+01 0.1509E+09 0.1137E+09 0.1707E+09
 prop travel 0.3421E+01proj travel 0.3956E+01
 0.9610E-02 0.1422E+06 0.1321E+04 0.4087E+01 0.1461E+09 0.1100E+09 0.1652E+09
 prop travel 0.3533E+01proj travel 0.4087E+01
 0.9710E-02 0.1376E+06 0.1335E+04 0.4220E+01 0.1414E+09 0.1064E+09 0.1599E+09
 prop travel 0.3646E+01proj travel 0.4220E+01
 0.9810E-02 0.1332E+06 0.1349E+04 0.4354E+01 0.1368E+09 0.1030E+09 0.1548E+09
 prop travel 0.3759E+01proj travel 0.4354E+01
 0.9910E-02 0.1289E+06 0.1362E+04 0.4490E+01 0.1325E+09 0.9970E+08 0.1498E+09
 prop travel 0.3873E+01proj travel 0.4490E+01
 deltat t 0.998000E-02 intg t 0.998000E-02
 PMAXMEAN Pa 0.301051E+09 time at PMAXMEAN sec 0.608000E-02
 PMAXBASE Pa 0.227560E+09 time at PMAXBASE sec 0.598000E-02
 PMAXBREECH Pa 0.337155E+09 time at PMAXBREECH sec 0.618000E-02
 muzzle VELOCITY m/s 0.136949E+04 time of muzzle velocity sec 0.997004E-02
 total initial energy available J = 0.483777E+08
 FOR PROPELLANT 1 MASSFRACT BURNT IS 0.966455E+00 at time in sec 0.000000E+00
 total energy remaining in gas J= 0.329127E+08
 energy loss from projectile translation J= 0.920305E+07
 energy loss from projectile rotation J= 0.000000E+00
 energy lost to gas and propellant motion J= 0.311553E+07
 energy lost to bore resistance J= 0.000000E+00
 energy lost to recoil J= 0.188116E-11
 energy loss from heat transfer J= 0.152361E+07
 energy lost to air resistance J= 0.000000E+00

APPENDIX 8:
INPUT DATA FOR XKTC

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GRADIENT TESTS 600 IN~ X6315A

TFFFFTTT 1 1 0 00

25 -3 0 3500

10.0 180.0 0.0001 2.0 0.05 0.01 0.0001 0.0001

300 100 1100 100 1500 100

6 0 0 5 0 0 1 2 0 0 0 8 0

0

529. 14.7 28.896 1.4

529.0

JA2 7PF LOT RH2-5 0.0 21.3 21.6 .06

7 .4845 0.020 1.25 7.

15000. 1.0 41754. .5

8000. .00030 1.000 100000. .000300 1.000 0.0 520.

19826000. 23.0000 1.2300 27.00

0.0 3.0279 18.3 3.0279 21.3 2.5 21.9 2.5

23.37 2.5 210.56 2.5

6.0 0. 6.25 0. 7.2 00. 7.8 0.

210.56 0.

7.77 0.0228 0.7

21.3 21.6 44.0 0.000

0.0 3. 12. 30. 40. 50. 80. 30.

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