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DYNAMIC INTERACTION OF ELASTIC COMPOSITES
AND AN ACOUSTIC FLUID

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The research work completed under the subject contract during the period stated is briefly described below. Also included is a list of publications which contain the results of research supported under this contract. Finally, a list of presentations at various scientific conferences, invited lectures, and workshops is also included.

WORK COMPLETED

1. One-Dimensional Response of Periodically Layered Composites in Contact with an Acoustic Fluid

The research program began with the investigation of a simple one-dimensional model of a two-phase, isotropic, periodically layered composite. Floquet theory was applied to formulate the dispersion equation for elastic compressional waves propagating normally to the layering. A closed form solution of this equation was established, and the frequency spectrum, showing the typical Brillouin structure of stopping and passing bands, was analyzed. The theory was then applied to investigate the one-dimensional dynamical behavior of this two-phase composite in contact with an acoustic fluid. Two geometries were investigated, namely a layered half-space and an infinite laminated plate. The results were presented in terms of the coefficient of reflection for waves normally incident from the fluid, as well as in terms of the complex impedance for the solid surface in contact with the fluid [1][†].

2. Applicability of Floquet Theory to Bounded Periodic Domains

The most significant contribution of this initial phase of the research program, was the realization that Floquet theory can be applied to the mathematical modeling of bounded periodically layered structures. This point was proven controversial, due to the common misconception that Floquet's theorem can only be applied to infinite domains [2]. The fact is, however, that this famous theorem applies in the first place to ordinary differential equations with periodic coefficients. It states that any solution of such type of equations can be written as a product of two functions. One is periodic, with the same period as the coefficients of the medium, while the other is a complex exponential whose argument

[†] Numbers in brackets refer to a list of references.

is multiplied by the so-called Floquet wave-number. The theorem also guarantees that a complete set of *linearly independent* solutions in this form can be found. In the case of bounded periodic domains, these linearly independent solutions can be combined in order to satisfy any well posed boundary conditions. Another way to show the validity of the Floquet representation for bounded media is to consider several Floquet waves in an unbounded periodic body at different angles and to adjust the amplitudes, wave-numbers, and frequencies in such a way that, for example, some arbitrary plane parallel to the layering be traction-free.

3. Two-Dimensional Waves in Anisotropic Layered Composites

The next step attempted was the extension of this formulation to two dimensions (plane-strain waves). Previous work in this direction by the Principal Investigator and colleagues [3-5], had shown that the straightforward method of solving the elastic equations for each layer in the unit cell, matching stresses and displacements at the interfaces, and using Floquet's theorem to extend the solution to the entire (periodic) domain, generated rather cumbersome determinantal equations. Although this method yielded very important results concerning the dispersion spectrum for two-phase periodic media, an extension to more general laminated composites, with more than two layers in the unit cell and including the effects of anisotropy, was very difficult. An alternative matrix formalism, capable of handling the additional difficulties introduced by accounting for the anisotropy of the layers, was then proposed.

This matrix formalism is an extension to multilayered media of Stroh's sextic theory [6], which has been explored in the context of surface waves and dislocations in crystals. An important feature of this six-dimensional theory is that it leads naturally to the idea of a propagator matrix [7]. In the sextic formalism the material symmetry of the anisotropic layers is reflected in the structure of certain fundamental matrices, which allows great simplification in the mathematical treatment of the problem. By exploring the structure of these matrices, one is able to obtain a closed form, algebraic solution for the dispersion equation of plane waves propagating in an infinite, periodically layered medium. This solution serves as a building block to treat boundary value problems involving layered bodies (not necessarily infinite nor periodic). Furthermore, since a closed form analytical solution

is employed, numerical problems encountered in other matrix formulations to multilayered media are bypassed, and it is possible to treat the problem in an extended range of frequencies. This approach is much more effective for composites in which the relative orientation of the anisotropic layers allows for the uncoupling of plane and anti-plane strain effects, which is the case of both cross-ply orthotropic and isotropic laminates [8]. When this decoupling is not possible, the sextic matrix formalism still yields good results, but now for a more limited range of frequencies. At large frequencies, numerical stability problems occur due to the ill-conditioning of the propagator matrix, which has pairwise reciprocal eigenvalues [8]. Another numerical limitation is the necessity of operating with very large numbers, which appear when computing the complex exponentiation of imaginary quantities. This matrix formalism was then applied to the study of the dispersion spectrum of plane waves propagating in the unbounded periodically layered medium. Again, as for the one-dimensional problem, the frequency spectrum was characterized by stopping and passing bands [8].

4. The Surface Impedance Tensor — Rayleigh Waves in Anisotropic Layered Composites

Another important feature of the sextic formalism for periodically layered composites, which makes it particularly attractive for the study of problems of fluid/solid interaction, is the quite natural definition of a surface impedance tensor [8]. This novel impedance operator for periodically layered media is an extension of the concept introduced by Ingebrigtsen and Tonnig [9] for homogeneous anisotropic media.

Next, the propagation of Rayleigh surface waves in a periodically layered half-space was investigated [10]. It was observed that, in contrast to a homogeneous anisotropic solid, these waves are dispersive and, in general, occur in more than one branch. Another significant result, was the finding that free waves can propagate in the half-space with vanishing displacements along its surface and amplitude decaying with the depth. Such surface waves have no counterpart in a homogeneous elastic medium.

5. Rayleigh-Lamb Waves in Anisotropic Layered Composites

The propagation of Rayleigh-Lamb waves in anisotropic periodically laminated plates was also investigated. A somewhat surprising result was obtained when the anisotropic layers were oriented in such a way that an uncoupling of anti-plane and plane motions occurs. In this case, the dispersion equation for the plane-strain waves has a form identical to the well-known Rayleigh-Lamb equation for a homogeneous isotropic plate [10]. Although some preliminary numerical results, concerning the dispersion curves for the Rayleigh-Lamb waves, were obtained, a more complete analysis of this problem has still to be performed.

6. Scholte-Gogoladze-Like Waves at the Interface Between a Fluid and an Anisotropic Layered Composite

Next, the matrix formalism was applied to the study of Scholte-Gogoladze-like waves, propagating along the plane interface between a periodically layered composite and an acoustic fluid. These are subsonic waves that propagate unattenuated parallel to the boundary, while decaying exponentially in both directions away from the fluid/solid interface. The dispersion equation for the Scholte-Gogoladze waves were obtained by the vanishing of the sum of the fluid impedance and the normal impedance of the layered half-space, which is given by $(\mathbf{n} \cdot \mathbf{Z}^{-1} \mathbf{n})^{-1}$, where $\mathbf{Z}$ denotes the surface impedance operator and $\mathbf{n}$ the unit vector normal to the wet interface. By contrast to their counterpart in a homogeneous solid, these waves are dispersive for the layered composite, and have more than one branch in the frequency-wave-number space [11].

7. An Algorithm for The Computation of The Surface Impedance Tensor of Anisotropic Layered Media

It has become clear, in the course of this investigation, that the surface impedance operator is a fundamental quantity in the study of fluid/solid interaction problems. This is a rank two tensor which associates the traction vector acting across the solid surface, with the value of the velocity field there. When the solid is in contact with an acoustic fluid, the important quantity is the normal impedance, which is given by $Z_n = (\mathbf{n} \cdot \mathbf{Z}^{-1} \mathbf{n})^{-1}$.

With the objective of providing means for a systematic study of fluid/laminated-plate interaction problems, a computational algorithm for the evaluation of the surface impedance tensor of anisotropic layered plates has then been devised. This recursive algorithm is an extension of the work of Hager and Rostamian [12] to anisotropic layered media. Based on this algorithm, a FORTRAN 77 subroutine to evaluate the normal impedance was developed. This subroutine evaluates Z_n for a given (frequency, wave-number) pair, for fixed material properties and geometry of the layers. The numerical stability problems mentioned above are completely eliminated, since the algorithm does not require the computation of the propagator matrix [13].

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1. Chow, S. L., and Herrmann, G., "Dynamic Response of a Ring Fabricated from a Non-Linear Elastic Material," *J. Sound and Vibration*, Vol. 124(3), pp. 397-413, 1988.
2. Braga, A. M. B. and Herrmann, G., "Plane Waves in Anisotropic Layered Composites," in *Wave Propagation in Structural Composites*, edited by A. K. Mal and T. C. Ting, ASME-AMD Vol. 90, pp 81-98, 1988.
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4. Braga , A. M. B. , and Herrmann , G. , " Rayleigh and Rayleigh-Lamb Waves in Anisotropic Laminated Composites," in *Proceedings of The Panamerican Congress of Applied Mechanics*, edited by Bevilacqua, L. and Steele, C., pp. 314-317 (1989).
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LIST OF PRESENTATIONS

1. " Dynamic Response of Submerged Layered Composites," paper presented at the 114th Meeting of the Acoustical Society of America, Miami, Florida, November 1987.
2. " Wave Propagation in Submerged Layered Composites," paper presented at the IU-TAM Symposium on Wave Propagation, Galway, Ireland, March 1988.
3. Presentation of Progress at the 8th Annual Meeting of the Industrial Affiliates of Stanford University in Applied Mechanics, May 1988.
4. "Plane Waves in Anisotropic Layered Composites," paper presented at the ASME-AMD Symposium on Wave Propagation in Structural Composites, Berkeley, California, June 1988.
5. " Scholte-Gogoladze-Like Waves at the Interface Between a Fluid and an Anisotropic Layered Composite," paper presented at the 116th Meeting of the Acoustical Society of America, Waikiki, Hawaii, November 1988.
6. Presentations of progress at the Review of ONR Solid-Fluid Interaction Accelerated Research Initiative, Austin, Texas, January 1989 and July 1989.
7. Presentation of Progress at the 9th Annual Meeting of the Industrial Affiliates of Stanford University in Applied Mechanics, May 1989.

8. Discussion of Fluid-Solid Interaction Problems at University of Paris VI, France, June, 1989.
9. Lecture at Ecole Polytechnique, Paris, on "Wave Propagation in Submerged Layered Composites," France, July 1989.
10. " Rayleigh and Rayleigh-Lamb Waves in Anisotropic Laminated Composites," paper presented at the Panamerican Congress of Applied Mechanics, Rio de Janeiro, Brazil, January 1989.
11. " Free Waves at a Fluid/Layered-Composite Interface," paper presented at the IUTAM Symposium on Elastic Wave Propagation and Ultrasonic Nondestructive Evaluation, Boulder, Colorado, July/August, 1989.
12. Presentations of progress at the Review of ONR Solid-Fluid Interaction Accelerated Research Initiative, Cambridge, Massachusetts, February 1990.