Annual Progress Report 1/1/90 - 9/30/90

# Pattern selection, wave formation, turbulence and vortex breakdown in spiral flows

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### I. Summary of research activities.

The major effort of the Principal Investigator during the period of the report has been devoted to a study of time-dependent transition solutions connecting steady state solutions of the Navier-Stokes equations and to the development of a theory of periodic waves and turbulence in rotating viscous fluids. The results obtained include an analytic description of the difference between primary and secondary flows of a viscous fluid, the existence of a continuum of periodic waves in rotating plane Couette flow, and an analytic description of a mechanism to generate the turbulent-like flows observed in experiments on rotating spiral flows.

### II. Publications and papers in preparation.

The following papers were completed and accepted for publication during the period of the report:

(1) "Waves in rotating plane Couette flow" (joint work with George H. Knightly, University of Massachusetts), Geometry, Topology and Number Theory, Ed. G. Rassias, World Scientific Publishing Company, Singapore, invited paper, to appear.

(2) "Bifurcation and stability problems in rotating plane Couette-Poiseuille flow" (joint work with George H. Knightly), Contemporary Math., 108 (1990), to appear.

The following papers are in the final stages of preparation (a * denotes joint work with George H. Knightly of the University of Massachusetts). The final versions of these papers have been delayed due to the necessity of establishing two major technical lemmas described at the end of this section. All of these papers will be completed and submitted for publication in 1990 or early in 1991.

(3) "Transition solutions in the Taylor problem." The paper contains the first results on the existence of transient solutions connecting stationary solutions of the Taylor problem. The paper also contains the first analytic description of the difference between a primary and a secondary flow of a viscous fluid.

(4*) "Periodic waves in rotating plane Couette flow." The paper contains the first results on the existence of periodic waves in spiral flows. Standard methods in bifurcation theory do not apply here because of the presence of a certain singular operator J described in the technical lemmas at the end of this report.

Because of the length of the paper and because the methods introduced are applicable to a variety of fluid problems, it is likely that this paper will be split into two papers, one paper describing the general operator theoretic approach and a second paper describing the application to rotating plane Couette flow.

(5*) "Transition solutions in Rayleigh-Bénard convection." The paper contains the first results on the existence of transition solutions connecting the basic conduction flow with steady state hexagonal flows. A second basic result here is that it is not possible to make a direct connection between the conduction solution and the roll solutions that exist only as secondary flows. Such a connection apparently can be made only by first connecting the conduction flow with a hexagonal flow via a transition solution and then connecting the hexagonal flow with a roll solution via an unstable stationary solution. Such results are the only theoretical results of this type for Rayleigh-Bénard convection and are in close agreement with experimental results on subcritical hexagonal flows.

(6*) "Turbulence in spiral Couette flow." The results in (4*) in the special case of axisymmetric flows show that a vanishingly small axial shear is sufficient to destabilize a stable swirling flow. In fact, such results essentially yield a continuum of periodic waves in any "imperfect" experiment, i.e., any experiment in which the basic spiral angle cannot be maintained exactly throughout the experiment. This leads to an analytic description of a mechanism to generate turbulent-like flows such as those obtained in experiments on rotating spiral flows. There seem to be no other theoretical results in the literature that explain the almost immediate transition to turbulence in such experiments.

The two major technical lemmas required to complete the papers (3) - (6) listed above are described briefly in the following paragraphs.

The first of these technical lemmas is concerned with whether or not the eigenfunctions of the classical Bénard problem are complete; the Bénard eigenvalue problem here represents the unperturbed part of some more complicated eigenvalue problem. It is stated incorrectly in most standard works that the eigenfunctions of the Bénard problem are complete whereas, in fact, they are not complete. The eigenfunctions obtained in the usual analysis of the classical Bénard problem represent only poloidal eigenfunctions with no vertical vorticity whereas toroidal eigenfunctions with no vertical velocity are also required to represent a general solenoidal fluid. The inclusion here of the toroidal eigenfunctions to complete the set of eigenfunctions for the classical Bénard problem is especially important in considering three-dimensional flows because only very special three-dimensional flows have vanishing vertical vorticity.

A second technical lemma required in all of the papers (3) - (6) involves showing that the inverse, $J^{-1}$ , of a "singular" operator $Ju = \gamma \frac{du}{dt} + Au$ exists and is uniformly bounded in $\gamma$ for $\gamma$ near $\gamma=0$. Here, in general, A is a nonselfadjoint linear operator on some Hilbert space. Standard analytic results for the Navier-Stokes equations do not apply to $J$ because of the necessity of a bound for $J^{-1}$ that is uniform in $\gamma$ . Nevertheless, such a lemma has now been established in suitable generality so as to apply to all of the problems in papers (3) - (6).

## III. Invited symposia lectures.

(1) Fluid Dynamic Stability, SIAM Minisymposium, Chicago, July 16-20, 1990 (25 minutes).

(2) Partial Differential Equations, MSI Workshop, Cornell University, October 5-7, 1990 (60 minutes).

(3) Nonlinear Dynamics in Mathematics and Science, AMS Special Session, Amherst, October 20-21, 1990 (20 minutes).

## IV. Research plans.

The immediate goal here will be to finalize the papers (3) - (6) in preparation. Since some of the results obtained in these papers may be viewed as preliminary steps in a detailed treatment of vortex breakdown, one of the short term research goals will be a study of axisymmetric and nonaxisymmetric spiral flows in vortex breakdown. In addition, since one of the basic sets of governing equations for Langmuir circulations has many features in common with the equations studied in papers (4) and (5), a second short term research goal will be the study of Langmuir circulations using the magnitude of the Stokes mass drift as a second parameter. One of the long term research goals will be the controllability of pattern selection and turbulence in rotating spiral flows.