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Ultrasonic Scattering and NDE of Surface Breaking Cracks

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ABSTRACT

Scattering of body waves, longitudinal and shear, by a normal surface breaking crack has been studied in this paper. A hybrid method that combines the integral representation of the scattered field with a finite element discretization of the near-field has been used to study the crack opening displacement (COD) and the free-surface displacement (SD). It is shown that both COD and SD are insensitive to the crack-tip field. In addition, at low frequencies the COD is found to be uniform over most of the crack length and thus can be approximated by a constant, which may be estimated from far-field observations.

KEY WORDS:

Ultrasonic scattering; surface breaking crack; crack opening displacement.

INTRODUCTION

Problems of elastic wave scattering by surface-breaking and near-surface cracks are of considerable current interest for ultrasonic nondestructive evaluation. Ultrasonic scattering by planar cracks near or at the free surface of a semi-infinite elastic isotropic homogeneous medium has been studied by many authors (Shah *et al.*, 1985, 1986; Achenbach *et al.*, 1984; Van der Hijden and Neerhoff, 1984; Shah *et al.*, 1987; Zhang and Achenbach, 1988a,b).

In this paper we analyze the problem of body wave (plane longitudinal and shear) scattering by a surface-breaking crack with particular emphasis on the crack opening displacement (COD) and the free surface displacement (SD) near the mouth of the crack. Our object here is to assess the dependence of these quantities on the representation of the near-the-crack-tip displacement field in a finite element formulation. It is now well established that in order to capture the crack-tip stress singularily in a finite element formulation it is necessary to use singular (quarter-point) elements at the tip of the crack.

However, for the scattering problem one is generally interested in the scattered field away from this region. For this purpose is it still important to capture the crack-tip singularity? This paper addresses this question.

FORMULATION AND SOLUTION

Consider a homogeneous, isotropic, and linearly elastic medium with a surface breaking crack of arbitrary orientation and shape as shown in Fig. 1. This figure shows the cross-section of the crack in the xz-plane. The stress-free crack occupies the region $(x,z) \in C', -\infty < y < \infty$. The displacement field $\underset{\sim}{u}(x,y,z;t)$ at any point (x,y,z) and time t is taken to be $\underset{\sim}{u}(x,z)e^{i\zeta y - i\omega t}$, where ω is the circular frequency and $2\pi/\zeta$ is the wavelength in the y-direction. In this paper incident plane body waves will be assumed to be propagating in a vertical plane that makes an angle ϕ with the x-axis. Thus ζ will be determined by the type of wave (longitudinal or shear) and the angle its direction of propagation make with the negative z-axis. This angle will be noted by θ.

The problem of scattering of incident body waves by the crack will be solved by a hybrid method which combines the advantages of the finite element technique and the boundary integral method. For this purpose we consider two artificial boundaries C and B (Fig. 2). The medium is now divided into two regions. The interior region R_1 is bounded by B, part of the free surface, and C'. The exterior region R_0 is bounded by the free surface and C and extends to infinity in the x and z directions. The area between C and B is shared by both regions.

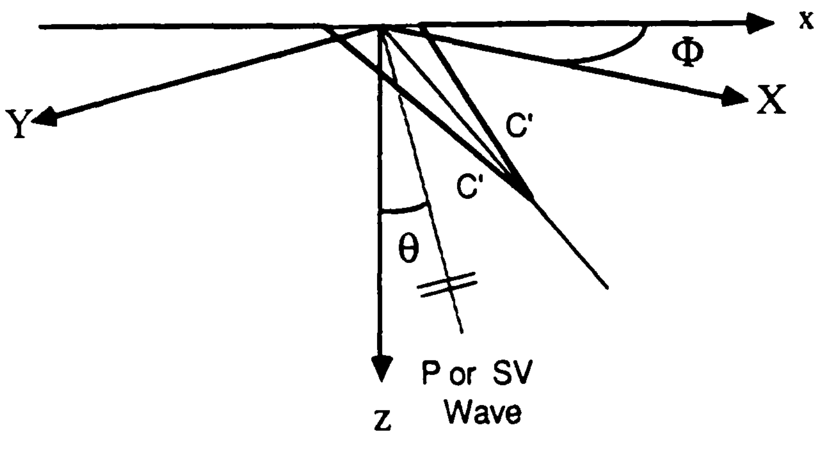

Fig. 1. Geometry of the surface breaking crack showing also the direction of the incident field.

In regions R_0 and R_1 the governing equation of elastic motion is written as

$$T_{ij,j} + \rho\omega^2 u_i = 0 \quad (i,j = 1,2,3) \tag{1}$$

where T_{ij} is the stress tensor, ρ the mass density and u_i the ith displacement component. Solution to (1) satisfying the stress-free boundary conditions along the surface of the half-space and the crack surface is sought.

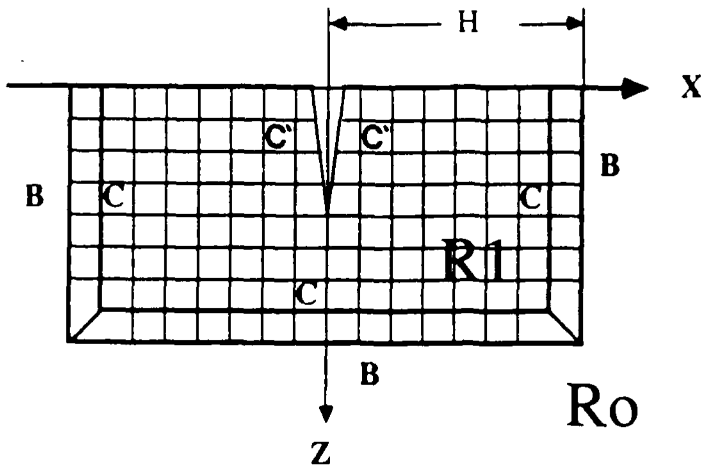

Fig. 2. Finite element grids around the crack.

Solution in the Exterior Region R_0

In this region the dispacement is composed of two parts.

$$u_i = u_i^{(o)} + u_i^{(s)} \quad (2)$$

where $u_i^{(o)}$ ($i = 1,2,3$) represents the free field displacement components (the incident field and its reflection from the free surface) and $u_i^{(s)}$ is the scattered field. The scattered displacement field in R_0 is represented by a surface integral (Khair *et al.*, 1988), after dropping the factor $e^{i\zeta y}$,

$$u_i^{(s)}(x', z') = \int_C (G_{ij}\ T_{jk} - u_j\ \Sigma_{ijk}) n_k\, dc \quad (3)$$

Here $G_{ij}\ (x,z;\ x',z')e^{i\zeta y}$ is the Green's displacement tensor for the half-space, $\Sigma_{ijk}\ e^{i\zeta y}$ is the corresponding stress tensor, and n_k defines the components of the outward unit normal vector to C. The integration along C is carried out in the clockwise direction. Expressions for $\underset{\sim}{G}$ and $\underset{=}{\Sigma}$ S have been derived before (Khair *et al.*, 1988).

The Interior Region R_I

This region is divided into finite elements having N_I number of interior nodes and N_B number of boundary nodes. For the finite element representation in region R_I the energy functional is taken to be

$$E = \frac{1}{2} \iint_{R_I} \left[\underset{\sim}{T} \cdot \underset{\sim}{\epsilon}^* - \rho\omega^2 \ \underset{\sim}{u} \cdot \underset{\sim}{u}^* \right] - \frac{1}{2} \int_B \left[\underset{\sim}{t}_B \cdot \underset{\sim}{u}_B^* + \underset{\sim}{t}_B^* \cdot \underset{\sim}{u}_B \right] ds \tag{4}$$

where * denotes complex conjugate and $\underset{\sim}{T}$, $\underset{\sim}{\epsilon}$ are stress and strain vectors defined as,

$$\underset{\sim}{T} = \left[T_{xx}, T_{yy}, T_{zz}, T_{yz}, T_{zx}, T_{xy}\right]^T \tag{5}$$

$$\underset{\sim}{\epsilon} = \left[\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{yz}, \epsilon_{zx}, \epsilon_{xy}\right]^T \tag{6}$$

where superscript T denotes transpose. $\underset{\sim}{t}_B$ and $\underset{\sim}{u}_B$ denote the traction and displacement on contour B, respectively. It is assumed that the displacement field within an element is represented in terms of the shape functions ϕ_ℓ (x,z) and elemental nodel displacements $u^{(e)}_{i;\ell}$ as

$$u_i^{(e)} = \sum_{\ell=1}^{n} \phi_\ell \ u^{(e)}_{i;\ell} \quad (i = 1,2,3) \tag{7}$$

The number of nodes in each element is given by n. $T_{ij}^{(e)}$ and $\epsilon_{ij}^{(e)}$ are computed by substituting (7) into strain-displacement relations and these, in turn, into the stress-strain relations. Substituting these in (4) and taking variation, the equation of motion for region R_I can be written as

$$\begin{bmatrix} S_{II} & S_{IB} \\ S_{BI} & S_{BB} \end{bmatrix} \begin{Bmatrix} \underset{\sim}{U}_I \\ \underset{\sim}{U}_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ \underset{\sim}{Y}_B \end{Bmatrix} \tag{8}$$

Elemental impedance matrix $[S]_e$ is given by

$$[S]_e = \iint_{A_e} \left[\left[B_e^*\right]^T [D] \ [B_e] - \rho\omega^2 \left[\phi_e\right]^T \ [\phi_e] \right] dxdz \tag{9}$$

$\underset{\sim}{Y}_B$ is the nodal force vector due to surface tractions on the boundary.

Using the top set of equations in (8) we get

$$\{\underset{\sim}{U}_I\} = - \left[S_{II}\right]^{-1} [S_{IB}] \{\underset{\sim}{U}_B\} \tag{10}$$

Matrices $[B_e]$ and $[D]$ have been derived before (Khair *et al.*, 1988). Note that in this derivation use has been made of the equation $\underset{\sim}{u}(x,y,z) = \underset{\sim}{u}(x,z)e^{i\zeta y}$.

Combining (3) and (11) the boundary displacement $\underset{\sim}{U}_B$ is found to be

$$\{\underset{\sim}{U}_B\} = \left[- [A_{BI}] \left[S_{II}\right]^{-1} [S_{IB}] + A_{BB} \right] \{\underset{\sim}{U}_B\} + \{\underset{\sim}{U}_B^{(0)}\} \tag{11}$$

Once $\{\underset{\sim}{U}_B\}$ is found by solving (11), $\{\underset{\sim}{U}_I\}$ is found from (10).

In the following we present the results for the COD and z-component of the surface displacement for incident longitudinal (P) and vertically polarized shear (SV) waves. In all the numerical results presented it is assumed that $\zeta = 0$, i.e., the waves are traveling in the xz-plane. Also, to keep the problem simple the crack is assumed to be planar and normal to the free surface. Results for cracks of different orientations and for non-planar cracks will be presented elsewhere.

NUMERICAL RESULTS AND DISCUSSION

The method described above was used to solve body wave (P and SV) scattering by a normal surface-breaking crack. The attention has been focused on the near-field displacements as they depend on near-the-crack-tip singular stress distribution. To examine this dependence the region around the crack was divided into finite elements with or without using the quarter-point singular elements near the crack tip. The Poisson's ratio of the medium is taken to be $\nu = 1/3$. Displacements at all the interior nodes were calculated for different angles of incident using (11). Crack opening displacements

$$\Delta_z = |u_z^+ - u_z^-| \tag{12}$$

$$\Delta_x = |u_x^+ - u_x^-| \tag{13}$$

were calculated. These are shown in Figs. 3-6 for different frequencies and for angle of incidence 45°. The lines denote the results obtained by using the crack-tip singular elements and the points without the use of these elements. As seen from these figures the COD's do not depend on the accurate representation of the crack-tip singular stress field. That the crack-tip field does not significantly influence the scattered field is also seen from the surface displacement field presented in Figs. 7 and 8. Here the amplitudes of the vertical surface displacements near the mouth of the crack are presented.

Figures 3-6 also show that at long wavelengths the COD's are nearly constant along most of the crack length changing rapidly only near the tip. This change is influenced by the way the crack-tip field is represented in the finite element shape functions in this region. However, as noted above, this local detailed field does not influence the scattered field much, even near the mouth of the crack.

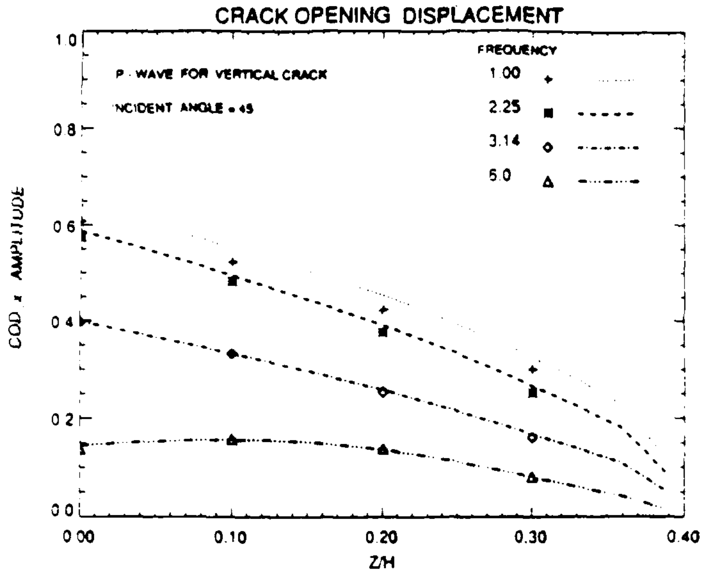

Fig. 3. Distribution of Δ_x along the crack length for P wave incident at 45° with the vertical. Lines show results using crack-tip quarter-point signular elements and the points without that.

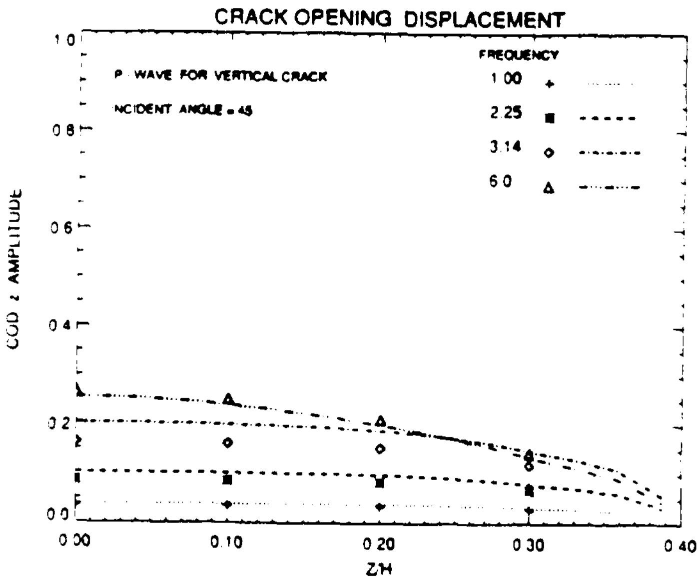

Fig. 4. Δ_z along the crack-length for the same case as in Fig. 3.

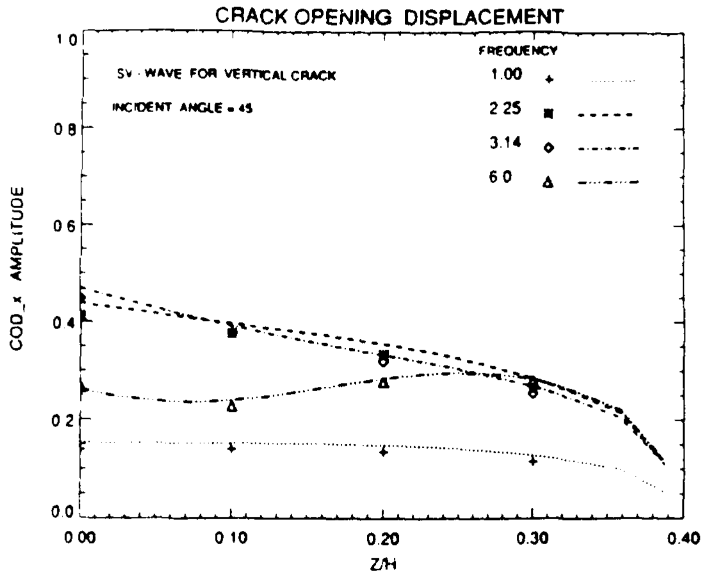

Fig. 5. Δ_x along the crack-length for SV wave incident at 45° with the vertical. Explanations are the same as in Fig. 3.

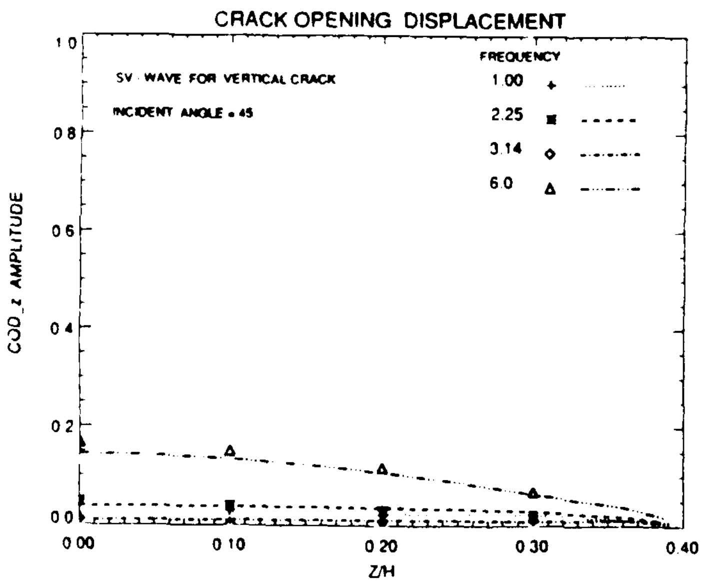

Fig. 6. Δ_z along the crack-length for the same case as in Fig. 5.

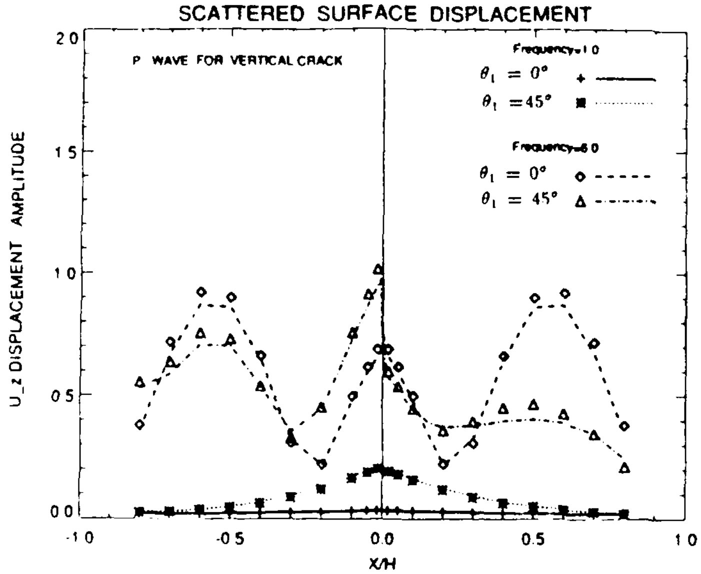

Fig. 7. Normalized scattered vertical displacement amplitude near the mouth of the crack for P wave incident at 45° with the vertical. Lines show results obtained using the crack-tip elements and points without them.

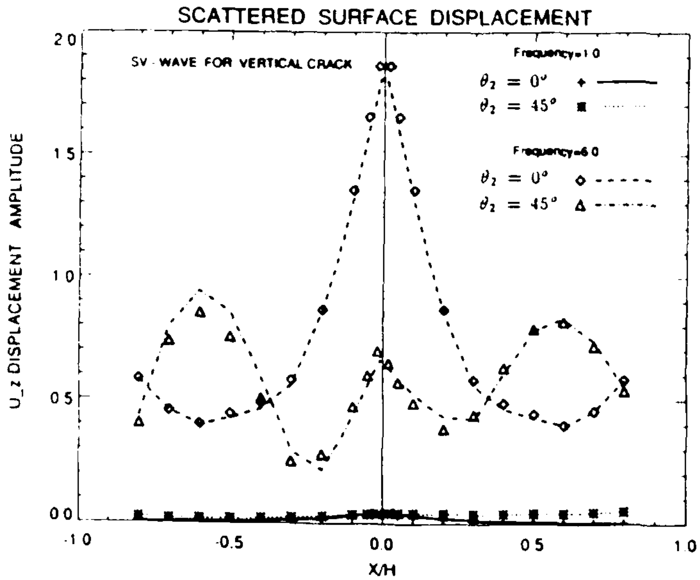

Fig. 8. Same as in Fig. 7 for SV wave incident at 45° with the vertical.

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REFERENCES

Achenbach, J. D., Y. C. Angel and W. Lin (1984). Scattering from surface-breaking and near-surface cracks. In: *Wave Propagation in homogeneous media and ultrasonic nondestructive evaluation* (G. C. Johnson, ed.), pp. 93-109. The American Society of Mechanical Engineers, New York.

Khair, K. R., S. K. Datta and A. H. Shah (1988). *Three dimensional amplification of seismic waves by alluvial valleys of arbitrary shape: I. Incident P and SV Waves.* CUMER-88-2, Department of Mechanical Engineering, University of Colorado, Boulder.

Shah, A. H., K. C. Wong and S. K. Datta (1985). Surface displacements due to elastic wave scattering by planar and non-planar cracks. *Wave Motion*, 7, 319-333.

Shah, A. H., K. C. Wong and S. K. Datta (1986). Dynamic stress intensity factors for buried planar and non-planar cracks. *Int. J. Solids Str.*, 22, 845-857.

Shah, A. H., Y. F. Chin and S. K. Datta (1987). Elastic wave scattering by surface-breaking planar and nonplanar cracks. *J. Appl. Mech.*, 109, 761-767.

Van der Hijden, J. H. M. T. and F. L. Neerhoff (1984). Diffraction of elastic waves by a sub-surface crack (in plane motion). *J. Acoust. Soc. Am.*, 75, 1694-1704.

Zhang, C. and J. D. Achenbach (1988a). Scattering of body waves by an inclined surface-breaking crack. *Ultrasonics*, 26, 132-138.

Zhang, C. and J. D. Achenbach (1988b). Numerical Analysis of surface-wave scattering by the boundary element methods. *Wave Motion*, 10, 365-374.