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NAVAL POSTGRADUATE SCHOOL

Monterey, California

A PROGRAM TO COMPUTE MAGNETIC ANOMALY DETECTION PROBABILITIES

R. N. FORREST

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(Revised March 1990)

Approved for public release; distribution is unlimited.

Prepared for:
Naval Postgraduate School
Monterey, CA 93943-5000

[redacted]

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IN REPLY REFER TO

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From: Superintendent, Naval Postgraduate School
To: Administrator, Defense Technical Information Center, Cameron Station, Alexandria, VA 22304-6145

Subj: TECHNOLOGY SCREENING OF UNCLASSIFIED/LIMITED REPORTS

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1. In reply to reference (a), the publication, "A Program to Compute Magnetic Anomaly Detection Probabilities", was referred to its author for his comments.

2. It is the author's opinion that the report should have unlimited distribution. Using the applicable paragraphs of the MCTL as reference the author opined that the application of the theory on which the report is based is well known, and, without data on magnetic noise and submarine magnetic moments of a classified nature the program which implements the theory could not be used as a tactical decision aid. In addition, without a complete revision of the two detection models on which the program is based, the program's predictive value would not be operationally superior to a simple inverse cube detection range model. The program is based on a noise model that describes magnetic noise as gaussian but does not describe the filter that would be required to achieve this nor the effect of such a filter on the magnetic dipole signals.

3. The author advises that the report has been revised and two copies of this revision were sent to your Center recently. The revision is date March 1990. The remarks in paragraph 2 above apply to the revised report as well.

B. J. FERRARA
By direction

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The report contains user instructions, a listing and documentation for a microcomputer BASIC program that can be used to compute an estimate of the probability that a magnetic anamoly detection (MAD) system such as the AN/ASQ-81 will detect a submarine during an encounter.

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The second revision of the report contains corrections to some errors that were contained in the original version as well as some that were introduced in the first revision. In addition, on Page 43 there is a discussion of the definition of sweep width as it relates to the lateral range functions that are generated by the detection models on which the program is based. As was noted in the earlier versions of the report, the program in the report is presented without representation or warranty of any kind.

Table of Contents

I. Introduction

This report contains user instructions, a listing and documentation for a microcomputer BASIC program that can be used to compute an estimate of the probability that a magnetometer based detection system such as the AN/ASQ-81 will detect a submarine magnetic dipole field during an encounter.

The program generates detection probabilities based on two encounter models. In the first encounter model, the detection system uses a crosscorrelation detector. In the second encounter model, the detection system uses a square law detector. Relative to operationally realizable values, probabilities based on the first model represent upper bounds and those based on the second model represent lower bounds. For both encounter models, the signal is proportional to the square of the magnitude of the projection of the dipole field on the earth magnetic field and the magnetic noise does not change with changes in the position of the magnetometer. Also, for both encounter models, an encounter is a straight line encounter with constant vertical separation.

The encounter models can be interpreted as models of a magnetic anomaly detection (MAD) system on an aircraft that is moving with constant course, speed and altitude in an encounter with a submarine moving with constant course, speed and depth. Or, they can be interpreted as models of a stationary magnetic anomaly detection system in an encounter with a submarine moving with constant course, speed and depth.

The program parameters include: encounter latitude and longitude, submarine induced magnetic moments, submarine permanent magnetic moments, submarine course, speed and altitude, magnetometer course, speed and depth, encounter lateral range (the horizontal range at the closest point of approach in a straight line encounter) and false alarm rate. (In the program, a false alarm is the event that the detection system classifies noise as a dipole signal.)

II. Program User Instructions

As listed in Appendix 10, the program can be run under a BASIC language that is compatible with IBM PC BASIC. If the listing is used to enter the program through the keyboard, then the program should be saved with the name MAD.BAS or the value of N$ on line 40 should be changed to the file name under which it is saved.

The program contains user instructions in the form of query and parameter limitation messages. As an example of the former, after starting the program under BASIC, the following message should appear:

Magnetic Anomaly Detection (MAD) Lateral Range Function

generate or print a program data file (g/p)?

By entering p, data can be printed from a program data file that was generated by the program. By entering g, a program data file can be generated for a set of user specified conditions. With either response, a sequence of additional queries is displayed. These queries require either an indicated response or a parameter value as the input. If the initial response is g, the sequence includes queries whose responses determine whether or not an auxiliary data file will be generated that can be used for future input of magnetic, processing or kinematic data. In particular, the first query in the sequence gives the option of using a combined magnetic, processing and kinematic data file. If the response indicates that it should be used and the file is available, the parameter values that remain to be entered in order to generate a program data file are the following: the false alarm rate, the magnetic noise, the maximum encounter lateral range and the lateral range step. The combined file should be used only if the effect of varying just one or more of these parameters is desired. If the response indicates that the file will not be used, queries concerning magnetic, kinematic and processing parameter values are displayed.

After all of the program parameter values have been entered, a query is displayed giving the option of generating the combined file. Then a query giving the option of generating a program data file, a query giving the option of printing the encounter parameter values and a query giving the option of printing lateral range function values are displayed. The lateral range function values are the encounter detection probabilities indexed by lateral range. The parameters maximum lateral range and lateral range step determine the index lateral ranges of the encounters for which probabilities of detection are computed.

The program generates magnetic signal values that correspond to points in time during an encounter. Following the lateral range function query, a query is displayed that gives the option of printing the magnetic signal values for an encounter. If the option is exercised, the option is repeated. When the option is not exercised, a query is displayed that gives the option of generating or printing a new program data file. If this option is not exercised, the program ends.

Some suggested guides for determining parameter values can be found in Section IV of this report.

III. Encounter Model Limitations

In the two encounter models, the signal is the unfiltered signal that is generated by a magnetic dipole that moves relative to a magnetometer. Depending on the input filtering, describing a ship magnetic anomaly as a dipole anomaly should not be a significant limitation for an encounter slant range at the closest point of approach (CPA) that is greater than one hull length. The detection decision is based on samples from a single time interval (window) that is centered on the CPA. The length of the sampling interval and the sampling rate are parameter values that are inputs to the program. In terms of signal-to-noise ratio, there is an optimum sampling interval length (integration time) and sample rate. Although dipole signal energy is not symmetrically distributed about the CPA time, for a given sampling interval, the difference between the signal energy for the optimum interval location and the CPA centered location should not be significant in most cases.

The encounter model magnetometer noise samples are determined by a gaussian random process. They are values of independent identically distributed normal random variables. The standard deviation of these random variables is referred to as the magnetic noise and the variance is the noise in the sense of the signal-to-noise ratio. The degree of correspondence between this process and operational noise depends on the nature of the dominant operational noise sources and on the magnetometer input filtering (noise whitening).

In the encounter models, the intervals are adjacent but not overlapping and a detection statistic corresponds to each sample interval and its value is determined by the sample values. If the value of the detection statistic for an interval equals or exceeds a threshold value, a detection is indicated. The threshold value is determined by the false alarm probability which in turn is determined by the false alarm rate and the sampling interval length.

A detector that used a moving sample interval that was generated by replacing the oldest sample by the newest one would correspond more closely to the detector in an operational detection system. In a model of the detector, since the sample windows overlap, the detection statistics would represent a sequence of random variables that were correlated over an interval equal to the width of the sample interval. Because of this dependence, it seems unlikely that the results that would be obtained with an encounter model based on an overlapping interval detector would differ significantly from those obtained with an encounter model based on a nonoverlapping interval detector.

For encounter lengths of the order of a few nautical miles or less, the straight line encounter condition should not be a significant limitation. In particular, this should be the case for a fixed magnetometer since, for a submarine (or surface ship) target, vertical separation and course changes should be less likely to occur.

Other models are available that can be used as the basis for computing an estimate of the probability that a magnetic anomaly detection system will detect a submarine during an encounter. For example, one that is described in Appendix 8 can be used to determine the slant ranges of straight line encounters for which the detection probability is equal to a specified value. The parameter values that are required to do this are an average submarine dipole moment, a detection system capability factor and a noise factor. Values for these parameters can be determined from operational data. However, the values are specific to averages over a particular set of encounter conditions. An advantage of the two encounter models relative to this model is their adaptability to different magnetic, processing and kinematic conditions.

IV. Parameter Values

The magnetic parameter value queries are generally explanatory with regard to the value that should be entered. This is also true of the kinematic parameter value queries. However, there is some ambiguity with respect the processing parameter value queries and the noise parameter value query. To reduce this ambiguity, a brief discussion of the common characteristics of the two encounter model processing parameters and the noise parameter is given below. This is followed by some guidelines for choosing these parameter values.

In both encounter models, a decision is made at the end of each sampling interval. The decision is either noise energy was present during the interval or noise energy plus signal energy was present during the interval. The sample intervals are adjacent, equal width, nonoverlapping time intervals. The number of samples that are input in a sample interval is determined by the sampling rate and the interval length.

The program default choice for the sampling rate is 2·MAXF where MAXF is a parameter that is labeled the maximum magnetic signal frequency. This sample rate is the Nyquist rate for an ideal low pass filter. However, the signal in a sample interval that is computed by the program represents an unfiltered dipole signal. This is a reasonable approximation if the signal energy that is associated with signal components greater than MAXF is relatively small. As discussed below, the noise energy in a sample interval should be considered to be proportional to MAXF in order to be consistent with the encounter models. Ideally, a default choice for MAXF would make the ratio of the signal energy to the noise energy a maximum for the sampling interval of an encounter. The program default choice for MAXF is 2·MAXVM/MINRO where MAXVM is a user estimated maximum encounter relative speed converted from knots to meters per second and MINRO is a user estimated maximum slant range at CPA in meters in terms of a just detectable target. (A precise definition of a just detectable target can be made in terms of a specified detection probability, false alarm probability and target dipole moment.) The default choice for MAXF is consistent with the observation in Reference 1 that if an optimum value for MAXF is determined for a minimum dipole moment target, then no significant increase in MAXF is required in order to maintain a required detection probability if the encounter lateral range is decreased even though the signal energy spectrum is shifted to higher frequencies.

Because of the detection statistics and the gaussian noise model, if there were no penalty for decision delay, a sample interval length for a signal should be chosen equal to the signal duration, since this would make the detection probability for an encounter a maximum. In the program, a default choice for the sample interval length (integration time) is 2·MAXRO/MINVM where MINVM is a user estimated minimum encounter relative speed converted from knots to meters per second and MAXRO is a user estimated maximum slant range at CPA in meters for a detectable encounter (in terms of a specific detection probability and false alarm probability) with a user specified maximum dipole moment target. This choice might be considered a balance between minimizing decision delay and maximizing detection probability. In the encounter models, the sample intervals are located so that the CPA time is at the center of a sample interval and this sample interval is the only one that contains signal energy. This characteristic is consistent with the default choice for the sample interval. For a given sample interval length, although in general the interval location is not the optimum one in terms of signal energy, it should be approximately so in most cases.

The program noise parameter is SIG. It represents the standard deviation σ associated with the magnetic noise process of the two encounter models. In terms of the ideal low pass filter implied by the encounter models, its square should be equal to $MAXF \cdot (SIG0)^2$ where SIG0 is the magnetic noise process constant power spectral density. The program does not enforce this relation. Therefore, in using the program, the implied relation between the two input parameters: magnetic noise and maximum magnetic signal frequency should be kept in mind. If an average value of the peak-to-peak magnetic noise for an encounter can be estimated, for example from a magnetometer trace, then the value for SIG should be chosen so that the estimate is 4 to 6 times this value.

Appendix 1. The Detection Statistics

In this appendix, $y_1, y_2, \cdots, y_M$ are sequential values (voltages) representing the sample values in a sample interval. They are the input to a magnetometer's detector. With these M sample values, the detector computes the value of a detection statistic. This value is represented by x and the detection decision corresponding to the sample interval is determined by the decision rule: If $x \geq x^*$, then the input during the sample interval was noise plus signal, otherwise, the input was noise.

For both encounter models, the detection probability and the false alarm probability are decreasing functions of x^* and the relation is one-to-one in both cases. In the program, the false alarm probability p_F is used to determine a unique value of the threshold x^*. This value is then used to determine a unique value of the detection probability p_D.

In the program, p_F is found using the relation $p_F = R \cdot \delta t$ where R is the false alarm rate in false alarms per second and δt is the sample interval length in seconds. This relation is based on the following argument: With no signal energy in a sample interval, $y_1 = n_1, y_2 = n_2, \cdots, y = n_M$ where $n_1, n_2, \cdots, n_M$ are noise values (voltages) input to a magnetometer's detector that are values of independent normal (gaussian) random variables, each with mean zero and standard deviation σ. Because of this, in the encounter models, values of x for different sample intervals are the values of independent random variables that determine two outcomes: $x \geq x^*$ or $x < x^*$.

Therefore, p_F is the same for each sample interval and, in terms of these outcomes, the sequence is a series of repeated independent Bernoulli trials and the expected number of trials between false alarms is $1/p_F$. Since the time between trials is δt, the expected number of seconds between false alarms is $\delta t/p_F$ and the expected number of false alarms per second is equal to $p_F/\delta t$. So, $R = p_F/\delta t$.

The determination of x^* depends on the encounter model statistic. For both encounter models, when there is a signal, $y_1 = n_1 + s_1$, $y_2 = n_2 + y_2$, $\cdots$, $y_M = n_M + s_M$ where s_1, s_2, $\cdots$, s_M are signal values (voltages) input to a magnetometer's detector. The models imply that the signal values $s = c \cdot H_S$ where c is a constant whose value is determined by the characteristics of the encounter magnetometer and where the H_S are dipole magnetic signal intensities. The models also imply that the noise values n are determined by a gaussian stochastic processes characterized by a standard deviation σ and that $n = c \cdot H_N + n'$ where the H_N are magnetic noise intensities and the n' are magnetometer instrument noise values (voltages). In the program, the magnitude of c is 1. Since, for both models, p_D depends only on the ratio of signal energy to noise energy for a sample interval, this is a satisfactory choice for the program if instrument noise process is assumed to be independent of the magnetic noise process and to be expressed in terms of an equivalent magnetic noise $H = (1/c) \cdot n'$.

For both encounter models, the signal (the average signal power) $S = (1/M) \cdot \Sigma s_K^2$ and the noise (the expected value of the average noise power) $N = \sigma^2$ so that the signal-to-noise ratio is $(1/M) \cdot \Sigma s_K^2/\sigma^2$ where the sum index $K = 1, 2, \cdots, M$.

The Crosscorrelation Detector Statistic: The statistic for the first encounter model is a crosscorrelation detector statistic that is defined by the sum

$$x = \Sigma y_K \cdot s_K$$

where the summation index $K = 1, 2, \cdots, M$ and the sum is over the values corresponding to a sample interval. For the first encounter model, the characteristics of both the noise and the signal are required in order to determine encounter detection probabilities. In particular, the signal values for an encounter are in the memory of the detector prior to the encounter. For the encounter conditions and a specified false alarm probability, the statistic is optimum in the sense that the encounter detection probability for this statistic is at least equal to that for any other statistic. Because of these considerations, encounter probabilities based on the crosscorrelation statistic can be considered to represent upper bounds on detection performance against dipole targets for magnetometers of the type described by the models.

For a sample interval without signal energy, x is the value of a normal random variable with a mean $\mu_x = 0$ and a variance $\sigma_x^2 = \sigma^2 \cdot \Sigma s_K^2$ where σ is the standard deviation associated with the noise process and the sum index $K = 1, 2, \cdots, M$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_F = 1 - P(x^*/\sigma_X)$$

where $P(z)$ is the standard normal cumulative distribution function. This relation is the basis for determining the threshold value x^*.

For the sample interval with signal energy, x is the value of a normal random variable with a mean $\mu_X = \Sigma\, s_K^2$ where the sum index $K = 1, 2, \cdots, M$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_D = 1 - P(v^* - d^{1/2})$$

where $v^* = x^*/\sigma_X$ and $d = \Sigma\, s_K^2/\sigma^2 = M \cdot S/N$. This relation is the basis for determining encounter detection probabilities for the first encounter model. The relation implies that for a specified false alarm probability p_F the detection probability p_D is an increasing function of the signal to-noise ratio S/N.

The Square Law Detector Statistic: The statistic for the second encounter model is a square law (energy) detector statistic that is defined by the sum

$$x = \Sigma\, y_K^2$$

where the sum index $K = 1, 2, \cdots, M$ and the sum is over the values corresponding to the sample interval. For the second encounter model, only the characteristics of the noise are required to determine encounter detection probabilities.

For a sample interval without signal energy, x/σ^2 is the value of a chi-square random variable with M degrees of freedom. This implies that

$$p_F = 1 - P(x^*/\sigma^2|M)$$

where $P(x^*/\sigma^2|M)$ is the chi-square cumulative distribution function for a chi-square random variable with M degrees of freedom and where σ is the standard deviation associated with the noise process. This relation is the basis for determining the threshold value x^*.

For the sample interval with signal energy, x/σ^2 is the value of a noncentral chi-square random variable with M degrees of freedom and noncentral parameter $\Sigma\, s_K^2/\sigma^2$ where the sum index $K = 1, 2, \cdots, M$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_D = 1 - P(x^*/\sigma^2|M,\ \Sigma\, s_K^2/\sigma^2)$$

where $P(x^*/\sigma^2|M,\ \Sigma\, s_K^2/\sigma^2)$ is the cumulative distribution function for a noncentral chi-square random variable with M degrees of freedom and noncentral parameter $\Sigma\, s_K^2/\sigma^2 = M \cdot S/N$. This relation is the basis for determining encounter detection probabilities for the second encounter model. The relation implies that for a specified false alarm probability p_F, the detection probability p_D is an increasing function of the signal-to-noise ratio S/N. This is made more evident by the following relation:

$$P(x^*/\sigma^2|M,\ \Sigma\, s_K^2/\sigma^2) = \Sigma\{(a^J/J!)\cdot \exp(-a)\cdot P[x^*/\sigma^2|(M + 2\cdot J)]\}$$

where the parameter $a = (1/2) \cdot \Sigma\, s_K^2/\sigma^2 = (M/2) \cdot (S/N)$, the sum index $J = 0, 1, 2, \cdots$ and the sum index $K = 1, 2, \cdots, M$. Note, since $P(x^*/\sigma^2 | M) \leq P[x^*/\sigma^2 | (M + 2 \cdot J)]$ for $J = 0, 1, 2, \cdots$, the relation $p_D \geq p_F$ is satisfied.

Appendix 2. Program Probability Calculations

The program evaluates the cumulative and inverse cumulative distribution functions using approximations described in Reference 2. These approximations are listed below.

The Standard Normal Cumulative Distribution Function Approximation:

$$P(z) = 1 - s \cdot t \cdot (b_1 + t \cdot (b_2 + t \cdot (b_3 + t \cdot (b_4 + t \cdot b_5))))$$

where $s = (1/(2 \cdot \pi)^{1/2}) \cdot \exp(-z^2/2)$ and $t = 1/(1 + b_0 \cdot z)$. And where

$b_0 = .2316419$, $b_1 = .319381530$, $b_2 = -.356563782$,

$b_4 = -1.821255978$, $b_5 = 1.330274429$,

and $z \geq 0$. For $z < 0$, $P(z) = 1 - P(|z|)$.

The Inverse Standard Normal Cumulative Distribution Function Approximation:

$$z(P) = t - (c_0 + t \cdot (c_1 + t \cdot c_2))/(1 + t \cdot (d_1 + t \cdot (d_2 + t \cdot d_3)))$$

where $t = (\ln(1/Q^2))^{1/2}$ and $Q = 1 - P$. And where

$c_0 = 2.515517$, $c_1 = .802853$, $c_2 = .010328$,

$d_1 = 1.432788$, $d_2 = .189269$, $d_3 = .001308$,

and $.5 \leq P < 1$. For $0 < P < .5$, $z(P) = -z(1 - P)$.

The Inverse Chi-Square Cumulative Distribution Function Approximation:

$$v(P|M) = M \cdot [1 - 2/(9 \cdot M) + z \cdot (2/(9 \cdot M))^{1/2}]^3$$

where $P(z) = P(v|M)$. In the program, the inverse standard normal cumulative distribution function approximation is used to determine z.

The Noncentral Chi-Square Cumulative Distribution Function Approximation:

$$P(w|M,\Sigma\ s_K^2/\sigma^2) = P(z)$$

where $z = [2 \cdot w/(1 + b)]^{1/2} - [2 \cdot a/(1 + b) - 1]^{1/2}$ with $a = M + \Sigma\ s_K^2/\sigma^2$, $b = (\Sigma\ s_K^2/\sigma^2)/(M + \Sigma\ s_K^2/\sigma^2)$ and the sum index $K = 1, 2, \cdots, M$. In the program, the standard normal cumulative distribution function approximation is used to determine $P(z)$.

Appendix 3. The Magnetic Signal

The encounter models are defined by the following conditions: A submarine magnetic anomaly field is a magnetic dipole field that is superimposed on an earth magnetic field that is constant over an encounter region. A magnetometer magnetic signal value is the magnitude of the projection of a dipole magnetic field on the earth magnetic field at the location of the magnetometer. The basis for determining magnetic signal values in the program is an expression that can be developed as follows: In the right handed rectangular coordinate system that is shown in Figure 1, a magnetic dipole is at the origin, the xy-plane is the horizontal plane at a representative point in an encounter region, the positive y-axis is in the direction of magnetic north, the positive x-axis is in the direction of magnetic east and the positive z-axis is positive upward. In this rectangular coordinate system, the constant earth magnetic field can be expressed by $\mathbf{H}_E = H_E \cdot (\mathbf{j} \cdot \cos \Phi + \mathbf{k} \cdot \sin \Phi)$ where Φ is a magnetic dip angle and H_E is a magnetic field magnitude that characterizes the earth field in an encounter region.

In a spherical coordinate system with the origin at the magnetic dipole and the polar axis in the direction of the dipole moment, $\mathbf{H}_d = (c \cdot p/r^3) \cdot (2 \cdot \mathbf{r} \cdot \cos \theta + \boldsymbol{\theta} \cdot \sin \theta)$ is the magnetic field of the dipole at a point whose spherical coordinates are (r, Γ, θ). In this expression p is the magnitude of the dipole moment and c is a constant whose value is determined by the choice of units. The magnetic signal for a magnetometer that is

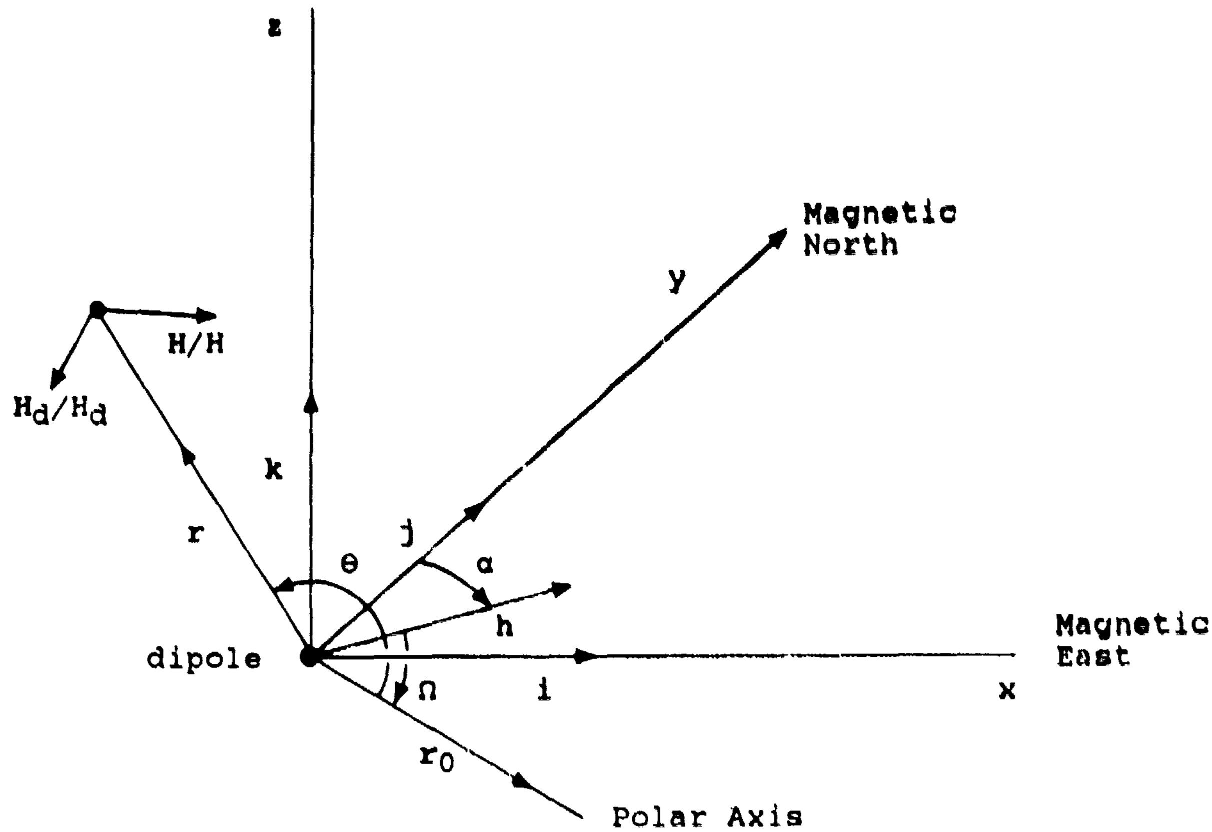

Figure 1. A unit vector in the direction of the earth field and a unit vector in the direction of the dipole field of a dipole at the origin are shown at a point that is a distance r from the dipole. The unit vector **h** is in the direction of the horizontal component of the dipole moment.

described by the encounter models is $H_s = \mathbf{H}_e \cdot \mathbf{H}_d / H_e$ when the magnetometer is at the point (r, Γ, Θ) and H_s can be expressed in terms of the rectangular coordinate system as follows: First, let $\mathbf{r}_0$ be a unit vector in the direction of the spherical coordinate system polar axis. Then the magnetic dipole field

$$\mathbf{H}_d = (c \cdot p/r^3) \cdot (3 \cdot \mathbf{r} \cdot \cos\Theta - \mathbf{r}_0)$$

since $\mathbf{r} = (\mathbf{i} \cdot x + \mathbf{j} \cdot y + \mathbf{k} \cdot z)/r$ where $r = (x^2 + y^2 + z^2)^{1/2}$, $\mathbf{\Theta} = [(\mathbf{r}_0 \times \mathbf{r}) \times \mathbf{r}]/\sin\Theta = [(\mathbf{r} \cdot \mathbf{r}_0) \cdot \mathbf{r} - (\mathbf{r} \cdot \mathbf{r}) \cdot \mathbf{r}_0]/\sin\Theta$, and the dot product $\mathbf{r} \cdot \mathbf{r}_0 = \cos\Theta$. The unit vector $\mathbf{r}_0$ can be expressed in the rectangular coordinates by noting that $\mathbf{r}_0 = \mathbf{p}/p$ and then expressing $\mathbf{p}$ in rectangular coordinates. To do this, let Ω be the depression angle of $\mathbf{p}$ from the xy-plane (the horizontal plane) with Ω positive downward and let α be the direction of $\mathbf{p}$ relative to magnetic north. Then the magnetic dipole moment $\mathbf{p} = p \cdot (\mathbf{h} \cdot \cos\Omega - \mathbf{k} \cdot \sin\Omega)$ in terms of the unit vector $\mathbf{k}$ and the unit vector $\mathbf{h} = \mathbf{i} \cdot \sin\alpha + \mathbf{j} \cdot \cos\Omega$ which has the direction of the horizontal component of $\mathbf{p}$. With those results, the unit vector $\mathbf{r}_0 = \mathbf{i} \cdot (\cos\Omega \cdot \sin\alpha) + \mathbf{j} \cdot (\cos\Omega \cdot \cos\alpha) - \mathbf{k} \cdot \sin\Omega$ and

$$H_s = (c \cdot p/r^3) \cdot [(3/r) \cdot \cos\theta \cdot (y \cdot \cos\phi - z \cdot \sin\phi) - (\cos\phi \cdot \cos\Omega \cdot \cos\alpha + \sin\phi \cdot \sin\Omega)]$$

where $\cos\theta = (1/r) \cdot (x \cdot \cos\Omega \cdot \sin\alpha + y \cdot \cos\Omega \cdot \cos\alpha - z \cdot \sin\Omega)$ since $\cos\theta = \mathbf{r} \cdot \mathbf{r}_0$. As can be seen from this expression, for a constant dipole moment magnitude and direction and a constant earth field magnitude and direction, the magnetic signal is only a function of the rectangular coordinates of the location of the magnetometer relative to the dipole. In the encounter models, both of these conditions are satisfied. However, by allowing p, Ω, α, H_s and ϕ to vary, the expression for H_s is applicable to more general encounter models.

Appendix 4. The Anderson Formulation

In the encounter models, the magnetic signal H_s at a sample point in a straight line encounter can be represented in a form described by Anderson in Reference 4. For convenience, the Anderson formulation is used in the program to determine values for H_s. It can be developed as follows: In Figure 2, the primed rectangular coordinate system is superimposed on the rectangular coordinate system of Figure 1 so that the origin is coincident with the origin of that system. A magnetometer is in a straight line encounter with a magnetic dipole that is located at the origin of the combined system. The combined system moves with the magnetic dipole with the x'-axis oriented so that it is parallel to and in the direction ϕ of magnetometer's relative to the magnetic dipole and the z'-axis oriented so that it is directed toward and passes through the CPA on that track. Let l, m, and n be the direction cosines of the dipole moment **p** and l_1, m_1 and n_1 be the direction cosines of the earth magnetic field $\mathbf{H}_E$ in the primed system. Then the unit vector $\mathbf{r}_0 = \mathbf{i}'\cdot l + \mathbf{j}'\cdot n + \mathbf{k}'\cdot m$. And, for points on the relative track, $x' = s'$, $y' = 0$ and $z' = R$ where R is the slant range of the dipole at CPA and where s' is the algebraic distance of the magnetometer from CPA on the relative track. (It is negative for points before CPA and positive for points after CPA.) This implies that magnetometer position vector in the moving coordinate system is $\mathbf{r} = \mathbf{i}'\cdot(s'/r) + \mathbf{k}'\cdot(R/r)$.

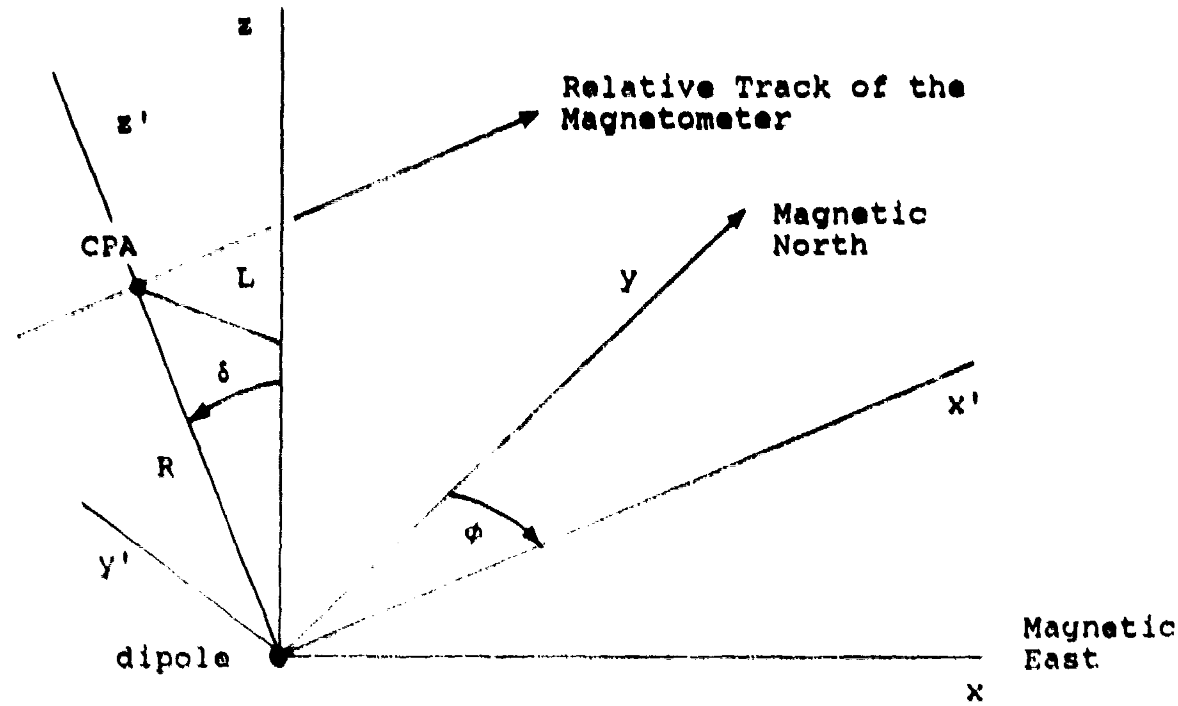

Figure 2. The dipole is at the origin in the unprimed and primed coordinate systems. In the primed coordinate system, the CPA is at (0, 0, R) and, for a time t during the encounter, the magnetometer is at [s'(t), 0, R].

From Appendix 3,

$$\mathbf{H}_d = (c \cdot p/r^3) \cdot (3 \cdot \mathbf{r} \cdot \cos\theta - \mathbf{r}_0).$$

With $\mathbf{r}$ and $\mathbf{r}_0$ expressed in terms of the primed unit vectors and $\cos\theta = \mathbf{r} \cdot \mathbf{r}_0 = l \cdot (s'/r) + n \cdot (R/r)$, this becomes

$$\mathbf{H}_d = (c \cdot p/r^3) \cdot [(3/r^2) \cdot (l \cdot s' + n \cdot R) \cdot (\mathbf{i}' \cdot s' + \mathbf{k}' \cdot R) - (\mathbf{i}' \cdot l + \mathbf{j}' \cdot m + \mathbf{k}' \cdot n)].$$

Then, since $\mathbf{H}_E/H_E = \mathbf{i}' \cdot l_1 + \mathbf{j}' \cdot m_1 + \mathbf{k}' \cdot n_1$ and $H_s = \mathbf{H}_E \cdot \mathbf{H}_d/H_E$,

$$H_s = (c \cdot p/r^3) \cdot [(2 \cdot l \cdot l_1 - m \cdot m_1 - n \cdot n_1) \cdot (s')^2 + 3 \cdot (n \cdot l_1 + l \cdot n_1) \cdot s' \cdot R + (2 \cdot n \cdot n_1 - l \cdot l_1 - m \cdot m_1) \cdot R^2].$$

The quantities

$$A_2 = 2 \cdot l \cdot l_1 - m \cdot m_1 - n \cdot n_1$$

$$A_1 = 3 \cdot (n \cdot l_1 + l \cdot n_1)$$

$$A_0 = 2 \cdot n \cdot n_1 - l \cdot l_1 - m \cdot m_1,$$

are the Anderson coefficients. With $r = [(s')^2 + R^2]^{1/2}$ and $\beta = s'/R$, H_s can now be expressed as follows:

$$H_s(\beta) = (c \cdot p/R^3) \cdot \Sigma\, A_J \cdot F_J(\beta)$$

where the $F_J(\beta) = \beta^J/(1 + \beta^2)^{5/2}$ for $J = 0, 1, 2$ are the Anderson functions. This is the Anderson formulation.

To relate the Anderson formulation for H_s to the formulation for H_s in Appendix 3, first note that

$$l = \mathbf{r}_0 \cdot \mathbf{i}'$$

$$m = \mathbf{r}_0 \cdot \mathbf{j}'$$

$$n = \mathbf{r}_0 \cdot \mathbf{k}'$$

and

$$l_1 = (\mathbf{H}_E/H_E)\cdot\mathbf{i}'$$

$$m_1 = (\mathbf{H}_E/H_E)\cdot\mathbf{j}'$$

$$n_1 = (\mathbf{H}_E/H_E)\cdot\mathbf{k}'.$$

Then express $\mathbf{r}_0$, $(\mathbf{H}_E/H_E)$ and the unit vectors $\mathbf{i}'$, $\mathbf{j}'$ and $\mathbf{k}'$ in terms of the unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ and take the indicated dot products. From Appendix 3,

$$\mathbf{r}_0 = \mathbf{i}\cdot(\cos\Omega\cdot\sin\alpha) + \mathbf{j}\cdot(\cos\Omega\cdot\cos\alpha) - \mathbf{k}\cdot\sin\Omega$$

and

$$\mathbf{H}_E/H_E = \mathbf{j}\cdot\cos\Phi + \mathbf{k}\cdot\sin\Phi.$$

To express $\mathbf{i}'$, $\mathbf{j}'$ and $\mathbf{k}'$ in terms of the unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, note that the unprimed coordinate system can be transformed to the primed coordinate system by two rotations that are defined as follows: First, rotate a coordinate system that is coincident with the unprimed coordinate system about the z-axis through the angle $(\phi - \pi/2)$ with positive angles clockwise (left hand rule) so that its x-axis is parallel to and in the direction of the relative track. Next, rotate this system about its x-axis through an angle δ with positive angles counterclockwise (right hand rule) so that the positive z-axis passes through the CPA. The angle δ is related to the vertical separation z_0 of the magnetometer and the dipole and the algebraic encounter lateral range L that is positive if the dipole is to the left of the relative track. With these sign definitions: $L = R\cdot\sin\delta$ and $z_0 = R\cdot\cos\delta$. After the rotation, the auxiliary coordinate system is coincident with the primed coordinate system.

These transformations can be described in terms of matrix equations as follows: Let (x'', y'', z'') be the coordinates of a point in the coordinate system that is coincident with the auxiliary coordinate system after the first rotation. Then the transformation from the unprimed coordinate system to this double primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} = \begin{vmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} .$$

And the transformation from the double primed coordinate system to the primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} .$$

Taking the product of the rotation matrices in the indicated order yields the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\delta\cdot\cos\phi & \cos\delta\cdot\sin\phi & \sin\delta \\ \sin\delta\cdot\cos\phi & -\sin\delta\cdot\sin\phi & \cos\delta \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

which defines the transformation from the unprimed to the primed coordinate system. The unprimed vector components of the unit vector **i'** can be found by transforming the coordinates (1,0,0) in the primed system to their corresponding coordinates in the unprimed system with the inverse of this matrix and then repeating this process for (0,1,0) and (0,0,1) in order to find the unprimed unit vector components of **j'** and **k'**. However, since the inverse transformation matrix is the transpose of this matrix, the elements of the row that corresponds to a primed unit vector are the magnitudes of the unprimed vectors that are its components. Consequently:

$$\mathbf{i}' = \mathbf{i}\cdot\sin\phi + \mathbf{j}\cdot\cos\phi$$

$$\mathbf{j}' = -\mathbf{i}\cdot\cos\cdot\delta\cdot\cos\phi + \mathbf{j}\cdot\cos\delta\cdot\sin\phi + \mathbf{k}\cdot\sin\delta$$

$$\mathbf{k}' = \mathbf{i}\cdot\sin\delta\cdot\cos\phi - \mathbf{j}\cdot\sin\delta\cdot\sin\phi + \mathbf{k}\cdot\cos\delta.$$

Then, taking the dot products between $\mathbf{r}$, $\mathbf{r}_0$ and these three unit vectors as indicate above gives:

$$l = \cos\Omega\cdot\cos(\phi - \alpha)$$

$$m = \cos\delta\cdot\cos\Omega\cdot\sin(\phi - \alpha) - \sin\delta\cdot\sin\Omega$$

$$n = -\sin\delta\cdot\cos\Omega\cdot\sin(\phi - \alpha) - \cos\delta\cdot\sin\Omega$$

and

$$l_1 = \cos\Phi\cdot\cos\phi$$

$$m_1 = \cos\delta\cdot\cos\Phi\cdot\sin\phi - \sin\delta\cdot\sin\Phi$$

$$n_1 = -\sin\delta\cdot\cos\Phi\cdot\sin\phi - \cos\delta\cdot\sin\Phi.$$

These are the relations that are used in the program to determine values for the Anderson coefficients.

Appendix 5. The Encounter Equations of Motion

In the double primed coordinate system that is defined in Appendix 4, the equations of motion of a magnetometer relative to a submarine (dipole) are:

$$x''(t) = s'(t)$$

$$y''(t) = -L$$

$$z''(t) = z_0$$

where L is the algebraic encounter lateral range that is defined in Appendix 4, z_0 is the vertical separation between the magnetometer and the submarine and $s'(t)$ is the distance of the magnetometer from the CPA on the relative track. With w the speed of the magnetometer relative to the submarine and t a relative time parameter, $s'(t) = w \cdot t$. These equations can be considered to be the ones used in the program to describe the motion of a magnetometer relative to a submarine. There, t is determined by $t = [J - (M-1)/2] \cdot \delta t$ where the index $J = 1, 2, \cdots, M$ and δt, a time step, is the time between samples. Note, when $t = 0$, the magnetometer is at the CPA.

In the coordinate system of Figure 1 in a straight line encounter as defined in the encounter models, the equations of motion of a magnetometer relative to a submarine can be written as follows:

$$x(t) = s'(t) \cdot \sin \phi \; + \; L \cdot \cos \phi$$

$$y(t) = s'(t) \cdot \cos \phi \; - \; L \cdot \sin \phi$$

$$z(t) = z_0,$$

since the transformation from the double primed coordinate system to the primed coordinate system is determined by the matrix equation

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \sin\phi & -\cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix},$$

$x'' = s'(t)$ and $y'' = -L$. The above equations and the expression for H_S in Appendix 3 could have been used in the program to evaluate the magnetic signal. In particular, with these equations of motion and with the two relations $L = R\cdot\sin\delta$ and $z_0 = R\cdot\cos\delta$, the expression for H_S in Appendix 3 can be written in terms of Ω, α, Φ, ϕ, δ and R so that it is identical in appearance to the Anderson formulation for H_S in terms of these quantities. The definition of δ in Appendix 4 in terms of a counterclockwise rotation results in a definition of the algebraic lateral range that is consistent with some that have been used elsewhere.

In the program, the relation $\mathbf{w} = \mathbf{v} - \mathbf{u}$ is the basis for determining the relative speed w. In this relation, $\mathbf{v}$ is the velocity of the magnetometer, $\mathbf{u}$ is the velocity of the submarine (dipole) and $\mathbf{w}$ is the velocity of the magnetometer relative to the submarine. This relation implies the following equations: $w_x = v_x - u_x$, $w_y = v_y - u_y$ and $w_z = v_z - u_z$ where the coordinates x, y and z refer to a fixed coordinate system with the same orientation as that of Figure 1. In the encounters of the models, $v_x = v \cdot \sin \sigma$, $v_y = v \cdot \cos \sigma$ and $v_z = 0$ where σ is the magnetic course and v is speed of the magnetometer. And, in addition, $u_x = u \cdot \sin \beta$, $u_y = u \cdot \cos \beta$ and $u_z = 0$ where β is the magnetic course and u is the speed of the submarine. The relative magnetic course ϕ and the relative speed of the magnetometer are defined by $w_x = w \cdot \sin \phi$ and $w_y = w \cdot \cos \phi$. In the program, ϕ and w are determined with a rectangular to polar conversion routine where $w = (w_x^2 + w_y^2)^{1/2}$ and where ϕ is determined by $\sin^{-1}(w_x/w)$ and $\cos^{-1}(w_y/w)$.

Appendix 6. The Submarine Magnetic Dipole

In the encounter models, the magnitude and direction of a submarine's dipole moment **p** are determined by first determining its components in the rectangular coordinate system of Figure 1. In that coordinate system,

$$p_x = (p_{LP} + p_{LI})\cdot\sin\beta + (p_{TP} + p_{TI})\cdot\cos\beta$$

$$p_y = (p_{LP} + p_{LI})\cdot\cos\beta - (p_{TP} + p_{TI})\cdot\sin\beta$$

$$p_z = -(p_{VP} + p_{VI})$$

where β is the submarine's magnetic course, p_{LP}, p_{TP} and p_{VP} are the permanent and p_{LI}, p_{TI} and p_{VI} are the induced longitudinal, transverse and vertical magnetic dipole moments of the submarine. These relations are based on the following sign convention:

p_L is positive when $\mathbf{p}_L$ is directed from stern to bow.

p_T is positive when $\mathbf{p}_T$ is directed from port to starboard.

p_V is positive when $\mathbf{p}_V$ is directed downward.

The permanent dipole moments are input parameters in the program and the induced dipole moments are determined in a way that is similar to one that is described in Reference 1. In the encounter models, a submarine is a ferromagnetic prolate ellipsoid with the major axis the submarine's longitudinal axis and the equal minor axes the submarine's transverse and vertical axes. And, the induced dipole moments are:

$$\mathbf{p}_{LI} = k_L\cdot\mathbf{H}_{EL}$$

$$\mathbf{p}_{TI} = k_T\cdot\mathbf{H}_{ET}$$

$$\mathbf{p}_{VI} = k_V\cdot\mathbf{H}_{EV}$$

where $\mathbf{H}_{EL}$, $\mathbf{H}_{ET}$ and $\mathbf{H}_{EV}$ are the vector components of the earth magnetic field in the rectangular coordinate system defined by the ellipsoid axes and the submarine magnetic moment sign convention. The magnitude of these vector components are:

$$H_{EL} = H_E \cdot \cos \Phi \cdot \cos \beta$$

$$H_{ET} = -H_E \cdot \cos \Phi \cdot \sin \beta$$

$$H_{EV} = H_E \cdot \sin \Phi .$$

The earth magnetic field dip angle Φ and the earth magnetic field magnitude H_E can each be chosen to be input parameters in the program or the can be computed by the program as described in Appendix 7. (Values for Φ and H_E can be found from magnetic data charts, for example, see References 5 and References 6.) By using the above relations,

$$p_x = H_E \cdot \cos \Phi \cdot (k_L - k_T) \cdot \sin \beta \cdot \cos \beta + (p_{LP} \cdot \sin \beta + p_{TP} \cdot \cos \beta)$$

$$p_y = H_E \cdot \cos \Phi \cdot (k_L \cdot \cos^2 \beta + k_T \cdot \sin^2 \beta) + (p_{LP} \cdot \cos \beta - p_{TP} \cdot \sin \beta)$$

$$p_z = -(H_E \cdot k_V \cdot \sin \Phi + p_{VP})$$

In the encounter models, the values of the permeability coefficients k_L, k_T and k_V are related to submarine displacement by the following relations:

$$k_L = f_L \cdot W$$

$$k_T = f_T \cdot W$$

$$k_V = f_V \cdot W$$

where W is the submarine displacement in tons and f_L, f_T and f_V are permeability factors that are determined by a submarine's magnetic characteristics. In the program, the units of H_E are gamma, the units of p are in oersted-centimeter^3, the units of k are oersted-centimeter^3/gamma and the units of f are oersted-centimeter^3/gamma-ton. If the units of H_E were gamma, but the units of p were gamma-foot^3, then the units of k would be foot^3 and the units of f would be foot^3/ton. To convert p in gamma-foot^3 to oersted-centimeter^3 or k in foot^3 to oersted-centimeter^3/gamma or f in foot^3/ton to oersted-centimeter^3/gamma-ton, divide by 3.53. (The program default values for f_L, f_T and f_V are values from Reference 1 in foot^3/ton that have been divided by 3.53 to give values in oersted-centimeter^3/gamma-ton.) To convert p in weber-meter to oersted-centimeter^3, multiply by $[1/(4 \cdot \pi)] \cdot 10^{10}$ and to convert p in ampere-meter^2 to oersted-centimeter^3, multiply by $1 \cdot 10^3$. Note, H_E in oersted equals 10^{-5} times H_E in gamma, H_E in gamma equals H_E in nanotesla and H_E in tesla equals H_E in weber/meter^2, so H_E in gamma equals 10^9 times H_E in weber/meter^2 and H_E in oersted equals 10^4 times H_E in weber/meter^2.

Appendix 7. The Earth Magnetic Field and Dip Angle

An auxiliary magnetic field model is described in this appendix. The model is the basis for a default choice for either the value of the earth magnetic field magnitude parameter H_E or the earth magnetic field dip angle parameter Φ. Relative to encounter model accuracy, the default values should be adequate in most cases.

In the model, the earth magnetic field is generated by a magnetic dipole that is located at the earth's center and the earth is a nonmagnetic sphere of radius r_E. With p_E the magnitude of the dipole moment, the magnitude of the earth field at a point is

$$H_E = (\mathbf{H}_E \cdot \mathbf{H}_E)^{1/2} = (c \cdot p_E / r^3) \cdot (3 \cdot \cos^2\theta + 1)^{1/2}$$

where θ is the polar angle of the point in a spherical coordinate system and c is a constant whose value is determined by the choice of units. The dipole moment is coincident with and in the direction of the polar axis which is directed toward the earth's southern hemisphere. In this coordinate system, at any point on the surface of the earth:

$$H_E = H_{E0} \cdot (3 \cdot \cos^2\theta + 1)^{1/2}$$

where H_{E0} is the value of H_E at the magnetic equator which is defined by the points on the earth where $\theta = 90°$. In terms of the dip angle, at any point on the earth's surface:

$$H_E = 2 \cdot H_{E0} \cdot (3 \cdot \cos^2\Phi + 1)^{-1/2}.$$

This expression can be obtained by noting that ϕ can be defined in terms of the or the r and θ components of H_t in the spherical coordinates as follows:

$$\sin \phi = -H_{tr}/H_t = -2 \cdot H_{t0} \cdot (\cos \theta)/H_t$$

and

$$\cos \phi = H_{t\theta}/H_t = H_{t0} \cdot (\sin \theta)/H_t .$$

Based on these relations,

$$[(\sin \phi)/2]^2 + \cos^2\phi = (H_{t0})^2$$

and

$$H_t = H_{t0} \cdot [(\sin^2)/4 + \cos^2\phi]^{1/2} = 2 \cdot H_{t0} \cdot (3 \cdot \cos^2\phi + 1)^{1/2}$$

The dip angle is determined from a magnetic latitude for the encounter region.

The magnetic latitude and longitude of the representative point of an encounter region can be defined in terms of its geographic latitude and longitude by using the following transformations: First, convert the latitude and longitude of the point to rectangular coordinates in a right-handed coordinate system whose origin is at the center of a spherical earth, whose positive z-axis passes through its north geographic pole and whose positive y-axis passes through the point on its equator with latitude 0° and longitude 0°. Next, rotate a coordinate system that is coincident with this system in a clockwise direction (left hand rule) about its z-axis so that its positive y-axis passes through the point with latitude 0° and with longitude equal to that of the north magnetic pole. Then, rotate the system in a clockwise direction about its x-axis so that its positive z-axis passes through the north magnetic pole. Next, transform the rectangular coordinates of the representative point in this system to its coordinates in the spherical coordinate system that is associated with it. Then, with Ω the polar angle of the representative point in this system, the magnetic latitude of the representative point is $L_M = 90° - \Omega$. Since $\tan \Phi = -2 \cdot \cot \Theta$ from above and $\Theta = 180° - \Omega$, the dip angle Φ is given by the following relation:

$$\Phi = \tan^{-1}(2 \cdot \tan L_M).$$

In the program, the transformations described above are accomplished in part by a rectangular to polar conversion routine. In particular, by rotating the final rectangular coordinate system about its z-axis so that the x-coordinate is zero, the polar angle of the polar coordinates of the representative point in the resulting yz-plane determines L_M.

Some values of Φ and H_E that are listed in Table 1 were generated using the program. The latitude and longitude of the magnetic pole that are in the program are 76° N and 100° W. Values of Φ from Reference 6 and of H_E from Peference 7 are also listed in Table 1. A comparison of the program values with these values gives an indication of the errors inherent in the procedure.

Encounter Latitude	Encounter Longitude	Program Dip Φ	Chart #30 Dip Φ	Program Field H_E	Chart #39 Field H_E
60°N	180°W	74°	70°	.63	.53
60°N	150°W	78°	73°	.66	.55
60°N	90°W	82°	84°	.68	.61
60°N	30°W	75°	74°	.64	.52
30°N	150°W	58°	50°	.51	.41
30°N	90°W	62°	60°	.55	.51
30°N	60°W	59°	59°	.53	.48
30°N	30°E	37°	43°	.41	.42
30°N	60°E	31°	45°	.39	.46
30°N	90°E	30°	44°	.39	.44
30°N	150°E	42°	40°	.43	.40
0°	90°W	26°	18°	.38	.32
0°	30°W	9°	-5°	.35	.28
0°	0°	-48°	-25°	.35	.31
0°	60°E	-25°	-20°	.38	.37
30°S	180°W	-45°	-55°	.44	.50
30°S	90°W	-30°	-34°	.39	.31
30°S	90°E	-62°	-66°	.55	.52
60°S	30°W	-69°	-56°	.60	.34

Table 1. Some corresponding values for the encounter dip angle Φ and the earth magnetic field intensity H_E in oersted. Chart #33 refers to Reference 5 and Chart #39 refers to Reference 6.

A small difference between a chart value and a computed value for the dip angle Φ at some location does not imply a small difference between the chart value and the computed value for the earth magnetic field intensity at that location. For example, note the values for 60°N latitude and 30°W longitude. (Positive values of Φ indicate the inclination or dip below the horizontal of the north seeking end of a dip needle. Negative values indicate the inclination below the horizontal of the south seeking end.)

An extension of the above procedure can be made for finding the magnetic variation at a location. However, the values generated by using the procedure are generally unsatisfactory. Magnetic variation values are charted in Reference 7.

Appendix 6. An Alternative Encounter Model

The alternative model that is referred to in Section III is described in more detail in this appendix. In the model, the detection range is the slant range R at the CPA for an encounter with a specified detection probability (usually .5) is defined by:

$$R = [c \cdot p/H_s]^{1/3}$$

where c and p are defined in Appendix 3 and H_s represents a minimum detectable average magnetic signal that is defined by:

$$H_S = (ORF) \cdot N_M$$

where ORF is a signal-to-noise ratio called the operator recognition factor and N_M represents the magnetic noise. Combining these two relations gives:

$$R = \{c \cdot p/[(ORF) \cdot N_M]\}^{1/3}.$$

The value for ORF depends both on the specified detection probability and on a specified or implied false alarm probability.

The Anderson formulation is consistent with these relations in an approximate sense if the average magnetic signal H is defined as a root mean square value such that $H = (c \cdot p/R^3) \cdot k$ where k is an encounter parameter defined by:

$$k = \{\Sigma\ [\Sigma\ A_J \cdot F_J(\beta_K)]^2\}^{1/2}$$

with the first sum index $K = 1, 2, \cdots, M$ and the second sum index $J = 0, 1, 2$. For a particular encounter geometry, k is constant and this suggests that the two encounter models could be used to determine an average value for k for an encounter region based on average submarine magnetic characteristics. Values for both k and H are generated by the program and such values can give an indication of the magnitude of the differences in detection range estimates that are based on this model and those that are based on either of the other two encounter models. A more detailed comparison of these encounter models is described in Reference 8.

Appendix 9. An Example of the Program Output

The program is designed to generate values for the following quantities: encounter parameters, lateral range functions for the crosscorrelation encounter model and for the square law encounter model, average magnetic signal, slant range at CPA, encounter parameter values and magnetic signal values. The values can be saved as a program data file and/or they can be printed. Tables 2 through 7 are examples of the program's printed output.

Figures 3 and 5 are plots of the lateral range function values that are listed in Tables 3 and 6 for the square law encounter model. For either the square law or the crosscorrelation encounter model, as the magnitude of the lateral range increases, lateral range function values (detection probabilities) do not approach zero but approach the value of the encounter false alarm probability. A necessary condition for the formal definition of sweep width given in Reference 9 to be meaningful is that the lateral range function approach zero as the lateral range increases without limit. Since this is not the case for either encounter model, the formal definition of sweep width must be modified with their use. Reference 10 provides an example of this.

Figures 4 and 6 are plots of the magnetic signal values that are listed in Tables 4 and 7.

program file name	MAD.BAS
program data file name	data.mad
magnetic data file name	data.mag
processing data file name	data.pro
kinematic data file name	data.kin
combined magnetic, processing & kinematic data file name	data.mpk
encounter latitude (decimal degrees)	45
encounter longitude (decimal degrees)	-45
encounter variation (decimal degrees)	-25
encounter dip angle (decimal degrees)	68.41608
encounter magnetic field intensity (gamma)	59035.05
permanent longitudinal moment (oersted-cm3)	0
permanent transverse moment (oersted-cm3)	0
permanent vertical moment (oersted-cm3)	0
target displacement (tons)	1800
target longitudinal permeability coefficient	13140
target transverse permeability coefficient	2880
target vertical permeability coefficient	2880
target longitudinal permeability factor	7.3
target transverse permeability factor	1.6
target vertical permeability factor	1.6
sampling period (seconds)	.5
integration time (seconds)	20
adjusted integration time (seconds)	20.5
number of samples per encounter	41
magnetometer course (decimal degrees)	290
magnetometer speed (knots)	180
magnetometer altitude (meters)	100
target course (decimal degrees)	20
target speed (knots)	10
target depth (meters)	100
magnetometer relative course (decimal degrees)	286.8202
magnetometer relative speed (knots)	180.2776
magnetometer-target vertical separation (meters)	200
target induced longitudinal dipole moment (oersted-cm3)	2.017795E+08
target induced transverse dipole moment (oersted-cm3)	-4.422564E+07
target induced vertical dipole moment (oersted-cm3)	1.58099E+08
magnetic dipole moment (oersted-cm3)	2.601272E+08
dipole moment azimuth (decimal degrees)	7.637512
dipole moment depression angle (decimal degrees)	37.42885
distance between samples on the relative track (meters)	46.3714
false alarm rate (false alarms per hour)	3
false alarm probability	1.666667E-02
magnetic noise (gamma)	.35
maximum lateral range (meters)	500
lateral range step (meters)	20
number of lateral range function values	51

Table 2. An example of an encounter parameter values printout.

[illegible] lateral range function values

L meters	[illegible]	p(sl)	H gamma	R meters	k
-500	.2155959	.0255824	.184247	538.5165	2.818488
-480	.2721803	.029692	.21034	520	2.879916
-460	.3464673	3.582633E-02	.2410144	501.5974	2.943744
-440	.4415205	4.529992E-02	.2785504	483.3218	3.009938
-420	.5573696	6.047795E-02	.3231911	465.1881	3.078418
-400	.6872897	.0856887	.3764825	447.2136	3.149025
-380	.8146549	.1287804	.4404623	429.4182	3.221497
-360	.9159235	.2030898	.5177304	411.8252	3.29544
-340	.9745625	.3272475	.6108596	394.4617	3.370274
-320	.9958122	.5141411	.7233319	377.3593	3.445185
-300	.9997211	.7388175	.8597726	360.5551	3.519032
-280	.9999951	.9177597	1.025158	344.093	3.590261
-260	1	.9896971	1.224529	328.0244	3.656775
-240	1	.999714	1.463644	312.41	3.71578
-220	1	.9999992	1.74981	297.3214	3.763606
-200	1	1	2.085394	282.8427	3.795517
-180	1	1	2.470496	269.0725	3.805514
-160	1	1	2.901352	256.125	3.786234
-140	1	1	3.359555	244.1311	3.72898
-120	1	1	3.819509	233.2381	3.624058
-100	1	1	4.239317	223.6068	3.461596
-80	1	1	4.532758	215.4066	3.232958
-60	1	1	4.618943	208.8061	2.932828
-40	1	1	4.44342	203.9608	2.5617
-20	1	1	3.97739	200.9975	2.128285
0	1	1	3.332139	200	1.651697
20	1	.9999993	2.540665	200.9975	1.165581
40	.99999[illegible]	.908713	1.76834	203.9608	.7394171
60	.9933111	.467368	1.110188	208.8061	.5637183
80	.9998733	.7874625	.9039734	215.4066	.7782836
100	.9999999	.9856169	1.086971	223.6068	1.134662
120	1	.9992399	1.254863	233.2381	1.490748
140	1	.9998478	1.308583	244.1311	1.811734
160	1	.9998431	1.29482	256.125	2.089946
180	1	.9994995	1.230946	269.0725	2.326072
200	1	.9974561	1.131703	282.8427	2.523877
220	1	.9872744	1.017684	297.3214	2.688154
240	.999999	.9510013	.9018685	312.41	2.823785
260	.9999747	.8646194	.7917129	328.0244	2.935307
280	.9996544	.7242736	.6909791	344.093	3.026728
300	.9973592	.5582789	.6011183	360.5551	3.10149
320	.9876337	.4037092	.5222269	377.3593	3.162495
340	.9608865	.2816961	.4536729	394.4617	3.212155
360	.9086407	.1948741	.3944811	411.8252	3.252463
380	.8301703	.1365131	.3443362	429.4182	3.285058
400	.7333605	9.820431E-02	.3014853	447.2136	3.311281
420	.6299387	.0731037	.2645045	465.1881	3.332228
440	.5302704	5.646636E-02	.2325951	483.3218	3.348799
460	.4408869	4.522933E-02	.205043	501.5974	3.361725
480	.3645446	3.747106E-02	.1812233	520	3.371603
500	.3013791	3.199235E-02	.1605954	538.5165	3.378925

Table 3. An example of a lateral range function values printout. The heading for the crosscorrelation values is p(cc) and the heading for the square law values is p(sl).

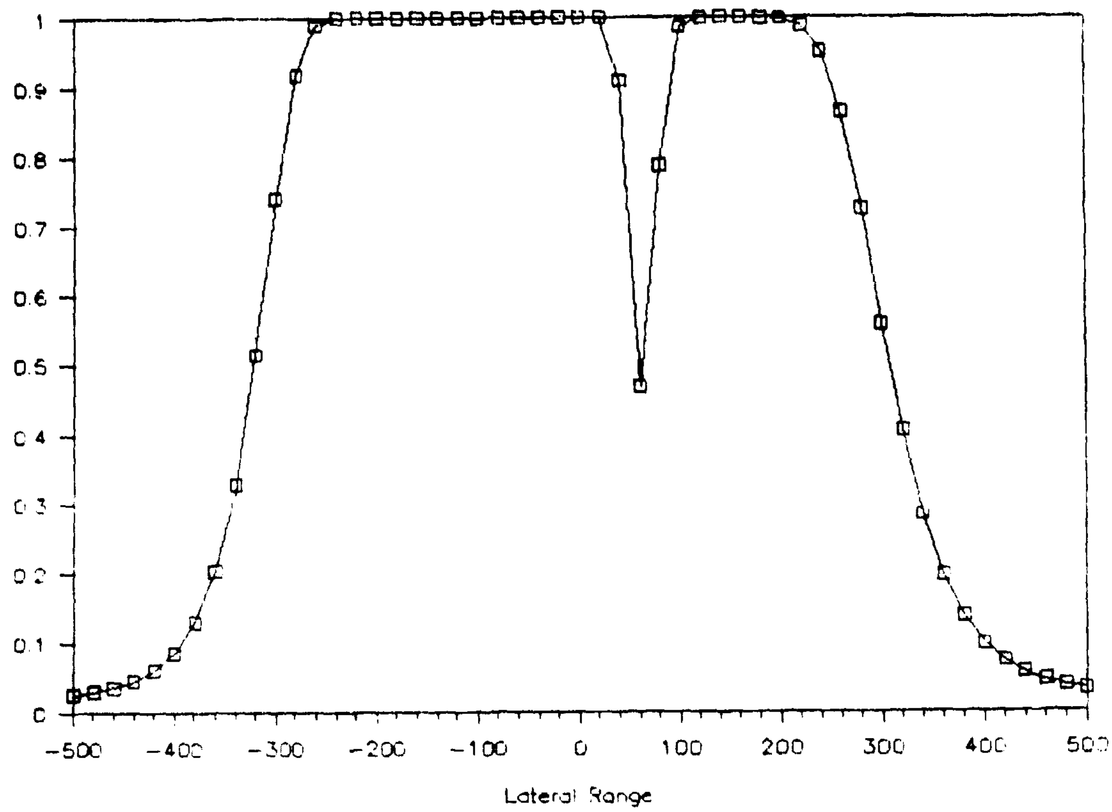

Figure 3. A plot of the square law lateral range function values that are listed in Table 3. The horizontal axis is encounter horizontal range at CPA in meters. The vertical axis is encounter detection probability.

data.mad magnetic signal values
for a lateral range of 40 meters

relative CPA distance s in meters	magnetic signal Hs in gamma
-927.4282	-1.619177E-02
-881.0568	-1.834315E-02
-834.6854	-2.086144E-02
-788.314	-2.381637E-02
-741.9426	-2.728804E-02
-695.5712	-.0313641
-649.1998	-3.613118E-02
-602.8283	-4.165412E-02
-556.4569	-4.792992E-02
-510.0855	-5.478963E-02
-463.7141	-6.169327E-02
-417.3427	-6.730653E-02
-370.9713	-.0686387
-324.5999	-5.933568E-02
-278.2285	-2.652314E-02
-231.8571	.0540608
-185.4856	.2215785
-139.1142	.5163918
-92.74282	.9104893
-46.37141	1.184586
0	1.003207
46.37141	.3837067
92.74282	-.2252113
139.1142	-.5323231
185.4856	-.5837545
231.8571	-.5168925
278.2285	-.4202569
324.5999	-.3304064
370.9713	-.2572838
417.3427	-.2007385
463.7141	-.1577988
510.0855	-.125285
556.4569	-.1005532
602.8283	-8.158605E-02
649.1998	-6.689598E-02
695.5712	-5.539922E-02
741.9426	-4.630719E-02
788.314	-3.904362E-02
834.6854	-3.318428E-02
881.0568	-.0284142
927.4282	-2.449729E-02

Table 4. An example of a magnetic signal values printout.

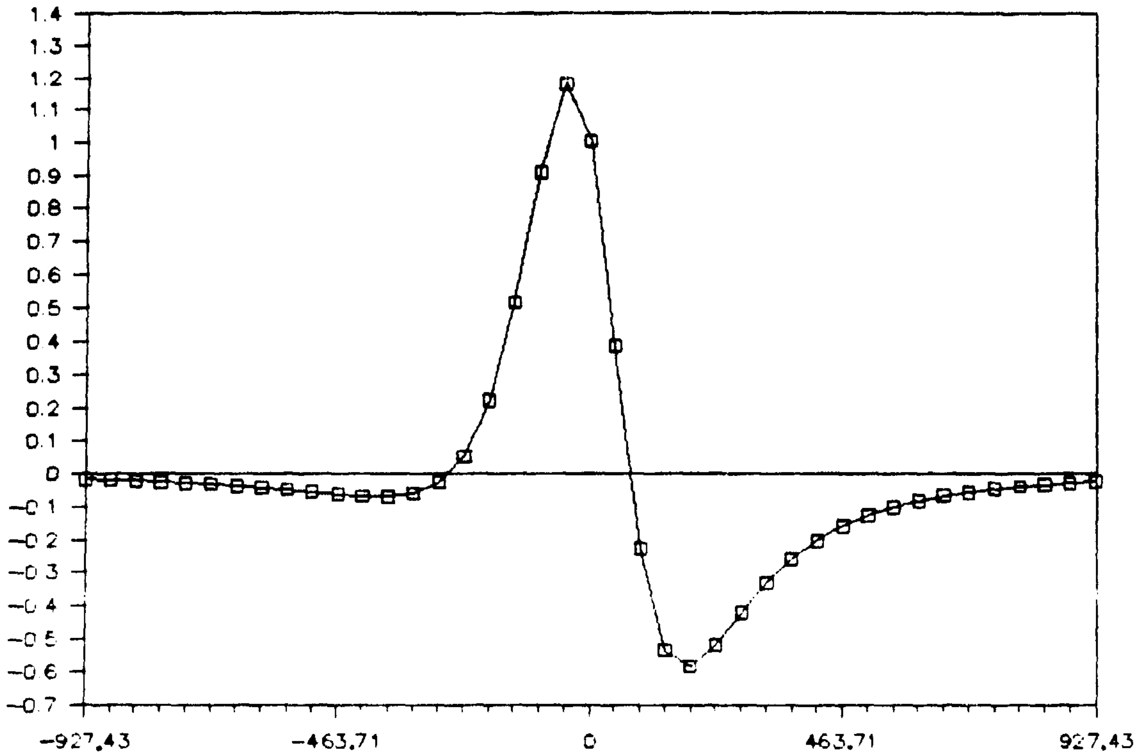

Figure 4. A plot of the magnetic signal values listed in Table 4. The horizontal axis is relative CPA distance in meters. The vertical axis is magnetic signal in gamma.

program file name	MAD.BAS
program data file name	data1.mad
magnetic data file name	data.mag
processing data file name	data.pro
kinematic data file name	data1.kin
combined magnetic, processing & kinematic data file name	data1.mpk
encounter latitude (decimal degrees)	45
encounter longitude (decimal degrees)	-45
encounter variation (decimal degrees)	-35
encounter dip angle (decimal degrees)	68.41608
encounter magnetic field intensity (gamma)	59038.05
permanent longitudinal moment (oersted-cm3)	0
permanent transverse moment (oersted-cm3)	0
permanent vertical moment (oersted-cm3)	0
target displacement (tons)	1800
target longitudinal permeability coefficient	13140
target transverse permeability coefficient	2880
target vertical permeability coefficient	2880
target longitudinal permeability factor	7.3
target transverse permeability factor	1.6
target vertical permeability factor	1.6
sampling period (seconds)	.5
integration time (seconds)	20
adjusted integration time (seconds)	20.5
number of samples per encounter	41
magnetometer course (decimal degrees)	290
magnetometer speed (knots)	180
magnetometer altitude (meters)	100
target course (decimal degrees)	273
target speed (knots)	10
target depth (meters)	100
magnetometer relative course (decimal degrees)	290.9828
magnetometer relative speed (knots)	170.462
magnetometer-target vertical separation (meters)	200
target induced longitudinal dipole moment (oersted-cm3)	1.339682E+08
target induced transverse dipole moment (oersted-cm3)	5.522353E+07
target induced vertical dipole moment (oersted-cm3)	1.58099E+08
magnetic dipole moment (oersted-cm3)	2.144584E+08
dipole moment azimuth (decimal degrees)	295.4022
dipole moment depression angle (decimal degrees)	47.49356
distance between samples on the relative track (meters)	43.84662
false alarm rate (false alarms per hour)	3
false alarm probability	1.666667E-02
magnetic noise (gamma)	.35
maximum lateral range (meters)	500
lateral range step (meters)	20
number of lateral range function values	51

Table 5. An example of an encounter parameter values printout.

data1.mad lateral range function values

L meters	p(cc)	p(sl)	H gamma	R meters	W
-500	8.718863E-02	1.815807E-02	.1162113	538.5165	2.11[illegible]
-480	.1097076	1.887764E-02	.1307483	520	2.088194
-460	.1282767	1.978802E-02	.1476663	501.5974	2.01[illegible]
-440	.1449347	2.088804E-02	.1677343	483.3218	1.97[illegible]
-420	.1701671	2.286687E-02	.1918806	465.1881	1.930378
-400	.2031047	2.473001E-02	.2196898	447.2136	1.89458
-380	.2467793	2.779396E-02	.2543082	429.4182	1.865873
-360	.3053632	3.231674E-02	.2951223	411.8252	1.845898
-340	.3841007	3.932704E-02	.3461031	394.4617	1.836884
-320	.4881344	5.082670E-02	.4088031	377.3593	1.840855
-300	.6188469	7.091362E-02	.4830141	360.5551	1.858016
-280	.7643011	.1082717	.5703262	344.093	1.89408
-260	.8944848	.1810074	.6693423	328.0244	1.947158
-240	.9731281	.3214172	.8450639	312.41	2.019043
-220	.9973629	.5584107	1.022772	297.3214	2.109683
-200	.9998472	.8329310	1.262045	282.8427	2.217957
-180	.9999999	.9784214	1.555326	269.0725	2.341425
-160	1	.9996887	1.906387	256.125	2.476081
-140	1	.9999999	2.315886	244.1311	2.616057
-120	1	1	2.774988	233.2381	2.753399
-100	1	1	3.289594	223.6068	2.878014
-80	1	1	3.848771	215.4066	2.978009
-60	1	1	4.384607	208.8061	3.040665
-40	1	1	4.738054	203.9608	3.054164
-20	1	1	4.921412	200.9975	3.008893
0	1	1	4.862248	200	2.904766
20	1	1	4.561756	200.9975	2.742697
40	1	1	4.06876	203.9608	2.534609
60	1	1	3.462592	208.8061	2.296925
80	1	1	2.876776	215.4066	2.049263
100	1	.9999861	2.341757	223.6068	1.812213
120	1	.9908414	1.862014	233.2381	1.60562
140	.9999632	.8492265	1.463262	244.1311	1.446881
160	.9973499	.55795	1.150695	256.125	1.348197
180	.9773154	.3394268	.9187382	269.0725	1.31296
200	.9311967	.2232309	.7327372	282.8427	1.334314
220	.8711751	.1618173	.5979505	297.3214	1.39856
240	.8071772	.1253278	.4948332	312.41	1.49089
260	.7414287	.1006817	.4163041	328.0244	1.599193
280	.6739562	8.250537E-02	.3566572	344.093	1.714996
300	.6054811	.0684997	.3130742	360.5551	1.832886
320	.5376865	5.753178E-02	.2806611	377.3593	1.949611
340	.4726234	4.891159E-02	.2530364	394.4617	2.063321
360	.4121077	4.213673E-02	.2280127	411.8252	2.173046
380	.3573765	3.681085E-02	.2054994	429.4182	2.27835
400	.3090144	3.261634E-02	.1853368	447.2136	2.379115
420	.2670518	.0293013	.1673323	465.1881	2.475412
440	.2311289	2.666843E-02	.151282	483.3218	2.567413
460	.2006615	2.456539E-02	.1369845	501.5974	2.655341
480	.1749714	.0228749	.12437	520	2.739442
500	.1533755	2.150718E-02	.1132715	538.5165	2.819965

Table 6. An example of a lateral range function values printout. The heading for the crosscorrelation values is p(cc) and the heading for the square law values is p(sl).

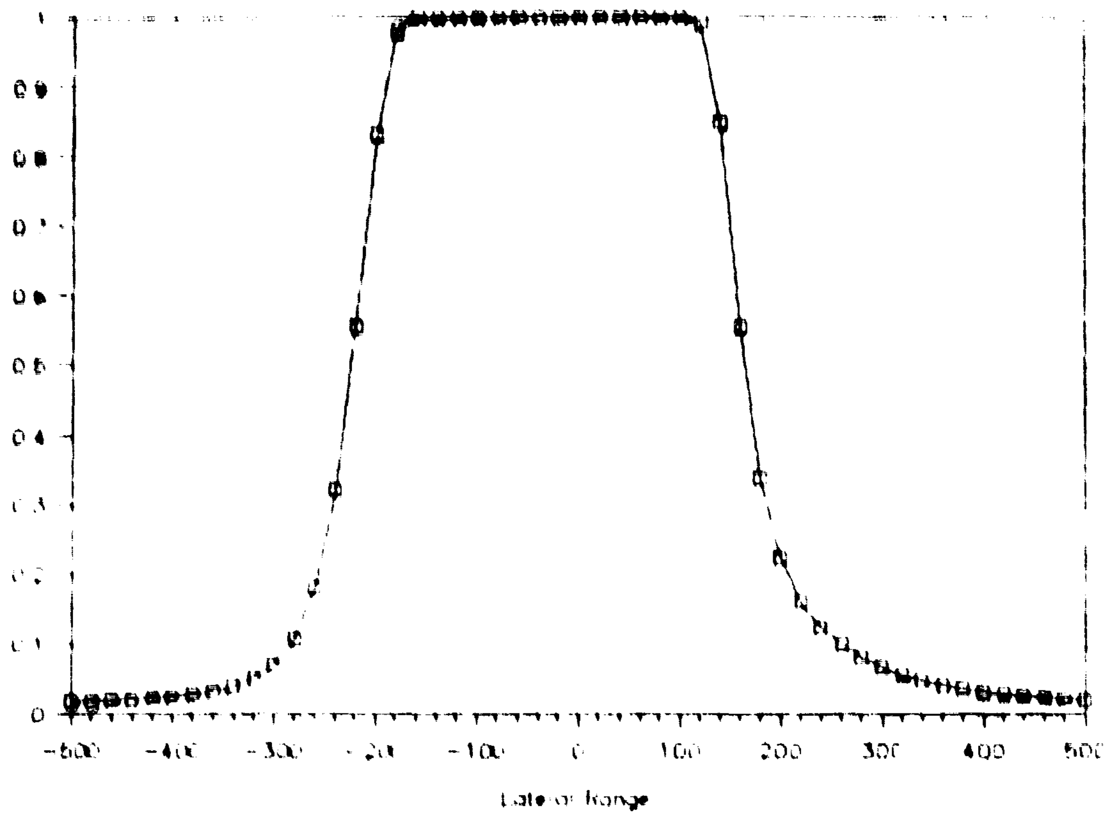

Figure 5. A plot of the square law lateral range function values that are listed in Table 6. The horizontal axis is encounter horizontal range at CPA in meters. The vertical axis is encounter detection probability.

data1.mad magnetic signal values
for a lateral range of 40 meters

relative CPA distance s in meters	magnetic signal Hs in gamma
-876.9324	6.869243E-03
-833.0858	8.988515E-03
-789.2391	1.181241E-02
-745.3925	1.561336E-02
-701.5459	2.078687E-02
-657.6993	2.791575E-02
-613.8527	3.787262E-02
-570.006	5.198577E-02
-526.1594	7.231193E-02
-482.3128	.1020894
-438.4662	.1464967
-394.6196	.2139163
-350.7729	.317984
-306.9263	.480659
-263.0797	.735856
-219.2331	1.130141
-175.3865	1.707647
-131.5399	2.448238
-87.69324	3.132554
-43.84662	3.263226
0	2.439828
43.84662	1.023829
87.69324	-.1487533
131.5399	-.7045798
175.3865	-.8055338
219.2331	-.7078842
263.0797	-.5631492
306.9263	-.4307948
350.7729	-.3256765
394.6196	-.2465582
438.4662	-.1881302
482.3128	-.1451051
526.1594	-.1132614
570.006	-8.948089E-02
613.8527	-7.152954E-02
657.6993	-5.782279E-02
701.5459	-4.723642E-02
745.3925	-3.898839E-02
789.2391	-3.244173E-02
833.0858	-2.723733E-02
876.9324	-2.304761E-02

Table 7. An example of a magnetic signal values printout.

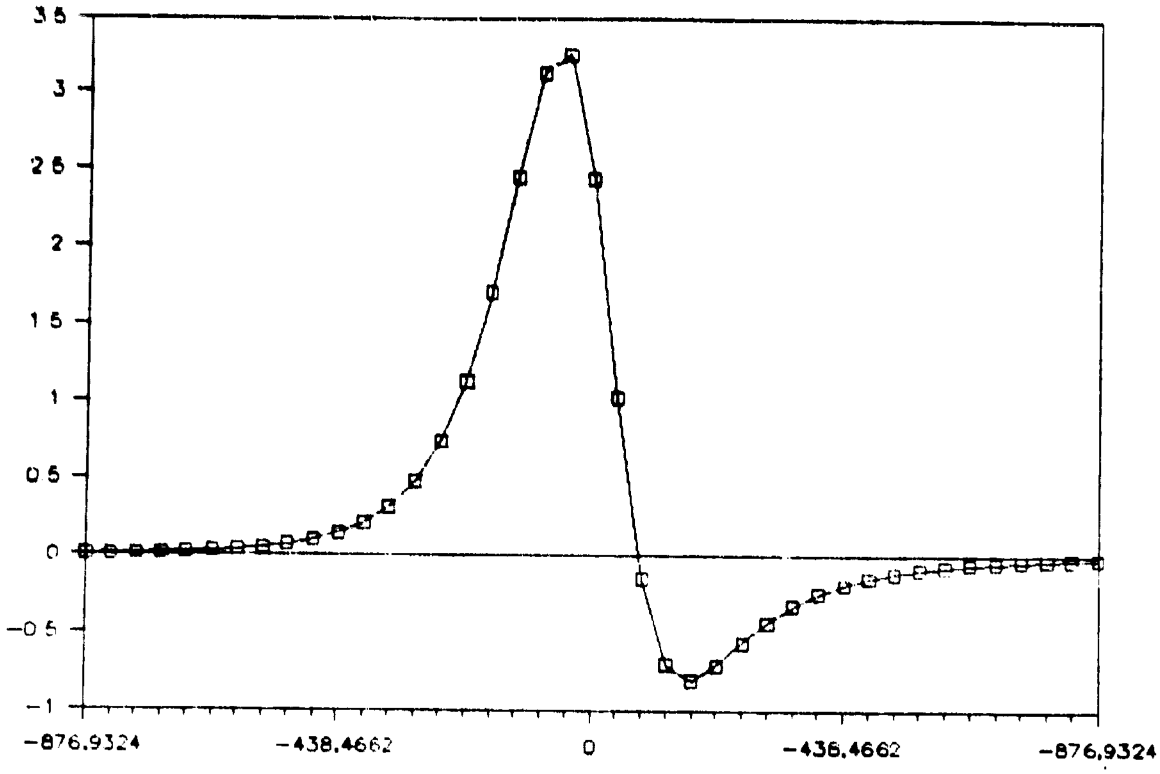

Figure 6. A plot of the magnetic signal values listed in Table 7. The horizontal axis is relative CPA distance in meters. The vertical axis is magnetic signal in gamma.

Appendix 10. A Program Listing

```
10 CLS : CLEAR
20 PRINT "Magnetic Anomaly Detection (MAD) Lateral Range Function Program"
30 DIM X0(70), R0(70), H(70), HS(70, 150), K(70), PDCC(70), PDSL(70)
40 PI = 3.141592654#: CON = 2 * PI / 360: KON = 1852 / 3600: N$ = "MAD.BAS"
50 Q0 = .2316419: Q1 = .31938153#: Q2 = -.356563782#: Q3 = 1.781477937#
60 Q4 = -1.821255978#: Q5 = 1.330274429#
70 I1 = 2.515517: I2 = .802853: I3 = .010328: I4 = 1.432788: I5 = .189269: I6 = .001308
80 PRINT
90 INPUT "generate data or print a program data file (g/p)"; A$
100 IF A$ = "G" OR A$ = "g" THEN GOTO 120
110 IF A$ = "P" OR A$ = "p" THEN GOTO 3070 ELSE GOTO 90
120 PRINT : A$ = "a"
130 INPUT "magnetic, processing & kinematic data entry by combined file (y/n)"; A$
140 IF A$ = "Y" OR A$ = "y" THEN GOTO 2120
150 IF A$ = "N" OR A$ = "n" THEN GOTO 160 ELSE GOTO 130
160 INPUT "magnetic data entry by file/keyboard (f/k)"; A$
170 IF A$ = "F" OR A$ = "f" THEN GOTO 780
180 IF A$ = "K" OR A$ = "k" THEN GOTO 190 ELSE GOTO 160
190 INPUT "latitude in decimal degrees (N +)"; LAT: LATR = LAT * CON
200 INPUT "longitude in decimal degrees (E +)"; LNG: LNGR = -LNG * CON
210 INPUT "variation in decimal degrees (E +)"; DEC: DECR = DEC * CON
220 A$ = "a"
230 INPUT "input dip angle (y/n)"; A$
240 IF A$ = "Y" OR A$ = "y" THEN GOTO 260
250 IF A$ = "N" OR A$ = "n" THEN GOTO 290 ELSE GOTO 230
260 INPUT "dip angle in decimal degrees (north magnetic hemisphere +)"; DIP
270 DIPR = DIP * CON
280 GOTO 390
290 LATPR = 76 * CON: LNGPR = 100 * CON
300 D = SIN(LNGR - LNGPR) * COS(LATR): E = COS(LNGR - LNGPR) * COS(LATR): F = SIN(LATR)
310 X = E: Y = F
320 GOSUB 4170
330 T = T - (90 * CON - LATPR): E = R * SIN(T): F = R * COS(T)
340 X = E: Y = D
350 GOSUB 4170
360 X = F: Y = R
370 GOSUB 4170
380 DIPR = ATN(2 * TAN(T)): DIP = DIPR / CON
390 A$ = "a"
400 INPUT "input encounter magnetic field intensity (y/n)"; A$
410 IF A$ = "Y" OR A$ = "y" THEN GOTO 430
420 IF A$ = "N" OR A$ = "n" THEN GOTO 450 ELSE GOTO 400
430 INPUT "encounter magnetic field intensity in gamma"; HE
440 GOTO 460
450 HE = 70000! / SQR(3 * COS(DIPR) * COS(DIPR) + 1)
460 PLM = 0: PTM = 0: PVM = 0
470 A$ = "a"
480 INPUT "input target permanent dipole moments (y/n)"; A$
490 IF A$ = "Y" OR A$ = "y" THEN GOTO 510
500 IF A$ = "N" OR A$ = "n" THEN GOTO 540 ELSE GOTO 480
510 INPUT "permanent longitudinal moment in oersted-cm3 (stern-to-bow +)"; PLM 520 INPUT "permanent
transverse moment in oersted-cm3 (port-to-starboard +)"; PTM
530 INPUT "permanent vertical moment in oersted-cm3 (downward +)"; PVM
540 INPUT "target displacement in tons"; WT
550 A$ = "a"
560 INPUT "input target permeability coefficients or factors (c/f)"; A$
570 IF A$ = "C" OR A$ = "c" THEN GOTO 590
580 IF A$ = "F" OR A$ = "f" THEN GOTO 630 ELSE GOTO 560
590 INPUT "longitudinal permeability coefficient in cgs units"; KL
600 INPUT "transverse permeability coefficient in cgs units"; KT
610 INPUT "vertical permeability coefficient in cgs units"; KV
620 FL = KL / WT: FT = KT / WT: FV = KV / WT: GOTO 710
630 INPUT "longitudinal displacement factor in cgs units"; FL
```

```
640 INPUT "transverse displacement factor in cgs units"; FT
650 INPUT "vertical displacement factor in cgs units"; FV
660 KL = FL * WT: KT = FT * WT: KV = FV * WT
670 A$ = "a"
680 INPUT "generate a magnetic data file (y/n)"; A$
690 IF A$ = "Y" OR A$ = "y" THEN GOTO 710
700 IF A$ = "N" OR A$ = "n" THEN GOTO 850 ELSE GOTO 680
710 INPUT "magnetic data file name"; M$
720 ON ERROR GOTO 730: GOTO 740
730 RESUME 710
740 OPEN "O", #1, M$
750 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV
760 CLOSE
770 GOTO 850
780 INPUT "magnetic data file name"; M$
790 ON ERROR GOTO 800: GOTO 810
800 RESUME 780
810 OPEN "I", #1, M$
820 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV
830 CLOSE
840 DECR = DEC * CON: DIPR = DIP * CON
850 A$ = "a"
860 INPUT "processing data entry by file/keyboard (f/k)"; A$
870 IF A$ = "F" OR A$ = "f" THEN GOTO 1450
880 IF A$ = "K" OR A$ = "k" THEN GOTO 890 ELSE GOTO 860
890 A$ = "a"
900 INPUT "input sampling period (y/n)"; A$
910 IF A$ = "Y" OR A$ = "y" THEN GOTO 930
920 IF A$ = "N" OR A$ = "n" THEN GOTO 960 ELSE GOTO 900
930 INPUT "sampling period in seconds"; DT
940 IF DT <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 930
950 GOTO 1090
960 A$ = "a"
970 INPUT "input maximum magnetic signal frequency (y/n)"; A$
980 IF A$ = "Y" OR A$ = "y" THEN GOTO 1000
990 IF A$ = "N" OR A$ = "n" THEN GOTO 1030 ELSE GOTO 970
1000 INPUT "maximum magnetic signal frequency in Hertz"; MAXF
1010 IF MAXF <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 1000
1020 GOTO 1080
1030 INPUT "minimum target slant range at CPA in meters"; MINRO
1040 IF MINRO <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 1030
1050 INPUT "maximum magnetometer relative speed in knots"; MAXVMK
1060 MAXVM = MAXVMK * KON: MAXF = 2 * MAXVM / MINRO
1070 IF MAXVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 1050
1080 DT = 1 / (2 * MAXF): REM low pass filter, Nyquist sampling rate
1090 A$ = "a"
1100 INPUT "input integration time (y/n)"; A$
1110 IF A$ = "Y" OR A$ = "y" THEN GOTO 1130
1120 IF A$ = "N" OR A$ = "n" THEN GOTO 1210 ELSE GOTO 1100
1130 INPUT "integration time in seconds"; IT
1140 IF IT >= DT THEN GOTO 1170
1150 PRINT : PRINT "IT = " + STR$(IT) + " seconds     minimum = " + STR$(DT) + " seconds": PRINT
1160 GOTO 1130
1170 NS = 2 * INT(IT / DT / 2) + 1: REM adj number of samples per integration time
1180 IF NS <= 151 THEN GOTO 1340
1190 PRINT : PRINT "IT = " + STR$(IT) + " seconds     maximum = " + STR$(150 * DT) + " seconds": PRINT
1200 GOTO 1130
1210 INPUT "maximum target slant range at CPA in meters"; MAXRO
1220 IF MAXRO <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 1210
1230 INPUT "minimum magnetometer relative speed in knots"; MINVMK
1240 MINVM = MINVMK * KON
1250 IF MINVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO 1230
1260 IT = 2 * MAXRO / MINVM
1270 IF IT >= DT THEN GOTO 1300
1280 PRINT : PRINT "IT = " + STR$(IT) + " seconds     minimum = " + STR$(DT) + " seconds": PRINT
```

```
1290 GOTO 1100
1300 NS = 2 * INT(IT / DT / 2) + 1: REM adjusted number of samples per integration time
1310 IF NS <= 151 THEN GOTO 1340
1320 PRINT : PRINT "IT = " + STR$(IT) + " seconds     maximum = " + STR$(150 * DT) + " seconds": PRINT
1330 GOTO 1100
1340 A$ = "a"
1350 INPUT "generate a processing data file (y/n)"; A$
1360 IF A$ = "Y" OR A$ = "y" THEN GOTO 1380
1370 IF A$ = "N" OR A$ = "n" THEN GOTO 1510 ELSE GOTO 1350
1380 INPUT "processing data file name"; P$
1390 ON ERROR GOTO 1400: GOTO 1410
1400 RESUME 1380
1410 OPEN "O", #1, P$
1420 WRITE #1, DT, IT, NS
1430 CLOSE
1440 GOTO 1510
1450 INPUT "processing data file name"; P$
1460 ON ERROR GOTO 1470: GOTO 1480
1470 RESUME 1450
1480 OPEN "I", #1, P$
1490 INPUT #1, DT, IT, NS
1500 CLOSE
1510 A$ = "a"
1520 INPUT "kinematic data entry by file/keyboard (f/k)"; A$
1530 IF A$ = "F" OR A$ = "f" THEN GOTO 1790
1540 IF A$ = "K" OR A$ = "k" THEN GOTO 1550 ELSE GOTO 1510
1550 INPUT "magnetometer course in decimal degrees (0 if at rest)"; CM
1560 INPUT "magnetometer speed in knots"; VMK
1570 INPUT "magnetometer altitude in meters (below 0 is -)"; AM
1580 INPUT "target course in decimal degrees"; CT
1590 INPUT "target speed in knots"; VTK
1600 INPUT "target depth in meters (above 0 is -)"; AT
1610 CMR = CM * CON: CTR = CT * CON
1620 WXK = VMK * SIN(CMR) - VTK * SIN(CTR)
1630 WYK = VMK * COS(CMR) - VTK * COS(CTR)
1640 Z = AM + AT: REM vertical separation (- for magnetometer below target)
1650 X = WXK: Y = WYK: GOSUB 4170
1660 CR = T: C = CR / CON: REM magnetometer relative course
1670 WK = R: REM magnetometer relative speed
1680 A$ = "a"
1690 INPUT "generate a kinematic data file (y/n)"; A$
1700 IF A$ = "Y" OR A$ = "y" THEN GOTO 1720
1710 IF A$ = "N" OR A$ = "n" THEN GOTO 1860 ELSE GOTO 1690
1720 INPUT "kinematic data file name"; K$
1730 ON ERROR GOTO 1740: GOTO 1750
1740 RESUME 1720
1750 OPEN "O", #1, K$
1760 WRITE #1, CM, VMK, AM, CT, VTK, AT, Z, C, WK
1770 CLOSE
1780 GOTO 1860
1790 INPUT "kinematic data file name"; K$
1800 ON ERROR GOTO 1810: GOTO 1820
1810 RESUME 1790
1820 OPEN "I", #1, K$
1830 INPUT #1, CM, VMK, AM, CT, VTK, AT, Z, C, WK
1840 CLOSE
1850 CMR = CM * CON: CTR = CT * CON: CR = C * CON
1860 MCMR = (CMR - DECR): REM magnetometer magnetic course in radians
1870 MCTR = (CTR - DECR): REM target magnetic course in radians
1880 ILM = KL * HE * COS(DIPR) * COS(MCTR): ITM = -KT * HE * COS(DIPR) * SIN(MCTR)
1890 IVM = KV * HE * SIN(DIPR)
1900 DMX = (PLM + ILM) * SIN(MCTR) + (PTM + ITM) * COS(MCTR)
1910 DMY = (PLM + ILM) * COS(MCTR) - (PTM + ITM) * SIN(MCTR)
1920 DMV = PVM + IVM
1930 X = DMX: Y = DMY
```

```
1940 GOSUB 4170
1950 OMLR = T: REM dipole azimuth relative to magnetic north
1960 X = DMV: Y = R
1970 GOSUB 4170
1980 DM = R: OMR = T: REM dipole depression angle
1990 OML = OMLR / CON: OM = OMR / CON
2000 A$ = "a"
2010 INPUT "generate a combined magnetic, processing & kinematic data file (y/n)"; A$
2020 IF A$ = "Y" OR A$ = "y" THEN GOTO 2040
2030 IF A$ = "N" OR A$ = "n" THEN GOTO 2210 ELSE GOTO 2010
2040 INPUT "combined magnetic, processing & kinematic data file name"; E$
2050 ON ERROR GOTO 2060: GOTO 2070
2060 RESUME 2040
2070 OPEN "O", #1, E$
2080 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV, DT, IT
2090 WRITE #1, NS, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML, OM, M$, P$, K$
2100 CLOSE
2110 GOTO 2210
2120 INPUT "combined magnetic, processing & kinematic data file name"; E$
2130 ON ERROR GOTO 2140: GOTO 2150
2140 RESUME 2120
2150 OPEN "I", #1, E$
2160 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV, DT, IT
2170 INPUT #1, NS, CM, VMK, AM, CT, VTK, AT, Z, C, WK, ILM, ITM, IVM, DM, OML, OM, M$, P$, K$
2180 CLOSE
2190 DECR = DEC * CON: DIPR = DIP * CON
2200 CR = C * CON: OMLR = OML * CON: OMR = OM * CON
2210 INPUT "required false alarm rate in false alarms per hour"; FAR
2220 PF = FAR * IT / 3600: REM false alarm probability
2230 Y = PF: IF PF > .5 THEN Y = 1 - Y: REM inverse normal approximation
2240 Y = SQR(LOG(1 / Y / Y))
2250 Y = Y - (I1 + Y * (I2 + I3 * Y)) / (1 + Y * (I4 + Y * (I5 + I6 * Y)))
2260 IF PF < .5 THEN Y = -Y
2270 ZP = -Y
2280 CHI = NS * (1 - 2 / 9 / NS + ZP * SQR(2 / 9 / NS)) ^ 3: REM inverse chi-square approximation
2290 INPUT "magnetic noise in gamma"; SIG
2300 INPUT "maximum lateral range in meters"; LRM
2310 INPUT "lateral range step in meters"; ST
2320 IF ST <= LRM THEN GOTO 2340
2330 PRINT : PRINT "maximum step is " + STR$(LRM) + " meters": PRINT : GOTO 2310
2340 NC = 2 * INT(LRM / ST) + 1: REM number of lateral range function values
2350 IF NC <= 71 THEN GOTO 2370
2360 PRINT : PRINT "minimum step is " + STR$(LRM / 35) + " meters": PRINT : GOTO 2310
2370 AIT = DT * NS: REM adjusted integration time
2380 W = WK * KON: REM magnetometer relative speed in meters/second
2390 DS = W * DT: REM distance between samples on the relative track
2400 XO = -(NC - 1) / 2 * ST
2410 FOR I = 0 TO NC - 1
2420 XO(I) = XO
2430 X = XO: Y = Z
2440 GOSUB 4170
2450 RO = R: RO(I) = R: REM target slant range at CPA in meters
2460 DELR = T: REM target depression angle complement at CPA in radians
2470 IF RO = 0 THEN GOTO 2800: REM zero lateral range and vertical separation 2480 DMF = DM / 10 / RO ^ 3:
REM dipole moment factor
2490 CMR = CR - DECR: REM target relative magnetic course
2500 L = COS(OMR) * COS(CMR - OMLR)
2510 M = COS(DELR) * COS(OMR) * SIN(CMR - OMLR) - SIN(DELR) * SIN(OMR)
2520 N = -SIN(DELR) * COS(OMR) * SIN(CMR - OMLR) - COS(DELR) * SIN(OMR)
2530 L1 = COS(DIPR) * COS(CMR)
2540 M1 = COS(DELR) * COS(DIPR) * SIN(CMR) - SIN(DELR) * SIN(DIPR)
2550 N1 = -SIN(DELR) * COS(DIPR) * SIN(CMR) - COS(DELR) * SIN(DIPR)
2560 A2 = 2 * L * L1 - M * M1 - N * N1: REM Anderson Function Coefficient
2570 A1 = 3 * (N * L1 + L * N1): REM Anderson Function Coefficient
2580 AO = 2 * N * N1 - L * L1 - M * M1: REM Anderson Function Coefficient
```

```
2590 SUM = 0: HMAX = 0: HMIN = 0
2600 FOR J = 0 TO NS - 1
2610 S = (J - (NS - 1) / 2) * DS: BA = S / RO: REM Anderson Function Argument 2620 AF = 1 / (1 + BA * BA) ^
2.5: REM Anderson Function Factor
2630 HSF = (A2 * BA * BA + A1 * BA + A0) * AF: REM magnetic signal factor
2640 HS(I, J) = DMF * HSF: REM magnetic signal value
2650 IF HS(I, J) > HMAX THEN HMAX = HS(I, J)
2660 IF HS(I, J) < HMIN THEN HMIN = HS(I, J)
2670 SUM = SUM + HSF * HSF
2680 NEXT J
2690 H(I) = HMAX - HMIN
2700 K(I) = SQR(SUM)
2710 VV = -ZP + DMF * SQR(SUM) / SIG
2720 LAM = DMF * DMF * SUM / (SIG * SIG): AA = NS + LAM: BB = 1 + LAM / (NS + LAM)
2730 ZN = -SQR(2 * CHI / BB) + SQR(2 * AA / BB - 1): X1 = VV
2740 GOSUB 4230
2750 IF Y1 > 1 THEN Y1 = 1
2760 PDCC(I) = Y1: X1 = ZN
2770 GOSUB 4230
2780 IF Y1 > 1 THEN Y1 = 1
2790 PDSL(I) = Y1
2800 X0 = X0 + ST
2810 NEXT I
2820 C = C / 360: C = (C - INT(C)) * 360
2830 IF C < 0 THEN C = 360 + C
2840 OML = (OML + DEC) / 360: OML = (OML - INT(OML)) * 360
2850 IF OML < 0 THEN OML = 360 + OML
2860 PRINT : A$ = "a"
2870 INPUT "generate a program data file (y/n)"; A$
2880 IF A$ = "Y" OR A$ = "y" THEN GOTO 2900
2890 IF A$ = "N" OR A$ = "n" THEN GOTO 3230 ELSE GOTO 2870
2900 INPUT "program data file name"; D$
2910 ON ERROR GOTO 2920: GOTO 2930
2920 RESUME 2900
2930 OPEN "O", #1, D$
2940 WRITE #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV, DT, IT, AIT
2950 WRITE #1, NS, CM, VMK, AM, CT, VTK, AT, ILM, ITM, IVM, Z, C, WK, DM, OML, OM, FAR, PF, SIG, ST
2960 WRITE #1, LRM, DS, NC, M$, P$, K$, E$
2970 FOR I = 0 TO NC - 1
2980 WRITE #1, X0(I), PDCC(I), PDSL(I), K(I), H(I), RO(I)
2990 NEXT I
3000 FOR I = 0 TO NC - 1
3010 FOR J = 0 TO NS - 1
3020 WRITE #1, HS(I, J)
3030 NEXT J
3040 NEXT I
3050 CLOSE
3060 GOTO 3230
3070 INPUT "program data file name"; D$
3080 ON ERROR GOTO 3090: GOTO 3100
3090 RESUME 3070
3100 OPEN "I", #1, D$
3110 INPUT #1, LAT, LNG, DEC, DIP, HE, PLM, PTM, PVM, WT, KL, KT, KV, FL, FT, FV, DT, IT, AIT
3120 INPUT #1, NS, CM, VMK, AM, CT, VTK, AT, ILM, ITM, IVM, Z, C, WK, DM, OML, OM, FAR, PF, SIG, ST
3130 INPUT #1, LRM, DS, NC, M$, P$, K$, E$
3140 FOR I = 0 TO NC - 1
3150 INPUT #1, X0(I), PDCC(I), PDSL(I), K(I), H(I), RO(I)
3160 NEXT I
3170 FOR I = 0 TO NC - 1
3180 FOR J = 0 TO NS - 1
3190 INPUT #1, HS(I, J)
3200 NEXT J
3210 NEXT I
3220 CLOSE
3230 PRINT : A$ = "a"
```

```
3240 INPUT "print encounter parameter values (y/n)"; A$
3250 IF A$ = "Y" OR A$ = "y" THEN GOTO 3270
3260 IF A$ = "N" OR A$ = "n" THEN GOTO 3780 ELSE GOTO 3240
3270 LPRINT
3280 LPRINT "program file name                                          " + N$
3290 LPRINT "program data file name                                     " + D$
3300 LPRINT "magnetic data file name                                    " + M$
3310 LPRINT "processing data file name                                  " + P$
3320 LPRINT "kinematic data file name                                   " + K$
3330 LPRINT "combined magnetic, processing & kinematic data file name " + E$
3340 LPRINT "encounter latitude (decimal degrees)                    "; SPC(2); LAT
3350 LPRINT "encounter longitude (decimal degrees)                   "; SPC(2); LNG
3360 LPRINT "encounter variation (decimal degrees)                   "; SPC(2); DEC
3370 LPRINT "encounter dip angle (decimal degrees)                   "; SPC(2); DIP
3380 LPRINT "encounter magnetic field intensity (gamma)              "; SPC(2); HE
3390 LPRINT "permanent longitudinal moment (oersted-cm3)             "; SPC(2); PLM
3400 LPRINT "permanent transverse moment (oersted-cm3)               "; SPC(2); PTM
3410 LPRINT "permanent vertical moment (oersted-cm3)                 "; SPC(2); PVM
3420 LPRINT "target displacement (tons)                              "; SPC(2); WT
3430 LPRINT "target longitudinal permeability coefficient            "; SPC(2); KL
3440 LPRINT "target transverse permeability coefficient              "; SPC(2); KT
3450 LPRINT "target vertical permeability coefficient                "; SPC(2); KV
3460 LPRINT "target longitudinal permeability factor                 "; SPC(2); FL
3470 LPRINT "target transverse permeability factor                   "; SPC(2); FT
3480 LPRINT "target vertical permeability factor                     "; SPC(2); FV
3490 LPRINT "sampling period (seconds)                               "; SPC(2); DT
3500 LPRINT "integration time (seconds)                              "; SPC(2); IT
3510 LPRINT "adjusted integration time (seconds)                     "; SPC(2); AIT
3520 LPRINT "number of samples per encounter                         "; SPC(2); NS
3530 LPRINT "magnetometer course (decimal degrees)                   "; SPC(2); CM
3540 LPRINT "magnetometer speed (knots)                              "; SPC(2); VMK
3550 LPRINT "magnetometer altitude (meters)                          "; SPC(2); AM
3560 LPRINT "target course (decimal degrees)                         "; SPC(2); CT
3570 LPRINT "target speed (knots)                                    "; SPC(2); VTK
3580 LPRINT "target depth (meters)                                   "; SPC(2); AT
3590 LPRINT "magnetometer relative course (decimal degrees)          "; SPC(2); C
3600 LPRINT "magnetometer relative speed (knots)                     "; SPC(2); WK
3610 LPRINT "magnetometer-target vertical separation (meters)        "; SPC(2); Z
3620 LPRINT "target induced longitudinal dipole moment (oersted-cm3)"; SPC(2); ILM
3630 LPRINT "target induced transverse dipole moment (oersted-cm3)  "; SPC(2); ITM
3640 LPRINT "target induced vertical dipole moment (oersted-cm3)    "; SPC(2); IVM
3650 LPRINT "magnetic dipole moment (oersted-cm3)                    "; SPC(2); DM
3660 LPRINT "dipole moment azimuth (decimal degrees)                 "; SPC(2); OML
3670 LPRINT "dipole moment depression angle (decimal degrees)        "; SPC(2); OM
3680 LPRINT "distance between samples on the relative track (meters)"; SPC(2); DS
3690 LPRINT "false alarm rate (false alarms per hour)                "; SPC(2); FAR
3700 LPRINT "false alarm probability                                 "; SPC(2); PF
3710 LPRINT "magnetic noise (gamma)                                  "; SPC(2); SIG
3720 LPRINT "maximum lateral range (meters)                          "; SPC(2); LRM
3730 LPRINT "lateral range step (meters)                             "; SPC(2); ST
3740 LPRINT "number of lateral range function values                 "; SPC(2); NC
3750 FOR I = 0 TO 15
3760 LPRINT
3770 NEXT I
3780 PRINT : A$ = "a"
3790 INPUT "print lateral range function values (y/n)"; A$
3800 IF A$ = "Y" OR A$ = "y" THEN GOTO 3820
3810 IF A$ = "N" OR A$ = "n" THEN GOTO 3900 ELSE GOTO 3790
3820 LPRINT D$; " lateral range function values"
3830 LPRINT : LPRINT
3840 LPRINT "L         p(cc)        p(sl)        H            R                k"
3850 LPRINT "meters                              gamma        meters"
3860 LPRINT
3870 FOR I = 0 TO NC - 1
3880 LPRINT X0(I); TAB(10); PDCC(I); TAB(24); PDSL(I); TAB(38); H(I); TAB(52); R0(I); TAB(70); K(I)
```

```
3890 NEXT I
3900 PRINT : A$ = "a"
3910 INPUT "print magnetic signal values (y/n)"; A$
3920 IF A$ = "Y" OR A$ = "y" THEN GOTO 3940
3930 IF A$ = "N" OR A$ = "n" THEN GOTO 4120 ELSE GOTO 3910
3940 PRINT : PRINT "identify signal by encounter lateral range index"
3950 PRINT "lateral range equals the index times " + STR$(ST) + " meters"
3960 PRINT "index values: -" + STR$((NC - 1) / 2) + " to " + STR$((NC - 1) / 2)
3970 INPUT "lateral range index"; K: I = K + (NC - 1) / 2
3980 LPRINT : LPRINT : LPRINT
3990 LPRINT D$; " magnetic signal values"
4000 LPRINT "for a lateral range of "; K * ST; " meters"
4010 LPRINT
4020 LPRINT "relative CPA distance"; TAB(35); "magnetic signal"
4030 LPRINT "s in meters"; TAB(35); "Hs in gamma"
4040 LPRINT
4050 FOR J = 0 TO NS - 1
4060 LPRINT (J - (NS - 1) / 2) * DS; TAB(35); HS(I, J)
4070 NEXT J
4080 PRINT : A$ = "a"
4090 INPUT "print magnetic signal values for a different lateral range (y/n)"; A$
4100 IF A$ = "Y" OR A$ = "y" THEN GOTO 3940
4110 IF A$ = "N" OR A$ = "n" THEN GOTO 4120 ELSE GOTO 4090
4120 PRINT : A$ = "a"
4130 INPUT "continue to use the program (y/n)"; A$
4140 IF A$ = "Y" OR A$ = "y" THEN GOTO 10
4150 IF A$ = "N" OR A$ = "n" THEN GOTO 4160 ELSE GOTO 4130
4160 END
4170 R = SQR(X * X + Y * Y): REM rectangular to polar conversion
4180 IF R = 0 THEN T = 0: RETURN
4190 IF ABS(X / R) = 1 THEN Q = SGN(X) * (PI / 2) ELSE Q = ATN(X / R / SQR(1 - X * X / R / R))
4200 IF ABS(Y / R) = 1 THEN T = (PI / 2) * (1 - SGN(Y)) ELSE T = (PI / 2) - ATN(Y / R / SQR(1 - Y * Y / R /
R))
4210 IF Q < 0 THEN T = 2 * PI - T
4220 RETURN
4230 Y1 = X1: IF X1 < 0 THEN Y1 = -Y1: REM normal approximation
4240 G = 1 / (1 + Q0 * Y1)
4250 Y1 = EXP(-Y1 * Y1 / 2) / SQR(2 * PI) * G * (Q1 + G * (Q2 + G * (Q3 + G * (Q4 + G * Q5))))
4260 IF X1 >= 0 THEN Y1 = 1 - Y1
4270 RETURN
```

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