5801-MA-01
F
(2)

ERO Final Report
Contract DAJA 45-88-C-009

DTIC FILE COPY

1990

AD-A225 750

SUMMARY

Introduction

I Nonlinear Vibrations, by I. Ekeland
II Viscosity solutions and applications, by P.L. Lions
III Wavelets and applications, by Y. Meyer
IV Variational methods for image segmentation, by J.M. Morel

Introduction

Since the beginning of the contract DAJA 45-88-C-009, considerable progress has been made on the four themes listed in the initial proposal. These themes correspond essentially to four different groups in Ceremade with various collaborations including several ones with american colleagues and their groups (R. Coifman at Yale Univ., I. Daubechies at Bell Labs, S. Mallat at Courant Institute, M.G. Crandall at UCSB, S. Osher at UCLA, W.H. Fleming and P.E. Souganidis at Brown University...)

As it will be seen from the more detailed description presented below of the research at Ceremade on each theme , progress has been made not only on the theoretical side but also on more practical issues which include computational aspects.

The support provided by the contract has been essential to reinforce the links between Ceremade and various american groups and also to the development of computing facilities at Ceremade.

90

Final Report
Contract DAJA 45-88-C-009

form 50

I. Nonlinear vibrations
by Ivar EKELAND

A-1

The team working on these topics consists of Ivar Ekeland (Professor, Ceremade), Claude Viterbo (C.N.R.S., Ceremade), Eric Séré (PhD student, Ceremade), Philippe Bolle (PhD. student, Ceremade), Nabil Sioufi (PhD. student, Ceremade). Most work was done in collaboration with Antonio Ambrosetti (Professor, Scuola Normale, Pisa), Vittorio Coti-Zelati (Assistant Professor, SISSA, Trieste), Helmut Hofer (Professor, Ruhr Universität, Bochum, West Germany) and Salem Mathlouti (Assistant Professor, Bizerte, Tunisia).

The program focused on the study of solutions of Hamiltonian systems and mainly on three aspects.

I. Periodic orbits and symplectic topology
II. Homoclinic orbits
III. Singular potentials

I. Periodic orbits and symplectic topology.

Until five years ago, the only energy levels known to carry a periodic orbit for the Hamiltonian flow were the starshaped ones. In 1986, C. Viterbo proved that any "contact type" energy level in euclidean space carries such an orbit, thus solving the so called "Weinstein conjecture" in this case.

The contact type condition is too technical to be explained here, but has the main advantage of being invariant by canonical transformation.

The result and method of proof was used by Ekeland and Hofer to construct a new family of symplectic invariants, the so called symplectic capacities.

At first, these capacities yielded simple proofs of Gromov's rigidity theorems, which had been obtained previously by different (and much harder) methods.

Since then however, Ekeland, Hofer and Viterbo showed that capacities can give results which have not been obtained by Gromov elliptic methods. One example is the study of Lagrange tori in $\mathbb{R}^{2n}$. Using the methods developed by these authors, Viterbo shows that the Maslov class of such a torus does not vanish, that such a torus cannot be squeezed in a arbitrarily small ball, and that there are infinitely many equivalent such tori with given periods.

However, we think that these methods may still give many new results in symplectic topology.

In another direction, Hofer and Viterbo generalized the proof of the Weinstein conjecture to other symplectic manifolds like the contigent bundles of compact manifolds, products $P \times \mathbb{C}$ where P is compact and has vanishing second homotopy group.

P. Bolle investigated a third direction. He proved in some special cases that if H_1, H_2 are Hamiltonians in involution, then the submanifold $\{H_1 = h_1, H_2 = h_2\}$ carries a periodic orbit of the Hamiltonian flow of H_3, where is a suitable linear combination of H_1 and H_2.

Finally, extensive numerical work has been done by S. Mathlouti : he found numerically the periodic solutions the existence of which had been proved theoretically, and computed the symplectic capacity of cubes.

II. Homoclinic orbits.

The following problem was studied by Coti-Zelati, Ekeland, Séré :

$$\begin{cases} \dot{x} = J\left(Ax + R'(t,x)\right) \\ \lim\limits_{t \to \pm\infty} x(t) = 0 \end{cases}$$

where JA has no imaginary eigenvalue, $R(t,x)$ is T-periodic and superquadratic.

First, Coti-Zelati, Ekeland and Séré proved existence of at least two geometrically distinct solutions. Then, Séré proved the existence of infinitely many such solutions.

These results were previuously known only in the very special case of a perturbation of a completely integrable system.

It seems that the method used by Séré can also be applied to the study of equations of the type

$$-\Delta u + u = f(x,u)$$

with f periodic in $x \in \mathbb{R}^n$.

III. Singular potentials.

This work mostly deals with equation

$$\begin{cases} \ddot{x} + \nabla V(t,x) = 0 \\ x(0) = x(T) \\ \dot{x}(0) = \dot{x}(T) \end{cases}$$

where V is a singular potential.
Most cases of interest deal with a potential which is either

$V(x) = \frac{1}{|x|^{\alpha}} + \varphi(x)$ where φ is a C^2 function

$V(x) = \frac{1}{d(x,\partial U)^{\alpha}} + \varphi(x)$ where U is a bounded set .

In the first case, existence and multiplicity results have been obtained by Ambrosetti, Ekeland and Coti-Zelati.

In the second case, results due to the above authors have been by P. Bolle to study periodic trajectories of the billiard problem.

ERO Final Report
Contract DAJA 45-88-C-009

II Viscosity solutions and applications (stochastic control, differential games, vision...) by P.L Lions

This report describes the work performed under contract DAJA 45-88-C-009 by the following group of people : P.L Lions (Professor, Ceremade), A. Bensoussan (Professor, Ceremade), B. Perthame (Professor, Ceremade and Univ. of Orléans), G. Barles (Professor, Ceremade) A. Sayah (Assistant Professor, Ceremade and Univ. of Rabat) B. Alziary (Assistant Professor, Ceremade), E. Rouy (PhD student, Ceremade) and A. Tourin (PhD student, Ceremade), with several collaborators : M.G. Crandall (UCSB), P.E. Souganidis (Brown Univ.), H. Ishii (Uhuo Univ, Tokyo), L.C. Evans (UC. Berkeley).

SUMMARY

1. Introduction

Since the initial discovery of viscosity solutions by M. G. Crandall and the author in 1981 for first order Hamilton Jacobi equations, many important contributions have been brought to the subject by R. Jensen, L.C. Evans, H Ishii...

The theory developed rapidly and allowed a rather complete theory for first-order Hamilton-Jacobi equations that is general fully nonlinear first-order scalar equations including existence, uniqueness, stability, boundary conditions. The link with optimal control and differential games was established even for optimal stochastic control where the relevant PDE's are known as Hamilton-Jacobi-Bellman equations (fully nonlinear, convex, possibly degenerate, scalar, second-order elliptic equations). However, extending the general theory to general fully nonlinear second-order degenerate elliptic questions remained an open question except for optimal stochastic control problems (the author, 1983). A breakthrough was achieved by R. Jensen (1986) who obtained the first rather general existence and uniqueness results. The report will describe below in section 2 some of the advances on the basic theory (existence, uniqueness, boundary conditions, stability). Section 3 will present some applications of the theory which were discovered during the period 1987-1990 while section 4 will illustrate the impact of that theory on numerical computations.

Let us finally mention that the author will talk about this theory and its applications as an invited speaker at ICIAM-91 (Seattle) -related invitations include ICM-82-83 (Warsaw) and ICM-90 (Kyoto).

2. Advances on the theory of viscosity solutions.

The extension of viscosity theory from first-order equations to general fully nonlinear degenerate elliptic second-order equations has been completed in H. Ishii and the author allowing for existence, uniqueness together with a complete treatment of boundary conditions. Even fully nonlinear first-order boundary conditions can be incorporated in the treatment (works by G. Barles and the author, H. Ishii and in preparation by G. Barles). Some regularity results have been obtained (H. Ishii and the author, G. Barles, N.S. Trudinger, L. Caffarelli) where both results and proofs rely on viscosity solutions theory. The interplay between uniqueness results and quadratic growth at infinity has also been settled by M. G. Crandall and the author.

Even fully nonlinear second order equations in infinite dimensions have been solved by the author including various infinite-dimensional stochastic control problems and in particular the optimal control of Zakai's equation. This last example is important for Stochastic Control theory since it corresponds to the optimal control of finite dimensional diffusion processes with partial observations.

A general theory of first-order Hamilton-Jacobi equations in infinite dimension as those corresponding to the Bellman's equations associated to the optimal control of partial differential equations has been developed by M.G. Crandall and the author in a series of papers with some work still in progress.

Another development of the theory is the possible treatment of fully nonlinear integro-differential equations (with maximum principle) with the application to the optimal control of jump diffusion processes (works in preparation by A. Sayah, A. Sayah and the author).

A very important addition to the theory has been the introduction by G. Barles and B. Perthame of a general method for passages to the limit assuming only local bounds on solutions. This method which relies upon discontinuous viscosity solutions has numerous applications that we mention below.

3. Applications of the theory.

We already mentioned several times above the application to optimal control theory or differential games (optimal stochastic control with complete informations in finite or infinite dimensions, optimal control of pde's, optimal control of Zakai's equation and optimal stochastic control with partial observations, stochastic differential games, optimal control of jump diffusion processes...). Other applications to optimal control have been made possible by the viscosity solutions treatment of boundary conditions for state-constraints problems and by uniqueness results for discontinuous value functions (work by G. Barles and B. Perthame).

Other important applications concern various asymptotic problems (homogeneization, large deviations, WKB type expansions) and we refer to various works by G. Barles, L.C. Evans, P.E. Souganidis, H.M. Soner... The "weak passage to the limit" method introduced by G. Barles and B. Perthame is one of the main tools to study such asymptotic problems. Another application of this method is the convergence of general numerical schemes (work by G. Barles and P.E. Souganidis). Let us also mention the recent by P.E. Souganidis and the author on the convergence of MUSCL schemes for conservation laws : an unexpected application to a classical open problem in numerical analysis that was solved by raising the problem to the Hamilton-Jacobi level and using viscosity theory.

Another unexpected application of the theory concerns artificial vision and the so-called shape-from-shading problem. This classical problem which consists in recovering a shape from a light intensity observation can be formulated as a first-order Hamilton-Jacobi equation. Viscosity theory has allowed a complete understanding of possible losses of uniqueness together with the construction of stable numerical schemes for the reconstitution (work by E. Rouy and A. Tourin).

4. Numerical computations

Our group has studied some model problems involving first-order or second-order equations and has devised some efficient and robust schemes based upon viscosity and optimal control theories. B. Alziary has solved pursuit games with constraints on both players and optimal control problems of trajectories (controls on the acceleration) with target type constraints and discontinuous value functions. In particular, B. Alziary and the author have introduced a general grid refinement method for control and games problems which can be used to reduce the size of calculations ; this method also can be used to combine minimum principle (Pontryagin) and dynamic programming in a numerical solution for value functions and optimal trajectories.

E. Rouy and A. Tourin have developed stable and efficient schemes for the shape-from-shading problem described in the preceding section based upon existence and uniqueness results following from the general theory and on optimal control considerations. E. Rouy has developed in particular a general acceleration method for the computation of value functions of control problems which has improved considerably the efficiency of the monotone schemes which we first used.

We are also currently developing second-order schemes with limiters for first-order "convex" Hamilton-Jacobi equations. Finally, Th. Zariphopoulou and A. Tourin have begun a program on the numerical solution of optimal stochastic control problems arising in Finance which involve transaction costs.

ERO Final Report June, 1990

III Wavelets and applications.
by Y. Meyer.

1. The team :

P. Auscher[(1)], G. Beylkin[(2)], A. Cohen[(3)], R. Coifman[(4)], I. Daubechies[(5)], J.C. Feauveau[(6)], S. Jaffard[(7)], S. Mallat[(8)], Y. Meyer[(3)], V. Rocklin[(4)], R.V. Wickerhauser[(9)].

(1) Dpt. of mathematics, Wash. Univ. St Louis, 63130 MO. USA.
(2) Schlumberger-Doll Research, Ridgefield, CT 06877 USA.
(3) CEREMADE, Univ. Paris Dauphine, 75775 Paris Cedex 16.
(4) Dpt. of mathematics, Yale University, CT. 06520, USA.
(5) Bell Labs, Murray Hill
(6) Univ. Paris Sud, Informatique théorique, 91405-ORSAY.
(7) Centre de mathematique appliquée, Ecole Polytechnique, 91128 Palaiseau.
(8) Courant Institute, New York, 251 Mercer Street
(9) Dpt. of mathematics, Univ of Georgia, Athenes, Georgia 30602.

2. The program.

Our program started in September 1985 when Y. Meyer constructed a "mother wavelet" ψ , belonging to the $\mathcal{S}$ class of Schwartz, such that the collection $2^{j/2}\psi(2^j x-k)$; $j \in \mathbb{Z}$, $k \in \mathbb{Z}$, is an orthonormal basis of $L^2(\mathbb{R})$.

Our goals have been (1) to find other orthonormal wavelets bases with the same structure, (2) to relate wavelets expansions to some previously known algorithms in numerical analysis, (3) to use these wavelets to improve the existing technologies in image processing and speech signal processing.

The three components of this program fortunately happened to be intimately related. S. Mallat unveiled the subtle and mysterious relations between (1) orthonormal wavelets expansions (2) quadrature mirror filters (as used in speech processing) and (3) pyramidal algorithms (as used in image processing). S. Mallat's program was completed in 1987 by I. Daubechies [3] and in 1989 by A. Cohen [1].

In 1987, I. Daubechies proved the existence for each integer of an orthonormal wavelet basis in which the mother wavelet ψ is compactly supported and of class C^m.

In 1989, A. Cohen found the necessary and sufficient condition to be satisfied by QMF in order to generate an orthonormal wavelet basis.

G. Beylkin, R. Coifman & V. Rockhlin [2] clarified the relations between wavelets expansions and multigrid methods. Following this line of investigation, they discovered striking fast algorithms for the evaluation of singular integral operators. Some pieces of this algorithm have been improved in [6].

A completely distinct line of investigation has been followed by S. Jaffard who proved that J.O. Strömberg's (1981) orthonormal basis of wavelets remained hidden inside Ph. Franklin's system (1927) and could be extracted through (1) and obvious rescaling and (2) a natural limiting process.

M. Barlaud, I. Daubechies and their colleagues switched from orthonormal wavelets to some more sophisticated schemes set up by A. Cohen, I. Daubechies et J.C Feauveau [1]. The main point is that these new schemes use symmetric filters which are convenient in image processing. The method proposed enables high compression bit rates while maintaining good visual quality.

A second striking application has been the construction by R. Coifman and Y. Meyer of wavelets packets. Wavelets packets are very similar to Gabor wavelets which have always been used in speech processing. A search algorithm based on an entropy criterion gives exceptional compression bit rates in speech processing [4].

3. International conferences.

(a) International congress in mathematical Physics, Swansea 1988, Wavelets & applications, Yves MEYER (one hour invited address).

(b) International congress of mathematics, Kyoto 1990, Wavelets & applications, Yves MEYER (45 minutes invited address).

(c) Conference on wavelets organized by Y. MEYER, in Marseille-Luminy (29 May-3June 1989), Proceedings to be published.

4. Publications

[1]. A. Cohen. PhD Dissertation, CEREMADE.

[2] G. Beylkin, R. Coifman, V. Rokhlin,
Fast wavelets transforms and numerical algorithms I
Research Report YALEU/DCS/RR-696.

[3]. I. Daubechies, Orthonormal bases of compactly supported wavelets,
Comm. Pure & Appl. Math. XLI (1988), 909-966.

[4]. R. Coifman, Y. Meyer & V. Wickerhauser,
Research being patented.

[5]. Y. Meyer, Ondelettes et Opérateurs, tome I (pp. 1-215),
tome II (pp. 216-381), tome III (to appear), Hermann , actualités
mathématiques, 1990.

[6]. Y. Meyer, Ondelettes sur l'intervalle, Cahiers du CEREMADE, 1990.

IV. Varational methods for image segmentation
by Jean-Michel Morel

During the years 87-90, a basic research activity in signal and image processing has been developped at Ceremade. The description and results of this team are described in the enclosed document "Equipe de traitement d'images". The most important part of this research, the work on the wavelets and "shape from shading" will be reported by Yves MEYER and Pierre-Louis LIONS. We shall here give a brief account of the research in the "variational theory of image segmentation". Details can be found in the enclosed papers.

A task which has always been considered essential in image processing is the "edge or boundary detection". The difficulty of the concept of edge, or boundary, is that it had until recently neither a universally admitted general definition, nor even a systematic classification. There has been probably more than thousand proposed definitions, each attached to a special algorithm, to a certain researcher in a certain period, and even worse, proved to work "well" only on a very few samples chosen by the researcher himself. However, in the last ten years, very serious attempts have been made to organize the field and define general methods and devices. One of the directions is the linear edge detection which leads to the wavelets theory. Another, begun by Marr, Witkin, Koenderink,... is the so-called "scale space", which has led in the last three years to the very interesting model of Perona and Malik, where the edge detection task is achieved by a nonlinear porous-medium-like equation.

The last approach, not opposed to the above mentioned, is to minimize some functional defined on the set of possible interpretations of pictures. The original, stochastic model seems to be due to Grenander, and it has been developped by (among others) Geman (Markov random field models for pictures). As showed by Mumford and Shah, this kind of models leads to the minimization of deterministic functionals in nonclassical functional spaces. The general functional proposed by Mumford and Shah (and used explicitly or implicitly by most researchs in vision, see for instance the Blake and Zissermann book at MIT press : "Visual Reconstruction") contains one dimensional terms, defined as integrals on the possible edges, and two dimensional terms, defined as integrals supported by the possible regions of the segmentation. (A segmentation is therefore a combination of edges and regions, the edges being conceived as the boundaries of the regions. The Mumford and Shah functional tends to minimize the autosimilarity of the regions (that is, in the simplest case, the variance of the gray level in each region) and the adequacy of edges (their regularity and location).

Our work has been mainly concerned with the proof of several conjectures proposed by these authors in their seminal paper of 1985. In collaboration with Blat, Dal Maso, Koepfler, Solimini, the author has obtained the following results (see the enclosed papers for more details).

1). Blat and the author have studied the mathematical justification of the existence of open edges in the segmentation, that is, the existence of edges finishing by a tip. They examine the relation of the segmentation with fracture propagation models studied by, among others, J. Knowles. This study was suggested by Mumford and Shah in their first paper.

2). Solimini and the author have proved the existence and regularity properties of the minimizing segmentation in the case of the simplest model proposed by Mumford and Shah, where the a priori model is that the picture is piecewise constant on each region. This proof is quite different from those proposed independently by Mumford and Shah and Yang Wang. The side effect of this proof is to suggest that the "region growing" methods, a very large class of segmentation methods in use, are sound if applied with an energy functional of the preceding class as "merging" criterion. Koepfler, Solimini and the author indeed proved that the class of the segmentations computed by such an adaptation of the "region growing" methods computes a compact class of segmentations. This result is no longer true in dimension greater than two.

3). Returning to the general model of Mumford and Shah where the a priori model of the image is that it is piecewise harmonic (harmonic on each region) : Dal Maso, Morel and Solimini proved the following properties of the weak solutions to the minimization problem found by De Giorgi, Ambrosio, Carriero and Leaci.

a- The set of minimizing segmentations is compact for the Hausdorff distance (one measures, from a very practical viewpoint, the distance of two segmentations by taking the Hausdorff distance between their edge sets).

b- The edges of a minimizing segmentation are "almost" a finite set of curves (that is, up to a set of arbitrary small length).

This theorem explains why concrete minimization procedures like Yvan Leclerc's or Richardson's give stable edges and why they look like curves. It also suggests that a minimization can be made, were the edges are a priori fixed to be a finite number of curves, like in the very popular "snakes" methods.

Solimini and the author have also obtained several results for the model in higher dimension (in course of redaction). They now prepare a book for Birkhäuser on these variational methods. The support of ERO helped the contributed to the numerical simulations for which several SUN stations were bought.