UNLIMITED

TR90018

(2)

TR 90018

ROYAL AEROSPACE ESTABLISHMENT

Technical Report TR 90018

February 1990

Orbit Determination and Analysis for Aureole 2 Rocket at 27:2 Resonance

by

A.N.Winterbottom

Procurement Executive, Ministry of Defence
Farnborough, Hampshire

UNLIMITED

ROYAL AEROSPACE ESTABLISHMENT

Technical Report 90018

Received for printing 26 February 1990

ORBIT DETERMINATION AND ANALYSIS FOR AUREOLE 2 ROCKET AT 27:2 RESONANCE

by

A. N. Winterbottom

SUMMARY

Aureole 2 rocket (1973-107B) was launched on 26 December 1973 into an orbit of inclination 74° and eccentricity 0.1 and has an estimated lifetime of 30 years. The orbit has been determined from observations for 90 epochs between September 1983 and December 1984, during which time the orbit was expected to be influenced significantly by the effects of 27:2 resonance with the Earth's gravitational field; exact resonance occurred on 28 April 1984. The observations numbered nearly 7400, of which 344 were from the Hewitt cameras of the University of Aston which are sited at Herstmonceux in England, and Siding Spring in Australia. The orbital inclination and eccentricity of the orbits derived had standard deviations corresponding on average to positional accuracies of 130 m cross-track and 80 m in perigee distance.

The variations in inclination and eccentricity have been analysed individually to determine values of two pairs of lumped harmonics of order 27 from each parameter; when these parameters were fitted simultaneously they gave three pairs of harmonics with standard deviations corresponding to accuracies of approximately 2.5 cm in geoid height.

Departmental Reference: SS 11

LIST OF CONTENTS

1 INTRODUCTION

When the satellite Aureole 2 was launched on 26 December 1973, its rocket, designated 1973-107B, entered an orbit with an estimated lifetime of 30 years. The rocket is cylindrical in shape, 7.4 m long with a diameter of 2.4 m, and has a mass of about 2200 kg. Its initial orbital parameters[1] were: inclination 74.01°, perigee and apogee heights 396 and 1965 km respectively, and nodal period 109.02 min.

In April 1984, Aureole 2 rocket passed through the condition of 27:2 resonance, *ie* the track over the Earth repeated every 2 days after 27 revolutions. If the passage through resonance of an orbiting body is slow enough, the effects of 27th-order harmonics in the geopotential can build up and result in an appreciable perturbation to some of the orbital elements. Thus measurement of these resultant perturbations provides a good method for accurately determining the appropriate lumped geopotential harmonics. The aim of this Report is to compute accurate orbits from observations made during the time when the 27:2 resonance with the geopotential was affecting the orbit and to evaluate lumped geopotential harmonics of order 27 from the changes they produced in the orbital inclination and eccentricity; this is the first occasion on which 27th-order harmonics have been obtained from resonant satellite orbit analysis. The orbit was determined between September 1983 and December 1984 from radar and optical observations using the RAE orbit refinement program PROP, in the PROP6 version[2].

2 OBSERVATIONS AND ORBITS

2.1 Data sources

The orbit of 1973-107B has been determined at 90 epochs between 18 September 1983 and 22 December 1984 from 7383 observations, not including those rejected in the orbit determinations.

These observations came from four different sources, the most accurate being those from the University of Aston's Hewitt cameras at the Royal Greenwich Observatory, Herstmonceux, and at Siding Spring in Australia; 344 of these observations were used in 27 of the 90 orbits. The second group consisted of 496 visual observations made by volunteer observers reporting to the Earth Satellite Research Unit at the University of Aston. The third and largest group, of 4200 radar observations, were made by the US Navy Navspasur system, kindly supplied by the US Naval Research Laboratory and the fourth group consisted of 2343 radar observations from the tracking station at RAF Fylingdales.

2.2 Observational accuracy

The rms residuals of the observations have been calculated using the RAE computer program ORES[3], and have been distributed to the observers. Table 1 gives the residuals for selected observing stations with at least five observations accepted in the final orbit determinations. The US Navy observations from station 29 are geocentric, and if they were given in the same form as the topocentric observations, their angular rms residuals would increase by a factor of

Table 1

Residuals for selected stations

Station		Number of accepted observations	Rms residuals			
			Range km	Minutes of arc		
				RA	Dec	Total
1	US Navy	497		1.5	1.3	2.0
2	US Navy	441		3.3	3.1	4.5
3	US Navy	456		3.0	2.3	3.8
4	US Navy	461		3.3	2.4	4.1
5	US Navy	481		2.2	1.9	2.9
6	US Navy	516		1.6	1.5	2.2
29	US Navy	1348	0.6	0.2*	0.2*	
414	Capetown	39(37)		1.2	1.4	1.8
2115	Yateley	15(13)		4.3	3.1	5.3
2122	Malvern 5	6(6)		2.4	2.1	3.2
2265	Farnham	61(55)		2.5	2.3	3.4
2392	Cowbeech	7(7)		0.6	1.7	1.8
2414	Bournemouth	173(152)		2.4	2.6	3.5
2418	Sunningdale	13(10)		0.9	2.2	2.4
2420	Willowbrae	82(75)		2.7	2.8	3.9
2430	Stevenage 4	13(11)		0.7	2.1	2.2
2539	Dymchurch	31(28)		1.3	1.2	1.7
2657	Bridgwater	14(13)		1.6	2.0	2.5
2659	Herstmonceux 3†	219(203)		0.06	0.05	0.08
4156	Apeldoorn	5(5)		1.7	2.2	2.7
4160	Achel 1	9(9)		5.4	3.7	6.5
8517	Sacramento	15(14)		2.9	3.5	4.6
9652	Siding Spring†	125(117)		0.07	0.07	0.10

* Geocentric † Hewitt cameras

NB Figures in brackets indicate the number of observations used to calculate the rms residuals; *ie* those observations with residuals less than twice the rms value.

between 5 and 10. In calculating the rms residuals for the visual observers, observations with residuals greater than twice the rms have been omitted, the numbers used being shown in brackets. This gives a truer impression of the normal accuracy of the observer, as it eliminates observations marred by poor visibility and possible deficiencies in orbital fitting.

The rms residuals of the Hewitt cameras are 5 seconds of arc from 203 observations by the Herstmonceux camera and 6 seconds of arc from 117 observations by the Siding Spring camera. Since the residuals combine the orbital and observational errors, and the orbital model is less accurate than the observations, the observational errors of the Hewitt cameras are likely to be less than their rms residuals, and 2 seconds of arc would be an accuracy consistent with the results.

2.3 Orbits and orbital accuracy

Orbits were determined using RAE's orbit refinement program PROP, in the PROP6 version, and orbital elements at the 90 epochs together with their standard deviations are listed in Table 2 on page 16. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 \quad , \tag{1}$$

where t is the time measured from epoch and the number of M-coefficients used depends on the drag. For 1973 107B, with orbital eccentricity approximately 0.1, and perigee and apogee heights of about 400 and 1700 km respectively, $M_0 - M_2$ were sufficient for 87 of the 90 orbits. The other three orbits required the use of coefficients $M_0 - M_3$.

The value of ε , the parameter which indicates the measure of fit of the observations to the orbit, varied between 0.33 and 0.83 with an average value of 0.56, showing that all of the orbits were fitted satisfactorily. The average number of observations in an orbit determination was 82, spread over a time interval averaging 4.9 days.

The average standard deviation in eccentricity e for the 90 orbits is 0.000011, equivalent to an error in perigee distance of 80 m; the average for the 27 orbits containing Hewitt camera observations was 0.000008. The perigee distances $Q = a(1 - e)$ from Table 2 are plotted in Fig 1, and exhibit the usual sinusoidal oscillations dependent on the argument of perigee, ω ; also plotted in Fig 1, but on a larger scale, are the values of Q', the perigee distance after removal of lunisolar and zonal harmonic perturbations.

The mean standard deviation in inclination for the 90 orbits is 0.0010°, corresponding to an error of 130 m in cross-track distance; for the 27 orbits containing Hewitt camera observations the accuracy is again better, the average standard deviation being 0.0008°.

3 THEORY FOR THE RESONANCE EFFECTS

The theory has been given in detail in Ref 4 and will only be summarized here. The longitude-dependent part of the geopotential at an exterior point (r,θ,λ) is written as[5]

$$\frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=1}^{\ell} \left(\frac{R}{r}\right)^{\ell} P_{\ell}^{m} (\cos \theta) \left\{\bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda\right\} N_{\ell m} \quad , \tag{2}$$

where r is the distance from the Earth's centre, θ is co-latitude, λ is longitude (positive to the east), μ is the gravitational constant for the Earth (398600 km^3/s^2), R is the Earth's equatorial radius (6378.1 km), $P_{\ell}^{m} (\cos \theta)$ is the associated Legendre function of order m and degree ℓ, and $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are the normalized tesseral harmonic coefficients, of which only those of order $m = 27$ are relevant here. The normalizing factor $N_{\ell m}$ is given by

$$N_{\ell m}^{2} = \frac{2(2\ell + 1)(\ell - m)!}{(\ell + m)!} \quad . \tag{3}$$

The rate of change of inclination i caused by a relevant pair of coefficients, $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$, near $\beta:\alpha$ resonance may be written[4] (ignoring terms of order e^2) as

$$\frac{di}{dt} = \frac{n}{\sin i} \left(\frac{R}{a}\right)^{\ell} \bar{F}_{\ell mp} G_{\ell pq} (k \cos i - m) \mathcal{R} \left[j^{\ell-m+1} (\bar{C}_{\ell m} - j\bar{S}_{\ell m}) \exp\{j(\gamma\Phi - q\omega)\} \right] \quad , \qquad \ldots\ldots(4)$$

where $\bar{F}_{\ell mp}$ is Allan's normalized inclination function[6], $G_{\ell pq}$ is a function of eccentricity for which explicit forms and a computer program are given in Ref 4, $\mathcal{R}$ denotes 'real part of' and $j = \sqrt{-1}$. The resonance angle Φ is defined by the equation

$$\Phi = \alpha(\omega + M) + \beta(\Omega - \nu) \quad , \tag{5}$$

where ω is the argument of perigee, M the mean anomaly, Ω the right ascension of the node and ν the sidereal angle. The indices γ, q, k and p in equation (4) are integers, with γ taking the values 1, 2, 3 ... and q the values 0, ±1, ±2, ...; the equations linking ℓ, m, k and p are: $m = \gamma\beta$; $k = \gamma\alpha - q$; $2p = \ell - k$.

Here $\beta = 27$ and $\alpha = 2$, and as we shall only consider the $\gamma = 1$ terms, which are usually dominant, we have $m = 27$ and $k = 2 - q$. The values of ℓ to be taken must be such that $\ell \geqslant m$ and $(\ell - k)$ is even. The successive coefficients which arise (for given γ and q) may be grouped into a lumped harmonic, written as

$$\bar{C}_m^{q,k} = \sum_{\ell} Q_{\ell}^{q,k} \bar{C}_{\ell m} , \qquad \bar{S}_m^{q,k} = \sum_{\ell} Q_{\ell}^{q,k} \bar{S}_{\ell m} , \tag{6}$$

where ℓ increases in steps of 2 from its minimum permissible value ℓ_0 , and the Q_ℓ are constant coefficients, with $Q_{\ell 0} = 1$. The values of the Q_ℓ can be obtained from equation (4), and R.H. Gooding has written a computer program PROF for their evaluation.

The rate of change of eccentricity produced by a relevant pair of coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ near $\beta{:}\alpha$ resonance may be written

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{\ell} \bar{F}_{\ell mp} G_{\ell pq} \left\{\frac{q - \frac{1}{2}(k + 3q)e^2}{e}\right\} \Re \left[j^{\ell-m+1} (\bar{C}_{\ell m} - j\bar{S}_{\ell m}) \exp j(\gamma\Phi - q\omega)\right] ,$$

......(7)

where terms of order e^2 have again been ignored.

As the $G_{\ell pq}$ are of order $e^{|q|}$, it turns out that, for orbits of eccentricity less than 0.1, the leading terms in equation (4) are those with $q = 0$ and $q = \pm 1$, while the main terms in (7) are those with $q = \pm 1$. These are the only terms that will be evaluated in the analysis of 1973-107B.

The explicit forms of equations (4) and (7) are given in Ref 4 for the 31:2 and 29:2 resonances, but not for the 27:2 resonance. The equation for di/dt at 27:2 resonance, with $q = 0$ and $q = \pm 1$, is

$$\frac{di}{dt} = n\left(\frac{R}{a}\right)^{27}\left[\left(\frac{R}{a}\right)\bar{F}_{28,27,13}G_{28,13,0}(27\ \mathrm{cosec}\ i - 2\cot i)\left\{\bar{S}_{27}^{0,2}\sin\Phi + \bar{C}_{27}^{0,2}\cos\Phi\right\}\right.$$
$$+ \bar{F}_{27,27,13}G_{27,13,1}(27\ \mathrm{cosec}\ i - \cot i)\left\{\bar{C}_{27}^{1,1}\sin(\Phi - \omega) - \bar{S}_{27}^{1,1}\cos(\Phi - \omega)\right\}$$
$$\left. + \bar{F}_{27,27,12}G_{27,12,-1}(27\ \mathrm{cosec}\ i - 3\cot i)\left\{\bar{C}_{27}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{27}^{-1,3}\cos(\Phi + \omega)\right\}\right] . \qquad \ldots\ldots(8)$$

A factor $(1 - e^2)^{-\frac{1}{2}}$ should be introduced on the right-hand side if terms of order e^2 are required. The equation for de/dt, with $q = \pm 1$ terms only, is

$$\frac{de}{dt} = \frac{n}{e}\left(\frac{R}{a}\right)^{27}\left[-\bar{F}_{27,27,13}G_{27,13,1}\left\{\bar{C}_{27}^{1,1}\sin(\Phi - \omega) - \bar{S}_{27}^{1,1}\cos(\Phi - \omega)\right\}\right.$$
$$\left. + \bar{F}_{27,27,12}G_{27,12,-1}\left\{\bar{C}_{27}^{-1,3}\sin(\Phi + \omega) - \bar{S}_{27}^{-1,3}\cos(\Phi + \omega)\right\}\right] . \qquad (9)$$

Here, with $\gamma\alpha = 2$ and $q \neq 0$, the factors that need to be introduced on the right-hand side of (9) to take account of terms of order e^2 are given by $\{1 - e^2(1 + 1/q) + O(e^4)\}$, from Ref 4. Thus for the first term in curly brackets in (9) $(q = 1)$, the factor is $\{1 - 2e^2 + O(e^4)\}$, and for the second term $(q = -1)$, the factor is $\{1 + O(e^4)\}$.

4 ANALYSIS OF THE VARIATIONS IN INCLINATION AND ECCENTRICITY

4.1 Progress through resonance

Variations in $\dot{\Phi}$ during the period covered by this study are shown in Fig 2. The increase in $\dot{\Phi}$, from about -10 deg/day through to +7 deg/day, proceeds quite steadily, but is rather slower after the exact resonance, which occurs on 28 April 1984. Progress through resonance is not as slow as would ideally be hoped for: it is 8 times faster than for the recent analysis[7] of 1968-40B at 29:2 resonance, so the resulting perturbations and derived coefficients cannot be expected to be so accurate as those obtained from 1968-40B.

4.2 Analysis of inclination

The raw values of inclination given in Table 2 need to be cleared of perturbations due to zonal harmonics and lunisolar effects: this has been done by use of the PROD computer program[8] with integration at 1-day intervals.

Perturbations due to the $J_{2,2}$ harmonic are recorded within each PROP run and have also been removed. Fig 3 shows the resulting values of i, with sd.

These values of inclination were then fitted with the computer program THROE[9], within which the effects of atmospheric rotation are also removed (the value used for the atmospheric rotation rate Λ was 1.00, in conformity with Ref 10). At first the fittings were made with all three pairs of values of (γ, q) in equation (4), that is, $(\gamma, q) = (1,0)$, $(1,1)$ and $(1,-1)$, as in equation (8). However, it was found that the values for $(\gamma, q) = (1,-1)$ were small and indeterminate, so these terms were dropped. It also became apparent from the THROE runs that a number of the orbits fitted badly, as indeed is obvious from Fig 3. Values of inclination which had residuals greater than 2ε, where ε is the overall measure of fit, were relaxed by doubling the standard deviation and, if necessary, quadrupling it. Also the first four orbits were omitted. As a result of these procedures 12 values had their standard deviations doubled and two values had quadrupled standard deviations. The fitting began at MJD 45628, covered 85 orbits and ended at MJD 46051, the last orbit also being omitted, as it was ill-fitting. The overall measure of fit, ε, was 1.17, which is quite satisfactory.

Although the simultaneous fitting of i and e will be preferred (see section 4.4), the values of the $(\gamma, q) = (1,0)$ terms from the fitting of i alone should be fairly satisfactory. They are:

$$10^9 \hat{C}_{27}^{0,2} = 22.4 \pm 12.4 \quad , \qquad 10^9 \hat{S}_{27}^{0,2} = 30.8 \pm 10.2 \quad . \tag{10}$$

The 'hats' over the lumped harmonics indicate that these are the values emerging from THROE, within which $G_{\ell pq}$ is replaced by an approximation $\hat{G}_{\ell pq}$. See Ref 4 and section 4.4. (The values of $(\bar{C}, \bar{S})_{27}^{1,1}$ are not given as they are subsidiary terms and are normally better determined from e.)

4.3 Analysis of eccentricity

As the eccentricity of 1973-107B is quite large, and decreases appreciably due to drag during the resonance, it is better to work with the perigee distance, which is much less affected by drag. The lower graph in Fig 1 gives the raw values of $a(1 - e)$, showing the characteristic oscillation due to odd zonal harmonics, which has an amplitude near 7 km. The upper graph in Fig 1 shows the values after removal of zonal harmonic and lunisolar perturbations. These values, denoted by Q', should show the effects of air drag and of resonance only. For the fitting by THROE, it is convenient to define a revised value of e, e_{rev} say, based on Q'. Thus

$$e_{rev} = 1 - Q'/\bar{a} \quad , \tag{11}$$

where $\bar{a}$ is the mean value of a during the orbit determinations. To a first approximation the decrease in Q' due to drag at any stage in the orbit determinations is given by $\frac{1}{2}H\ell n(e_0/e)$, where H is the density scale height, e_0 the initial eccentricity and e its current value (corrected to $\omega = \omega_0$). On comparing values of e at similar values of ω in Table 1, the decrease in e between MJD 45595 and 45983 is found to be by a factor 1.0112, and, on taking $H = 50$ km, this gives a decrease of 0.28 km in Q' , which translates into an average increase of almost exactly 10^{-7} per day in e . Thus a new value of e , e^* say, cleared of air drag, may be defined as

$$e^* = e_{rev} - 10^{-7}(t - t_0) \quad . \tag{12}$$

(In practice the correction was taken as $-\frac{1}{2}N \times 10^{-6}$, where N is the orbit number.)

The resulting values of e^* were fitted by THROE with $(\gamma,q) = (1,1)$ and $(1,-1)$. As with i , the first four values were omitted, and a number of the standard deviations were relaxed. The two anomalous values at MJD 46033 and 46037 (see Fig 1) were allocated standard deviations of 0.0001, and eight other standard deviations which exceeded 2ε were relaxed by a factor of 2. The overall measure of fit, ε , then had the value 2.66: this is rather high, but not unusually so, because values of ε near 2 often arise when fitting eccentricity[7].

4.4 Fitting of i and e simultaneously

As both inclination and eccentricity required the $(\gamma,q) = (1,1)$ term, it seemed best to fit them simultaneously with R.H. Gooding's SIMRES program. The SIMRES fitting was made with $(\gamma,q) = (1,0),(1,1)$ and $(1,-1)$ and the eccentricity fitting was given a lower weighting, in accordance with the ratio of the values of ε in the THROE fittings for i and e : the weighting factor was 1.848. The overall measure of fit for the SIMRES fitting was 1.362, and the individual values of ε for i and e were 1.32 and 2.96, as compared with 1.17 and 2.66 for the individual fittings. Thus the fittings are not much worse, and the combined fitting is to be recommended because the $(\gamma,q) = (1,1)$ term is significant for both i and e , and SIMRES provides an 'average'. The values of the harmonics that emerge are:

$$
\begin{aligned}
10^9 \hat{\hat{C}}_{27}^{0,2} &= 18.4 \pm 15.8 \ , & 10^9 \hat{\hat{S}}_{27}^{0,2} &= 26.3 \pm 12.3 \\
10^9 \hat{\hat{C}}_{27}^{1,1} &= 14.4 \pm 15.1 \ , & 10^9 \hat{\hat{S}}_{27}^{1,1} &= -9.1 \pm 13.3 \\
10^9 \hat{\hat{C}}_{27}^{-1,3} &= 14.5 \pm 8.7 \ , & 10^9 \hat{\hat{S}}_{27}^{-1,3} &= 2.4 \pm 11.2
\end{aligned} \qquad . \qquad (13)
$$

It may be noted that the values of $(\hat{\hat{C}},\hat{\hat{S}})_{27}^{0,2}$ in equations (10) are consistent with those in (13).

The fittings of inclination and eccentricity are shown in Figs 4 and 5, where the standard deviations indicated are those after relaxation. It will be seen that in the combined fitting some of the residuals exceed 2ε : for example the residual for i at MJD 45734 is $3.1 (= 2.3\varepsilon)$. Further readjustment of the relaxations was not attempted.

The values of C and S above have been given 'hats' (^) to indicate that they are the values emerging from SIMRES, in which the values of the G functions in equations (8) and (9) are replaced by an approximation $\hat{G}$. Thus if $\bar{C}_m^{q,k}$ is the correct value we have

$$
\hat{G}_{\ell_0 p_0 q} \hat{\hat{C}}_m^{q,k} = G_{\ell_0 p_0 q} \bar{C}_m^{q,k} \ . \qquad (14)
$$

Thus the values of $\hat{\hat{C}}$ and $\hat{\hat{S}}$ in equations (13) have to be divided by $G_{\ell_0 p_0 q}/\hat{G}_{\ell_0 p_0 q}$, values of which (always >1) have been obtained from the computer program GQUAD[4]. For an orbit of such high eccentricity as 1973-107B and such high degree, the corrections are large: the three coefficients arising need to be divided by 3.323, 2.014 and 2.009 respectively.

Thus the values given in equations (13) may be rewritten as

$$
\begin{aligned}
10^9 \bar{C}_{27}^{0,2} &= 5.5 \pm 4.8 \ , & 10^9 \bar{S}_{27}^{0,2} &= 7.9 \pm 3.7 \\
10^9 \bar{C}_{27}^{1,1} &= 7.1 \pm 7.5 \ , & 10^9 \bar{S}_{27}^{1,1} &= -4.5 \pm 6.6 \\
10^9 \bar{C}_{27}^{-1,3} &= 7.2 \pm 4.3 \ , & 10^9 \bar{S}_{27}^{-1,3} &= 1.2 \pm 5.6
\end{aligned} \qquad . \qquad (15)
$$

These are the final values of the lumped harmonics derived from analysis of 1973-107B.

For eccentricity, further THROE runs were made in which the value of the third zonal harmonic J_3 was adjusted to minimize the value of ε, and ε was substantially reduced, from 2.66 to 1.85. However, this 'optimum-J_3' run for e and the THROE run for i, when combined in SIMRES, gave larger standard deviations for the lumped harmonics than those in (15): so the values (15) are preferred.

5 LUMPED HARMONICS IN TERMS OF INDIVIDUAL COEFFICIENTS $\bar{C}_{\ell m}, \bar{S}_{\ell m}$

The lumped harmonics $(\bar{C},\bar{S})_{27}^{q,k}$ are expressible in terms of the individual coefficients $(\bar{C},\bar{S})_{\ell m}$ by equations (6), and the computer program PROF evaluates the Q functions with adequate accuracy if e is very small. For 1973-107B, however, $e \triangleq 0.087$ and it is necessary to multiply each $Q_\ell^{q,k}$ by a correction factor ξ, say, where[11]

$$\xi = \frac{G_{\ell pq}}{\hat{G}_{\ell pq}} \cdot \frac{\hat{G}_{\ell_0 p_0 q}}{G_{\ell_0 p_0 q}} = f\,\frac{G_{\ell pq}}{\hat{G}_{\ell pq}} , \tag{16}$$

and f is the factor by which the lumped harmonics had to be *multiplied*, namely 0.3009, 0.4965 and 0.4978 respectively, for the three pairs of harmonics in equations (15). Values of $G_{\ell pq}/\hat{G}_{\ell pq}$ have been obtained from the computer program GQUAD for values of ℓ up to 48 with $e = 0.087$ and the resulting values of ξ are given in Table 3. It is apparent that ξ departs greatly from 1 for

Table 3

Values of ξ for 1973-107B with $e = 0.087$

$q = 0$		$q = \pm 1$	
ℓ	ξ	ℓ	ξ
28	1.000	27	1.000
30	1.152	29	1.098
32	1.331	31	1.212
34	1.543	33	1.341
36	1.792	35	1.490
38	2.085	37	1.662
40	2.432	39	1.858
42	2.840	41	2.084
44	3.321	43	2.344
46	3.889	45	2.642
48	4.560	47	2.986

high values of ℓ . The resulting expressions for the lumped harmonics in terms of the individual coefficients, after the values of Q from PROF have been multiplied by ξ , are as follows:

$$\bar{C}_{27}^{0,2} = \bar{C}_{28,27} + 0.139\bar{C}_{30,27} - 0.319\bar{C}_{32,27} - 0.395\bar{C}_{34,27} - 0.256\bar{C}_{36,37} - 0.061\bar{C}_{38,27} + 0.089\bar{C}_{40,27} + 0.156\bar{C}_{42,27} + 0.144\bar{C}_{44,27} \quad , \tag{17}$$

$$\bar{C}_{27}^{1,1} = \bar{C}_{27,27} - 1.411\bar{C}_{29,27} - 0.785\bar{C}_{31,27} + 0.094\bar{C}_{33,27} + 0.541\bar{C}_{35,27} + 0.535\bar{C}_{37,27} + 0.278\bar{C}_{39,27} - 0.013\bar{C}_{41,27} - 0.204\bar{C}_{43,27} - 0.255\bar{C}_{45,27} \quad , \tag{18}$$

$$\bar{C}_{27}^{-1,3} = \bar{C}_{27,27} - 0.442\bar{C}_{29,27} - 0.695\bar{C}_{31,27} - 0.508\bar{C}_{33,27} - 0.188\bar{C}_{35,27} + 0.090\bar{C}_{37,27} + 0.244\bar{C}_{39,27} + 0.268\bar{C}_{41,27} + 0.196\bar{C}_{43,27} + 0.083\bar{C}_{45,27} \quad . \tag{19}$$

Similar equations apply for the S coefficients. Equation (17) has been terminated after 9 terms at $\ell = 44$, after which no numerical coefficient exceeds 0.1. Ten terms have been included for equations (18) and (19): in the neglected terms ($\ell > 45$), no numerical coefficient exceeds 0.2.

6 DISCUSSION

6.1 Comparison with comprehensive gravity-field models

A number of recent comprehensive models of the gravity field give values of $\bar{C}_{28,27}$, $\bar{C}_{30,27}, \ldots \bar{C}_{36,27}$ which can be substituted into equation (17) to evaluate the lumped harmonic $\bar{C}_{27}^{0,2}$, and similarly for S . The models chosen for comparison are, as in previous such comparisons[7,11], the Goddard Earth Model 10B (GEM 10B, Ref 12), the 1981 model of Rapp[13], and the European GRIM3-L1 (Ref 14). The newer models GEM T1 and GEM T2 are not included, because the values for

ℓ = 28 to 36 are believed to be less reliable than for the other three models, being based on satellite data only. The values of the lumped harmonics obtained are given in Table 4.

Table 4

Values of lumped harmonics from 1973-107B and comprehensive models

	$10^9\bar{C}_{27}^{0,2}$	$10^9\bar{S}_{27}^{0,2}$	$10^9\bar{C}_{27}^{1,1}$	$10^9\bar{S}_{27}^{1,1}$	$10^9\bar{C}_{27}^{-1,3}$	$10^9\bar{S}_{27}^{-1,3}$
1973-107B	5.5 ± 4.8	7.9 ± 3.7	7.1 ± 7.5	-4.5 ± 6.6	7.2 ± 4.3	1.2 ± 5.6
GEM 10B	-13.4	3.1	9.3	-1.0	6.5	0.6
Rapp 1981	-5.6	6.2	17.6	-19.5	10.4	-8.7
GRIM 3-L1	-8.6	4.4	28.7	-16.8	15.6	-3.6

The estimated standard deviations of the values from the models in Table 4 are mostly near ±4 , slightly lower than those from the resonance. The values in Table 4 vary rather widely, but over 50% differ by less than the sum of the standard deviations, and the resonance values and GEM10B agree particularly well, apart from the first coefficient ($\bar{C}_{27}^{0,2}$). The values from Rapp 1981 and GRIM3-L1 are very similar to each other (probably due to using similar terrestrial gravity data in the solution), and in general do not agree well with the resonance values, except for $\bar{S}_{27}^{0,2}$, though all agree to within about twice the sum of the standard deviations. Thus it can be said that the values from 1973-107B are broadly consistent with the models, but are not accurate enough to provide any significant improvement on the models as a whole.

6.2 Geoid height accuracy

The error in geoid height implied by the standard deviations σ of the lumped harmonics may be roughly estimated as $R\sigma/\bar{Q}$, where R is the Earth's radius and $\bar{Q} = \{\sum(Q_\ell^{q,k}\,\ell_0^2/\ell^2)^2\}^{\frac{1}{2}}$, the summation running from ℓ_0 up to the maximum ℓ considered (44 or 45). For $\bar{C}_{27}^{0,2}$ and $\bar{S}_{27}^{0,2}$ the value of $\bar{Q}$ is 1.09 and, with $\sigma = 4.3 \times 10^{-9}$ as the average, the error in geoid height is about 2.5 cm. For $\bar{C}_{27}^{1,1}$ and $\bar{S}_{27}^{1,1}$ the value of $\bar{Q}$ is 1.75 and, with $\sigma = 7.1 \times 10^{-9}$ as the average, the error in geoid height is about 2.6 cm. For $\bar{C}_{27}^{-1,3}$ and $\bar{S}_{27}^{-1,3}$ the value of $\bar{Q}$ is 1.26 and, with $\sigma = 5.0 \times 10^{-9}$ as the average, the error in geoid height is about 2.5 cm.

6.3 General discussion

This analysis of 1973-107B is the first known attempt at determining lumped harmonics of order 27 from an orbit passing through 27:2 resonance. However the satellite was not ideal for the purpose, because perigee height was below 400 km and hence drag effects were strong enough to carry the orbit through resonance rather rapidly - about 8 times faster than in Walker's recent analysis of 1968-40B at 29:2 resonance[7]. The accuracy achieved here was therefore expected to be considerably poorer than in Walker's analysis, especially as the effects of the resonance on an orbit are smaller when the satellite's altitude is greater. This expectation is confirmed by the results: in Walker's analysis the best geoid height accuracy was 0.5 cm (though the errors in some harmonics were much larger); here the values are all near 2.5 cm.

The high eccentricity of the orbit also causes problems, and it is possible that the $q = \pm 2$ terms in equations (4) and (7) may be significant.

The results therefore show that analysis of 27:2 resonance is feasible, but that a less eccentric orbit of considerably lower drag is needed to obtain values of the lumped harmonics that are much better than those available from the comprehensive models of the gravity field.

7 CONCLUSIONS

The orbit of 1973-107B has been analysed at 90 epochs from nearly 7400 observations, as it passed through 27:2 resonance between September 1983 and December 1984. The orbital accuracy was good, corresponding to positional accuracies of 130 m cross-track and 80 m in perigee distance.

The variations of inclination and eccentricity have been analysed to determine three pairs of lumped harmonics of order 27, with accuracies equivalent to approximately 2.5 cm in geoid height. These accuracies are of the same order as those of values obtainable from comprehensive gravity field models: to improve on the latter values calls for an orbit of considerably lower drag than 1973-107B, *ie* having a perigee height well above 400 km.

Acknowledgments

I thank Desmond King-Hele for his help with the detailed analysis, Martin Suttie for his work on the early orbit determinations and Bob Gooding for providing values of $G_{\ell pq}$.

Table 2

ORBITAL PARAMETERS FOR AUREOLE 2 ROCKET AT THE 90 EPOCHS, WITH STANDARD DEVIATIONS

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
1	45595	1983 Sep 18	7419.1189	0.086729	73.9893	135.976	275.76	154.67	4891.9201	0.0079	–	0.35	56	6.6	6775.67
			4	6	11	1	1	1	4	2					
2	45601	Sep 24	7419.0346	0.086722	73.9862	126.110	264.54	346.59	4892.0034	0.0066	–	0.45	75	6.0	6775.64
			4	7	12	1	1	1	4	2					
3	45609	Oct 2	7418.9127	0.086744	73.9844	112.954	249.60	243.23	4892.1239	0.0096	0.0004	0.46	93	9.4	6775.37
			5	7	10	1	1	1	5	1	<1				
4	45619	Oct 12	7418.6315	0.086848	73.9828	96.506	230.96	205.88	4892.4022	0.0196	0.0006	0.33	92	9.9	6774.34
			4	6	7	1	<1	1	4	1	<1				
5	45628	Oct 21	7418.2300	0.087018	73.9854	81.702	214.17	319.47	4892.7996	0.0194	–	0.41	79	7.9	6772.71
			3	11	11	1	1	1	3	1					
6	45635	Oct 28	7417.9554	0.087169	73.9882	70.187	201.15	10.09	4893.0715	0.0202	–	0.44	57	5.5	6771.34
			5	14	14	1	1	1	5	3					
7	45641	Nov 3	7417.6556	0.087291	73.9889	60.318	190.01	209.39	4893.3684	0.0278	–	0.43	54	4.6	6770.16
			6	16	15	1	<1	1	6	5					
8	45647	Nov 9	7417.3310	0.087413	73.9924	50.450	178.90	50.56	4893.6898	0.0288	–	0.48	62	7.5	6768.96
			3	15	12	1	<1	<1	3	2					
9	45654	Nov 16	7416.9347	0.087590	73.9922	38.934	165.98	107.67	4894.0328	0.0183	–	0.36	49	5.5	6767.33
			3	12	11	1	<1	<1	3	2					
10	45662	Nov 24	7416.7884	0.087794	73.9893	25.772	151.23	20.81	4894.2270	0.0077	–	0.55	71	7.5	6765.64
			4	13	11	1	1	1	4	2					
11	45672	Dec 4	7416.6454	0.088019	73.9868	9.320	132.85	3.67	4894.3686	0.0061	–	0.59	63	6.5	6763.84
			3	12	13	1	1	1	3	2					
12	45680	Dec 12	7416.5237	0.088122	73.9838	356.154	118.13	278.97	4894.4840	0.0081	–	0.52	63	7.9	6762.97
			2	10	12	1	1	1	2	1					
13	45687	Dec 19	7416.4332	0.088207	73.9851	344.633	105.33	340.56	4894.5787	0.0056	–	0.63	63	4.9	6762.25
			6	9	12	1	1	1	6	3					
14	45691	Dec 23	7416.3981	0.088228	73.9859	338.050	98.01	118.86	4894.6135	0.0052	–	0.78	86	4.9	6762.06
			6	9	11	1	1	1	6	4					
15	45697	Dec 29	7416.3319	0.088199	73.9856	328.178	87.01	326.63	4894.6791	0.0069	–	0.83	71	6.9	6762.21
			5	10	11	2	2	1	5	3					
16	45704	1984 Jan 5	7416.2214	0.088144	73.9875	316.655	74.14	29.65	4894.7885	0.0084	–	0.64	95	7.9	6762.53
			2	9	8	1	1	1	2	1					
17	45712	Jan 13	7416.0857	0.088032	73.9890	303.493	59.46	308.34	4894.9230	0.0099	–	0.69	92	7.6	6763.23
			3	11	9	1	1	1	3	1					
18	45719*	Jan 20	7415.9481	0.087893	73.9906	291.974	46.56	13.20	4895.0592	0.0116	–	0.65	100	5.6	6764.14
			2	5	6	1	<1	<1	2	1					
19	45724*	Jan 25	7415.8314	0.087779	73.9885	283.744	37.36	8.72	4895.1748	0.0134	–	0.64	99	4.6	6764.88
			3	5	7	1	<1	<1	3	2					
20	45729*	Jan 30	7415.6855	0.087648	73.9891	275.515	28.16	4.89	4895.3192	0.0138	–	0.62	88	4.7	6765.72
			6	4	7	1	<1	<1	6	2					

Table 2 (continued)

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
21	45734*	1984 Feb 4	7415.5462 2	0.087484 2	73.9886 1	267.286 <1	18.93 <1	1.80 <1	4895.4571 2	0.0143 1	–	0.67	100	3.8	6766.80
22	45738*	Feb 8	7415.4443 3	0.087383 3	73.9880 2	260.700 <1	11.53 <1	143.83 <1	4895.5580 3	0.0114 3	–	0.58	84	4.0	6767.46
23	45743	Feb 13	7415.3238 8	0.087239 13	73.9834 11	252.466 1	2.29 1	141.88 1	4895.6771 8	0.0141 4	–	0.66	79	4.9	6768.42
24	45748	Feb 18	7415.2102 9	0.087034 18	73.9844 13	244.234 1	353.05 1	140.57 1	4895.7896 8	0.0080 5	–	0.67	74	4.9	6769.83
25	45753	Feb 23	7415.1228 9	0.086934 20	73.9855 14	235.998 1	343.77 1	139.74 1	4895.8762 9	0.0092 8	–	0.67	69	4.0	6770.50
26	45758	Feb 28	7415.0253 10	0.086735 19	73.9837 13	227.762 2	334.48 1	139.38 1	4895.9727 10	0.0113 6	–	0.72	71	4.9	6771.88
27	45763	Mar 4	7414.9035 8	0.086587 18	73.9844 13	219.529 1	325.16 1	139.59 1	4896.0934 8	0.0107 5	–	0.68	85	4.9	6772.87
28	45769*	Mar 10	7414.7713 8	0.086476 13	73.9898 11	209.646 1	313.97 1	356.63 1	4896.2246 8	0.0106 5	–	0.54	84	4.9	6773.57
29	45774	Mar 15	7414.6731 8	0.086329 14	73.9867 12	201.411 1	304.62 1	358.09 1	4896.3216 8	0.0090 5	–	0.65	79	4.8	6774.57
30	45779	Mar 20	7414.5647 6	0.086234 12	73.9867 11	193.178 1	295.25 1	0.05 1	4896.4290 6	0.0115 3	–	0.65	90	5.8	6775.18
31	45784	Mar 25	7414.4439 7	0.086173 11	73.9916 12	184.943 1	285.93 1	2.56 1	4896.5490 7	0.0117 5	–	0.63	84	4.9	6775.52
32	45789	Mar 30	7414.2902 11	0.086113 11	73.9924 15	176.707 2	276.55 1	5.75 2	4896.7013 11	0.0197 10	–	0.68	75	3.9	6775.82
33	45793*	Apr 3	7414.1571 11	0.086147 10	73.9941 10	170.118 2	269.05 1	152.93 1	4896.8333 11	0.0123 8	–	0.75	90	4.0	6775.45
34	45797*	Apr 7	7414.0318 9	0.086094 11	73.9904 8	163.528 2	261.55 1	300.58 1	4896.9573 9	0.0173 8	–	0.64	80	3.9	6775.73
35	45801*	Apr 11	7413.9180 9	0.086141 11	73.9934 9	156.938 1	254.07 1	88.72 1	4897.0702 9	0.0136 7	–	0.57	95	3.9	6775.28
36	45804*	Apr 14	7413.8536 19	0.086189 6	73.9923 9	151.995 1	248.43 1	20.12 1	4897.1339 19	0.0063 14	–	0.65	100	2.9	6774.86
37	45807	Apr 17	7413.7898 20	0.086224 11	73.9890 14	147.049 1	242.80 1	311.68 1	4897.1971 19	0.0091 16	–	0.61	57	2.7	6774.54
38	45810*	Apr 20	7413.7279 11	0.086254 6	73.9884 7	142.106 1	237.23 1	243.35 1	4897.2584 11	0.0024 14	–	0.63	100	1.9	6774.26
39	45812*	Apr 22	7413.6870 16	0.086283 3	73.9890 5	138.810 1	233.52 1	317.90 1	4897.2990 16	0.0023 21	–	0.54	62	2.0	6774.01
40	45814*	Apr 24	7413.6579 20	0.086345 8	73.9886 9	135.513 1	229.78 1	32.55 1	4897.3278 20	0.0081 24	–	0.65	77	1.9	6773.53

Table 2 (continued)

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ε	N	D	a(1 - e)
41	45817*	1984 Apr 27	7413.5794 14	0.086369 12	73.9862 6	130.568 1	224.16 1	324.70 1	4897.4056 14	0.0136 13	-	0.60	87	2.9	6773.28
42	45820*	Apr 30	7413.5186 8	0.086442 14	73.9864 9	125.625 2	218.54 1	257.09 1	4897.4659 8	0.0092 9	-	0.61	73	3.9	6772.68
43	45824	May 4	7413.4519 9	0.086515 16	73.9828 13	119.034 1	211.05 1	47.17 1	4897.5318 9	0.0063 7	-	0.54	75	3.9	6772.08
44	45828	May 8	7413.4027 8	0.086626 17	73.9823 15	112.443 1	203.60 1	197.43 1	4897.5806 8	0.0057 5	-	0.55	73	4.9	6771.21
45	45833*	May 13	7413.3298 7	0.086780 15	73.9867 7	104.201 1	194.30 1	205.53 1	4897.6531 7	0.0069 6	-	0.53	83	4.0	6770.00
46	45838	May 18	7413.2483 6	0.086851 18	73.9864 14	95.962 1	185.01 1	214.00 1	4897.7339 6	0.0090 3	-	0.61	82	5.8	6769.40
47	45844	May 24	7413.1175 12	0.087041 16	73.9885 11	86.075 1	173.89 1	80.77 1	4897.8637 12	0.0096 2	-0.0012 2	0.56	73	5.7	6767.87
48	45850*	May 30	7413.0081 6	0.087153 13	73.9912 11	76.189 1	162.77 1	308.26 <1	4897.9723 6	0.0098 3	-	0.53	76	4.6	6766.94
49	45855	Jun 4	7412.9249 5	0.087246 17	73.9893 15	67.949 1	153.50 1	318.32 1	4898.0548 5	0.0093 3	-	0.64	86	5.7	6766.18
50	45861	Jun 10	7412.8327 4	0.087415 13	73.9907 12	58.064 1	142.44 1	186.87 1	4898.1463 4	0.0073 2	-	0.50	85	5.7	6764.84
51	45868	Jun 17	7412.7120 3	0.087565 11	73.9907 12	46.528 1	129.54 1	274.20 1	4898.2660 3	0.0113 2	-	0.60	89	7.6	6763.62
52	45875	Jun 24	7412.5564 3	0.087653 10	73.9897 12	34.992 1	116.65 1	2.51 1	4898.4203 3	0.0108 2	-	0.61	83	6.9	6762.82
53	45882	Jul 1	7412.4064 4	0.087720 8	73.9883 12	23.456 1	103.81 1	91.84 1	4898.5689 4	0.0103 2	-	0.49	69	5.9	6762.19
54	45888*	Jul 7	7412.2863 3	0.087730 5	73.9879 9	13.565 1	92.77 1	323.50 1	4898.6881 3	0.0104 2	-	0.45	67	6.0	6762.01
55	45894	Jul 13	7412.1625 3	0.087716 6	73.9875 9	3.676 1	81.77 1	195.87 1	4898.8108 3	0.0114 2	-	0.46	81	6.0	6762.00
56	45900	Jul 19	7412.0500 4	0.087646 8	73.9886 11	353.785 1	70.75 1	69.00 1	4898.9224 4	0.0063 3	-	0.63	78	5.8	6762.41
57	45906	Jul 25	7411.9951 3	0.087569 8	73.9894 10	343.894 1	59.72 1	302.61 1	4898.9768 3	0.0041 2	-	0.50	80	5.8	6762.93
58	45912	Jul 31	7411.9501 4	0.087436 11	73.9901 10	334.003 1	48.70 1	176.50 1	4899.0214 4	0.0039 2	-	0.52	80	5.8	6763.88
59	45918	Aug 6	7411.8903 3	0.087317 10	73.9904 8	324.111 1	37.63 1	50.75 1	4899.0807 3	0.0038 2	-	0.45	90	5.8	6764.71
60	45924	Aug 12	7411.8365 4	0.087136 14	73.9939 10	314.221 1	26.57 1	285.31 1	4899.1342 4	0.0049 2	-	0.53	86	6.0	6766.00

Table 2 (continued)

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
61	45929	1984 Aug 17	7411.7873	0.087006	73.9925	305.979	17.31	301.08	4899.1829	0.0043	-	0.51	92	4.9	6766.92
			3	12	9	1	1	<1	3	2					
62	45934	Aug 22	7411.7487	0.086849	73.9917	297.734	8.05	317.08	4899.2211	0.0035	-	0.64	82	4.9	6768.05
			7	19	12	1	1	1	7	5					
63	45939	Aug 27	7411.7055	0.086703	73.9910	289.491	358.78	333.28	4899.2639	0.0050	-	0.55	88	4.9	6769.09
			5	16	10	1	1	1	5	4					
64	45944	Sep 1	7411.6514	0.086610	73.9920	281.246	349.46	349.76	4899.3176	0.0048	-	0.60	82	3.9	6769.73
			7	21	13	1	1	1	7	6					
65	45948	Sep 5	7411.5934	0.086426	73.9888	274.649	342.03	147.13	4899.3749	0.0128	-	0.51	87	4.0	6771.04
			7	17	11	1	1	1	7	6					
66	45953	Sep 10	7411.5053	0.086286	73.9869	266.404	332.70	164.30	4899.4621	0.0078	-	0.39	100	4.9	6772.00
			4	12	8	1	<1	1	4	3					
67	45957	Sep 14	7411.4477	0.086165	73.9861	259.804	325.22	322.32	4899.5192	0.0067	-	0.41	100	3.9	6772.84
			6	12	9	1	1	1	6	5					
68	45961	Sep 18	7411.4005	0.086111	73.9862	253.207	317.76	120.52	4899.5660	0.0064	-	0.53	100	4.9	6773.20
			7	13	11	1	1	1	7	4					
69	45966*	Sep 23	7411.3273	0.086034	73.9883	244.957	308.41	138.59	4899.6387	0.0080	-	0.52	83	3.9	6773.70
			7	7	7	<1	<1	<1	7	6					
70	45970*	Sep 27	7411.2584	0.085951	73.9876	238.359	300.96	297.31	4899.7070	0.0087	-	0.59	86	3.9	6774.25
			9	7	14	1	1	1	9	8					
71	45974	Oct 1	7411.2021	0.085857	73.9880	231.763	293.42	96.38	4899.7628	0.0055	-	0.51	93	4.9	6774.90
			8	10	13	1	1	1	8	5					
72	45979	Oct 6	7411.1450	0.085801	73.9892	223.515	284.05	115.43	4899.8195	0.0055	-	0.34	80	3.7	6775.26
			6	6	9	1	1	1	6	6					
73	45983	Oct 10	7411.0966	0.085765	73.9900	216.918	276.56	274.88	4899.8676	0.0041	-	0.45	76	3.7	6775.48
			7	8	12	1	1	1	7	8					
74	45988	Oct 15	7411.0516	0.085779	73.9920	208.672	267.19	294.44	4899.9122	0.0031	-	0.48	98	4.7	6775.34
			4	6	4	1	1	1	4	4					
75	45992	Oct 19	7411.0201	0.085794	73.9940	202.071	259.68	94.24	4899.9436	0.0050	-	0.52	72	2.7	6775.20
			9	9	4	1	1	1	10	15					
76	45995	Oct 22	7410.9822	0.085821	73.9936	197.124	254.03	34.21	4899.9812	0.0039	-	0.60	100	3.7	6774.96
			7	9	5	1	1	1	7	9					
77	45999	Oct 26	7410.9444	0.085828	73.9915	190.524	246.51	194.32	4900.0186	0.0040	-	0.65	100	3.7	6774.88
			9	13	7	2	1	1	9	8					
78	46003	Oct 30	7410.9200	0.085901	73.9903	183.928	239.02	354.53	4900.0428	0.0015	-	0.56	83	3.7	6774.31
			8	12	8	1	1	1	8	7					
79	46007	Nov 3	7410.8909	0.085983	73.9910	177.329	231.53	154.82	4900.0717	0.0050	-	0.52	100	4.6	6773.68
			6	14	11	1	1	1	6	4					
80	46012	Nov 8	7410.8446	0.086034	73.9889	169.082	222.19	175.35	4900.1176	0.0052	-	0.43	83	4.3	6773.26
			7	11	11	1	1	1	7	5					

Table 2 (concluded)

	MJD	Date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ε	N	D	a(1 − e)
81	46016*	1984 Nov 12	7410.8039	0.086197	73.9901	162.478	214.69	335.98	4900.1580	0.0037	-	0.58	100	4.0	6772.01
			6	7	9	1	1	1	6	7					
82	46019*	Nov 15	7410.7795	0.086238	73.9884	157.533	209.12	276.49	4900.1822	0.0016	-	0.50	65	2.0	6771.69
			8	10	9	1	<1	<1	8	17					
83	46022	Nov 18	7410.7142	0.086264	73.9876	152.584	203.53	217.16	4900.2470	0.0078	-	0.46	81	4.9	6771.44
			6	11	12	1	<1	<1	6	4					
84	46028	Nov 24	7410.6131	0.086468	73.9871	142.683	192.34	98.98	4900.3473	0.0094	-	0.49	98	6.0	6769.83
			4	15	11	1	<1	<1	4	2					
85	46033*	Nov 29	7410.5041	0.086435	73.9894	134.435	183.00	121.00	4900.4555	0.0082	-	0.69	96	3.8	6769.98
			9	9	9	1	<1	<1	9	7					
86	46037*	Dec 3	7410.4170	0.086522	73.9900	127.835	175.54	283.01	4900.5420	0.0080	-	0.78	82	3.8	6769.25
			4	5	8	1	<1	<1	4	9					
87	46041*	Dec 7	7410.3448	0.086812	73.9870	121.235	168.24	85.23	4900.6137	0.0087	-	0.54	90	4.8	6767.04
			8	7	8	1	<1	<1	8	4					
88	46046*	Dec 12	7410.2718	0.086960	73.9883	112.987	158.97	108.47	4900.6862	0.0089	-	0.53	89	4.7	6765.87
			7	11	8	1	<1	<1	7	4					
89	46051*	Dec 17	7410.1937	0.087030	73.9902	104.738	149.72	132.06	4900.7637	0.0097	-	0.57	79	3.7	6765.28
			7	5	9	1	<1	<1	7	8					
90	46056	Dec 22	7410.1385	0.087171	73.9935	96.491	140.48	155.99	4900.8188	0.0044	-	0.58	80	5.7	6764.19
			4	11	11	1	1	1	4	3					

Key:
- * = orbits containing Hewitt camera observations
- MJD = modified Julian day
- a = semi major axis (km)
- e = eccentricity
- i = inclination (degrees)
- Ω = right ascension of ascending node (degrees)
- ω = argument of perigee (degrees)
- M_0 = mean anomaly at epoch (degrees)
- M_1 = mean motion n (degree/day)
- M_2, M_3 = later coefficients in the polynomial for M
- ε = measure of fit
- N = number of observations accepted
- D = time covered by the observations (days)

REFERENCES

No.	Author	Title, etc
1	D.G. King-Hele D.M.C. Walker J.A. Pilkington A.N. Winterbottom H. Hiller G.E. Perry	*The RAE table of Earth satellites 1957-1986.* Macmillan, London (1987)
2	R.H. Gooding	The evolution of the PROP6 orbit determination program, and related topics. RAE Technical Report 74164 (1974)
3	D.W. Scott	ORES: a computer program for the analysis of residuals from PROP. RAE Technical Report 69163 (1969)
4	R.H. Gooding D.G. King-Hele	Explicit forms of some functions arising in the analysis of resonant satellite orbits. *Proc. Roy. Soc. Lond.*, A <u>422</u>, 241-259 (1989)
5	W.M. Kaula	*Theory of satellite geodesy.* Blaisdell; Waltham, Mass ('966)
6	R.R. Allan	Resonant effect on inclination for close satellites. RAE Technical Report 71245 (1971) [Also *Planet. Space Sci.*, <u>21</u>, 205-225 (1973)]
7	D.M.C. Walker	Orbit determination and analysis of Cosmos 220 rocket at resonance to evaluate 29th-order harmonics. *Planet. Space Sci.*, <u>37</u>, 351-362 (1989) RAE Technical Report 88055 (1988)
8	G.E. Cook	PROD, a computer program for predicting the development of drag-free satellite orbits. Part 1: Theory. RAE Technical Report 71007 (1971) *Celestial Mechanics*, <u>7</u>, 301-314 (1973)
9	R.H. Gooding	Lumped geopotential coefficients $\bar{C}_{15,15}$ and $\bar{S}_{15,15}$ obtained from resonant variation in the orbit of Ariel 3. RAE Technical Report 71068 (1971)

REFERENCES (concluded)

No.	Author	Title, etc
10	D.G. King-Hele D.M.C. Walker	Upper-atmosphere zonal winds from satellite orbit analysis: an update. *Planet. Space Sci.*, 36, 1085-1093 (1988)
11	D.G. King-Hele D.M.C. Walker	Evaluation of 15th- and 30th-order geopotential harmonic coefficients from 26 resonant satellite orbits. *Planet. Space Sci.*, 37, 805-823 (1989) RAE Technical Report 88045 (1988)
12	F.J. Lerch B.H. Putney C.A. Wagner S.M. Klosko	Goddard Earth Models for oceanographic applications (GEM 10B and 10C). *Marine Geodesy*, 5, 145-187 (1981)
13	R.H. Rapp	The Earth's gravity field to degree and order 180 using Seasat altimeter data, terrestrial gravity data, and other data. Ohio State University, *Department of* Geodetic Science and Surveying, Report No.322 (1981)
14	C. Reigber G. Balmino H. Müller W. Bosch B. Moynot	GRIM gravity model improvement using Lageos (GRIM3-L1). *J. Geophys. Res.*, 90, 9285-9299 (1985)

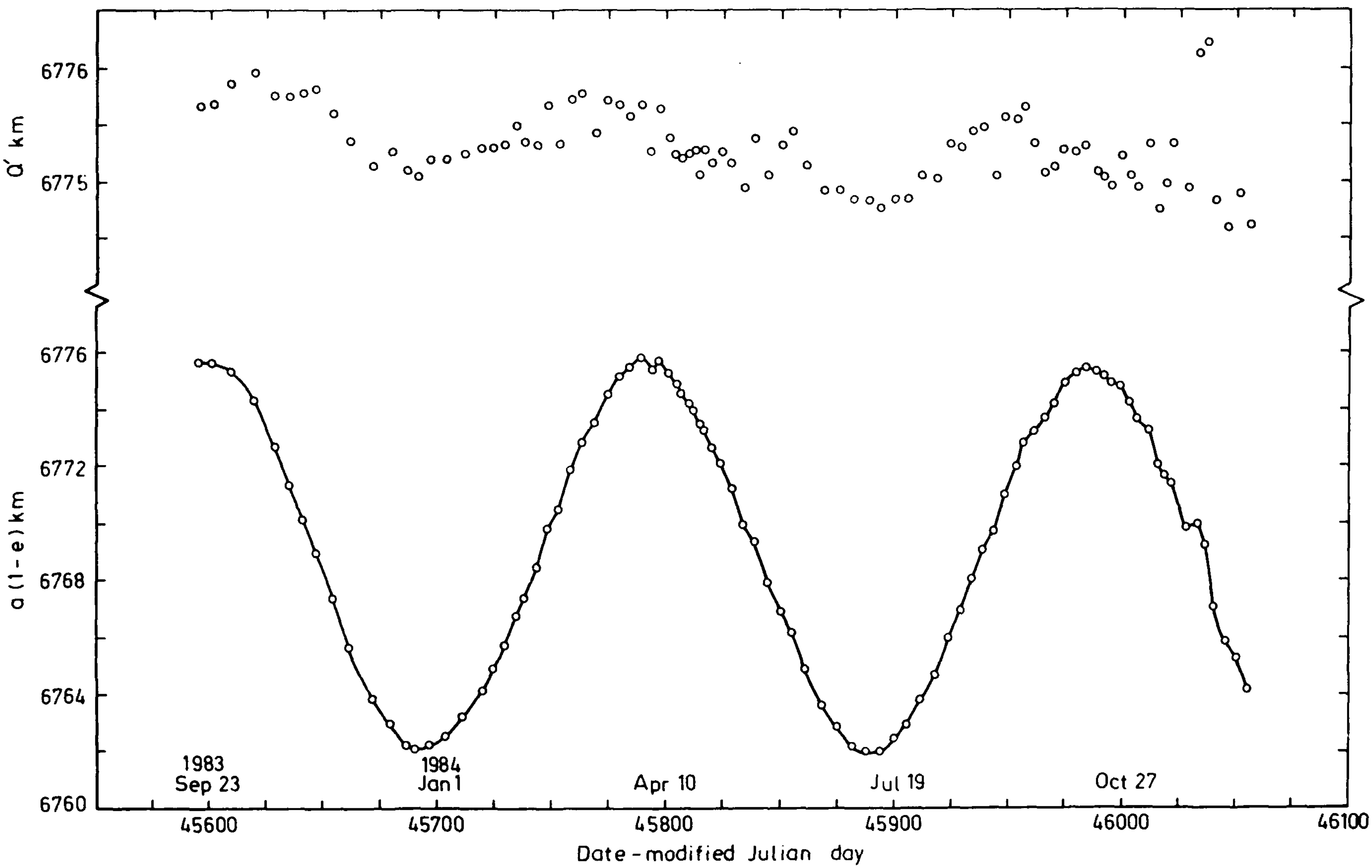

Fig 1 Values of a(1-e) from Table 2, and after removal of lunisolar and zonal harmonic perturbations, Q'

Fig 2

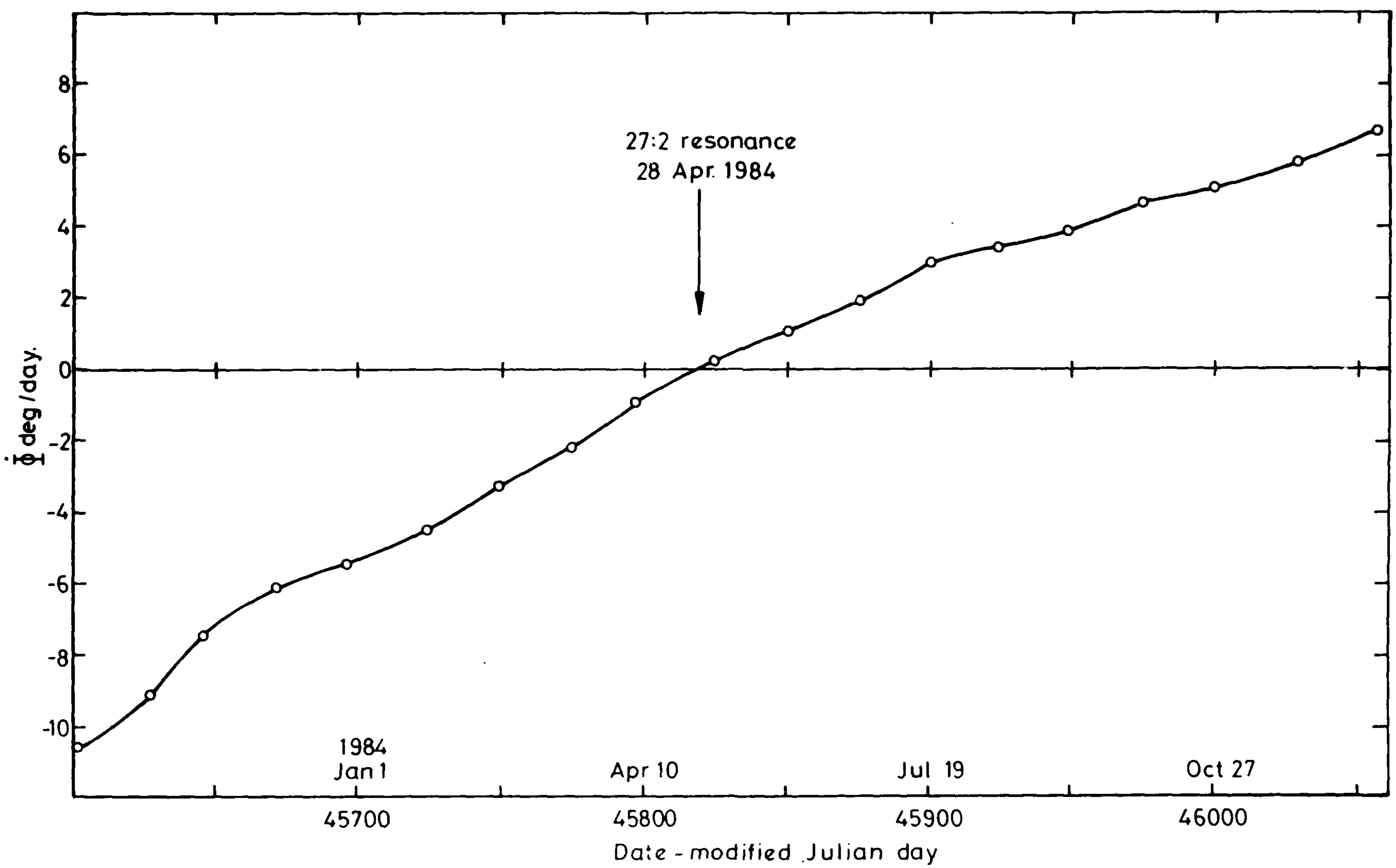

Fig 2 Variation of $\dot{\Phi}$

TR 90018

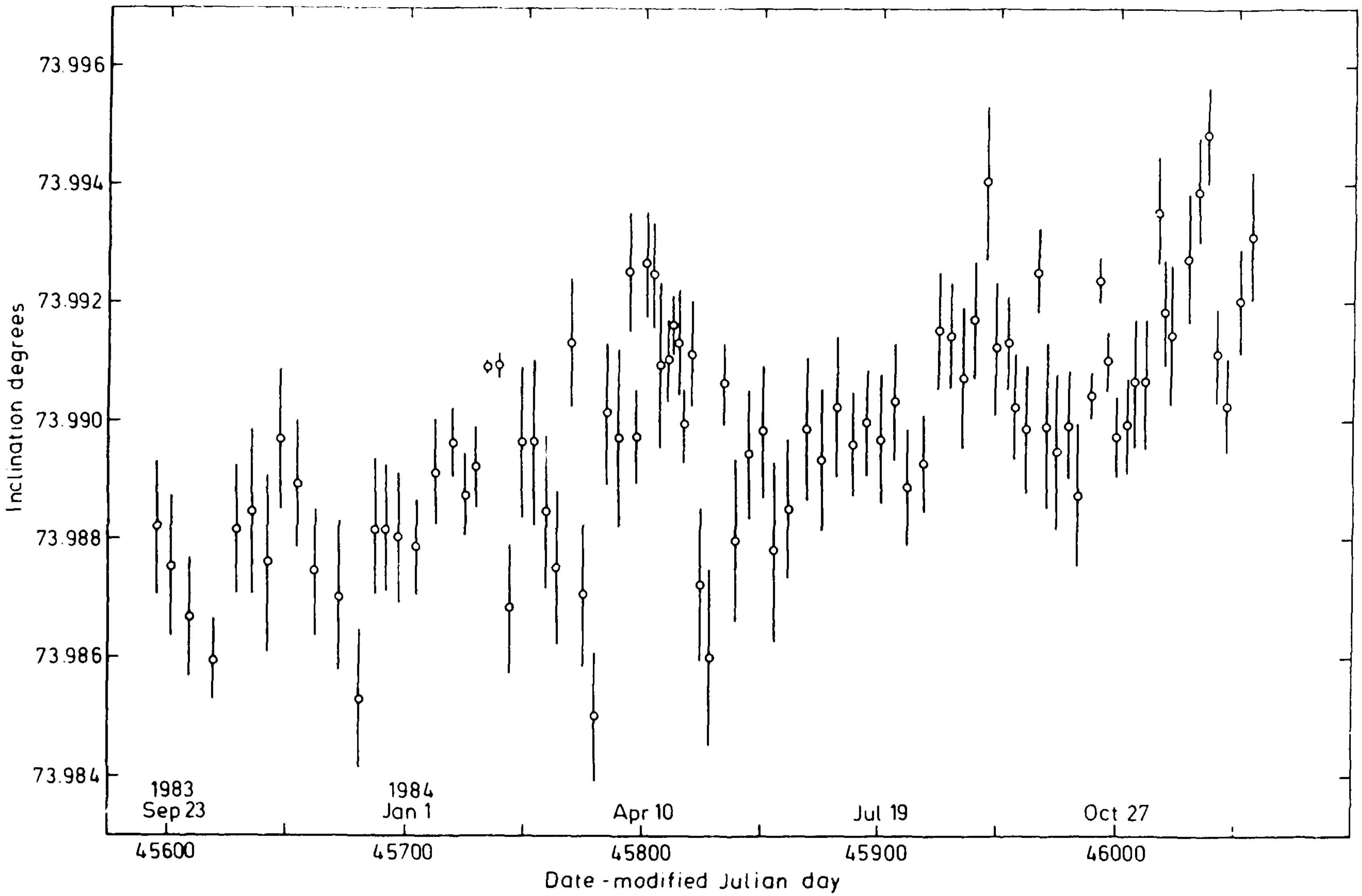

Fig 3 Values of inclination after removal of lunisolar, zonal harmonic and $J_{2,2}$ perturbations

Fig 4

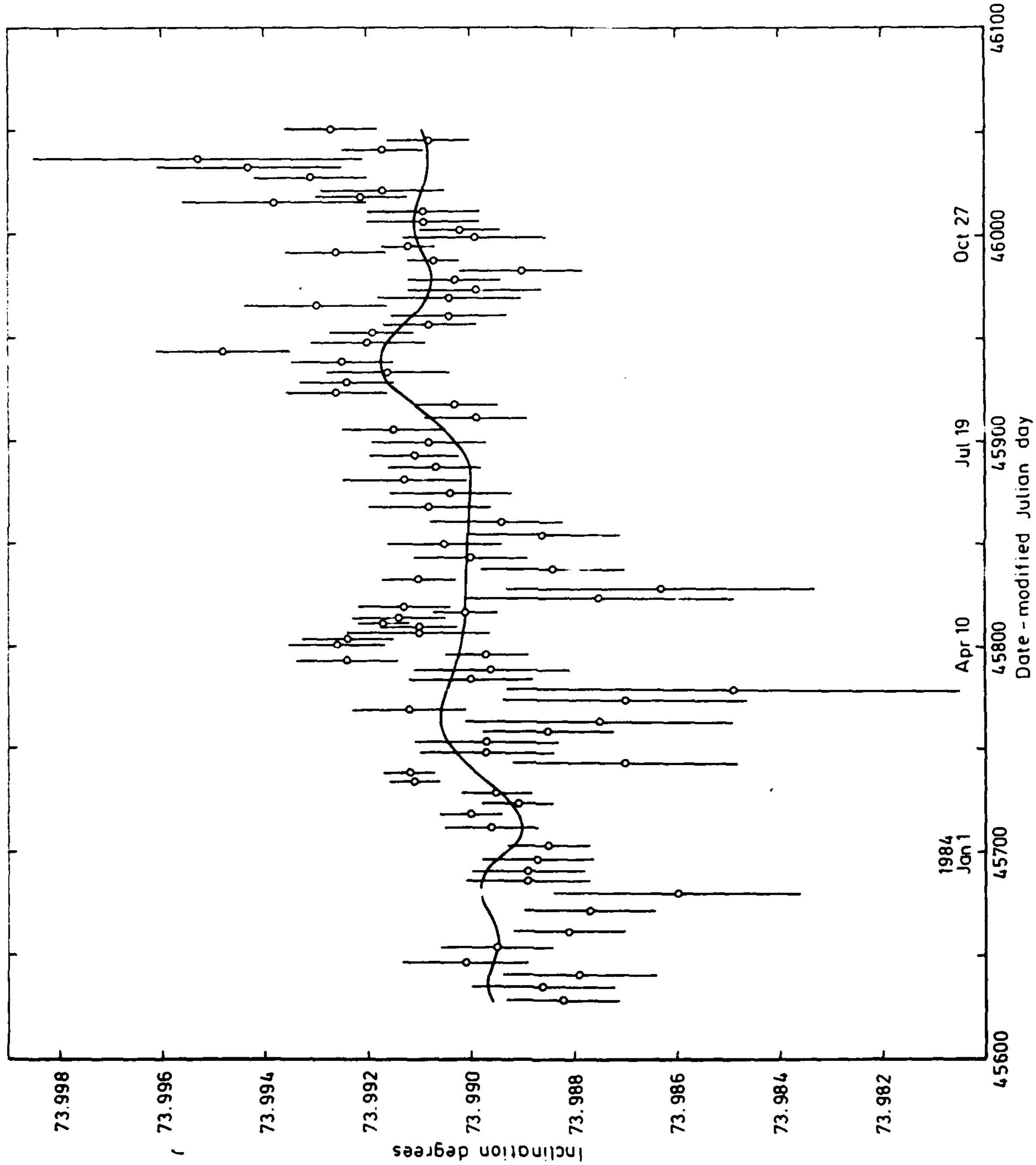

Fig 4 Values of inclination with fitted curve in SIMRES fitting of i and e with (γ,q) = (1,0),(1,1),(1,-1)

TR 90018

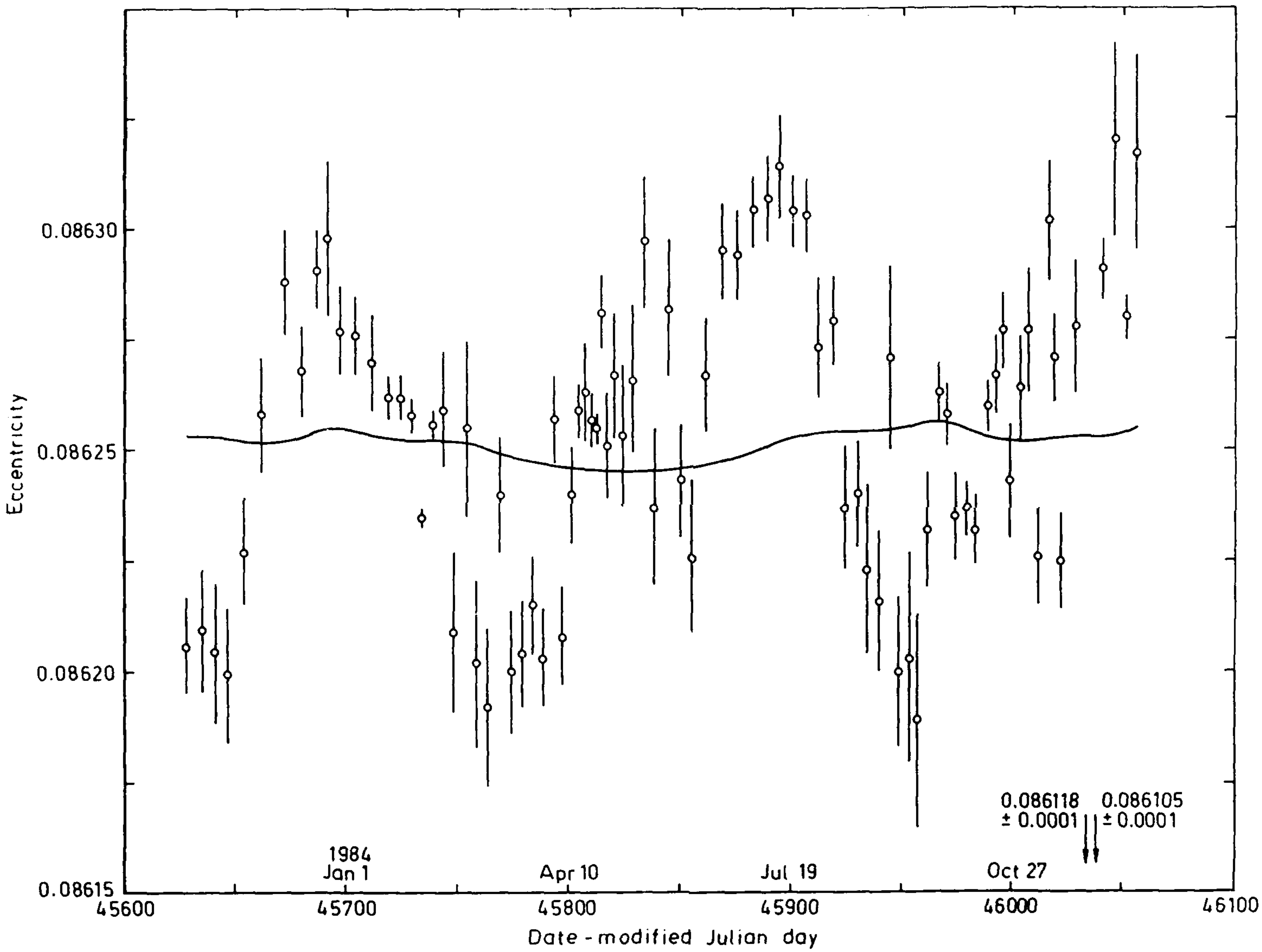

Fig 5 Values of eccentricity with fitted curve in SIMRES fitting of i and e with $(\gamma, q) = (1,0),(1,1),(1,-1)$

REPORT DOCUMENTATION PAGE

Overall security classification of this page

UNLIMITED

As far as possible this page should contain only unclassified information. If it is necessary to enter classified information, the box above must be marked to indicate the classification, e.g. Restricted, Confidential or Secret.

1. DRIC Reference (to be added by DRIC)	2. Originator's Reference RAETR 90018	3. Agency Reference	4. Report Security Classification/Marking UNLIMITED

5. DRIC Code for Originator	6. Originator (Corporate Author) Name and Location
7673000W	Royal Aerospace Establishment, Farnborough, Hants, UK
5a. Sponsoring Agency's Code	6a. Sponsoring Agency (Contract Authority) Name and Location

7. Title Orbit determination and analysis for Aureole 2 rocket at 27:2 resonance

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials	9a. Author 2	9b. Authors 3, 4	10. Date	Pages	Refs
Winterbottom, A.N.			February 1990	27	14

11. Contract Number	12. Period	13. Project	14. Other Reference Nos.
			SS 11

15. Distribution statement

(a) Controlled by –

(b) Special limitations (if any) –

If it is intended that a copy of this document shall be released overseas refer to RAE Leaflet No.3 to Supplement 6 of MOD Manual 4.

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Orbital determination. Orbit analysis. Geopotential harmonics. Satellite orbits. Resonance.

17. Abstract

Aureole 2 rocket (1973-107B) was launched on 26 December 1973 into an orbit of inclination 74° and eccentricity 0.1 and has an estimated lifetime of 30 years. The orbit has been determined from observations for 90 epochs between September 1983 and December 1984, during which time the orbit was expected to be influenced significantly by the effects of 27:2 resonance with the Earth's gravitational field: exact resonance occurred on 28 April 1984. The observations numbered nearly 7400, of which 344 were from the Hewitt cameras of the University of Aston which are sited at Herstmonceux in England, and Siding Spring in Australia. The orbital inclination and eccentricity of the orbits derived had standard deviations corresponding on average to positional accuracies of 130 m cross-track and 80 m in perigee distance.

The variations in inclination and eccentricity have been analysed individually to determine values of two pairs of lumped harmonics of order 27 from each parameter; when these parameters were fitted simultaneously they gave three pairs of harmonics with standard deviations corresponding to accuracies of approximately 2.5 cm in geoid height.

F5910/1