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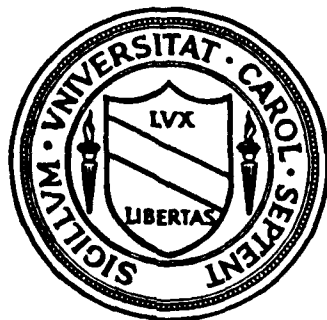
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CHARACTERIZATIONS OF ONE-SIDED FRACTIONAL LÉVY MOTIONS

by

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Technical Report No. 289

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CHARACTERIZATIONS OF ONE-SIDED LINEAR FRACTIONAL LÉVY MOTIONS*

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ABSTRACT

We show that the only self-similar stable processes with stationary increments whose left-equivalent (resp. right-equivalent) stationary processes are nonanticipating (resp. fully anticipating) moving averages are the left (resp. right) linear fractional Lévy motions.

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1. Introduction and characterizations

A stochastic process $X = (X(t))_{t \in \mathbb{R}}$ is self-similar with parameter $H > 0$ (H-ss) if $X(c \cdot) \stackrel{d}{=} c^H X(\cdot)$ for all $c > 0$, and has stationary increments (si) if $X(\cdot + b) - X(b) \stackrel{d}{=} X(\cdot)$ for all $b \in \mathbb{R}$, where $\stackrel{d}{=}$ means the equality of all finite-dimensional distributions. Every H-ss si process is stochastically continuous [V1]. A real-valued stochastic process X is α -stable if all finite-dimensional distributions are α -stable where $0 < \alpha \leq 2$, and when $\alpha = 2$, it is Gaussian.

The only H-ss si Gaussian process is the fractional Brownian motion (see [MV]). On the other hand, when $0 < \alpha < 2$, many H-ss si α -stable processes have been recognized in the past ten years (see [KM]). Among these, the linear fractional Lévy motion seems to be the most important example of an H-ss si α -stable process for $0 < \alpha < 2$, and its properties have been well studied (see, e.g., [CM], [ST], [ALT]).

As discussed in our previous paper [CM], to any stochastically continuous si α -stable process $X = (X(t))_{t \in \mathbb{R}}$ with $1 < \alpha \leq 2$ correspond two stationary α -stable processes $Y_L = (Y_L(t))_{t \in \mathbb{R}}$ and $Y_R = (Y_R(t))_{t \in \mathbb{R}}$ in the following way:

$$(1.1) \quad Y_L(t) = \int_{-\infty}^0 e^u [X(t) - X(t+u)] du = X(t) - \int_{-\infty}^t e^{u-t} X(u) du, \quad t \in \mathbb{R},$$

and

$$(1.2) \quad Y_R(t) = \int_0^{\infty} e^{-u} [X(t) - X(t+u)] du = X(t) - \int_t^{\infty} e^{t-u} X(u) du, \quad t \in \mathbb{R}.$$

A straightforward calculation gives for all $s < t$,

$$(1.3) \quad X(t) - X(s) = Y_L(t) - Y_L(s) + \int_s^t Y_L(u) du$$

and

$$(1.4) \quad X(t) - X(s) = Y_R(t) - Y_R(s) - \int_s^t Y_R(u) du.$$

((1.3) is given in (2.2) on p. 307 in [CM], where the minus sign before the integral should be replaced by a plus sign as in (1.3)). (1.4) is derived from (1.2) in the same way as (1.3) is derived from (1.1) in [CM].

The existence of the integrals in (1.1) and (1.2), and the derivation of (1.3) and (1.4) from (1.1) and (1.2), respectively, are justified only when $1 < \alpha \leq 2$, (but both are expected to be valid for $0 < \alpha \leq 1$ as well). Therefore, whenever Y_L or Y_R is considered, we assume $1 < \alpha \leq 2$. The correspondence between X and Y_L is one-to-one and so is the correspondence between X and Y_R . Furthermore, it follows from (1.1) and (1.3) that either one of X and Y_L is expressed linearly in terms of past values of the other, so they are left-equivalent in the sense that if $\mathcal{L}_L(X, t)$ is the closure in probability of all finite linear combinations of the values to the left of t : $\{X(u), u \leq t\}$, then

$$\mathcal{L}_L(X, t) = \mathcal{L}_L(Y_L, t), \text{ for any } t \in \mathbb{R}.$$

Similarly it follows from (1.2) and (1.4) that if $\mathcal{L}_R(X, t)$ is the closure in probability of all finite linear combinations of the values to the right of t : $\{X(u), u \geq t\}$, then

$$\mathcal{L}_R(X, t) = \mathcal{L}_R(Y_R, t), \text{ for any } t \in \mathbb{R}.$$

We will call Y_L and Y_R the left-equivalent and right-equivalent stationary process of the si process X .

One of the most important classes of stationary stable processes consists of moving averages:

$$(1.5) \quad Y(t) = \int_{-\infty}^{\infty} g(t-u) dM_{\alpha}(u), \quad t \in \mathbb{R},$$

where $0 < \alpha \leq 2$, $g \in L^{\alpha}(\mathbb{R})$, M_{α} is α -stable Lévy motion (Brownian motion when $\alpha = 2$), i.e., has stationary independent α -stable increments with

$$(1.6) \quad E[\exp\{i\theta[M_{\alpha}(t)-M_{\alpha}(s)]\}] = \begin{cases} \exp\{-|t-s|\theta^2/2\} & \text{if } \alpha = 2, \\ \exp\{-|t-s||\theta|^{\alpha}[1 - i\beta \operatorname{sgn}(\theta) \tan(\pi\alpha/2)]\} & \text{if } 0 < \alpha < 2, \alpha \neq 1, \\ \exp\{-|t-s||\theta|[1 + i\beta(2/\pi)\operatorname{sgn}(\theta)\ln|\theta|]\} & \text{if } \alpha = 1, \end{cases}$$

where $|\beta| \leq 1$. We note that for each $\alpha \in (0, 2]$, M_{α} has $1/\alpha$ -ss increments.

When $g(x) = 0$ for $x < 0$, Y is a nonanticipating moving average process as it is expressed in terms of the past increments of the stable Lévy motion M_{α} : $Y(t) = \int_{-\infty}^t g(t-u) dM_{\alpha}(u)$; and when $g(x) = 0$ for $x > 0$, Y is fully anticipating moving average as it is expressed in terms of the future increments of the stable Lévy motion M_{α} : $Y(t) = \int_t^{\infty} g(t-u) dM_{\alpha}(u)$.

The linear fractional Lévy motion $(\Delta_{\alpha, H}(a, b; t))_{t \in \mathbb{R}}$ is an H -ss si α -stable process defined for $0 < H < 1$, $0 < \alpha \leq 2$, $H \neq 1/\alpha$, $a, b \in \mathbb{R}$, by

$$(1.7) \quad \Delta_{\alpha, H}(a, b; t) = \int_{-\infty}^{\infty} \{a[(t-u)_{+}^{H-1/\alpha} - (-u)_{+}^{H-1/\alpha}] + b[(t-u)_{-}^{H-1/\alpha} - (-u)_{-}^{H-1/\alpha}]\} dM_{\alpha}(u),$$

with the convention $0^{\gamma} = 0$ even for $\gamma < 0$, where we assume $\beta = 0$ in (1.6) if $\alpha = 1$. When $\alpha = 2$, the processes $\Delta_{2, H}(a, b; \cdot)$ are multiples of the fractional Brownian motion for all a and b . However, when $0 < \alpha < 2$, different lines through the origin of the (a, b) plane give different processes ([CM], [ST]).

We now assume $1 < \alpha < 2$. As shown in [CM], the left-equivalent stationary α -stable process $Y_L(a, b; \cdot)$ of the linear fractional Lévy motion $\Delta_{\alpha, H}(a, b; \cdot)$ is a moving average

$$(1.8) \quad Y_L(a,b;t) = \int_{-\infty}^{\infty} g_{a,b}(t-u) dM_{\alpha}(u), \quad t \in \mathbb{R},$$

where

$$(1.9) \quad g_{a,b}(x) = a \left[x_+^{H-1/\alpha} - e^{-x} \int_0^{x_+} e^v v^{H-1/\alpha} dv \right] \\ + b \left[x_-^{H-1/\alpha} - e^{-x} \int_{x_-}^{\infty} e^{-v} v^{H-1/\alpha} dv \right] \\ =: a g_1(x) + b g_2(x).$$

When $Y_L(a,b;\cdot)$ is a nonanticipating moving average of M_{α} , namely $g_{a,b}(x) = 0$ for $x < 0$, then $b = 0$, since for $x < 0$, $g_1(x) = 0$ but $g_2(x) \neq 0$. The linear fractional Lévy motion with $b = 0$, $\Lambda_{\alpha,H}(a,0;\cdot)$, is the one introduced in [TW] and is called left linear fractional Lévy motion. Hence, given a linear fractional Lévy motion, if its left-equivalent stationary process Y_L is a nonanticipating moving average, then it is a left linear fractional Lévy motion.

In this paper we generalize this characterization, replacing the class of linear fractional Lévy motions, by the class of all H -ss si α -stable processes whose left-equivalent stationary stable processes are nonanticipating moving averages.

A similar situation prevails for the right-equivalent stationary process Y_R . As in (1.8) and (1.9), the right equivalent stationary α -stable process Y_R of the linear fractional Lévy motion $\Lambda_{\alpha,H}$ is a moving average

$$(1.10) \quad Y_R(a,b;t) = \int_{-\infty}^{\infty} h_{a,b}(t-u) dM_{\alpha}(u),$$

where

$$\begin{aligned}
 (1.11) \quad h_{a,b}(x) &= a \left[x_+^{H-1/\alpha} - e^x \int_{x_+}^{\infty} e^{-v} v^{H-1/\alpha} dv \right] \\
 &\quad + b \left[x_-^{H-1/\alpha} - e^x \int_0^{x_-} e^v v^{H-1/\alpha} dv \right] \\
 &=: a h_1(x) + b h_2(x).
 \end{aligned}$$

When $Y_R(a,b;\cdot)$ is a fully anticipating moving average of M_α , namely $h_{a,b}(x) = 0$ for $x > 0$, then $a = 0$, since for $x > 0$, $h_2(x) = 0$ but $h_1(x) \neq 0$. Thus, given a linear fractional Lévy motion, if its right-equivalent stationary process Y_R is fully anticipating moving average, then it is a right linear fractional Lévy motion; and we also generalize this characterization to general H-ss si α -stable process as follows.

Theorem 1. Fix $1 < \alpha < 2$, $0 < H < 1$, $H \neq 1/\alpha$. Let X be a nondegenerate H-ss si α -stable process.

(i) If its left-equivalent stationary process Y_L is a nonanticipating moving average, then

$$X(\cdot) \stackrel{d}{=} \Lambda_{\alpha,H}(a,0;\cdot) \quad \text{for some } a \neq 0.$$

(ii) If its right-equivalent stationary process Y_R is a fully anticipating moving average, then

$$X(\cdot) \stackrel{d}{=} \Lambda_{\alpha,H}(0,b;\cdot) \quad \text{for some } b \neq 0.$$

We suspect that the linear fractional Lévy motions $\Lambda_{\alpha,H}$ of (1.7) are the only H-ss si α -stable processes whose left- (or right-) equivalent stationary stable processes are moving averages. However, we are not able to prove this characterization at present.

It is seen from (1.9)-(1.12) that the left-equivalent stationary process Y_L of a nondegenerate linear fractional Lévy motion cannot be a fully

anticipating moving average, because if $g_{a,b}(x) = 0$ for $x > 0$, then $a = b = 0$ and $g_{a,b} \equiv 0$; and, likewise, the right-equivalent stationary process Y_R cannot be a nonanticipating moving average.

A related characterization of the linear fractional Lévy motions would be as the only H-ss si α -stable processes X of the form

$$(1.12) \quad X(t) = \int_{-\infty}^{\infty} [G(t-u) - G(-u)] dM_{\alpha}(u), \quad t \in \mathbb{R},$$

where $G(t - \cdot) - G(-\cdot) \in L^{\alpha}(\mathbb{R})$ for each $t \in \mathbb{R}$. Expressed in this way, this characterization does not involve the corresponding stationary process Y , and hence is stated for all $0 < \alpha < 2$. Vervaat [V2] derives such a characterization under a self-similarity assumption on the kernel of (1.12) of the following form: for all $t, u \in \mathbb{R}, c > 0$,

$$(1.13) \quad G(c(t-u)) - G(-cu) = c^{\beta} \{G(t-u) - G(-u)\}$$

and $G(0) = 0$. Indeed, (1.13) implies immediately that $G(x) = a x_{+}^{\beta} + b x_{-}^{\beta}$. This self-similarity of the kernel G also clearly implies the self-similarity of the process X . Theorem 1 (or more precisely, its proof below) establishes this characterization when the self-similarity of G is replaced by that of X , in the case where $G(x) = 0$ for $x < 0$ or $G(x) = 0$ for $x > 0$. We state this characterization in the following.

Theorem 2. Fix $0 < \alpha < 2, 0 < H < 1, H \neq 1/\alpha$. Let X be a nondegenerate H-ss si α -stable process of the form (1.12). If G vanishes on some half line, then X is a one-sided linear fractional Lévy motion.

2. Proof of Theorem 1.

Since the proofs are similar, we only prove (i) of Theorem 1. Also for notational simplicity, we write Y for Y_L and M for M_α .

Since $Y(t) = \int_{-\infty}^{\infty} g(t-u) dM(u)$ is a nonanticipating moving average, $g(x) = 0$ for $x < 0$. Since $\int_0^{\infty} |g(x)|^\alpha dx > 0$ (otherwise $Y \equiv 0$ and thus X is degenerate), if we define

$$t_0 = \inf\{t \geq 0 \mid \text{Leb}\{x \in (t, t+\delta); g(x) \neq 0\} > 0 \text{ for any } \delta > 0\},$$

then $0 \leq t_0 < \infty$. We then have

$$\begin{aligned} Y(t) &= \int_{-\infty}^{\infty} g(t-u) dM(u) = \int_{-\infty}^{\infty} g(t-(v-t_0)) dM(v-t_0) \\ &\stackrel{d}{=} \int_{-\infty}^{\infty} g_{t_0}(t-v) dM(v), \end{aligned}$$

where $g_{t_0}(x) = g(x + t_0)$ and we have used the fact that M has stationary increments. By the definition of t_0 ,

$$(2.1) \quad \text{Leb}\{x \in (0, \delta); g_{t_0}(x) \neq 0\} > 0 \text{ for any } \delta > 0,$$

and $g_{t_0}(x) = 0, x < 0$. Hereafter we write g for g_{t_0} .

From (1.3) and (1.5), we have for $s < t$,

$$\begin{aligned} (2.2) \quad X(t) - X(s) &\stackrel{d}{=} \int_{-\infty}^{\infty} [g(t-u) - g(s-u) + \int_{s-u}^{t-u} g(v) dv] dM(u) \\ &= \int_{-\infty}^{\infty} [G(t-u) - G(s-u)] dM(u), \end{aligned}$$

where

$$G(x) = g(x) + \int_0^x g(v) dv, \quad x \in \mathbb{R}.$$

Note that $G(x) = 0, x < 0$, since $g(x) = 0, x < 0$.

Fix $t > 0$. Then we have for any $a_1, \dots, a_N, x_1, \dots, x_N \in \mathbb{R}$ and $c > 0$,

$$\begin{aligned}
 & \sum_{n=1}^N a_n [X(c(t + x_n)) - X(cx_n)] \\
 & \stackrel{d}{=} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^N a_n [G(c(t + x_n) - u) - G(cx_n - u)] \right\} dM(u) \\
 & = \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^N a_n [G(c(t + x_n - v)) - G(cx_n - v)] \right\} dM(cv) \\
 & \stackrel{d}{=} c^{1/\alpha} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^N a_n [G(c(t + x_n - v)) - G(cx_n - v)] \right\} dM(v)
 \end{aligned}$$

(where we have used the $1/\alpha$ -self-similarity of M).

$$\begin{aligned}
 & \sum_{n=1}^N a_n [X(t + x_n) - X(x_n)] \\
 & \stackrel{d}{=} \int_{-\infty}^{\infty} \left\{ \sum_{n=1}^N a_n [G(t + x_n - v) - G(x_n - v)] \right\} dM(v).
 \end{aligned}$$

Hence by the self-similarity of X and

$$|E[\exp\{i \int_{-\infty}^{\infty} f(v) dM(v)\}]| = \exp\{-\int_{-\infty}^{\infty} |f(v)|^\alpha dv\},$$

we have for each $c > 0$,

$$\begin{aligned}
 & c \int_{-\infty}^{\infty} \left| \sum_{n=1}^N a_n [G(c(t + x_n - v)) - G(cx_n - v)] \right|^\alpha dv \\
 & = c^{\alpha H} \int_{-\infty}^{\infty} \left| \sum_{n=1}^N a_n [G(t + x_n - v) - G(x_n - v)] \right|^\alpha dv,
 \end{aligned}$$

namely, the functions $c^{1/\alpha-H}[G(c(t-\cdot)) - G(c(-\cdot))]$ and $G(t-\cdot) - G(-\cdot)$ have equal L_α -norms of all linear combinations of their translates. Then, by Kanter's result [K], for each fixed $c > 0$,

$$(2.3) \quad c^{1/\alpha-H}[G(c(t-u)) - G(-cu)] = \epsilon_c[G(t-u-\tau_c) - G(-u-\tau_c)] \quad \text{for a.a. } u,$$

where $\epsilon_c \in \{-1, 1\}$, $\tau_c \in \mathbb{R}$, and they may depend on c . We shall show that $\epsilon_c = 1$ and $\tau_c = 0$ for all $c > 0$.

Suppose $\tau_c > 0$. Then for u satisfying $\max\{0, t - \tau_c\} < u < t$, (2.3)

reduces to

$$c^{1/\alpha - H} G(c(t-u)) = 0, \quad \text{a.a. } u,$$

since $G(x) = 0$, $x < 0$. Namely,

$$G(x) = 0 \quad \text{for a.a. } x \in (0, \min\{ct, c\tau_c\}) =: I_c,$$

and thus

$$g(x) = \int_0^x g(v) dv \quad \text{for a.a. } x \in I_c.$$

The function $f(x) := \int_0^x g(v) dv$ is continuous and satisfies $f(x) = \int_0^x f(v) dv$ for any $x \in I_c$. Thus $f \equiv 0$ on I_c , implying $g = 0$ a.e. on I_c , which contradicts (2.1).

When we suppose $\tau_c < 0$, it is enough to take u satisfying $\max\{t, -\tau_c\} < u < t - \tau_c$ in order to get a contradiction. Namely, for such u , we have $G(t-u-\tau_c) = 0$, so that $G(x) = 0$ for a.a. $x \in (0, \min\{t, -\tau_c\})$. Thus $\tau_c = 0$ for any $c > 0$, and we obtain

$$c^{1/\alpha - H} [G(c(t-u)) - G(-cu)] = \epsilon_c [G(t-u) - G(-u)] \quad \text{for a.a. } u.$$

We thus have for a.a. $u \in (0, t)$, $c^{1/\alpha - H} G(c(t-u)) = \epsilon_c G(t-u)$, namely

$$(2.4) \quad G(cx) = \epsilon_c c^{H-1/\alpha} G(x) \quad \text{for a.a. } x \in (0, t).$$

Choose $0 < a < b < t$ and integrate (2.4); then

$$\int_a^b G(cx) dx = \epsilon_c c^{H-1/\alpha} \int_a^b G(x) dx$$

so that

$$(2.5) \quad \int_{ca}^{cb} G(x) dx = \epsilon_c c^{H-1/\alpha+1} \int_a^b G(x) dx.$$

If $\int_a^b G(x) dx = 0$ for any $0 < a < b < t$, then $G(x) = 0$ a.e. on $(0, ct)$. We thus have the same contradiction as before for each $c > 0$. If $\int_a^b G(x) dx \neq 0$ for some $0 < a < b < t$, then we see from (2.5) that ϵ_c is a continuous function of $c > 0$ and thus $\epsilon_c = \epsilon_1 = +1$ for all $c > 0$.

Therefore, we obtain

$$(2.6) \quad G(cx) = c^{H-1/\alpha} G(x) \quad \text{for all } c > 0 \text{ and a.a. } x \in (0, t).$$

By the same reasoning as before, we see that $G(x_0) \neq 0$ for some $x_0 > 0$ independent of $c > 0$. We thus have from (2.6),

$$(2.7) \quad G(x) = (x/x_0)^{H-1/\alpha} G(x_0) = a x^{H-1/\alpha}, \quad \text{for all } x > 0,$$

where $a = G(x_0) x_0^{1/\alpha-H} \neq 0$. We know that $G(x) = 0$ for all $x < 0$, and assuming $G(0) = 0$ does not change the distribution of X . So, we have

$$(2.8) \quad G(x) = a x_+^{H-1/\alpha}, \quad \text{for all } x \in \mathbb{R}.$$

Noting that any H -ss si process with $H > 0$ satisfies $X(0) = 0$ a.s. (see, e.g., [V1]), we conclude from (2.2) and (2.8) that

$$X(t) = a \int_{-\infty}^{\infty} [(t-u)_+^{H-1/\alpha} - (-u)_+^{H-1/\alpha}] dM(u),$$

which completes the proof.

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- 267. H.L. Hurd, Correlation theory of almost periodically correlated processes, June 89.
- 268. O. Kallenberg, Random time change and an integral representation for marked stopping times, June 89. *Probab. Th. Rel. Fields*, accepted.
- 269. O. Kallenberg, Some uses of point processes in multiple stochastic integration, Aug. 89.
- 270. W. Wu and S. Cambanis, Conditional variance of symmetric stable variables, Sept. 89.
- 271. J. Mijneer, U-statistics and double stable integrals, Sept. 89.
- 272. O. Kallenberg, On an independence criterion for multiple Wiener integrals, Sept. 89.
- 273. G. Kallianpur, Infinite dimensional stochastic differential equations with applications, Sept. 89.
- 274. G.W. Johnson and G. Kallianpur, Homogeneous chaos, p-forms, scaling and the Feynman integral, Sept. 89.
- 275. T. Hida, A white noise theory of infinite dimensional calculus, Oct. 89.
- 276. K. Benhenni, Sample designs for estimating integrals of stochastic processes, Oct. 89. (Dissertation)
- 277. I. Rychlik, The two-barrier problem for continuously differentiable processes, Oct. 89.
- 278. G. Kallianpur and R. Selukar, Estimation of Hilbert space valued parameters by the method of sieves, Oct. 89.

279. G. Kallianpur and R. Selukar, Parameter estimation in linear filtering, Oct. 89.
280. P. Bloomfield and H.L. Hurd, Periodic correlation in stratospheric ozone time series, Oct. 89.
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298. T.S. Chiang, G. Kallianpur and P. Sundar, Propagation of chaos and the McKean-Vlasov equation in duals of nuclear spaces, May 90.