

AD-A226 257

DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188TIC
ECTE

AUG 29 1990

2a. SECURITY CLASSIFICATION AUTHORITY
N/A
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE
N/A

1b. RESTRICTIVE MARKINGS

3. DISTRIBUTION/AVAILABILITY OF REPORT

Approved for Public Release;
Distribution Unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

Technical Report No. 288

5. MONITORING ORGANIZATION REPORT NUMBER(S)

AFOSR-TR- 90 0856

6a. NAME OF PERFORMING ORGANIZATION
University of North Carolina
Center for Stochastic Processes6b. OFFICE SYMBOL
(If applicable)

7a. NAME OF MONITORING ORGANIZATION

AFOSR/NM

6c. ADDRESS (City, State, and ZIP Code)
Statistics Department
CB #3260, Phillips Hall
Chapel Hill, NC 27599-3260

7b. ADDRESS (City, State, and ZIP Code)

Bldg. 410
Bolling Air Force Base, DC 20332-64488a. NAME OF FUNDING/SPONSORING
ORGANIZATION
AFOSR8b. OFFICE SYMBOL
(If applicable)
NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

F49620 85C 0144

8c. ADDRESS (City, State, and ZIP Code)

Bldg. 410
Bolling AFB, DC 20332-6448

10. SOURCE OF FUNDING NUMBERS

PROGRAM
ELEMENT NO.

6.1102F

PROJECT
NO.

2304

TASK
NO.

AS

WORK UNIT
ACCESSION NO.

11. TITLE (Include Security Classification)

Trimmed sums of mixing triangular arrays with stationary rows

12. PERSONAL AUTHOR(S)

Maejima, M. and Morita, Y.

13a. TYPE OF REPORT

preprint

13b. TIME COVERED

FROM _____ TO _____

14. DATE OF REPORT (Year, Month, Day)

1990, March

15. PAGE COUNT

18

16. SUPPLEMENTARY NOTATION

None.

17. COSATI CODES

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

FIELD GROUP SUB-GROUP

XXXXXXXXXXXXXXXXXXXX

N/A

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT

☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION

UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL

Professor Eytan Barouch Jon A. Sjogren

22b. TELEPHONE (Include Area Code)

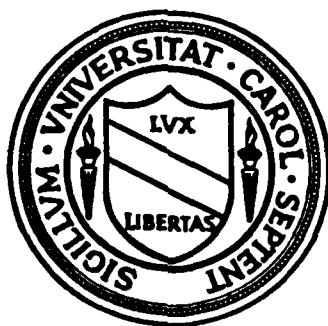
(202) 767-5026

22c. OFFICE SYMBOL

AFOSR/NM

CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
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Chapel Hill, North Carolina



Trimmed sums of mixing triangular arrays with stationary rows

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Technical Report No. 288

March 1990

90 08 27 024

Trimmed sums of mixing triangular arrays with stationary rows



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Unannounced	☐
Justification	
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Distribution /	
Availability Codes	
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A-1	

Abstract:

Limit laws of trimmed sums are studied for triangular arrays of rowwise stationary random variables. It is shown that if the marginal distribution of the array belongs to the domain of attraction of an infinitely divisible law without Gaussian component, the trimmed sum converges weakly to a nondegenerate random variable under some mixing and local dependence conditions.

*Research completed during a visit to the Center for Stochastic Processes, University of North Carolina at Chapel Hill supported by the Air Force Office of Scientific Research Contract No. F49620 85C 0144.

1. Introduction and results

Let $\{X_{nj}, 1 \leq j \leq n; n=1,2,\dots\}$ be a triangular array of rowwise stationary real valued random variables, where stationarity means that for any $1 \leq i_1, \dots, i_k, i_1+h, \dots, i_k+h \leq n$, $(X_{n,i_1}, \dots, X_{n,i_k})$ and $(X_{n,i_1+h}, \dots, X_{n,i_k+h})$ have the same distribution. For each n , let $X_n^{(k)}$ be the k -th largest among $\{X_{n1}, \dots, X_{nn}\}$ in the sense of absolute value such that $|X_n^{(1)}| \geq \dots \geq |X_n^{(n)}|$. Let $S_n = \sum_{j=1}^n X_{nj}$ and define the modulus trimmed sum by

$$^{(r)}S_n = S_n - \sum_{k=1}^r X_n^{(k)},$$

where $r \geq 1$ is a fixed number of trimmed terms. In this paper, we shall study the asymptotic behaviour of $^{(r)}S_n$.

There are two types of modulus trimming. In light trimming r is fixed, and in full trimming $r = r_n$ depends on n and in most cases $r_n \rightarrow \infty$ and $r_n/n \rightarrow 0$ are assumed. Light trimming has been studied by [Dar], [AB], [H], [Ma] and [Mo]. Full trimming has been investigated by, e.g., [KL], [GP1] and [HK].

Natural order trimming is based on $X_{n1}^* \leq \dots \leq X_{nn}^*$, the order statistics of $\{X_{n1}, \dots, X_{nn}\}$, and the naturally trimmed sum is

$$S_n(r_n, s_n) = S_n - \sum_{k=1}^{r_n} X_{nk}^* - \sum_{k=1}^{s_n} X_{n,n-k+1}^*.$$

$^{(2r)}S_n$ and $S_n(r, r)$ are expected to have similar asymptotic behaviour, but this is not necessarily true (cf. [GP2] p. 1188). Natural trimming has been studied by, e.g., [CHoM], [GP1], [CHaM], [GP2]. We shall briefly mention this trimming at the end of the paper.

In all papers mentioned above, the random variables are i.i.d. Here we

relax the condition of independence. Namely, our random variables are stationary, but not necessarily independent. However, we treat only triangular arrays whose marginal distributions belong to the domain of attraction of an infinitely divisible law without Gaussian component.

Our assumptions are as follows.

We first describe the assumptions on the marginal distribution function. Let F_n be the distribution function of X_{nj} , $1 \leq j \leq n$, and let $\{\hat{X}_{nj}, 1 \leq j \leq n; n=1,2,\dots\}$ be the associated triangular array of $\{X_{nj}\}$ with independent random variables, namely for each n , $\{\hat{X}_{nj}\}$ are i.i.d. and $\hat{X}_{nj}$ has distribution F_n .

(A.1) there exist $\{c_n\}$ such that $\sum_{i=1}^n \hat{X}_{ni} - c_n$ converges in law to an infinitely divisible random variable without Gaussian component whose characteristic function is given by

$$(1.1) \quad f(\theta) = \exp\left\{ \int_{\mathbb{R} \setminus \{0\}} h(\theta, x) \nu(dx) \right\}$$

where $h(\theta, x) = e^{i\theta x} - 1 - i\theta x 1_{\{|x| \leq 1\}}$ and $\nu(\cdot)$ is a Lévy measure on $\mathbb{R} \setminus \{0\}$ satisfying $\int_{\mathbb{R} \setminus \{0\}} (x^2 \wedge 1) \nu(dx) < \infty$.

It is known (e.g., see [P]) that (A.1) is equivalent to the following two conditions:

$$(1.2) \quad \begin{cases} n(1 - F_n(x)) \rightarrow \nu((x, \infty)), & x > 0, \\ nF_n(x) \rightarrow \nu((-\infty, x]), & x < 0, \end{cases}$$

at all continuity points x of ν , and

$$(1.3) \quad \lim_{\epsilon \rightarrow 0} \lim_{n \rightarrow \infty} n \left\{ \int_{|x| < \epsilon} x^2 dF_n(x) - \left(\int_{|x| < \epsilon} x d(F_n(x)) \right)^2 \right\} = 0.$$

(A.2) $\nu(\cdot)$ has a polar decomposition of product type such that for another Lévy measure $\rho(\cdot)$ on $(0, \infty)$ and for $0 \leq p, q \leq 1$ with $p + q = 1$,

$$\nu(A) = p \rho(A \cap (0, \infty)) + q \rho((-A) \cap (0, \infty)), \quad A \in \mathcal{B}(\mathbb{R} \setminus \{0\}).$$

Hence, (1.2) can be written as

$$(1.4) \quad \begin{cases} n(1 - F_n(x)) \rightarrow p \rho((x, \infty)), & x > 0, \\ nF_n(-x) \rightarrow q \rho([x, \infty)), & x > 0. \end{cases}$$

We next explain the assumptions on the joint distribution functions, namely some mixing, local dependence and other dependence conditions. Our conditions are those originally formulated by Leadbetter [Le] and then modified by Davis [Dav2], (also see [JK]).

(D.1) For every set B which is a finite union of disjoint intervals of the form

$$\bigcup_{i=1}^d (a_i, b_i] \text{ whose closure does not contain } 0, \text{ as } n \rightarrow \infty$$

$$\sup |P\{X_{nj} \in B^c, p < j \leq r\} - P\{X_{nj} \in B^c, p < j \leq q\} P\{X_{nj} \in B^c, q < j \leq r\}| \rightarrow 0,$$

where supremum is taken over all p, q, r such that $0 \leq p \leq q \leq r \leq n$.

(D.2) For all $x > 0$,

$$\lim_{k \rightarrow \infty} \limsup_{n \rightarrow \infty} n \sum_{j=2}^{[n/k]} P\{|X_{n1}| > x, |X_{nj}| > x\} = 0.$$

$$(D.3) \quad \lim_{\epsilon \rightarrow 0} \limsup_{n \rightarrow \infty} n \sum_{j=2}^n \max\{0, \text{Cov}(X_{n1}^1\{ |X_{n1}| < \epsilon\}, X_{nj}^1\{ |X_{nj}| < \epsilon\})\} = 0.$$

To state our theorem, we need more notation. Let $\{\delta_j\}_{j=1}^{\infty}$ be a sequence of i.i.d. random variables such that $P\{\delta_1 = 1\} = p$ and $P\{\delta_1 = -1\} = q$, where p and q are defined in assumption (A.2). Let $\{\Gamma_j\}_{j=1}^{\infty}$ be Poisson arrival times with unite rate, namely $\Gamma_j = e_1 + \dots + e_j$, where $\{e_j\}_{j=1}^{\infty}$ are i.i.d. random variables having the exponential distribution with mean 1. We assume $\{\delta_j\}$ and $\{\Gamma_j\}$ are independent of each other. Let $Y_{nk} = |X_n^{(k)}|$ and $\delta_{nk} = \text{sign}(X_n^{(k)})$ so that $X_n^{(k)} = \delta_{nk} Y_{nk}$ and $(r)S_n = \sum_{j=r+1}^n \delta_{nj} Y_{nj}$.

Our main theorems are the following:

Theorem 1. Assume (A.1), (A.2), (D.1), (D.2) and (D.3), and define

$$R(u) = \inf\{x > 0: \rho((x, \infty)) \leq u\}, \quad u > 0,$$

as the right continuous inverse of the function $x \mapsto \rho((x, \infty))$. Then we have as $n \rightarrow \infty$,

$$({}^r)S_n - ({}^r)c_n \xrightarrow{w} ({}^r)Z,$$

where

$$({}^r)c_n = \sum_{j=r+1}^n E[\delta_{nj} Y_{nj} 1_{\{Y_{nj} \leq 1\}}],$$

$$({}^r)Z = \sum_{j=r+1}^{\infty} \{\delta_j R(\Gamma_j) - (p-q) E[R(\Gamma_j) 1_{\{R(\Gamma_j) \leq 1\}}]\}.$$

$\xrightarrow{w}$ denotes weak convergence, and the infinite series in $({}^r)Z$ converges almost surely.

Theorem 2. The characteristic function of $({}^r)Z$ is given by

$$E[e^{i\theta({}^r)Z}] = \frac{1}{(r-1)!} \int_0^{\rho((0, \infty))} u^{r-1} \exp\{-u + \int_{0 < |x| \leq R(u)} h(\theta, x) \nu(dx)\} du$$

$$(1.5) \quad + \frac{1}{(r-1)!} \int_{\rho((0, \infty))}^{\infty} u^{r-1} e^{-u} du.$$

Remark 1. Hall [H] studied the same problem for a sequence of independent random variables belonging to the domain of attraction of a non-normal stable law and obtained the limiting characteristic function. Later Mori [Mo] examined the limiting characteristic function of modulus trimmed sums when the law of the subsequential limit of sums of independent random variables is infinitely divisible. The limiting characteristic function in (1.5) is an extension of that in [H] where $\rho((x, \infty)) = x^{-\alpha}$, $\rho((0, \infty)) = 0$, $R(u) = u^{-1/\alpha}$, and also is the

same as in Theorem 2 of [Mo].

2. Preliminary lemmas (I)

The proof of Theorem 1 can be carried out by the same idea as in Davis [Da2]. He applied the LePage representation (the infinite series representation) of stable random variables to prove the stable limit theorem for dependent random variables. Here we shall use the infinite series representation of infinitely divisible random variables recently developed by Rosinski [R].

Let M_n^k and W_n^k be, respectively, the k -th largest and smallest among $\{X_{n1}, \dots, X_{nn}\}$, and denote $\hat{M}_n^k$ and $\hat{W}_n^k$ for the associated i.i.d. sequence $\{\hat{X}_{nj}\}$ similarly. (Of course $M_n^k = X_{n,n-k+1}^*$ and $W_n^k = X_{n,k}^*$ by the notation in Introduction.)

Lemma 2.1. ([Lo]) Under (1.2), as $n \rightarrow \infty$,

$$P\{\hat{M}_n^1 \leq x\} \rightarrow e^{-v([x, \infty))}, \quad x > 0,$$

$$P\{\hat{W}_n^1 > x\} \rightarrow e^{-v((-\infty, x])}, \quad x < 0.$$

Lemma 2.2. Under (1.2), (D.1) and (D.2),

$$P\{M_n^1 \leq x\} \rightarrow e^{-v([x, \infty))}, \quad x > 0,$$

$$P\{W_n^1 > x\} \rightarrow e^{-v((-\infty, x])}, \quad x < 0.$$

Proof. This is an analogue of Theorem 3.4.1 in [LLR] and it is enough to modify its proof slightly to the case of triangular arrays by using Lemma 2.1.

Lemma 2.3. Under (1.2), (D.1) and (D.2), the joint limiting distribution of two vectors

$$(M_n^1, \dots, M_n^k) \text{ and } (W_n^1, \dots, W_n^k)$$

is the same as that of

$$(\hat{M}_n^1, \dots, \hat{M}_n^k) \text{ and } (\hat{W}_n^1, \dots, \hat{W}_n^k)$$

for each positive integer k .

Proof. The case where $v((-\infty, 0)) > 0$ and $v((0, \infty)) > 0$ is an analogue of Theorem 1 of Davis [Da2]. He proved the corresponding result in the case of single sequences by using Lemma 2.2 together with Theorem 4.2 and Remarks 1 and 2 in [Da1], which remain valid in our case of triangular arrays. Details are omitted.

The case where $v((-\infty, 0)) = 0$ or $v((0, \infty)) = 0$ is an analogue of Lemma 1 in [Da2]. We only prove the case $v((0, \infty)) = 0$. The case $v((-\infty, 0)) = 0$ is similar. It follows from (1.2) that when $v((0, \infty)) = 0$, for any $\epsilon > 0$,

$$n(1 - F_n(\epsilon)) \rightarrow 0, \quad F_n(-\epsilon) \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Hence for $1 \leq \ell \leq k$ and for any $\epsilon > 0$,

$$\begin{aligned} P\{ |M_n^\ell| > \epsilon \} &\leq P\{ M_n^k < -\epsilon \} + P\{ M_n^1 > \epsilon \} \\ &\leq P\left\{ \bigcup_{j=1}^k \{X_{nj} < -\epsilon\} \right\} + P\left\{ \bigcup_{j=1}^n \{X_{nj} > \epsilon\} \right\} \\ &\leq k F_n(-\epsilon) + n(1 - F_n(\epsilon)) \rightarrow 0 \end{aligned}$$

as $n \rightarrow \infty$. Thus, the vector $(M_n^1, \dots, M_n^k)$ has a degenerate limit while $(W_n^1, \dots, W_n^k)$ has the same limit as $(\hat{W}_n^1, \dots, \hat{W}_n^k)$. This completes the proof. $\square$

We end this section with remarking the almost sure convergence of the infinite series in $(r)_Z$. Recently, Rosinski [R] studied the infinite series

representation of infinitely divisible random variables without Gaussian components. Our Lévy measure $\nu(\cdot)$ satisfying assumption (A.2) is a special case of those he treated in his paper. It follows from Corollary 4.8 of [R] that

$$\sum_{j=1}^n \delta_j R(\Gamma_j) - (p-q) \int_0^n R(u) 1_{\{R(u) \leq 1\}} du$$

converges a.s. as $n \rightarrow \infty$ to an infinitely divisible random variable whose characteristic function is given by (1.1) with $\nu(dx) = p 1_{\{x>0\}} \rho(dx) + q 1_{\{x<0\}} \rho(dx)$. On the other hand, it is easy to see that as $n \rightarrow \infty$,

$$\int_0^n R(u) 1_{\{R(u) \leq 1\}} du = \sum_{j=1}^n E[R(\Gamma_j) 1_{\{R(\Gamma_j) \leq 1\}}] + o(1).$$

Hence

$$\sum_{j=1}^{\infty} \{\delta_j R(\Gamma_j) - (p-q) E[R(\Gamma_j) 1_{\{R(\Gamma_j) \leq 1\}}]\}$$

converges a.s., and so does $(r)_Z$ for any $r \geq 1$.

3. Preliminary lemmas (II)

Let $H_n(x) = P\{|X_{nj}| > x\}$, $x > 0$. Then (1.4) implies

$$(3.1) \quad n H_n(x) \rightarrow \rho((x, \infty)), \quad x > 0, \quad \text{as } n \rightarrow \infty.$$

Define $\hat{Y}_{nk}$ and $\hat{\delta}_{nk}$ for the associated i.i.d. sequence $\{\hat{X}_{nj}\}$ as Y_{nk} and δ_{nk} for $\{X_{nj}\}$. We see that

$$(3.2) \quad (H_n(\hat{Y}_{nn}), \dots, H_n(\hat{Y}_{n1})) \stackrel{d}{=} \left(\frac{\Gamma_1}{\Gamma_{n+1}}, \dots, \frac{\Gamma_n}{\Gamma_{n+1}} \right),$$

where $\stackrel{d}{=}$ denotes the equality in distribution (see, e.g., Breiman [Br], Section 13.6).

Let us define the following infinite-dimensional random vectors.

$$Z^n = (Y_{n1}, \dots, Y_{nn}, 0, 0, \dots) \in [0, \infty)^\infty.$$

$$\hat{Z}^n = (\hat{Y}_{n1}, \dots, \hat{Y}_{nn}, 0, 0, \dots) \in [0, \infty)^\infty.$$

$$Z = (R(\Gamma_1), R(\Gamma_2), \dots) \in [0, \infty)^\infty.$$

$$\delta^n = (\delta_{n1}, \dots, \delta_{nn}, 1, 1, \dots) \in \{-1, 1\}^\infty.$$

$$\hat{\delta}^n = (\hat{\delta}_{n1}, \dots, \hat{\delta}_{nn}, 1, 1, \dots) \in \{-1, 1\}^\infty$$

and

$$\delta = (\delta_1, \delta_2, \dots) \in \{-1, 1\}^\infty.$$

Lemma 3.1. Under (1.4), $\hat{Z}^n \xrightarrow{w} Z$.

Proof. This is an analogue of Lemma 1 in [LWZ]. The same proof can be carried out by using (3.2). $\square$

Lemma 3.2. Under (1.4), (D.1) and (D.2), $Z^n \xrightarrow{w} Z$.

Proof. This is an analogue of Lemma 2 in [Dav2]. Under the assumptions for $\{X_{nj}\}$, $\{|X_{nj}|\}$ satisfies the same assumptions. Thus we can apply Lemma 2.3 for $p=1$ to $\{|X_{nj}|\}$ to conclude that Z^n and $\hat{Z}^n$ have the same limiting distribution. The statement thus follows from Lemma 3.1. $\square$

Lemma 3.3. Under (1.4), $\hat{\delta}^n \xrightarrow{w} \delta$, and $\hat{\delta}^n$ is asymptotically independent of $\hat{Z}^n$.

Proof. This is an analogue of Lemma 2 in [LWZ] and the proof is quite similar, so we omit it. $\square$

Lemma 3.4. Under (1.4), (D.1) and (D.2), $\delta^n \xrightarrow{w} \delta$, and δ^n is asymptotically independent of Z^n .

Proof. This is an analogue of Lemma 3 in [Dav2] and the same proof is carried

out after replacing Theorem 1 in [Dav2] and Lemma 2 in [LWZ] by our Lemmas 2.3 and 3.3 above. $\square$

4. Proofs of the theorems.

Proof of Theorem 1. The argument of the proof is the same as those in [LWZ] and [Dav2]. For notational convenience, let $[x; A] = x 1_A(x)$ for an interval A .

For $0 < \epsilon < \lambda$, let

$$^{(r)}S_n(\epsilon, \lambda] = \sum_{j=r+1}^n \delta_{nj}[Y_{nj}; (\epsilon, \lambda]]$$

and

$$^{(r)}S(\epsilon, \lambda] = \sum_{j=r+1}^{\infty} \delta_j[R(\Gamma_j); (\epsilon, \lambda]].$$

Then

$$^{(r)}S_n - ^{(r)}c_n = ^{(r)}S_n(0, \epsilon] - E[^{(r)}S_n(0, \epsilon]] + ^{(r)}S_n(\epsilon, \infty) - E[^{(r)}S_n(\epsilon, 1]].$$

To prove the theorem, it is enough to show (cf. Theorem 4.2 in [Bi]) that

$$(4.1) \quad \lim_{\epsilon \rightarrow 0} \limsup_{n \rightarrow \infty} \text{Var}[^{(r)}S_n(0, \epsilon]] = 0,$$

$$(4.2) \quad ^{(r)}S_n(\epsilon, \infty) \xrightarrow{w} ^{(r)}S(\epsilon, \infty) \quad \text{as } n \rightarrow \infty,$$

$$(4.3) \quad E[^{(r)}S_n(\epsilon, 1]] \rightarrow E[^{(r)}S(\epsilon, 1]] \quad \text{as } n \rightarrow \infty$$

and

$$(4.4) \quad ^{(r)}S(\epsilon, \infty) - E[^{(r)}S(\epsilon, 1]] \xrightarrow{w} ^{(r)}Z \quad \text{as } \epsilon \rightarrow 0.$$

To show (4.1), observe that

$$\begin{aligned} \text{Var}[^{(r)}S_n(0, \epsilon]] &= \sum_{j=r+1}^n E[|\delta_{nj}[Y_{nj}; (0, \epsilon]] - E[\delta_{nj}[Y_{nj}; (0, \epsilon]]|^2] \end{aligned}$$

$$\begin{aligned}
& + \sum_{j=r+1}^n \sum_{\substack{k=r+1 \\ k \neq j}}^n \text{Cov}(\delta_{nj}[Y_{nj}; (0, \epsilon)], \delta_{nk}[Y_{nk}; (0, \epsilon)]) \\
& \leq \sum_{j=1}^n E[|\delta_{nj}[Y_{nj}; (0, \epsilon)] - E[\delta_{nj}[Y_{nj}; (0, \epsilon)]]|^2] \\
& \quad + \sum_{j=1}^n \sum_{\substack{k=1 \\ k \neq j}}^n \max\{0, \text{Cov}(\delta_{nj}[Y_{nj}; (0, \epsilon)], \delta_{nk}[Y_{nk}; (0, \epsilon)])\} \\
& \leq n \left\{ \int_{|x| < \epsilon} x^2 dF_n(x) - \left(\int_{|x| < \epsilon} x dF_n(x) \right)^2 \right\} \\
& \quad + 2n \sum_{j=2}^n \max\{0, \text{Cov}([X_{n1}; [-\epsilon, \epsilon]], [X_{nj}; [-\epsilon, \epsilon]])\},
\end{aligned}$$

where we have used the stationarity of $\{X_{n1}, \dots, X_{nn}\}$. We thus have (4.1) by (1.3) and (D.3).

To show (4.2), define the subspace W of $[0, \infty)^\infty$ consisting of all $w = (w_1, w_2, \dots)$ for which $w_1 \geq w_2 \geq \dots \geq 0$, and a function ϕ_r on $W \times \{-1, 1\}^\infty$ by

$$\phi_r(w, d) = \sum_{j=r+1}^{\infty} d_j [w_j; (\epsilon, \infty)]$$

for $d = (d_1, d_2, \dots) \in \{-1, 1\}^\infty$ and $w = (w_1, w_2, \dots) \in W$ for which $w_j \rightarrow 0$ as $j \rightarrow \infty$ and $\phi_r(w, d) = 0$ for other values of w . Then as shown in [LWZ], ϕ_r is continuous in the product topology of $W \times \{-1, 1\}^\infty$ at a.e. (w, d) with respect to the distribution of Z and δ , and so by Lemmas 3.2 and 3.4,

$${}^{(r)}S_n(\epsilon, \infty) = \phi_r(Z^n, \delta^n) \xrightarrow{w} \phi(Z, \delta) = {}^{(r)}S(\epsilon, \infty)$$

as $n \rightarrow \infty$ for $\epsilon > 0$, concluding (4.2). Note that the above proof also assures

$${}^{(0)}S_n(\epsilon, 1] \xrightarrow{w} {}^{(0)}S(\epsilon, 1].$$

Also as in [LWZ], $\hat{{}^{(0)}S}_n(\epsilon, 1]$, $n=1, 2, \dots$, are uniformly integrable, because

$P(|\hat{X}_{n1}| > \epsilon) \sim n^{-1} \nu((\epsilon, \infty))$, where $\hat{S}_n$ is defined for $\{\hat{X}_{nj}\}$. Then we have

$$E^{(0)}[S_n(\epsilon, 1)] = E^{(0)}[\hat{S}_n(\epsilon, 1)] \rightarrow E^{(0)}[S(\epsilon, 1)].$$

Therefore, to get (4.3) it suffices to show that

$$(4.5) \quad E[\delta_{nj}[Y_{nj}; (\epsilon, 1)]] \rightarrow E[\delta_j[R(\Gamma_j); (\epsilon, 1)]]$$

for $1 \leq j \leq r$. However, we see from Lemma 3.4 that

$$\delta_{nj}[Y_{nj}; (\epsilon, 1)] \xrightarrow{P} \delta_j[R(\Gamma_j); (\epsilon, 1)]$$

and those random variables are bounded by 1. The bounded convergence theorem concludes (4.5).

Finally, to show (4.4), note that

$$\begin{aligned} (r)_Z &= \{(r)_S(\epsilon, \infty) - E[(r)_S(\epsilon, 1)]\} \\ &= \sum_{j=r+1}^{\infty} \{\delta_j[R(\Gamma_j); (0, \epsilon)] - E[\delta_j[R(\Gamma_j); (0, \epsilon)]]\}. \end{aligned}$$

Noting $\sum_{i=1}^{\infty} E[R(\Gamma_j)^2] < \infty$, we have that the variance of the above random variable is at most

$$\sum_{j=1}^{\infty} E[[R(\Gamma_j); (0, \epsilon)]^2] \leq \sum_{j=1}^{\infty} \min\{\epsilon^2, E[R(\Gamma_j)^2]\}$$

which tends to zero as $\epsilon \rightarrow 0$. This completes the proof of Theorem 1 $\square$

Remark 2. It may not be needed to assume (D.3), in particular, for the case when $\int_0^{\infty} (x \wedge 1) \rho(dx) < \infty$, but our proof requires it.

Proof of Theorem 2. Although the form of the characteristic function of $(r)_Z$ can be given from Theorem 2 of [Mo], we give below a simpler proof. Note that

$$\begin{aligned}
({}^r)Z &= \sum_{j=r+1}^{\infty} \{ \delta_j R(\Gamma_j) - (p-q) E[R(\Gamma_j); (0,1]] \} \\
&\stackrel{d}{=} \sum_{j=1}^{\infty} \{ \delta_j R(\Gamma_r + \tilde{\Gamma}_j) - (p-q) E[R(\Gamma_r + \tilde{\Gamma}_j); (0,1]] \}.
\end{aligned}$$

where $\{\tilde{\Gamma}_j\}$ is an independent copy of $\{\Gamma_j\}$. It follows from Rosinski's result explained at the end of Section 2 that for each $u > 0$, the random variable

$$Z(u) := \sum_{j=1}^{\infty} \{ \delta_j R(u + \tilde{\Gamma}_j) - (p-q) E[R(u + \tilde{\Gamma}_j); (0,1]] \}$$

is infinitely divisible and has the characteristic function of the form (1.1) with the Lévy measure ν_u whose right continuous inverse is $(R(u + \cdot))$, namely,

$$\nu_u(dx) = \begin{cases} \nu(dx), & \text{if } u \leq \rho((0,\infty)) \text{ and } |x| \leq R(u) \\ 0, & \text{otherwise.} \end{cases}$$

Thus we can write

$$E[e^{i\theta Z(u)}] = \exp\left\{ \int_{0 < |x| \leq R(u)} h(\theta, x) \nu(dx) \right\},$$

if we interpret $\int_{0 < |x| \leq R(u)}$ to be zero when $u > \rho((0,\infty))$, equivalently, $R(u) = 0$. Hence we conclude that $({}^r)Z$ is a mixture of infinitely divisible random variables and

$$E[e^{i\theta ({}^r)Z}] = \frac{1}{(r-1)!} \int_0^{\infty} u^{r-1} e^{-u} E[e^{i\theta Z(u)}] du,$$

implying (1.5). □

5. Related results

The argument in this paper up to here can be applied to prove other results related to extreme values. As an example, we show a result on natural

trimming.

We have been assuming that the Lévy measure ν has a special polar decomposition (A.2). We do not need this restriction if we consider natural trimming based on the order statistics of $\{X_{n1}, \dots, X_{nn}\}$. Recall that $X_{n1}^* \leq \dots \leq X_{nn}^*$, and set

$$S_n(r, s) = \sum_{j=r+1}^{n-s} X_{nj}^*.$$

We rewrite (1.1) as

$$f(\theta) = \exp\left\{ \int_{-\infty}^{0-} h(\theta, x) dM(x) + \int_{0+}^{\infty} h(\theta, x) dN(x) \right\},$$

where $M(x) = \nu((-\infty, x])$, $x < 0$ and $N(x) = -\nu((x, \infty))$, $x > 0$. Let for $u > 0$,

$$R_1(u) = \inf\{X < 0; M(x) > u\}$$

and

$$R_2(u) = \inf\{x > 0; -N(x) > u\}.$$

Let $\{\Gamma_j^{(1)}\}$ and $\{\Gamma_j^{(2)}\}$ be two independent copies of $\{\Gamma_j\}$. We have the following.

Theorem 3. Assume (A.1), (D.1), (D.2) and (D.3). Then we have

$$\begin{aligned} S_n(r, s) &= \sum_{j=r+1}^{n-s} E[X_{nj}^*; [-1, 1]] \\ \xrightarrow{w} Z(r, s) &:= \sum_{j=r+1}^{\infty} \{R_1(\Gamma_j^{(1)}) - E[R_1(\Gamma_j^{(1)}); [-1, 0]]\} \\ &\quad + \sum_{j=s+1}^{\infty} \{R_2(\Gamma_j^{(2)}) - E[R_2(\Gamma_j^{(2)}); (0, 1]]\}. \end{aligned}$$

The characteristic function of $Z(r, s)$ is given by

$$E[e^{i\theta Z(r, s)}] = \frac{1}{\{(r-1)!\}^2} \left[\int_0^{M(0-)} u^{r-1} \exp\{-u + \int_{R_1(u)}^{0-} h(\theta, x) dM(x)\} du \right]$$

$$\begin{aligned}
& + \int_{M(0-)}^{\infty} u^{r-1} e^{-u} du] \\
& \times [\int_0^{-N(0+)} u^{r-1} \exp\{ -u + \int_{0+}^{R_2(u)} h(\theta, x) dN(x) \} du \\
& + \int_{-N(0+)}^{\infty} u^{r-1} e^{-u} du].
\end{aligned}$$

Proof of Theorem 3. Some minor changes in the proof of Theorem 1 are enough to prove the first part of the theorem.

Instead of Z^n , δ^n , etc. in Section 3, we consider the following infinite-dimensional random vectors.

$$Z_{(1)}^n = ([X_{n1}^*; (-\infty, 0)], \dots, [X_{n, n-s}^*; (-\infty, 0)], 0, 0, \dots) \in (-\infty, 0]^\infty.$$

$$\hat{Z}_{(1)}^n = ([\hat{X}_{n1}^*; (-\infty, 0)], \dots, [\hat{X}_{n, n-s}^*; (-\infty, 0)], 0, 0, \dots) \in (-\infty, 0]^\infty.$$

$$Z_{(1)} = (R_1(\Gamma_1^{(1)}), R_1(\Gamma_2^{(1)}), \dots) \in (-\infty, 0]^\infty.$$

$$Z_{(2)}^n = ([X_{nn}^*; (0, \infty)], \dots, [X_{n, r+1}^*; (0, \infty)], 0, 0, \dots) \in [0, \infty)^\infty.$$

$$\hat{Z}_{(2)}^n = ([\hat{X}_{nn}^*; (0, \infty)], \dots, [\hat{X}_{n, r+1}^*; (0, \infty)], 0, 0, \dots) \in [0, \infty)^\infty.$$

$$Z_{(2)} = (R_2(\Gamma_1^{(2)}), R_2(\Gamma_2^{(2)}), \dots) \in [0, \infty)^\infty.$$

Lemma 5.1. ([J]) Under (A.1),

$$\hat{Z}_{(1)}^n \xrightarrow{w} Z_{(1)} \quad \text{and} \quad \hat{Z}_{(2)}^n \xrightarrow{w} Z_{(2)}.$$

By Lemmas 2.3 and 5.1, we get

Lemma 5.2. Under (A.1), (D.1) and (D.2),

$$Z_{(1)}^n \xrightarrow{w} Z_{(1)} \quad \text{and} \quad Z_{(2)}^n \xrightarrow{w} Z_{(2)}.$$

To prove Theorem 3, as in the proof of Theorem 1, we have

$$\begin{aligned}
S_n(r,s) &= \sum_{j=r+1}^{n-s} E[X_{nj}^*; [-1,1]] \\
&= \sum_{j=r+1}^{n-s} (E[X_{nj}^*; [-\epsilon, \epsilon]] - E[X_{nj}^*; [-\epsilon, \epsilon]]) \\
&\quad + \sum_{j=r+1}^{n-s} (E[X_{nj}^*; (-\infty, -\epsilon)] - E[X_{nj}^*; [-1, -\epsilon]]) \\
&\quad + \sum_{j=r+1}^{n-s} (E[X_{nj}^*; (\epsilon, \infty)] - E[X_{nj}^*; (\epsilon, 1)])
\end{aligned}$$

and need to show that

$$(5.1) \quad \lim_{\epsilon \rightarrow 0} \limsup_{n \rightarrow \infty} \text{Var} \left[\sum_{j=r+1}^{n-s} X_{nj}^*; [-\epsilon, \epsilon] \right] = 0.$$

$$(5.2) \quad \sum_{j=r+1}^{n-s} [X_{nj}^*; A_k] \xrightarrow{w} \sum_{j=\delta_k+1}^{\infty} [R_k(\Gamma_j^{(k)}); A_k], \quad k=1,2,$$

where $A_1 = (-\infty, -\epsilon)$, $A_2 = (\epsilon, \infty)$ and $\delta_1 = r$, $\delta_2 = s$.

$$(5.3) \quad \sum_{j=r+1}^{n-s} E[X_{nj}^*; B_k] \rightarrow \sum_{j=\delta_k+1}^{\infty} E[R_k(\Gamma_j^{(k)}); B_k], \quad k=1,2,$$

where $B_1 = [-1, -\epsilon]$, $B_2 = (\epsilon, 1]$, and

$$\begin{aligned}
(5.4) \quad &\sum_{j=\delta_k+1}^{\infty} \{ [R_k(\Gamma_j^{(k)}); A_k] - E[R_k(\Gamma_j^{(k)}); B_k] \} \\
&\xrightarrow{w} \sum_{j=\delta_k+1}^{\infty} \{ R_k(\Gamma_j^{(k)}) - E[R_k(\Gamma_j^{(k)}); [-1, 0]] \}.
\end{aligned}$$

$k=1,2$, as $\epsilon \rightarrow 0$.

(5.1) can be shown as in (4.1). To show (5.2) for $k=1$, define the subspace W of $(-\infty, 0]^\infty$ consisting of all $w = (w_1, w_2, \dots)$ for which $w_1 \leq w_2 \leq \dots \leq 0$, and a function ϕ_r on W by

$$\phi_r(w) = \sum_{j=r+1}^{\infty} [w_j; (-\infty, -\epsilon)]$$

for $w \in W$ for which $w_j \rightarrow 0$ as $j \rightarrow \infty$ and $\phi_r(w) = 0$ for other values of w . Then it is shown that ϕ_r is continuous in W at a.e. w with respect to the distribution of $Z_{(1)}$, and so by Lemma 5.2,

$$\sum_{j=r+1}^{n-s} [X_{nj}^*; (-\infty, -\epsilon)] = \phi_r(Z_{(1)}^n)$$

$$\xrightarrow{W} \phi_r(Z_{(1)}) = \sum_{j=r+1}^{\infty} [R_1(\Gamma_j^{(1)}); (-\infty, -\epsilon)]$$

as $n \rightarrow \infty$ for $\epsilon > 0$. A similar argument proves (5.2) for $k = 2$. As to (5.3), we can show it by the same reasoning as in (4.3). Also, (5.4) is given by the same way as in (4.4), if we note that for $k=1,2$,

$$\sum_{j=1}^{\infty} E[(R_k(\Gamma_j^{(k)}); [-\epsilon, \epsilon]]^2] \leq \sum_{j=1}^{\infty} \min\{\epsilon^2, E[R_k(\Gamma_j^{(k)})^2]\},$$

where $\sum_{j=1}^{\infty} E[R_k(\Gamma_j^{(k)})^2] < \infty$ because of the property of Lévy measures. This completes the proof of the first part of Theorem 3.

The form of the characteristic function of $Z(r,s)$ is given by Theorem 2, if we regard $Z(r,s)$ as a sum of two independent infinitely divisible random variables, one of which is distributed as $^{(r)}Z$ with $p=0$ and another is as $^{(s)}Z$ with $p=1$ in Theorem 2. □

Acknowledgement

The authors are very grateful to A. Janssen who sent us his unpublished manuscript.

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