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Research Report Submitted to the Office of Naval Research

for year 1 of grant # N00014-90-J-1194 on

COMPUTER SIMULATIONS OF DYNAMICAL FORMATION OF STRUCTURES IN SPACE AND TIME

DATES 10/1/89 through 9/30/90

ABSTRACT

We report the research done under year 1 of ONR Contract No. N00014-90-J-1194. The work involves using computer simulations to understand the formation of complex structures by dynamical systems, particularly hydrodynamic ones.

Narrative

For the last year we have performed computer simulation of the production of space-time structures under ONR Contract No. N00014-90-J-1194. This work is described below.

1. Viscous Fingering and Singularity Formation

Recently, much attention has been given to moving free boundary problems, both because of their relevance to interesting physical systems, and because of some of the surprising and elegant answers that have been found to long standing puzzles. The growth of dendritic solids from liquid melt,[1] directional solidification,[2] and diffusion limited aggregation[3] are a few such systems. The shape of the boundary between two different phases- liquid/gas, nematic/isotropic, occupied/unoccupied - is studied as one phase is driven to grow at the expense of another. The problem of fingering in two dimensional viscous fluid flow has connections to a number of different topics arising in the free boundary studies, and is a central example, because it is in some way simpler than the others (for example, the governing field satisfies Laplace's equation instead of the diffusion equation), while it retains many of the complexities. We have devoted considerable effort to the study of the problem of selection in viscous fingering.

During the present research period, we have performed a considerable amount of simulational work which has advanced the understanding of this field. In particular, Su-Min Zhou, Wei-Shen Dai and Leo Kadanoff have performed computer simulations of viscous fingering patterns with particular regard to the kinds of singularities in the complex plane which can be used to describe these situations. Our work is to be published in the Proceedings of a NATO Cargese conference on Non-Linear Phenomena related to Growth and Form[4]

In this regard, Peter Constantin and Leo Kadanoff have constructed a mathematical theory which can serve as a basis for this singularity discussion. This theory has two remarkable results: (1). that 2/3 power branch cuts seem to be the principal singularity structure. and (2). that the integrable equation called the Harry Dym equation describes the local behavior of the singularities.[5]

2. Dissipative Dynamical Systems

The phase space orbit of chaotic dynamical systems lie on fractal structures called strange attractors.[6] They have structure on ever smaller length scales and show self similarity locally. In this grant period, Emily Ching has looked at a chain of linked maps. Thus there is both space and time dependence. In her particular model, in which information is transferred in only one direction down the chain. Our goal is to understand how the dimension of the observed signal depends upon distance down the chain. So far we have set up the problem, but have gotten bogged down in finding an efficient calculation of dimension. We hope that we can beat this problem.

3. Lagrangian Description of Magnetic Bubbles

The boundary between dissipative and conservative dynamics is somewhat artificial. It lies mostly in the special techniques and results which have been obtained for the Hamiltonian cases, which, though singular from the mathematical viewpoint, are very important in many physical situations, when dissipative losses can be neglected. It is well known, however, that any dissipative dynamics can be realized in a Hamiltonian system by doubling the dimension of phase space, by a simple mathematical trick. A particular aspect of this, however, lies in that it is not possible to go from the Hamiltonian formulation to the Lagrangian one, making this mechanism exclusively Hamiltonian.

We have encountered the equivalent Lagrangian mechanism while studying a model for the dynamical behavior of a system of magnetic bubbles. This model was derived in Lagrangian form because the positions and velocities of the bubbles could be easily measured in a real situation, while the associated canonical momenta could not, and were expected not to be transparent mathematical constructions.

The model starts from a microscopic Langrangian for a spin system on a lattice. It consists of ferromagnetic Ising terms, the kinetic energy terms associated with these when they are derived from a free scalar field theory and magnetic dipole- dipole interactions at all lengths. Then, a macroscopic Lagrangian is obtained by constraining the spin configurations to a magnetic bubble system.

The macroscopic Lagrangian is in some senses highly singular. The kinetic energy terms vanish when trying to construct the Hamiltonian, thereby eliminating the possibility of a Hamiltonian formulation of the problem. Furthermore, it violates the conditions under which Lagrangian dynamics is constrained to be conservative. This aspect is an interesting one, since systems of magnetic bubbles behave dissipatively, what is usually taken into account by ad-hoc assumptions, while this model shows "built in" dissipative dynamics out of conservative microscopic assumptions.

The techniques we have used to deal with this problem are mainly computer aided algebraic, due to the enormous complication of the forms of the intermediate results. We now have a good form of the dynamical equations, and we are studying them by computer simulations. We are particularly interested in the topological properties of the network of bubbles. This work will form the basis of M. Magnasco's Ph.D. thesis.

4. Thermal Plumes

It is well known from everyday experience that whenever there is a sufficiently heated spot in a body of fluid, convective current occur. When the surrounding fluid is reasonably still, then a thin column of hot fluid, called a thermal plume, is seen to rise from the heated area.

Understanding the behavior and interaction of plumes seems to be an important issue in understanding the more general problem of turbulence in thermal convection systems. Plumes are seen to build, rise and detach from the boundary layers to enter the mixing region.

Calculating the development of a thermal plume in a general situation is a task of forbidding computational complexity. We then resort to a situation which is as simple as possible, although we firmly believe that its simplicity will not disallow rich and interesting phenomena and patterns of behavior to come to light. Vered Rom Kedar, Michael Shelley, and Michael Vinson have constructed codes to analyze the instabilities which lead to plume-like objects in a stratified Rayleigh-Taylor system. Both *thermal conductivity and viscosity are neglected in* these first calculations.The first results give an excellent comparison to the instabilities observed by Libchaber, Zocchi, and Moses[7] and in particular give a dispersion relation which looks like theirs. Now, we are in the process of seeing how this instability leads to non-linear effects This work will form a major portion of Vinson's PhD thesis.

PEOPLE

The principle investigator for this work is Leo P. Kadanoff. During the Summer, he has been joined by Itamar Procaccia (a Visiting Professor) who has played a major role in directing the research. Assistant Professor Michael Shelley, from the Mathematics Department, will direct many of the computational aspects of the program. The full theoretical group which will be and has been connected with this project includes:

Faculty

Leo P. Kadanoff*
Itamar Procaccia (summer visitor)*
Michael Shelley (Assistant Professor Mathematics)*

Research Associates and Other Visitors

Gemunu Gunaratne
Vered Rom-Kedar*
Ray Goldstein (NSF postdoctoral fellow)
Maria Vieira

Graduate Students:

Bruce Shaw* (Ph.D. 6/89)
Guy McNamara (Ph.D. 6/89, now scientist at Livermore National Lab)
Michael Vinson (PhD expect 10/90. He then goes to a postdoc at Syracuse)
Marcelo Magnasco*
Wei-Shen Dai*
Su-Min Zhou *
Emily Ching*

A star (*) indicates past ONR support,

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