

FINAL REPORT

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Outline

Abstracts of papers published under this contract (DAAG 29-84-K-0093) are enclosed in this final report. The first entry concerning Energy Dispersion Relations in N-well Superlattice Configurations is to be published in *Phys. Rev. B*.

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Exact Energy dispersion relations for N -well superlattice configurations

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and

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Abstract

Exact energy dispersion relations for N coupled quantum wells are obtained where $N \geq 1$. Each such relation is a composite function of transcendental forms which in turn determine the eigenenergies of the system. This relation is explicitly given for the cases of $N = 2, 3, 4$, for an even potential with arbitrary barrier and well widths. For the more standard case of constant barrier and well widths, a band structure emerges with the number of bound states in the outermost band varying from 1 to N . For arbitrary N , results reduce to that of a well of width a or Na in the limits of infinite and zero barrier widths, respectively, where a is the fundamental well width. With variation in well parameters, the number of states at a given value of N , varies from a total of one state, to a band structure with N states per band. The manner in which the ground state of the configuration varies with N is found. Plots of the dispersion relations for arbitrary N reveal the manner in which these curves merge to the single-well result with increase in barrier width. All dispersion relations are found to be asymptotic, in the limit of large decay wavenumber, to the same asymptotes as for the single finite well, independent of f , where f represents the ratio of barrier-to-well widths. Zeros of the dispersion relation merge to zeros of a single well of width a , for large f , and to a well of width Na for small f .

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PLASMA DOMAINS IN EXTRINSIC GaAs AND InP

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Abstract—Various parameters are introduced relevant to criteria for physical domains in solid-state plasmas. Application is made to extrinsic GaAs and InP at 300 K and varying charge-carrier concentrations. At concentrations less than $\sim 10^{15} \text{ cm}^{-3}$, charge-carrier plasmas for both p- and n-type semiconductors, respectively, are classical and weakly coupled. At a concentration of 10^{16} cm^{-3} the plasmas grow degenerate. At a concentration of 10^{17} cm^{-3} both p-type materials approach a degenerate state whereas both n-type materials are weakly-coupled degenerate.

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**Analytic distribution for charge carriers in a semiconductor dominated
by equivalent intervalley scattering**

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The transport of charge carriers in silicon immersed in an electric field is studied with use of the quasiclassical Boltzmann equation. Strain-acoustic and equivalent intervalley electron-phonon interactions are taken into account. A nonlinear difference-differential equation for the distribution function of charge carriers is obtained. An approximate analytic solution to this equation is constructed, from which an expression for drift velocity is derived. Comparison of values obtained from this expression with experimental measurement is found to give very good agreement for electric fields up to 10^4 V/cm.

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Solution of a new nonlinear equation for the distribution of charge carriers in a semiconductor

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The solution of a recently obtained nonlinear differential equation for the distribution function of charge carriers in a semiconductor in an electric field is derived. It is given by $f_{SL}(x) = \{1 + B[s/(x+s)]e^x\}^{-1}$. This solution represents the symmetric part of the total distribution function. The nondimensional energy and applied electric field are x and $\sqrt{s}$, respectively, and B is a constant determined by normalization. The total distribution is given by the above and its derivative and is found to be rotationally symmetric about the electric field. This distribution reduces to the shifted Fermi-Dirac distribution for small s and to the Druyvesteyn distribution in the classical limit. An analytic expression for electrical conductivity is derived together with an approximate expression for the chemical potential in the small-electric-field limit. A generalized criterion for the classical versus quantum domains is discussed relevant to the present study. It is found that otherwise quantum domains become classical for sufficiently large applied electric fields.

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QUASICLASSICAL MOBILITY FOR EXTRINSIC SEMICONDUCTORS

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Abstract—A quasiclassical formulation for mobility in extrinsic semiconductors is presented based on scattering from ionized impurity atoms. Quantum theory enters the otherwise classical Chapman-Enskog expansion of the Boltzmann equation through incorporation of the Thomas-Fermi interaction potential together with the Born approximation for evaluation of scattering integrals. The following expression results for mobility μ_i (cgs):

$$\mu_i = \frac{3}{2} \frac{\epsilon^2}{n_i e^3 m^{*1/2}} \left(\frac{k_B T}{2\pi} \right)^{1/2} \frac{1}{f(\gamma)},$$

$$f(\gamma) = [(1 + \gamma) e^\gamma E_1(\gamma) - 1],$$

where n_i is impurity concentration, m^* is effective mass, $E_1(\gamma)$ is the exponential integral, ϵ is dielectric constant and γ is dimensionless Thomas-Fermi energy. The structure of the dimensional factor in the preceding expression for μ_i agrees with previous expressions for this parameter.

Keywords: mobility, quasi-classical, impurity atoms, extrinsic semiconductor, Chapman-Enskog expansion, Thomas-Fermi interaction, Born approximation.

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Fluctuations and quantum domains in solid-state microdevices

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The validity of employing classical macroscopic equations of motion to describe degenerate plasma domains is examined. It is argued that such analyses are inconsistent when fluctuations in charge-carrier number grows comparable to mean values. The following expression for relative mean-square fluctuation of charge-carrier number away from the mean was applied to micro-solid-state samples. $\delta N = (\kappa_e / n^{5/6}) (\bar{m}^* T / 300 V)^{1/2}$. In this expression n is number density of charge carriers, $\bar{m}^*$ is effective mass divided by electron mass, V is volume, and κ_e is a constant. Employing this formula it was concluded that for n -type GaAs and InP at charge-carrier density $= 10^{17} \text{ cm}^{-3}$ and temperature 300 K, classical equations fail at dimensions less than $\approx 0.15 \text{ }\mu\text{m}$. For p -type GaAs and InP, under the same conditions, the critical length is $\approx 0.29 \text{ }\mu\text{m}$.

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Distribution functions and fluid variables in a semiconductor

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Fluid dynamic variables for charge-carrier transport in a semiconductor in the presence of an electric field are constructed from a recently derived distribution. This distribution is relevant to processes in a semiconductor where the deformation-potential interaction dominates. Fluid variables thus found are compared to those obtained from a shifted Fermi-Dirac distribution. In the limit of zero electric field both distributions give identical results. Analytic corrections to Fermi-Dirac variables are obtained by expanding the new variables about small electric field. Corrections at higher electric field are found numerically. Among other results it is found that at sufficiently high electric field, drift velocity grows insensitive to charge-carrier concentration. A discussion is included of the appropriate expression for electron temperature in a semiconductor.

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Criteria for physical domains in laboratory and solid-state plasmas

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Physical domains relevant to laboratory and solid-state plasmas are described in terms of relevant characteristic parameters. Strongly- and weakly-coupled classical plasmas are divided according to the plasma parameter Γ , whereas quantum and classical domains are separated according to the thermal DeBroglie wavelength λ , nondimensionalized through mean interparticle spacing. These parameters are found to obey the relation $\lambda^2 = (\pi/16)^{1/3} (k_B T / R^*) \Gamma^{4/3}$, where T is temperature and the Rydberg constant R^* includes the dielectric constant of the medium and effective mass of charge carriers. The weakly-coupled degenerate plasma is described in terms of the quantum compression parameter r_s , which represents interparticle spacing measured in Bohr radii. An alternative description of this domain is given in terms of a new quantum parameter (labeled Γ_Q) whose definition includes the Thomas-Fermi screening length in place of the Debye length in the classical plasma parameter. A graphical display in terms of appropriately nondimensionalized particle number density and temperature, respectively, reveals that all nonrelativistic, nonmagnetic plasma domains are included over the unit area of this graph. Application of these findings is made to GaAs and InP at 300 and 1000 °K in the intrinsic domain. Incorporating recent empirical expressions for effective mass, energy gap, and Fermi energy, it is found that at the lower temperature, the conducting solid-state plasmas of these semiconductors are weakly coupled and classical. At the higher temperature, due primarily to increased carrier concentration, the plasmas grow degenerate.

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Nonlinear electrical conductivity for a strongly coupled plasma

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A nonlinear analysis of electrical conductivity in a plasma is given, stemming from the Uehling-Uhlenbeck equation. Anisotropy due to an applied electric field is incorporated through a Legendre polynomial expansion of the distribution function. The plasma is comprised of ions, electrons, and a neutral component. The electron-ion interaction is described by a shielded Debye potential at high energy and a cutoff Coulomb potential at low energy. A nonlinear equation for the distribution function is solved and yields $\tilde{f}_{SL}(x) = 1/(1 + Be^{A(x)})$ for the symmetric part of the solution. The nondimensional energy is x , B is a normalization constant, and $A(x)$ is an explicit integral dependent on the electric field and specifics of the interaction. The resulting nondimensional conductivity $\tilde{\sigma}$ is given by $\tilde{\sigma} = \frac{1}{2} (2/\pi)^{3/2} [a_c (Z + 1)^{1/2} / \Lambda_Q \Gamma_D] \int_0^\infty \tilde{f}_{SL}(x) (d/dx)(x/\tilde{Q}) dx$, where Z is the effective ionization, a_c is the ratio of charge to total heavy-particle density, $\tilde{Q}$ is the dimensionless, weighted cross section, and Λ_Q and Γ_D are quantum and plasma parameters, respectively. Application is made to an aluminum plasma and plots of conductivity versus electric field are obtained. These plots exhibit three distinct regions; with an increase in field strength these are the Ohmic, Coulomb-dominated, and neutral-dominated regions.

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