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13. ABSTRACT (Maximum 200 words)

A final report describes the results obtained under the contract title, "Matrix Algorithms in Signal Processing." A number of algorithms are described which have the prperties that they are numerically stable, robust, and useful. A basic tool for many of these algorithms is the Lanczos class of algorithms. These algorithms have the virtue that they can be used in a dynamic environment and require little storage. Abstracts of several of the reports are included.

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Final Report

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2. Period covered by report: **18 May 1987 – 17 May 1990**

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5. Name of institution: **Stanford University**

6. Authors of report: **Gene H. Golub**

7. List of manuscripts submitted or published under ARO sponsorship during this reporting period, including journal references.

[1] (with P.Arbenz), *On the spectral decomposition of Hermitian matrices modified by low rank perturbations with applications*, SIAM J. MATRIX ANAL. APPL. 9, 40–58 (1988).

[2] (with P. Arbenz and W. Gander), *Restricted rank modification of the symmetric eigenvalue problem: theoretical considerations*, LAA 104, 75–95 (1988).

[3] (with Howard C. Elman), *Iterative methods for cyclically reduced non-self-adjoint linear systems*, published in Math. Comp.

[4] (with Pierre Comon), *Tracking a Few Extreme Singular Values and Vectors in Signal Processing*, accepted by the IEEE.

[5] W.Gander, G. Golub and U.Von Matt, *A constrained eigenvalue problem*, LAA 114/115, 815-839 (1989).

[6] S. Elhay, G. Golub and J. Kautsky, *Updating and downdating of orthogonal polynomials with data fitting applications*, to be published in SIAM J. MATRIX ANAL. APPL.

[7] B. De Moor and G. Golub, *Generalized singular value decompositions: a proposal for a standardized nomenclature*, Numerical Analysis Project Report NA-89-05.

[8] B. De Moor and G. Golub, *The restricted singular value decomposition: properties and applications*, submitted to SIAM J. MATRIX ANAL. APPL.

[9] B. De Moor, *On the structure and geometry of the product singular value decomposition*, Numerical Analysis Project Report NA-89-06.

[10] (with M. Gutknecht), *Modified moments for indefinite weight functions* , published in Numerische Mathematik.

[11] (with D. Boley, S. Makar, N. Saxena, E. McCluskey) *Backward error assertions for checking solutions to systems of linear equations*, Numerical Analysis Project, Computer Science Dept., Stanford University, NA-89-12 (1989).

[12] (with D. Boley, S. Makar, N. Saxena, E. McCluskey) *Backward error assertions for checking solutions to systems of linear equations*, Numerical Analysis Project, Computer Science Dept., Stanford University, NA-89-12 (1990).

[13] (S. Van Huffel and H. Zha) *The restricted total least squares problem: formulation, algorithm and properties*, Numerical Analysis Project, Computer Science Dept. Stanford University, NA-89-12 (1990).

[14] (with Urs Von Matt) *Quadratically constrained least squares and quadratic problems*, Numerical Analysis Project, Computer Science Dept. Stanford University, NA-90-04 (1990).

[15] (with S. Elhay and J. Kautsky) *Jacobi matrı...s for sums of weight functions*,Numerical Analysis Project, Computer Science Dept. Stanford University, NA-90-05 (1990).

[16] (with D. Boley) *The nonsymmetric Lanczos algorithm and controllability*, Numerical Analysis Project, Computer Science Dept. Stanford University, NA-90-06 (1990).

[17] (with W. Ferng and R. Plemmons) *Adaptive Lanczos methods for recursive condition estimation*,Numerical Analysis Project, Computer Science Dept. Stanford University, NA-90-07 (1990).

8. Scientific personnel supported by this project.

- Gene H. Golub

  Professor Golub was elected to the National Academy of Engineering in February, 1990.

9. Degrees awarded during this reporting period.

Mark Kent, received his degree in June, 1988. He is currently working for Integrated Systems Inc. in Santa Clara, CA.

Raymond Tuminaro received his degree in June, 1989. He is currently on a fellowship at the Sandia Laboratory in Albuquerque, New Mexico.

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Research Activities

1 Singular value decompositions

The singular value decomposition (**SVD**) has proved to be a useful tool in signal processing. Many of the most powerful methods such as ESPRIT which is used for directional of arrival estimation in sensor array processing use the SVD as a basic building tool. We have gone on to generalize this decomposition for two or more matrices and these generalizations are very useful in many applications arising in signal processing and systems theory.

As an example, the *restricted singular value decomposition* (**RSVD**) is the factorization of a given matrix, relative to two other given matrices. It can be interpreted as the *ordinary singular value decomposition* with different inner products in row and column spaces. Its properties and structure have been investigated in detail as well as its connection to generalized eigenvalue problems, canonical correlation analysis and other generalizations of the singular value decomposition.

Applications include the analysis of the extended shorted operator, unitarily invariant norm minimization with rank constraints, rank minimization in matrix balls, the analysis and solution of linear matrix equations, rank minimization of a partitioned matrix and the connection with generalized Schur complements, constrained linear and total linear least squares problems, with mixed exact and noisy data, including a generalized Gauss-Markov estimation scheme.

A constructive proof of the **RSVD** based upon the ordinary and the product singular value decomposition has been derived.

(Joint work with Bart De Moor)

2 Chebyshev, Krylov and Lanczos

We consider functions that are orthogonal with respect to a given symmetric inner-product. A new derivation of the fundamental relationship between orthogonal functions and their representation in non-orthogonal bases is given.

Given a basis of linearly independent polynomials, the modified Chebyshev algorithm constructs the set of orthogonal polynomials corresponding to the inner-product. We present a characterization of the algorithm in terms of transformations between polynomial bases. This leads to generalized Krylov sequences and demonstrates the equivalence of the modified Chebyshev algorithm and the Lanczos algorithm for determining eigenvalues of linear operators.

Block generalized Krylov sequences (of which the block Lanczos algorithm is a special case) are shown to produce matrix polynomials whose linearization is the matrix used to generate the Krylov sequence.

This theoretical work leads to a better understanding of the important Lanczos algorithm and its variants. Immediate applications include determining optimal parameters and error bounds for certain iterative methods for solving systems of linear equations.

(Thesis of Mark Kent)

3 Quadratic problems with constraints

In many situations in signal processing, one needs to minimize a quadratic form subject to a quadratic constraint or to linear constraints which are inhomogeneous. The quadratic constraint problem requires the solution of a lagrange multiplier that satisfies a quadratic eigenvalue problem. The difficulty in solving the lagrange multiplier is that the eigenvalues of the original matrix must be known precisely. We have recently developed an elegant algorithm which yields upper and lower bounds on the lagrange multiplier. The key idea is to approximate the secular equation by an integral and then bound the integral using the ideas of Gauss-Radau integration.

The Lanczos algorithm is used for determining the orthogonal polynomials which are required when computing the quadrature rule. The details of this approach will soon appear in a report, "A Constrained Least Squares Problem", which is being co-authored by Golub and Von Matt.

4 Orthogonal Polynomials

Orthogonal polynomials occur in many contexts in applied mathematics, but are especially relevant to data fitting. They have the property of stabilizing the delicate problem of fitting polynomials to data in least squares problems. In many situations, the data arrives in a sequential manner, and this requires the updating of the coefficients of the three term recursions used in generating the polynomials.

In some cases, one has a sliding window so that a new point is added as one point of data is deleted. We have developed algorithms which allow for performing these calculations in a very satisfactory manner. Our study of updating orthogonal polynomials has led to algorithms for handling data where several processors are available. That is, we are able to subdivide a large set of data into subsets and, then after computing the recursion relations on each subset, we are able to combine the coefficients in an efficient manner. This leads to a highly parallel algorithm which is also quite efficient.

5   Beam Forming

In various applications, it is necessary to keep track of a low-rank approximation of a covariance matrix, R(t), slowly varying with time. It is convenient to track the left singular vectors associated with the largest singular values of the triangular factor, L(t), of the Cholesky factorization. These algorithms are referred to as "square root". The drawback to those applications is the volume of the computational burden of the Eigenvalue Decomposition or the Singular Value Decomposition (SVD). Various numerical methods carrying out this task have been performed, and we show why this point is in fact questionable in numerous situations and should be revised. Indeed, the complexity per eigenpair is generally a quadratic function of the problem size, but there exist faster algorithms whose complexity is linear. Finally in order to make a choice among the large and fuzzy set of available techniques, we have made comparisons based on computer simulations in the relevant signal processing context.

A report, "Tracking a Few Singular Values and Vectors in Signal Processing", by Pierre Comon and Gene Golub has recently been completed and submitted for publication.

6   Least squares with a quadratic constraint.

We consider the following problem: Compute a vector x such that $\|Ax - b\|_2 = \min$, subject to the constraint $\|x\|_2 = \alpha$. A new approach to this problem based on Gauss quadrature has been derived. The method is especially well suited where the dimensions of A are large and the matrix is sparse.

The heart of the new method consists in the computation of a partial bidiagonalization of the matrix A, which is often all one can do in these settings.

(Joint work with Urs Von Matt).

7   Recursive condition estimation.

Estimates for the condition number of a matrix are required in many application areas of scientific computing, including: optimization, least squares computations, eigenanalysis, and general nonlinear problems solved by linearization techniques. Our purpose is to develop some adaptive condition estimators, based on the Lanczos algorithm, and test them on recursive least squares (RLS) computations arising in control and signal processing. RLS algorithms are known to suffer from numerical instability problems under finite word-length conditions.

We have provided an adaptive Lanczos schemes for estimating the smallest and largest singular values of R, $\sigma_{min}(R)$ and $\sigma_{max}(R)$, respectively, for each recursive update or downdate step for R (or R^{-1} in the covariance method). The computations are adaptive in the sense that estimates at time t are used to obtain estimates at time $t + 1$.

(Joint Work with W. Ferng and R. Plemmons)

8 The Lanczos algorithm and controlability

We derive a non-symmetric Lanczos algorithm that does not require strict bi-orthogonality among the generated vectors. We show how the vectors generated are algebraically related to "Controllable Space" and "Observable Space" for a related linear dynamical system.

The Lanczos Algorithm was originally proposed by Lanczos as a method for the computation of eigenvalues of symmetric and nonsymmetric matrices. The idea was to reduce a general matrix to tridiagonal form, from which the eigenvalues could be easily determined. The nonsymmetric Lanczos Algorithm has received much less attention than the symmetric case. Besides some numerical stability problems, the method suffers from the possibility of a breakdown from which the only way to "recover" was to restart the whole process from the beginning with different starting vectors . This problem was not solved until comparatively recently.

The Lanczos Algorithm is an example of a method that generates bases for Krylov subspaces starting with a given vector. In previous paper , we have examined how another closely related method, the Arnoldi Algorithm, may be used to compute the controllable space for a linear time-invariant dynamical system. The Arnoldi Algorithm can be thought of as a "one-sided" method, which generates one sequence of vectors that span the controllable space. We have extended this idea to the use of a two-sided method, the non-symmetric Lanczos Algorithm, which generates two sequences of vectors spanning the left and right Krylov spaces corresponding to the controllable and the observabt(le spaces. We have demonstrated how the vectors are generated in such a way that we obtain bases not only for the left and right Krylov spaces, but also for the intersections of these spaces and the complementary spaces.