David Taylor Research Center
Bethesda, MD 20084–5000

DTRC–90/029 October 1990

Ship Systems Integration Department
Research and Development Report

SCATSPHERE2 — A Computation for the Plane–Wave Scattering from a Submerged, Elastic, Spherical, Evacuated or Fluid–Filled, Thin Shell

by
Raymond S. Cheng
Francis M. Henderson

DTRC–90/029 SCATSPHERE2 — A Computation for the Plane–Wa Scattering from a Submerged, Elastic, Spherical, Evacuated or Fluid–Filled, Thin Shell

MAJOR DTRC TECHNICAL COMPONENTS

CODE 011	DIRECTOR OF TECHNOLOGY, PLANS AND ASSESSMENT
12	SHIP SYSTEMS INTEGRATION DEPARTMENT
14	SHIP ELECTROMAGNETIC SIGNATURES DEPARTMENT
15	SHIP HYDROMECHANICS DEPARTMENT
16	AVIATION DEPARTMENT
17	SHIP STRUCTURES AND PROTECTION DEPARTMENT
18	COMPUTATION, MATHEMATICS & LOGISTICS DEPARTMENT
19	SHIP ACOUSTICS DEPARTMENT
27	PROPULSION AND AUXILIARY SYSTEMS DEPARTMENT
28	SHIP MATERIALS ENGINEERING DEPARTMENT

DTRC ISSUES THREE TYPES OF REPORTS:

1. **DTRC reports, a formal series,** contain information of permanent technical value. They carry a consecutive numerical identification regardless of their classification or the originating department.

2. **Departmental reports, a semiformal series,** contain information of a preliminary, temporary, or proprietary nature or of limited interest or significance. They carry a departmental alphanumerical identification.

3. **Technical memoranda, an informal series,** contain technical documentation of limited use and interest. They are primarily working papers intended for internal use. They carry an identifying number which indicates their type and the numerical code of the originating department. Any distribution outside DTRC must be approved by the head of the originating department on a case-by-case basis.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) DTRC-90/029	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION David Taylor Research Center 6b. OFFICE SYMBOL (If applicable) Code 1282	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Bethesda, Maryland 20084-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Defense Advanced Research Projects Agency (DARPA) 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) 1400 Wilson Blvd. Arlington, Virginia 22209	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: 63569E PROJECT NO.: TASK NO.: WORK UNIT ACCESSION NO.:

11. TITLE (Include Security Classification)

SCATSPHERE2 – A Computation for the Plane-Wave Scattering from a Submerged, Elastic, Spherical, Evacuated or Fluid-Filled, Thin Shell

12. PERSONAL AUTHOR(S)

Cheng, R. S. and F. M. Henderson

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (YEAR, MONTH, DAY)	15. PAGE COUNT
Final	FROM 8910 TO 9005	1990 October	30

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

Scattering, Radiation, Spherical Shell, Helmholtz Equation ,

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Program SCATSPHERE2, a modification of program SCATSPHERE ~~by F.M. Henderson~~, computes the series solution for the time-harmonic plane-wave scattering by a submerged, elastic, spherical, evacuated or fluid-filled, thin shell. For program SCATSPHERE2, this report presents the basic scattering theory, the user's instructions, and three examples involving underwater plane-wave scattering by (1) an evacuated steel spherical thin shell, (2) a fluid-filled aluminum spherical thin shell, and (3) a fluid-filled aluminum spherical thin shell.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Raymond S. Cheng and Francis M. Henderson	22b. TELEPHONE (Include Area Code) (301) 227-1938 22c. OFFICE SYMBOL Code 1282

DD FORM 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

0102-LF-014-6602

CONTENTS

FIGURES

TABLES

Page

ABSTRACT

Program SCATSPHERE2, a modification of program SCATSPHERE by F.M. Henderson, computes the series solution for the time-harmonic plane-wave scattering by a submerged, elastic, spherical, evacuated or fluid-filled thin shell. For program SCATSPHERE2, this report presents the basic scattering theory, the user's instructions, and three examples involving underwater plane-wave scattering by (1) an evacuated steel spherical thin shell, (2) a fluid-filled steel spherical thin shell, and (3) a fluid-filled aluminum spherical thin snell.

ADMINISTRATIVE INFORMATION

The work was sponsored by the DARPA structural acoustics project at the David Taylor Research Center funded under DARPA Funding Document GD3210890T06660, Amendment 11, Program Element 63569E. The DTRC program manager was J.R. Peoples, Jr (Code 1930.6). The DARPA program manager was Dr. A. Tucker.

INTRODUCTION

A few years ago, the NASHUA structural acoustics procedure [1–5] was created to model the radiation/scattering problem for submerged elastic obstacles by using the boundary element method for the exterior fluid and the finite element method for the structure. The NASHUA procedure could also model interior fluids using structural finite elements by using analogies between the equations for dynamic elasticity and acoustics. [6–7] To verify the NASHUA procedure, three benchmark problems, where the analytical series solutions [8–11] are known, were chosen: (1) time-harmonic uniformly-driven radiation by a submerged elastic spherical thin shell; (2) time-harmonic sector-driven radiation by a submerged elastic spherical thin shell; and (3) time-harmonic plane-wave scattering by a submerged elastic spherical thin shell. Programs RADSPHERE [12] and SCATSPHERE were developed by F.M. Henderson to compute the series solutions for the second and third benchmark problems.

Recently, the NASHUA procedure was enhanced [13] to model structural acoustic problems for submerged fluid-filled structures by using the boundary element method for both the exterior and interior fluids, and the finite element method for the structure. For interior fluids, using the boundary element method instead of the finite element method reduces the matrix computation significantly for fine meshes. To verify the new procedure, the time-harmonic plane-wave scattering by a submerged elastic fluid-filled spherical thin shell was chosen as a benchmark problem since the analytical series solution can be derived and computed. Here, a thin shell assumption is used to simplify the analytical series solution. Program SCATSPHERE2, a modification of program SCATSPHERE, was developed to compute the series solution for scattering by a submerged elastic spherical thin shell that is either evacuated or fluid-filled.

In the next section, the time-independent scattering theory is introduced. A brief review of the plane-wave scattering by a submerged evacuated spherical thin shell [11] is presented. Then the series solution for the plane-wave scattering by a submerged fluid-filled spherical thin shell is derived. Next, the user's instructions are outlined for program SCATSPHERE2. This section includes discussions of the program's inputs and outputs, as well as its graphics capabilities. Finally, the three examples presented are underwater plane-wave scattering by (1) an evacuated steel spherical thin shell; (2) a fluid-filled steel spherical thin shell; and (3) a fluid-filled aluminum spherical thin shell. The first two examples verify the updated NASHUA/NASTRAN procedure while the last example is compared with (and shown to agree with) both the thick shell theory [14] and experimental results. [15–16]

UNDERWATER SCATTERING THEORY

The three-dimensional wave in the fluid is modeled by the homogeneous Helmholtz equation,

$$\Delta p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0, \quad \mathbf{x} \text{ in } R^3, \tag{1}$$

where Δ is the Laplacian operator, p is the pressure, and k is the wavenumber. Here, the time-dependent term $\exp(-i\omega t)$ has been suppressed. Using separation of variables in spherical coordinates, [17–18] we substitute $p(r,\theta,\phi)=R(r)\Theta(\theta)\Phi(\phi)$, where r is the radius, θ is the angle from the z-axis, and ϕ is the angle in the x-y plane, into Eq. 1 to obtain the spherical Bessel equation,

$$\frac{d}{dr}(r^2 \frac{dR}{dr}) + (k^2 r^2 - (n^2+n))R = 0, \tag{2}$$

the associated Legendre equation,

$$\frac{\sin(\theta)}{\Theta(\theta)} \frac{d}{d\theta}[sin(\theta) \frac{d\Theta}{d\theta}] + (n^2+n)\sin^2(\theta) = m^2, \tag{3}$$

and the one-dimensional wave equation,

$$\frac{-1}{\Phi(\phi)} \frac{d^2\Phi}{d\phi^2} = m^2. \tag{4}$$

The ranges of ϕ and θ restrict the general solution to those with integer eigenvalues, m and n. The general solution of Eq. 2 is expressed in terms of $j_n(kr)$, the spherical Bessel function of the first kind, and $h_n(kr)$, the spherical Hankel function of the first kind, both of order n. Note that $j_n(kr)$ is finite at the origin and unbounded in the infinite space, while $h_n(kr)$ is unbounded at the origin and satisfies the Sommerfeld radiation condition in the infinite space for the assumed $\exp(-i\omega t)$ time dependence. The general solution of Eq. 3 is expressed in terms of $P_n^m(cos\theta)$, the associated Legendre polynomials of order m and n. The general solution of Eq. 4 is the trivial one-dimensional wave solution. Note that m is the number of great nodal circles intersecting the z-axis, and n is the number of nodal circles coaxial with the z-axis. The plane-wave scattering by spherical obstacles is axisymmetric, and therefore, without loss of generality, $d\Phi/d\phi=0$, $m=0$, and Θ solves the Legendre equation.

Let the notation "J&F" indicate a formula in *Sound, Structures, and Their Interaction* by Junger and Feit. [11] By an addition theorem, [19 (section 10.1.47)] the free-space Green's function in spherical coordinates (J&F, section 6.10) is

$$G(\mathbf{x},\mathbf{x}_0) = \frac{exp(i\,k\,|\mathbf{x}-\mathbf{x}_0|)}{4\,\pi\,|\mathbf{x}-\mathbf{x}_0|} \tag{5}$$

$$= \frac{-i\,k}{4\,\pi} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \epsilon_m \frac{(n-m)!}{(n+m)!} (2n+1)\, P_n^m(cos\theta)\, P_n^m(cos\theta_0)\, cos\,(m\,(\phi-\phi_0))\, h_n(kr)\, j_n(kr_0),$$

$$r \geq r_0, \quad \mathbf{x},\mathbf{x}_0 \text{ in } R^3,$$

where the Neumann factor ϵ_m is 1 for $m=1$ and 2 for $m>1$. By the same addition theorem, the incident plane-wave pressure (J&F, section 10.13) is

$$p_0(r,\theta) = \Phi_0 \sum_{n=0}^{\infty} (2n+1)\, i^n\, P_n(cos\theta)\, j_n(kr), \tag{6}$$

where Φ_0 is the incident pressure magnitude.

EVACUATED SPHERICAL THIN SHELL

In this section, the series solution for the plane-wave scattering by a submerged evacuated spherical thin shell is reviewed. Assume that the shell is thin enough and the frequency is low enough that flexural stresses can be ignored as compared to membrane stresses. The plane-wave scattering by the submerged evacuated spherical thin shell requires finding (1) the shell impedance, (2) the fluid radiation impedance on the exterior surface using the Green's function for the exterior region, (3) the "rigid-body" scattered field, and (4) the radiated scattered field (due to shell vibration).

Using the equations for nontorsional axisymmetric motions (J&F, sections 7.102 and 7.103), the shell impedance for the nth axisymmetric mode (J&F, section 7.121) is

$$Z_n = -\frac{i\,\rho_s c_p}{\Omega} \frac{h}{a} \frac{[\Omega^2-(\Omega_{n1})^2][\Omega^2-(\Omega_{n2})^2]}{[\Omega^2-(1+\beta^2)(\nu+\lambda_n-1)]}, \tag{7}$$

where ρ_s is the shell density, $c_p=\sqrt{E/[\rho_s(1-\nu^2)]}$, E is the Young's modulus, h is the shell thickness, a is the shell mean radius, $\Omega=\omega a/c_p$ is the dimensionless frequency, $\beta=h/(a\sqrt{12})$, ν is the Poisson's ratio, and

$\lambda_n = n\,(n+1)$. The quantities Ω_{n1} and Ω_{n2} are the upper and lower shell resonance dimensionless frequencies, respectively. They are the solutions of the characteristic equation (J&F, section 7.114)

$$\Omega^4 - [1+3\nu+\lambda_n-\beta^2(1-\nu-\lambda_n{}^2-\nu\lambda_n)]\,\Omega^2 + (\lambda_n-2)(1-\nu^2) + \beta^2[\lambda_n{}^3-4\lambda_n{}^2+\lambda(5-\nu^2)-2(1-\nu^2)] = 0. \tag{8}$$

The exterior fluid impedance (the fluid radiation impedance at the exterior surface) is found by first finding the exterior region's Green's function which must satisfy the Sommerfeld radiation condition and the Neumann boundary condition The former condition implies that the exterior region's Green's function is in terms of $h_n(kr)$. The latter condition and the Wronskian relation [19 (sections 8.14.11 and 3.14.14)] implies that the Green's function for the exterior region specialized to the spherical surface (J&F, section 6.15) is

$$G(r,\theta,\phi\,|\,a,\theta_0,\phi_0) = \frac{1}{4\pi ka^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\epsilon_m\frac{(n-m)!}{(n+m)!}(2n+1)\cos(m(\phi-\phi_0))\,P_n^m(\cos\theta)\,P_n^m(\cos\theta_0)\,\frac{h_n(kr)}{h_n'(ka)}, \quad r \geq a. \tag{9}$$

The Green's identity for the exterior region and the problem's axisymmetry imply that the fluid pressure (J&F, section 6.16 and 6.19) is

$$\begin{aligned} p(r,\theta,\phi) &= \frac{\rho_f}{i\omega}\int_S G(r,\theta,\phi\,|\,a,\theta_0,\phi_0)\,w'(\theta_0,\phi_0)\sin(\theta_0)\,dS \\ &= i\,\rho_f c_f\sum_{n=0}^{\infty} w_n{}'\,P_n(\cos\theta)\,\frac{h_n(kr)}{h_n'(ka)}, \quad r>a, \end{aligned} \tag{10}$$

where ρ_f is the fluid density, w' is the shell surface normal velocity, and c_f is the fluid sound speed. The exterior fluid impedance of axisymmetric mode n (J&F, section 6.29) is

$$z_n = \frac{p_n(a,\theta,\phi)}{w_n{}'(a,\theta,\phi)} = i\,\rho_f c_f\,\frac{h_n(ka)}{h_n'(ka)}. \tag{11}$$

The "rigid-body" scattered pressure field (J&F, section 10.16) is found by using the Neumann boundary condition and the incident plane-wave pressure. Using the asymptotic expansion for the Bessel function, the far-field "rigid-body" scattered pressure field (J&F, section 10.17) is

$$p_{sr}(r,\theta,\phi) = -\frac{i\, e^{ikr}\, \Phi_0}{kr} \sum_{n=0}^{\infty} (2n+1)\, P_n(\cos\theta)\, \frac{j_n'(ka)}{h_n'(ka)}, \quad r \gg a. \tag{12}$$

From (J&F, section 11.25) to (J&F, section 11.28), the shell velocity response to the incident and "rigid-body" scattered pressure is

$$w'(r,\theta,\phi) = -\sum_{n}^{\infty} \frac{p_n\, P_n(\cos\theta)}{(Z_n + z_n)} \tag{13}$$

$$= -\frac{\Phi_0}{k^2 a^2} \sum_{n=0}^{\infty} \frac{i^{n+1}\, (2n+1)\, P_n(\cos\theta)}{(Z_n + z_n)\, h_n'(ka)},$$

where $p_n = (Z_n + z_n)\, w_n'$ is used. From Eqs. 10 and 13 and the asymptotic expansion for the Bessel function, the far-field radiated pressure due to the shell motion (J&F, section 11.29) is

$$p_r(r,\theta,\phi) = -\frac{i\, \rho_f c_f e^{ikr}\, \Phi_0}{k\, r} \sum_{n=0}^{\infty} \frac{(2n+1)\, P_n(\cos\theta)}{(Z_n + z_n)\, [k\, a\, h_n'(ka)]^2}, \quad r \gg a. \tag{14}$$

The total far-field scattered pressure (J&F, section 11.30), which is the sum of the "rigid-body" scattered pressure and the radiated pressure, is

$$p_{se}(r,\theta,\phi) = -\frac{i\, e^{ikr}\, \Phi_0}{k\, r} \sum_{n=0}^{\infty} \frac{(2n+1)\, P_n(\cos\theta)}{h_n'(ka)} \left[j'_n(ka) - \frac{\rho_f c_f}{(Z_n + z_n)\, (k\, a)^2\, h_n'(ka)} \right], \quad r \gg a. \tag{15}$$

For more in-depth theoretical presentation, see *Sound, Structures, and Their Interaction*, by Junger and Feit. [11]

FLUID-FILLED SPHERICAL THIN SHELL

In this section, the analytical plane-wave scattering solution for the submerged fluid-filled spherical thin shell is derived by extending the theory from the previous section. The extended solution requires finding (1) the interior fluid impedance using the Green's function for the interior region, and (2) the radiated pressure field due to shell vibration.

The Green's function for the interior region is expressed in terms of $j_n(kr)$ because it must be finite inside the sphere. Using the Neumann boundary condition to find the unknown coefficient, the Green's function for the interior region is

$$G(r,\theta,\phi\,|\,a,\theta_0,\phi_0) = \frac{1}{4\pi k a^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\epsilon_m \frac{(n-m)!}{(n+m)!}(2n+1)\cos(m(\phi-\phi_0))\,P_n^m(cos\theta)\,P_n^m(cos\theta_0)\,\frac{-j_n(kr)}{j_n{}'(ka)}, \quad (16)$$

$$r \leq a.$$

The Green's identity for the interior region and the problem's axisymmetry imply

$$p(r,\theta,\phi) = \frac{\rho_g}{i\,\omega}\int_S G(r,\theta,\phi\,|\,a,\theta_0,\phi_0)\,w'(\theta_0,\phi_0)\sin(\theta_0)\,dS \quad (17)$$

$$= i\,\rho_g\,c_g\sum_{n=0}^{\infty} w_n{}'\,P_n(cos\theta)\,\frac{-j_n(kr)}{j_n{}'(ka)}, \quad r \leq a,$$

where ρ_g is the interior fluid density, and c_g is the interior fluid sound speed. Thus, the interior fluid impedance of mode n is

$$\varsigma_n = \frac{p_n}{w_n{}'} = i\,\rho_g\,c_g\,\frac{-j_n(ka)}{j_n{}'(ka)}. \quad (18)$$

The only difference between computing the evacuated and fluid-filled cases is that the interior fluid impedance ς_n needs to be added to the shell modal impedance Z_n. Thus, the far-field radiated pressure is

$$p_r(r,\theta,\phi) = -\frac{i\,\rho_f c_f e^{ikr}\,\Phi_0}{k\,r}\sum_{n=0}^{\infty}\frac{(2n+1)\,P_n(cos\theta)}{(Z_n+z_n+\varsigma_n)\,[k\,a\,h_n'(ka)]^2},\quad r >> a. \tag{19}$$

The total far-field scattered pressure (which is the sum of the "rigid-body" scattered pressure and the radiated pressure) is

$$p_{se}(r,\theta,\phi) = -\frac{i\,e^{ikr}\,\Phi_0}{k\,r}\sum_{n=0}^{\infty}\frac{(2n+1)\,P_n(cos\theta)}{h_n'(ka)}\left[j'_n(ka) - \frac{\rho_f c_f}{(Z_n+z_n+\varsigma_n)\,(k\,a)^2\,h_n'(ka)}\right],\quad r >> a. \tag{20}$$

PROGRAM IMPLEMENTATION

Program SCATSPHERE2 computes the spherical Bessel function values after using IMSL library calls for the cylindrical Bessel function values. [19 (section 10.1.1)] The Legendre polynomials are computed using a recursion relation. [20] The infinite series is approximated by a finite number of terms. More terms are added until either the relative error criterion is met or the maximum number of terms allowed is used.

USER'S INSTRUCTIONS

SCATSPHERE2 input data consist of 11 record types for 25 parameters which are illustrated in Table 1. All variables are read in as unformatted (i.e., separated by commas). Records 7a and 7b are read in only if the field pressure calculations are done, i.e., NFLAG = 2 or 3. Record 7b consists of an array of radii whose dimension depends on NRADII. After NINT is recorded,the next record, Record 11, is read NINT times. The dimensionless frequency in Table 1 is defined as $\Omega = k\,a\,c_f/c_p$. The complex Young's modulus is $E\,(1+i\,\eta)$.

The series is computed for as many terms as needed until the relative error criterion is satisfied or the maximum number of terms is used. Recommended values are NTERMS = 40 and ERRCRT = 1.0 x 10^{-6}. If erratic behavior occurs, then increase NTERMS and decrease ERRCRT. Overall, the computation is quite inexpensive even for small ERRCRT. The maximum value for NTERMS is 201, and the maximum number of

Table 1. SCATSPHERE2 input data.

Record No.	Parameter	Description
1	TITLE	Title Card (up to 75 characters)
2	Φ_0	Incident Pressure Magnitude
3a	a	Shell Radius
3b	h	Shell Thickness
3c	ρ_s	Shell Density
3d	ν	Shell Poisson's Ratio
3e	E	Shell Young's Modulus
3f	η	Shell Damping (or Loss) Factor
4a	ρ_f	Exterior Fluid Density
4b	c_f	Exterior Fluid Sound Speed
5a	ρ_g	Interior Fluid Density (= 0 for evacuated shell)
5b	c_g	Interior Fluid Sound Speed (= 0 for evacuated shell)
6a	IPRINT	Print Control Flag (0-5) 0 = Minimal Printing 1 = Normal Printing 2 = Above Normal Printing 3 = Low Debug Printing 4 = Debug Printing 5 = Excess Printing
6b	NFLAG	Calculation Type Flag (1-5) 1 = Surface Pressure Calculation Only 2 = Surface and Field Pressure Calculation 3 = Field Pressure Calculation Only 4 = Surface Pressure and Radial Velocity Calculation 5 = Surface Radial Velocity Calculation Only

Table 1 (Continued)

Record No.	Parameter	Description
7a	NRADII	Number of Observation Radii: If NFLAG = 2 or 3
7b	r	Radii Array: 1 to NRADII: If NFLAG = 2 or 3
8a	NTERMS	Maximum Number of Series Terms
8b	ERRCRT	Relative Error Criterion
9a	θ_1	Starting Angle from z-Axis
9b	$\Delta\theta$	Delta Angle from z-Axis (> 0)
9c	θ_2	Ending Angle from z-Axis (> θ_1)
10	NINT	Number of Frequency Runs
11a	Ω_1	Starting Dimensionless Frequency: 1 to NINT
11b	$\Delta\Omega$	Delta Dimensionless Frequency: 1 to NINT
11c	Ω_2	Ending Dimensionless Frequency: 1 to NINT

samples, which is the number of ka's times the number of angles, is 2001.

Program SCATSPHERE2 produces two output files, tapes 11 and 13. Tape 11 contains the input data and a table for the nondimensional pressure $|pr/p_0 a|$. Tape 13 contains an input X-Y PLOT file which is processed by the LOGOS graphics program. [21] Tape 12 is not currently used, but the user could restore this tape for input to F.M. Henderson's program PLOTTER, which creates polar plots and uses the software DISSPLA. [22] For more information on using program PLOTTER, see F.M. Henderson's report on RADSPHERE. [12]

The setup for a typical CRAY COS job is as follows:

```
JOB,JN=SCAT,T=60,US=BUDGET. RUN SCATSPHERE
ACCOUNT,AC=xxxxxxxxxx,US=yyyy,UPW=zzzz.
CFT77,ON=AXM.
ACCESS,DN=IMSL,OWN=PUBLIC.
SEGLDR,CMD='LIB=IMSL',GO.
```

```
DISPOSE,DN=TAPE11,...
DISPOSE,DN=TAPE12,...
DISPOSE,DN=TAPE13,...
/EOF
      PROGRAM SCATSPHERE2
      .
      .
      .
      END
/EOF
Forward Scattering for a Fluid-Filled Steel Spherical Shell
20.
5.,0.15,7669.,0.3,2.07E11,0.01
1000.,1524.
1000.,1524.
1,3
1,100.
80,1.E-8
0.,30.,0.
1
0.005,0.005,5.0
```

Note that the code is in FORTRAN-77 and IMSL library is used for the Bessel function calculations.

NUMERICAL EXAMPLES

In this section, we present three examples: underwater plane-wave scattering by (1) an evacuated steel spherical thin shell, (2) a fluid-filled steel spherical thin shell, and (3) a fluid-filled aluminum spherical thin shell. The first two examples verify the updated NASHUA procedure, while the last example is compared to and agrees with the numerical implementation of the thick shell theory [14] and experimental results. [15–16] All the examples are for a thin shell; that is, the shell thickness is no more than 5% of the shell radius.

EVACUATED STEEL SPHERICAL THIN SHELL

This example was used to verify the updated NASHUA procedure and to check program SCATSPHERE2. Figure 1 illustrates the time-harmonic plane-wave scattering problem for a submerged evacuated steel spherical thin shell. Table 2 presents the shell and fluid properties used. Note that the interior fluid properties ρ_g and c_g are set to zero.

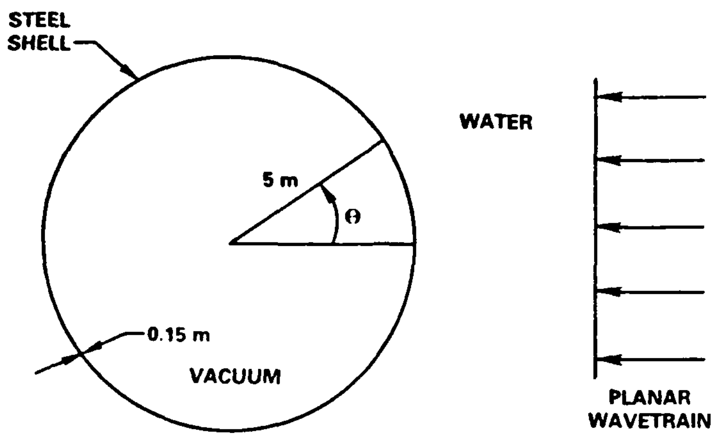

Fig. 1. Plane-wave scattering from a submerged elastic spherical shell.

Table 2. Spherical thin steel shell and fluid properties.

Description	Parameter	Measurement
Shell Radius	a	5.0 m
Shell Thickness	h	0.15 m
Shell Density	ρ_s	7669.0 kg/m^3
Shell Poisson Ratio	ν	0.3
Shell Young Modulus	E	2.07×10^{11} N/m^2
Shell Loss Factor	η	0.0
Exterior Fluid Density	ρ_f	1000 kg/m^3
Exterior Fluid Sound Speed	c_f	1524 m/s
Interior Fluid Density	ρ_g	0 kg/m^3
Interior Fluid Sound Speed	c_g	0 m/s

For the NASHUA finite element model, 40 axisymmetric conical shell elements spanning the 180 degrees between the two spherical poles were used, making a total of 205 structural and 41 fluid degrees of freedom (DOF) for the exterior fluid. The nondimensional frequency increment was about $ka = 0.02$ in the range $0 < ka < 5$.

For SCATSPHERE2: the maximum number of series terms NTERMS = 40, the relative error criterion ERRCRT = 1.0×10^{-4}, NFLAG = 3 for the field pressure calculation only, NINT = 1 for one frequency sweep, and NRADII = 1 for one radial observation per frequency point. The nondimensional frequency range $0 < ka < 5$ was swept using a frequency increment of $ka = 0.005$.

Figures 2 and 3 show the forward and back-scattered pressure fields vs. the dimensionless frequency. These figures show that NASHUA and SCATSPHERE2 agree well.

FLUID-FILLED STEEL SPHERICAL THIN SHELL

This example was used to verify the updated NASHUA procedure. The properties are the same as in the previous example except that the spherical thin shell is filled with the same interior fluid as the exterior fluid.

For the NASHUA finite element model, 40 axisymmetric conical shell elements spanning the 180 degrees between the two spherical poles were used, making a total of 205 structure DOF and 41 fluid DOF for each fluid region. The nondimensional frequency increment was about $ka = 0.05$ in the nondimensional frequency range $0 < ka < 5$.

For SCATSPHERE2, the maximum number of series terms NTERMS = 80, the relative error criterion ERRCRT = 1.0×10^{-8}, NFLAG = 3 for the field pressure calculation only, NINT = 1 for one frequency sweep, and NRADII = 1 for one radial observation per point. Note that NTERMS and ERRCRT are different from those used in the evacuated case. These changes were made to avoid convergence problems due to the Bessel function computations. The nondimensional frequency increment was $ka = 0.005$ in the nondimensional frequency range $0 < ka < 5$.

Figures 4 and 5 plot the frequency response of the forward and back-scattered nondimensional pressure $|pr/p_0 a|$. Both figures show very good agreement between the two approaches.

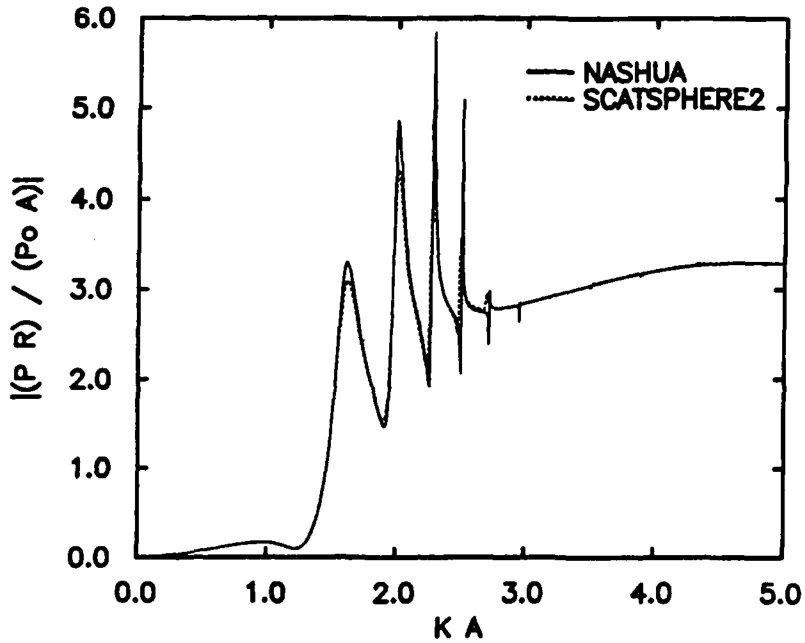

Fig. 2. Forward-scattered field by a submerged evacuated steel spherical shell.

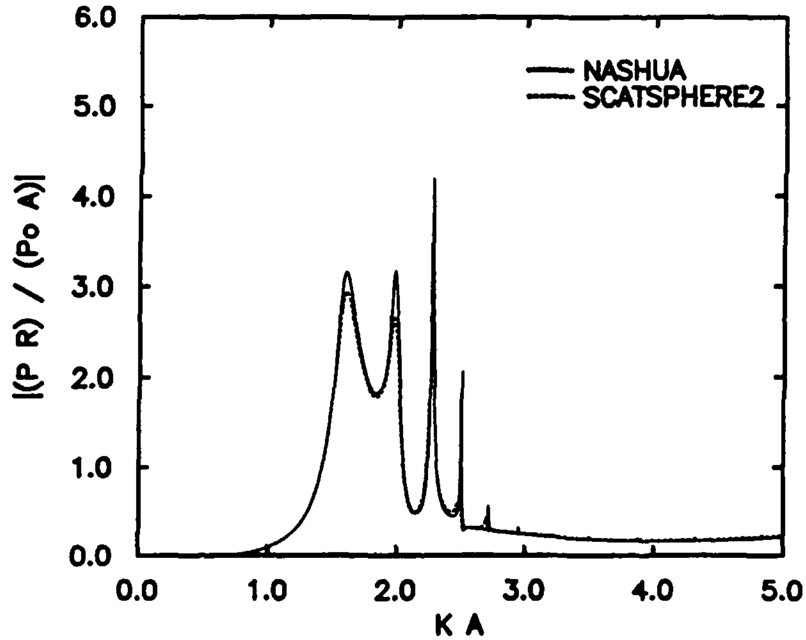

Fig. 3. Back-scattered field by a submerged evacuated steel spherical shell.

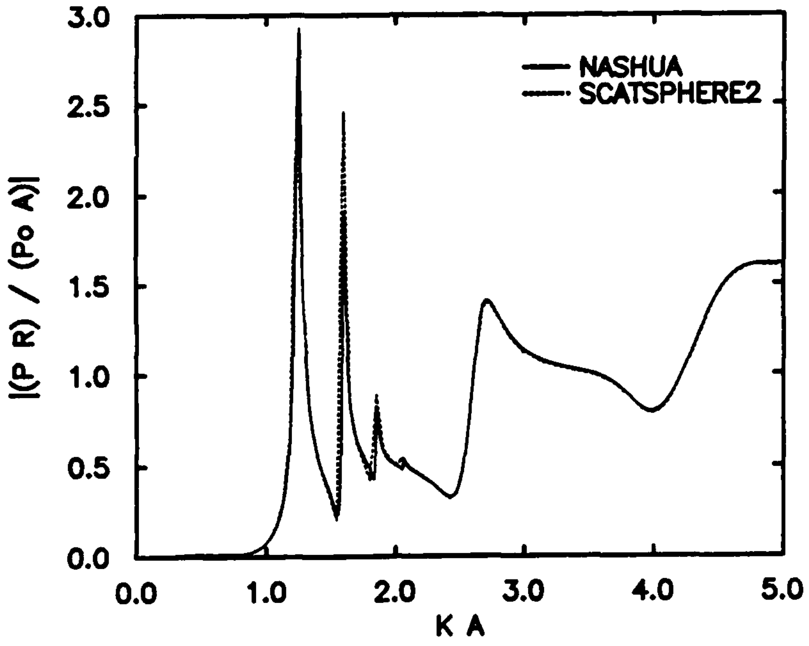

Fig. 4. Forward-scattered field by a submerged fluid-filled steel spherical shell.

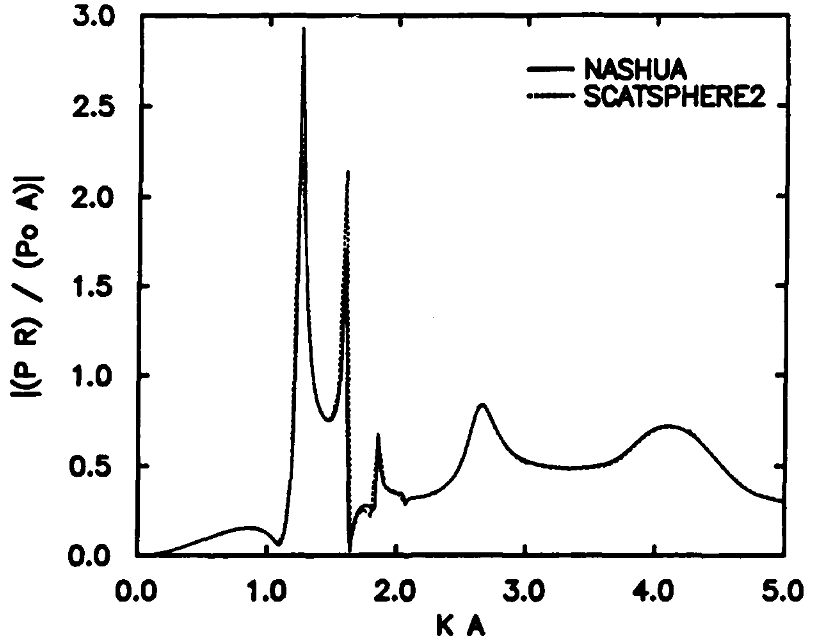

Fig. 5. Back-scattered field by a submerged fluid-filled steel spherical shell.

Table 3. Spherical thin aluminum shell and fluid properties.

Description	Parameter	Measurement
Shell Radius	a	4.875 m
Shell Thickness	h	0.25 m
Shell Density	ρ_s	2700.0 kg/m^3
Shell Poisson Ratio	ν	0.355
Shell Young Modulus	E	0.675×10^{11} N/m^2
Shell Loss Factor	η	0.00
Exterior Fluid Density	ρ_f	1000 kg/m^3
Exterior Fluid Sound Speed	c_f	1410 m/s
Interior Fluid Density	ρ_g	1000 kg/m^3
Interior Fluid Sound Speed	c_g	1410 m/s

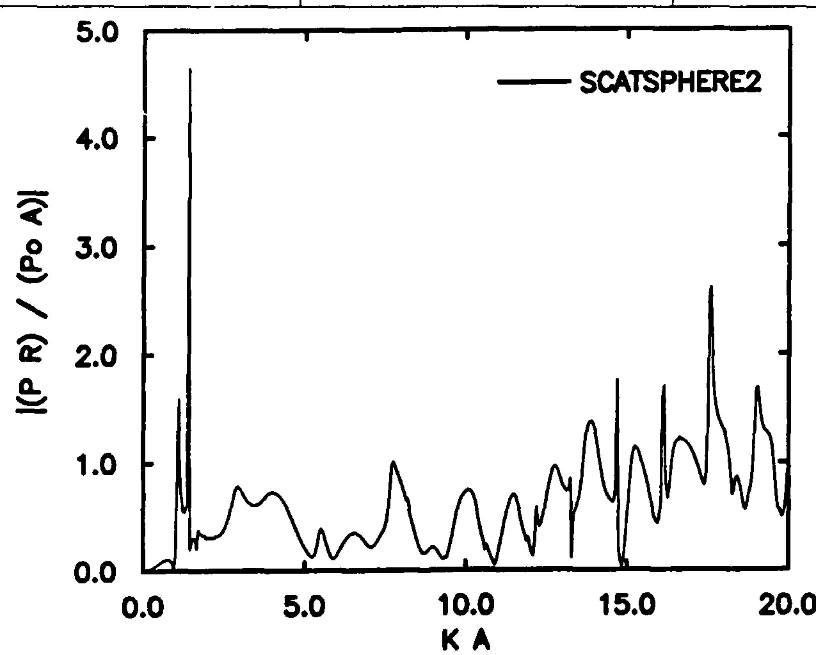

Fig. 6. Back-scattered field by a submerged fluid-filled aluminum spherical shell.

FLUID-FILLED ALUMINUM SPHERICAL THIN SHELL

This example was chosen from earlier papers [14–15] as another example to verify program SCATSPHERE2. Table 3 presents the shell and fluid properties used. Figure 6 gives the back-scattered nondimensional absolute pressure for nondimensional frequency $0<ka<20$. Close comparison of Fig. 6 with Fig. 4b in Hickling's paper [14] and Fig. 1b in Diercks's and Hickling's paper, [15] shows good agreement between SCATSPHERE2 computations and results obtained by Diercks and Hickling.

SUMMARY

Program SCATSPHERE2 solves the time-harmonic plane-wave scattering by a submerged elastic spherical thin shell that is evacuated or fluid-filled. This code was verified by the updated NASHUA procedure, [24] the numerical implementation of the thick shell theory, [14] and experimental results. [15] This program allows the user to examine the surface velocity at any angle, and the scattered pressure field at any radius and angle. Program SCATSPHERE2 also allows the user to create polar plots, tables, and X-Y plots. In the future, program SCATSPHERE2 could be modified for other axisymmetric problems such as including additional interior mediums (solids and/or fluids) or spherical-wave scattering.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Gordon Everstine for his guidance in this project.

THIS PAGE INTENTIONALLY LEFT BLANK

REFERENCES

1. G.C. Everstine, F.M. Henderson, E.A. Schroeder, and R.R. Lipman, "A General Low Frequency Acoustic Radiation Capability for NASTRAN," **Proceedings of the Fourteenth NASTRAN Users' Colloquium,** National Aeronautics and Space Administration, Washington, DC, NASA CP-2419, pp. 293-310 (1986).
2. G.C. Everstine, F.M. Henderson, and L.S. Schuetz, "Coupled NASTRAN/Boundary Element Formulation for Acoustic Scattering," **Proceedings of the Fifteenth NASTRAN Users' Colloquium,** National Aeronautics and Space Administration, Washington, DC, NASA CP-2481, pp. 250-265 (1987).
3. G.C. Everstine, "Treatment of Static Preload Effects in Acoustic Radiation and Scattering," **Proceedings of the Sixteenth NASTRAN Users' Colloquium,** National Aeronautics and Space Administration, Washington, DC, NASA CP-2505, pp. 138-152 (1988).
4. G.C. Everstine, "Calculation of Low Frequency Vibrational Resonances of Submerged Structures," **Proceedings of the Seventeenth NASTRAN Users' Colloquium,** National Aeronautics and Space Administration, Washington, DC, NASA CP-3029, pp. 247-261 (1989).
5. G.C. Everstine and F.M. Henderson, "Coupled Finite Element/Boundary Element Approach for Fluid-Structure Interaction," **J. Acoust. Soc. Amer.,** Vol. 87, No. 5 (1990).
6. G.C. Everstine, "Structural Analogies for Scalar Field Problems," **Int. J. Num. Meth. in Engrg.,** Vol. 17, No. 3, pp. 471-476 (1981).
7. G.C. Everstine, "Structural-Acoustic Finite Element Analysis, with Application to Scattering," **Proc. 6th Invitational Symposium on the Unification of Finite Elements, Finite Differences, and Calculus of Variations,** ed. H. Kardestuncer, Univ. of Connecticut, Storrs, Conn., pp. 101-122 (1982).
8. M.C. Junger, "Sound Scattering by Thin Elastic Shells," **J. Acoust. Soc. Amer.,** Vol. 24, No. 4 (1952).
9. M.C. Junger, "Vibration of Elastic Shells in a Fluid Medium and the Associated Radiation of Sound," **J. Appl. Mech.,** Vol. 74, pp. 439-445 (1952).

REFERENCES (Continued)

10. S. Hayek, "Vibration of a Spherical Shell in an Acoustic Medium," **J. Acoust. Soc. Amer.**, Vol. 40, No. 2 (1966).

11. M.C. Junger and D. Feit, **Sound, Structures, and Their Interaction,** 2nd ed., The MIT Press, Cambridge, Mass. (1986).

12. F.M. Henderson, "RADSPHERE - A Computer Program for Calculating the Steady-State, Axially Symmetric, Forced Response and Radiation Field of a Submerged Spherical Shell," Report 87/031, David Taylor Research Center, Bethesda, Md. (1987).

13. G.C. Everstine and R.S. Cheng, "Coupled BE/FE/BE Approach for Scattering from Fluid-Filled Structures," **Proceedings of the Eighteenth NASTRAN Users' Colloquium,** National Aeronautics and Space Administration, Washington, DC (Apr 1990).

14. R. Hickling, "Analysis of Echoes from a Hollow Metallic Sphere in Water," **J. Acoust. Soc. Amer.**, Vol. 36, No. 6 (1964).

15. K.J. Diercks and R. Hickling, "Echoes from Hollow Aluminum Spheres in Water," **J. Acoust. Soc. Amer.**, Vol. 41, No. 2 (1967).

16. D.J. Shirley and K.J. Diercks, "Analysis of the Frequency Response of Simple Geometric Targets," **J. Acoust. Soc. Amer.**, Vol. 48, No. 5 (1970).

17. P.M. Morse and H. Feshbach, **Methods of Theoretical Physics,** McGraw-Hill Book Company, New York, N.Y. (1953).

18. P.M. Morse and K.U. Ingard, **Theoretical Acoustics,** McGraw-Hill Book Company, New York (1968).

19. **Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,** ed. M. Abramowitz and I.A. Stegun, National Bureau of Standards, Gaithersburg, Md., (1972).

20. R.V. Churchill, **Fourier Series and Boundary Value Problems,** 2nd ed., McGraw-Hill Book Company, New York, N.Y. p. 206 (1969).

21. **LOGOS User's Manual**, Version 5, Department of Civil Engineering, University of Illinois, Urbana-Champaign, (1989).

22. DISSPLA (Display Integrated Software System and Plotting Language), ISSCO, Visual Information System Software, User's Manual Version 10.0 (1981).

INITIAL DISTRIBUTION

Copies	
17	NAVSEA
	1 05R
	1 08
	1 502
	1 55N
	1 55N A. Kukk
	1 55W33
	1 55Y
	1 55Y2 R. Provencher
	1 55Y22 D. Nichols
	1 55Y3 R. Chiu
	1 55Y32
	1 921N
	1 92R
	1 PMS 393
	1 PMS 394
	1 PMS 396
	1 Lib
1	CNO/OP-21T3 A. Bisson
7	ONR/ONT
	1 21 E. Zimet
	1 23 A. Faulstich
	1 121 R. Hansen
	1 233 G. Remmers
	1 1132
	1 1132SM R. Barsoum
	1 Lib
1	DARPA/A. Tucker
1	NADC/Lib
1	USNA/Lib
2	NAVPGSCOL
	1 M.E. Dept.
	1 Lib
1	NAVSSES (Phila)/Lib
1	NCEL/Lib
1	NCSC/Lib

Copies	
1	NORDA/Lib
2	NOSC
	1 701 H. Schenck
	1 Lib
2	NRL
	1 5130 J. Bucaro
	1 Lib
3	NRL/Orlando
	1 5970 R.W. Timme
	1 5980 A.L. Van Buren
	1 Lib
3	NSWC/White Oak
	1 R14 H. Huang
	1 R31 G. Gaunaurd
	1 Lib
1	NSWC/Dahlgren/Lib
6	NUSC/New London
	1 44 A. Carlson
	1 44 J. Patel
	1 44 A. Kalinowski
	1 3234 D. Porter
	1 3332 D. Lee
	1 Lib
1	NUSC/Newport/Lib
1	NWC/Lib
8	NAVSHIPYD/
	1 Charleston/Lib
	1 Long Beach/Lib
	1 Mare Island/Lib
	1 Norfolk/Lib
	1 Pearl Harbor/Lib
	1 Philadelphia/Lib
	1 Portsmouth/Lib
	1 Puget Sound/Lib
1	WPAFB/V. Venkayya

INITIAL DISTRIBUTION (Continued)

Copies	
12	DTIC
1	Alcoa Def. Systems/J. Bock
1	APL/Johns Hopkins Univ.
1	ARL/Penn. State Univ./Lib
2	AT&T Bell Labs (Whippany) 1 D. Burnette 1 Lib
1	Bettis/Lib
2	Bolt, Beranek, and Newman (New London) 1 R. Haberman 1 H. Allik
4	Cambridge Acoust. Assoc. 1 M. Junger 1 J. Garrelick 1 K. Martini 1 R. Martinez
1	Cambridge Collaborative
1	Catholic Univ./H. Uberall
1	Colorado Univ./T. Geers
1	Clarkson Univ./J. Kane
1	CSAR/R. Narayanaswami
1	Donanco, Inc./D. Ross
1	Draper Lab/A. Edsall
4	General Dynamics (EB) 1 443 (J. Wilder) 1 443 (S. Gordon) 1 Lib
1	Georgia Tech/J. Ginsberg

Copies	
2	Global Associates, Ltd. 1 P.W. Sparks 1 E. O'Neill
1	GM Research Lab./S. Sung
1	Iowa State Univ./P. Goswami
1	J.G. Eng. Res. Assoc.
1	Knolls Atomic Power Lab./Lib
1	Lockheed Aero. Sys. (Burbank)/ B. Thorn
1	Lockheed Palo Alto Res. Lab./ J. DeRuntz
1	Los Alamos National Lab/S. Girrens
1	Martin Marietta (Baltimore)/Lib
3	Newport News Shipbuilding 1 E-12 (T. Heldreth) 1 E-92E (W. Floyd) 1 Lib
2	NKF Engineering Assoc. 1 M. Pakstys 1 R. Miller
2	Penn State Univ. 1 S. Hayek 1 D.N. Arnold
1	Polytechnic Inst. of NY/J. Klosner
1	Sandia/D. Lobitz
1	Sperry Marine (Charlottesville)/ J. Brazell
1	Stanford Univ./P. Pinsky
1	Tracor Applied Sciences Inc. (Cabin John)

INITIAL DISTRIBUTION (Continued)

Copies	
2	Univ. of Delaware
	1 G.C. Hsiao
	1 R.E. Kleinman
1	Univ. of Illinois/F. Rizzo
1	Univ. of Kentucky/A.F. Seybert
1	Univ. of Texas/J. Bennighof
1	Univ. of Wisc. (Milwaukee)/ T.-C. Lin
1	VPI/C. Fuller
2	Weidlinger Associates
	1 M. Baron
	1 D. Ranlet

CENTER DISTRIBUTION

Copies	Code	Name
1	011	
1	0112	B. Douglas
1	0113	D.L. Winegrad
1	0114	Baker
1	0117	
1	0118	
1	0204	
1	12	
1	1205	
1	122	
1	1234	O.K. Ritter
1	1234	E.D. Wolfe
1	1234	H.J. Howe
1	125	
1	128	
1	1281	
1	1282	
10	1282	R.S. Cheng
1	1282	T. Moyer
1	1282	A.J. Quezon
1	1282	E.A. Schroeder
1	1282	M.S. Marcus
1	1282	G.C. Everstine

Copies	Code	Name
1	1282	S.A. Hambric
1	1282	F.M. Henderson
1	129	
1	14	
1	15	
1	1504	
1	1508	
1	1522	
1	154	
1	1542	
1	1542	Y.T. Shen
1	1544	
1	17	
1	1702	
1	1703	
1	172	
1	1720.1	
1	1720.1	D.E. Lesar
1	1720.1	C.L. McNamara
1	1720.1	M.G. Costello
1	19	
1	1901	
1	1902	
1	1903	
1	1904	
1	1905.1	
1	1905.2	
1	1905.3	
1	1905.4	
1	1906	
1	1908	
1	191	
1	192	
1	193	
1	1930.1	
1	1930.2	
1	1930.6	J.R. Peoples
1	194	
1	1940.3	M.L. Rumerman
1	1941	
1	1941	J. Su
1	1941	R. Vasudevan

CENTER DISTRIBUTION (Continued)

Copies	Code	Name
1	1941	W.H. Vogel
1	1942	
1	1942	Y.-F.Hwang
1	1943	
1	1944	
1	1945	
1	1945	K. Jones
1	27	
1	2704	
1	2704.1	
1	274	
1	2741	
1	2740	D.E. Goldsmith
1	2742	
1	2742	H.C. Neilson
1	2743	
1	2744	
1	2749	
1	28	
1	342.1	(TIC)
1	342.2	(TIC)
10	3432	Reports Control
1	35	S.E. Willner
1	93	