AD-A229 004

...MENTATION PAGE

DTIC FILE COPY

Form Approved
OMB No. 0704-0188

...REPORT DATE

3. REPORT TYPE AND DATES COVERED
Final Report, 1 Aug 88 to 31 Jul 90

4. TITLE AND SUBTITLE
OBSERVABILITY OF SYSTEMS WITH COMPLICATED DYNAMICS

5. FUNDING NUMBERS
AFOSR-88-0254
61102F 2304 A1

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AFOSR-TR- 90 1090

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-88-0254

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Significant advances have been made in understanding the observability problem for systems with chaotic or otherwise complicated dynamics. Rigorous connections have been established between the theory of stochastic noise and observations of deterministic dynamical systems which are chaotic or otherwise display a complicated dynamical structure. New techniques have been developed for implementing state estimation of chaotic dynamical systems in the presence of observational noise. A general sufficient condition has been established for the observability of a benchmark class of chaotic dynamical systems, the Anosov diffeomorphisms.

14. SUBJECT TERMS

15. NUMBER OF PAGES
7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

Final Technical Report
"Observability of Systems with Complicated Dynamics"
AFOSR grant number 88-0254
October 23, 1990

Summary.

In the two years comprising the period of AFOSR funding of the proposal "Observability of Systems with Complicated Dynamics", we have made significant advances in understanding the observability problem for systems with chaotic or otherwise complicated dynamics. We have established rigorous connections between the theory of stochastic noise and observations of deterministic dynamical which are chaotic or otherwise displaying a complicated dynamical structure. We have developed new techniques for implementing state estimation of chaotic dynamical systems in the presence of observational noise, and continue our efforts in this area. Most recently, we have established a quite general sufficient condition for the observability of a benchmark class of chaotic dynamical systems, the Anosov diffeomorphisms.

Research Objectives.

A To Construct State Estimators for Morse-Smale-like dynamical systems

B To Characterize Observability of Functions on Subshifts of Finite Type

C. To Study the Differentiable Observability of Chaotic Systems

D. To Study Properties of Observations of Chaotic Systems in the Presence of Noise

1. To develop explicitly computable models for observed chaotic systems with noisy perturbations.

2. To study the idea that perturbation by "noise" is the result of coupling to a chaotic dynamical system.

Research Status.

We discuss the status of our research, beginning with the more recent results first, and in most detail, as these results are still in preparation for publication.

In the last several months we have substantially fulfilled one of the research goals put forth in our original proposal, as listed below under the Papers in Preparation section, items number 7 and 8 . In particular, we have developed a new sufficient condition for a given real analytic function to globally observe a *hyperbolic toral automorphism.* These maps comprise an important sub-class of *Anosov diffeomorphisms* and their properties tend to be representative of all Anosov maps. This class of dynamical systems was one of the premier examples of chaotic dynamical systems and is perhaps the best studied and understood of chaotic maps. Our work in this area represents a considerable development in the understanding of the observability properties of chaotic dynamical systems, particularly as via the interplay real analyticity and the topological properties of chaotic dynamical systems, we are able to reduce the explicit consideration of the observability of (the countable dense set of) periodic orbits to the consideration of only a small finite number of such orbits. Our results in this area rather exceed the our expectations of one year ago as illustrated by our remarks in our Annual Report of August 31, 1989. The results depend on a crucial way on the use of real analyticity; neither the techniques nor results are valid in the C^{∞} or topological categories.

The prospects for extending these results to other Anosov dynamical systems are encouraging, and even the extension to observability in neighborhood of chaotic attractors seems not beyond reach, provided that the unstable manifolds of the attractor are real analytic.

As mentioned in our Annual Report last year, we have developed new techniques for implementing state estimation of chaotic dynamical systems in the presence of observational noise. To date, this work includes a paper, in collaboration with Professor DiMasi of the University of Padova, Italy, on new techniques for the implementation of non-linear estimation techniques. We continue to work in this area, concentrating on the development of robust non-linear filtering methods for dynamical systems observed in the presence of noise. This continuing work is based on the our recent observation that here, in the absence of state noise, the Kushner-Stratonovic and Zakai equations, while still intrinsically infinite dimensional when viewed as bilinear stochastic partial differential equations, they are never-the-less now first order hyperbolic partial differential equations rather than parabolic. In this circumstance the robust filtering concept developed by Mark Davis provided a method of transforming the filtering equations into new equations which are sufficiently solvable for quite efficient simulation and perhaps even on-line recursive filtering.

We have established rigorous connections between the theory of stochastic noise and observations of deterministic dynamical which are chaotic or otherwise displaying a complicated dynamical structure. Our advances here are in two directions. The first is to show that observations of chaotic dynamical systems obey a sort of central limit theorem or invariance principle of Donsker type, with the result that when the speed of the chaotic dynamical system is very fast, observations or terms coupling the fast chaos to other systems approximate a gaussian white noise in a rigorous sense. Our other area of investigation, with

Professor Giorgio Picci, of the Electrical Engineering Department, University of Padova, which we had not suspected at the time of our original proposal more than two years ago, has shown that even observations of linear systems may approximate stochastic behavior when the dimension is large and the frequency response obeys suitable conditions. These results suggest intriguing ideas for stochastic model reduction of complicated systems,particularly of distributed parameter systems which low frequency modes or resonances are not widely separated in frequency from other nearby modes. Professor Picci and I are currently in the process of developing these ideas.

Publications.

A. Articles Published

1. "Systems with fast chaotic components," by T. Taylor, in *Computation and Control*, Bowers and Lund, Eds., pp. 333-341, Birkhauser, 1989.

2. "On the Dynamical Origins of Stochastic Motion," by T. Taylor, in *Aportaciones Matematicas 4 'Symposium on Probability and Stochastic Processes'*, Sociedad Matematica Mexicana, 1989, pp. 122-136.

3. "Stochastic Aggregation for Linear Hamiltonian Systems in the Microcanonical Ensemble," by G. Picci and T. Taylor, in *Realization Theory and Modelling in Systems Theory Proc. 1989 International Symposium on the Mathematical Theory of Networks and Systems (MTNS-89).* Kaashoek, van Schuppen and Ran, Eds., Birkhauser 1990, pp. 513-520

B. Articles to Appear

4. "On Observations of Noisy Chaos and Observable Entropy," by T. Taylor
—To appear, *Proceedings 2nd Summer Conference on Computation and Control*

C. Articles in Preparation

5. "Stochastic Aggregation for Linear Hamiltonian Systems in the Microcanonical Ensemble," by G. Picci and T. Taylor
—To be submitted (*Stochastics* is a likely choice)

6. "Nonlinear Filtering and Noncommutative Harmonic Analysis," by G. DiMasi and T. Taylor
—To be submitted to *Systems and Control Letters*.

7. "Conditions for Observability of Chaotic Circle Endomorphisms," by M. Nerurkar and T. Taylor
—To be Submitted (*Conference on Decision and Control*?)

8. "Sufficient Conditions for Observability of the Arnold Cat map and Other Torus Mappings" by M. Nerurkar and T. Taylor (*SIAM J. Control*?)

9. "Higher order Lyapunov exponents," by U. Dressler, D. Farmer and T. Taylor, (*Nonlinearity* or *Physics Review*?)

D. Articles Planned

10. " A Class of Chaotic Infinite Dimensional Linear Dynamical Systems, and Applications to the Study of Statistical Properties of Flows", with D. Herrero.

Interactions.

A. Meetings and Conferences

1. Summer Conference on Computation and Control, Bozeman, Montana, August 1988,
"Systems with fast chaotic components."

2. Midwest Differential Equations Conference, Ames, Iowa, October 1988
"On the connection between chaos and stochasticity."

3. 4th Southwest Symposium on Control, Tempe, Arizona, December 1988
"Brownian representation of functions on Hyperbolic flows."

4. Conference on Probability and Stochastic Processes, Center for Investigation in Mathematics,Guanajuato, Mexico, December 1988
"On the dynamical origins of stochastic motion"

5. International Symposium on the Mathematical Theory of Networks and Systems (MTNS-89), Amsterdam, Holland, June 1989
"Stochastic Aggregation for Linear Hamiltonian Systems in the Microcanonical Ensemble."

6. SIAM Conference on Dynamical Systems, Orlando, Florida, May 1990
" On the Generation of Noise by Deterministic Systems with Complicated Dynamics"

7. 2nd Summer Conference on Computation and Control, Bozeman, Montana, July 1990
"On Observations of Noisy Chaos and Observable Entropy,"

B. Seminars

1. University of Southern California, Department of Mathematics Los Angeles, California, March 1989, "On the dynamical origins of stochastic motion"

2. Centro Nazionale di Ricerche and Universitá di Padova, Padova, Italy May and June 1989 — A series of talks on observations of chaotic systems and stochastic processes

3. Institute of Mathematics, National Autonomous University of Mexico, Mexico City, June 1990, "On stochastic realization theory"