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Geopotential Harmonics of Order 15 and 30 from Analysis of 50 Orbit Determinations of 1967-102B

by

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GEOPOTENTIAL HARMONICS OF ORDER 15 AND 30 FROM ANALYSIS OF 50 ORBIT DETERMINATIONS OF 1967-102B

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SUMMARY

The orbital parameters of the satellite 1967-102B, Cosmos 184 rocket, have been determined at 50 epochs from some 3900 observations. For 21 of these determinations, Hewitt camera observations were available.

During the time of the orbit determinations the satellite passed through 15th-order resonance. The variations in inclination and eccentricity have been analysed to obtain six lumped 15th-order geopotential harmonics, with accuracies equivalent to between 0.2 and 3.3 cm in geoid height and four lumped 30th-order geopotential harmonics, with accuracies better than 1 cm in geoid height.

The lumped harmonics obtained in this Report have been compared with those from the GEM 10B and 10C models and with those from GRIM3-L1 and the RAPP 1981 models. A comparison has also been made with the lumped harmonics obtained from a new PGS-3337 model published by NASA, and the agreement with this model is very good.

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1 INTRODUCTION

The satellite 1967-102B is the final-stage rocket which put Cosmos 184 into orbit. It entered orbit in October 1967 and initially had the following elements[1]: inclination 81.20°; nodal period 97.27 min; apogee and perigee heights, 711 and 544 km respectively; and eccentricity 0.012.

In 1986, after some 18 years in orbit, 1967-102B was approaching 15th-order resonance, *ie* when the satellite makes 15 revolutions while the Earth spins once, relative to the orbital plane. In this Report the orbit of 1967-102B has been determined from radar and optical observations during 1986 and 1987, using the RAE orbit determination program PROP in the PROP 6 version[2]. The satellite during this time was placed high on the priority list for observing. The inclination and eccentricity have been analysed over the time of the orbit determination to evaluate lumped harmonics of order 15 and 30 in the geopotential.

2 THE OBSERVATIONS AND ORBITS

2.1 The observations

The orbit of 1967-102B has been determined at 50 epochs, from some 3900 observations, between 1986 June 14 and 1987 November 13. The number of observations used in each orbit determination is given in Table 1 and the source of the observations is also indicated.

There were three groups of observations available. Those listed first in Table 1 were the most accurate, being made by the University of Aston's Hewitt cameras at the Royal Greenwich Observatory site at Herstmonceux and at the Siding Spring Observatory site in Australia; these observations usually have an accuracy of a few seconds of arc. The second consisted of visual observations made by volunteer observers who used to report to the University of Aston but now report to the Royal Greenwich Observatory, Herstmonceux; these observations usually have accuracies between 2 and 5 min of arc in good observing conditions. The final and largest group is made up of Navspasur observations kindly supplied by the US Naval Research Laboratory, with accuracies of about 2 min of arc.

Table 1

Sources of the observation used in each orbit

Orbit No.	Source of observation			Total	Orbit No.	Source of observation			Total
	Hewitt Camera	Visual	US Navy			Hewitt Camera	Visual	US Navy	
1	16(S)		77	93	26	6(S)		73	79
2		1	90	91	27			86	86
3			88	88	28		3	93	96
4			76	76	29	5(H)	26	51	82
5		5	60	65	30	6(H)	16	41	63
6	5(H)	6	58	69	31		10	85	95
7		21	82	103	32	2(S)		88	90
8	10(H)	3	84	97	33	12(S)		65	77
9		11	95	106	34		3	84	87
10		4	83	87	35			69	69
11		5	71	76	36		5	66	71
12	12(S)		38	50	37	15(H)	2	60	77
13			65	65	38	5(H)	5	59	69
14			64	64	39			71	71
15			67	67	40	6(S)		73	79
16		5	61	66	41			96	96
17	4(H)4(S)	18	58	84	42		1	72	73
18		11	61	72	43		1	91	92
19	6(S)	3	70	79	44			78	78
20	12(S)		45	57	45			90	90
21	4(S)		52	56	46		11	55	66
22	17(S)		40	57	47	6(S)	22	55	83
23	30(S)		40	70	48	18(S)	22	53	93
24		2	52	54	49		18	80	98
25		19	46	65	50	6(S)		72	78

S = camera at Siding Spring, Australia. H = camera at Herstmonceux, UK

2.2 The orbits

The orbits were determined at approximately 10-day intervals with the aid of the RAE orbit refinement program PROP in the PROP 6 version, and the orbital elements at each epoch are listed in Table 2, with the standard deviations below each value. The epoch for each orbit is at 00 hours on the day indicated, and the PROP program fits the mean anomaly M by a polynomial of the form:

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5 , \qquad (1)$$

where t is the time measured from epoch, and the number of M coefficients used depends on the drag. For 1967-102B, which was in a nearly circular orbit with a perigee height of 470 km, only $M_0 - M_2$ were needed.

For all 50 orbits listed in Table 2, the standard deviations in inclination were between 0.0001° and 0.0014°, the average being 0.0007°, which is equivalent to about 90 m in distance. The values of standard deviation in eccentricity varied between 0.000002 and 0.000015, and the average value was 0.000007, corresponding to 50 m in distance. The value of ε , which is the parameter indicating the measure of fit of the observations to the orbit, ranged between 0.26 and 0.92, the average value being 0.51. The standard deviation for the right ascension of the node was never more than 0.001°.

Table 2

Values of orbital parameters at 50 epochs with standard deviations

MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	ε	N	D
1 46595	1986 Jun 14	6918.0984	0.009752	81.1560	348.775	178.40	171.057	5433.3205	0.0013	0.85	91	7.9
		2	4	1	<1	1		3	1			
2 46607	1986 Jun 26	6918.0621	0.010478	81.1548	334.953	139.91	171.419	5433.3632	0.0020	0.43	89	9.9
		1	7	7	1	4		1	1			
3 46617	1986 Jul 6	6918.0249	0.010807	81.1570	323.431	109.86	112.178	5433.4071	0.0019	0.31	85	9.5
		1	3	5	<1	3		1	<1			
4 46627	1986 Jul 16	6917.9883	0.010800	81.1588	311.914	80.41	53.359	5433.4503	0.0025	0.29	73	7.6
		1	3	5	<1	4		1	1			
5 46635	1986 Jul 24	6917.9526	0.010535	81.1592	302.700	56.67	294.648	5433.4924	0.0033	0.39	63	7.5
		2	7	8	1	5		2	1			
6 46643	1986 Aug 1	6917.9042	0.010074	81.1675	293.488	32.16	176.341	5433.5496	0.0032	0.59	66	7.7
		3	9	6	1	5		3	2			
7 46654	1986 Aug 12	6917.8280	0.009261	81.1615	280.826	356.57	149.453	5433.6392	0.0043	0.45	99	7.0
		1	10	7	1	2		2	1			
8 46662	1986 Aug 20	6917.7661	0.008685	81.1658	271.616	328.33	32.378	5433.7123	0.0036	0.73	93	5.6
		3	7	2	<1	1		4	2			
9 46670	1986 Aug 28	6917.7038	0.008283	81.1645	262.411	298.99	275.879	5433.7858	0.0044	0.53	96	8.0
		2	6	8	1	5		2	<1			
10 46678	1986 Sep 5	6917.6477	0.008154	81.1626	253.204	268.15	159.959	5433.8518	0.0035	0.42	84	7.9
		2	4	8	1	6		2	1			
11 46687	1986 Sep 14	6917.5976	0.008399	81.1653	242.851	233.89	75.086	5433.9109	0.0034	0.54	75	9.0
		2	8	11	1	8		3	1			
12 46695	1986 Sep 22	6917.5608	0.008903	81.1721	233.646	205.10	320.070	5433.9545	0.0031	0.63	49	6.0
		3	7	14	1	5		3	2			
13 46703	1986 Sep 30	6917.5316	0.009455	81.1729	224.449	177.69	205.357	5433.9890	0.0017	0.57	64	9.6
		2	13	12	1	4		2	1			
14 46716	1986 Oct 13	6917.4958	0.010338	81.1788	209.504	136.60	244.447	5434.0314	0.0015	0.29	59	9.6
		1	5	6	1	3		1	<1			
15 46729	1986 Oct 26	6917.4583	0.010839	81.1781	194.561	98.05	284.066	5434.0756	0.0026	0.47	66	9.0
		1	5	8	1	6		2	1			
16 46742	1986 Nov 8	6917.3866	0.010751	81.1725	179.614	60.02	324.556	5434.1600	0.0011	0.56	65	8.5
		2	8	10	1	5		3	1			
17 46752	1986 Nov 18	6917.3728	0.010219	81.1693	168.111	30.24	273.135	5434.1761	0.0007	0.57	74	9.7
		1	4	3	1	2		1	<1			
18 46762	1986 Nov 28	6917.3483	0.009634	81.1657	156.605	358.44	221.872	5434.2049	0.0017	0.51	64	8.9
		2	12	8	1	3		3	1			

Table 2 (continued)

	MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	ε	N	D
19	46774	1986 Dec 10	6917.3370 1	0.008712 7	81.1618 6	142.791 <1	317.05 2	232.723	5434.2180 2	0.0001 1	0.55	74	9.9
20	46787	1986 Dec 23	6917.3284 1	0.008308 5	81.1636 2	127.824 <1	269.02 2	274.558	5434.2282 1	0.0006 1	0.40	55	9.0
21	46803	1987 Jan 8	6917.3119 1	0.008745 3	81.1624 4	109.403 1	209.50 3	49.415	5434.2476 1	0.0004 1	0.41	56	9.5
22	46816	1987 Jan 21	6917.2983 1	0.009567 4	81.1603 1	94.431 <1	165.58 2	91.670	5434.2637 1	0.0007 <1	0.52	57	8.5
23	46824	1987 Jan 29	6917.2947 1	0.010070 8	81.1569 1	85.215 <1	140.23 2	339.326	5434.2678 2	-0.0013 1	0.76	70	5.5
24	46835	1987 Feb 9	6917.2973 1	0.010482 3	81.1555 5	72.542 <1	107.06 3	319.851	5434.2647 1	-0.0002 <1	0.26	50	9.4
25	46846	1987 Feb 20	6917.2960 1	0.010474 5	81.1527 8	59.862 1	74.66 5	300.344	5434.2662 2	0.0003 1	0.39	56	9.9
26	46865	1987 Mar 11	6917.2694 1	0.009602 2	81.1534 3	37.954 <1	16.23 2	168.721	5434.2975 1	0.0012 <1	0.31	78	9.6
27	46878	1987 Mar 24	6917.2365 1	0.008752 6	81.1546 7	22.968 <1	332.24 3	211.773	5434.3362 1	0.0021 <1	0.33	84	9.6
28	46890	1987 Apr 5	6917.2014 1	0.008282 4	81.1563 6	9.134 1	288.13 5	224.358	5434.3777 1	0.0016 1	0.37	95	9.7
29	46900	1987 Apr 15	6917.1652 3	0.008427 7	81.1519 10	357.602 1	250.08 7	175.228	5434.4202 3	0.0031 1	0.68	74	7.8
30	46907	1987 Apr 22	6917.1298 4	0.008713 15	81.1547 14	349.530 1	224.60 6	33.153	5434.4621 5	0.0031 3	0.65	59	5.6
31	46915	1987 Apr 30	6917.0918 1	0.009146 9	81.1503 8	340.305 1	196.71 3	282.553	5434.5068 1	0.0028 <1	0.42	92	9.6
32	46925	1987 May 10	6917.0239 1	0.009907 6	81.1473 10	328.771 1	164.19 2	234.875	5434.5868 2	0.0051 1	0.56	84	9.9
33	46935	1987 May 20	6916.9248 1	0.010445 3	81.1476 13	317.236 1	133.67 4	188.207	5434.7037 1	0.0051 1	0.92	73	9.1
34	46946	1987 May 31	6916.8065 1	0.010725 4	81.1506 8	304.553 1	100.75 5	174.245	5434.8433 2	0.0061 1	0.48	81	9.9
35	46957	1987 Jun 11	6916.7061 1	0.010588 6	81.1565 8	291.876 1	68.51 6	161.771	5434.9618 2	0.0048 1	0.46	69	9.5
36	46969	1987 Jun 23	6916.6068 1	0.009936 10	81.1621 9	278.051 1	32.30 4	182.290	5435.0790 2	0.0045 1	0.49	70	9.4

Table 2 (concluded)

	MJD	Date	a	e	i	Ω	ω	ω + M_0	M_1	M_2	ε	N	D
37	46978	1987 Jul 2	6916.5386 <1	0.009316 4	81.1667 1	267.682 <1	2.67 2	108.560	5435.1596 <1	0.0039 3	0.87	77	6.1
38	46987	1987 Jul 11	6916.4793 3	0.008640 13	81.1709 9	257.321 1	331.96 5	35.502	5435.2296 3	0.0037 1	0.65	68	9.1
39	46997	1987 Jul 21	6916.4094 2	0.008179 6	81.1683 8	245.811 1	294.84 7	355.074	5435.3119 2	0.0040 1	0.48	71	9.6
40	47007	1987 Jul 31	6916.3267 2	0.008238 5	81.1640 11	234.299 1	256.35 5	315.540	5435.4093 3	0.0044 1	0.64	76	9.9
41	47019	1987 Aug 12	6916.2424 1	0.008783 8	81.1693 7	220.492 1	212.34 3	341.298	5435.5090 1	0.0041 <1	0.40	94	9.4
42	47030	1987 Aug 23	6916.1605 1	0.009567 11	81.1739 9	207.837 1	175.13 3	335.969	5435.6057 1	0.0045 1	0.49	72	9.1
43	47044	1987 Sep 6	6916.0401 1	0.010413 7	81.1804 7	191.743 1	131.77 4	69.045	5435.7479 1	0.0048 1	0.45	91	8.6
44	47058	1987 Sep 20	6915.8866 2	0.010693 4	81.1866 8	175.657 1	90.67 7	164.323	5435.9291 2	0.0063 1	0.45	73	7.5
45	47067	1987 Sep 29	6915.7792 1	0.010492 6	81.1890 8	165.319 1	64.09 5	98.406	5436.0560 2	0.0071 1	0.46	90	9.6
46	47077	1987 Oct 9	6915.6687 3	0.009910 12	81.1897 12	153.832 1	33.77 5	66.481	5436.1863 3	0.0064 1	0.50	65	7.7
47	47085	1987 Oct 17	6915.5695 3	0.009298 5	81.1946 9	144.640 1	8.05 6	329.909	5436.3034 4	0.0071 2	0.72	78	7.6
48	47092	1987 Oct 24	6915.4993 <1	0.008834 3	81.1880 1	136.593 <1	344.02 2	201.126	5435.3860 <1	0.0053 1	0.57	85	6.0
49	47101	1987 Nov 2	6915.4150 1	0.008389 9	81.1853 9	126.251 1	311.99 6	139.185	5436.4854 1	0.0059 1	0.53	94	9.9
50	47112	1987 Nov 13	6915.2953 2	0.008142 4	81.1859 8	113.608 1	270.47 6	144.790	5436.6265 2	0.0070 1	0.31	57	7.9

KEY

MJD modified Julian day
a semi major axis (km)
e eccentricity
i inclination (deg)
Ω right ascension of ascending node (deg)
ω argument of perigee (deg)
M_0 mean anomaly at epoch (deg)
M_1 mean motion n (deg/day)
M_2 second coefficient in the polynomial for M
ε measure of fit
N number of observations used
D time covered by the observations (days)

3 THE EQUATIONS FOR 15th-ORDER RESONANCE

The theory for general $\beta:\alpha$ resonance has been given before, in Ref 3, where all the parameters used are defined. The theoretical equations at 15th-order resonance for the variation of inclination and eccentricity are as follows:

$$\frac{di}{dt} = \frac{n(1-e^2)^{-\frac{1}{2}}}{\sin i}\left(\frac{R}{a}\right)^{15}\Bigg[(15 - \cos i)\bar{F}_{15,15,7}G_{15,7,0}\left\{\bar{C}_{15}^{0,1}\sin\Phi - \bar{S}_{15}^{0,1}\cos\Phi\right\}$$

$$+ 15\left(\frac{R}{a}\right)\bar{F}_{16,15,8}G_{16,8,1}\left\{\bar{S}_{15}^{-1,0}\sin(\Phi-\omega) + \bar{C}_{15}^{-1,0}\cos(\Phi-\omega)\right\}$$

$$+ (15 - 2\cos i)\left(\frac{R}{a}\right)\bar{F}_{16,15,7}G_{16,7,-1}\times$$

$$\times\left\{\bar{S}_{15}^{-1,2}\sin(\Phi+\omega) + \bar{C}_{15}^{-1,2}\cos(\Phi+\omega)\right\}$$

$$+ \text{terms in}\left\{\frac{(\ell e/2)^{|q|}}{(|q|)!}\,{}^{\cos}_{\sin}(\gamma\Phi - q\omega)\right\}\Bigg], \qquad (2)$$

and, on using equation (21) of Ref 3 after omission of e^2 terms that are negligible,

$$\frac{de}{dt} = n\left(\frac{R}{a}\right)^{15}\Bigg[\Bigg[\frac{e}{2}\bar{F}_{15,15,7}G_{15,7,0}\left(\bar{C}_{15}^{0,1}\sin\Phi - \bar{S}_{15}^{0,1}\cos\Phi\right)$$

$$-e^{-1}\left(\frac{R}{a}\right)\bar{F}_{16,15,8}G_{16,8,1}\left\{\bar{S}_{15}^{-1,0}\sin(\Phi-\omega) + \bar{C}_{15}^{-1,0}\cos(\Phi-\omega)\right\}$$

$$+e^{-1}\left(\frac{R}{a}\right)\bar{F}_{16,15,7}G_{16,7,-1}\left\{\bar{S}_{15}^{-1,2}\sin(\Phi+\omega) + \bar{C}_{15}^{-1,2}\cos(\Phi+\omega)\right\}$$

$$+ \text{terms in}\left[\frac{(\ell/2)^{|q|}e^{|q|-1}}{(|q|)!}\{q - \tfrac{1}{2}(k+3q)e^2\}\,{}^{\cos}_{\sin}(\gamma\Phi - q\omega)\right]\Bigg]\Bigg] \qquad (3)$$

The resonance angle Φ is given by

$$\Phi = \omega + M + 15(\Omega - \nu) \quad , \tag{4}$$

where ν is the sidereal angle, and at exact resonance $\dot{\Phi} = 0$. In equations (2) and (3) only the three terms with $(\gamma,q) = (1,0)(1,1)$ and $(1,-1)$ are given explicitly.

The quantities such as $\bar{C}_{15}^{0,1}$ in equations (2) and (3) are lumped geopotential harmonics of order 15, which can be expressed as linear sums of individual coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$,

$$\bar{C}_m^{q,k} = \sum_\ell Q_\ell^{q,k} \bar{C}_{\ell m} \quad \text{and} \quad \bar{S}_m^{q,k} = \sum_\ell Q_\ell^{q,k} \bar{S}_{\ell m} \quad , \tag{5}$$

and these individual coefficients can be evaluated when enough lumped harmonics from satellites at different inclinations are available. The summation for ℓ in equation (5) is in steps of 2, beginning at the lowest ℓ , which is either 15 or 16 for the lumped harmonics in equations (2) and (3), see Ref 3; and for 15th-order resonance $k = \gamma - q$.

The orders of magnitude of the terms in the equations for di/dt and de/dt can be estimated, as the $\bar{C}_{\ell m}$ (or $\bar{S}_{\ell m}$) are expected to be of order $10^{-5}/\ell^2$, so the value of $\bar{C}_m^{q,k}$ (or $\bar{S}_m^{q,k}$) can be taken to be of order $\left\{\sum (Q_\ell \times 10^{-5}/\ell^2)^2\right\}^{1/2}$, the Q coefficients being obtained for specific values of (γ,q) using the RAE computer program PROF.

Estimating the orders of magnitude of the terms in equation (2) for inclination suggests that, if just the $(\gamma,q) = (1,0)$ and $(2,0)$ terms are used, the likely error is 6% from the neglect of the $(1,\pm 1)$ terms. In equation (3) the $(\gamma,q) = (1,1)$ and $(1,-1)$ terms are dominant, the $(\gamma,q) = (1,0)$ term only contributing about 0.5%; but the $(\gamma,q) = (2,\pm 1)$ terms may need to be considered.

4 ANALYSIS OF INCLINATION

Cosmos 184 rocket passed through exact 15th-order resonance on 1987 July 30 and its orbit has been determined either side of resonance, with the rate of change of resonance angle, $\dot{\Phi}$, increasing from -2.1 to +1.2 deg/day. The variation of $\dot{\Phi}$ and Φ , the resonance angle, given by equation (4) are shown in Fig 1.

In order to analyse the changes in inclination due to resonance, all other known perturbations must be removed. The 50 values of inclination in Table 2 were therefore cleared of lunisolar and zonal harmonic perturbations, by using the computer program PROD[4] with 1-day integration steps, and the perturbation due to the $J_{2,2}$ tesseral harmonic, which is recorded on each PROP run, was also removed. When the standard deviation for inclination was less than 0.0005°, it was increased to 0.0005° to allow for the perturbation due to the neglected effects of earth and ocean tides.

These revised values of inclination, with the standard deviations quoted in Table 2 except where they have been increased to 0.0005°, were fitted with equation (2) in integrated form using the RAE THROE[5] computer program. This program removes the further perturbations due to atmospheric rotation and luni-solar precession of the Earth's axis. The density scale height H was taken as 55 km, appropriate to a height of 530 km, 1.1 H above perigee[6], and the atmospheric rotation rate, Λ, was taken as 1.2 rev/day[7]. The values of M_2 were altered to mean values, $\bar{M}_2$, by the technique described in Ref 8.

The first fitting by THROE, with $(\gamma,q) = (1,0)(2,0)$ gave $\varepsilon = 2.9$, where ε is the measure of fit parameter. On inspection of the PROP runs for some of the ill-fitting values of inclination, it was found that where PROP runs had more than one set of Hewitt camera observations, relaxation of the standard deviations of the sets away from the epoch of the PROP run gave better-fitting values of inclinatic· The need for relaxation is justifiable because of limitations in the accu. ·y . the PROP orbital model.

With these relaxations made, the value of ε obtained was 1.9. A final THROE run, with the standard deviations increased by a factor of two for three of the values of inclination to keep all weighted residuals less than 2ε, gave $\varepsilon = 1.64$ and the values of the lumped harmonics were as follows:

$$10^9\bar{C}^{0,1}_{15} = -18.04 \pm 0.44 \qquad 10^9\bar{S}^{0,1}_{15} = -1.62 \pm 0.33$$

$$10^9\bar{C}^{0,2}_{30} = 0.35 \pm 0.67 \qquad 10^9\bar{S}^{0,2}_{30} = 8.80 \pm 1.08 \,. \tag{5}$$

A further THROE run, with the $(\gamma,q) = (1,\pm1)$ terms added, was tried, but these were not well determined and did not improve the fit.

The values of inclination, cleared of all known perturbations except those due to resonance, are plotted in Fig 2. The theoretical curve derived from the THROE fitting with $(\gamma,q) = (1,0)(2,0)$, that gave the values in equation (5), is also shown as a full line.

5 ANALYSIS OF ECCENTRICITY

The 50 values of eccentricity in Table 2 were fitted with equation (3) in integrated form using THROE. The eccentricity values were cleared of zonal harmonic and lunisolar perturbations, and variations due to air drag, within THROE before the remaining variation due to resonance is fitted by the program.

Before fitting the values of eccentricity, the M_2 values were altered to mean values, $\bar{M}_2$, as for the inclination, and 16 values of eccentricity with standard deviations less than 0.000005, had them increased to that value in case there was a significant effect from the neglect of earth and ocean tides.

In some previous fittings of eccentricity with THROE, it has been found that adjustment of the odd zonal harmonics was needed. The same method has been used here as in Ref 9, *ie* to add an increment ΔJ_3 to the J_3 value used in the PROP model.

With these adjustments made, the fitting with THROE was undertaken. After experimenting with pairs of coefficients, the best result was obtained with $(\gamma,q) = (1,1)(1,-1)$ and $(2,1)$, giving $\varepsilon = 4.6$. Other (γ,q) terms tried were $(2,-1)(1,2)$ and $(1,-2)$ and these resulted in undetermined coefficients. During the various fittings it was found that a linear term improved the fit, so the value of scale height was optimized to give a better fit. The final run, with $(\gamma,q) = (1,1)(1,-1)$ and $(2,1)$, ΔJ_3 optimized at -0.237, H optimized at 60 km and standard deviations increased by a factor of two on seven of the values to keep all weighted residuals less than 2ε, resulted in the following values of lumped harmonics, with $\varepsilon = 3.310$.

$$10^9 \bar{C}_{15}^{1,0} = -41.6 \pm 1.9 \qquad 10^9 \bar{S}_{15}^{1,0} = -50.8 \pm 1.8$$

$$10^9 \bar{C}_{15}^{-1,2} = -68.3 \pm 15.4 \qquad 10^9 \bar{S}_{15}^{-1,2} = -90.4 \pm 16.3 \tag{6}$$

$$10^9 \bar{C}_{30}^{1,1} = 8.3 \pm 1.4 \qquad 10^9 \bar{S}_{30}^{1,1} = -12.3 \pm 0.8 .$$

The values of eccentricity, cleared of all known perturbations larger than the standard deviations of the value, except those due to resonance, are plotted in Fig 3. The fitted curve is that given by the THROE fitting which produced the values in equations (6).

6 INCLINATION AND ECCENTRICITY FITTED SIMULTANEOUSLY

The values of inclination and eccentricity fitted separately by THROE can be fitted simultaneously using the RAE computer program SIMRES developed by Dr R.H. Gooding. This program combines the results from a number of THROE runs, with the same set of (γ, q) terms, and produces a single set of coefficients to fit the data. For this SIMRES fitting, the results from THROE runs with $(\gamma, q) = (1,0)(2,0)(1,1)(1,-1)$ and $(2,1)$ were used. The SIMRES program allows a choice of weighting, so the contributing THROE runs can be given more or less weight according to their accuracy of fit, which is indicated by the value of ε.

The THROE fittings of inclination and eccentricity with $(\gamma, q) = (1,0)(2,0)(1,1)(1,-1)$ and $(2,1)$ gave $\varepsilon = 1.458$ and 3.321 respectively. For the SIMRES fitting, therefore, the weighting of eccentricity was down-graded by a factor equal to the ratio of the values of ε on the THROE fittings, namely 2.278(=3.321/1.458). The values of the lumped harmonics given by the SIMRES fitting are:

$$
\begin{aligned}
10^9\bar{C}_{15}^{0,1} &= -19.37 \pm 0.45 & \qquad 10^9\bar{S}_{15}^{0,1} &= -2.13 \pm 0.34 \\
10^9\bar{C}_{30}^{0,2} &= 2.48 \pm 0.67 & \qquad 10^9\bar{S}_{30}^{0,2} &= 5.98 \pm 1.06 \\
10^9\bar{C}_{15}^{1,0} &= -41.2 \pm 2.1 & \qquad 10^9\bar{S}_{15}^{1,0} &= -50.2 \pm 1.9 \qquad (7) \\
10^9\bar{C}_{15}^{-1,2} &= -70.1 \pm 16.7 & \qquad 10^9\bar{S}_{15}^{-1,2} &= -85.0 \pm 17.6 \\
10^9\bar{C}_{30}^{1,1} &= 9.0 \pm 1.5 & \qquad 10^9\bar{S}_{30}^{1,1} &= -12.3 \pm 0.8 .
\end{aligned}
$$

It is not clear whether these values are better than those determined for inclination and eccentricity separately, equations (5) and (6): the standard deviations are nearly the same. The fittings, if shown pictorially in Figs 2 and 3, would be indistinguishable from the curves given by the THROE fittings.

7 EQUATIONS FOR INDIVIDUAL COEFFICIENTS

The lumped coefficients in equation (7) can be expressed as linear sums of individual coefficients: see equation (5). The Q coefficients in these equations depend on the ratios[3] of the eccentricity functions $G_{\ell pq}$, and can be evaluated using the RAE computer program PROF. However, a correction factor has to be applied to the values of Q, as it was assumed in the PROF program that the e^2 terms in the function $G_{\ell pq}$ could be neglected (see Ref 3). This leads to large errors in Q if e is large; here, with e approximately 0.01, the correction is small, being less than 4%. The values of the lumped harmonics from THROE and SIMRES also need a small correction, which has been made in equations (5) to (7).

The equations given below for the individual coefficients have been terminated when the expected contribution from the coefficients permanently falls to less than 5% of the largest contribution.

The resulting ten equations are as follows:

$$\bar{C}_{15}^{0,1} = \bar{C}_{15,15} + 0.441\bar{C}_{17,15} + 0.173\bar{C}_{19,15} - 0.002\bar{C}_{21,15} - 0.116\bar{C}_{23,15} - 0.180\bar{C}_{25,15} - 0.211\bar{C}_{27,15} - 0.210\bar{C}_{29,15} \; ; \qquad (8)$$

$\bar{S}_{15}^{0,1}$: the equation is the same as (8), with S instead of C ; (9)

$$\bar{C}_{30}^{0,2} = \bar{C}_{30,30} + 0.302\bar{C}_{32,30} - 0.011\bar{C}_{34,30} - 0.186\bar{C}_{36,30} - 0.272\bar{C}_{38,30} - 0.298\bar{C}_{40,30} - 0.280\bar{C}_{42,30} - 0.235\bar{C}_{44,30} - 0.176\bar{C}_{46,30} \; ; \qquad \ldots\ldots (10)$$

$\bar{S}_{30}^{0,2}$: the equation is the same as (10) with S instead of C ; (11)

$$\bar{C}_{15}^{1,0} = \bar{C}_{16,15} + 0.910\bar{C}_{18,15} + 0.671\bar{C}_{20,15} + 0.384\bar{C}_{22,15} + 0.106\bar{C}_{24,15}$$

$$- 0.132\bar{C}_{26,15} - 0.308\bar{C}_{28,15} - 0.415\bar{C}_{30,15} - 0.455\bar{C}_{32,15}$$

$$- 0.434\bar{C}_{34,15} - 0.368\bar{C}_{36,15} ; \qquad (12)$$

$\bar{S}_{15}^{1,0}$: the equation is the same as (12) with S instead of C ; (13)

$$\bar{C}_{15}^{-1,2} = \bar{C}_{16,15} + 1.688\bar{C}_{18,15} + 2.276\bar{C}_{20,15} + 2.707\bar{C}_{22,15} + 2.939\bar{C}_{24,15}$$

$$+ 2.955\bar{C}_{26,15} + 2.759\bar{C}_{28,15} + 2.380\bar{C}_{30,15} + 1.863\bar{C}_{32,15}$$

$$+ 1.262\bar{C}_{34,15} + 0.636\bar{C}_{36,15} + 0.040\bar{C}_{38,15} - 0.480\bar{C}_{40,15}$$

$$- 0.888\bar{C}_{42,15} - 1.163\bar{C}_{44,15} - 1.299\bar{C}_{46,15} - 1.302\bar{C}_{48,15}$$

$$- 1.191\bar{C}_{50,15} - 1.001\bar{C}_{52,15} ; \qquad (14)$$

$\bar{S}_{15}^{-1,2}$: the equation is the same as (14) with S instead of C ; (15)

$$\bar{C}_{30}^{1,1} = \bar{C}_{31,30} + 0.799\bar{C}_{33,30} + 0.494\bar{C}_{35,30} + 0.194\bar{C}_{37,30} - 0.054\bar{C}_{39,30}$$

$$- 0.233\bar{C}_{41,30} - 0.338\bar{C}_{43,30} - 0.374\bar{C}_{45,30} - 0.354\bar{C}_{47,30}$$

$$- 0.293\bar{C}_{49,30} - 0.208\bar{C}_{51,30} ; \qquad (16)$$

$\bar{S}_{30}^{1,1}$: the equation is the same as (16) with S instead of C ; (17)

8 APPROXIMATE ACCURACY IN GEOID HEIGHT

Equations (8) to (17) are useful in allowing an approximate assessment of the accuracy of the lumped harmonics evaluated here, to be expressed as an accuracy in geoid height. If σ is the error in the lumped harmonic, the corresponding geoid height error, σ_{geoid}, is given approximately by $R\sigma/Q^*$, where R is the Earth's radius and

$$Q^* = \left\{ \sum \left(Q_\ell^{q,k} \ell_0^2 / \ell^2 \right)^2 \right\}^{\frac{1}{2}} .$$

The values of σ_{geoid} for each lumped harmonic in equation (7) are as follows:

lumped harmonic	σ_{geoid} (cm)	lumped harmonic	σ_{geoid} (cm)
$\bar{C}_{15}^{0,1}$	0.3	$\bar{S}_{15}^{0,1}$	0.2
$\bar{C}_{30}^{0,2}$	0.4	$\bar{S}_{30}^{0,2}$	0.6
$\bar{C}_{15}^{1,0}$	1.0	$\bar{S}_{15}^{1,0}$	0.9
$\bar{C}_{15}^{-1,2}$	3.1	$\bar{S}_{15}^{-1,2}$	3.3
$\bar{C}_{30}^{1,1}$	0.7	$\bar{S}_{30}^{1,1}$	0.4

9 LUMPED HARMONICS FROM 1967-102B COMPARED WITH THOSE OBTAINED FROM COMPREHENSIVE MODELS

It is interesting to compare the lumped harmonics obtained from 1967-102B with those from comprehensive gravity-field models. The comparison is made in Tables 3 and 4, the values in equation (7) being cited. The five models used for comparison are: GEM 10B and GEM 10C[10]; the model produced in 1981 by R.H. Rapp[11]; GRIM3-L1[12]; and a new Goddard Model, PGS-3337[13], a preliminary version of GEM-T3.

The Goddard Earth Model GEM 10B extends to order and degree 36, and GEM 10C consists of the GEM 10B solution up to degree 36, together with some 31000 coefficients of order and degree up to 180, derived from analysis of altimeter measurements over the oceans. Equations (10), (11) and (14)-(17) require coefficients above 36, some as high as degree 52, but the lumped harmonics in Tables 3 and 4 for GEM 10B are truncated at degree 36. The expected accuracy of the GEM 10B coefficients has been assumed[14] to be 3×10^{-9}. The GEM 10C accuracy (above degree 36) is certainly poorer[15] and is rather arbitrarily taken as 5×10^{-9}. The standard deviations for the GEM 10B and GEM 10C values in Table 4 have been assessed using these accuracies for the individual coefficients.

The comprehensive geopotential model produced by R.H. Rapp at the Ohio State University in 1981 gives the individual coefficients to order and degree 180, and is derived from Seasat altimeter data, terrestrial gravity measurements and other data. An accuracy estimate for each coefficient is also given, so the standard deviations for the lumped harmonics in Tables 3 and 4 have been assessed using these accuracies. The lumped harmonics obtained by using the individual coefficients from GRIM3-L1 are also given in Tables 3 and 4. This model only extends to degree and order 36, so all the equations have to be truncated at this value, as with GEM 10B. The standard deviations are calculated from the accuracies given for the individual coefficients. Finally the lumped harmonics obtained from a new Goddard solution PGS-3337, which is complete in harmonic coefficients to degree and order 50, together with accuracies for each coefficient, are included in Tables 3 and 4.

Table 3

Values of 15th-order lumped harmonics from 1967-102B and comprehensive geoid models

	$10^9 \bar{C}_{15}^{0,1}$	$10^9 \bar{S}_{15}^{0,1}$	$10^9 \bar{C}_{15}^{1,0}$	$10^9 \bar{S}_{15}^{1,0}$	$10^9 \bar{C}_{15}^{-1,2}$	$10^9 \bar{S}_{15}^{-1,2}$
1967-102B	-19.4 ± 0.5	-2.1 ± 0.3	-41 ± 2	-50 ± 2	-70 ± 17	-85 ± 18
GEM 10B	-22 ± 3	-4 ± 3	-60 ± 5	-41 ± 5	-94 ± 22	-74 ± 22
GEM 10C	-22 ± 3	-4 ± 3	-60 ± 5	-41 ± 5	-94 ± 26	-63 ± 26
GRIM3-L1	-20 ± 5	-1 ± 5	-52 ± 7	-57 ± 7	-148 ± 27	-89 ± 27
RAPP	-18 ± 4	-6 ± 4	-50 ± 4	-45 ± 3	-141 ± 19	-91 ± 19
PGS 3337	-17 ± 1	-5 ± 1	-51 ± 2	-55 ± 2	-120 ± 12	-88 ± 12

Table 4

Values of 30th-order lumped harmonics from 1967-102B and comprehensive geoid models

	$10^9\bar{C}_{30}^{0,2}$	$10^9\bar{S}_{30}^{0,2}$	$10^9\bar{C}_{30}^{1,1}$	$10^9\bar{S}_{30}^{1,1}$
1967-102B	2.5 ± 0.7	6.0 ± 1.1	9 ± 2	-12 ± 1
GEM 10B	-5 ± 3	11 ± 3	0 ± 4	-19 ± 4
GEM 10C	-7 ± 4	13 ± 4	1 ± 6	-14 ± 6
GRIM3-L1	-1 ± 3	6 ± 3	-4 ± 4	-21 ± 4
RAPP	-5 ± 2	7 ± 2	-4 ± 2	-13 ± 3
PGS-3337	-2 ± 2	4 ± 2	-3 ± 3	-15 ± 3

In Table 3 the values of $\bar{C}_{15}^{0,1}$ and $\bar{S}_{15}^{0,1}$ from 1967-102B are nominally much more accurate than those from the models: the agreement is good, with eight of the ten values differing from the 1967-102B values by less than the sum of their standard deviations. The mean difference for all 30 model values in Table 3 from those of 1967-102B is 1.1 times the sum of the standard deviations.

In Table 4 the values for GEM 10B and GRIM3-L1 should not be taken too seriously, as both models only go to degree and order 36, whereas equations (10) and (11) require coefficients to degree 46 and equations (16) and (17) coefficients to degree 51. The values from the other 3 models agree on average to within 1.4 times the sum of the standard deviations, with those from 1967-102B.

The values of $\bar{C}_{30}^{0,2}$ and $\bar{S}_{30}^{0,2}$ obtained here have been used to determine an improved set of values of individual harmonic coefficients of order 30 and even degree: see Appendix.

10 CONCLUSIONS

The orbit of 1967-102B has been determined at 50 epochs from some 3900 observations, between 1986 June and 1987 November while the satellite was passing through 15th-order resonance. The average accuracy of the inclination and eccentricity for all 50 epochs was equivalent to 90 m and 50 m in distance respectively.

The variations in inclination and eccentricity have been analysed; six 15th-order and four 30th-order lumped harmonics have been evaluated. These ten values of lumped harmonics have standard deviations corresponding to between 0.2 cm and 3.3 cm in geoid height.

The lumped harmonics have been compared with those obtained from five comprehensive models of the gravity field, and have been used in a new evaluation of individual harmonic coefficients of order 30 and even degree: see Appendix.

Appendix

NEW VALUES FOR INDIVIDUAL HARMONIC COEFFICIENTS OF ORDER 30 AND EVEN DEGREE, OBTAINED WITH THE AID OF THE RESULTS FROM 1967-102B

When values of lumped harmonics of a particular order are determined from resonant orbits having a wide variety of orbital inclinations, it is possible to solve for the individual harmonic coefficients of that order. Such solutions obtained previously for order 30 and even degree[16] have used accurate results from several satellites at inclinations between 50° and 74°, and also one at 90°; but the only previous results from an orbit at an inclination between 74° and 90° were at 80° from Ariel 3, a satellite of relatively high drag. The new results from 1967-102B, at 81° inclination, are much more accurate (nominally by a factor of about 5) and should lead to an improved solution for individual harmonics of order 30 and even degree.

The previous solutions, for six individual C(or S) coefficients, of degree 30, 32, 34,..40, were derived from 11 satellite equations of the form:

$$\bar{C}_{30,30} + Q_{32}\bar{C}_{32,30} + Q_{34}\bar{C}_{34,30} + \cdots Q_{40}\bar{C}_{40,30} = \bar{C}_{30}^{0,2} \pm \sigma \qquad \text{(A-1)}$$

where the numerical values of the Q coefficients depend on the inclination, and $\bar{C}_{30}^{0,2}$ is the lumped harmonic determined from analysis of the resonance, with standard deviation σ. These satellite equations were supplemented by six constraint equations of the form:

$$\bar{C}_{\ell,30} = 0 \pm \frac{10^{-5}}{\ell^2}, \qquad \text{(A-2)}$$

which express the expectation that most of the $\bar{C}_{\ell,30}$ values will be less than $10^{-5}/\ell^2$. There were similar equations for the S coefficients.

If 1967-102B is to be added to these equations, the standard deviations of the lumped harmonics need to be increased to allow for the neglected coefficients of degree 42, 44, 46, ... This was done on the assumption that, for $\ell \geq 42$, $\bar{C}_{\ell,30} = 0.5 \times 10^{-5}/\ell^2$, and similarly for $\bar{S}_{\ell,30}$, giving a total extra contribution to σ of 1.1×10^{-9}. When this is added to the existing errors, equations (10) and (11), truncated at degree 40, give

$$\bar{C}_{30,30} + 0.302\bar{C}_{32,30} - 0.011\bar{C}_{34,30} - 0.186\bar{C}_{36,30}$$

$$- 0.272\bar{C}_{38,30} - 0.298\bar{C}_{40,30} = (0.35 \pm 1.29) \times 10^{-9} \qquad \text{(A-3)}$$

and similarly for S, with the term on the rhs as: $(8.80 \pm 1.54) \times 10^{-9}$. Note that the harmonics of equation (5) are used here.

The new values for C(or S) coefficients are obtained from 12 satellite equations - the previous 11 and equations (A-3) - and the six constraint equations (A-2). As before, the constraints were relaxed when necessary to ensure that the weighted residuals did not exceed 1.0, and the standard deviations of the lumped harmonics were doubled for any satellite for which the weighted residual exceeded 1.4. For the S coefficients, no relaxations or doublings were required, except for those previously made. For the C-coefficient solution the standard deviation of $\bar{C}_{30}^{0,2}$ for Ariel 3 and for 1967-102B had to be doubled, and two of the constraint equations had to be slightly relaxed. The new values of the individual harmonic coefficients obtained, with their standard deviations are given in Table A.1.

Table A.1

New values of even-degree $\bar{C}_{\ell,30}$ and $\bar{S}_{\ell,30}$

ℓ	$10^9\bar{C}_{\ell,30}$	$10^9\bar{S}_{\ell,30}$
30	-1.8 ± 0.8	7.6 ± 0.7
32	-7.6 ± 1.6	4.7 ± 1.6
34	-16.4 ± 1.9	-5.9 ± 1.8
36	-11.2 ± 2.6	4.8 ± 2.3
38	-0.9 ± 2.5	3.0 ± 2.1
40	0.0 ± 2.2	-4.5 ± 2.1

The standard deviations of the values in this new solution are on average 25% lower than before, so the addition of 1967-102B does have a beneficial effect. The average standard deviation is 1.85×10^{-9}, equivalent to 1.2 cm in geoid height.

The values of the C coefficients differ appreciably from the previous set[16]: two of them (for ℓ = 36 and 38) have changed by more than the sum of the standard deviations. The S values are not significantly altered.

The weighted residuals for the 12 satellite equations and six constraint equations are given in Table A.2, where the symbol R denotes a relaxation of the $10^{-5}/\ell^2$ constraint. The overall measure of fit ε was 0.82 for C and 0.85 for S .

Table A.2

Weighted residuals

	Satellite equations			Constraint equations		
i deg	Satellite	$\bar{C}_{30}^{0,2}$	$\bar{S}_{30}^{0,2}$	ℓ	$\bar{C}_{\ell,30}$	$\bar{S}_{\ell,30}$
50.6	74-34A	0.13	0.46	30	0.16	-0.68
56.1	68-70A	-0.53	-1.26	32	0.78	-0.48
58.2	63-24B	0.72	0.30	34	1.00R	0.68
65.0	65-14A	0.14	-0.33	36	1.00R	-0.62
65.8	71-10B	-0.49	0.98	38	0.12	-0.44
74.0	70-111A	-0.38	-0.31	40	-0.01	0.72
74.0	71-13B	1.09	0.16			
80.2	67-42A	-1.34	-1.20			
81.2	67-102B	0.76	0.08			
90.2	71-54A	-0.13	0.07			
98.7	64-52B(H)	-0.57	0.81			
98.7	64-52B(B)	0.56	1.14			

Table A.3 gives a comparison of the new values with the corresponding values in four comprehensive geopotential models, GEM 10B[10], PGS-3337[13], GRIM3-L1[12] and Rapp's 1981 model[11]. The values from these models are believed to be completely independent of those from resonances, apart from Rapp's value for ℓ = 30, which utilizes some resonant results. The errors of GEM 10B, PGS-3337, GRIM3-L1 and Rapp(1981) are assessed as 3,2,3 and 2.5 × 10^{-9} respectively. As before[16], the comparison is terminated at ℓ = 36 , which is as far as GEM 10B and GRIM3-L1 go.

Table A.3

Comparison of new 30th-order values with comprehensive models, for $\ell \le 36$

ℓ	$10^9\bar{C}_{\ell,30}$					$10^9\bar{S}_{\ell,30}$				
	GEM 10B	PGS-3337	GRIM3-L1	Rapp	New values	GEM 10B	PGS-3337	GRIM3-L1	Rapp	New values
30	-5.2	0.0	-0.6	(-3.3)	-1.8 ± 0.8	11.1	5.4	7.1	(7.5)	7.6 ± 0.7
32	-0.6	-5.7	-6.9	-6.7	-7.6 ± 1.6	-0.2	-0.1	-1.0	0.5	4.7 ± 1.6
34	-11.9	-18.6	-23.0	-22.9	-16.4 ± 1.9	1.2	-3.1	0.7	-0.6	-5.9 ± 1.8
36	-3.9	-8.2	-7.6	-6.0	-11.2 ± 2.6	-0.9	3.4	6.4	4.8	4.8 ± 2.3

The new values agree well with PGS-3337: the mean difference between them is 2.5×10^{-9} and seven of the eight values are within the sum of their standard deviations. The mean difference between the new values in Table A.3 and the corresponding values in GEM 10B, GRIM-L1 and Rapp (1981) are 5.4, 3.3 and 3.7×10^{-9} respectively (excluding $\ell = 30$ for Rapp). As the average standard deviation of the new values in Table A.3 is 1.7×10^{-9}, the differences are consistent with the error estimates already quoted for the models. All five sets of values in Table A.3 agree in having $\bar{C}_{34,30}$ as the numerically largest value.

The new values should be better than those previously obtained from resonances, but they cannot be regarded as completely reliable because of the sparsity of data at inclinations between 74° and 90°.

Until now there have been considerable differences in the values of 30th-order coefficients from different sources: the agreement between the new values from resonance and the completely independent PGS-3337 model is encouraging.

Author's note

This Appendix has been written in collaboration with Dr D.G. King-Hele and a version of it has been published in *Planetary and Space Science* (Vol 38, pp 407-409 (1990)). The results given here include NASA's latest PGS model for comparison and this gives much better agreement than GEM-T2, which was used in the published version.

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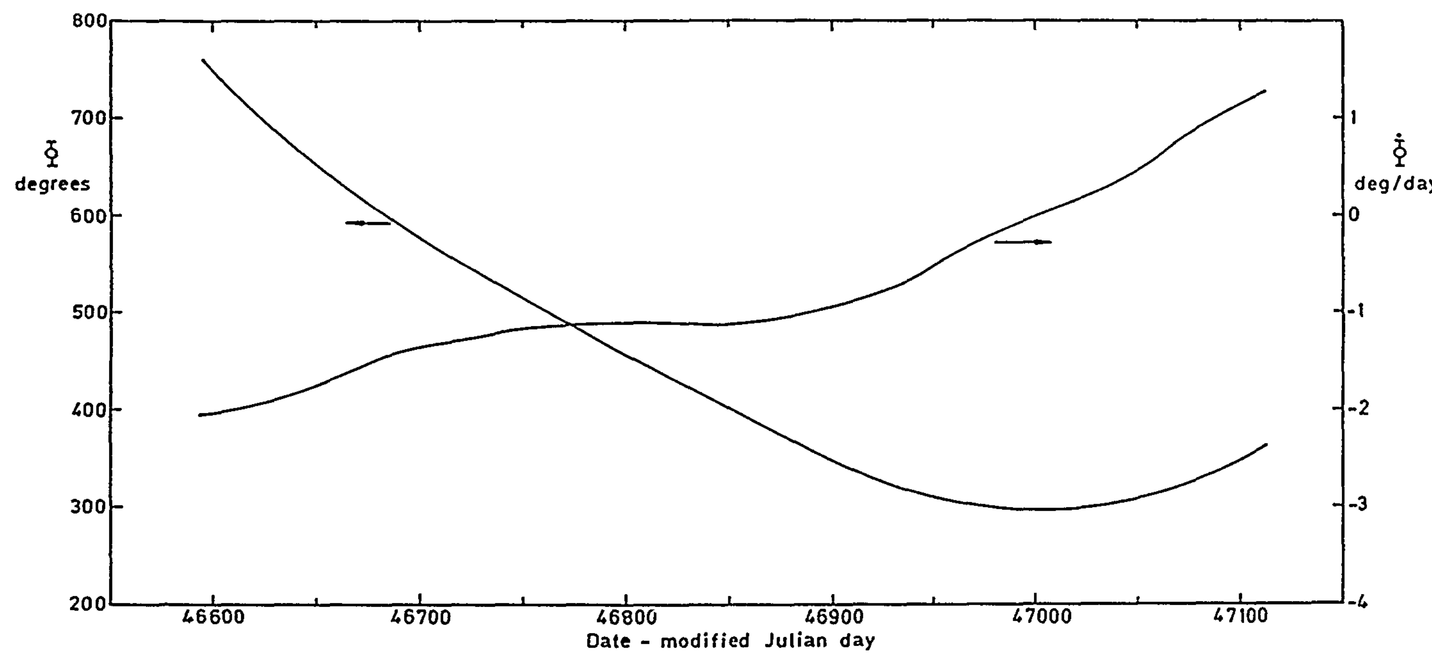

Fig 1 Variation of Φ and $\dot{\Phi}$ near 15th-order resonance

Fig 2.

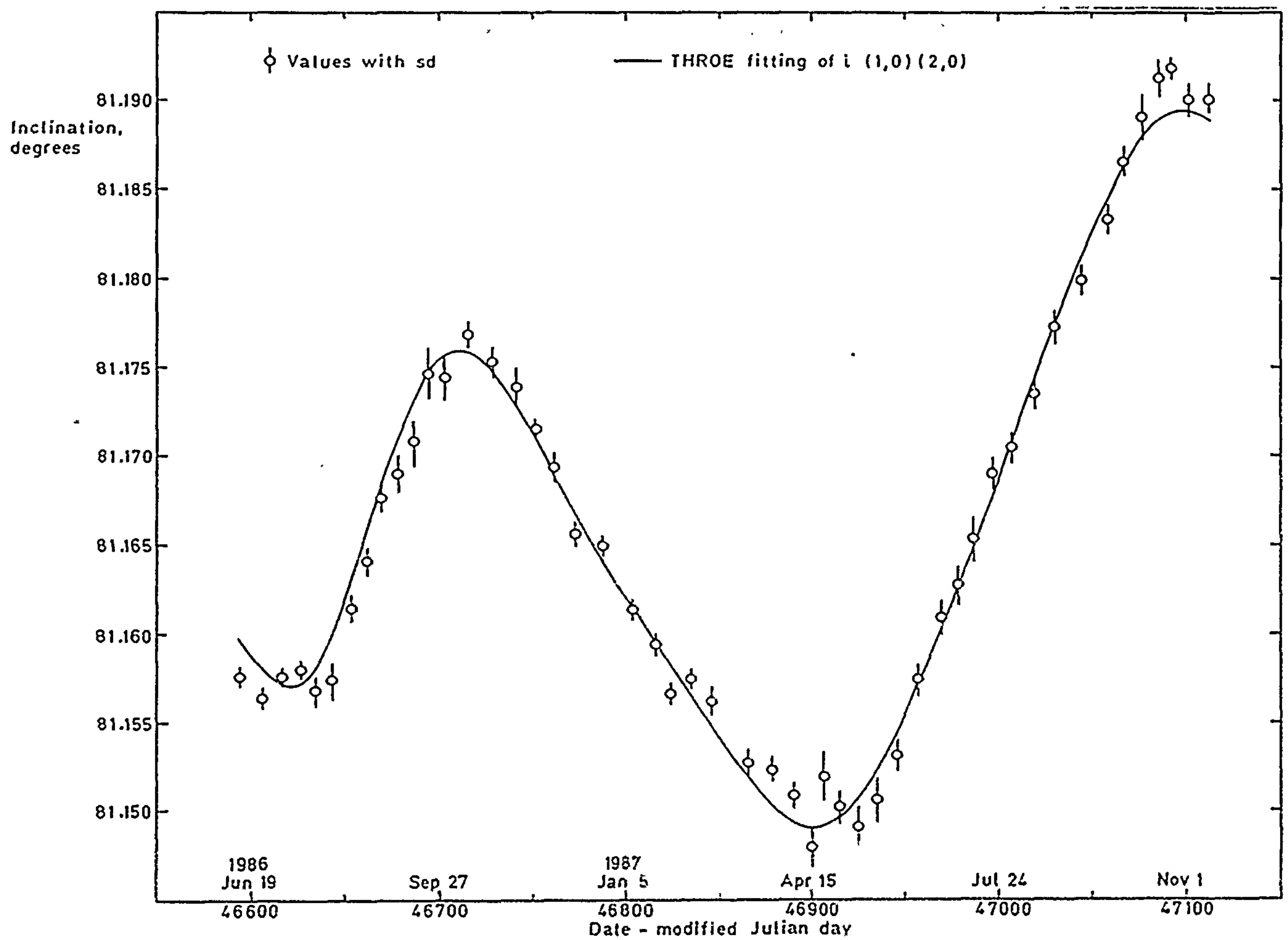

Fig 2 Values of inclination near 15th-order resonance, with fitted theoretical curve

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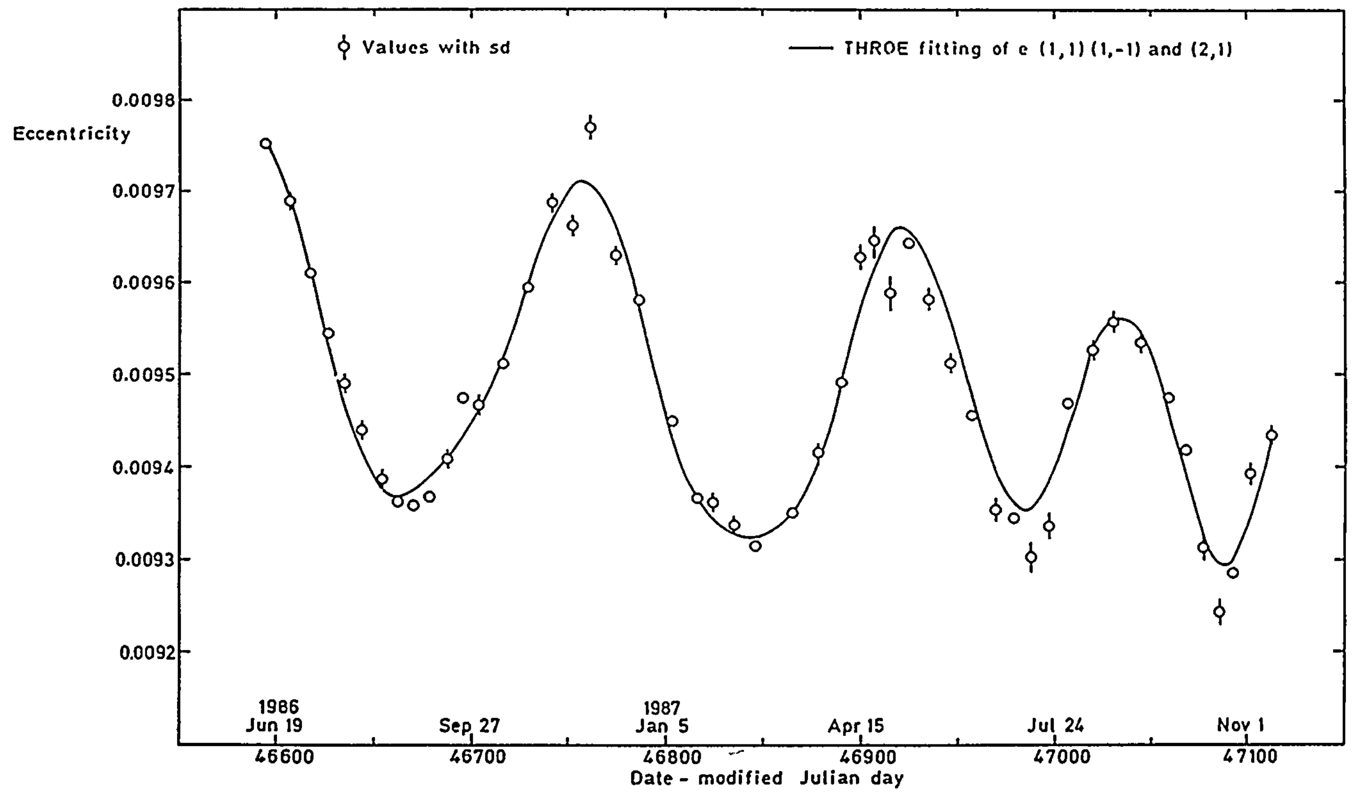

Fig 3 Values of eccentricity near 15th-order resonance, with fitted theoretical curve

Fig 3

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16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Orbit determination. Orbit analysis. Geopotential harmonics. Resonance. Satellite orbits.

17. Abstract

The orbital parameters of the satellite 1967-102B, Cosmos 184 rocket, have been determined at 50 epochs from some 3900 observations. For 21 of these determinations, Hewitt camera observations were available.

During the time of the orbit determinations the satellite passed through 15th-order resonance. The variations in inclination and eccentricity have been analysed to obtain six lumped 15th-order geopotential harmonics, with accuracies equivalent to between 0.2 and 3.3 cm in geoid height and four lumped 30th-order geopotential harmonics, with accuracies better than 1 cm in geoid height.

The lumped harmonics obtained in this Report have been compared with those from the GEM 10B and 10C models and with those from GRIM3-L1 and the RAPP 1981 models. A comparison has also been made with the lumped harmonics obtained from a new PGS-3337 model published by NASA, and the agreement with this model is very good.

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