

NPS-AS-91-001

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**LEARNING CURVE AND RATE ADJUSTMENT
MODELS: COMPARATIVE PREDICTION
ACCURACY UNDER VARYING CONDITIONS**

O. Douglas Moses

November 1990

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Prepared for: Naval Sea Systems Command
Cost Estimating and Analysis Division
Washington, DC 20362

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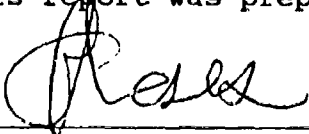
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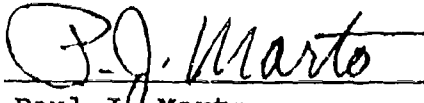
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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b RESTRICTIVE MARKINGS	
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for Public Release, Distribution Unlimited	
2b DECLASSIFICATION / DOWNGRADING SCHEDULE		5 MONITORING ORGANIZATION REPORT NUMBER(S)	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) NPS-54-91-001		7a NAME OF MONITORING ORGANIZATION Naval Sea Systems Command	
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School	6b OFFICE SYMBOL (If applicable)	7b ADDRESS (City, State, and ZIP Code) Code 017 Washington, DC 20362	
6c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943		9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER O&MN, Direct Funding	
8a NAME OF FUNDING / SPONSORING ORGANIZATION Naval Postgraduate School	8b OFFICE SYMBOL (If applicable)	10 SOURCE OF FUNDING NUMBERS	
8c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943		PROGRAM ELEMENT NO	PROJECT NO
		TASK NO	WORK UNIT ACCESSION NO
11 TITLE (Include Security Classification) Learning Curve and Rate Adjustment Models: Comparative Prediction Accuracy under Varying Conditions.			
12 PERSONAL AUTHOR(S) O. Douglas Moses			
13a TYPE OF REPORT Technical Report	13b TIME COVERED FROM _____ TO _____	14 DATE OF REPORT (Year, Month, Day) 1990 November	15 PAGE COUNT 50
16 SUPPLEMENTARY NOTATION			
17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
		Cost Estimation, Cost Analysis, Learning Curve, Production Rate, Prediction, Forecasting	
19 ABSTRACT (Continue on reverse if necessary and identify by block number) Learning curve models have gained widespread acceptance as a technique for analyzing and forecasting the cost of items produced from a repetitive process. Considerable research has investigated augmenting the traditional learning curve model with the addition of a production rate variable, creating a rate adjustment model. This study compares the predictive accuracy of the learning curve and rate adjustment models. A simulation methodology is used to vary conditions along seven dimensions. Forecast errors are analyzed and compared under the various simulated conditions using ANOVA. Overall results indicate that neither model dominates; each is more accurate under some conditions. Conditions under which each model tends to result in lower forecast errors are identified and discussed. Keywords: Learning curves, cost estimates, cost models, cost analysis, Production rate, Predictions, Forecasting. (RWS)			
20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USES		21 ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a NAME OF RESPONSIBLE INDIVIDUAL O. Douglas Moses		22b TELEPHONE (Include Area Code) 408-646-3218	22c OFFICE SYMBOL AS/No

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This work was sponsored by the Cost Estimating and Analysis Division of the Naval Sea Systems Command and the Naval Postgraduate School.

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ABSTRACT

Learning curve models have gained widespread acceptance as a technique for analyzing and forecasting the cost of items produced from a repetitive process. Considerable research has investigated augmenting the traditional learning curve model with the addition of a production rate variable, creating a rate adjustment model. This study compares the predictive accuracy of the learning curve and rate adjustment models. A simulation methodology is used to vary conditions along seven dimensions. Forecast errors are analyzed and compared under the various simulated conditions using ANOVA. Overall results indicate that neither model dominates; each is more accurate under some conditions. Conditions under which each model tends to result in lower forecast errors are identified and discussed.



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INTRODUCTION

Learning curves have gained widespread acceptance as a tool for planning, analyzing, explaining, and predicting the behavior of the unit cost of items produced from a repetitive production process. (See Yelle, 1979, for a review.) Cost estimation techniques for planning the cost of acquiring weapon systems by the Department of Defense, for example, typically consider the role of learning in the estimation process. The premise of learning curve analysis is that cumulative quantity is the primary driver of unit cost. Unit cost is expected to decline as cumulative quantity increases.

There is general acknowledgement that cumulative quantity is not the only factor that influences unit cost and that the simple learning curve is not a fully adequate description of cost behavior. Hence prior research has attempted to augment learning curve models by including additional variables (e.g., Moses, 1990). Most attention has been focused on the addition of a production rate term.¹ The resulting augmented model is usually referred to as a rate adjustment model.

Conceptually, production rate should be expected to affect unit cost because of the impact of economies of scale. Higher

¹One review of the literature pertaining to learning curves (Cheney, 1977) found that 36% of the articles reviewed attempted to augment the learning curve model in some manner by the inclusion of production related variables.

production rates may lead to several related effects: greater specialization of labor, quantity discounts and efficiencies associated with raw materials purchases, and greater use of facilities permitting fixed overhead costs to be spread over a larger output quantity. Together, these effects work to increase efficiency and reduce production cost (Bemis, 1981; Boger and Liao, 1990; Large, et. al., 1974; Linder and Wilbourn, 1973). However, higher production rate does not guarantee lower cost. When production rate exceeds capacity, such factors as over-time pay, lack of skilled labor, or the need to bring more facilities online may lead to inefficiencies and increased unit cost. In short, production rate may be associated with both economies and diseconomies of scale.

PRIOR RESEARCH

Numerous studies, using data on actual production cost elements, have been conducted to empirically examine the impact of production rate on unit cost. The broad objective of the research has been to document rate/cost relationships and determine if consideration of production rate leads to improvements in cost explanation or prediction. Results have been inconsistent and general findings inconclusive. Various studies (e.g., Alchian, 1963; Cochran, 1960; Hirsh, 1952; Large, Campbell and Cates, 1976) found little or no significance for rate variables. Other studies did document significant rate/cost relationships (e.g., Bemis, 1981; Cox and Gansler, 1981). Some research found significant results only for particular individual cost elements, such as labor

(Smith, 1976), tooling (Levenson, et. al., 1971) or overhead (Large, Hoffmayer, and Kontrovich, 1974). But rate/cost relationships for these same cost elements were not consistently evident in other studies. When significant, estimates of the rate/cost slope varied greatly and the direction of the relationship was sometimes negative and sometimes positive (e.g., Moses, 1990). In reviewing the existing research on production rate, Smith (1980) concluded that a rate/cost relationship may exist but that the existence, strength and nature of the relationship varies with the item produced and the cost element examined.

Several explanations for these varying, inconclusive empirical results can be offered:

(a) Varying results are to be expected because rate changes can lead to both economies and diseconomies of scale.

(b) Production rate effects are difficult to isolate empirically because of colinearity with cumulative quantity (Gulledge and Womer, 1986).

(c) Researchers have usually used inappropriate measures of production rate leading to misspecified models (Boger and Liao, 1990).

(d) The impact of a production rate change is dominated by other uncertainties (Large, Hoffmayer, and Kontrovich, 1974), particularly by cumulative quantity (Asher, 1956). Alchian (1963), for example, was unable to find results for rate adjustment models that improved on the traditional learning curve without a rate

parameter.

OBJECTIVE OF THE STUDY

The prior research suggests that consideration of production rate sometimes improves cost explanation, but not always. The prior research suggests that a traditional learning curve model sometimes is preferable to a rate adjustment model, but not always. The prior research provides little guidance concerning the circumstances under which explicit incorporation of production rate into a learning curve model is likely to lead to improved explanation or prediction. This issue is important in a number of cost analysis and cost estimation situations. Dorsett (1990), for example, describes the current situation facing military cost estimators who, with the military facing budget reductions and program stretchouts, are required to rapidly develop weapon system acquisition cost estimates under many different quantity profiles. One choice the cost analyst faces is between using a rate adjustment model or a traditional learning model to develop estimates.²

The objective of this paper is to address the following broad issue: Under what circumstances is it beneficial to explicitly

²Two other techniques for making cost estimates when production rate changes are also mentioned by Dorsett: curve rotation, which involves an ad hoc upward or downward revision to the slope of the learning curve, and the use of repricing models (e.g., Balut, 1981; Balut, Gullledge, and Womer, 1989) which adjust learning curve estimates to reflect a greater or lesser application of overhead cost. Dorsett criticized curve rotation for being subjective and leading to a compounding of error when the prediction horizon is not short. He criticized repricing models because they must be plant-specific to be effective.

consider production rate in cost analysis? That broad issue is addressed by focusing more narrowly on the following research question: Under what circumstances do rate adjustment models outperform traditional learning curve models? Performance here means the ability to provide accurate predictions of future cost.³

RESEARCH APPROACH

Operationally the research question implies a comparison of the predictive accuracy of two competing cost estimation models. The two competing models were as follows:

The traditional learning curve model, which predicts unit cost as a function of cumulative quantity⁴:

$$C_L = aQ^b \quad (1)$$

where

- C_L = Unit cost of item at quantity Q (i.e., with learning considered).
- Q = Cumulative quantity produced.
- a = Theoretical first unit cost.
- b = Learning curve exponent (which can be converted to a learning slope by slope = 2^b).

³For many of the studies cited in the section on prior research, performance was assessed in terms of a model's ability to ex post, statistically account for the variance in an actual cost series. The problem of "overfitting" a model to data always exists when regression techniques are used to ex post explain a cost series. In fact, if higher R^2 is achieved by overfitting, predictive ability can be reduced (Wetherill, 1986).

⁴Note that this is an incremental unit cost model rather than a cumulative average cost model. Liao (1988) discusses the differences between the two approaches and discusses why the incremental model has become dominant in practice. One reason is that the cumulative model weights early observations more heavily and, in effect, "smooths" away period-to-period changes in average cost.

The most widely used rate adjustment model, which modifies the traditional learning curve model with the addition of a production rate term:

$$C_R = aQ^bR^c \quad (2)$$

where

- C_R = Unit cost of item at quantity Q and production rate per period R (i. e., with production rate as well as learning considered).
- Q = Cumulative quantity produced.
- R = Production rate per period measure.
- a = Theoretical first unit cost.
- b = Learning curve exponent.
- c = Production rate exponent (which can be converted to a production rate slope by slope = 2^c).

A simulation approach was used to address the research question. In brief, cost series were generated under varying simulated conditions. The learning curve model and the rate adjustment model were separately fit to the cost series to estimate model parameters. The estimated models were then used to separately predict future cost. The relative accuracy of the two alternative future cost predictions was determined. Finally, an analysis (ANOVA) was conducted relating relative prediction accuracy (dependent variable) to the simulated conditions (independent variables).

There are three main benefits gained from the simulation approach. First, factors hypothesized to influence prediction accuracy can be varied over a wider range of conditions than would be encountered in any one (or many) sample(s) of actual cost data. Second, explicit control is achieved over the manipulation of

factors. Third, noise caused by factors not explicitly investigated is removed. Hence simulation provides the most efficient way of investigating data containing a wide variety of combinations of the factor levels while controlling for the effects of other factors not explicitly identified.

RESEARCH CHOICES

There were five choices that had to be made in conducting the simulation experiment:

(1) The form of the rate adjustment (RA) model whose performance was to be compared to the learning curve (LC) model.

(2) The functional form of the cost model used to generate the simulated cost data.

(3) The conditions to be varied across simulation treatments.

(4) The cost objective (what cost was to be predicted).

(5) The measure of prediction performance.

Items (1), (2), (4) and (5) deal with methodological issues. Item (3) deals with the various conditions simulated. The discussion of item (3) encompasses the major variables examined in the study and hence the major issues addressed. Each item will be discussed in turn.

1. The Rate Adjustment Model. Various models, both theoretical and empirical, have been suggested for incorporating production rate into the learning curve (Balut 1981; Balut, Gullledge, and Womer, 1989; Linder and Willbourn, 1973; Smith, 1980, 1981; Washburn, 1972; Womer, 1979). The models vary with respect to tradeoffs made between theoretical completeness and empirical

tractability. Equation 2, described above, was the specific rate adjustment model analyzed in this study, for several reasons: First, it is the most widely used rate adjustment model in the published literature. Second, it is commonly used today in the practice of cost analysis (e.g., Dorsett, 1990). Third, in addition to cost and quantity data (needed to estimate any LC model), equation 2 requires only production rate data. Thus equation 2 is particularly appropriate for examining the incremental benefit of attention to production rate. (Other RA models offered in the literature require knowledge of still additional variables).⁵ In short, equation 2 is the most widely applicable and most generally used rate adjustment model.

2. The Cost Generating Function: A "true" cost function for an actual item depends on the item, the firm, the time period and all the varying circumstances surrounding actual production. It is likely that most manufacturers do not "know" the true cost function underlying goods they manufacture. Thus the choice of a cost function to generate simulated cost data is necessarily ad hoc. The objective here was to choose a "generic" cost function which had face validity, which included components (parameters and variables) that were generalizable to all production situations, and which resulted in a unit cost that depended on both learning

⁵The equation 2 model is particularly applicable in situations where a cost analyst or estimator does not have ready access to or sufficient knowledge about the cost structure and cost drivers of a manufacturer. Examples include the Department of Defense procuring items from government contractors in the private sector, or prime contractors placing orders with subcontractors.

and production rate factors. The following explanation of the cost function used reflects these concerns.

At the most basic level the cost of any unit is just the sum of the variable cost directly incurred in creating the unit and the share of fixed costs assigned to the unit, where the amount of fixed costs assigned depend on the number of units produced.

$$UC = VC + \frac{FC}{PQ} \quad (3)$$

where

UC = Unit cost.
 VC = Variable cost per unit.
 FC = Total fixed costs per period.
 PQ = Production quantity per period.

The original concept of "learning" (Wright, 1936) involved the reduction in variable cost per unit expected with increases in cumulative quantity produced. (By definition, fixed costs are assumed to be unaffected by volume or quantity.) To incorporate the effect of learning, variable cost can be expressed as:

$$VC_0 = VC_1(Q^d) \quad (4)$$

where

Q = Cumulative quantity.
 VC₀ = Variable cost of the Qth unit.
 VC₁ = Variable cost of the first unit.
 d = Parameter, the learning index.

Substituting into equation 3:

$$UC_0 = VC_1(Q^d) + \frac{FC}{PQ} \quad (5)$$

Additionally, assume the existence of a "standard" ("benchmark," "normal," "planned") production quantity per period (PQ_s).

Standard fixed cost per unit (SFC) at the standard production quantity would be:

$$SFC = \frac{FC}{PQ_s} \quad (6)$$

The production rate (PR) for any period can then be expressed as a ratio of the production quantity to the standard quantity:

$$PR = \frac{PQ}{PQ_s} \quad (7)$$

The second term of equation (6) can then be rewritten as:

$$\frac{FC}{PQ} = \frac{SFC}{PR} \quad (8)$$

and equation 5 rewritten as:

$$UC_0 = VC_1(Q^d) + SFC (PR^{-1}) \quad (9)$$

In this formulation it can be seen that total cost per unit is the sum of variable cost per unit (adjusted for learning) plus standard fixed cost per unit (adjusted for production rate). This model incorporates the two factors presumed to impact unit costs that have been most extensively investigated: cumulative quantity (Q) and production rate per period (PR).⁶ It is consistent with both the theoretical and empirical literature which sees the primary impact of learning to be on variable costs and the primary

⁶Smith (1980, 1981), for example, used a model similar to equation 9 to explore the effect of different production rates on unit cost. Balut (1981) and Balut, Gullledge and Womer (1989) construct models based on learning and production quantity to assist in "redistributing" overhead and "repricing" unit costs when changes in production rate occur. The Balut and Balut, Gullledge and Womer models differ in that they determine a learning rate for total (not variable) unit cost and then apply an adjustment factor to allow for the impact of varying production quantity on the amount of fixed cost included in total cost.

impact of production rate to be on the spreading of fixed costs (Smith, 1980). Simulated cost data in this study was generated using equation 9, while varying values for the variables and parameters on the right hand side of the equation to reflect differing conditions.

3. The Simulated Conditions: The general research hypothesis is that the relative predictive accuracy of the LC and RA models will depend on the circumstances in which they are used. What conditions might be hypothesized to affect prediction accuracy? Seven different factors (independent variables) were varied during the simulation. In the following paragraphs, each factor is discussed. A label for each is provided, along with a discussion of why the factor may be relevant to prediction accuracy and how the factor was operationalized in the simulation. Table 1 summarizes the seven factors.

i) Data History (DATAHIST): The number of data points available to estimate parameters for a model should affect the accuracy of a model. More data available during the "estimation period" should be associated with greater accuracy for both the LC and the RA model.⁷ But increased data may not impact the accuracy of the two models in the same manner. The RA model requires the estimation of an additional parameter, and consequently is likely to require more data points to avoid unreliable parameter estimates and resultant prediction inaccuracy. Hence it may be hypothesized

⁷There are, of course, cost/benefit tradeoffs. The marginal benefits of increased prediction accuracy for any model must be weighed against the marginal costs of additional data collection.

that the RA model will be more sensitive (less accurate) when data is lean.

In the simulation, data history was varied from four to seven to ten data points available for estimating model parameters. This simulates having knowledge of costs and quantities for four, seven or ten production lots. Four is the minimum number of observations needed to estimate the parameters of the RA model by regression. The simulation focuses on lean data availability both because the effects of marginal changes in data availability should be most pronounced when few observations are available and because many real world applications (e.g., cost analysis of Department of Defense weapon system procurement) occur under lean data conditions.

ii) Variable Cost Learning Rate (LEARNRAT): In the cost generating function, learning affects total unit cost by affecting variable cost per unit. Past research (Smunt, 1986) has shown that the improvement in prediction accuracy from including a learning parameter in a model (when compared to its absence) depends on the degree of learning that exists in the underlying phenomena being modeled; the greater the degree of learning, the more benefit gained from inclusion of a learning rate term. This suggests that prediction accuracy may depend on the learning rate and hence that relative prediction accuracy of the LC and RA models may also depend on the learning rate. In the simulation, variable cost learning rate (reflected in parameter d in equation 9) was varied from 75% to 85% to 95%. Generally, complex products or labor

TABLE 1
INDEPENDANT VARIABLES

<u>Concept</u>	<u>Label</u>	<u>Levels</u>		
Data History	DATAHIST ¹	4	7	10
Variable Cost Learning Rate	LEARNRAT	75%	85%	95%
Fixed Cost Burden	BURDEN ²	15%	33%	50%
Production Rate Trend	PROTREND ³	Level	Growth	
Production Rate Instability/Variance	RATEVAR ⁴	.05	.15	.25
Cost Noise/Variance	COSTVAR ⁵	.05	.15	.25
Future Production Level	FUTUPROD ⁶	Low	Same	High

¹Number of data points available during the model estimation period; simulates the number of past production lots.

²Standard per unit fixed cost as a percentage of cumulative average per unit total cost, during the model estimation period.

³A level trend means production at 100% of standard production for each lot during the estimation period. A growth trend means production rate gradually increasing to 100% of standard production during the estimation period. The specific growth pattern depends on the number of production lots in the estimation period, with sequences as follows (expressed as a % of standard): For DATAHIST = 4: 33%, 67%, 100%, 100%. For DATAHIST = 7: 20%, 40%, 60%, 80%, 100%, 100%, 100%. For DATAHIST = 10: 10%, 20%, 35%, 50%, 70%, 90%, 100%, 100%, 100%, 100%.

⁴Coefficient of variation of production rate. (Degree of instability of production rate around the general production rate trend.)

⁵Coefficient of variation of total per unit cost.

⁶"Same" means production rate at 100% of standard for each lot produced within the prediction zone. "Low" means production rate at 50%. "High" means production rate at 150%.

total unit cost made up of fixed cost.⁹ Three percentages were used in the simulation: 15%, 33%, and 50%. The different percentages can be viewed as simulating different degrees of operating leverage, of capital intensiveness, or of plant automation. The 15% level reflects the average fraction of price represented by fixed overhead in the aerospace industry, as estimated at one time by DOD (Balut, 1981).¹⁰ The larger percentages are consistent with the trend toward increased automation (McCullough and Balut, 1986).

iv) Production Rate Trend (PROTREND): When initiating a new product, it is not uncommon for the production rate per period to start low and trend upward to some "normal" level. This may be due both to the need to develop demand for the output or the desire to start with a small production volume, allowing slack for working bugs out of the production process. Alternatively, when a "new" product results from a relatively small modification of an existing product, sufficient customer demand or sufficient confidence in the

⁹Operationally this is a bit complex, since both per unit variable and per unit fixed cost depend on other simulation inputs (cumulative quantity and production rate per period). The process of relating fixed cost to total cost was as follows: First, a cumulative average per unit variable cost for all units produced during the estimation period was determined. Then a standard fixed cost per unit was set relative to the cumulative average per unit variable cost. For example, if standard fixed cost per unit was set equal to cumulative average variable cost per unit, then "on average" fixed cost would comprise 50% of total unit cost during the estimation period. Actual fixed cost per unit may differ from standard fixed cost per unit if the production rate (discussed later) was not at 100% of standard.

¹⁰In the absence of firm-specific cost data, the Cost Analysis Improvement Group in the Office of the Secretary of Defense treats 15% of the unit price of a defense system as representing fixed cost (Pilling, 1990).

production process may be assumed and full scale production may be initiated rapidly. In short, two different patterns in production volume may be exhibited early on when introducing a new item: a gradual growing trend toward full scale production or a level trend due to introduction at full scale production volume.

How might the production rate trend during the estimation period affect the relative prediction accuracy of the LC and the RA models? Two contrary arguments seem relevant. On the one hand, the purpose of an RA model is to explain variance in cost due to variance in production rate. If production rate is virtually level during a model estimation period, there would be little period-to-period differences in unit cost caused by the spreading of fixed costs over varying outputs. Any rate effects would likely be swamped by other random impacts on cost. This suggests greater incremental benefit to using an RA model when the production rate has not been level, i.e., when growth occurred during the model estimation period. On the other hand, a growing production rate from period to period results in statistical problems. Within the RA model, there will be greater colinearity between cumulative quantity (necessarily growing each period) and production rate. Empirically, this colinearity has been observed by many cost analysts (e. g., Gullledge and Womer, 1986). The colinearity makes production rate a somewhat redundant variable and causes unreliable parameter estimates. This suggests less incremental benefit to using an RA model if production rate has been growing.

Which argument holds is an empirical question. The simulation

created two production trends during the model estimation period: "level" and "growth." These represented general trends (but, as will become clear momentarily, variance around the general trend was introduced). The level trend simulated a production rate set at a "standard" 100% each period during model estimation. The growth trend simulated production rate climbing gradually to 100%. Details of the trends are in table 1.

v) Production Rate Instability/Variance (RATEVAR): Numerous factors, in addition to the general trend in output discussed above, may operate to cause period-to-period fluctuations in production rate. Manufacturers typically do not have complete control over either demand for output or supply of inputs. Conditions in either market can cause instability in production rate. (Of course, unstable demand, due to the uncertainties of annual budget negotiations, is claimed to be a major cause of cost growth during the acquisition of major weapon systems by the DoD). The concern here is with the likely effect of rate instability on the relative accuracy of LC and RA models. If production rate is highly stable across periods, there would be little variance in cost outcomes on which to estimate a rate parameter. Rate effects would be dominated by other impacts on cost. This suggests greater benefit to using an RA model when production rate is unstable over time.

Production rate instability was simulated by adding random variance to each period's production rate during the estimation period. The amount of variance ranged from a coefficient of

variation of .05 to .15 to .25. For example, if the production trend was level and the coefficient of variation was .05 then "actual" production rates simulated were generated by a normal distribution with mean equal to the standard production rate (100%) and sigma equal to 5%.

vi) Cost Noise/Variance (COSTVAR): From period to period there will be unsystematic, unanticipated, non-recurring, random factors that will impact unit cost. Changes in the cost, type or availability of input resources, temporary increases or decreases in efficiency, and unplanned changes in the production process are all possible causes. Conceptually such unsystematic factors can be thought of as adding random noise to unit cost. While unsystematic variation in cost cannot (by definition) be controlled, it is often possible to characterize different production processes in terms of the degree of unsystematic variation; some processes are simply less well-understood, more uncertain, and less stable than others.

Does the relative predictive accuracy of LC and RA models depend on the stability of the process underlying cost? To investigate this question, random variance was added to the simulated costs generated from the cost function. The amount of variance ranged from a coefficient of variation of .05 to .15 to .25. For example, when the coefficient of variation was .25, then "actual" unit costs simulated were generated by a normal distribution with mean equal to cost from equation 9 and sigma equal to 25%.

vii) Future Production Level (FUTUPROD): Once a model is

constructed (from data available during the estimation period), it is to be used to predict future cost. The production rate planned for the future may vary from past levels. Further growth may be planned. Cutbacks may be anticipated. Will the level of the future production rate affect the relative predictive accuracy of the LC and RA models? Is one model more accurate if cutbacks in production are anticipated and another if growth is planned? Conflicting scenarios can be argued. For example, on the one hand, inclusion of a rate term might be expected to increase prediction accuracy when production rate changes significantly (i. e., either growth or decline in the future period). On the other hand, if growth during the future prediction period merely continues a growth trend established during the estimation period, then production rate will once again tend to be correlated with cumulative quantity, and the incremental benefit of an RA model over an LC model may be less obvious.

In the simulation, future production was set at three levels: low (50% of standard), same (100% of standard) and high (150% of standard). These simulate conditions of cutting back, maintaining or increasing production relative to the level of production existing at the end of the model estimation period.

4. The Cost Objective: What is to be predicted? Up to this point the stated purpose of the study has been to evaluate accuracy when predicting future cost. But which future cost? Three alternatives were examined.

i) Next period average unit cost: As the label suggests this

is the average per unit cost of items in the production "lot" manufactured in the first period following the estimation period. Here the total cost of producing the output for the period is simply divided by the output volume to arrive at unit cost. Attention to this cost objective simulates the need to predict near term unit cost.

ii) Total cost over a finite production horizon: The objective here is to predict the total cost of all units produced during a fixed length production horizon. Three periods were used as the length of the production horizon (one production lot produced each period). If the future production rate is low (high) then relatively few (many) units will be produced during the finite production horizon. Attention to this cost objective simulates the need to predict costs over some specific planning period, regardless of the volume to be produced during that planning period.

iii) Total program cost: The objective here is to predictive total cost for a specified number of units. If the future production rate is low (high) then relatively more (fewer) periods will be required to manufacture the desired output. The simulation was constructed such that at a low (same, high) level of future production six (three, two) future periods were required to produce the output. Attention to this cost objective simulates the need to predict total cost for a particular production program, regardless of the number of future periods necessary to complete the program.

Examining each of these three cost objectives was deemed

necessary to provide a well-rounded investigation of predictive accuracy. However, the findings were substantially the same across the three cost objectives. In the interests of space, the remainder of this paper will discuss the analysis and results only for the first cost objective, the average cost per unit for the next period's output.

5. The Measures of Prediction Performance: In order to compare prediction accuracy between LC and RA models, two kinds of measures were required. First, a model prediction error (ERR) was determined separately for each (LC or RA) model as follows:

$$ERR = |PUC - AUC| \div AUC$$

where

PUC = Predicted unit cost from either the learning curve or the rate adjustment model.

AUC = Actual unit cost as generated by the cost function.

This is a commonly used error measure, the absolute percentage error.

Next a simple difference in errors (ERRDIFF) between the LC prediction and the RA prediction was calculated.

$$ERRDIFF = ERR_{LC} - ERR_{RA}$$

where

ERR_{LC} = Absolute percentage error using the learning curve model prediction of cost.

ERR_{RA} = Absolute percentage error using the rate adjustment model prediction of cost.

Positive values for ERRDIFF mean that the rate adjustment model produced smaller prediction errors than the learning curve model

and consequently imply increased accuracy from incorporating a production rate variable into the prediction process. Negative values mean the learning curve model is more accurate. ERRDIFF represents the dependent variable in the statistical analysis. The research question then becomes: What factors or conditions explain variance in ERRDIFF?

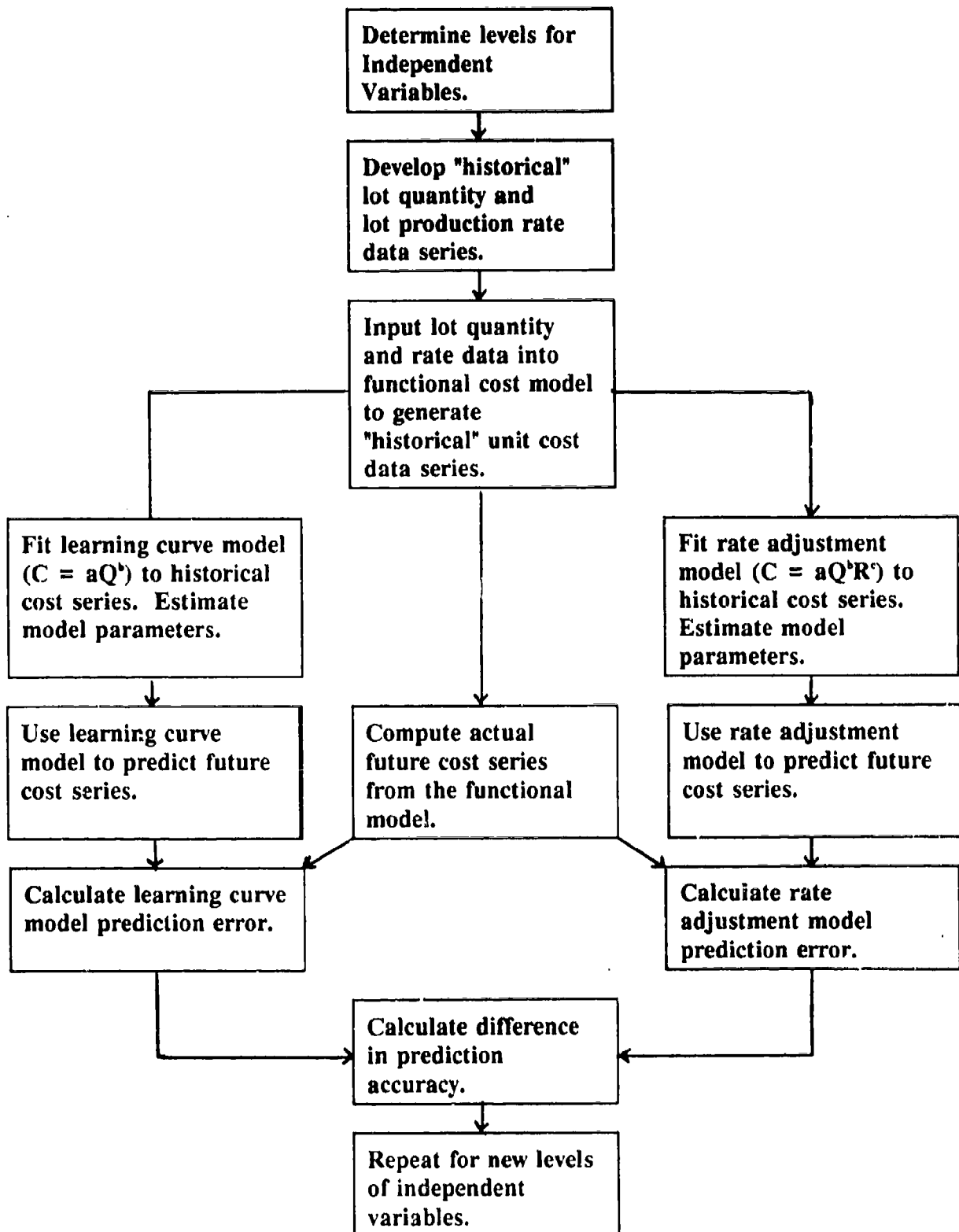
Figure 1 summarizes the complete simulation process leading up to the determination of ERRDIFF. The simulation was run once for each possible combination of treatments. Given seven factors varied and three possible values for each factor (except for PROTREND which had two), there were $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 2 = 1458$ combinations. Thus the simulation generated 1458 observations and 1458 values for ERRDIFF.¹¹

ANALYSIS AND FINDINGS

The simulation results were evaluated using analysis of variance (ANOVA) to conduct tests of statistical significance. All main effects are discussed in the following section. First order (pairwise) interactions were also tested. Those significant at .01

¹¹In the simulation, just as in the real practice of cost analysis, it is possible for a model estimated on limited data to be very inaccurate, leading to extreme values for ERR (or ERRDIFF). If such outlier values were to be used in the subsequent analysis, findings would be driven by the outliers. Screening of the observations for outliers was necessary. During the simulation, if a model produced an ERR value in excess of 100%, then that value was replaced with 100%. This truncation has the effect of reducing the impact of an outlier on the analysis while still retaining the observation as one that exhibited poor accuracy. Alternative approaches to the outlier problem included deletion instead of truncation and use of a 50% ERR cutoff rather than the 100% cutoff. Findings were not sensitive to these alternatives.

FIGURE 1
SIMULATION FLOWCHART



are also discussed. Table 2 provides the ANOVA results.

Main Effects: As indicated in table 2, all main effects, except LEARNRAT, are significant at .0005, indicating that values for ERRDIFF are significantly influenced by the treatment conditions. Table 3 summarizes ERRDIFF values under the various experimental conditions.

The major findings are perhaps made most obvious by a plot of ERRDIFF by treatments. Figure 2 shows a sample plot of ERRDIFF against DATAHIST. Two points are of interest. First, the relative accuracy of the LC and RA models clearly does depend on the number of observations available for model estimation. Second, both negative and positive value for ERRDIFF are present. This implies that under some conditions (few observations) the LC model is more accurate than the RA model. But under different conditions (larger number of observations) the RA model is more accurate. The crossover point occurs at about seven observations.

Figure 3 shows a similar plot of ERRDIFF with all (significant) variables superimposed. In this plot, 1, 2, and 3 on the x-axis reflect low, medium, and high values for the independent variables (which are taken from the left, middle and right columns of Table 3). Figure 3 makes several points. First for each the six variables plotted, ERRDIFF varies significantly depending on the value (treatment) for the independent variable. This simply implies that the experimental conditions do reflect factors relevant to model accuracy.

Second for all six independent variables, at some point over

TABLE 2
ANALYSIS OF VARIANCE RESULTS

<u>SOURCE</u>	<u>DF</u>	<u>SUM OF SQUARES</u>	<u>MEAN SQUARE</u>	<u>F VALUE</u>
Model	85	33.10921	.3895	9.03
Error	1372	59.18006	.0431	PR>F:
Corrected Total	1457	92.28927		.0001

<u>R²</u>	<u>CV</u>	<u>ERRDIFF MEAN</u>
.3588	1236.81	-.0168

<u>SOURCE</u>	<u>DF</u>	<u>ANOVA SS</u>	<u>F VALUE</u>	<u>PR>F</u>
DATAHIST	2	3.6780	42.63	.0001
LEARNRAT	2	.0510	.59	.5536
BURDEN	2	.7383	8.56	.0002
PROTREND	1	1.6259	37.70	.0001
RATEVAR	2	5.4728	63.44	.0001
COSTVAR	2	9.3918	108.87	.0001
FUTUPROD	2	.6662	7.72	.0005
DATAHIST*LEARNRAT	4	.0699	.41	.8050
DATAHIST*BURDEN	4	.1772	1.03	.3920
DATAHIST*PROTREND	2	.3217	3.73	.0243
DATAHIST*RATEVAR	4	.0844	.49	.7436
DATAHIST*COSTVAR	4	.9813	5.69	.0002
DATAHIST*FUTUPROD	4	.7531	4.37	.0016
LEARNRAT*BURDEN	4	.2444	1.42	.2260
LEARNRAT*PROTREND	2	.0092	.11	.8979
LEARNRAT*RATEVAR	4	.3933	2.28	.0588
LEARNRAT*COSTVAR	4	.2407	1.40	.2332
LEARNRAT*FUTUPROD	4	.3818	2.21	.0655
BURDEN*PROTREND	2	.0806	.93	.3931
BURDEN*RATEVAR	4	.2325	1.35	.2501
BURDEN*COSTVAR	4	.7304	4.23	.0021
BURDEN*FUTUPROD	4	1.0539	6.11	.0001
PROTREND*RATEVAR	2	2.1867	25.35	.0001
PROTREND*COSTVAR	2	.1022	1.19	.3059
PROTREND*FUTUPROD	2	.4751	5.51	.0041
RATEVAR*COSTVAR	4	.3163	1.83	.1199
RATEVAR*FUTUPROD	4	1.7734	10.28	.0001
COSTVAR*FUTUPROD	4	.8760	5.08	.0005

TABLE 3

PREDICTION ERROR DIFFERENCE BETWEEN
LEARNING CURVE AND RATE ADJUSTMENT MODELS:
MAIN EFFECTS

<u>Independent Variable</u>	<u>ERRDIFF for Each Level</u>		
DATAHIST Value:	4	7	10
ERRDIFF Mean:	-.87	.007	.030
LEARNRAT Value:	75%	85%	95%
ERRDIFF Mean:	-.009	-.019	-.023
BURDEN Value:	15%	33%	50%
ERRDIFF Mean:	-.047	-.009	.006
PROTREND Value:	level	-	growth
ERRDIFF Mean:	-.050		.017
RATEVAR Value:	.05	.15	.25
ERRDIFF Mean:	-.095	-.010	.054
COSTVAR Value:	.05	.15	.25
ERRDIFF Mean:	.082	-.019	-.114
FUTUPROD Value	low	same	high
ERRDIFF Mean:	.003	-.007	-.046

FIGURE 2

PLOT OF PREDICTION ERROR DIFFERENCE
BY DATA HISTORY

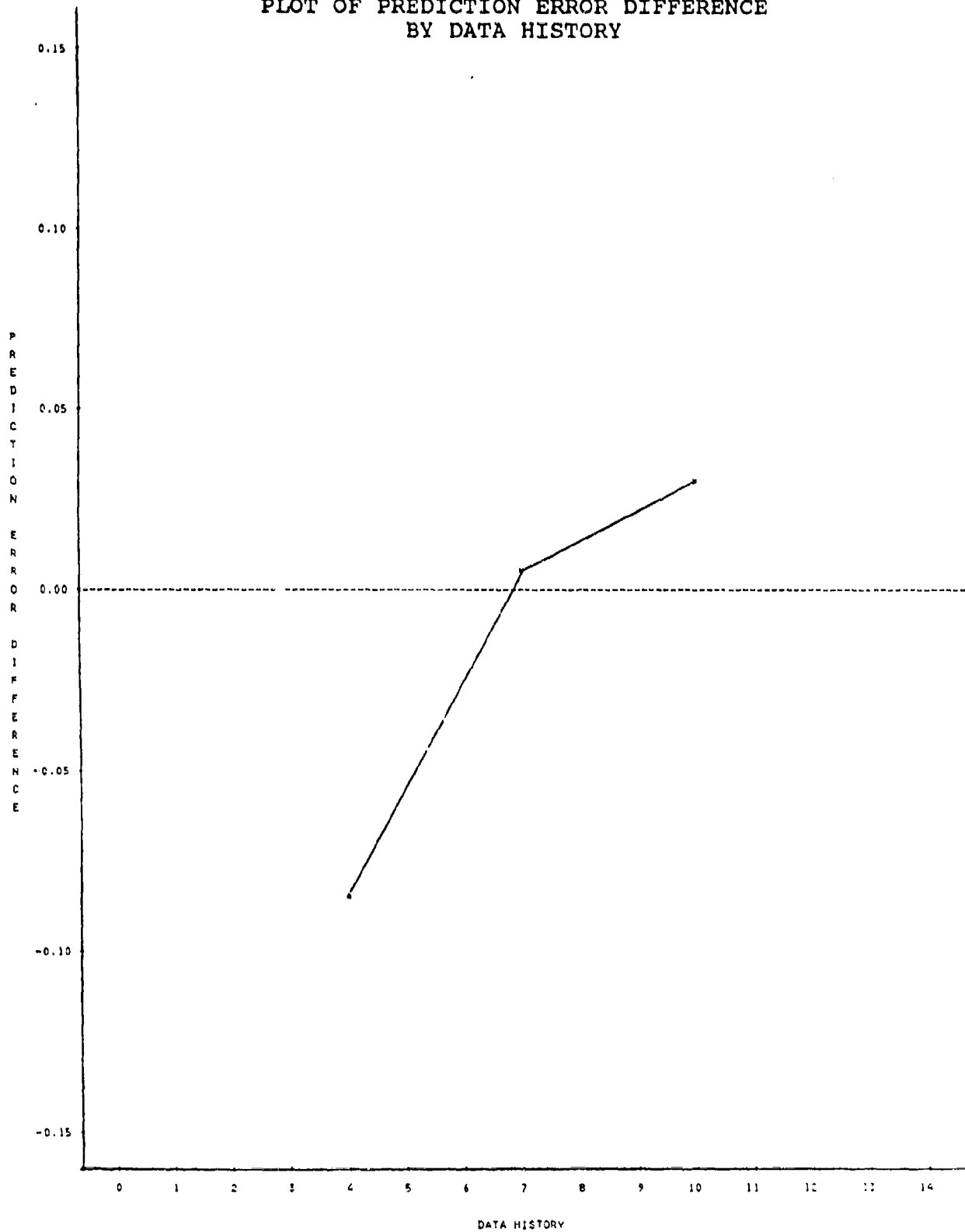
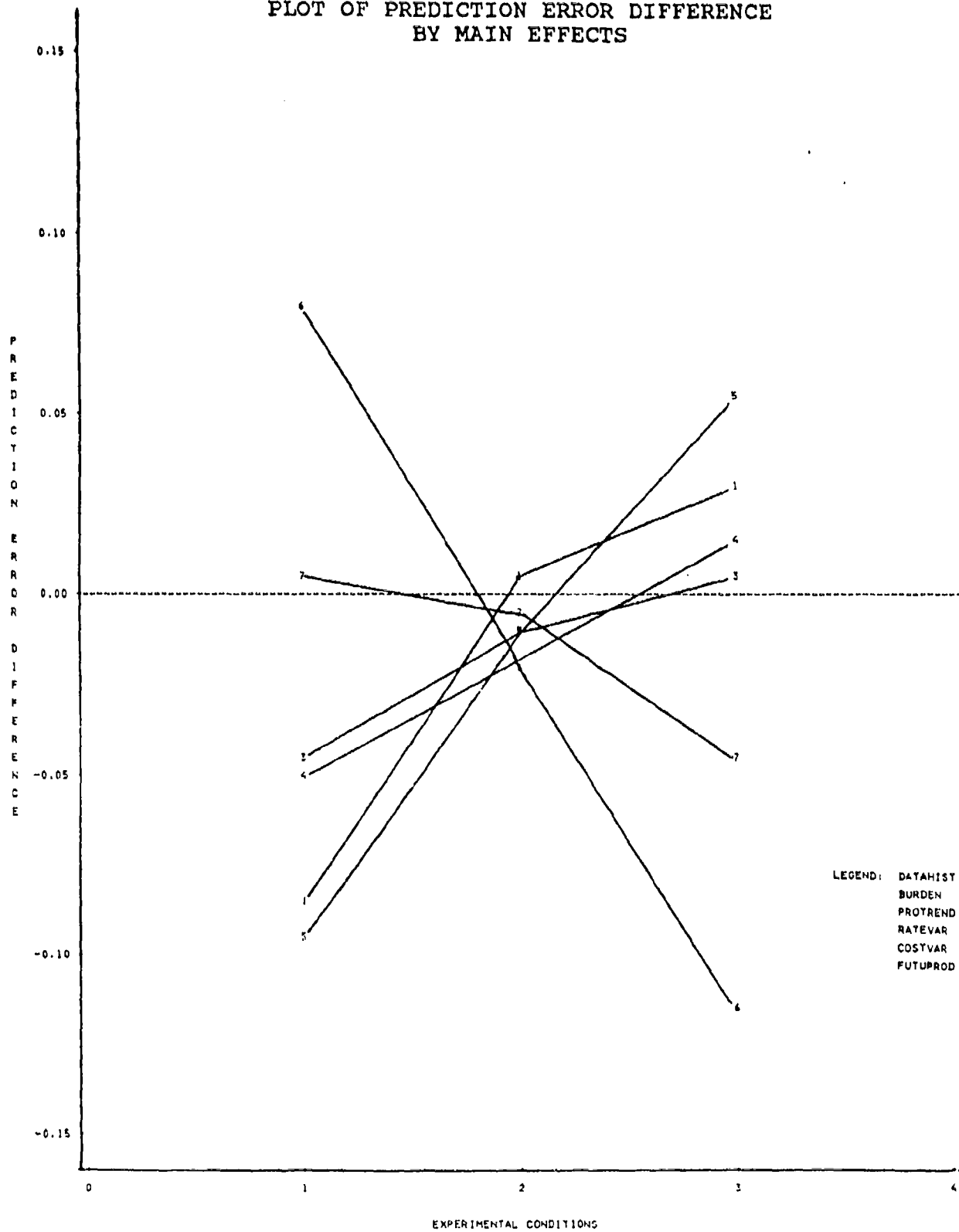


FIGURE 3

PLOT OF PREDICTION ERROR DIFFERENCE
BY MAIN EFFECTS



the range of these variables, both positive and negative values for ERRDIFF exist. This means that neither the LC model nor the RA model dominates. Each model is more accurate under some experimental conditions.

Third, some factors apparently have a greater impact on relative prediction accuracy than others. For example, the plot shows that both the most extreme values for ERRDIFF (highest positive and lowest negative) are associated with different treatments on COSTVAR. The F-value for COSTVAR is also the highest and most significant in the ANOVA. This suggests that the greatest impact on relative predictive accuracy is due to variation in the amount of unsystematic noise impacting the measure of cost. A relatively large effect is also apparent due to variation in RATEVAR; while relatively small (but still significant) effects are associated with variations in BURDEN and FUTUPROD. These observations are interesting but must be interpreted with caution. COSTVAR does have the greatest impact on ERRDIFF over the particular range of treatments examined by the simulation. But if the ranges over which the individual independent variables were allowed to vary were altered, a different factor could appear to be most important.

Fourth, values for ERRDIFF are either monotonically increasing or decreasing for the various independent variable treatments. Hence some general conclusions about the impact of the experimental conditions on relative prediction errors are possible:

- a) Data History: As indicated above the RA model outperforms

the LC model when the number of observations for model estimation is relatively greater. This is expected; the benefit from greater precision due to the additional term in the RA model can only be realized when the data are sufficient to produce reliable parameter estimates.

b) Fixed Cost Burden: As expected the RA model tends to outperform the LC model as the proportion of total unit cost comprised of fixed cost increases. This implies that the benefit derived from use of a rate adjustment model depends on the capital intensiveness of the production process. Surprising, perhaps, was the fact that the change in ERRDIFF from the low (15%) fixed cost treatment to the high (50%) was relatively small.

c) Past Production Trend: The RA model tends to outperform the LC model if the production rate was growing during the model estimation period. This finding suggests that it is necessary to have a changing production rate from period-to-period in order for the model estimation process to reliably capture the effect of rate on cost. It also suggests that relatively higher correlation between cumulative quantity and production rate (which will occur when rate has been growing) does not imply that cumulative quantity will be a sufficient variable. Cumulative quantity does not adequately capture the impact of rate changes.

d) Production Rate Instability: The RA model tends to outperform the LC model when there is greater instability in production rate. This suggests that it is desirable to have large period-to-period variation in production rate during the time of

model estimation. In retrospect this conclusion seems obvious. If the variance in cost caused by rate changes is small, because the rate changes themselves are small, then the effect will be swamped by other effects and model parameters will be unreliable.

e) Cost Noise/Variance: As the random noise in unit cost increases, the RA model will perform more poorly relative to the LC model. This suggests that when there are unsystematic, unpredictable factors that influence cost, use of the RA model is less beneficial. The noise leads to unreliable estimates of the production rate effect and poorer cost predictions.

f) Future Production Level: The degree to which the RA model outperforms the LC model appears to be inversely associated with the production rate in the prediction period. When cutbacks in production are anticipated, cost is more accurately predicted by the RA model. When growth is anticipated, cost is more accurately predicted by the LC model. This suggests that the RA model is most beneficial when a reversing of the production trend from the estimation period to the prediction period is expected.

Interaction Effects: Eight first order interactions were also found to be significant in the ANOVA. Note that five of these interactions involve FUTUPROD as one of the interacting variables. This means that the impact of other variables on relative prediction accuracy is particularly sensitive to the anticipated level of production in the prediction period. Examining these eight interactions individually leads to further insight concerning the conditions that impact relative prediction accuracy. They are

illustrated in Figures 4 through 11.

Figure 4, the interaction of Data History with Future Production Level, demonstrates again that the RA model tends to outperform the LC model as the data history becomes richer. But this effect is most pronounced when a low future production level is planned. If a cutback in production is projected and few observations are available for model estimation, the LC model is substantially more accurate than the RA model (the most negative value for ERRDIFF occurs under these conditions). If a cutback in production is projected and relatively many observations are available for model estimation, the RA model is substantially more accurate than the LC model. (The most positive value for ERRDIFF occurs under these conditions.)

Figure 5, the interaction of Fixed Cost Burden and Future Production Level, demonstrates a somewhat analogous effect. The RA model tends toward better accuracy than the LC model as fixed cost burden increases. But again this effect is most pronounced when low levels of production in the future are planned. The analyst's concern for the impact of burden percentage on model accuracy should be greatest when volume cutbacks are expected.

Figure 6, the interaction of Production Rate Instability and Future Production Level also shows the importance of the future production level. The RA model tends to outperform the LC model as the degree of variability in the production rate increases. But the impact of rate variability is most pronounced when a cutback in production volume is planned. If the production rate was stable

FIGURE 4

INTERACTION OF DATA HISTORY
WITH FUTURE PRODUCTION LEVEL

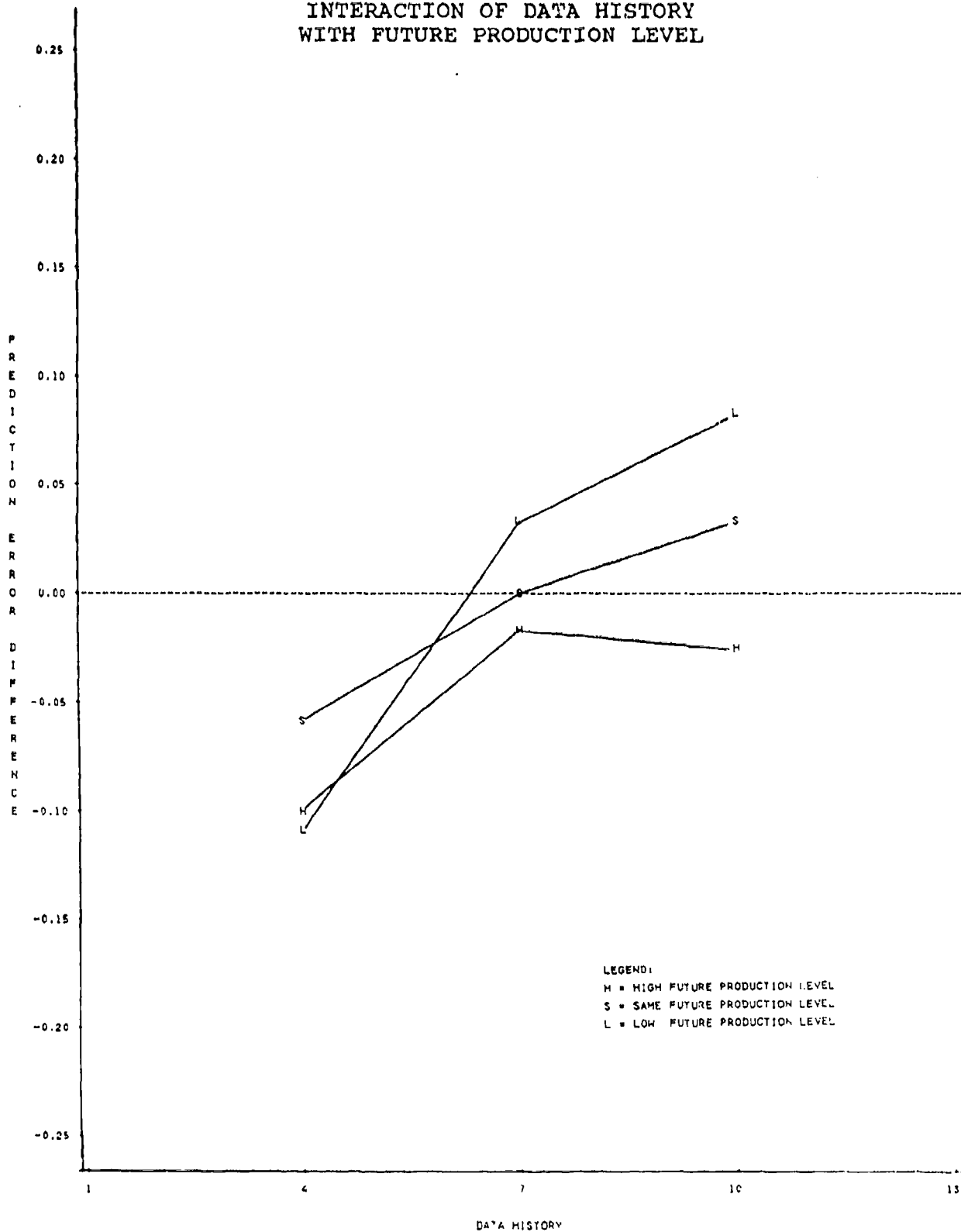


FIGURE 5

INTERACTION OF FIXED COST BURDEN
WITH FUTURE PRODUCTION LEVEL

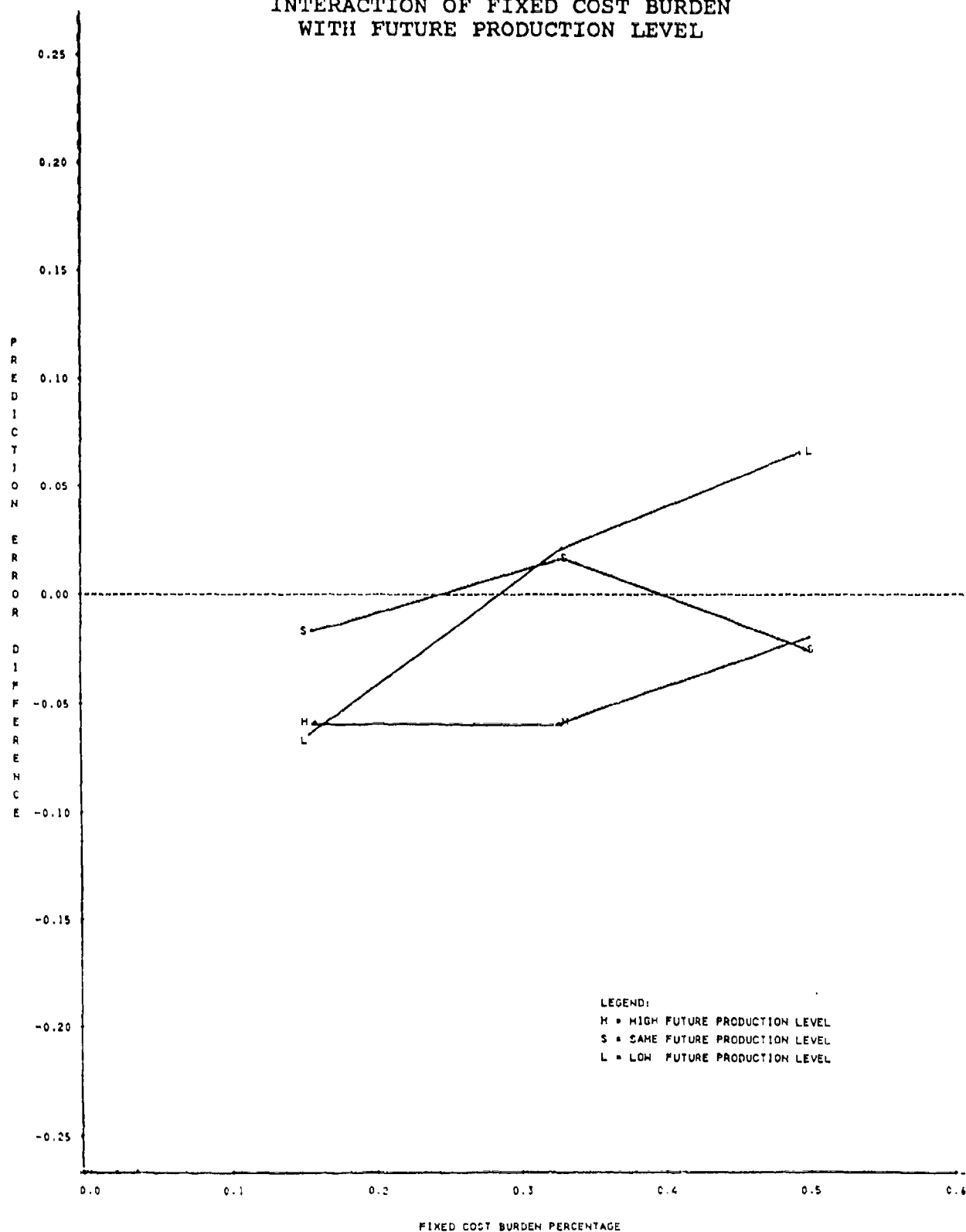


FIGURE 6

INTERACTION OF PRODUCTION RATE INSTABILITY
WITH FUTURE PRODUCTION LEVEL

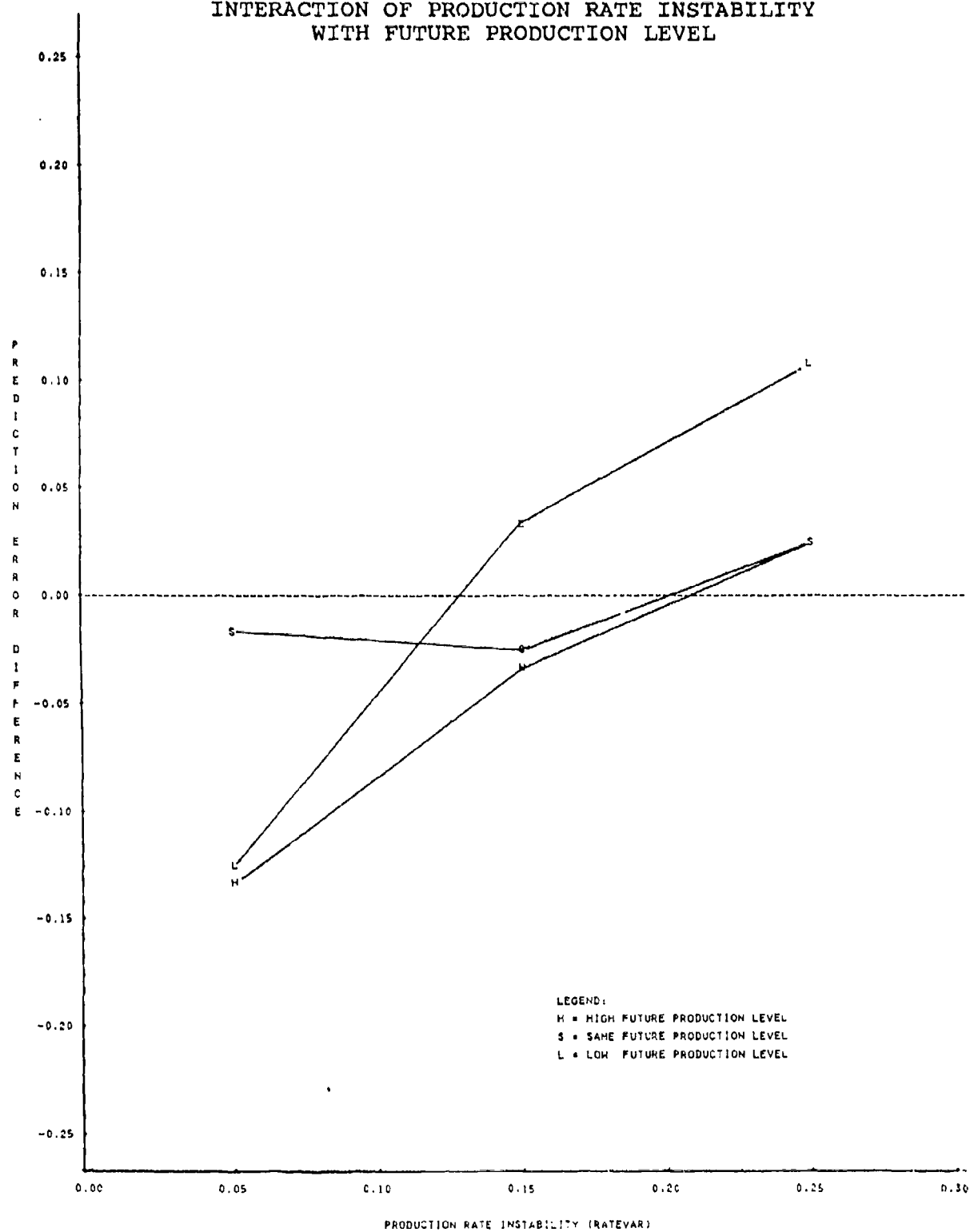
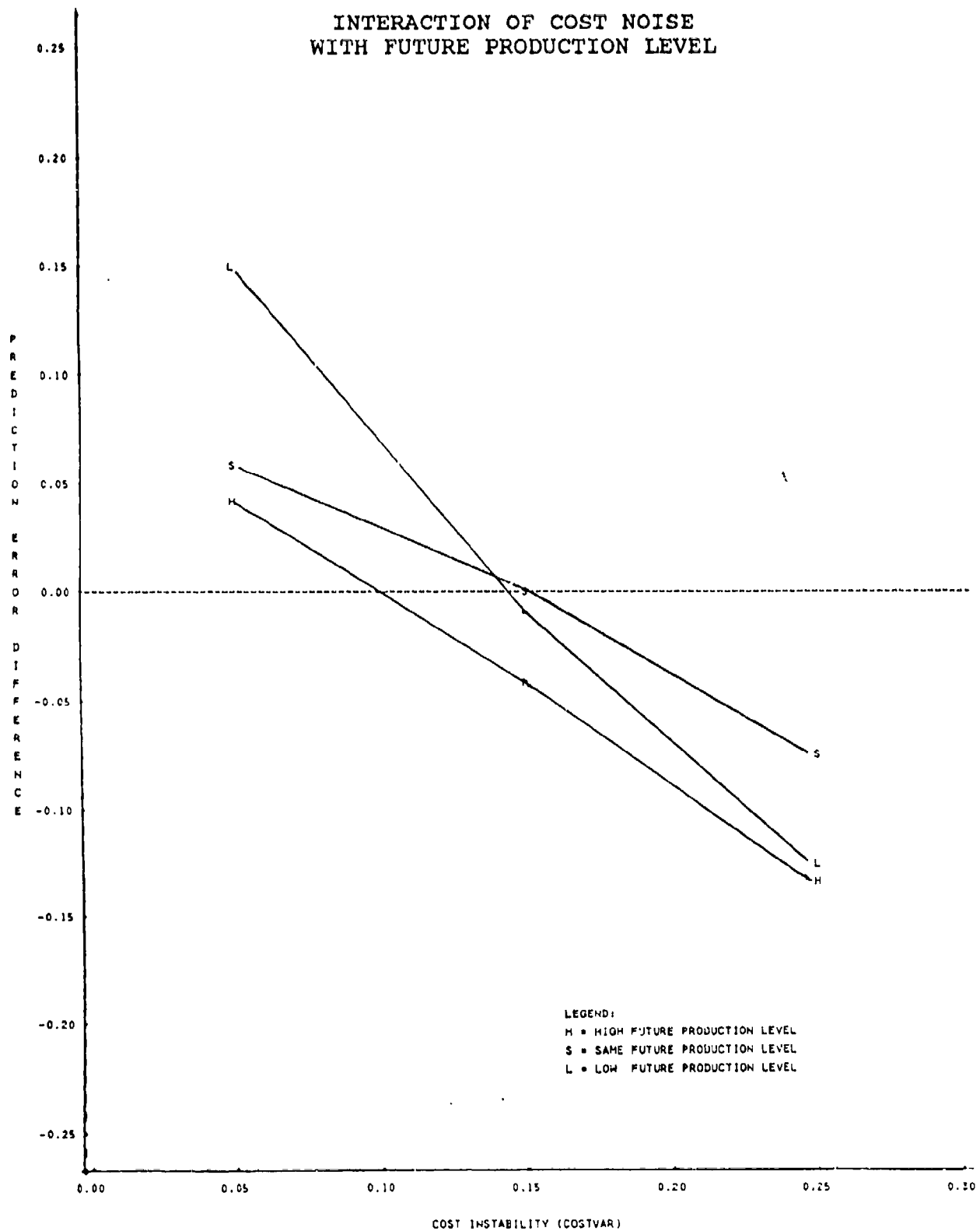


FIGURE 7

INTERACTION OF COST NOISE
WITH FUTURE PRODUCTION LEVEL



during the model estimation period, the LC model is more accurate. If the production rate was fluctuating and a cutback in volume is planned, then the RA model substantially outperforms the LC model.

Figure 7, the interaction of Cost Noise with Future Production Level, again illustrates how the future production volume impacts relative prediction accuracy. For all treatments, as unsystematic factors make cost more noisy, the RA model tends to lose its relative advantage over the simpler LC model. But this deterioration in predictive accuracy due to noise for the RA model is more evident when a cutback in future production volume is anticipated.

Figure 8 shows the interaction of the Production Trend during the estimation period and the Future Production Level during the prediction period. Again the steepest curve occurs when the future production level is low. This plot demonstrates that if production rate is relatively level during the model estimation period, the simple LC model tends to outperform the RA model. But the RA model outperforms the LC model when there is a shift or change in direction of the production trend from the estimation period to the prediction period. This follows from the positive value for ERRDIFF when a growing past production trend is coupled with a decline in future production level.

Figure 9 shows an interesting interaction between Cost Noise and Fixed Cost Burden. The plot shows that under all conditions, increasing cost noise reduces, and then eliminates, any benefit from using the RA model. But the deterioration is most dramatic if

the percentage of fixed cost in total cost is high. If burden is high and cost is not subject to unsystematic noise, then the RA model is substantially superior. But if burden is high and cost is subject to unsystematic noise, the RA model is substantially inferior.

Figure 10, the interaction between Cost Noise and Data History, illustrates a somewhat additive effect of two factors. Either increasing cost noise or reducing the number of observations causes deterioration in the performance of the RA model relative to the LC model. But combining the two effects magnifies the deterioration. If there is high cost noise and few observations on which to estimate model parameters, the RA model becomes very unreliable and the LC model is strongly superior.

Finally, Figure 11 shows the interaction between Production Rate Instability and Production Rate Trend. These variables reflect two aspects of production rate during the model estimation period. The plot illustrates that when there is little trend in rate, coupled with little variability in rate relative to trend, then there is little basis on which to estimate a rate parameter. Hence the LC model strongly outperforms the RA model.

SUMMARY AND CONCLUSIONS

The central research question addressed in the study was: under what conditions does consideration of production rate and incorporation of a rate variable into a learning curve analysis of cost lead to a more accurate prediction of future cost? The

FIGURE 8

INTERACTION OF PRODUCTION RATE TREND
WITH FUTURE PRODUCTION LEVEL

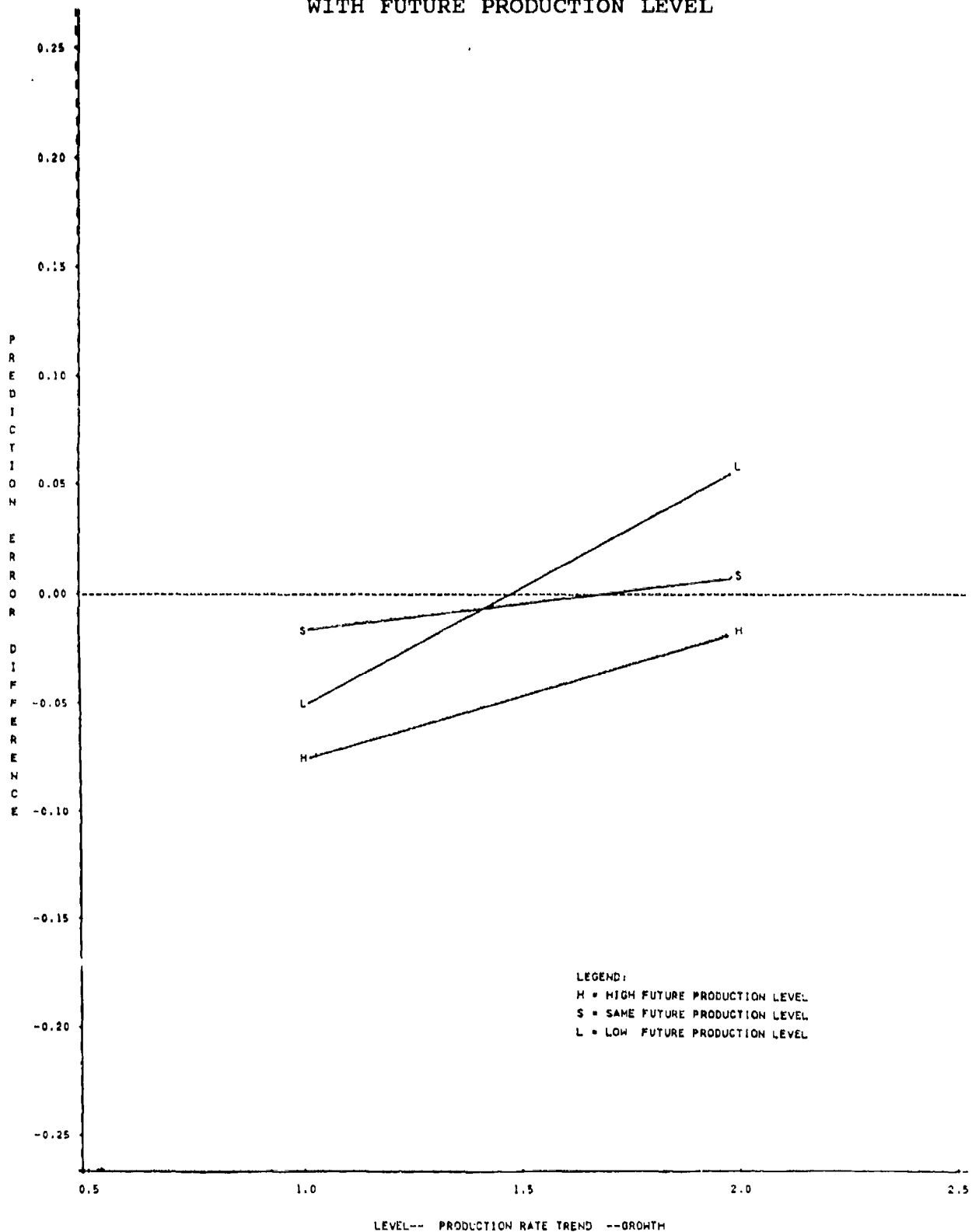


FIGURE 9
INTERACTION OF COST NOISE
WITH FIXED COST BURDEN

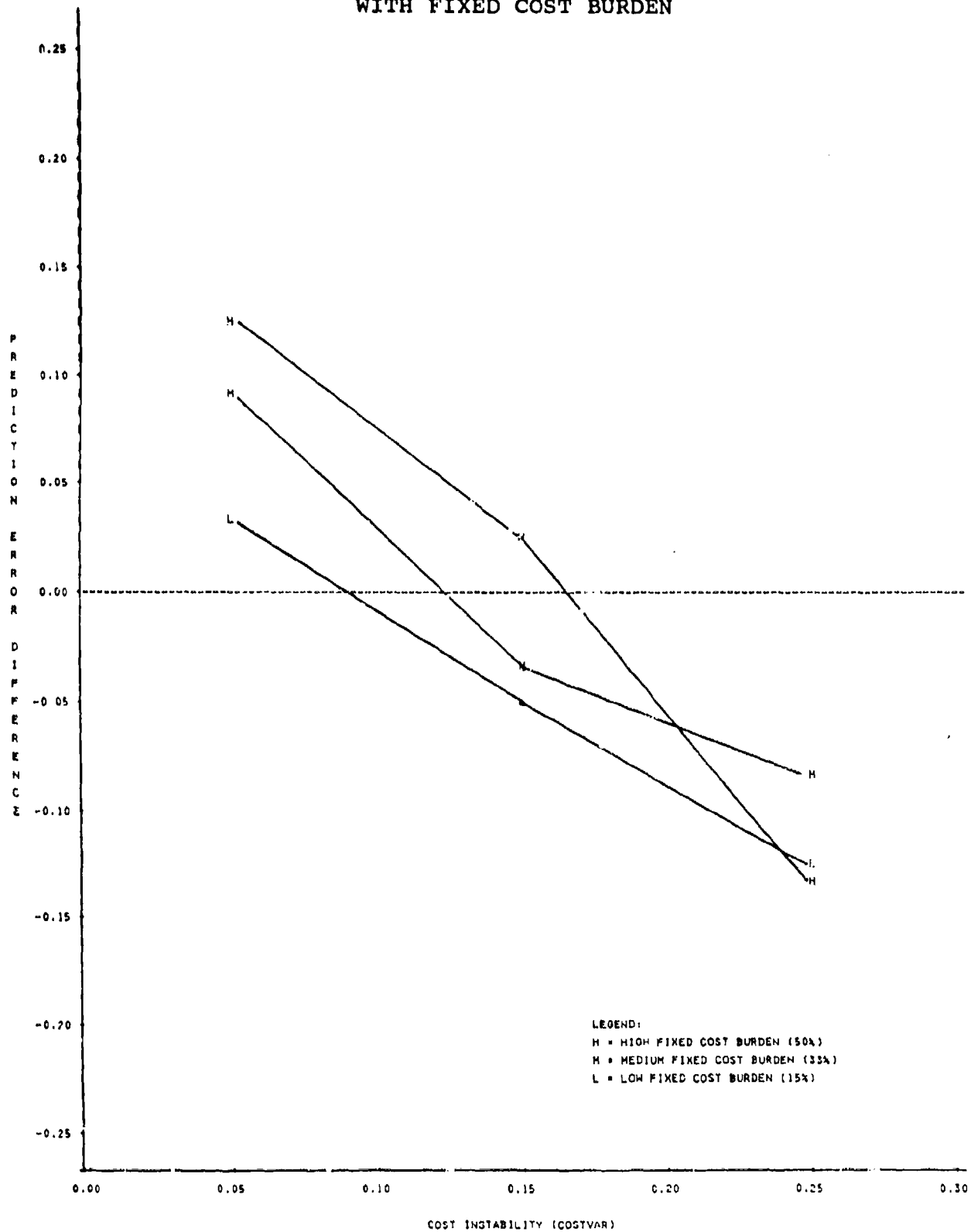


FIGURE 10
INTERACTION OF COST NOISE
WITH DATA HISTORY

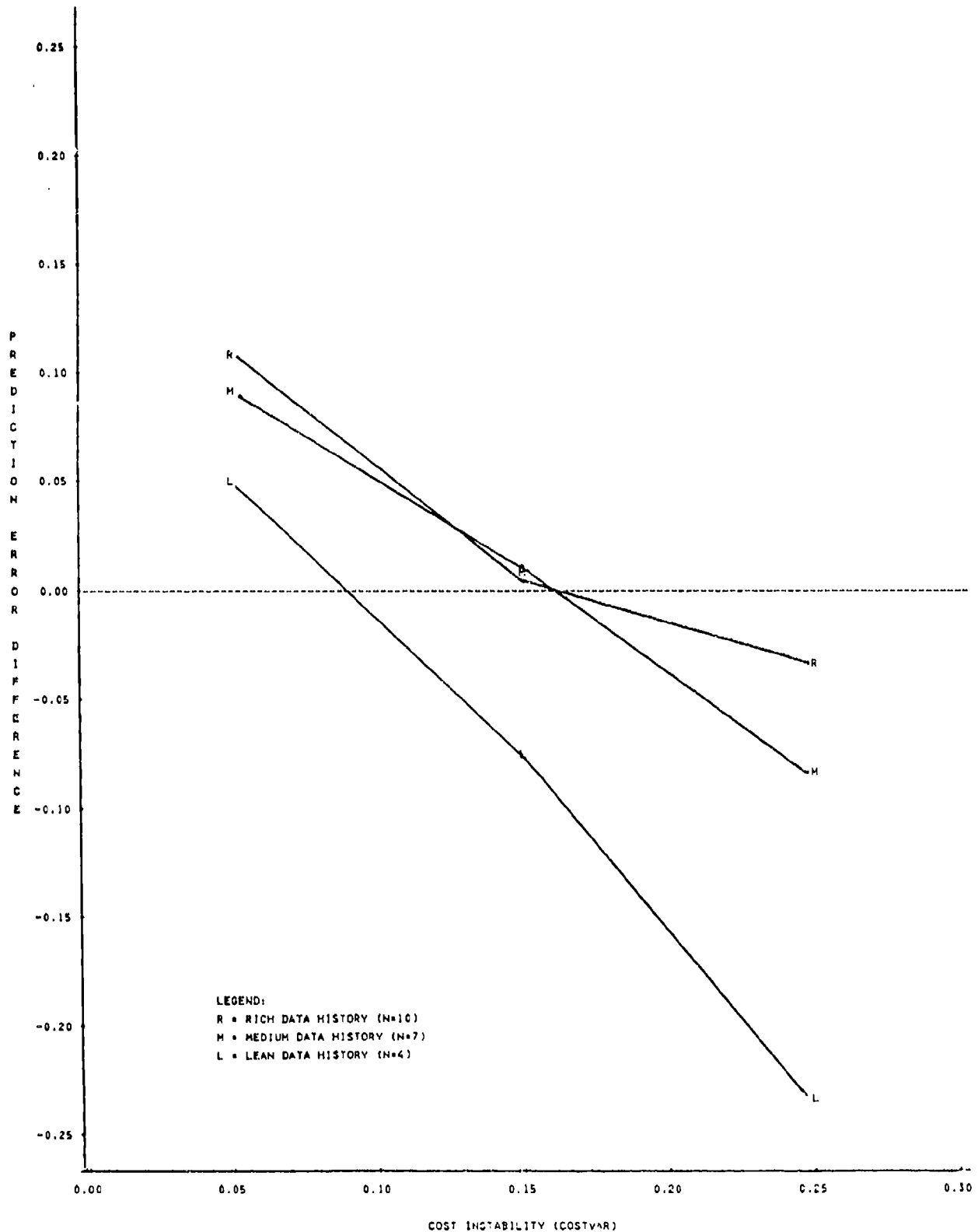
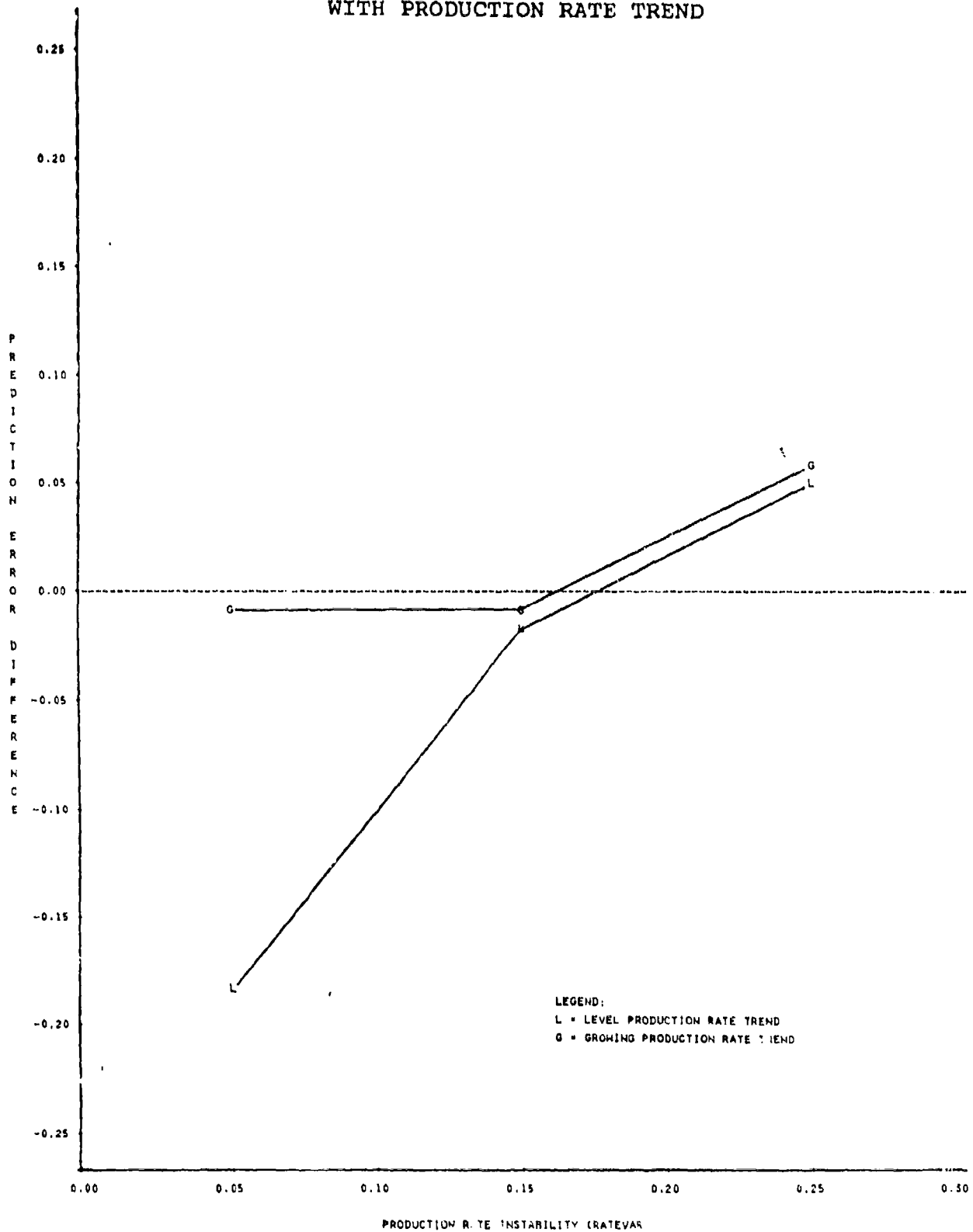


FIGURE 11

INTERACTION OF PRODUCTION RATE INSTABILITY
WITH PRODUCTION RATE TREND



analysis simulated prediction for both a traditional learning curve and a rate adjustment model and compared prediction accuracy under various conditions. One central finding was that neither a traditional learning curve analysis nor an analysis which considers the impact of production rate on cost is inherently superior. Neither models dominates; both outperform the other under some conditions.

General tendencies were evident. Considering production rate in the analysis lead to reduction in prediction error and improved accuracy when

- The number of observations available for the analysis was relatively rich.
- The amount of fixed cost in total cost was relatively high.
- The production rate trend had been growing during the model estimation period.
- The period-to-period variability in production rate was relatively large.
- Random noise in cost due to unsystematic factors impacting cost was relatively low.
- Production volume was expected to be cutback in the future periods for which cost predictions were being made.

Each of these findings suggests that researchers or cost analysts, engaged in a cost prediction or cost analysis problem, may benefit from attending to such factors when deciding on the form of model or analysis they might bring to bear.

Numerous interacting impacts of combinations of factors on prediction accuracy were also evident, but a broad "theme" was apparent in the interactions, suggesting a general conclusion:

- The greatest impact (of changes in the various factors) on relative prediction accuracy (of the learning curve approach and the rate adjustment approach) occurs when cutbacks in future production are anticipated.

This means that researchers and cost analysts, attempting to predict future cost in an environment where future production volume is declining, will find the choice of an approach to be most critical. The relative accuracy of the learning curve approach or the rate adjustment approach is particularly sensitive to changes in data richness, fixed cost burden, production rate trend and stability, and cost noise when cutbacks are anticipated.

Other interactions collectively suggest a second general conclusion:

- The impacts of factors on the relative prediction accuracy of the two approaches tend to be additive.

This means that if the presence of one factor (say, lean data history) and the presence of another (say, high cost noise) both reduce the relative accuracy of the rate adjustment approach, then the presence of both will magnify the effect.

The conclusions of any study must be tempered by any limitations. The most prominent limitation of this study is the use of simulated data. Use of the simulation methodology was justified by the need to create a wide range of treatments and maintain control over extraneous influences. This limitation suggests some directions for future research.

- Re-analyze the research question while altering aspects of the simulation methodology. For example, are findings sensitive to the cost function assumed?
- Address the same research question using actual cost and production rate data. Are the same findings

evident when using "real-world" data?

Providing confirmation of the findings by tests using alternative approaches would be beneficial.

Additional future research may be directed toward new, but related, research questions.

- Investigate other qualities of predictive performance, for example, bias. Do learning curves or rate adjustment models systematically provide predictions that are biased toward under or over estimation of future cost? Under what conditions?
- Investigate the magnitude of prediction errors using either a learning curve or rate adjustment approach. How large are average prediction errors under varying circumstances?
- Investigate other competing models or approaches to cost prediction. Perhaps accuracy can be improved by using some version of a "moving average" prediction model. Can such a model outperform both the learning curve and the rate adjustment approach? If so, under what circumstances?

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