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(a) Convenient stability criteria for difference approximations to hyperbolic initial-boundary value problems.
(b) Multiplicativity and stability of matrix and operator norms

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STABILITY ANALYSIS OF FINITE DIFFERENCE APPROXIMATIONS TO HYPERBOLIC SYSTEMS, AND PROBLEMS IN APPLIED AND COMPUTATIONAL MATRIX AND OPERATOR THEORY

Period: 1 May 1988 - 30 November 1990

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STABILITY CRITERIA FOR DIFFERENCE APPROXIMATIONS TO HYPERBOLIC SYSTEMS, AND MULTIPLICATIVITY OF MATRIX AND OPERATOR NORMS

Principal Investigator: Moshe Goldberg

ABSTRACT

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STABILITY CRITERIA FOR DIFFERENCE APPROXIMATIONS TO HYPERBOLIC SYSTEMS, AND MULTIPLICATIVITY OF MATRIX AND OPERATOR NORMS

Principal Investigator: Moshe Goldberg

1. Convenient Stability Criteria for Difference Approximations to Hyperbolic Initial-Boundary Value Problems

Consider the first order system of hyperbolic partial differential equations

$$\partial \mathbf{u}(x,t)/\partial t = A\partial \mathbf{u}(x,t)/\partial x + B\mathbf{u}(x,t) + \mathbf{f}(x,t), \quad x \geq 0, \quad t \geq 0, \tag{1.1a}$$

where $\mathbf{u}(x,t) = (u^{(1)}(x,t), \ldots, u^{(n)}(x,t))'$ is the unknown vector (prime denoting the transpose), $\mathbf{f}(x,t) = (f^{(1)}(x,t), \ldots, f^{(n)}(x,t))'$ is a given n-vector, and A and B are fixed $n \times n$ matrices such that A is diagonal of the form

$$A = \begin{bmatrix} A^{I} & 0 \\ 0 & A^{II} \end{bmatrix}, \quad A^{I} > 0, \quad A^{II} < 0, \tag{1.2}$$

with A^{I} and A^{II} of orders $k \times k$ and $(n - k) \times (n - k)$, respectively.

The solution of (1.1a) is uniquely determined if we prescribe initial values

$$\mathbf{u}(x,0) = \mathring{\mathbf{u}}(x), \quad x \geq 0, \tag{1.1b}$$

and boundary conditions

$$\mathbf{u}^{II}(0,t) = S\mathbf{u}^{I}(0,t) + \mathbf{g}(t), \quad t \geq 0, \tag{1.1c}$$

where S is a fixed $(n - k) \times k$ coupling matrix, $\mathbf{g}(t)$ a given $(n - k)$-vector, and

$$\mathbf{u}^{I} = (u^{(1)}, \ldots, u^{(k)})', \quad \mathbf{u}^{II} = (u^{(k+1)}, \ldots, u^{(n)})' \tag{1.3}$$

a partition of $\mathbf{u}$ into its outflow and inflow components, respectively, corresponding to the partition of A in (1.2).

In [GT7, 8] E. Tadmor and I extended our results in [GT4-6] and obtained versatile, easily checkable stability criteria for a wide class of finite difference approximations to the above initial-boundary value problem.

More specifically, introducing a mesh size $\Delta x > 0$, $\Delta t > 0$, such that $\lambda \equiv \Delta t/\Delta x$ is a constant, and using the notation $\mathbf{v}_j(t) = \mathbf{v}(j\Delta x, t)$, we approximate (1.1a) by a general, basic difference scheme -- explicit or implicit, dissipative or not, two-level or multilevel -- of the form

$$Q_{-1}\mathbf{v}_j(t + \Delta t) = \sum_{\sigma=0}^{s} Q_\sigma \mathbf{v}_j(t - \sigma\Delta t) + \Delta t\mathbf{b}_j(t), \quad j = r, r+1, \ldots,$$

$$Q_\sigma = \sum_{\tau=-r}^{p} A_{\sigma\tau} E^{\tau}, \quad E\mathbf{v}_j = \mathbf{v}_{j+1}, \quad \sigma = -1, \ldots, s, \tag{1.4}$$

where the $n \times n$ coefficient matrices $A_{\sigma\tau}$ are polynomials in λA and ΔtB, and the n-vectors $\mathbf{b}_j(t)$ depend on $\mathbf{f}(x,t)$ and its derivatives.

The difference equations in (1.4) have a unique solution $\mathbf{v}_j(t + \Delta t)$ if we provide initial values

$$\mathbf{v}_j(\sigma\Delta t) = \mathring{\mathbf{v}}_j(\sigma\Delta t), \quad \sigma = 0, \ldots, s, \quad j = 0, 1, 2, \ldots, \tag{1.5}$$

and specify, at each time level $t = \sigma\Delta t$, $\sigma = s, s + 1, \ldots$, boundary values $\mathbf{v}_j(t + \Delta t)$, $j = 0, \ldots, r - 1$. Such boundary values are determined by boundary conditions of the form

$$T^{(j)}_{-1}\mathbf{v}_j(t+\Delta t) = \sum_{\sigma=0}^{q} T^{(j)}_{\sigma}\mathbf{v}_j(t-\sigma\Delta t) + \Delta t\mathbf{d}_j(t), \quad j = 0, \ldots, r-1,$$

$$T^{(j)}_{\sigma} = \sum_{\tau=0}^{m} C^{(j)}_{\sigma\tau} E^{\tau}, \quad \sigma = -1, \ldots, q, \tag{1.6a}$$

where the $n \times n$ matrices $C^{(j)}_{\sigma\tau}$ depend on A, ΔtB and S, and the n-vectors $\mathbf{d}_j(t)$ are functions of $\mathbf{f}(x,t)$, $\mathbf{g}(t)$ and their derivatives.

Our intention was to interpret the difficult and often stubborn Gustafsson-Kreiss-Sundström stability criterion in [GKS] in order to obtain convenient, simple stability criteria for approximation (1.4) - (1.6a). While we were unable to meet this goal for general boundary conditions of type (1.6a), we managed to achieve rather satisfactory results under the further assumption that, in accordance with the partition of A in (1.2), the matrices $C^{(j)}_{\sigma\tau}$ can be written as

$$C^{(j)}_{\sigma\tau} = \begin{bmatrix} C^{I(j)}_{\sigma\tau} & C^{I\,II(j)}_{\sigma\tau} \\ C^{II\,I(j)}_{\sigma\tau} & C^{II(j)}_{\sigma\tau} \end{bmatrix}, \tag{1.6b}$$

where for B = 0:

$$\text{the } C^{I(j)}_{\sigma\tau} \text{ are independent of } j, \tag{1.6c}$$

$$\text{the } C^{I(j)}_{\sigma\tau} \text{ are diagonal,} \tag{1.6d}$$

$$\text{the } C^{I\,II(j)}_{\sigma\tau} \text{ vanish ,} \tag{1.6e}$$

$$\text{the } C^{II(j)}_{\sigma\tau} \text{ vanish for } \tau>0 \text{ or } \sigma>-1. \tag{1.6f}$$

The essence of (1.6c)-(1.6e) is that for B = 0, the outflow boundary conditions are *translatory* (i.e., determined at all boundary points by the same coefficients), *separable* (i.e., split into independent scalar conditions for the different outflow unknowns), and *independent of outflow values*. Assumption (1.6f) implies that for $B = 0$, the inflow values at the boundary depend essentially on the outflow.

It should be pointed out that our outflow boundary conditions are quite general, despite the apparent restrictions in (1.6c)-(1.6e). Indeed, (1.6c) is not much of a restriction, since in practice the outflow boundary conditions are often translatory. In particular, if the numerical boundary consists of a single point, then the boundary conditions are translatory by *definition*, so (1.6c) holds automatically. The restrictions in (1.6d) and (1.6e) pose no great difficulties either, since they are satisfied by all reasonable boundary conditions, where for $B = 0$ the $C_{\sigma\tau}^{I\,(j)}$ usually reduce to polynomials in the diagonal block A^I, and the $C_{\sigma\tau}^{I\,II\,(j)}$ vanish.

We realize that in view of the restriction in (1.6f) our inflow boundary conditions are not quite as general as the outflow ones. They can, however, be constructed to any degree of accuracy (see [GT2]); and they can be conveniently extended as shown in [GT8]. We note that if the boundary consists of a single point, then such conditions can be achieved in a trivial manner, simply by duplicating the analytic condition (1.1c), i.e.,

$$\mathbf{v}_0^{II}(t + \Delta t) = S\mathbf{v}_0^{I}(t + \Delta t) + \mathbf{g}(t + \Delta t).$$

Throughout our work we assume, of course, that the basic scheme (1.4) is stable for the pure Cauchy problem, and that the additional assumptions which guarantee the validity of the Gustafsson-Kreiss-Sundström theory in [GKS], hold.

The first step in our analysis was to reduce the above stability question to that of a scalar, homogeneous problem. This is obtained by considering the outflow scalar equation

$$\partial u(x,t)/\partial t = a\partial u(x,t)/\partial x, \quad x \geq 0, \quad t \geq 0, \quad a = \text{constant} > 0, \tag{1.7}$$

for which the basic scheme (1.4) reduces to the homogeneous scheme

$$Q_{-1} v_j(t + \Delta t) = \sum_{\sigma=0}^{s} Q_\sigma v_j(t - \sigma\Delta t)$$

$$Q_\sigma = \sum_{\tau=-r}^{p} a_{\sigma\tau} E^\tau, \quad \sigma = -1, \ldots, s, \tag{1.8a}$$

and the boundary conditions (1.6) reduce to translatory conditions of the form

$$T_{-1} v_j(t + \Delta t) = \sum_{\sigma=0}^{q} T_\sigma v_j(t - \sigma\Delta t)$$

$$T_\sigma = \sum_{\tau=0}^{m} c_{\sigma\tau} E^\tau, \quad \sigma = -1, \ldots, q, \tag{1.8b}$$

where $a_{\sigma\tau}$ and $c_{\sigma\tau}$ are scalar coefficients.

Referring to (1.8) as the *basic approximation*, we proved:

Theorem 1.1 [GT7, 8]. *Approximation* (1.4)-(1.6) *is stable if and only if the basic approximation* (1.8) *is stable for every eigenvalue* a > 0 *of* A^I. *That is, approximation* (1.4)-(1.6) *is stable if and only if the scalar outflow components of its principal part are all stable.*

This reduction theorem implies that from now on we may restrict our stability study to the basic approximation (1.8).

In order to introduce our stability criteria for the basic approximation, we use the coefficients of the basic scheme (1.8a) to define the *basic characteristic function*

$$P(z,\kappa) = \sum_{\tau=-r}^{p} \left[a_{-1,\tau} - \sum_{\sigma=0}^{s} a_{\sigma\tau} z^{-\sigma-1} \right] \kappa^\tau .$$

Similarly, using the coefficients of the boundary conditions in (1.8b) we define the *boundary characteristic function*

$$R(z,\kappa) = \sum_{\tau=0}^{m} \left[c_{-1,\tau} - \sum_{\sigma=0}^{q} c_{\sigma\tau} z^{-\sigma-1} \right] \kappa^{\tau}.$$

Now putting

$$\Omega(z,\kappa) \equiv | P(z,\kappa) | + | R(z,\kappa) |,$$

we proved:

Theorem 1.2 [GT7, 8]. *The basic approximation* (1.8) *is stable if either*

$$\frac{\partial P(z,\kappa)}{\partial z} \cdot \frac{\partial P(z,\kappa)}{\partial \kappa} \bigg|_{z=\kappa=-1} < 0 \qquad (1.10a)$$

or

$$\Omega(z=-1, \kappa=-1) > 0\,; \qquad (1.10b)$$

and in addition

$$\Omega(z,\kappa) > 0 \text{ for all } |z| = |\kappa| = 1,\ \kappa \neq 1,\ (z,\kappa) \neq (-1,-1), \qquad (1.10c)$$

$$\Omega(z, \kappa=1) > 0 \text{ for all } |z| = 1,\ z \neq 1, \qquad (1.10d)$$

$$\Omega(z,\kappa) > 0 \text{ for all } |z| \geq 1,\ 0 < |\kappa| < 1. \qquad (1.10e)$$

The advantage of this setting of Theorem 1.2 is clarified by the following lemma, in which we provide helpful sufficient conditions for each of the four inequalities in (1.10b - e) to hold:

Lemma 1.3 [GT7, 8].

(i) *Inequalities* (1.10b,c) *hold if either the basic scheme* (1.8a) *or the boundary conditions* (1.8b) *are dissipative.*

(ii) *Inequality* (1.10d) *holds if any of the following is satisfied:*

(a) *The basic scheme is two-level.*

(b) *The basic scheme is three-level and*

$$\Omega(z = -1, \kappa = 1) > 0. \tag{1.11}$$

(c) *The boundary conditions are at most two-level and at least zero-order accurate as an approximation to equation* (1.7).

(d) *The boundary conditions are three-level, at least zero-order accurate as an approximation to* (1.7), *and* (1.11) *is satisfied.*

(iii) *Inequality* (1.10e) *holds if the boundary conditions fulfill the von Neumann condition, and are either explicit or satisfy*

$$T_{-1}(\kappa) \equiv \sum_{\tau=0}^{m} c_{-1,\tau} \kappa^{\tau} \neq 0 \quad \forall\ 0 < |\kappa| \leq 1.$$

The stability criteria obtained in Theorem 1.2 depend both on the basic scheme and the boundary conditions, but not on the intricate and often complicated interaction between the two. Consequently, Theorem 1.2, aided by Lemma 1.3, provide in many cases a convenient alternative to the celebrated stability criteria of Kreiss [K2] and of Gustafsson, Kreiss and Sundström [GKS].

Having the new criteria, we easily established stability for a host of examples that incorporate and generalize most of the cases studied in recent literature; e.g., [Go1, Go2, GGT, GKS, GO, GT1, GT2, GT4-6, K1, KO, Ol, Osh, Sk1, Sk2, SK, Ta, Th,Tr1]. To mention just a few of our examples, we obtained stability for:

(a) Any stable basic scheme, with boundary conditions generated by either the explicit or implicit one-sided Euler schemes.

(b) Any stable two-level basic scheme, with boundary conditions generated by either horizontal extrapolation or by the one-sided three-level Euler scheme.

(c) Any stable dissipative basic scheme, with boundary conditions generated by oblique extrapolation or by the Box scheme.

(d) The Crank-Nicolson, Backward-Euler, Leap-Frog and Lax-Friedrichs schemes (all *nondissipative*), with boundary conditions generated by either oblique extrapolation or by the one-sided Weighted Euler scheme.

We drew great satisfaction from the fact that our theory and examples in [GT4-6] were used already by a number of authors, including Berger [Bm], LeVeque [Le], South, Hafez and Gottlieb [SHG], Thuné [Th1, 2], Trefethen [Tr1, 2], and Yee [Y]. Thuné, in his effort to provide a software package for stability analysis of finite difference approximations to hyperbolic initial-boundary value problems, says in [Th1]: "...Another approach has been to derive new criteria, based on the Gustafsson-Kreiss-Sundström theory but more convenient for practical use... The most far-reaching work along these lines has been made by Goldberg and Tadmor [GT1, 2, 4] ..." In [Th2] he says: "A conceptually different approach has been suggested by Goldberg and Tadmor [GT6]. They have treated a fairly general class of difference approximations of problems in one space dimension. By exploring the properties of this class they have been able to ... simplify the stability analysis considerably". Talking about the ultimate goal of his project, Thuné [Th2] adds: "...the black box design ought to be abandoned. A flexible tool box design would be preferable. For example, the environment shoud include tools for checking the convenient stability conditions of Goldberg and Tadmor".

We were also pleased to learn that part of our theory in [GT7] was taught already in several institutions including UCLA, NYU, and the University of Paris VI.

2. Norms, Seminorms and Multiplicativity Factors

Let $\mathcal{A}$ be an algebra over the complex field $\mathbf{C}$. As usual, a real-valued *function*

$$S : \mathcal{A} \to \mathbf{R}$$

is called a *seminorm* if for all x, y $\in \mathcal{A}$ and $\lambda \in \mathbf{C}$:

$$S(x) \geq 0,$$

$$S(\lambda x) = |\lambda| \cdot S(x),$$

$$S(x + y) \leq S(x) + S(y).$$

If in addition, S is positive definite, i.e.,

$$S(x) > 0, \ \ x \neq 0,$$

then S is a *norm*. We call a seminorm S *proper* if $S \neq 0$ and $S(x) = 0$ for some $x \neq 0$. Finally, we say that S is *submultiplicative* (or simply, *multiplicative*) if

$$S(xy) \leq S(x)S(y), \ \ \forall x,y \in \mathcal{A}.$$

Of special interest are, of course, operator algebras. Here, we have a normed vector space **V** over **C** and an algebra $\mathcal{B}(\mathbf{V})$ of bounded linear operators on **V**.

The first example that comes to mind of a multiplicative norm on an operator algebra is the ordinary operator norm.

$$\|A\| = \sup\left\{ |Ax| : x \in \mathbf{V}, \ |x| = 1 \right\}, \tag{2.1}$$

where $|\cdot|$ is the vector norm on **V**.

If **V** is a Hilbert space, then a well known example of a nonmultiplicative norm on $\mathcal{B}(\mathbf{V})$ is the numerical radius (e.g., [Bc, Go3, GT3, Ha, P]):

$$r(A) = \sup\left\{ |(Ax, x)| : x \in \mathbf{V}, \ |x| \equiv (x, x)^{1/2} = 1 \right\}, \tag{2.2}$$

which plays an important role in stabil. : analysis of finite difference schemes for multi-space-dimensional hyperbolic initial-value problems [GT3, Li, LW, Tu].

Another example of considerable interest is the l_p norm, $1 \le p \le \infty$, defined on $\mathbf{C}_{n \times n}$, the algebra of $n \times n$ complex matrices:

$$|A|_p = \left(\sum_{i,j=1}^{n} |\alpha_{ij}|^p \right)^{1/p}, \quad A = (\alpha_{ij}) \in \mathbf{C}_{n \times n}. \tag{2.3}$$

Ostrowski [Ost] has shown that this norm is multiplicative if and only if $1 \le p \le 2$.

Given a seminorm S on an arbitrary algebra $\mathcal{A}$, and a fixed constant $\mu > 0$, then obviously $S_\mu \equiv \mu S$ is a seminorm too. Clearly, S_μ may or may not be multiplicative. If it is, we call μ a *multiplicativity factor* for S. That is, *μ is a multiplicativity factor for S if and only if*

$$S(xy) \le \mu S(x)S(y), \quad \forall x,y \in \mathcal{A}.$$

Evidently, if μ_0 is a multiplicativity factor of S, then so is any μ with $\mu \ge \mu_0$. Thus, having a seminorm S, the question is whether S has multiplicativity factors; and if so, is there a best (least) one?

This question can be easily answered as follows:

Theorem 2.1 ([GS1], [AG1]). *Let $\mathcal{A}$ be an algebra, and let $S \ne 0$ be a seminorm on $\mathcal{A}$. Then:*

(a) *S has multiplicativity factors if and only if* Ker S *is an ideal in $\mathcal{A}$ and*

$$\mu_{\inf} \equiv \sup\{S(xy) : x, y \in \mathcal{A},\ S(x) = S(y) = 1\} < \infty. \tag{2.4}$$

(b) *If S has multiplicativitiy factors and $\mu_{\inf} > 0$, then μ is a multiplicativity factor if and only if $\mu \ge \mu_{\inf}$.*

(c) *If S has multiplicativitiy factors and $\mu_{\inf} = 0$, then μ is a multiplicativity factor if and only if $\mu > 0$.*

If $\mathcal{A}$ is finite dimensional and S is a norm, then Ker S = {0} is an ideal in $\mathcal{A}$, and a standard compactness argument implies that μ_{inf} in (2.4) is finite; so by Theorem 2.1, S has multiplicativity factors. *This is not always the case if S is a proper seminorm.*

In the infinite dimensional case, it was shown by Straus, Arens and myself [GS2, AG1], that norms as well as proper seminorms may or may not have mutiplicativity factors.

While Theorem 1.1 seems to settle the question of characterizing multiplicativity factors, the quantity μ_{inf} in (2.4) is often difficult to compute. A more practical approach toward checking whether a given constant $\mu > 0$ is the best (least) multiplicativity factor for a given seminorm S, is by verifying that

$$S(xy) \le \mu S(x)S(y) \quad \forall x, y \in \mathcal{A},$$

with equality for some x_0 and y_0 for which $S(x_0) \neq 0$, $S(y_0) \neq 0$.

Using this observation, Holbrook [Ho], and independently Straus and myself [GS1], showed that if **V** is a Hilbert space over **C** of dimension at least 2, and r is the numerical radius in (2.2), then the best multiplicativity factor for r is $\mu = 4$.

Similarly, Maitre [M], and Straus and I [GS3], showed that the best multiplicativity factor for the l_p norm on $\mathbf{C}_{n \times n}$ defined in (2.3), is:

$$\mu = \begin{cases} 1 & , \quad 1 \le p \le 2 \\ n^{1-2/p} & , \quad 2 \le p \le \infty . \end{cases}$$

Often, when the least multiplicativity factor remains unknown, one may try to obtain multiplicativity factors through the following version of a theorem by Gastinel.

Theorem 2.2 [Ga, GS2, AG1]. *Let* S, T *be seminorms on an algebra* $\mathcal{A}$. *Let* T *be multiplicative, and let* $\tau \ge \sigma > 0$ be constants such that

$$\sigma T(x) \le S(x) \le \tau T(x), \quad \forall x \in \mathcal{A}.$$

Then any $\mu \ge \tau / \sigma^2$ *in a multiplicative factor for* S.

This result was utilized by Straus and myself [GS2, 4] to obtain multiplicativity factors for certain generalizations of the numerical radius, called C-numerical radii.

In [AG1] Arens and I investigated multiplicativity factors in terms of the kernels of S. In particular we proved:

Theorem 2.3 [AG1]. *If $\mathcal{A}$ is a simple algebra then there are no multiplicative proper seminorms on $\mathcal{A}$.*

Since $\mathbf{C}_{n \times n}$ is simple (e.g. [BM]), we immediately obtain from Theorem 2.3 the following result that was directly proved by Straus and myself in [GS1]:

Theorem 2.4 [AG1]. *There are no multiplicative proper seminorms on $\mathbf{C}_{n \times n}$.*

Arens and I [AG1, 2] specialized our study to function algebras and to seminorms generated by the sup norm, where we gave the following three characterizations of multiplicativity factors:

Theorem 2.5 [AG1, 2]. *Let $\mathbf{T}$ be a set and let $\mathcal{A}$ be the algebra of bounded functions*

$$x: \mathbf{T} \to \mathbf{C},$$

with the usual multiplication

$$xy(t) = x(t)y(t); \quad x, y \in \mathcal{A}; \quad t \in \mathbf{T}.$$

For a fixed element c, $0 \neq c \in \mathcal{A}$, *define the seminorm*

$$S_c(x) = \sup_{t \in \mathbf{T}} |c(t)x(t)| . \tag{2.5}$$

Then:

(a) S_c *has multiplicativity factors if and only if*

$$\varepsilon \equiv \inf\{|c(t)| : t \in \mathbf{T},\ c(t) \neq 0\} > 0.$$

(b) *If* $\varepsilon > 0$, *then* $\mu > 0$ *is a multiplicativity factor for* S_c *if and only if* $\mu \geq \varepsilon^{-1}$.

Theorem 2.6 [AG1, 2]. *Let* $\mathbf{T}$ *be a topological space and let* $\mathcal{A}$ *be the algebra of bounded continuous functions*

$$x : \mathbf{T} \to \mathbf{C}.$$

For a fixed c, $0 \neq c \in \mathcal{A}$, *let* S_c *be the seminorm in* (2.5). *Then conclusions* (a) *and* (b) *of Theorem* 2.5 *hold.*

Theorem 2.7 [AG1, 2]. *Let* $\mathbf{T}$, $\mathcal{A}$, c, and S_c *be as in Theorem* 2.6, *and suppose* $\mathbf{T}$ *is connected. Then*

(a) *The following are equivalent:*

(i) S_c *has the multiplicativity factors.*

(ii) $\delta \equiv \inf\{|c(t)| : t \in \mathbf{T}\} > 0.$

(iii) c *is invertible.*

(iv) S_c *is a norm on* $\mathcal{A}$.

(b) *If* $\delta > 0$ *then* $\mu > 0$ *is a multiplicativity factor for* S_c *if and only if* $\mu \geq \delta^{-1}$.

For example, consider l^∞, the algebra of bounded sequences $x = \{\xi_j\}_{j=1}^{\infty}$ over $\mathbf{C}$, with the usual Hadamard multiplication

$$xy = \{\xi_j \eta_j\}, \qquad x = \{\xi_j\},\ y = \{\eta_j\} \in l^\infty.$$

Fix an element $c = \{\gamma_j\} \in l^\infty$, $c \neq 0$, and define the seminorm,

$$S_c(x) = \sup_j |\gamma_j \xi_j|, \quad x = \{\xi_j\} \in l^\infty .$$

Obviously, S_c is a norm on l^∞ if and only if

$$\gamma_j \neq 0, \quad j = 1, 2, 3, \ldots .$$

Otherwise S_c is a proper seminorm.

By Theorem 2.5 (here $\mathbf{T} = \mathbf{Z}^+ = \{1,2,3, \ldots\}$), S_c has multiplicativity factors if and only if

$$\varepsilon \equiv \inf_{\gamma_j \neq 0} |\gamma_j| > 0 ,$$

and if $\varepsilon > 0$ then the best (least) multiplicativity factor for S_c is $\mu_{min} = \varepsilon^{-1}$.

The four simple selections

$$\gamma_j = 1, \quad j = 1,2,3, \ldots$$
$$\gamma_j = j^{-1}, \quad j = 1,2,3, \ldots$$
$$\gamma_1 = 0; \quad \gamma_j = 1, \quad j = 2, 3, 4, \ldots$$
$$\gamma_1 = 0; \quad \gamma_j = j^{-1}, \quad j = 2, 3, 4, \ldots$$

show, as indicated before, that in the infinite dimensional case, both norms and proper seminorms *may* or *may not* have multiplicativity factors.

Next, let S be a seminorm on $\mathbf{C}^n$ and consider the real-valued function S^+ defined by

$$S^+(x) = S(x^+), \quad x \in \mathbf{C}^n,$$

where

$$x^+ = (|\xi_j|) \in \mathbf{R}^n$$

is the vector obtained by taking the absolute values of the components of $x = (\xi_j)$.

As in [Go9], we call S *quasimonotonic* if

$$x, y \in \mathbf{R}^n \text{ with } 0 \leq x \leq y \text{ implies } S(x) \leq S(y),$$

where the inequalities $0 \leq x \leq y$ are construed componentwise.

With this definition we can prove:

Theorem 2.8 [Go9]. *Let S be a seminorm (norm) on $\mathbf{C}^n$. Then S^+ is a seminorm (norm) on $\mathbf{C}^n$ if and only if S is quasimonotonic.*

For example, it was shown in [Go8] that the numerical radius r in (2.2) is quasimonotonic on $\mathbf{C}_{n \times n}$. Since r is a norm, so is r^+ by Theorem 2.8.

In order to discuss multiplicativity factors for S^+ let us assume that $\mathbf{C}^n$ has been given a structure of an algebra over $\mathbf{C}$. This can be done, for instance, by taking $\mathbf{C}^n$ with the usual Hadamard multiplication

$$xy = (\xi_j \eta_j), \qquad x = (\xi_j),\ y = (\eta_j) \in \mathbf{C}^n,$$

or $\mathbf{C}_{n \times n}$ with usual matrix multiplication.

We can now prove the following result that associates multiplicativity factors of S with those of S^+:

Theorem 2.9 [Go9]. *Let* S *be a quasimonotonic seminorm on* $\mathbf{C}^n$ *with a multiplicativity factor* μ, *and suppose that* $\mathbf{C}^n$ *has been given a structure of an algebra such that*

$$(xy)^+ \leq x^+y^+ \qquad \forall\, x, y \in \mathbf{C}^n. \tag{2.6}$$

Then:

(a) μ *is a multiplicativity factor for* S^+.

(b) *If* S *has a least multiplicativity factor, then so does* S^+, *and these least factors satisfy*

$$\mu_{min}(S) \geq \mu_{min}(S^+). \tag{2.7}$$

Let us point out that condition (2.6) holds for all common multiplication rules on $\mathbf{C}^n$, including Hadamard's multiplication on $\mathbf{C}^n$ and the standard matrix multiplication on $\mathbf{C}_{n \times n}$ where we have

$$(xy)^+ = x^+y^+ \qquad \forall\, x, y \in \mathbf{C}^n$$

and

$$(AB)^+ \leq A^+B^+ \qquad \forall\, A, B \in \mathbf{C}_{n \times n},$$

respectively.

We further remark that the numerical radius in (2.2) satisfies

$$\mu_{min}(r) = \mu_{min}(r^+) = 4,$$

so equality in (2.7) is possible. It was shown, however, in [Go8] that in general the ratio

$$\mu_{min}(S) / \mu_{min}(S^+)$$

can be arbitrarily large.

The above concepts of multiplicativity and multiplicativity-factors can be extended as follows:

Definition. Let **U**, **V**, and **W** be normed vector spaces over **C**; and let $\mathcal{B}_1 = \mathcal{B}(\mathbf{U}, \mathbf{W})$, $\mathcal{B}_2 = \mathcal{B}(\mathbf{V}, \mathbf{W})$, and $\mathcal{B}_3 = \mathcal{B}(\mathbf{U}, \mathbf{V})$ be the spaces of bounded linear operators from **U** into **W**, **V** into **W**, and **U** into **V**, respectively. If S_1, S_2, and S_3 are seminorms on $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}_3$, respectively, and $\mu > 0$ is a constant such that

$$S_1(AB) \le \mu S_2(A) S_3(B) \quad \forall\, A \in \mathcal{B}_2,\ \ B \in \mathcal{B}_3,$$

then we say that μ is a *multiplicativity factor for* S_1 *with respect to* S_2 *and* S_3.

For example, if **V** is a Hilbert space, and if $\|\cdot\|$ and r are the operator norm and numerical radius in (2.1) and (2.2), then it is not hard to see that

$$r(AB) \ \le\ 2r(A)\| B \| \quad \forall\ A, B \ \in\ \mathcal{B}(\mathbf{V}),$$

with equality for certain operators $A \ne 0$ and $B \ne 0$. Thus, $\mu = 2$ is the best multiplicativity factor for r with respect to r and $\|\cdot\|$.

This example employs only a single vector space and two norms. In order to demonstrate the idea of mixed multiplicativity to its full extent, consider, for $1 \le p \le \infty$, the l_p norm of an $m \times n$ matrix $A = (a_{ij}) \in \mathbf{C}_{m \times n}$:

$$|A|_p = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |\alpha_{ij}|^p \right)^{1/p}. \tag{2.8}$$

Defining

$$\lambda_{pq}(m) = \begin{cases} 1, & p \ge q \\ m^{1/p - 1/q}, & q \ge p, \end{cases}$$

We have:

Theorem 2.10 [Go4]. *Let* p, q, r *satisfy* $1 \le p, q, r \le \infty$, *and let* q' *be the conjugate of* q (*i.e.*, $1/q + 1/q' = 1$). *Then the best multiplicativity factor for the* l_p *norm on* $\mathbf{C}_{m \times n}$ *with respect to the* l_q *norm on* $\mathbf{C}_{m \times k}$ *and the* l_r *norm on* $\mathbf{C}_{k \times n}$ *is*

$$\mu = \lambda_{pq}(m)\, \lambda_{pr}(n)\, \lambda_{q'r}(k).$$

That is, for all $A \in \mathbf{C}_{m \times k}$ *and* $B \in \mathbf{C}_{k \times n}$:

$$| AB |_p \le \lambda_{pq}(m)\, \lambda_{pr}(n)\, \lambda_{q'r}(k)\, | A |_q\, | B |_r , \tag{2.9}$$

where this inequality is sharp.

Theorem 2.10 has quite a few applications. For example (see [Go6, 7]), taking (2.9) with $m = n = 1$, we get an upper bound for the standard inner product (x, y) on $\mathbf{C}^n$ in terms of $| x |_p$ and $| y |_q$; and if we further set $r = q'$ we obtain the classical Hölder inequality.

Another application of Theorem 2.10 concerns the ordinary l_p operator-norm on $\mathbf{C}_{m \times n}$:

$$\| A \|_p = \max \left\{ | Ax |_p \ : \ x \in \mathbf{C}^n, \ | x |_p = 1 \right\}, \tag{2.10}$$

for which we obtain:

Theorem 2.11 [Go7]. *Let* p, q, r *satisfy* $1 \le p, q, r, \le \infty$. *Then for all* $A \in \mathbf{C}_{m \times k}$, $B \in \mathbf{C}_{k \times n}$,

$$\| AB \|_p \le \lambda_{pq}(m)\, \lambda_{qp}(k)\, \lambda_{pr}(k)\, \lambda_{rp}(n)\, \| A \|_q\, \| B \|_r ,$$

where the inequality is sharp if either $q \le p \le r$ *or* $r \le p \le q$.

A third consequence of (2.9) describes the equivalence relations between the norms in (2.8) and (2.9):

Theorem 2.12 [Go5]. *Let* p, q *satisfy* $1 \le p, q \le \infty$, *and let* q' *be the conjugate of* q. *Then for all* $A \in \mathbf{C}_{m \times n}$,

$$| A |_p \le \lambda_{pq}(mn) | A |_q ,$$

$$\| A \|_p \le \lambda_{pq}(m)\, \lambda_{qp}(n)\, \| A \|_q ,$$

$$\| A \|_p \le \lambda_{pq}(m)\, \lambda_{q'p}(n)\, | A |_q ,$$

$$| A |_p \le (mn)^{1/p} \| A \|_q ,$$

where the first three inequalities are sharp.

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Academic Degrees:

1965	B.Sc., Applied Mathematics, Tel Aviv University
1970	M.Sc. (magna cum laude), Applied Mathematics, Tel Aviv University
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Military Service: Israel Defense Force, 1965-1968, Captain

Academic Appointments:

1972- 1974	Instructor, Department of Mathematics, Tel Aviv University, Tel Aviv, Israel
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1980- 1986	Associate Research Mathematician, Institute for the Interdisciplinary Application of Algebra and Combinatorics, University of California, Santa Barbara.
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Professional Experience:

Guest in Mathematics (honorary appointment), University of California, Los Angeles, Summers of 1981-88

Visiting Associate Mathematician, Department of Mathematics, California Institute of Technology, Pasadena, California, Fall 1985 and Summer 1990.

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Editorial Activities:

Editor, "Linear Algebra and its Applications", Elsevier Science Publishing Co., New York

Editor,"Linear and Multilinear Algebra", Gordon and Breach Science Publisher, New York

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Referee for several mathematical journals and for "Letters in Physics"

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Selected Administrative Activities:

Officer, Executive Committee, Society for Industrial and Applied Mathematics (SIAM), Southern California Section, 1975-78

Organizing Committee, Joint AMS-MAA-SIAM Meeting, Pomona College, Claremont, California, October 19-21, 1978

Organizer, Minisymposium on Finite Difference Approximations to Hyperbolic Initial-Boundary Value Problems, as part of the First International Conference on Industrial and Applied Mathematics, Paris June 29 - July 4, 1987

Organizer, Workshop on Numerical Methods for Solving Partial Differential Equations, as part of the 1988 Annual Meeting of the Israel Mathematical Union, Tel Aviv University, Tel Aviv, March 29, 1988

Treasurer, Israel Mathematical Union, 1988 - 1990

Organizing Committee, The Fifth and Sixth Haifa Matrix Theory Conferences, Technion, Haifa, Israel, January 2-4, 1989, and June 11-14, 1990

Organizing Committee, The 1989 and 1990 Israel Mathematical Union Annual Conferences, Technion, Haifa, May 24, 1989, and May 22, 1990.

Memberships:

American Mathematical Society

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International Linear Algebra Society

Grants and Awards:

1976-80	Principal Investigator, U.S. Air Force Grant AFOSR-76-3046
1979-83	Principal Investigator, U.S. Air Force Grant AFOSR-79-0127
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Graduate Students:

Professor Eitan Tadmor, M.Sc., 1975
Thesis: "The Numerical Radius and Power Boundedness"
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Thesis: "Seminorms, Norms and Equivalence Constants"

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Publications: See attached list.

TALKS AT CONFERENCES AND MEETINGS

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1. Invited speaker, International Conference on Computational Methods in Nonlinear Mechanics, The Texas Institute of Computational Mechanics, The University of Texas at Austin, Austin, Texas, September 1974, title: "Stable approximations for hyperbolic systems with moving boundary conditons".

2. Principal speaker, The 104th Regular Meeting of the Association for Computer Machinery (ACM), Los Angeles Chapter, Special Interest Group on Numerical Mathematics, Los Angeles, California, April 1975, title: "Stable approximations for hyperbolic systems with moving internal boundaries".

3. Invited speaker, American Mathematical Society 1975 Summer Meeting, Special Session on Numerical Ranges, Western Michigan University, Kalamazoo, Michigan, August 1975, title: "Inclusion relations between certain sets of matrices".

4. Speaker, The 1977 Army Numerical Analysis and Computer Conference, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin, March 1977, title: "On boundary extrapolation and dissipative schemes for hyperbolic problems".

5. Invited speaker, The 746th American Mathematical Society Meeting, Special Session on Matrix Theory, California State University, Hayward, California, April 1977, title: "Some inclusion relations for c-numerical ranges".

6. Speaker, The 1977 Dundee Biennial Conference on Numerical Analysis, University of Dundee, Dundee, Scotland, June 1977, title: "Dissipative schemes for hyperbolic problems and boundary extrapolation".

7. Principal speaker, The National Science Foundation Conference on Linear and Multilinear Algebra, University of California, Santa Barbara, California, December 1977, title: "Numerical ranges and numerical radii".

8. Speaker, The Eighth U.S. National Congress of Applied Mechanics, University of California, Los Angeles, California, June 1978, title: "Spectral analysis of hydroelastic problems" (with R.S. Chadwick).

9. Invited speaker, The Second International Conference on General Inequalities, Mathematical Research Institute, Oberwolfach, West Germany, August 1978, title: "Some combinatorial inequalities and C–numerical radii".

10. Principal speaker, Workshop Series, Five One-Hour Talks, Institute for the Interdisciplinary Applications of Algebra and Combinatorics, University of California, Santa Barbara, California, September 1979, title: "Numerical ranges and numerical radii".

11. Principal speaker, The October 1979 Meeting of the Association for Computing Machinery (ACM), Los Angeles Chapter, Special Interest Group in Numerical Mathematics, Los Angeles, California, October 1979, title: "Stability theory for difference approximations of hyperbolic partial differential equations".

12. Invited speaker, The 1980 Annual Meeting of the Israeli Society for the Applications of Mathematics, Safad, Israel, May 1980, title: "Boundary-dependent stability criteria for difference approximations of hyperbolic initial-boundary value problems".

13. Speaker, The 1981 International Conference on Convexity and Graph Theory, University of Haifa, Haifa, Israel, March 1981, title: "On the convexity of numerical ranges".

14. Invited speaker, The 1981 Annual Meeting of the Israeli Society for Applications of Mathematics, The Weizmann Institute of Science, Rehovot, Israel, April 1981, title: "Scheme-independent stability criteria for difference approximations of hyperbolic initial-boundary value problems".

15. Invited speaker, The Third International Conference on General Inequalities, Mathematics Research Institute, Oberwolfach, West Germany, May 1981, title: "Better stability bounds for Lax-Wendroff schemes in several space dimensions".

16. Invited speaker, The Toeplitz Memorial Conference, Tel Aviv University, Tel Aviv, Israel, May 1982, title: "The numerical radius: from Toeplitz to modern numerical analysis" (with G. Zwas).

17. Invited speaker (two talks), The Fourth International Conference on General Inequalities, Mathematics Research Institute, Oberwolfach, West Germany, May 1983, titles (two talks): "New inequalities for ℓ_p norms of matrices", and "In memoriam Edwin Beckenbach".

18. Invited speaker, The AMS-SIAM Summer Seminar on Large-scale Computations in Fluid Mechanics, Scripps Institute of Oceanography, University of California, La Jolla, California, June-July 1983, title: "New stability criteria for difference approximations of hyperbolic initial-boundary value problems".

19. Invited speaker, The 1984 Annual Meeting of the Israel Mathematical Union, Applied Mathematics Session, Tel Aviv University, Tel Aviv, Israel, April 1984, title: "Convenient stability criteria for difference approximations of hyperbolic initial-boundary value problems".

20. Speaker, The Gatlinburg IX Conference on Numerical Algebra, Univesity of Waterloo, Waterloo, Canada, July 1984, titles (two talks): "Generalizations of the Perron-Frobenius Theorem", and "Norms and multiplicativity".

21. Invited attendee, The U.S.-Israel Binational Workshop on the Impact of Supercomputers on the Next Decade of Computational Fluid Dynamics, Jerusalem, Israel, December 1984, Panel Discussion.

22. Invited speaker, The 1984 Haifa Conference on Matrix Theory, Technion - Israel Institute of Technology and the University of Haifa, Haifa, Israel, December 1984, title: "Submultiplicativity of matrix norms and operator norms".

23. Invited speaker, Joint French-Israeli Mathematical Symposium on Linear and Nonlinear Partial Differential Equations, Numerical Analysis, and Geometry of Banach Spaces, The Israel Academy of Sciences and Humanities, Jerusalem, Israel, March 1985, title: "Stability criteria for difference approximations of hyperbolic initial-boundary value problems".

24. Principal speaker, Mathematics Research Conference, California Institute of Technology, Pasadena, California, October 1985, title: "Submultiplicativity of matrix norms and operator norms".

25. Principal speaker, Southern California Functional Analysis Seminar (SCFAS), California State University, Los Angeles, California, October 1985, title: "Submultiplicativity of matrix norms and operator norms".

26. Invited speaker, The 1985 Haifa Conference on Matrix Theory, Technion - Israel Institute of Technology and the University of Haifa, Haifa, Israel, December 1985, title: "Submultiplicativity and mixed submultiplicativity of matrix norms and operator norms".

27. Principal speaker, The 187th Meeting of the Association for Computing Machinery (ACM), Los Angeles Chapter, Special Interest Group in Numerical Analysis, Los Angeles, California, February 1986, title: "Stability criteria for finite difference approximations of hyperbolic initial-boundary value problems".

28. Invited speaker, The Fifth International Conference on General Inequalities, Mathematics Research Institute, Oberwolfach, West Germany, May 1986, title: "Multiplicativity and mixed-multiplicativity of operator norms and matrix norms".

29. Speaker, SIAM Conference on Linear Algebra in Signals, Systems and Control, Boston, Massachusetts, August 1986, title: "Mixed multiplicativity for l_p norms of matrices".

30. Invited speaker, The Third Haifa Matrix Theory Conference, Technion - Israel Institute of Technology, Haifa, Israel, January 1987, title: "Equivalence constants for l_p norms of matrices".

31. Invited speaker and Session Chairman, First International Conference on Industrial and Applied Mathematics, Minisymposium on Finite Difference Approximations to Hyperbolic Initial-boundary Value Problems, Paris, June-July 1987, title: "Convenient stability criteria for difference approximations of hyperbolic initial-boundary value problems".

32. Invited speaker, Meeting on Numerical Problems for Initial and Initial-boundary Value Problems, Mathematics Research Institute, Oberwolfach, West Germany, August 1987, title: "Stability criteria for finite difference approximations to hyperbolic initial-boundary value problems".

33. Invited speaker, The Fourth Haifa Matrix Theory Conference, Technion - Israel Institute of Technology, Haifa, Israel, January 1988, title: "On monotone and semi-monotone matrix functions".

34. Speaker and Session Chairman, Second International Conference on Hyperbolic Problems, RWTH Aachen, Aachen, West Germany, March 1988, title: "Convenient stability criteria for difference approximations of hyperbolic initial-boundary value problems".

35. Invited speaker and Session Chairman, The 1988 Annual Meeting of the Israel Mathematical Union, Tel Aviv University, Tel Aviv, Israel, March 1988, title: "Simple stability criteria for difference approximations of hyperbolic initial-boundary value problems".

36. Invited speaker and Session Chairman, The Fifth Haifa Matrix Theory Conference, Technion-Israel Institute of Technology, Haifa, Israel, January 1989, title: "Multiplicativity factors for seminorms".

37. Invited speaker, Technion Linear Algebra Summer Workshop, Technion - Israel Institute of Technology, Haifa, Israel, June 1989, title: "On a class of absolute norms".

38. Invited speaker and Session Chairman, Inaugural Conference of the International Linear Algebra Society, Brigham Young University, Provo, Utah, August 1989, title: "Norms, seminorms and multiplicativity factors".

39. Speaker, Third International Conference on Hyperbolic Problems, Uppsala, Sweden, June 1990, title: "Convenient stability criteria for difference approximations to initial-boundary value problems"

40. Speaker, Householder Symposium XI, Halmstad, Sweden, June 1990, title: "Norms, seminorms and submultiplicativity"

PUBLICATIONS

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Theses:

1. M.Sc. Thesis, "Quasi-conservative hyperbolic systems", Tel Aviv University, Tel Aviv, Israel, 1970.

2. Ph.D. Thesis, "Stable approximations for hyperbolic systems with moving internal boundary conditions", Tel Aviv University, Tel Aviv, Israel, 1973.

Published Papers:

1. Numerical solution of quasi-conservative hyperbolic systems - The cylindrical shock problem (with S. Abarbanel), Journal of Computational Physics 10 (1972), 1-21.

2. A note comparing the root condition and the resolvent condition (with W.L. Miranker), Information Sciences 4 (1972), 285-288.

3. A note on the stability of an iterative finite-difference method for hyperbolic systems, Mathematics of Computation 27 (1973), 41-44.

4. Stable approximations for hyperbolic systems with moving internal boundary conditions (with S. Abarbanel), Mathematics of Computation 28 (1974), 413-447.

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PROBLEMS IN APPLIED AND COMPUTATIONAL MATRIX AND OPERATOR THEORY

Principal Investigator: Marvin Marcus

ABSTRACT

Research completed under Grant AFOSR-88-0175 by Marvin Marcus during the period 5/1/88 - 11/30/90 consists of the following topics:

(a) Hadamard Products and Powers

(b) Inequalities for Tensors

(c) Inequalities for Generalized Matrix Functions

(d) Inequalities for Eigenvalues and Singular Values

(e) Distance Matrices

(f) Numerical Range

(g) Determinants of Sums

In the following report papers are listed that were partially or entirely completed during the reporting period.

EIGENVALUE AND SINGULAR VALUE INEQUALITIES, TENSOR SPACES, AND THE NUMERICAL RANGE

Principal Investigator: Marvin Marcus

Each of the sections that follow begin with the current disposition of the research paper named in the section heading. A description of the research and appropriate references follow each section heading.

1. Hadamard Square Roots

This paper is currently in press in the SIAM Journal of Matrix Analysis. The proof sheets have been corrected and returned to the editor during the period of this report.

If A is an n-square positive semi-definite hermitian matrix of rank 1 then the Hadamard square root of A is the n-square matrix obtained by replacing each entry of A by the principal value of its square root. It is proved that if A has no zero or negative entries then the Hadamard square root has odd rank and all odd ranks are possible.

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2. Multilinear Methods in Linear Algebra

This paper is currently in press in the journal Linear Algebra and its Applications. It is a written version of a one hour invited address at the first meeting of the International Linear Algebra Society held at Brigham Young University, Provo Utah, 8/12/89- 8/16/89.

Several classical and new results are presented in which multilinear algebra has proven to be an effective tool. Some of the topics covered are: Weyl's inequalities; the Hadamard product; mappings on tensor spaces; Gram matrices in tensor spaces; strong non-singularity and the LDU theorem.

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3. Experiments with Entrywise Square Roots

This paper is currently scheduled to appear in the next issue of The Math Works Newsletter. The Math Works Inc. distributes the linear algebra package, MATLAB.

In this paper MATLAB™ was used to compute the entrywise square root of an n-square complex matrix A and to investigate some of the usual invariants, e.g., rank (sqrt(A)). A particularly important class of matrices are the n-square positive semi-definite hermitian A of rank 1 (such matrices are the heart of the matter for the normal spectral decomposition theorem). It is easy to generate random complex A of this kind:

```
x = 2 * rand (1,n) - ones (1,n);
y = 2 * rand (1,n) - ones (1,n);
u = x + i * y;
A = u' * u;
```

Of course, if A has positive entries there is no difficulty in confirming that sqrt(A) has rank 1. If A has a negative entry then sqrt(A) need not be hermitian, nor of rank 1:

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

In fact, there are $A \geq 0$ (positive semi-definite hermitian) with positive entries for which sqrt(A) is indefinite [2, p. 462]:

$$A = \begin{bmatrix} 100 & 9 & 4 & 1 \\ 9 & 100 & 0 & 81 \\ 4 & 0 & 100 & 16 \\ 1 & 81 & 16 & 100 \end{bmatrix}.$$

The eig(A) command produces

1.020535443538800e+02
9.771678596998484e+01
1.832428791889355e+02
1.698679048719973e+01

whereas eig(sqrt(A)) yields

1.033573194164571e+01
9.027566724798694e+00
-1.873373948049415e-01
2.082403872836054e+01

The MATLAB™ code exhibited above can easily be embedded in a script to compute and tabulate rank(sqrt(A)) for a large number of complex n-square $A \geq 0$, rank $(A) = 1$. What is surprising is that for all n tested, and significantly large samples (250 random A), rank(sqrt(A)) was computed as an odd integer in every case. These experimental results led to the conjecture and proof of the following result.

> If A is an n-square, rank 1, positive semi-definite hermitian matrix with no negative entries then rank(sqrt(A)) is always an odd integer. Moreover, if ν is any odd integer, $1 \leq \nu \leq n$, there exists an n-square $A \geq 0$, rank$(A) = 1$, with no zero or negative entries, for which rank(sqrt(A)) = ν.

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4. Bessel's Inequality in Tensor Space

This paper appeared during the period of the current report: Linear and Multilinear Algebra (1988) Vol. 23, pp. 233-249.

Let A be an n-square complex matrix and define $\mathcal{A}_A$ to be the n!-square matrix whose entries are $\prod_{i=1}^{n} a_{\sigma(i),\tau(i)}$, where σ and τ run lexicographically over S_n. If A is positive definite hermitian and χ is a unit n!-tuple then

$$(\mathcal{A}_A\chi,\chi) \geq \det(A) + \left|\sum \chi(\sigma)\right|^2 c(A)$$

where c(A) is the largest of the numbers $\prod_{i=1}^{n} |a_{ij}|^2 / a_{jj}^n$, $j = 1, \ldots, n$, and the summation is over $\sigma \in S_n$. For n = 3, if A is not permutation similar to a direct sum and χ is a unit n!-tuple then $(\mathcal{A}_A\chi,\chi) = \det(A)$ iff χ is a multiple of the alternating character. The relationships among recent results of Bapat and Sunder, Chollet, and Gregorac and Hentzel are also discussed.

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5. A Unified Exposition of Some Classical Matrix Theorems

This paper appeared during the period of the current report: Linear and Multilinear Algebra, (1989), Vol. 25, pp. 137-147.

There are several interesting and elegant theorems about hermitian matrices that can be unified by a very simple inequality for inner products.

The concepts contained in this paper are all "name" theorems: the Hadamard determinant theorem; the Fischer inequality; the Kantorovich inequality; Weyl's inequalities. What may be less widely known is the observation that, except for Weyl's inequalities, these results are, in fact, equivalent to one another, and in turn, equivalent to an elementary inequality usually referred to as the "obvious" lower bound in the Kantorovich inequality.

The penultimate section of this note incorporates a brief, self-contained discussion of compound matrices. This old subject is less well known than it should be, and, as it turns out, provides additional insight into the theorems listed above.

In the final section, the important inequalities of H. Weyl that relate singular values and eigenvalues are discussed.

The statements of all of the theorems appearing in the sequel are found in [2], with details concerning their origins. The analysis of equality in Weyl's inequalities contained in the last section of the paper are new.

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6. A Note on the Determinants and Eigenvalues of Distance Matrices

This paper appeared during the period of the current report: Linear and Multilinear Algebra, (1989), Vol. 25, pp. 219-230.

A set of n vectors in a real s-dimenaional Euclidean space define an n-square matrix D whose i,j entry is the distance between the ith and the jth vectors. This symmetric matrix has 0 down the main diagonal and positive off diagonal entries. Such matrices are called distance matrices. Distance matrices arise in a class of techniques known as multidimensional scaling (MDS). The purppose of MDS is to reconstruct data concerning the vectors from an examination of their distance matrix. Thus the invertibility of distance matrices (and related matrices) is an important necessary condition on the vectors which is directly discernable from their distance matrix. The purposes of the present paper are: to examine the behavior of the determinant of distance matrices; to obtain explicit formulas for the eigenvalues of a distance matrix when the vectors from a regular polygon in real 2-dimensional space; and to present in simplified from certain classical results on the rank and signatutre properties of matrices whose entries are squared distances.

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7. Symmetry Properties of Higher Numerical Ranges

This paper appeared during the period of the current report: Linear Algebra and Appl., 1988, Vol. 104, pp. 141-164

Let A be a linear operator on a finite dimensional unitary space V of dimension n. The k^{th} higher numerical range of A, denoted by $W_k(A)$, is the totality of complex numbers tr(PAP) where P runs over all k-dimensional orthogonal projections on V. In this paper it is proved that $W_k(A)$ is a polygon with the real axis as a line of symmetry, k = 1, ..., n, if and only if A is normal with a real characteristic polynomial. Several non-normal examples are constructed in order to investigate the extent to which the symmetry of all of the $W_k(A)$ is required.

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8. Lower Bounds for the Norms of Decomposable Symmetrized Tensors

This paper appeared during the period of the current report: Linear and Multilinear Algebra, (1989), Vol. 25, pp. 269 - 274.

Lower bounds are given for the difference of two decomposable symmetrized tensors. The first bound uses a norm which makes the component vectors in a decomposable symmetrized tensor part of an orthonomral basis. The second bound holds only for decomposable elements of symmetry classes whose associated characters are linear.

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3. R. C. Thompson, Prinicpal submatrices, IX: Interlacing inequalities for singular values of submatrices, *Lin. Alg. Appl.* ***5*** (1972), 1 - 12.

VITA

Marvin Marcus

PERSONAL BACKGROUND

Academic Degrees:

1950	B.A. (highest honors in Mathematics) University of California, Berkeley
1953	Ph.D., Mathematics, University of California, Berkeley

RECENT PROFESSIONAL EXPERIENCE

1987 - present	Professor of Computer Science, UCSB
1983 - 1987	Professor of Mathematics and Computer Science, University of California, Santa Barbara

GRADUATE STUDENTS

M. Marcus has supervised the Ph.D. thesis work of 20 students. He has also supervised the M.S. thesis work of two students.

SELECTED ACADEMIC AWARDS AND DISTINCTIONS

1950	Graduated highest honors in mathematics, University of California, Berkeley
1954	Fulbright Award
1956-57	National Research Council, National Science Foundation, Post-doctoral Research Fellowship
1956, 1958-60,	National Science Foundation Research Grants
1962	Certificate of Award for Distinguished Service, U.S. Department of Commerce, National Bureau of Standards
1962 - present	Principal Investigator on Air Force Office of Scientific Research grants
1965	Mathematical Association of America Editorial Prize for the article entitled: "Linear Transformations on matrices"
1966	L.R. Ford Memorial Prize awarded by the Mathematical Association of America for the article, "Permanents"
1989	Outstanding Computer Scientist Award, UCSB, 1989

RECENT SELECTED INVITED PAPERS

1984	Invited contribution Special Issue of Linear Algebra and Its Applications honoring Helmut Wielandt
1986	University of California, Riverside
1986	University of California, San Diego
1986	Conference on Computers and Mathematics, Stanford University
1986	Western Educational Computing Consortium, Irvine, California
1988	Linear Algebra Conference, Santa Barbara, California
1988	Society for Industrial and Applied Mathematics, Linear Algebra Conference, Madison, Wisconsin
1988	Invited one-hour lecture, Fall, American Mathematical Society/Mathematical Association of America, joint meeting, Claremont, California

1989 Invited one hour lecture, Inaugural meeting of the International Linear Algebra Society, Provo, Utah, 8/12/89 - 8/16/89

1989 Invited one hour address, Southern California Linear Algebra Conference, San Diego, CA, 11/11/89.

1990 Invited principal speaker, Directions in Matrix Theory Conference, Auburn University, Auburn, AL, 3/20/90 - 3/23/90

MEMBERSHIP IN LEARNED SOCIETIES

American Mathematical Society
Mathematical Association of America
American Association of University Professors
Sigma Psi; Pi Mu Epsilon
American Association for the Advancement of Science
Washington Academy of Science
Society for Industrial and Applied Mathematics
Society for Technical Communication
Association for Computing Machinery

RECENT EXTRAMURAL SUPPORT

A Program In Quantitative Decision Making
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Eigenvalues, Numerical Ranges, Stability Analysis
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Air Force

Questions in Numerical Analysis / Associated Problems
M. Marcus, M. Goldberg
05/01/83 - 04/30/84 $57,515
Department of Education

A Program In Quantitative Decision Making
M. Marcus
05/01/83 - 09/14/84 $ 6,035
Air Force

Stability Analysis of Finite Difference Schemes
M. Marcus, M. Goldberg
05/01/84 - 04/30/85 $56,953
Air Force

Stability Analysis of Finite Difference Schemes
M. Marcus, M. Goldberg
05/01/85 - 04/30/86 $64,444

Air Force
Stability Analysis of Finite Difference Schemes
M. Marcus, M. Goldberg
05/01/86 - 04/30/87 $75,030

Air Force
Stability Analysis of Finite Difference Approximations to Hyperbolic Systems, and Problems in Applied and Computational Linear Algebra
M. Marcus, M. Goldberg
5/1/8787 - 4/30/88 $73,693

National Science Foundation
Computing and Algorithmic Mathematics for Secondary School Teachers
M. Marcus, J. Bruno
3/11/87 - 8/31/89 $516,999

National Science Foundation
A National Institute for Secondary School Teachers for the Dissemination of Computer Science and Algorithmic Mathematics
M. Marcus, J. Bruno, R. Mayer
2/1/88 - 8/31/89 $509,560

Augmentation to
National Science Foundation
A National Institute for Secondary School Teachers for the Dissemination of Computer Science and Algorithmic Mathematics
M. Marcus, J. Bruno
8/31/89 - 11/30/90 $36,000

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Stability Analysis of Finite Difference Approximations to Hyperbolic Systems, and Problems in Applied and Computational Matrix Theory
M. Marcus, M. Goldberg
5/1/88 - 10/31/88 $78,114

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Stability Analysis of Finite Difference Approximations to Hyperbolic Systems, and Problems in Applied and Computational Matrix Theory
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Institute for Scientific Computing Research
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The Geometry of the Numerical Range with Applications to Linear Numerical Analysis and Stability of Feedback Systems
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15 copies of *Mathematica*
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Visualization of the Numerical Range
M. Marcus
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PUBLICATIONS LIST

Marvin Marcus

RESEARCH PAPERS

1955

1. Field convexity of a square matrix (with B.N. Moyls), Proc. Amer. Math. Soc. 6 (1955), 981-983.

2. A remark on a norm inequality for square matrices, Proc. Amer. Math. Soc. 6 (1955), 117-119.

3. Some results on the asymptotic behavior of linear systems, Canad. J. Math. 7 (1955), 531-538.

4. Boundedness of a continuous function, Amer. Math. Monthly 62 (1955).

1956

5. An invariant surface theorem for a non-degenerate system. Contributions to non-linear oscillations, Annals of Math. Study 36 (1956), 243-256.

6. A note on the existence of periodic solutions of differential equations (with S.P. Diliberto), Annals of Math. Study 36 (1956), 237-241.

7. Repeating solutions for a degenerate system, Annals of Math. Study 36 (1956), 261-268.

8. On the optimum gradient method for systems of linear equations, Proc. Amer. Math. Soc. 1 (1956), 77-81.

9. Extramural properties of Hermitian matrices (with J. McGregor), Canad. J. Math. 8 (1956), 524-531.

10. An eigenvalue inequality for the product of normal matrices, Amer. Math. Monthly 63 (1956), 173-174.

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11. On the maximum principle of Ky Fan (with B. Moyls), Canad. J. Math. 9 (1957), 313-320.

12. Inequalities for symmetric functions and Hermitian matrices (with L. Lopes), Canad. J. Math. 9 (1957), 304-312.

13. A note on symmetric functions of eigenvalues (with R.C. Thompson), Duke Math. J. 24 (1957), 43-46.

14. A note on the values of a quadratic form, J. Wash. Acad. Sci. 47 (1957), 97-99.

15. Some extreme value results for indefinite Hermitian matrices I. (with B. Moyls and R. Westwick), Illinois J. Math. 1 (1957), 449-457.

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17. A determinantal inequality of H.P. Robertson II, J. Wash. Acad. Sci. 47 (1957), 264-266.

18. Maximum and minimum values for the elementary symmetric functions of Hermitian forms (with B. Moyls), J. Lond. Math. Soc. 32 (1957), 375-377.

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23. On the minimum of the permanent of a doubly stochastic matrix (with M. Newman), Duke J. Math. 26 (1959), 61-72.

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25. Linear transformations on algebras of matrices (with B. Moyls), Canad. J. Math. 11 (1959), 61-66.

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29. A note on the Hadamard product (with N. Khan), Canad. Math. Bull. 2 (1959), 81-83.

30. Transformations on tensor product spaces (with B. Moyls), Pacific J. Math. 9 (1959), 1215-1221.

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1963

63. Disjoint pairs of sets and incidence matrices (with H. Minc), Illinois J. Math 7 (1963), 137-147.

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1967

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1968

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99. Extensions of the Minkowski inequality (with S. Pierce), Linear Algebra and Appl. 1 (1968), 13-27.

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101. Elementary divisors of associated transformations (with S. Pierce), Linear Algebra and Appl. 2 (1969), 21-35.

102. Matrices of Schur functions (with S. Katz), Duke Math. J. 36 (1969), 343-352.

103. Singular value inequalities, J. London Math. Soc. 44 (1969),118-120.

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1970

109. Solution to advanced problem 63-2, SIAM Rev., 1970.

110. A generalization of the unitary group (with W.R. Gordon), Linear Algebra and Appl. 3 (1970), 225-247.

111. Inequalities for submatrices, article in Inequalities-II, Proceedings of the Second Symposium on Inequalities held at the U.S. Air Force Academy, Colorado, August 14-22, 1967, Academic Press, New York, 1970, 223-240.

112. Some results on unitary matrix groups (with M. Newman), Linear Algebra and Appl. 3 (1970), 173-178.

113. An analysis of equality in certain matrix inequalities I (with W.R. Gordon), Pacific J. Math 34 (1970), 407-413.

114. The structure of bases in tensor spaces (with W.R. Gordon), Amer. J. Math. 92 (1970), 623-640.

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1971

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1972

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132. Derivations, Plücker relations, and the numerical range, Indiana Univ. Math. J. 22 (1973), 1137-1149.

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