COMPUTER GENERATED HOLOGRAPHY AS A THREE-DIMENSIONAL DISPLAY MEDIUM

THESIS

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Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Computer Systems

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December, 1990

Preface

"I'm afraid I can't put it more clearly", Alice replied very politely, "for I can't understand it myself, to begin with."

LEWIS CARROL, *Alice in Wonderland*

I would like to thank several people that were invaluable to the success of this project: My thesis advisor, Major Phil Amburn, for his insight, inspiration, and encouragement. Captain Sean Kelly, WRDC/KTD (AFSC) who provided the necessary charts for this computer scientist to navigate the murky waters of optics. My thesis committee, Dr Matt Kabrisky who made learning fun and Major Steve Rogers who kept me on my toes. The other members of the *graphics group*, Captains Dave Dahn, Phil Platt, and Bill DeRouche. They provided the essential support and humor necessary for survival. My wife, Melissa, who had the thankless task of proof reading the document, being an AFIT widow, and providing all the love and encouragement necessary for me to complete this project. And finally, to my son, David, for gracefully accepting the absence of his dad.

Bryant L. Stuart

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Abstract

A process was developed to produce three dimensional images using computer generated holography (CGH). This process consisted of a series of steps that began with a geometric description of an object and concluded with a three dimensional holographic image being computed from a synthetic wavefront. The objects used in this series of steps (or CGH pipeline) were described geometrically as a collection of three dimensional points. The modular nature of the CGH pipeline provided a flexible platform from which to evaluate various object geometries, interference calculations algorithms, and interference pattern recording and reduction techniques. This system was implemented with general purpose computer workstations to compute the interference patterns, a postscript laser printer to record the patterns, and standard photographic reduction techniques to generate transmission holograms. Optical density filters were used to allow a hologram's virtual image to be safely viewed through the transmission hologram down the bore of the laser. The hologram's real image was observed as a collection of the object's planes captured on a white card. The reference beam was modeled as a plane wave normal to the recording surface with a constant phase angle of zero radians at all locations of the recording surface. The major constraint of this system was the limited spatial resolution of the laser printer which limited the geometric placement of the objects to be recorded.

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AS A THREE-DIMENSIONAL DISPLAY MEDIUM

I. Introduction

1.1 Background

"Computer graphics is concerned with the synthesis of pictures of real or imaginary objects" (8:3). In computer graphics, objects are defined mathematically and then displayed. The traditional output medium for computer graphics has been the computer display screen. Although improvements in the computer display have dramatically improved the image quality, the display device is limited by its two dimensional bounds. A computer display screen is a single planar device with only two spatial dimensions. Three dimensional objects that are mapped to the computer screen must be subjected to a data reduction process. That is, two of the object's spatial dimensions can be directly mapped to the display screen, but the object's third dimension must be mapped to one or both of two dimensions of the display screen. If the mapping is good, the three dimensional image appears to be realistic even though it is being displayed on a two dimensional plane. There are many techniques that allow three dimensional objects to be mapped to two dimensional computer display screens that preserve the essence of the three dimensions of the object in the mind of an observer. One technique is to provide depth cues using perspective projection; that is, objects farther away from the viewer appear smaller than objects up close. Other techniques include, but are not limited to, stereopsis (twin images from slightly varying perspectives corresponding to the separation of human eyes), and kinetic depth effect (far images appear to move faster and farther than closer images)(4:5). Although these and other techniques do a very good job of representing three dimensional objects realistically on two dimensional display screens, a *true* three dimensional display would be better. One has only to compare a two dimensional photograph of a human face to the face of a person sitting next to you to note the differences. Our eyes receive much more information from the *real image* of the person next to us than from the *virtual image* we see in the photograph no matter how life-like the latter may appear. Because most of the objects of interest in computer graphics are three dimensional and the information content of true three dimensional images is so much greater than two dimensional mappings of three dimensional images, methods to exceed the limitations of two dimensional display devices must be found.

1.2 True Three Dimensional Displays

Because true three dimensional display is so important, much research has focused on it. Three technologies currently offer hope for the effective and efficient display of three dimensional data. Texas Instruments first demonstrated a multiplanar display device in 1988. It modulates a laser beam onto an angled spinning disk and projects images within a transparent dome. The laser beam scans an image on the x and y axis of the display volume and the spinning disk is synchronized with the modulation of the laser beam to provide the z axis mapping. This is a true three dimensional display device because it can address points throughout its display volume. Three different colored lasers can be mixed to provide color images. The current prototype has a 36-inch diameter dome, but the design could be scaled up to 10 feet. The problems of this system are that it is still experimental, it requires expensive and delicate equipment, and only one company is currently developing the technology (25).

Another technology that provides true three dimensional displays is vibrating varifocal mirrors. A thin sheet of mylar film is distorted to form a concave or convex mirror by pneumatically changing the back pressure on the film. This distortion changes the focal length of the mirror which causes a corresponding change in the position of the reflected image. A series of two dimensional images are reflected on an object screen resulting in an autostereoscopic, or true three dimensional, images. The advantages of this system are that it produces true three dimensional images and that the hardware is simple, relatively inexpensive, and reliable. One of the limitations of this system is the persistence of the two dimensional images. These transparent, or *phantom*, images thwart an important three dimensional depth cue of hidden surfaces. Another anomaly of this process is that as images approach the viewer, they get smaller which is the opposite of how perspective projection functions in a real world viewing system(20).

Computer generated holography appears to be another promising process to provide true three dimensional displays. Computer generated holography for image display simulates optical holography by mathematically modeling an object and the interaction of light with that object. In optical holography a single coherent light source is split into two beams. One beam, the object beam, is directed at the object being recorded. The second beam, the reference beam, is directed to a recording plane, the film. As the object beam makes contact with the object, light rays are deflected. If the surface of the object is diffuse, the rays are deflected in all directions. Some of the deflected light rays reach the recording surface where they interfere with the rays from the reference beam. This interference is recorded as a set of gratings on the film that represents the intensity of the combined object and reference beams. Figure 1.1 illustrates the optical holography recording process. For a more detailed discussion on interference and diffraction, refer to Chapter 3.

A reconstruction of the original object can be obtained by illuminating the developed film with the original reference beam (7:7-16). Figure 1.2 depicts the process of reconstructing an image with optical holography. The resulting *real* image will appear to float in space on the side of the hologram opposite of the light source while the *virtual* image will appear on the same side of the hologram as the light

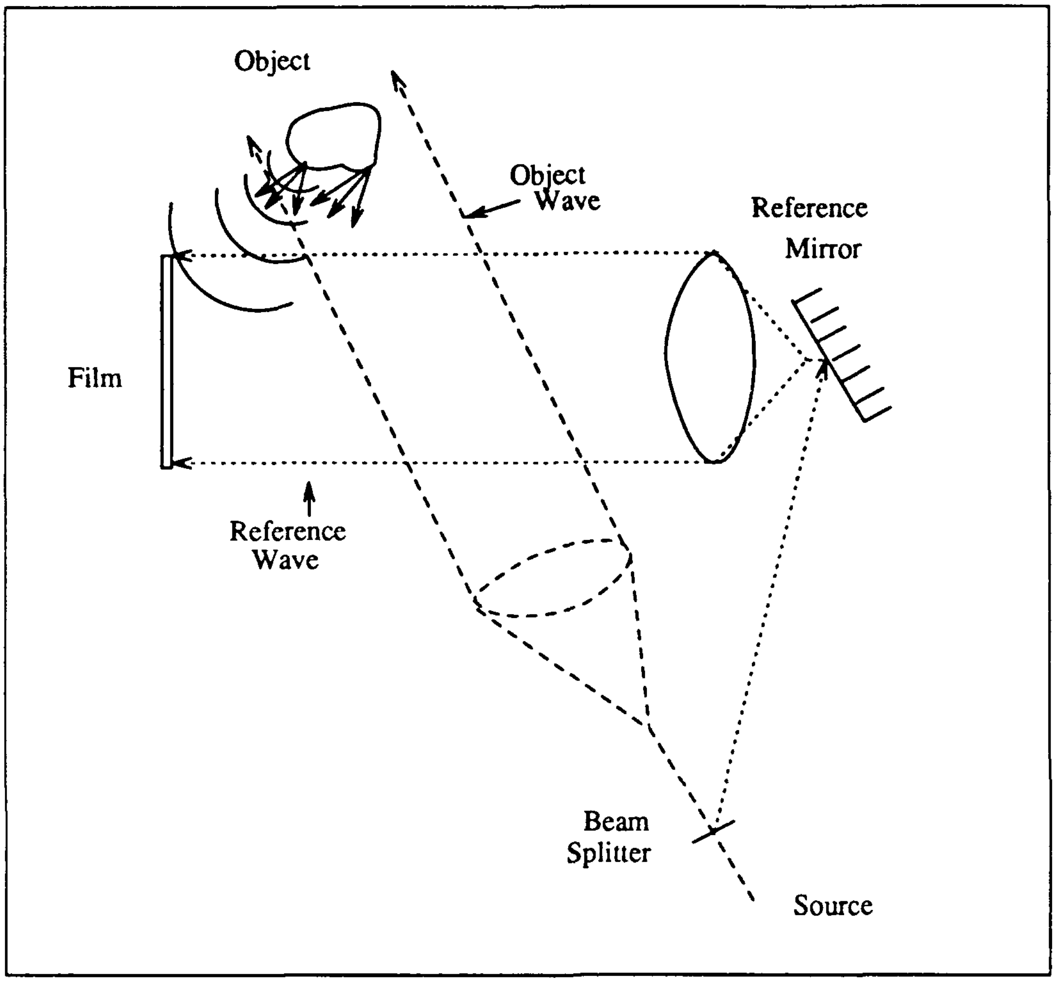

Figure 1.1. Optical Holography Image Recording

source. The *virtual* image can be observed by looking though the hologram in the direction of the light source. The *real* image is observable by looking in the direction of the hologram and light source or by capturing with a sheet of paper a single image plane of the object. It is worthwhile noting that all planes of the image are observable in the *real* image, but only one plane can be focused on the paper at a time. The image is visible because light diffracted by the interference pattern is behaving just as it would if light were reflected by the original object(12:1-2).

Computer generated holography has an advantage over optical holography in that the object being recorded need not really exist. A three dimensional image can

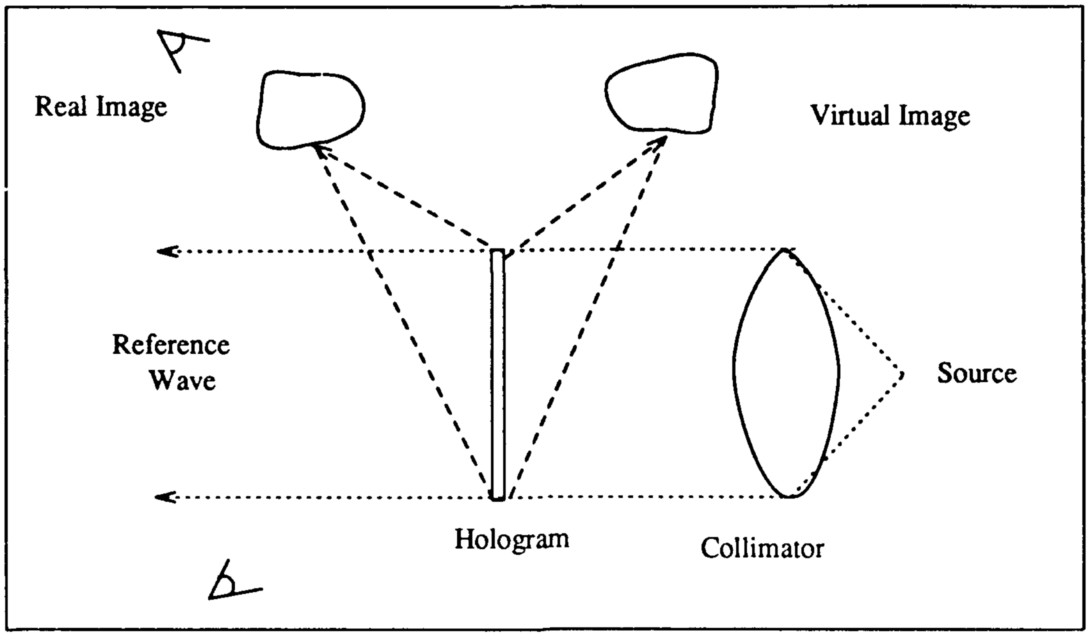

Figure 1.2. Optical Holography Image Reconstruction

be created from a mathematical description of an object using this process(26:1). Computer generated holography is a complex process that requires the mathematical modeling of light waves as they interact with physical objects. The approaches used to date in computer generated holography have required extensive computer processing time and excessive amounts of data storage(15:673). The process of computer generated holography consists of defining an object, producing a holographic interference pattern, recording the interference pattern, reducing the interference pattern, and reconstructing the image. Figure 1.3 depicts the computer generated holography process.

The most direct approach to model optical holography is to calculate the intensity distribution for all locations of the film resulting from the combination of the object and reference beams. The intensity distribution of the film is a combination of amplitude and phase at each location in the recording plane. The amplitude and

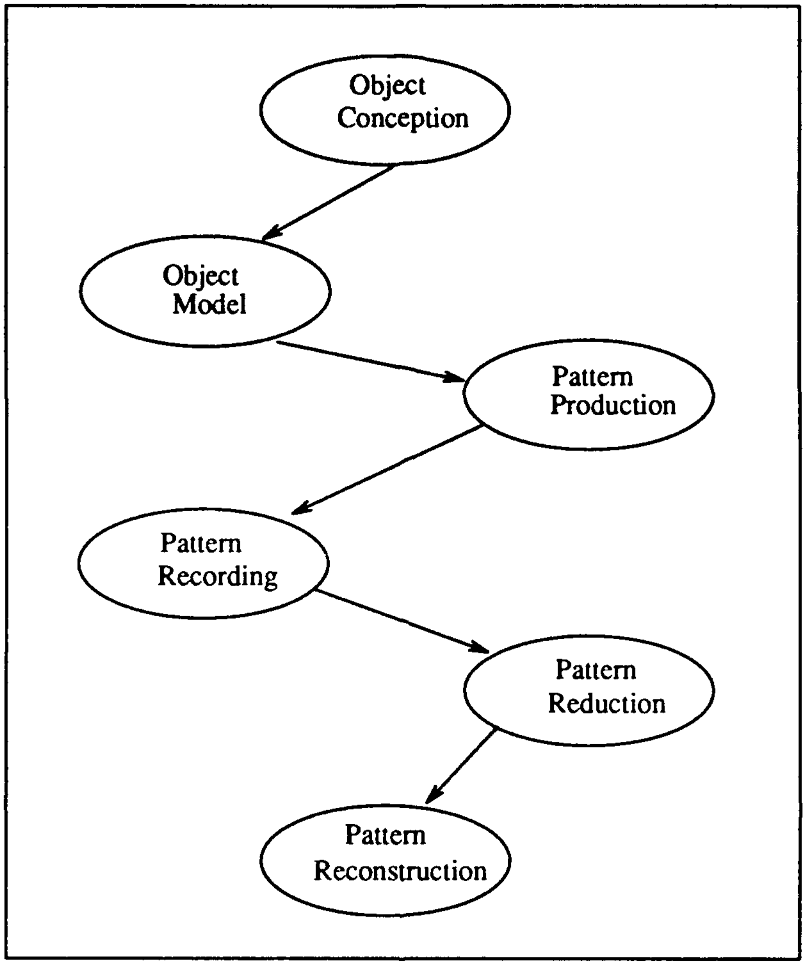

Figure 1.3. Computer Generated Holography Pipeline

phase distribution for each location is given by

$$I_f(x,y) = A_c(x,y)\, e^{-i\phi_c(x,y)} \tag{1.1}$$

where $I_f(x,y)$ represents the complex field at each location, $A_c(x,y)$ is the combined amplitude of the object and reference beams at each location, and $e^{-i\phi_c(x,y)}$ is the combined phase of the object and reference beams. The amplitude of a light beam is a real number denoted by A and its phase is a complex number given by $e^{-i\phi\lambda t}$, where λ represents the wave length of the light rays and t is time. In other words, the phase of a light wave is a function of its wavelength, time, and x and y.

Conceptually, objects can be described as a collection of modeling primitives such as points, lines, polygons, or surfaces. These primitives could then be defined as a set of locations in a three dimensional coordinate system at some distance from the recording plane to represent the placement of an object relative to the recording surface. Although any modeling primitive can be used to define objects, points have been used extensively in computer generated holography. This is because the use of points, or resolution points, as modeling primitives greatly simplifies the calculations of the phase and amplitude contributions from every location on the object to every location on the recording surface. Christian Huygens in 1678 proposed a principal that every point on a light wave front can be considered as a source of secondary waveletes. An extension of Huygens principal states that each resolution point (or surface element) is capable of deflecting incoming light rays and will become, in effect, a new and distinct point source emitter of spherical light waves(11:30-31). Some of these new light waves will reach the surface of the film and interfere with the light waves of the reference beam. However, because the surfaces of real world objects are continuous, exact mathematical modeling would require that objects be defined by an infinite number of points, or resolution points. A major problem in computer synthesis of the optical holography process is the large number of calculations and storage required to calculate the interference from an infinite (or extremely large) number of resolution points at each location of the recording plane. That is

$$\sum_{i=1}^{\infty}\sum_{j=1}^{x}\sum_{k=1}^{y} A_i(j,k)\, e^{-i\phi_i(j,k)} + A_R(j,k)\, e^{-i\phi_R(j,k)} \tag{1.2}$$

where the i represents the object resolution points and j and k are the column and row locations of the recording plane (film).

The purpose of this research was to find methods to decrease the processing and storage requirements of computer generated holography in an attempt to satisfy the question, *is computer generated holography a suitable output medium for three* *dimensional computer graphics?* This question was much too broad to be completely resolved in a single research cycle; however, this research did work toward that goal by focusing on reducing the computational complexity and storage requirements involved in the process. My research did not exhaustively survey or evaluate the major steps in the computer generated holography process. Instead, it was limited to an evaluation of a few potential approaches in each area.

Computer generated holography for three dimensional display has two major parameters: the computer resources necessary to produce an interference pattern and the quality of the resulting image. Image quality, to some extent, is inversely related to the amount of computer resources required to produce the interference pattern. Some of the major problems with computer generated holography as a three dimensional display medium is the time it takes to produce an image, the memory requirements to store the image, and the poor quality of the image. To be effective, the image quality must be improved, and to be efficient, the computer resource requirements must be reduced. In order to effectively balance this relationship, it is necessary to objectively evaluate the effects of algorithmic modifications on on image quality and computer resources. Computer processing times and memory storage requirements are easily evaluated, but image quality is a bit more difficult to objectively judge. An ideal situation would be to have a metric or set of metrics to objectively evaluate the quality of a resulting holographic image. There are, however, no such metrics currently available. This forced the use of such vague and subjective descriptors as *reasonable* and *good enough* to characterize image quality through out this research. Clearly, this is not an idealized approach to research. Although computer generated holography is a complex process which has many facets that could benefit from improved methods, it offers great promise to computer graphics as a true three dimensional display medium.

1.3 Problem Statement

It is possible to create a computer generated hologram that will faithfully reproduce three dimensional objects in a *reasonable* amount of time on a general purpose workstation. This research focused on the following questions. Can the number of calculations and amount of data storage to produce a computer generated hologram be reduced so that a hologram can be produced on a general purpose computer in a matter of days that will result in a three dimensional image with reasonable image quality? Can a computer generated hologram be produced using standard computer printers and photoreduction techniques that will allow the reconstructed three dimensional image to be visible to unaided eyes?

1.4 Approach

The primary goal of this research was to produce computer generated holograms of three dimensional objects without using extreme amounts of computer processing time and data storage. The foundation for this research was the work of Captain Tom Mouser. Captain Mouser developed an initial computer generated holography capability at AFIT using a form of equation (2). He modeled two faces of a 50,000 by 50,000 by 50,000 micron cube with 55 points at a distance of 50,000 microns from the recording plane. The calculations for the holographic interference pattern of this object required 59.5 hours of computer processor time on a supercomputer (Cray II), and 1.3 billion bytes of computer storage. The interference pattern was transferred to a glass plate using electron beam lithography thus eliminating the reduction step (18:28).

Captain Mouser's work was used as a baseline from which various approaches and methods were explored with the goal of reducing the computational complexity and computer storage requirements while maintaining acceptable image quality. The steps taken in this research consisted of exploring methods of calculating a holographic interference pattern, transferring the pattern to an output medium, and reducing the pattern so that it was able to diffract light. Image reconstruction was achieved using a 20 mWatt Helium-Neon gas laser. By limiting the size of the hologram and the size of the object and its distance from the recording surface, the processing requirements were relaxed so that the patterns could be produced on a computer workstation and output on a 300 dot per inch laser printer. Memory requirements were reduced by limiting the size of the hologram and by breaking it into cells which could be independently processed. Additional processing savings were achieved by assuming a constant amplitude for all points on the object and by arbitrarily assigning a phase of zero to the reference beam at the recording surface. Various photoreduction techniques were used to reduce the patterns. Other areas explored were varying the number of object resolutions points, object placement relative to the recording surface, object hidden surface issues, three dimensional perspective views, and additional processing and memory reduction techniques.

Although unbiased image quality metrics do not exist, the relative quality and three dimensionality of each image, as well as the computer processing time and storage required to produce each hologram were used to evaluate each method. The major focal points of this research included a review of computer generated holography literature to establish the current knowledge baseline; interviews with experts in the fields of optical engineering, physics, and computer graphics to solidify the theoretical approach and crystallize the direction; the development of a computer program library to generate interference patterns, control the laser printers, and determine object resolution point placement; and the evaluation of relative image quality balanced with computer processing and storage requirements.

1.5 Scope

Much research has been done in the field of computer generated holography; however, very little work has been done using holography to produce three dimensional images. This research will not answer all of the questions that need to be addressed to satisfy the issue of whether computer generated holography is a viable method for three dimensional computer graphics output. This research focused on only a few methods to calculate and record the interference pattern with the objective of reducing the processing and storage requirements.

There are many approaches that can be taken to model light and produce an interference pattern. The approach taken in the calculations of the interference pattern was to consider light to be propagating to the hologram plane from a series of points used to define an object (object resolution points). Only photoreduction techniques were considered to record the pattern. That is, the interference pattern was recorded on paper using a laser printer and then photographically reduced to diffract light for image reconstruction.

The effect of this research was to establish the capability of generating three dimensional computer generated holographic images with general purpose and readily available computer equipment. Although it appears likely that an improvement in processing time would be possible using a parallel processing approach, it was not explored in this research and a serial processing model was assumed in all algorithms. Absolute minimums were not established for processing times, storage requirements, or spatial resolutions of the holograms. Simple objects with relatively smooth surfaces that could be defined with few resolution points were used in these experiments. The effects of using more complex objects on processing times and storage requirements were not evaluated.

1.6 Equipment Required

The software developed for this research was written in the C computer programming language and implemented on Sun Microsystems Sun 4 and Silicon Graphics Iris 4D workstations. The printer used to record the interference pattern was a 300 dot per inch Apple LaserWriter II^{tm} using the Postscript language. The photoreduction of the laser printer output was accomplished using the NCR $Ultrafiche^{tm}$

process. In addition, standard photoreduction was used with Kodak Technical Pan film and a Nikon F3 35 millimeter camera with a 200 millimeter telephoto lens. A Spectra-Physics INC model 106-1 helium-neon gas laser was used in the reconstruction of the holographic images.

II. Current Knowledge

2.1 Introduction

This chapter provides an introduction to holography through a general review of its history, methods, types, and applications with a particular slant towards computer generated holography. In addition, some of the current work in computer generated holography for three dimensional image display is summarized. The motivation for this review was twofold: to provide a theoretical foundation upon which to base this research and to provide a research focus. The results and conclusions from previous work in these areas were reassessed in light of the current advances in computer technology and computer generated holography methodology as a part of this review. A generally accepted impression of computer generated holography as a display medium for three dimensional images is that it is not very useful because of the time required to produce an image and the relatively poor quality of the images compared to current imaging techniques (8, 22, 12:550,343,146-161).

2.2 History

In 1948 Denis Gabor formed the first hologram in an attempt to reduce the aberrations of electron microscopes (23:1). The Gabor, or *in-line*, holography process requires the object being recorded to be a transparency with opaque details. A coherent monochromatic light wave is directed through the transparency at a photographic plate. The opaque areas of the transparency scatter a portion of the light waves and the transparent portion of the object allows the remaining light waves to pass through to the photographic plate. The portion of the light wave that passes through the object is the reference beam, while the scattered light forms a coherent secondary wave, the object beam. The object beam and the reference beam combine at the surface of the photographic plate and interfere with each other. This interference is then recorded on the film. Illuminating the developed film with a

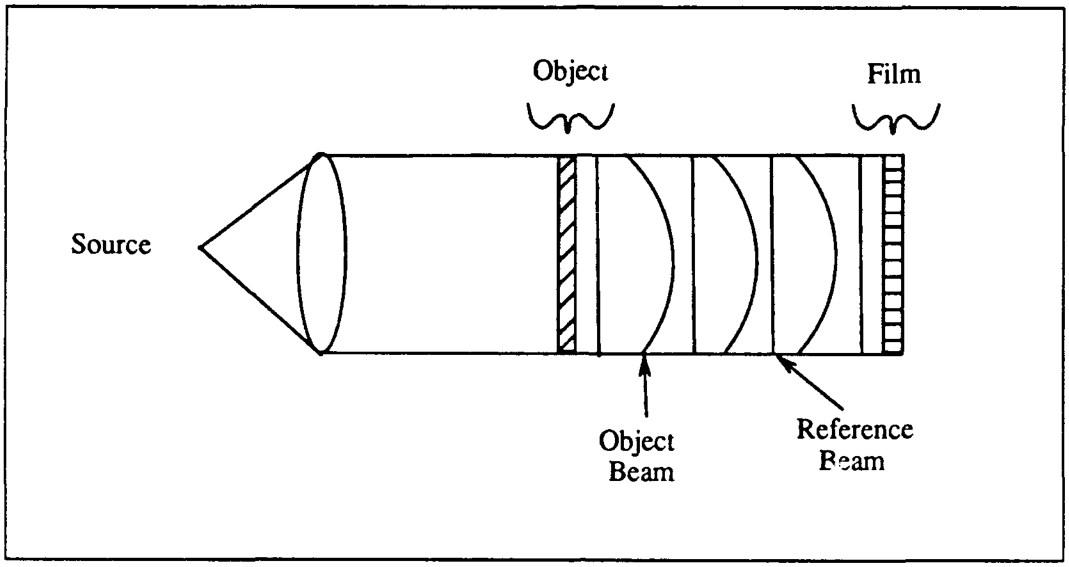

Figure 2.1. Gabor Holography Recording Process

coherent light source will produce twin images of the original object on the same axis as the light source. These images can be viewed by looking through the film at the light source (9). Figure 2.1 depicts the Gabor holography recording process and figure 2.2 illustrates the Gabor holography image reconstruction process. One of the problems with the Gabor holography process is that the object is restricted to a two dimensional transparency. Another, and perhaps the largest, problem is that two inseparable images are produced (11:208).

Leith and Upatnieks modified Gabor's approach by using an off-axis reference beam (14). Figure 2.3 depicts the Leith and Upatnieks holographic recording method. The major advantages of the Leith-Upatnieks, or *off-set reference*, hologram are that the object is not restricted to a two dimensional transparency due to a separate and distinct reference beam, and that the process produces *real* and *virtual* images that do not lie on the same axis (11:208).

In 1966 Brown and Lohmann first developed the concept of a computer generated hologram. They modeled light using Fraunhofer diffraction theory to produce the phase and amplitude information of the refracted light from the object at the

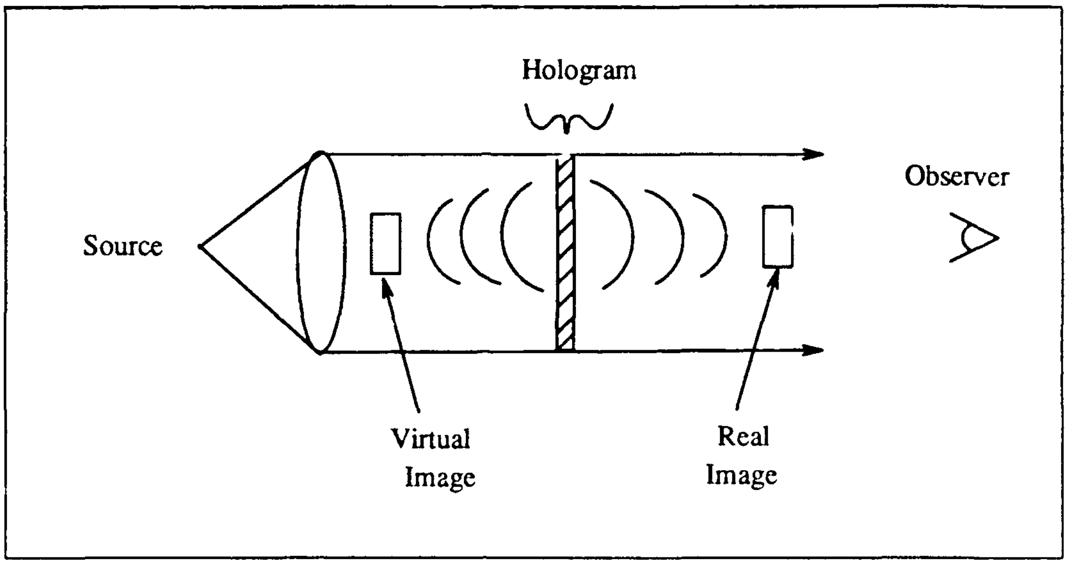

Figure 2.2. Gabor Holography Image Reconstruction Process

hologram plane (5:967-968). Waters was the first to publish the reconstruction of a three dimensional image using a computer generated hologram. Using Fresnel diffraction theory, he produced a binary hologram of a three dimensional tetrahedron modeled as a collection of points (24). Refer to Chapter 3 for more information on diffraction and interference.

Several techniques for computer generated holography developed following Brown and Lohmann's discovery. Although the early methods differed in the mathematical techniques used to compute holograms, they all faced the common problem of outputting the patterns. Brown and Lohmann initially drew their patterns by hand (5), but most of the other early methods (and eventually Brown and Lohmann's approach as well) used mechanical plotters. Other approaches used photographic plotters. These outputs (hand and plotter generated) were photographically reduced to a resolution capable of diffracting light waves (15). Electron beam lithography is currently the method of choice for recording the interference patterns. Using electron beam lithograph equipment, the interference pattern is etched directly on a glass plate at the proper resolution thus eliminating the need for further reduction

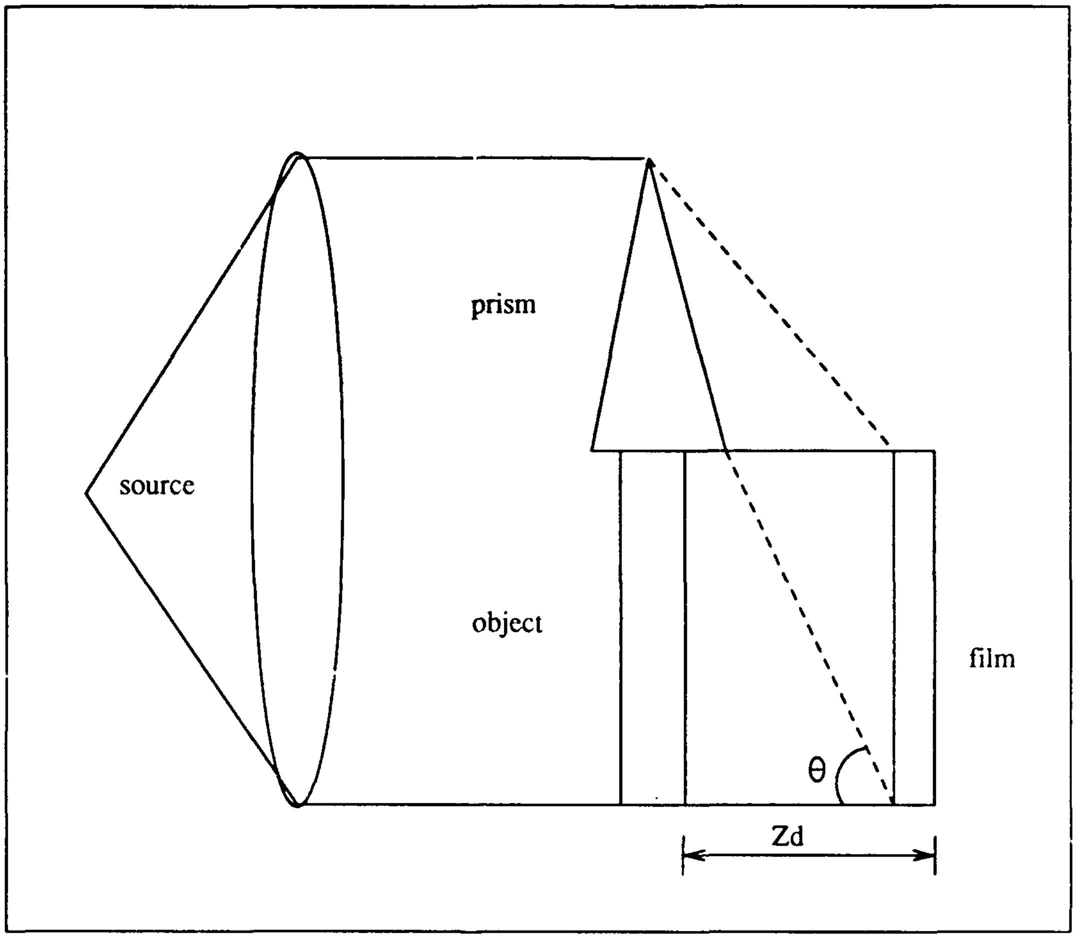

Figure 2.3. Leith-Upatnieks Holography Recording Process

(2:805-806).

2.3 Methods

The process of computer generated holography for three dimensional display involves an object that is usually defined by a series of points in a three dimensional cartesian coordinate system. These resolution points are considered to be light sources that radiate light waves toward a hologram. The hologram occupies a single plane at some finite distance from the object. In optical holography the hologram plane is a set of storage locations (film). In computer generated holography, the hologram plane is modeled as a two dimensional matrix of storage locations.

Increasing the number of storage locations in a hologram will increase the viewing dimensions of the reconstructed image, but will also increase the computer processing and storage requirements. The interference pattern resulting from the light from a single planar object and a reference beam can be quickly calculated using Fourier analysis methods. Consequently, the majority of computer generated holograms that have been created have used single plane (two dimensional) objects (15:673). The phase and intensity of light waves emanating from each of the resolution points are calculated at each location in the hologram. The hologram, then, represents a summation of the amplitude and phase of each of the resolution points combined with the amplitude and phase from the reference beam for all locations on the hologram.

In the most general sense, holograms are categorized into two major groups: those that require a coherent monochromatic light source for image reconstruction, and those that will reconstruct images using normal white light. Additionally, holograms are grouped into types based on the techniques used to create them. For example, holograms constructed using the Gabor process are categorized as *Gabor, or in-line, holograms*, while holograms constructed using the Leith-Upatnieks process are categorized as *Leith-Upatnieks, or off-axis reference, holograms*. Other types of holograms are reflection, phase, image plane, slit transfer (rainbow), achromatic transfer, Fourier, stereograms, achromatic transfer, alcove, and interferograms. An exception to grouping based on creation techniques is the category of transmission holograms which are so named because the reconstruction beam transmits the *real* image out of the hologram. All holograms capable of producing a real image are transmission holograms (22:40-59).

Of particular interest in computer generated holography are Fraunhöfer, Fresnel, and Fourier holograms. They are categorized based on assumptions concerning the object size and location relative to the recording surface. For example, a *Fraunhöfer*, or *far-field*, hologram is constructed by placing the object very far from the recording surface. A Fresnel, or *near-field*, hologram is constructed with the object close to the recording surface. If the size of the object is limited relative to the size of the hologram, and the distance from the recording surface is restricted, then a Fourier hologram can be constructed (11:61-87). Computer generated holography has at least five categories of techniques for modeling the holography process.

2.3.1 Categories. Brown and Lohmann called their computer generated hologram a *detour phase* hologram because of the manner that light waves were deflected, or *detoured*, by the grating structure of the hologram. The detouring changed the phase of the light wave. The hologram was represented as a set of equally sized cells. Each cell contained an aperture with variable width or height. They computed the Fourier transform of the pattern they were modeling, (the letter E) and represented the transform values as a set of transparent slits on an opaque background. The size and lateral positions of the slits were dependent on the transform's magnitude and phase at the center of each cell. The hologram was produced on a plotter and then photographically reduced (5).

Burch noticed that the amplitude, the $A_c(x, y)$ term in equation (1.1), did not contribute to the reconstruction of the image from the hologram and therefore did not need to be calculated. This insight reduced the spatial frequency content of the hologram and reduced the processing complexity. Burch also used the Fourier transform process to generate his modified off-axis hologram (6).

Lessem, Hirsch, and Jordan developed the concept of a kinoform. The kinoform is a computer generated wavefront reconstruction device, *hologram*, that records only the phase information from the object. It has a higher diffraction efficiency than conventionally produced computer generated holograms because all the light from the reconstruction beam is used to produce an image. The phase of the object wavefront is recorded at the hologram surface using Fourier transform procedures. The computed phase values are gray scaled and photographically reduced. The reduced hologram is bleached to create a transparency with areas of varying amounts of surface relief. (16).

2.3.2 Applications. Until 1966 when the first computer generated hologram was developed, holography was a process used entirely by optical engineers and physicists (5:967). Computer generated holography techniques have been used in many applications since that time. Some of the applications of computer generated holography include laser optics, pattern recognition, neural networks, optical computing, signal processing, and imaging (23:4355-4356).

2.3.2.1 Three Dimensional Displays. The problems with computer generated holography for three dimensional displays are considerable. Computer based three dimensional display holograms require excessive amounts of computing resources. Three dimensional objects have surface areas that can be hidden from the hologram by other areas of its surface. These hidden surfaces, must be taken into account when calculating the interference pattern. The full three dimensional nature of a reconstructed image can be difficult to display. In spite of these difficulties, there has been several reports of three dimensional displays that were produced with computer generated holography. In these approaches, three dimensional objects were considered as a set of many two dimensional cross sections. The contributions from each of cross section were calculated at the hologram plane. The results from all the cross section were then summed to get the hologram transmittance of the entire object (27:2722).

In 1968, Lesem and Hirsch created a three dimensional hologram consisting of two million locations using a Fourier transform process. This hologram took about thirty minutes to calculate on a general purpose computer. The reconstructed image was so small that a microscope was necessary to see it (15:673).

Yatagai developed a composite hologram for three dimensional image reconstruction using holographic stereogram techniques. A sequence of perspective projections (two dimensional views) were produced of a three dimensional object. The interference pattern of each of these views were calculated using a Fourier transform process. The interference patterns from each of these views were then processed in the order of the viewpoints (for example, from left to right). And finally, the composite hologram was photographically reduced for image reconstruction (27).

Benton developed an alternate approach to create three dimensional images with computer generated holography (3). He called his process synthetic holographic stereograms. This process uses a series of two dimensional viewpoints of an object which are overlapped onto a sheet of film using standard computer graphic and computer generated holographic techniques. The collection of these perspectives results in a three dimensional image that allows the viewer to see around the object by moving to different viewing locations.

In 1989, Captain Mouser explored the concept of producing a computer generated hologram using the basic physics of light to produce an interference pattern. He modeled a three dimensional cube using 55 resolution points located 50,000 microns from the recording surface. The recording surface was modeled with 1,296,000,000 storage locations. Captain Mouser's calculations required 59.5 hours on a special purpose computer (Cray 2) and 1.3 billion bytes of computer storage. Captain Mouser avoided the reduction step by using electron beam lithography to directly record his interference pattern on a quartz plate at a resolution of about one half of a micron separation (18:6).

2.4 Conclusion

The research indicates that it is possible to use computer generated holography to create images of three dimensional objects. But there are at least three major areas that need to be addressed in order for this technology to be a viable medium for three dimensional display: the holographic image quality needs to be improved, the computing resource requirements (processing time and memory storage) must be reduced, and more accessible methods to record the pattern without excessively limiting spatial frequencies must be identified. Image quality is a function of the number of points used to model the geometry (using the point geometry model), the placement of the object relative to the recording surface, and the size of the recording surface. As the object is moved closer to the recording surface, the spacing of the interference pattern fringes becomes smaller. This necessitates a higher spatial frequency output device. Increasing the number of points and size of the recording surface improves the image quality and parallax. However, these improvements will increase the computer processor time required to calculate the pattern, the computer memory space necessary to store the data, and the resolution requirements of the output device to record the pattern.

A significant improvement in the effectiveness of computer generated holography as a display medium for three dimensional computer graphics could be achieved if the process of calculating the interference pattern could be accomplished with less processor time using more readily available, general purpose computers. Towards this goal, it appears that limiting the number of locations in the hologram dramatically decreases the time required to compute the interference pattern. However, this decreases the viewing angle of the reconstructed image and thereby limits the three dimensional perspective of the image. The trade-off between processing time and viewing angle needs to be established experimentally, so that a hologram can be produced in a reasonable amount of time and viewed with unaided eyes. Another factor that drives the processing requirements is the large number of calculations necessary to compute an interference pattern. For example, a hologram with 350,000 locations of a simple object defined by 30 resolution points computed from equation 1.2 would require over 130,000,000 calculations. If another approach could be identified or an existing approach modified that would eliminate or simplify some of the calculations, then the processing time could be greatly reduced.

The recording step in the computer generated holography process is another likely area for investigation. Although the literature indicates that electron beam lithography is the method of choice for recording holographic interference patterns, there are only a few locations in the United States with this expensive equipment. Wright-Patterson Air Force Base is fortunate enough to be one of the few sites in the country that has this equipment. However, the demand for the use of this equipment is considerable and processing time on it is difficult to get. With the advances in laser printer resolution, photoreduction of the interference pattern may be as effective as electron beam lithography for recording the interference pattern. Certainly, it would be a much more accessible method.

III. Design Methodologies

3.1 System Decision Criteria

Conceptually, a hologram is like a window consisting of many smaller panes. An observer looking through a window can see the contents of a room. But if all of the panes are covered except one, then only part of the room is visible. Each cell of a hologram represents a perspective of the object under consideration much like a window pane represents a perspective of the contents of a room. Increasing the number of cells in the hologram (assuming all the cells are of equal size) increases the viewing angle of the hologram. This increase allows the hologram to *see* more surface area of an object, which provides a better three dimensional view of that object. However, increasing the number of cells in a hologram also increases the computational time and storage requirements necessary to produce the interference pattern. Refer to figure 3.1 for an illustration of this process.

Finding an adequate balance between the hologram size and the computer resources required to produce the interference pattern was just one problem to address in this research. Other areas to be addressed were:

1. Determining the recording medium for the patterns.
2. Selecting the pattern reduction process.
3. Determining which laser to be used for image reconstruction.
4. Selecting the interference calculation algorithm.
5. Determining the type and size of objects to model.
6. Selecting the class of computers to perform the interference calculations.

A preliminary decision which influenced most of the remaining decisions throughout this research was to opt for generally accessible equipment and processes where

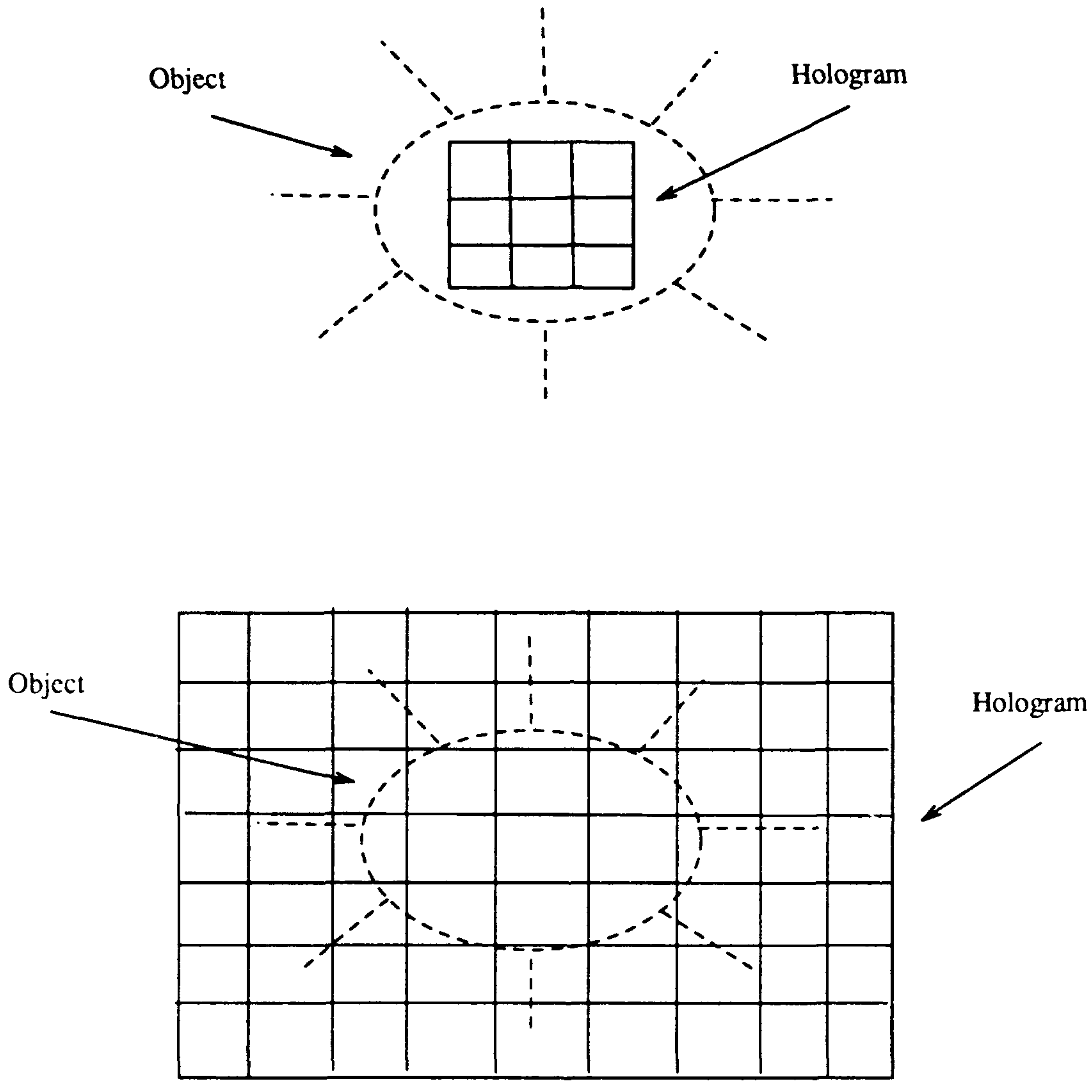

Figure 3.1. Perspective Effects of Increasing the Hologram Size

possible. That is, the target computers for the processing were general purpose workstations (Sun Microsystem Sun4s and Silicon Graphics SGI 4Ds). In addition, photographic reduction of plotted interference patterns was selected in lieu of electron beam lithography. Targeting the system for generally accessible equipment provided a more adaptable environment for experimentation by reducing the turn-around time for each trial. If computer generated holograms could be produced with a class of computers that are generally available and if the interference patterns could be recorded and reduced using more accessible methods, then this work could provide a broader base for further research.

A cornerstone of this research was the evaluation of the components in the computer generated holography process to assess the impact of processing and storage reduction efforts on image quality. A system that encapsulated the entire computer generated holography process was needed. This system would take a mathematical model of an object and generate a hologram that would create the wavefront that would be generated from light deflected from the modeled object, if that object were to exist. In the reconstructed wavefront the modeled object would be visible. To provide flexibility for experimentation, the system should consist of a set of modules that could be swapped or modified. Therefore, the process of generating an image with computer generated holography was abstracted into a system consisting of the following steps:

1. Model an object as a set of points in a three dimensional coordinate system.
2. Model the hologram as a matrix of storage locations at the origin of the coordinate system.
3. Compute the interference pattern of the object at the hologram plane.
4. Record the interference pattern on paper as a binary plot.
5. Photographically reduce the plot to a scale capable of diffracting light.
6. Illuminate the photograph negative with a collimated laser beam for image reconstruction.

This system was a computer generated holography pipeline in that an object model entered at one end and an image emerged at the other. The plan was to develop this pipeline and test it with models of very simple objects, such as an object consisting of a single point or two dimensional objects defined by only a few points. This testing served as a verification of the computer generated holography paradigm. In addition, testing of the system established the operational limits of the system's components. The operational limits focused on during the testing of the pipeline were:

1. Sampling rates:
   (a) Object sampling.
   (b) Printer sampling.
   (c) Hologram sampling.
2. Photographic parameters:
   (a) Exposure rates.
   (b) Lighting contrast levels.
   (c) Image focusing.
   (d) Emulsion storage requirements.
3. Computer resource requirements:
   (a) Processing time.
   (b) Internal memory size.
   (c) Off-line storage size.
4. Reconstructed image quality:
   (a) Image size.
   (b) Image clarity.
   (c) Image three dimensionality.
   (d) Variance between model and image.

3.2 Spatial Resolution Considerations

The spatial resolution of a computer generated hologram is limited by the resolution of the output device used to record the interference pattern. Before considering the spatial resolution requirements of a hologram, and consequently the resolution requirements of the output device used to record the interference pattern, the characteristics of light must be considered.

3.2.1 Sinusoidal Waveforms. The nature of light is not a trivial item to define. Through the years many theories have tried to explain it. For the purposes of this research, light will be considered as traveling, transverse, electromagnetic waves. As such, the Huygens wave model and Maxwell's electromagnetic model can be used to predict the behavior of light waves (22:21). Light waves are commonly described in terms of sinusoidal waveforms. Sinusoidal waves have three major components: amplitude, frequency, and phase. Amplitude is a measure of the height and depth of the wave. Figure 3.2 depicts two waves with amplitudes of one and two units, respectively. The wavelength of a sinusoidal wave consists of a complete cycle of

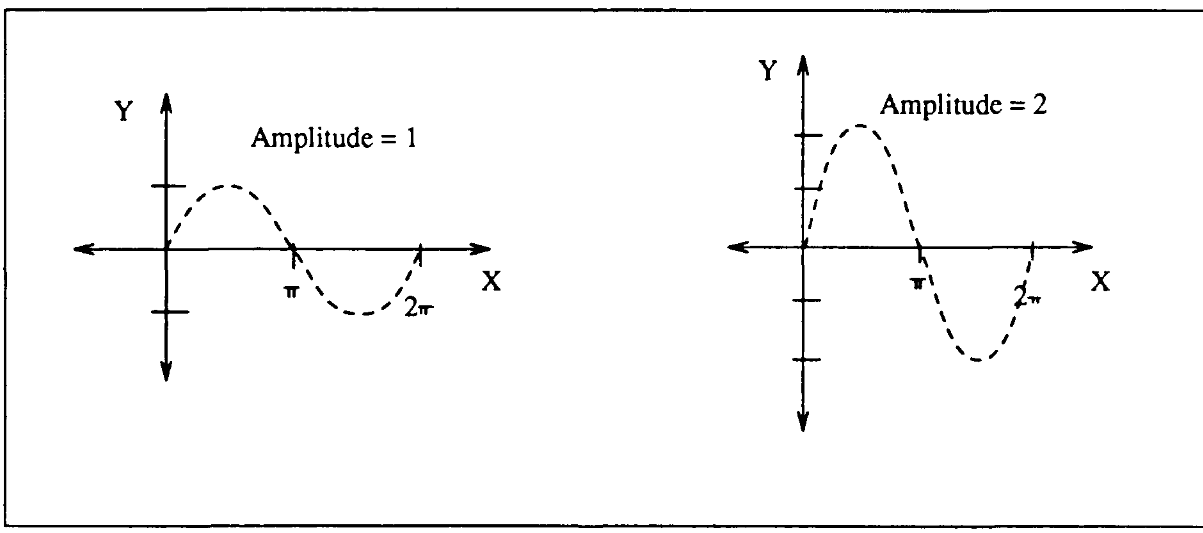

Figure 3.2. Amplitude Relationships

the wave through all its positive and negative amplitude values. The frequency of a sinusoidal wave is a measure of the number of cycles of the wave in a unit of measurement. Figure 3.3 contrasts two waves, one with twice the frequency of the other. A sinusoidal wave also has an attribute of phase. The phase of a wave is

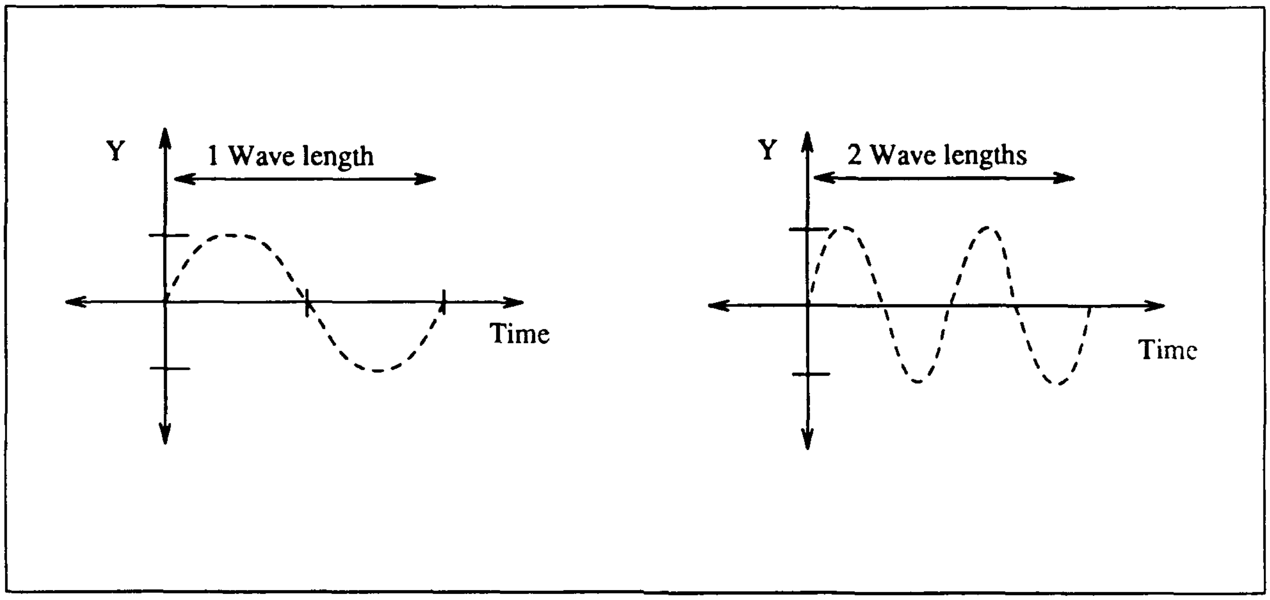

Figure 3.3. Frequency Relationships

an angle that specifies a relative position on the wave. It is normally specified in radians. For example, a phase of zero radians indicates the beginning of a wave cycle and a phase of 2 π radians is the end of a wave cycle. Phase is a convenient term to compare relative positions of waves and to indicate a wave's cycle position with respect to time (or distance). Figure 3.4 illustrates two waves that are 180 degrees (or π radians) different in phase. Encoding the phase of light is the most important characteristic of holography that differentiates it from photography. A beam of light is a collection of sinusoidal waves. If the light waves emanate from a single point source then they are spatially coherent. If the light waves have the same phase and temporal frequency, they are temporally coherent. If adjacent waves in a beam of light were connected with a line, then the beam could be represented as a series of parallel lines perpendicular to its source. If the lines connecting the waves were at the points of maximum amplitude on each wave, then the set of parallel lines

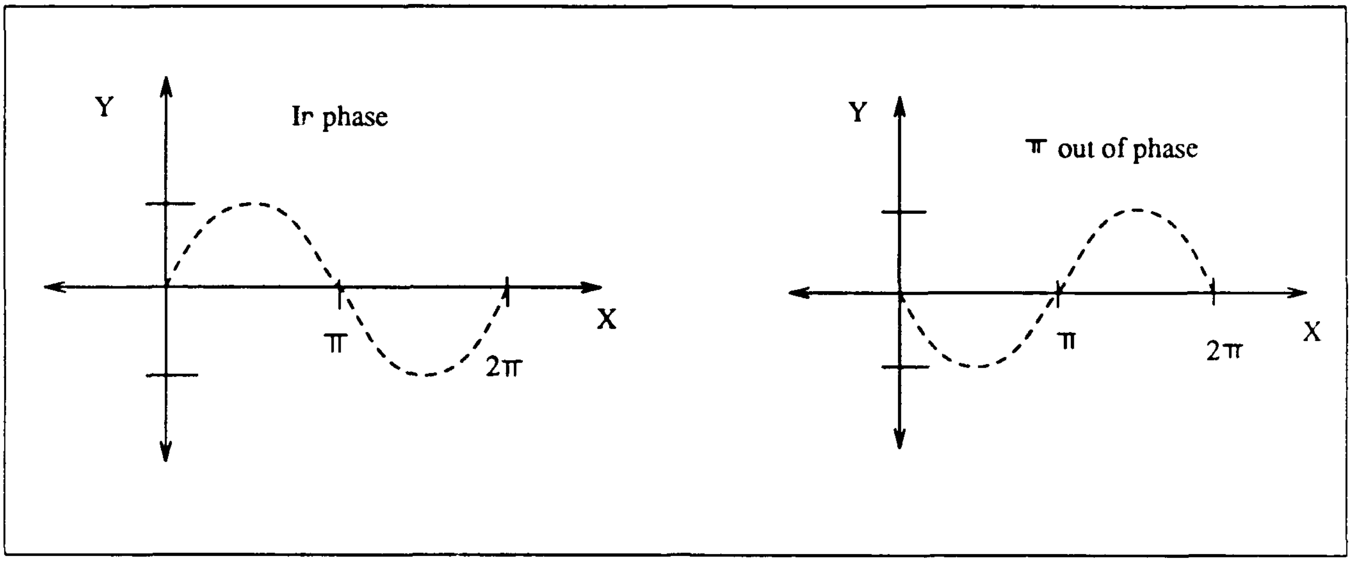

Figure 3.4. Phase Relationships

representing a beam of light would be separated by one half wavelength of light and would indicate regions of maximum amplitude of a traveling wavefront. Because the regions of maximum amplitude of a wave occur at the relative phase angles of $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ radians, these lines would also indicate the phase of the light waves. A single point source of light emits waves in all directions. Figure 3.5 illustrates how lines connecting relative phase angles of $\frac{\pi}{2}$ radians on waves emitted from a single point would appear two dimensionally as a set of concentric circles. In this figure each circle represents one wavelength of the light emitted from a single point source. These lines could be thought of as wavefronts traveling outward from their source. Wavefronts of this nature are referred to as spherical waves. As the spherical wavefronts travel from their source, their diameters increase. At a large distance from the source, segments of the spherical waves appear as a set of parallel straight lines. Wavefronts of this nature are referred to as plane waves, and can be thought of as emanating from a single point source very far away. Figure 3.6 depicts a plane wave as the reference beam and a spherical wave emanating from an object point meeting at the surface of a hologram.

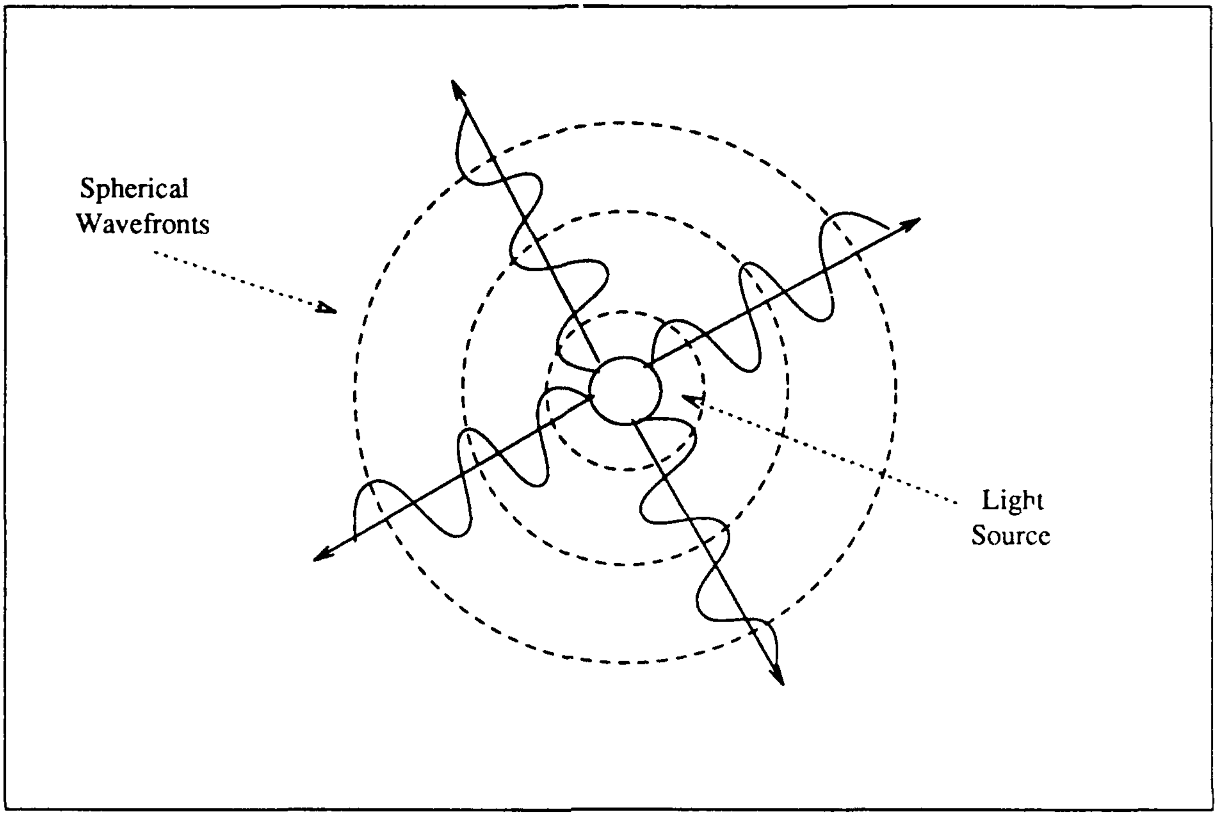

Figure 3.5. Point Source Emitting Spherical Wavefronts

3.2.2 Interference Grating Structures. A grating structure is a series of alternating light and dark bands. In optical holography, the interference between light waves at the recording surface produces a grating structure. Light wave interference can be considered as a summation process involving each wave's amplitude. This interference can be constructive or destructive. Constructive interference occurs when waves combine and produce a net increase in amplitude. Destructive interference occurs when waves combine and produce a net decrease in amplitude. Figure 3.7 illustrates constructive and destructive interference between two waves. In effect, the amplitudes of waves are added together when constructive interference occurs and cancel each other out when destructive interference occurs.

Figure 3.7 indicates light waves with equal phase angles for constructive interference and light waves phase angles differing by π radians for destructive in-

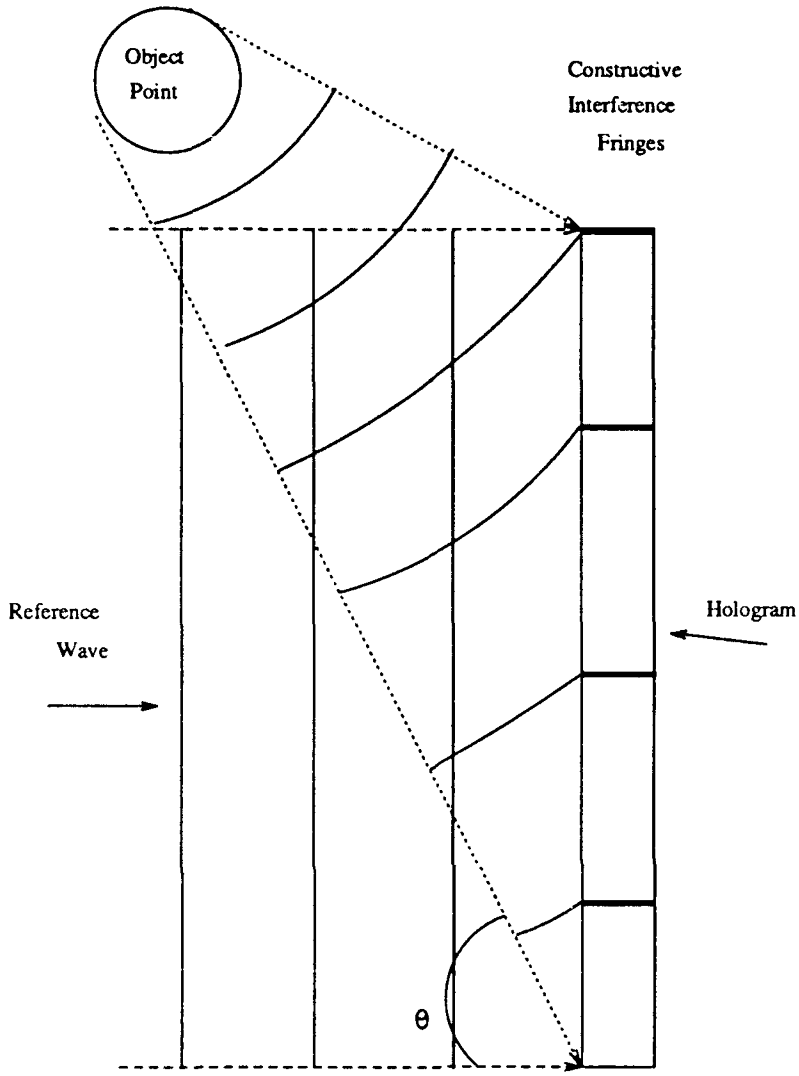

Figure 3.6. Recording Light Interference

terference. This example represents only extremes of constructive and destructive interference. Interference can also occur at intermediate angles of phase. In optical holography when light interference occurs at the hologram plane, it is recorded by the film. Film can only record light intensity. Because light intensity is proportional to the square of the amplitude of light waves, only the amplitudes of the combined wavefronts affect the recording (13:44). A hologram can be thought of as a recording of the various levels of intensity (ranging from zero to maximum intensity) that results from the interference of light waves.

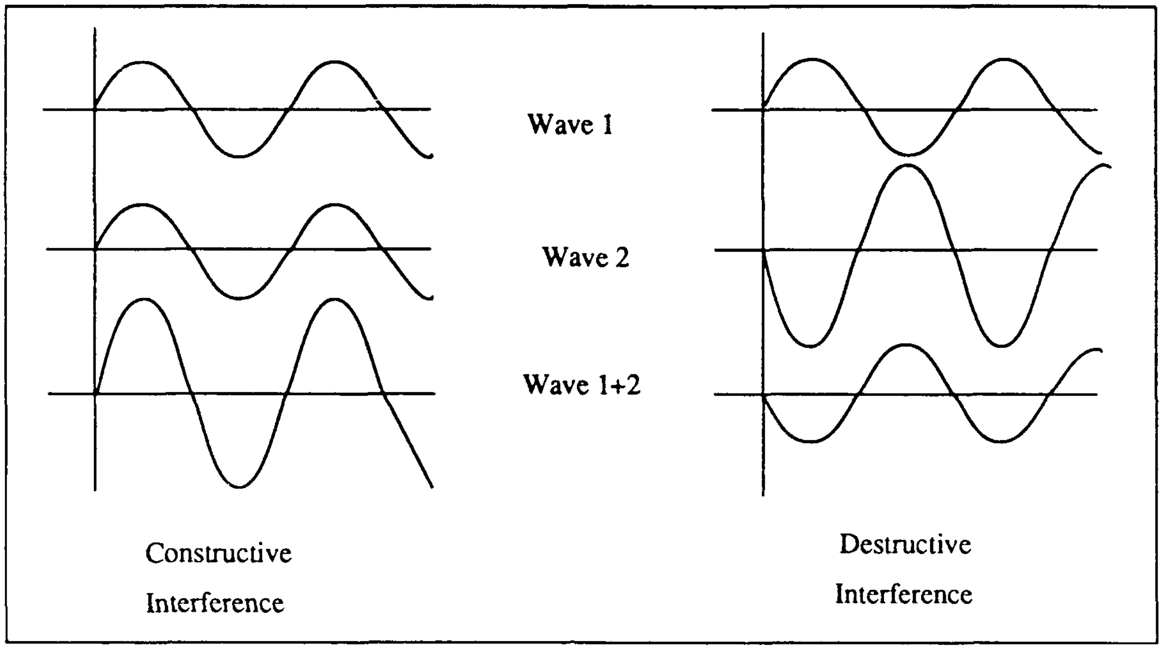

Figure 3.7. Constructive and Destructive Interference

Computer generated holography can model this process by recording the appropriate intensity values at the hologram plane. These intensity values could then be plotted on paper using various gray scaling levels to record the levels of intensity. A binary recording of a hologram could be produced if a threshold for intensity values was established. Intensity values below the threshold could be considered as zero values and intensities above the threshold could be maximum intensity. Maximum intensity values could be represented as a dark mark on the paper and zero intensity values could be left blank (or white). Figure 3.6 provides an example of the grating structure that results from the interference between two beams of light. The solid straight lines represent the regions of maximum amplitude of the plane wave emanating from reference source, and the solid curved lines represent the regions of maximum amplitude values of the spherical wave emanating from an object point. The dark lines on the hologram correspond to high intensity values resulting from the interference between the two beams of light. These dark lines, or fringes, on the hologram continue in a direction perpendicular to the page. The white areas on the hologram indicate areas of low intensity that result from destructive interference.

The spatial frequency of the grating structure is a function of the angle between the two waves. As the angle between the two waves increases, the spatial frequency of the grating structure increases (or the bands get closer together). Figure 3.8 provides an illustration of the angle between two light plane waves. The fringe spacing (FS), or spatial frequency of the grating structure, can be determined by the following equation if the wavelength (λ) of the light waves and the angle (θ) between them is known:

$$FS = \frac{\lambda}{sin(\theta)} \tag{3.1}$$

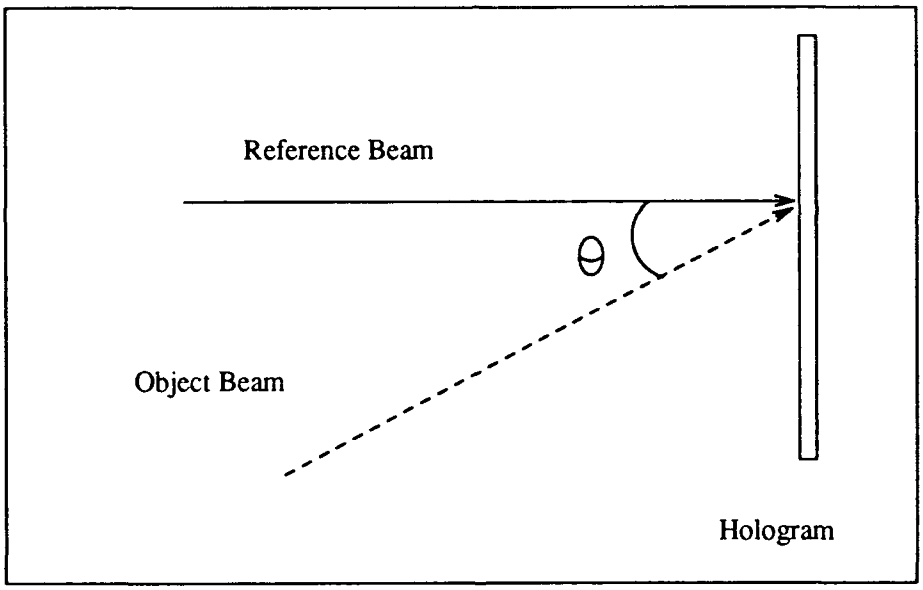

Figure 3.8. Angle Between Two Light Waves

3.2.3 Resolution Requirements. If the wavelength of the two light waves is .6328 microns and the angle between them is 60 degrees, then the fringe spacing of the hologram is .7307 microns. That is, the distance from the beginning of one dark band to the beginning of the next one is less than one micron. The maximum spatial frequency of a printer is determined by its dot pitch (the minimum diameter of dots it can generate). That is, the maximum number of non-overlapping dots that a printer can generate in given amount of space is limited by the printers minimum dot pitch. A standard 300 dot-per-inch (dpi) laser printer has a dot pitch of .0067 inches. In other words, one dot every 26.25 microns is the maximum spatial frequency of a 300 dpi printer. In order to record the grating structure from two interfering light waves with a 300 dpi laser printer working at its spatial frequency limits, the maximum angle between the two waves must be less than 26.3 degrees. From equation 3.1 the relationship between the wavelength of the light waves and the angle between them defines the fringe spacing of the interference pattern. However, the output device can limit the fringe spacing which will then limit the angle separating the light waves. The limit of the angle separating the two light waves corresponds to the maximum deflection angle of the grating structure. In other words, the object beam must originate in the region defined by the angle between the two beams in order for the grating structure to recreate the object beam without distortion. Figure 3.9 represents a valid domain for the object light source when the reference light beam is normal to the hologram surface. The valid object domain is actually a cone shaped volume beginning at a distance from the hologram that is defined by the angle θ.

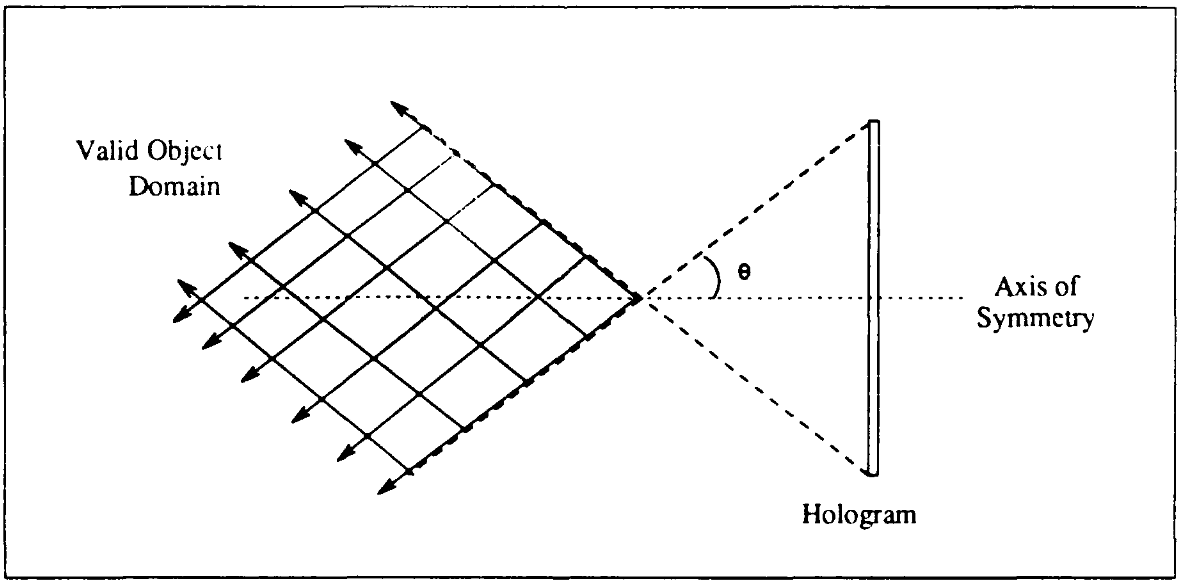

Figure 3.9. Valid Object Domain

If the minimum fringe spacing is a constant, then the valid object domain parameters would be a function of λ and θ in equation 3.1. The reduction factor of the hologram directly influences λ which then defines θ. The size of the hologram and θ define the distance from the hologram that the valid object domain begins. Tables 3.1 through 3.5 summarize changes in the valid object domain that result from various hologram sizes and reduction factors. This data is based on a minimum fringe spacing of 339 microns and hologram cell sizes of 169.34 by 169.34 microns. The location of the real image is determined by dividing the modeled object distance by the reduction factor. A sheet of paper from the laser printer contains 1200 by 1200 hologram cells. Table 3.1 represents a matrix of 12 by 12 sheets and table 3.5 represents 1 sheet. Tables 3.2 through 3.4 contain various combinations of sheets.

Original Size	Reduction Factor	Maximum θ	Minimum Object Distance	Reduced Size	Real Image Location
2.44 X 2.44 m	150	16.26°	5.91 m	1.63 X 1.63 cm	3.94 cm
2.44 X 2.44 m	100	10.8°	9.07 m	2.44 X 2.44 cm	9.07 cm
2.44 X 2.44 m	30	3.21°	30.74 m	8.13 X 8.13 cm	102.47 cm
2.44 X 2.44 m	16	1.71°	57.7 m	15.24 X 15.24 cm	360.65 cm
2.44 X 2.44 m	8	.86°	115.45 m	30.48 X 30.48 cm	1443.09 cm

Table 3.1. Limited Spatial Resolution Effects: 14400 by 14400 Cells

Original Size	Reduction Factor	Maximum θ	Minimum Object Distance	Reduced Size	Real Image Location
1.63 X .813 m	150	16.3°	3.12 m	1.08 X .54 cm	2.08 cm
1.63 X .813 m	100	10.76°	4.78 m	1.63 X .81 cm	4.78 cm
1.63 X .813 m	30	3.21°	16.2 m	5.42 X 2.71 cm	54.01 cm
1.63 X .813 m	16	1.71°	30.41 m	10.16 X 5.08 cm	190.08 cm
1.63 X .813 m	8	.86°	60.85 m	20.32 X 10.16 cm	760.58 cm

Table 3.2. Limited Spatial Resolution Effects: 9600 by 4800 Cells

Original Size	Reduction Factor	Maximum θ	Minimum Object Distance	Reduced Size	Real Image Location
1.22 X .813 m	150	16.3°	2.51 m	.81 X .54 cm	1.67 cm
1.22 X .813 m	100	10.76°	3.86 m	1.22 X .81 cm	3.86 cm
1.22 X .813 m	30	3.21°	13.06 m	4.06 X 2.71 cm	43.54 cm
1.22 X .813 m	16	1.71°	24.52 m	7.62 X 5.08 cm	153.25 cm
1.22 X .813 m	8	.86°	49.06 m	15.24 X 10.16 cm	613.2 cm

Table 3.3. Limited Spatial Resolution Effects: 7200 by 4800 Cells

Original Size	Reduction Factor	Maximum θ	Minimum Object Distance	Reduced Size	Real Image Location
.813 X .813 m	150	16.3°	1.97 m	.54 X .54 cm	1.31 cm
.813 X .813 m	100	10.76°	3.02 m	.81 X .81 cm	3.02 cm
.813 X .813 m	30	3.21°	10.25 m	2.71 X 2.71 cm	34.16 cm
.813 X .813 m	16	1.71°	19.23 m	5.08 X 5.08 cm	120.22 cm
.813 X .813 m	8	.86°	38.48 m	10.16 X 10.16 cm	481.03 cm

Table 3.4. Limited Spatial Resolution Effects: 4800 by 4800 Cells

3.3 Geometric Modeling

Computer generated holography models the optical holography process. Images being recorded are defined by modeling primitives such as points, lines, polygons, or surfaces. The recording surface, or hologram, can be represented as a two dimensional matrix of storage locations. Sinusoidal waveforms can describe the object and reference beams. The entire system can be contained in a three dimensional coordinate system. One approach to model the computer generated holography process could be to define the hologram as a plane on the X and Y axis centered at the origin. The object would be defined as a set of points in a volume at some distance on the Z axis from the hologram. The points would function as point source emitters of spherical waves in the direction of the hologram. The reference beam would be a plane wave that is perpendicular to the hologram that originated at infinity on the

Original Size	Reduction Factor	Maximum θ	Minimum Object Distance	Reduced Size	Real Image Location
.20 X .20 m	150	16.26°	.49 m	.14 X .14 cm	.33 cm
.20 X .20 m	100	10.76°	.76 m	.20 X .20 cm	.76 cm
.20 X .20 m	30	3.21°	2.56 m	.68 X .68 cm	8.54 cm
.20 X .20 m	16	1.71°	4.81 m	1.27 X 1.27 cm	30.05 cm
.20 X .20 m	8	.86°	9.62 m	2.54 X 2.54 cm	120.26 cm

Table 3.5. Limited Spatial Resolution Effects: 1200 by 1200 Cell

Z axis. With this configuration, all that would be needed to generate a hologram would be to compute the interference between the object beam and the reference beam at all locations of the hologram

3.3.1 Hologram Modeling. The hologram was modeled as matrix of 169.34 by 169.34 micron sized cells. The hologram size could be adjusted to any number of 1200 by 1200 cell combinations. The lower bound of 1200 by 1200 cells per hologram and the size of each cell were a result of the limited spatial resolution of the output device used to record the patterns. For more information on the output device constraints refer to the Pattern Recording section in this chapter. The hologram was located in the XY plane at the origin of a left-handed, three dimensional coordinate system. Figure 3.10 illustrates a hologram at the origin of a left-hand coordinate system. With this orientation, the hologram represents the eyes of the system looking in the positive Z direction at the object and reference beams.

3.3.2 Object Modeling. The objects modeled through this research were considered to be solids with opaque surfaces. These objects were sampled at uniform intervals and represented with a set of resolution points that defined the object. Figure 3.11 represents the sampling of an object with resolution points. Here the object consists of the letters *A F I T* defined in eight XY planes on the Z axis. This figure indicates the hidden surface problem associated with using points (or lines) to

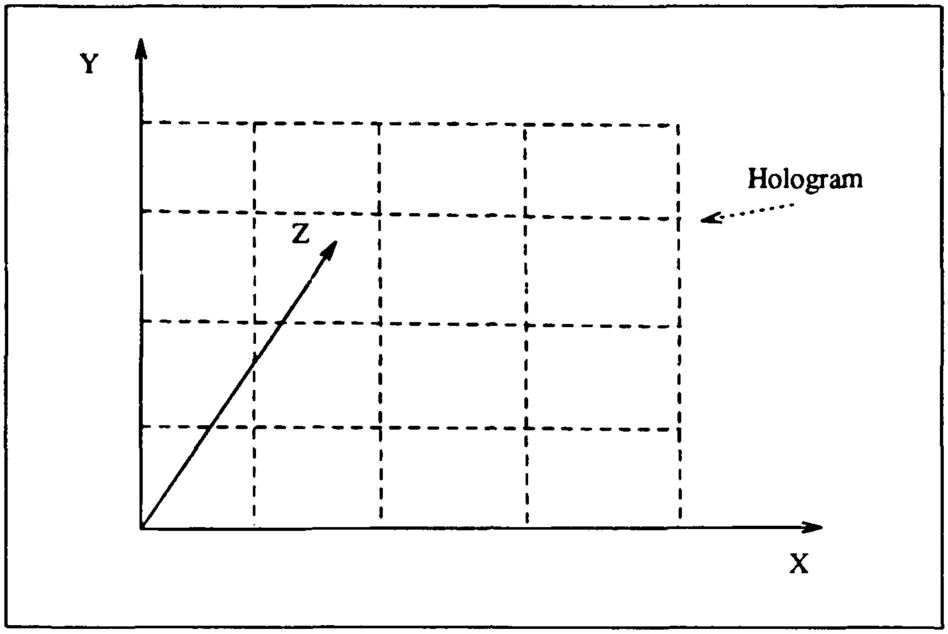

Figure 3.10. Hologram Modeled at the Origin of a Left-Hand Coordinate System

represent solid surfaces. That is, surfaces that would normally be blocked from the view of the hologram are visible through the gaps between the points representing the surfaces of the object nearer to hologram. This hidden surface problem was addressed by simply omitting the points that represented surfaces that would not be visible to the hologram.

Although the objects could have been modeled with any number of modeling primitives, points were selected because they simplified the interference calculations. Points can be considered as single point sources of light emitting spherical waves. This simplified the task of computing the interference pattern to simply tracing the path from the object resolution points to the hologram in order to calculate the contribution of the points to the grating structure of the hologram.

The following objects were modeled in this research:

1. One point.

2. Two points.

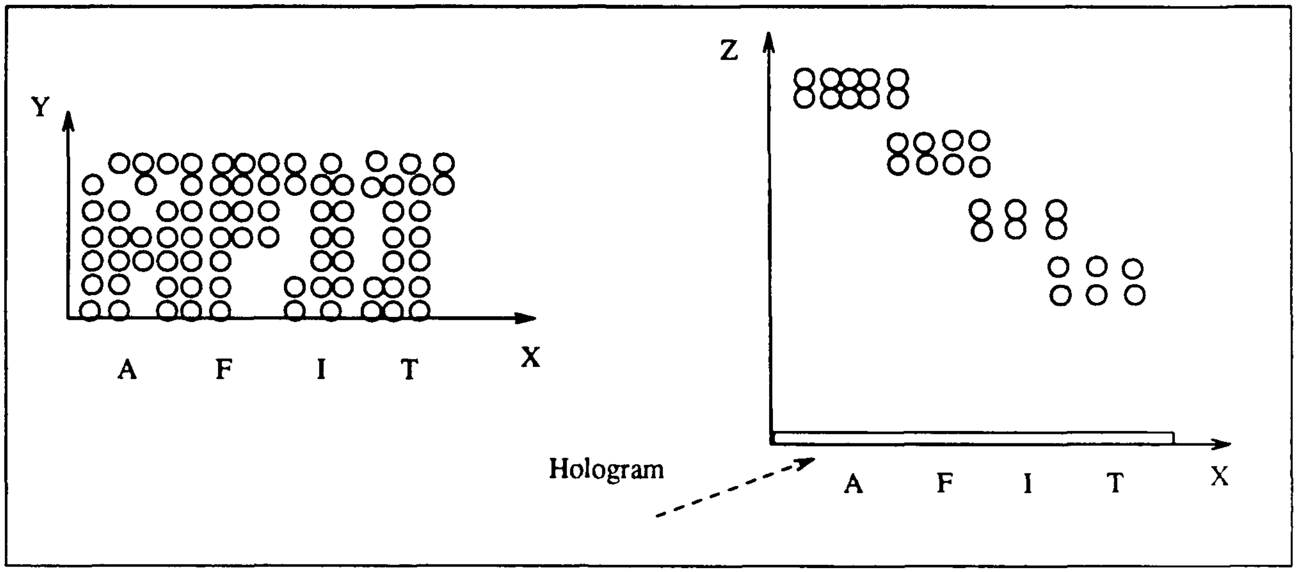

Figure 3.11. Object and Hologram Geometry Placement

3. A three-leaf clover defined in one plane with 41 resolution points.
4. The letters A F I T defined in 4 overlapping, off-set planes with 161 resolution points.
5. The letters A F I T defined in 4 non-overlapping, centered planes with 184 resolution points.

Figure 3.12 is a scaled version of the geometry for the letters *A F I T* defined in 4 planes by 184 resolution points. The dimensions of this geometry are approximately 1.3 meters on the X axis, 0.75 meters on the Y axis, and 1.5 meters on the Z axis.

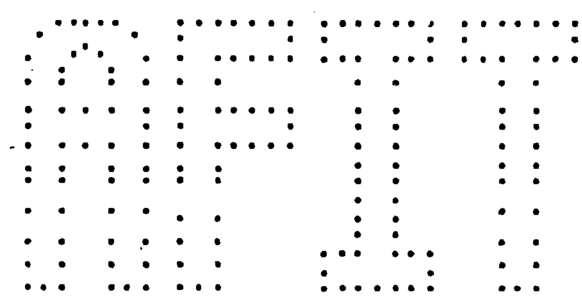

Figure 3.12. Geometry Description of A F I T with 184 points

3.3.3 Object Size and Placement. The limited spatial resolution of the output device and the reduction factors directly affect the location of the valid object domain. These factors dictate how close an object can be to the hologram. Because the valid object domain is a cone shaped volume, the size of the object also affects the distance the object must be located from the hologram (refer to figure 3.9). Determining the minimum distance an object can be located from the hologram is a matter of simple trigonometry. The sides of the cone are defined with the angle separating the object and reference beams (θ in equation 3.1). The distance, D, from the peak of the cone along the axis of symmetry and the cone's radius, R, are related by the following equation. Figure 3.13 illustrates this relationship.

$$D = \frac{R}{Tan(\theta)} \tag{3.2}$$

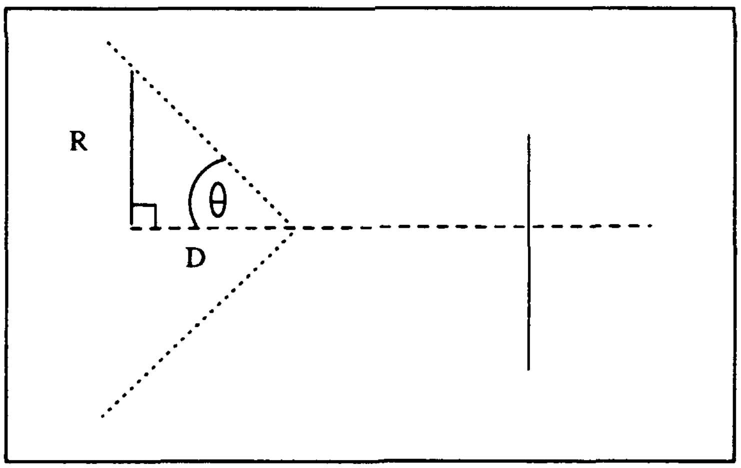

Figure 3.13. Distance Relationships Within Valid Object Domain

If, for example, the hologram size was 1200 by 1200 cells (one sheet) and the reduction factor was 30 percent, then (from table 3.5) the maximum angle separating the object and reference beams would be 3.21 degrees and the peak of the cone defining the valid object domain must be located at least 2.56 meters from the hologram along the Z axis. If the object was a sphere with a radius of one half meter, then the minimum distance on the Z axis the object could be defined would be:

$$11.48 \; meters = 2.56 \; meters + \frac{.5 \; meter}{Tan(3.21 \; degrees)} \qquad (3.3)$$

If an object was defined outside of its valid domain (for example, too close to the hologram) the following would result:

1. The output device would be forced to exceed its spatial frequency limits.
2. Aliasing would occur in the interference pattern.
3. The hologram's deflection angles would exceed the grating pattern's ability to diffract light.
4. Secondary (false) images would distort the primary real and virtual reconstructed images

3.3.4 Reference Beam Modeling. The reference beam was modeled as a spatially and temporally coherent plane wave normal to the hologram. Modeling the reference beam in this manner greatly simplified the calculations for the interference with the object beam. With the reference beam so defined, its phase would be constant at all locations in the hologram. This meant that only the phase of the waves from the object resolution points would need to be calculated to determine whether constructive or destructive interference occurred at each location on the hologram.

3.4 Pattern Calculations

3.4.1 Background. This research built on Captain Mouser's thesis work (18). The computer storage and processing requirements of his computer generated holography process were used as benchmarks. Captain Mouser calculated the interference pattern of a three dimensional cube defined by 55 resolution points at a distance of 5 centimeters from the hologram. His hologram consisted of 36,000 by 36,000 storage locations, and he defined the reference beam to be a coherent plane wave normal to the hologram. Using equation 3.4 as a theorical basis, Captain Mouser calculated the phase and amplitude of the light waves emanating from each of the object resolution points (X_o, Y_o, Z_o) for each location on the hologram (X_h, Y_h, Z_h).

$$H(X_h, Y_h) = \int\int\int \frac{e^{\frac{i2\pi}{\lambda}\sqrt{(X_o-X_h)^2+(Y_o-Y_h)^2+(Z_o-Z_h)^2}}}{\sqrt{(X_o - X_h)^2 + (Y_o - Y_h)^2 + (Z_o - Z_h)^2}} \, dx_o \, dy_o \, dz_o \qquad (3.4)$$

Captain Mouser simulated the integration in equation 3.4 by summing the contributions from the resolution points at all locations on the hologram. The contribution of phase and amplitude from a single object resolution point (X_1, Y_1, Z_1) at one location in the hologram $H(X_j, Y_k)$ is calculated using equation 3.5.

$$H(X_j, Y_k) = 2\,\frac{cos(\frac{2\pi}{\lambda})\,DT}{DT} + 2\,\frac{sin(\frac{2\pi}{\lambda})\,DT}{DT} \qquad (3.5)$$

The Euclidean distance, DT, between a point and the location in the hologram is determined by equation 3.6.

$$DT = \sqrt{(X_1 - X_j)^2 + (Y_1 - Y_k)^2 + (Z_1 - 0)^2} \qquad (3.6)$$

The real part of the complex number in equation 3.4 was represented by the cosine function and the imaginary part was represented by the sine function in equation 3.5. The values were squared to get the complex conjugate so that the result would be a nonnegative, real number. The distance in this equation is the Euclidean distance from each resolution point to each location in the hologram.

The phase and amplitude of the light from an object resolution point are encoded at a location of the hologram using equation 3.5. The process of creating a hologram with this equation would involve implementing it at all locations of the hologram. for all resolution points. The values from each point would be added together to generate the total contribution of all the resolution points at every location of the hologram.

These computed values would then be scaled to values between zero and one and compared to a scaled value that represents the phase and amplitude of the reference beam. (note: The reference beam has only one value for phase and amplitude for all locations on the hologram because it is a coherent uniform amplitude plane wave that is normal to the hologram). If the computed value at a cell was greater than (or equal to) the value from the reference beam, then that cell was assigned a value of maximum contrast, else it was left at zero contrast. Captain Mouser's approach required 59.5 processor hours on a Cray 2 computer and occupied 1.3 gigabytes of storage (18, 17).

3.4.2 Approach. In order to reduce processing and memory storage requirements, the following assumptions were made:

1. The interference calculations could be performed with real numbers.
2. The results of these calculations did not require scaling.
3. All object resolution points were of equal intensity.
4. The phase of the reference beam at the hologram was equal to zero.
5. Only the phase of the object resolution points contributes to the image (6).
6. The Euclidean distance calculations could be approximated.
7. The interference patterns could be binary rather than gray scaled.
8. Holograms, no matter how large, can be generated 1,440,000 cells at a time (holograms larger that 1,440,000 cells would be printed and combined off-line).

Because all object resolution points are assumed to be equal in intensity, the amplitude of the hologram is only effected by the phase of the light at the hologram from each point and the light from the reference beam. This can be easily accomplished with equation 3.7. The sine of the value computed from equation 3.7 will yield a number representing the position of the wave that intersected a cell in the hologram for a particular object resolution point. This number can be summed with the numbers from the other object resolution points to represent the contribution of phase of all the resolution points for each cell of the hologram. A value of zero or greater in a cell would imply constructive interference, and that cell would be assigned a value of one. If the value for a cell was less than zero, a value of zero would be assigned to that cell implying that destructive interference occurred.

$$Amplitude = \frac{2\pi}{\lambda} * DT \tag{3.7}$$

The Euclidean distance between an object resolution point and a hologram cell is determined by using equation 3.6. Because the distance calculations were required for each point at every cell of the hologram, this equation was an attractive target to optimize. Jack Ritter has suggested a fast approximation to 3D Euclidean distance calculations (10:432). His method uses no multiplications, divisions, or square roots and is performed entirely with fixed point arithmetic. This method has three levels of accuracies. The fastest option is accurate within plus or minus 13 percent. The next option is slower than the first but is accurate within plus or minus 9 percent. The final option is the slowest of the three but yields an accuracy of plus or minus 8 percent. This approximation was substituted in place of equation 3.6 for the calculations of single point holograms and compared to the exact calculations in terms of processing speed and image quality. The cell values of the hologram were computed using 64-bit, double precision variables to ensure the proper precision for the distance calculations. To reduce the storage requirements the data representing each cell was compress to a single bit.

The computer processing part of this computer generated holography pipeline was segmented into two programs: a routine that generated the interference patterns (holo.c), and a routine that transformed the interference patterns into postscript for later printing (psout.c). Only one block of 1200 by 1200 hologram cells could be processed during a single run of holo.c. With multiple runs of this program, holograms containing any multiple of these 1200 by 1200 cell blocks could be built. Holograms could be considered as matrices of these 1200 by 1200 cell blocks. Individual blocks would be specified to the program with command line parameters as matrix locations. For example, a 2 by 2 block hologram (2400 by 2400 cells) would require 4 runs of holo.c using the parameters of *0 0, 0 1, 1 0, and 11* to describe the particular block of 1200 by 1200 cells being processed. Holo.c was written in standard C and should be portable to any system supporting this language. The object resolution points are described with three-dimensional coordinates in a geometry file. Holo.c reads the geometry file, calculates a binary interference pattern, and outputs the binary data as integers to an ASCII text file. This file is the input data for psout.c which converts the integer ones and zeros to a postscript imagemask (bit mapped format) file that can be printed on any postscript printer. The Z-axis coordinate position of the points in the geometry file are relative depth positions for the points within the valid object domain. The distance along the Z axis to the start of the valid object domain is specified in holo.c. The light wavelength can be adjusted in holo.c to correspond with the laser being modeled and the reduction factor of the hologram.

3.5 Pattern Recording

3.5.1 Considerations. Captain Mouser's output method was electron beam lithography which eliminated the need for reduction (18). This process is very efficient and usually timely. But due to the system's workload and the relatively low priority assigned to Captain Mouser's work, his hologram has yet to be processed after

12 months. Considering the experimental nature of this research, this turn-around time was considered much too long. Consequently, electron beam lithography was eliminated as a pattern recording alternative. Due to their general availability and reasonable resolution, lase. printers were selected as the output device to record the interference patterns.

A high resolution, laser printer was desired because computer generated holograms are limited by the spatial frequency of the output device. The laser printers available were a 300 dot-per-inch Laserwriter II and and a 400 dot-per-inch NeXT printer. Although the NeXT printer was rated at a higher resolution, the dot pitch of both printers was the same. For the purposes of this research, both printers were equal in that the maximum density they could print non-overlapping dots was 150 per inch. With paper sized at $8\frac{1}{2}$ *by* 11 inches, the maximum imaging area of the printer would be $8\frac{1}{12}$ *by* $10\frac{1}{2}$ inches. To provide uniformly sized hologram blocks and to allow space for block identifiers on each sheet, 8 by 8 inches was selected as the size for each printed block. At 150 dots-per-inch an 8 by 8 inches block would allow 1200 by 1200 hologram cells per block.

Postscript was selected as the printer description language because of its versatility and portability. The desire for versatility and portability was also the driving factors in the decision to output the results of holo.c as integers in an ASCII text file.

3.5.2 Approach. Initially the move-to and draw-to capabilities of postscript were to form the basis for the output of the interference patterns. However, the postscript limit of 1500 points per path was too much of a constraint. Therefore the imagemask feature of postscript was used to create a bit-map of the interference data. In addition,. the imagemask operator provided a facility for generating positive or negative images of the data with a simple switch. This feature was used to evaluate the effects of positive and negative images in the computer generated holography process.

The process of psout.c was simple. It read the binary data from holo.c, generated the necessary header information, bit mapped the integer input data, generated the necessary trailer information, and output a postscript file. This file could be sent to any postscript printer for printing. This program is versatile enough to handle any number of cells per block, provided the block dimensions (number of cells per block) are provided. The spatial dimensions of the block are constant (8 by 8 inches), but the number of the cells per block can be varied. The number of cells per block determine the output resolution of the printed pattern. For example:

- 1200 by 1200 cells per block equates to 150 dots-per-inch
- 1600 by 1600 cells per block equates to 200 dots-per-inch
- 2000 by 2000 cells per block equates to 250 dots-per-inch
- 2400 by 2400 cells per block equates to 300 dots-per-inch
- 2800 by 2800 cells per block equates to 350 dots-per-inch
- 3200 by 3200 cells per block equates to 400 dots-per-inch

3.6 Pattern Reduction and Image Generation

3.6.1 Considerations. A 20 mWatt helium-neon (HeNe) gas laser was chosen as the light source to reconstruct the computer generated holography images. This laser was chosen because it was readily available and because it provided a relatively safe and easy method to evaluate the reconstructed images. The light from this laser has a wavelength (λ) of .6328 microns. Various multiples of this wavelength, corresponding to the hologram reduction factor, were used as the λ value in the equations used through out the modeling process.

Because the patterns were output on paper, they had to be transformed into transmission holograms capable of reconstructing images. The first transformation required was to reduce the patterns by the amount specified by the reduction factor. The reductions were accomplished by photographing the patterns at the appropriate distances according to a reduction formula. The reduction factors used were 8, 16, 30, 100, and 150. The next transformation that was required was to translate the image into transparent and opaque areas corresponding to the dark and light areas of the pattern.

3.6.2 Approach. The following alternatives were used to transform the printed interference patterns into transmission holograms:

1. The NCR Ultrafische process.
2. A photographic processing lab using step reduction equipment.
3. A 35 mm camera using a reduction formula.

The National Cash Register (NCR) company has developed a microfische process named Utlrafische. This process can reduce images up to 150 times and combine them with other images onto a transparent plate. This process was used to generate holograms because it could combine several sheets and produce a very large hologram. Four by four and eight by four block holograms were generated using this process. Some of the holograms were reduced 100 ıes from their original size while others were reduced 150 times. Figure 3.14 illustrates an NCR ultrafische plate with a four by four block hologram.

Another transformation process involved a photographic processing lab (ASD Technical Photography). A negative was produced from a photograph of several sheets (blocks) of the interference patterns laid out on foamboard. Step reduction photographic equipment was used to achieve the necessary reduction. Four by four block (16 sheets) holograms were produced with a reduction factor of 30. The final transformation process used a 35 mm camera and the following reduction formula:

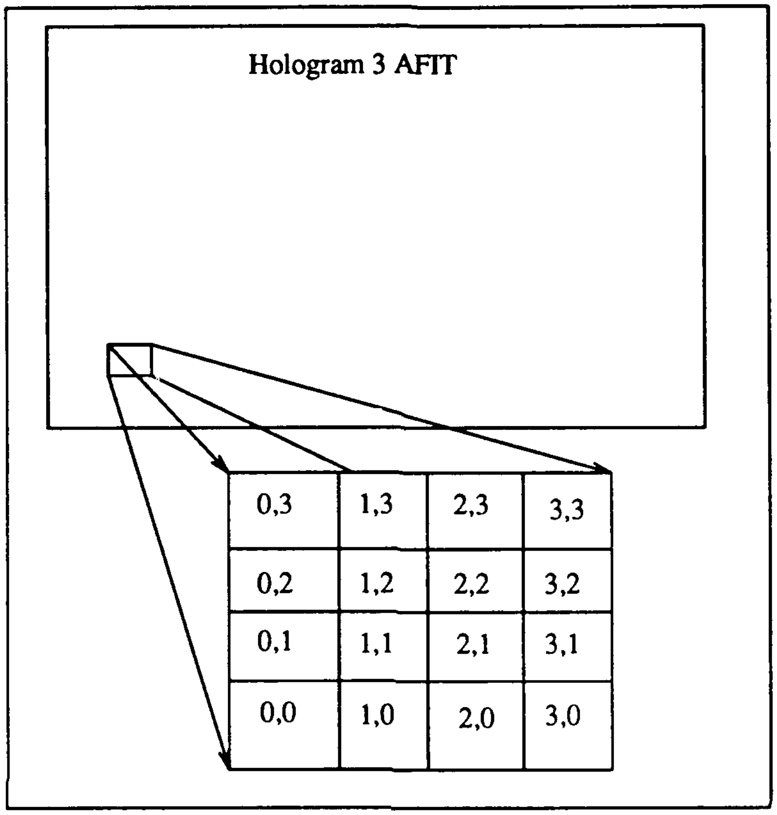

Figure 3.14. NCR Ultrafische Plate

$$R = \frac{D - F}{F} \tag{3.8}$$

The reduction factor (R) is determined by the distance (D) from the subject and the focal length (F) of the lens used (1:5) A 200 millimeter lens was used to photograph 1 by 1, 2 by 2, and 4 by 4 block hologram. Reduction factors of 8 (D = 5.9 feet), 16 (D = 11.2 feet), and 30 (D = 20.3 feet) were used to generate transmission holograms. Kodak technical pan film was used to photograph the patterns. This film was chosen because of its resolving power (400 line pairs per millimeter). The developed film negatives were mounted in slide holders and used to generate images by illuminating them with a collimated HeNe laser beam normal to the film. The transmitted *real* image was observed using a white background to capture successive planes of the image at the predicted focal length of the hologram. The image focal length is the modeled distance of the object divided by the reduction factor.

IV. *Results and Conclusions*

The computer generated holography pipeline functioned well. The pipeline was implemented with general purpose computer workstations, standard laser printers, and photographic reduction techniques. Multiple geometry files were used to generate several images. Compared to conventional computer graphics images, the computer generated holography images were crude, but they were identifiable and reasonably clear. The three dimensional nature of the images was difficult to exploit. The multiplanar images were focused one plane at time. The photographs of the images in this chapter used this approach to depict the three dimensional nature of the images.

4.1 Results

4.1.1 Aliasing. The first patterns calculated were single point holograms. The purposes of these holograms were to test the computer generated holography pipeline and to verify the sampling rate limits of the printer. The recording surface (or hologram) consisted of a single block of 1200 by 1200 cells located at the origin. The coordinates of the recording surface were (1200,1200,0). The object being recorded was a single point located in the center of the recording surface at (600,600,0). The distance from the recording surface to the point (the Z axis distance) was varied from 30,000 units (about 5 meters) to 100,000 units (about 17 meters). The reduction factor of this hologram was 30 which established the minimum distance to the valid object domain from the recording surface at 2.56 meters. This distance corresponded to the highest spatial resolution of the printer (150 unique dots per inch).

As the distance between the object point and the recording surface increased, the spatial resolution requirements of the printer decreased. The object at a distance of 2.56 meters from the recording surface would force the printer to work at its maximum resolution (or maximum sampling rate). In order to prevent aliasing, the printer's sampling rate must be less than its maximum. Figure 4.1 is the interference pattern of this point at a distance of about 5 meters. At this distance, the printer is sampled at about one half of its maximum sampling rate. Notice the secondary sets of concentric rings. These appear as additional point source emitters of spherical waves, but in fact are results of aliasing from the printer. Figure 4.2 is the interference

Figure 4.1. Single Point 30X at 5 Meters

pattern from this same point but at a distance of about 8 meters. This distance corresponds to a sampling rate of one third of the printer's maximum sampling rate. The secondary sets of rings are dimmer that the ones in figure 4.1. In figure 4.3 the point is located about 17 meters from the recording surface. This distance corresponds to a sampling rate of one sixth of the printer's maximum sampling rate. Notice that the secondary sets of concentric rings have disappeared.

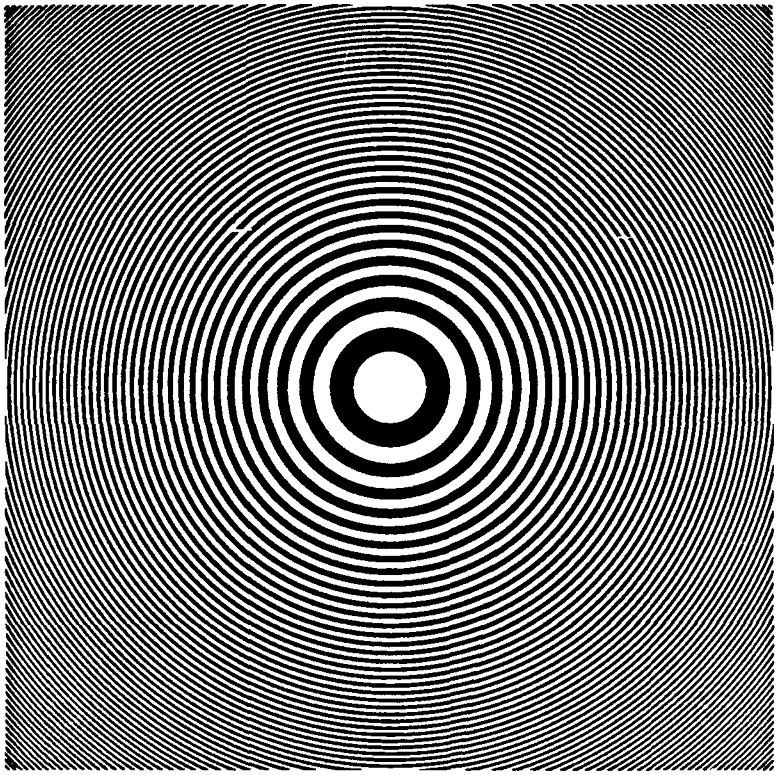

Figure 4.2. Single Point 30X at 8 Meters

4.1.2 Processing and Storage Requirements. The pattern calculations were tried on several computer workstations. Table 4.1 contains the elapased (wall clock) times to calculate an interference pattern for a single point with a recording surface of 1200 by 1200 cells and for an object with 184 points with a recording surface of 16 blocks of 1200 by 1200 cells. Each block of 1200 by 1200 cells required about 10 minutes to print on a Laserwriter II laser printer. Each 1200 by 1200 cells block

System	Elpased Time per Point/Block	Elpased Time for 184 Points/16 Blocks
S.G. Iris 4D 320VGX	14 seconds	11.5 hours
S.G. Iris 4D 85GT	26 seconds	21.1 hours
Sun 4	2 minutes	98.2 hours

Table 4.1. Processing Time Requirements

required 368,000 bytes of storage regardless of the number of points in the object or

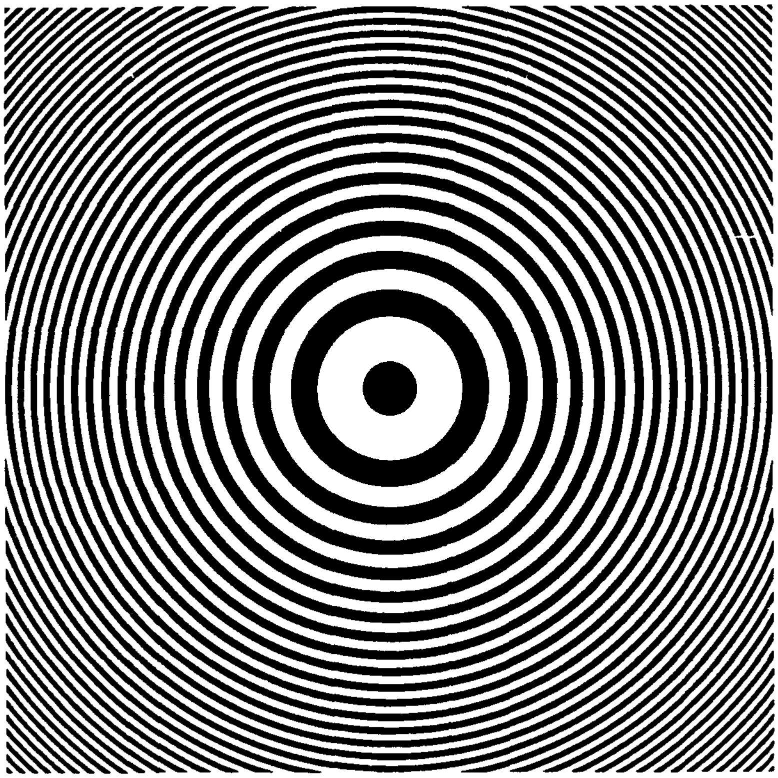

Figure 4.3. Single Point 30X at 16.933 Meters

the system used. A 16 block hologram requires 5,888,000 bytes of storage space.

The Silicon Graphics Iris 4D 320VGX performed the best in terms of processing times. This system which consisted of two 33 MHZ processors was the primary development system for this pipeline. Table 4.2 shows the elpased time and storage requirements of three of the holograms created using the Silicon Graphics Iris 4D 320VGX.

Hologram	Elpased Processing Time	Memory Space
41 Point Clover	9.6 minutes	1 Block/368,000 Bytes
161 Point AFIT	10 hours	16 Blocks/5,888,000 Bytes
184 Point AFIT	11.5 hours	16 Blocks/5,888,000 Bytes

Table 4.2. Relative Processing and Storage Requirements

4.1.3 Processing Speed Improvement. The major decrease in processing speed over Captain Mouser's approach was achieved through an algorithm change. The algorithm used in this research was inspired from an approach used to generate holograms on a personal computer (19). Additional processor time savings were explored using an approximation of the calculations of the Euclidean distances (equation 3.6) in the innermost loop of interference calculation program (holo.c) (10:432). Although this approximation algorithm had three levels of accuracy (maximum error rate), the most accurate approach, maximum error rate of plus or minus eight percent, was used. Two holograms were produced with the same geometry. In one hologram, the exact Euclidean distances were calculated using equation 3.6. In the second hologram, the Euclidean distances were approximated using Ritter's method (10:432).

Figure 4.4 illustrates the 41-point clover geometry (the center point is really two points on top of each other) used for these holograms. Figure 4.5 is the interference pattern generated from the clover geometry (figure 4.4. Exact Euclidean distances calculations were used to compute this pattern. Figure 4.6 is a photograph of the reconstructed image of the clover that was constructed with exact distance calculations.

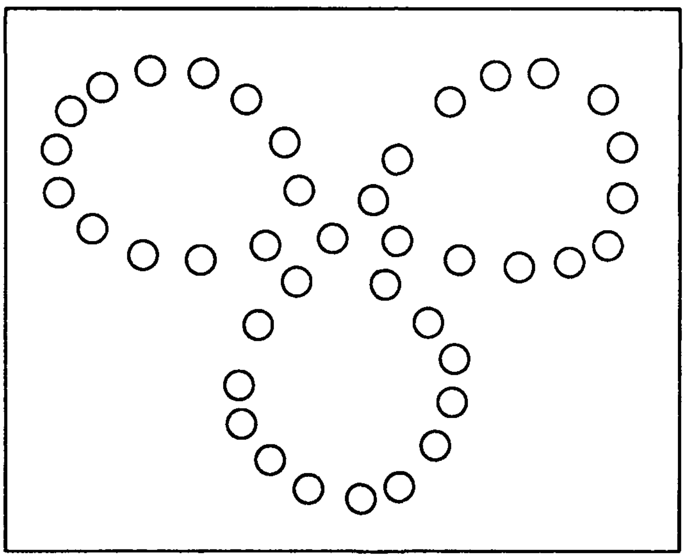

Figure 4.4. Clover Geometry Described with 41 Resolution Points

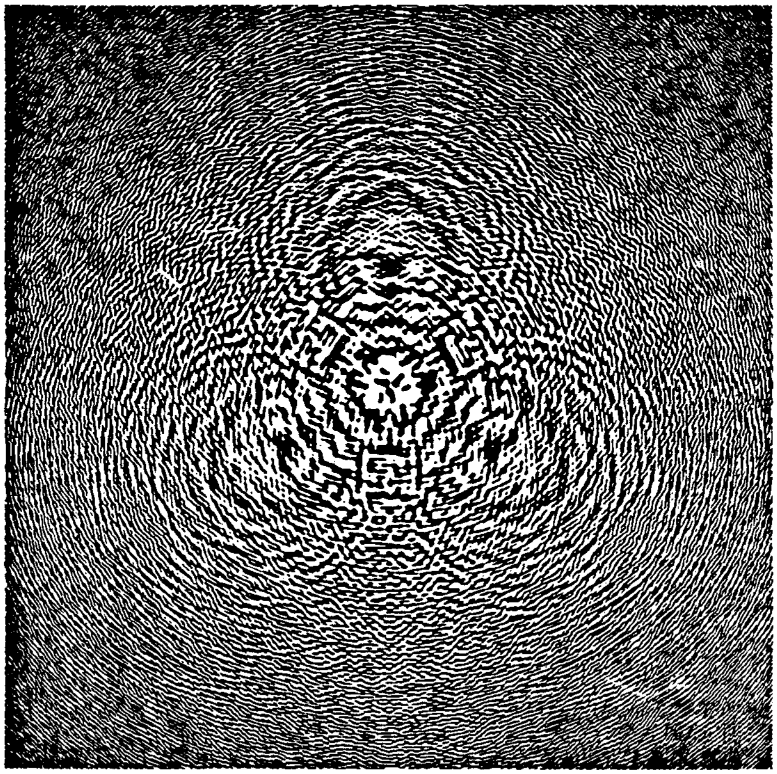

Figure 4.5. Interference Pattern With Exact Distance Calculations

The interference pattern of the clover geometry that was calculated with approximated Euclidean distances is shown in figure 4.7. No image was formed from this pattern. The approximated distance hologram calculations were about ten percent faster than the exact distance hologram calculations. Even though the processing time savings were significant, apparently the eight percent error (plus or minus) factor was significant enough to prevent image formation. The remainder of the holograms were produced with exact distance calculations.

4.1.4 Images. Two and three dimensional images were formed. Compared to conventional graphics, however, the images were quite crude. Essentially, the reconstructed holographic images were a series of dots that were focused on a white backstop. All the points of the clover geometry (reference figure 4.4) were in the same plane on the Z-axis. This image was captured by focusing only a single plane

Figure 4.6. Photograph of the Reconstructed Clover Image Created with Exact Distance Calculations

as illustrated in figure 4.6. Exploiting the three-dimensional nature of the images, on the other hand, was a problem. Three dimensional images were focused one plane at a time. The camera used to record the reconstructed images was not moved between each plane of an image. The backstop was adjusted in distance from the hologram and a photograph was taken of the images reflected on it. Figures 4.8 through 4.10 are photographs of holographic images produced from the AFIT geometry depicted in figure 3.12. This geometry created the letters A F I T with 184 points in four planes on the Z-axis. This geometry was modeled at 24 meters from the recording surface with a reduction factor of 8. The recording surface consisted of 1200 by 1200 cells. The geometry occupied a volume of about .4 meters wide by .2 meters tall by 1.5 meters deep. Each letter occupied a plane, and each plane was separated by

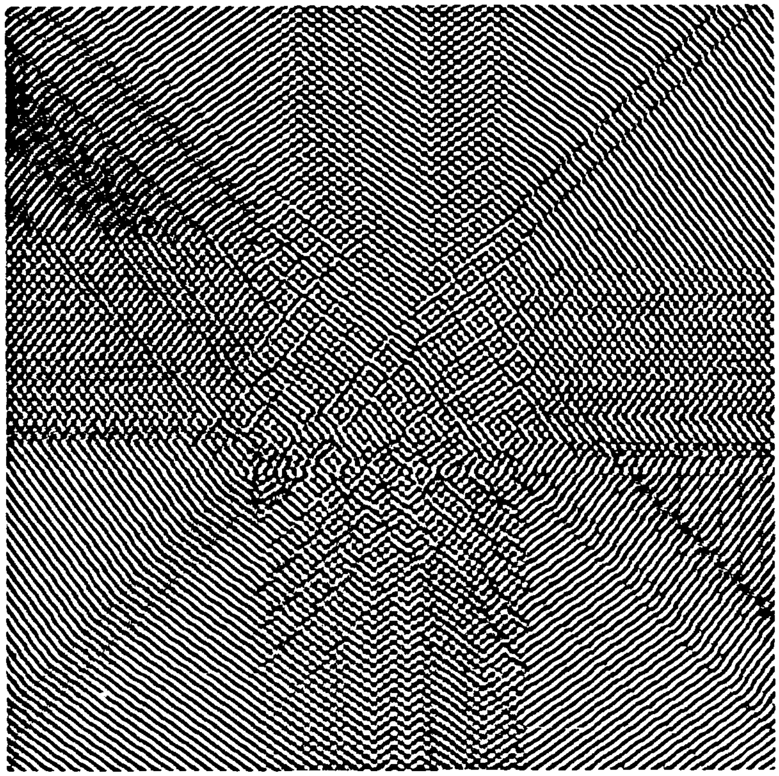

Figure 4.7. Interference Pattern with Approximated Distance Calculations

about one half meter. The four reconstructed letters were a total of 5 centimeters wide and 2.5 centimeters tall. The first letter (A) of the reconstructed image came into focus at 300 centimeters from the hologram and the last letter (T) remained in focus until about 390 centimeters from the hologram. Figure 4.8 is a photograph of a reconstructed image plane at 290 centimeters from the hologram. It illustrates that all four letters are out of focus. Figure 4.9 is a photograph of the reconstructed image of the AFIT hologram at 310 centimeters. It shows the letter A in focus with the remaining letters out of focus. Figure 4.10 is a photograph of this reconstructed image at 380 centimeters from the hologram. Here the letter T is in focus but the remaining letters are out of focus. This hologram was recorded with a laser printer. The interference pattern was then put on a wall, and photographed from a distance of 5.9 feet using a 35 mm camera and a 200 mm lens. The holograms produced at the ASD Technical Photography Lab were unable to reconstruct images. The

Figure 4.8. Photograph of the Reconstructed AFIT Image at 290 Centimeters

NCR Ultrafische process holograms were able to reconstruct parts of their original geometry. This process is normally used for reducing document pages for later reading with special Ultrafische equipment. It was reported that in this process, individual sheets could be placed at arbitrary close distances to each other. However, in practice all of the sheets (each sheet was a block of 1200 by 1200 hologram cells) had a gap between them. These gaps hindered the reconstruction of the images. Figure 4.11 is a photograph of the reconstructed image of a hologram produced with the NCR Ultrafische process. This hologram was produced from a 161 point geometry that defined the letters AFIT in 4 planes. These letters were 1.3 meters wide and .8 meters tall and were located 133 centimeters to the left of the center of the recording surface. The recording surface was defined as 8 blocks of 1200 by 1200 cells wide and 4 blocks of 1200 by 1200 cells tall (or 32 blocks). The reduction

Figure 4.9. Photograph of the Reconstructed AFIT Image at 310 Centimeters

factor was 100 and the Z-axis distance to the first letter (A) was 11.8 meters. This photograph shows the incoherent set of dots that should have represented the letters AFIT. This photograph was taken at 12 centimeters.

4.2 Analysis

The time lag from image modeling to image reconstruction while improved from the previous thesis work, was still too excessive for real-time applications. A reasonably sized hologram consisting of 16 blocks could be computed of a simple object consisting of less than 200 resolution points in less than 12 hours. The interference patterns can be printed, assembled, and photographed in less than four hours. The development of the film takes from two days to a week to be accomplished. The processing by NCR or the ASD Technical Photography lab requires about a week

Figure 4.10. Photograph of the Reconstructed AFIT Image at 380 Centimeters

to ten days. Once the film is developed, the image generation is a simple matter of illuminating the film with a collimated laser beam and capturing the image (or a plane of the image) at the appropriate focal length. However, capturing a full three dimensional view of the image at one time is another problem. The power output from the 20 milliwatt HeNe collimated laser used for image reconstruction was measured as 160 microwatts per square centimeter. According to Ready (21:325) less than 180 microwatts per square centimeter is not dangerous. Additionally AFOSH Standard 161-10, appendix III, specifies that a person can safely look into a laser beam for 20 seconds or less provided the power output is 850 microwatts per square centimeter or less. This standard also indicates that the laser can be safely observed for up to 3 hours provided its power output is 170 microwatts per square centimeter or less. To provide an additional margin of safety, a 1.0 optical density filter was

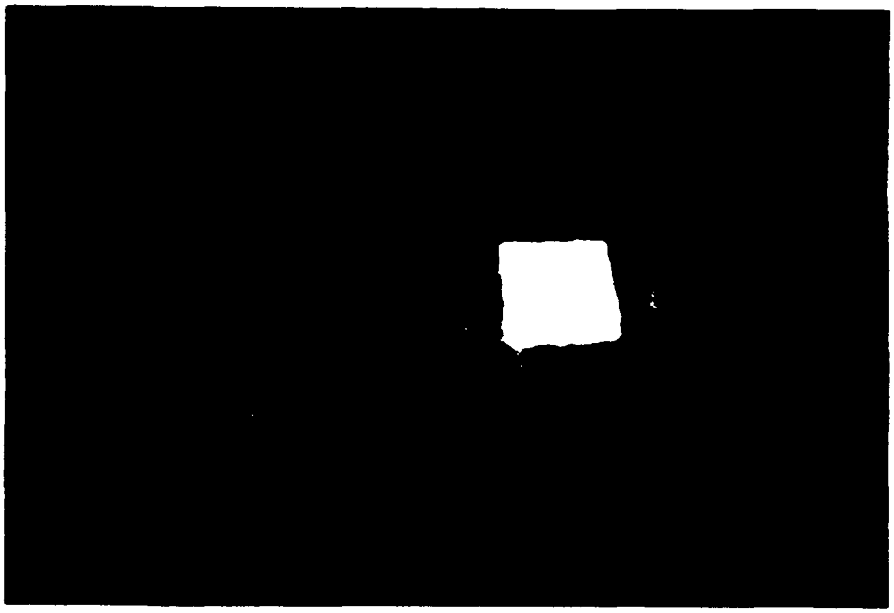

Figure 4.11. Photograph of the Reconstructed AFIT Image Using the NCR Ultrafische Process

used to lower the power output of the laser to 16 microwatts per square centimeter. The virtual images were then observed looking through the holograms down the bore of the laser. The small diameter of the holograms restricted the perspective of the images, but nevertheless the three dimensionality of the images were observable. Another approach to three dimensionally view the object would be to focus each plane of the image at once using adjustable backstops. However, this is not practicable as a general display technique. This display problem will continue to be a problem with this approach.

The 35mm camera approach for generating transmission holograms was the most effective for experimentation. The other methods will allow for larger holograms, but require more time to be processed. Additionally, the steps within these processes are outside the control of the researcher. This results in yet another variable being added to the process of creating, modifying, and evaluating computer generated holograms. The hologram produced at ASD Technical Photography never generated an image. The holograms developed by NCR generated images, but they were so distorted by the gaps between the blocks of cells that the images did not resemble the geometry.

This research confirmed that holograms, no matter how large, could be generated as a series of blocks of cells and later combined. This ability reduced the dynamic memory requirements to produce an interference pattern. The algorithm change from Captain Mouser's approach allowed general purpose computer workstations to be used to compute interference patterns in a reasonable amount of time. In terms of processing time and memory space required to store the images, the benefit of these images does not appear to be practical at this point. However, it is much too early in this research to draw far-reaching conclusions. Much more work is required to determine whether or not computer generated holography is a viable medium for three dimensional displays.

The use of easily accessible computer equipment provided an open environment in which to experiment. This allowed more freedom to try different approaches, like the Euclidean distance approximation attempt. The feedback from these experiments, even the ones that failed, proved to be a tremendous learning tool that instantiated abstract concepts with concrete results.

Due to the time lag in processing, photographing, and developing the holograms, much of this research was performed in parallel. While this optimized the time of the researcher, it generated problems with keeping track of the data. Early on, it became apparent that a system to track the various process and results was necessary. The approach chosen for this research consisted of separating each major approach (experiment) and copious amounts of notes. Each approach was maintained in a separate subdirectory on the computer file system. Charts were developed to track the multiple block holograms, to ensure all the patterns were generated and that they were not mixed with patterns from other holograms.

4.3 Recommendations

Clearly, this work has just begun. This pipeline will provide an adequate vehicle for future exploration of this area. The modular nature of the system will allow the calculations program, holo.c, to be modified or completely replaced without effecting the output software, psout.c. More complex geometry files can be developed and processed, and a preprocessing step to remove hidden surfaces (points representing surfaces) can be added without changing the rest of the pipeline.

A three dimensional view of the images may be more feasible if other approachs to calculating the interference patterns were tried. For example, if the reference beam was tilted rather than normal to the recording surface, the virtual image would be separated from the real image and from the noise (DC term) of the reconstruction beam.

Another approach to solve this three dimensional viewing problem might be holographic stereograms. In this approach a perspective view could be computed for each cell on the recording surface. The interference pattern calculations would be accomplished for each cell based on that view of the object. The reconstructed image should demonstrate its three dimensionality through changes in its perspective views as a viewer moves their eyes or head.

In addition to adding a preprocessing step to eliminate hidden surfaces in geometry files, other modeling primitives such as polygons could be used to model objects to be recorded. If the object could be modeled as a set of surfaces rather than points representing surfaces, more realistic appearing images might be produced. Of course, modeling objects as a set of surfaces, would also involve modifying (or completely changing) the calculations algorithm.

Appendix A. *Fourier Transform Filters*

Modeling sinusoidal waves with digital waves produces some errors. These errors show up in the reconstructed computer generated holographic images as noise (or a bright spot) in the center of the image. This noise can be filtered out with a Fourier transform filter. Figure A.1 illustrates this filtering process. L1 is a lens that performs a Fourier transform of the plane wave input. The filter is a transparent sheet with a small opaque dot in the center. This dot will block the low frequency (noise) components of the Fourier transform. The second lens, L2, transforms the plane wave from the frequency domain back to its original domain. The effect of this transform-filter-transform process is to remove the noise from the optical signal and the reverse the image. That is the image will be upside down.

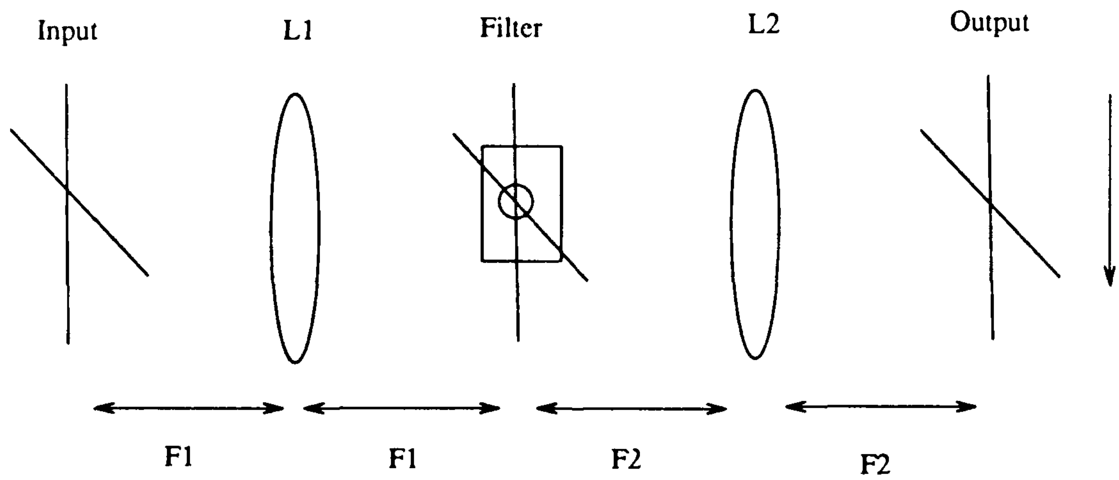

Figure A.1. Fourier Transform Filter

Bibliography

1. anonymous. *Techniques of Microphotography*. Rochester, New York: Eastman Kodak Company, 1967.
2. Arnold, Steven M. "Electron Beam Fabrication of Computer Generated Holograms," *Optical Engineering*, *24*(5):803–807 (September/October 1985).
3. Benton, Steven A. "Survey of Holographic Stereograms," *SPIE*, *367*(12):15–19 (December 1982).
4. Brooks, Frederick P. Jr. "Grasping Reality Through Illusion - Interactive Graphics Serving Science," *University of North Carolina at Chapel Hill* (March 1988).
5. Brown B. R. and Lohmann, A. W. "Complex Spatial Filtering with Binary Masks," *Applied Optics*, *5*(6):967–969 (June 1966).
6. Burch Jack J. "A Computer Algorithm for the Synthesis of Spatial Frequency Filters," *Proceedings of the IEEE*, *55*(4):599–601 (April 1967).
7. DeVelis, J. B. and Reynolds, G. O. *Theory and Application of Holography*. Reading, Massachusetts: Addison-Wesley, 1967.
8. Foley J. D. and Van Dam A. *Fundementals of Interactive Graphics*. Reading, Massachusetts: Addison-Wesley, 1984.
9. Gabor, D. "Microscopy by Reconstructed Wave-Fronts," *Proceedings of the Royal Society of London*, *197*(A 1048):454–487 (May 1949).
10. Glassner Andrew S. ,Editor. *Graphics Gems*. San Diego, California: Academic Press, 1990.
11. Goodman, J. W. *Introduction to Fourier Optics*. San Francisco, California: McGraw-Hill, 1968.
12. Hariharan, P. *Optical Holography*. Cambridge: Cambridge University Press, 1984.
13. Hecht, Eugene. *Optics*. Reading, Massachusetts: Addison-Wesley, 1988.
14. Leith, E. N. and Upatnieks, J. "Wavefront Reconstruction with Diffused Illumination and Three-dimensional Objects," *Journal of the Optical Society of America*, *54*:1295 (1964).
15. Lesem, L. B. and Hirsch, P. M. "Computer Synthesis of Holograms for 3-D Display," *Communications of the ACM*, *11*(10):661–674 (October 1968).
16. Lesem, L. B., et al. "The Kinoform: A New Wavefront Reconstruction Device," *IBM Journal of Research and Development*, *13*:150–155 (March 1969).

17. Mouser, Tom., et al. "Non-Fourier Computer Generated Holography for 3-D Display," *Practical Holography IV, Stephen A. Benton, Editor, Proceedings of the SPIE*, *1212*:325–333 (January 1990).

18. Mouser, Tommy A. *Non-Fourier Computer Generated Holography for 3-D Display*. MS thesis, AFIT/GCS/ENG/89D-13. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, 1989.

19. Nassar, Dale. "Computer-Generated Holographic Images," *Circuit Cellar INK*, pages 23–37 (April/May 1990).

20. Rawson, Eric G. "Vibrating Varifocal Mirrors for 3-D Imaging," *IEEE Spectrum*, *6*(9):37 (September 1969).

21. Ready, John, F. *Effects of High-Power Laser Radiation*. New York, New York: Academic Press, 1971.

22. Saxby, Graham. *Practical Holography*. Englewood Cliffs, New Jersey: Prentice Hall, 1988.

23. Tricoles, G. "Computer Generated Holograms: An Historical Review," *Applied Optics*, *26*(20):4351–4360 (October 1987).

24. Waters, J. P. "Holographic Image Synthesis Utilizing Theoretical Mehtods," *Applied Physics Letters*, *9*(11):405–407 (1966)).

25. Williams, Rodney D. and Garcia, Felix Jr. "Volume Visualization Displays," *Information Display*, *5*(4):8–10 (April 1989).

26. Yaroslavskii, L. P. and Merzlyakov, N.S. *Methods of Digital Holography*. New York, New York: Consultants Bureau, 1980.

27. Yatagai, Toyohiko. "Stereoscopic Approach to 3-D Display Using Computer Generated Holograms," *Applied Optics*, *15*(11):2722–2729 (November 1976).

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY (*Leave blank*)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1990	***Master's Thesis***

4. TITLE AND SUBTITLE
Computer Generated Holography As A Three-Dimensional Display Medium

5. FUNDING NUMBERS

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Institute of Technology, WPAFB OH 45433-6583

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/GCS/ENG/90D-14

9. SPONSORING MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
Approved for Public Release; Distribution Unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

Abstract

A process was developed to produce three dimensional images using computer generated holography (CGH). This process consisted of a series of steps that began with a geometric description of an object and concluded with a three dimensional holographic image being computed from a synthetic wavefront. The objects used in this series of steps (or CGH pipeline) were described geometrically as a collection of three dimensional points. The modular nature of the CGH pipeline provided a flexible platform from which to evaluate various object geometries, interference calculations algorithms, and interference pattern recording and reduction techniques. This system was implemented with general purpose computer workstations to compute the interference patterns, a postscript laser printer to record the patterns, and standard photographic reduction techniques to generate transmission holograms. Optical density filters were used to allow a hologram's virtual image to be safely viewed through the transmission hologram down the bore of the laser. The hologram's real image was observed as a collection of the object's planes captured on a white card. The reference beam was modeled as a plane wave normal to the recording surface with a constant phase angle of zero radians at all locations of the recording surface. The major constraint of this system was the limited spatial resolution of the laser printer which limited the geometric placement of the objects to be recorded.

14. SUBJECT TERMS
Computer Generated Holography (CGH), Three Dimensional Displays, Limited Spatial Resolution CGH, Fresnel Diffraction

15. NUMBER OF PAGES
78

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL