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The Estimation of Geoid Profiles in the Northwest Atlantic From Simultaneous Satellite Altimetry and Airborne Expendable Bathythermograph Sections

JIM L. MITCHELL

Naval Oceanographic and Atmospheric Research Laboratory, Stennis Space Center, Mississippi

JAN M. DASTUGUE

Sverdrup Technology, Incorporated, Stennis Space Center, Mississippi

WILLIAM J. TEAGUE AND ZACHARIAH R. HALLOCK

Naval Oceanographic and Atmospheric Research Laboratory, Stennis Space Center, Mississippi

In the NW Atlantic, the temporal mean circulation results in large amplitude surface topography precluding the reasonable use of mean altimetric surfaces or profiles for precise estimation of the geoid. We examine a procedure which (1) uses airborne expendable bathythermograph (AXBT) sections collected along altimeter ground tracks to estimate the single-pass or instantaneous surface dynamic topography, (2) employs concurrent or simultaneous satellite altimeter overflights to provide a measure of instantaneous sea level, and (3) arrives at a precise estimate of the alongtrack geoid profile by differencing the instantaneous altimetric sea level and the AXBT-derived dynamic topography. This technique is applied to several Geosat Exact Repeat Mission (ERM) ground tracks in the area of the NW Atlantic Regional Energetics Experiment (REX) using AXBT survey data collected in April and July 1987. Geoid profile estimates are repeatable between these two independent data sets to within 10–20 cm rms. This is near the estimated noise level due to barotropic variability in the Gulf Stream region. These AXBT Geosat-derived profiles provide a more consistent estimate of the alongtrack geoid gradient than do available gravimetric geoids. Analysis of the statistics of the ensemble of collinear passes provides an estimate of the geographically uncorrelated orbit error of approximately 1–2 m. The total surface topography derived as the difference between adjusted instantaneous sea level profiles and the AXBT Geosat-derived geoid profiles does not suffer significant contamination by either geographically uncorrelated or correlated orbit error.

1. INTRODUCTION

The fundamental objective of the NW Atlantic Regional Energetics Experiment (REX) is a measurement of the mean and mesoscale eddy available potential energy (APE) and kinetic energy (KE) in the Gulf Stream (from Cape Hatteras to beyond the New England Seamount Chain (NESC)). Three emerging technologies are employed to meet this objective: (1) satellite altimetry from the U.S. Navy's Geosat satellite, particularly from the collinear Geosat Exact Repeat Mission (ERM) [see *Born et al.*, 1987]; (2) extensive airborne expendable bathythermograph (AXBT) surveys and time series from bottom-moored arrays of inverted echo sounders with pressure gauges (IES/PGs); and (3) simulation of the circulation and associated energetics of the Gulf Stream through primitive-equation numerical models of the NW Atlantic. The data collected as part of REX are designed to be both complementary to [see *Hallock et al.*, 1989] and easily assimilated into numerical models for ultimate quantification of the mesoscale energetics in a dynamically consistent framework (see *Mitchell et al.* [1987] for an overview of REX).

Toward these objectives it is desirable to use Geosat, particularly Geosat ERM, altimetry as a means of measuring

the surface topography associated with mesoscale structure in the NW Atlantic. Throughout this paper we adopt the nomenclature of *Tapley et al.* [1982] (see their Figure 1) in which sea level relative to a reference ellipsoid consists of the sum of surface topography, tides, geoid, and errors (both environmental and instrumental). Numerous investigators have been remarkably successful in measuring the surface topographic variability associated with the Gulf Stream using satellite altimetry [e.g., *Thompson et al.*, 1983; *Hallock et al.*, 1989]. However, the use of the satellite altimeter as a means of directly measuring surface topography (i.e., that component of sea level, both temporal mean and variable, due to ocean currents) remains elusive owing chiefly to relatively large, unknown sea level gradients in the background geoid (upon which the topography is superimposed) and to the limitations of the space-time sampling characteristics of the single nadir-beam altimeter. The assimilation of surface topography into numerical models of the regional circulation promises a solution to the latter problem.

Previous attempts to compute absolute surface topography from satellite altimetry have estimated the background geoid using either a gravimetric surface [*Cheney*, 1982], a mean altimeter surface or alongtrack profiles [*Menard*, 1983], or a hybrid altimetric mean-gravimetric surface [*Lybanon and Crout*, 1987]. These attempts have been less than completely satisfactory owing either to the limited precision

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of the gravimetric surfaces (e.g., errors in the Marsh-Chang gravimetric surface are estimated to be of the order of the Gulf Stream surface topography [Marsh and Chang, 1978]) or to the contamination of the mean surfaces by large-amplitude temporal mean surface topography [Marsh et al., 1984].

We describe a straightforward (somewhat brute force) process for using AXBT sections collected along collinear altimeter ground tracks in conjunction with concurrent satellite overflights. This process will provide alongtrack geoid profile estimates which appear to be very precise (repeatable to within 10–20 cm rms) and are not contaminated by the temporal mean topography. Our primary use of these geoid profiles is the computation of surface topography from Geosat ERM sea level measurements along ground tracks. This estimated surface topography is not absolute in the strictest sense (i.e., we are not able to compute surface topography relative to some inertial reference ellipsoid tied to the center of mass of the Earth). Both long-wavelength orbit errors in the altimetry and seasonal steric adjustments made to the AXBT data make any attempt at such an absolute estimate impossible at present.

Section 2 provides a step-by-step description of the procedure for using AXBT sections and concurrent (hereinafter referred to as "simultaneous") Geosat ERM overflights to estimate alongtrack geoid profiles. Errors inherent in the procedure are assessed in section 3. In section 4 the rate of approach to stationarity by the ensemble mean sea level as repeat revolutions (revs) (i.e., orbits or passes) occur along a given ground track is used to assess both the order of magnitude of orbit determination error (or, simply, orbit error) and its wavelength inherent in the Geosat ERM ephemeris. Conclusions and suggestions for future application of the procedure are discussed in section 5.

2. PROCEDURE FOR ESTIMATING GEOID PROFILES

A schematic diagram of the procedure used to provide alongtrack estimated geoid profiles is shown as Figure 1. It must be emphasized that herein we are not directly interested in improving the regional geoid of the NW Atlantic (i.e., a two-dimensional surface); rather, we are only concerned with estimates of the geoid profile along Geosat ERM ground tracks for the purpose of computing regional surface topography from Geosat ERM sea level measurements. Our objectives are oceanographic, not geophysical.

Our original plan was to collect AXBT sections using U.S. Navy P-3 aircraft of the U.S. Naval Research Laboratory Flight Support Detachment along roughly 1200 km ground track segments of 13 Geosat ERM ascending ground tracks over the Gulf Stream region of the NW Atlantic from Cape Hatteras to east of the New England Seamounts. We number these Geosat ERM ground tracks as A239 to A7, where ground track A1 (for "ascending 1") is that ground track overflying Bermuda and ground tracks are numbered consecutively to the east along the equator. Three separate sets of AXBT surveys to complete this purpose were carried out (December 1986, April 1987, and July 1987). Optimistic plans called for the repeat (in triplicate) of each of these 13 sections. Problems with receivers on board the aircraft prevent us from using the data collected in December 1986 in this study. Weather and aircraft problems also took their toll on the volume of data collected in April 1987 and July 1987.

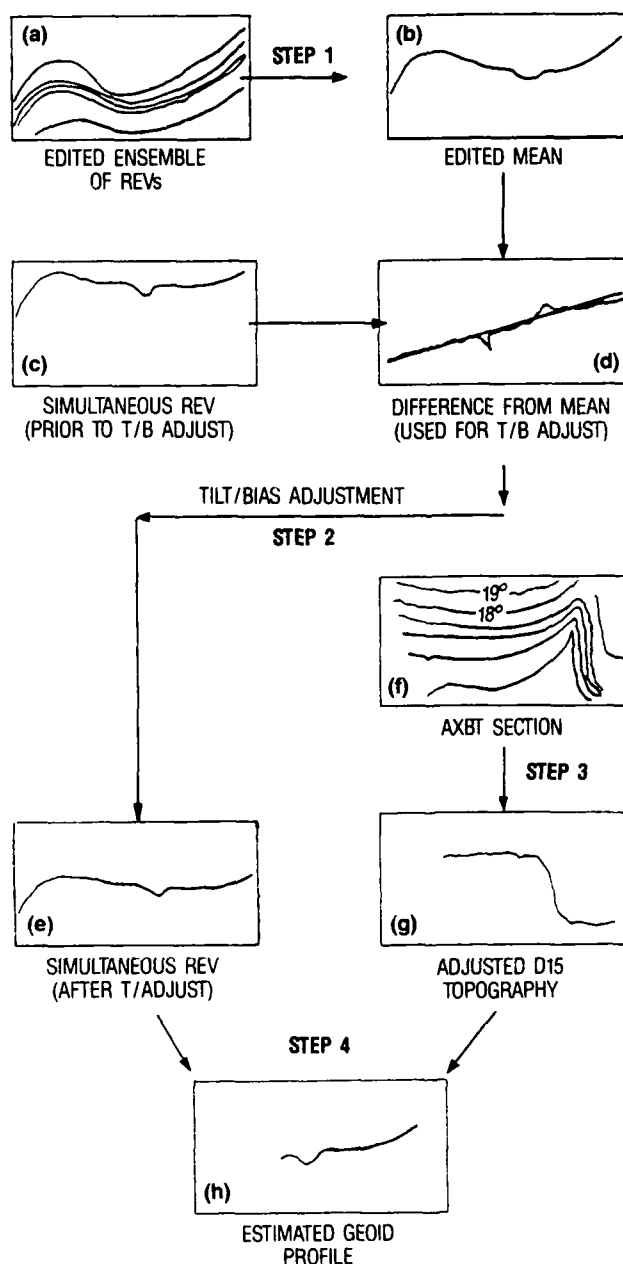


Fig. 1. Steps for estimating alongtrack geoid profiles from simultaneous altimetry and AXBT sections. Steps diagrammed in Figures 1a–1h correspond to those plotted in Figures 3a–3h.

Figure 2 posts the actual AXBT drop locations used in this study. Note that simultaneous AXBT sections and Geosat ERM overflights (as listed in Table 1) were obtained for nine separate ground tracks (three of these with usable repeated sections). Descending (NE to SW) altimeter tracks were excluded in this study owing to significant data dropouts over the NW Atlantic on these tracks.

Before detailing the steps of the geoid profile estimation procedure as diagrammed in Figure 1, and as illustrated by example in Figure 3, we briefly examine the pedigree of the altimeter input data (referred to here as step 0):

Step 0: Production of Binned Sea Level From Geosat-ERM Altimetry Data Records

Altimeter data input into the geoid estimation process consists of individual Geosat ERM passes of time-binned sea

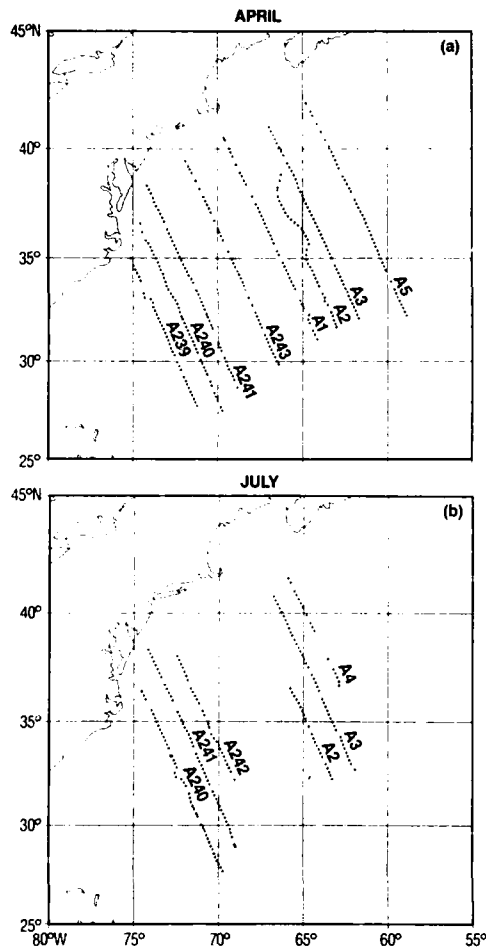


Fig. 2. Posting of AXBT drop locations during (a) April 1987 and (b) July 1987. Sections are numbered by Geosat ERM ground track number. A1 is the ascending ground track that overflies Bermuda.

level along a given ground track. These data are derived from NORDA (Naval Ocean Research and Development Activity) data records (NDRs) of Geosat altimetry, consisting of time tags and altimeter-measured satellite ranges (i.e., distance from the altimeter's antenna to the ocean's surface) and other geophysical and engineering parameters, as archived at the Naval Oceanographic and Atmospheric Re-

search Laboratory (NOARL, previously NORDA). Following a playback of Geosat's onboard recorded data, the NDRs are transmitted in near-real time from the Geosat ground station at Johns Hopkins Applied Physics Laboratory (APL) to NOARL's Satellite Data Reception and Processing System (SDRPS; see *Lybanon and Crout* [1987] for a more complete discussion). As part of the daily SDRPS processing of incoming Geosat altimetry, the following steps are performed.

1. Raw ranges are objectively edited for spurious values [West, 1986].
2. Ten-hertz range samples are smoothed to 1-Hz ranges R using a running mean.
3. The NDR is merged with an ephemeris (i.e., files consisting of satellite height r above a reference ellipsoid and the latitude and longitude of the subspacecraft point at each 1 min interval) using the time tags in the NDR.
4. The 1-Hz ranges are subtracted from the satellite height r , as interpolated using a 10-point Lagrangian interpolator, to provide a raw sea level (η_{obs}) measurement (relative to a specified reference ellipsoid):

$$r - R = \eta_{\text{obs}}$$

5. Open ocean tide heights η_t computed from the *Schwiderski* [1980] tide model and a wave height or EM-bias plus wave skewness correction totaling 5% of the altimeter-measured significant wave height (SWH) are removed from this raw sea level [Lybanon and Crout, 1987] to form a synoptic (hereinafter referred to as "instantaneous") sea level along the track (η).

$$\eta = \eta_{\text{obs}} - \eta_t + 0.05 * \text{SWH}$$

6. Linear interpolation of the alongtrack instantaneous sea level η provides a profile of sea level binned into 1-s (about 7 km alongtrack) averaging bins (η^*).

These binned sea level profiles constitute the input data for the geoid estimation procedure diagrammed in Figure 1. A step-by-step description of this procedure, along with tacit assumptions made at each step, is as follows.

Step 1: Computation of Binned Alongtrack Mean Sea Level

An edited ensemble of binned sea level values (η_i^* , where index i refers to a particular pass or rev along the specified ground track) is assembled over the first year of the Geosat ERM, using acceptable altimeter passes from the period November 9, 1986, to November 6, 1987. Each of the instantaneous (i.e., single rev) binned Geosat ERM passes provides a profile of sea level whose value within each bin (η_i^*) consists of

$$\eta_i^* = G + \zeta_i + q_i$$

where G is geoid height, ζ_i is instantaneous sea surface topography, and q_i is instantaneous (or single rev) orbit error.

For the moment, all errors other than instantaneous orbit error have been neglected (see section 3 for an error analysis of the procedure). Here acceptable passes are considered to be those that do not have obvious problems in the measurement of sea level (such as unexplained steps of several meters!). Of the approximately 21 repeats of the ERM's

TABLE 1. Summary of AXBT Sections and Revs Used in the Estimation of Geoid Profiles From Simultaneous AXBT Sections and Geosat ERM Overflights

Ground Track	No. of Revs	Simultaneous Rev	Date of Rev	Date of AXBT Section
A239	12	10924	April 15, 1987	April 17, 1987
A240	17	10967	April 18, 1987	April 20, 1987
		12187	July 12, 1987	July 13, 1987
A241	17	11010	April 21, 1987	April 22, 1987
		12230	July 15, 1987	July 15, 1987
A242	16	12273	July 18, 1987	July 17, 1987
A1	18	10938	April 16, 1987	April 17, 1987
A2	16	10981	April 19, 1987	April 20, 1987
A3	20	11024	April 22, 1987	April 22, 1987
		12244	July 16, 1987	July 15, 1987
A4	19	12287	July 19, 1987	July 17, 1987
A5	17	10866	April 11, 1987	April 10, 1987

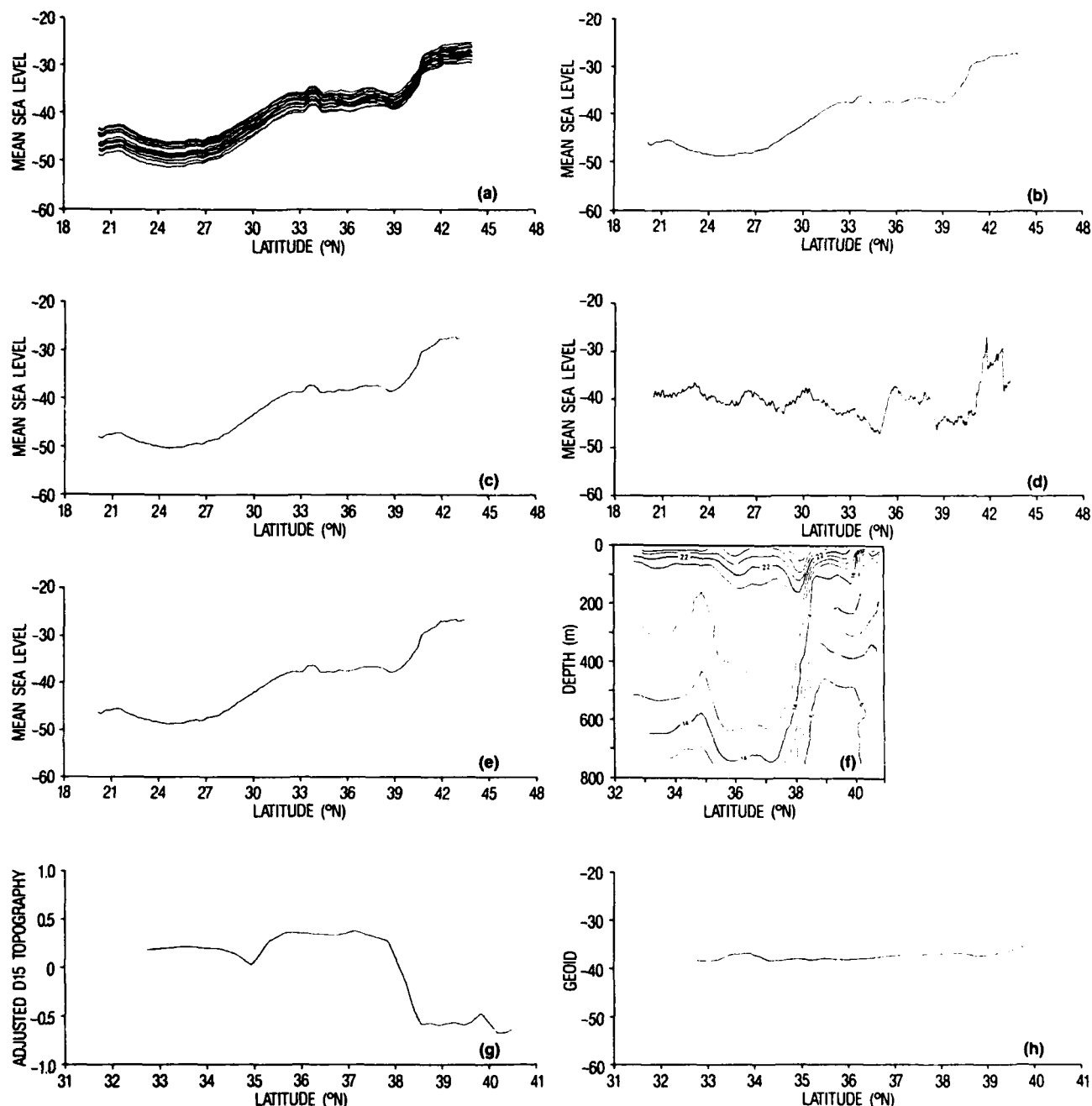


Fig. 3. Example of steps in an estimate of a geoid profile from simultaneous altimetry and an AXBT section for Geosat ERM ground track A3. In this case the simultaneous pass is provided by rev 12244 (July 16, 1987), roughly concurrent with the AXBT section (Figure 3f collected on July 15, 1987). Figures 3a-3h refer to the procedure as diagrammed in Figure 1. Though not obvious on the scale of these panels, unadjusted instantaneous sea level (Figure 3c) differs from adjusted instantaneous sea level (Figure 3e) by a bias of nearly 3 m (as estimated from a linear least squares fit to the residual profile in Figure 3d).

collinear orbit during 1 year, as few as 12 individual revs and as many as 21 individual revs were used in computing the alongtrack mean sea level profiles (see Table 1). Obvious spikes (i.e., large excursions in apparent sea level which occur over only a few 1-Hz data points) are removed, and linear interpolation is performed across the resulting gap (no more than a few seconds wide). True data gaps (i.e., sea level dropouts of more than a few seconds) are eliminated from the averaging process with care being taken that no resulting artificial step function is introduced into the ensemble mean. This editing is admittedly subjective, but we

believe that it provides cleaner, more realistic mean sea level profiles than do purely objective filtering approaches, none of which, to date, have provided a straightforward, fail-safe means of editing altimeter sea level data.

The ensemble mean sea level H over N repeats of a given ground track is computed in each bin as the arithmetic mean of the bin:

$$H = 1/N \sum_{i=1}^N (\eta_i^*) \quad (1)$$

where η_i^* is the binned sea level value for an individual satellite pass, or rev, i . The ensemble mean sea level H so formed is given by

$$H = G + Z + Q$$

where G is geoid height, Z is mean topography (sum of ζ_i/N), and Q is mean (or ensemble residual) orbit error (sum of q_i/N).

To this point we have used the terminology "orbit error" to refer only to that component of the total orbit error which is geographically uncorrelated. Actually, total orbit error has both geographically uncorrelated and correlated components. For the moment we continue to ignore the geographically correlated component and note that from one pass to the next repeat along the same track, which occurs 244 revs later, instantaneous estimated orbit error q_i is uncorrelated. Thus we assume that the summation used to compute H is performed over enough uncorrelated orbits that $Q \approx 0$ (i.e., residual geographically uncorrelated mean orbit error will be "very small"; see section 4).

Step 2: Computation of Instantaneous Alongtrack Sea Level

The binned sea level profile ($\eta_{i=m}^*$) for that instantaneous pass occurring nearly simultaneously ($i = m$; see Table 1) with the AXBT underflight survey is extracted from the ensemble. This individual pass suffers from residual orbit error which we estimate according to the tilt and bias (TB) of this pass relative to the mean sea level profile. We anticipate biases as large as 3 m in association with the quick-look Naval Astronautics Group (NAG) ephemeris used herein.

To reduce the orbit error in the individual simultaneous pass of sea level, the residual ($\eta_{i=m}'$), where

$$\eta_{i=m}' = \eta_{i=m}^* - H$$

is computed in each bin. Thus

$$\eta_{i=m}' = (\zeta_{i=m} - Z) + (q_{i=m} - Q)$$

or, assuming $Q = 0$,

$$\eta_{i=m}' = (\zeta_{i=m} - Z) + q_{i=m}$$

Since the residual orbit error is very long wavelength relative to the typical length of an altimeter pass over the Gulf Stream region (i.e., 2000 km), a reasonable estimate (see section 4) of residual orbit error for the simultaneous pass is obtained from the linear trend (tilt and bias) in the residual (η') profile given by

$$TB_{i=m} = (\eta_{i=m}')_{\text{LONG}} = (\zeta_{i=m} - Z)_{\text{LONG}} + (q_{i=m})_{\text{LONG}}$$

such that

$$\begin{aligned} \eta_{i=m}' - TB_{i=m} &= (\zeta_{i=m} - Z)_{\text{SHORT}} + (q_{i=m})_{\text{SHORT}} \\ &= (\zeta_{i=m} - Z)_{\text{SHORT}} \end{aligned}$$

Subscripts LONG and SHORT refer to linear and higher-order alongtrack trends, respectively, in the subscripted quantity. Thus η' estimates only time-variable, short-wavelength components of the surface topography. Such an approach is taken by Thompson *et al.* [1983] in which the

TABLE 2. Regression for Depths of 15°C and 17°C Isotherms and Dynamic Topography Based Upon CTD Data of Hogg *et al.* [1985]

$T, ^\circ\text{C}$	δ_0, m	$D_0, \text{dyn cm}$	Standard Error in $D, \text{dyn cm}$	$\chi, \text{dyn cm/m}$
17	317	227	± 7.1	0.191 ± 0.004
15	411	227	± 7.3	0.156 ± 0.003

ensemble mean sea level H is used to estimate the geoid. Without ancillary in situ data, this is often the only option.

The ancillary data provided by the simultaneous AXBT section allow us to proceed in a somewhat different and more inclusive fashion. Here we will use the binned mean profile to estimate residual orbit error in each of the individual revs and subsequently perform a tilt and bias adjustment to each of the instantaneous binned sea level (η_i^*) profiles along a given ground track. The best measure of sea level along a ground track at the time of the AXBT section is provided by the adjusted simultaneous pass of sea level ($\eta_{i=m}'$) given by

$$\begin{aligned} \eta_{i=m}' &= \eta_{i=m}^* - TB_{i=m} = G + \zeta_{i=m} + q_{i=m} \\ &\quad - (\zeta_{i=m} - Z)_{\text{LONG}} - (q_{i=m})_{\text{LONG}} \\ &= G + (\zeta_{i=m})_{\text{SHORT}} + (Z)_{\text{LONG}} + (q_{i=m})_{\text{SHORT}} \quad (2) \end{aligned}$$

Thus the adjusted estimate of sea level along the ground track ($\eta_{i=m}'$) is formed by the short-wavelength instantaneous topography superimposed upon the sum of the geoid and the long-wavelength mean topography (ignoring for the moment short-wavelength orbit error; see section 4).

Step 3: Estimation of Instantaneous Dynamic Topography From the AXBT Section

A complete description of the December 1986, April 1987, and July 1987 AXBT data and their analysis is provided by Teague *et al.* [1989], Dastugue *et al.* [1988], and Mitchell *et al.* [1988]. Typical AXBT sections used in this analysis represent tracks of 648 n. mi length (1200 km) with AXBTs spaced evenly at 12 n. mi (22 km) intervals along the track. U.S. Navy standard or shallow (400 m) AXBTs were dropped over the continental shelf region with alternate deep (800 m) and shallow probes employed in the deep water to the south. Figure 2 depicts the positions of AXBT stations from April 1987 and July 1987, respectively.

The depths δ of selected AXBT isotherms (15°C, 17°C) were converted to dynamic height (3 dbar over 3000 dbar) with the equation

$$D = D_0 + (\delta - \delta_0) \chi$$

where D_0 , δ_0 , and χ were determined by linear regression from historical (1983) conductivity-temperature-depth (CTD) data from the REX region [see Hogg *et al.*, 1985]. Table 2 summarizes these results. The AXBT-derived topography (D15) based upon the depth of the 15°C isotherm is typically a bit less noisy than that based upon the 17°C isotherm (i.e., D17), although the D15 profile may be somewhat shorter (on account of the AXBT's failure to reach the deeper 15°C isotherm in the Sargasso). Generally, the two profiles are very similar.

It is important to note that there is an unknown offset

between the dynamic reference level (3000 dbar) and the ellipsoid to which the altimetric level is referenced. We assume that this offset is arbitrary for regional-scale purposes and remove it by adjusting the AXBT-derived topographic profiles downward by 2.3 m (i.e., the mean D15 heights relative to 3000 dbar). In essence, we have moved the 3000-dbar level to an arbitrary zero level relative to the altimetry. By so doing, we necessarily limit the use of our estimated geoid profiles to shorter (i.e., nonlinear trend) wavelength anomalies alongtrack. Thus surface topography computed as the differenced between subsequent altimeter passes of sea level and these estimated geoid profiles will, in a strict sense, represent only the alongtrack anomalies in the surface topography relative to the local geoid profile. We must be particularly careful that estimates of mean topography based upon these geoid profile estimates are reasonable (e.g., as compared with mean topography computed from a circulation model).

Step 4: Estimation of the Binned Alongtrack Geoid Profile

The binned estimated geoid profile along each ground track ($g_{i=m}^*$) is computed as the adjusted instantaneous sea level ($\eta_{i=m}^*$) minus the adjusted AXBT-derived dynamic topography ($10/g^*D_{i=m}$) from the accompanying AXBT section:

$$g_{i=m}^* = \eta_{i=m}^* - 10/g^*D_{i=m} \quad (3)$$

Having an estimated instantaneous geoid profile ($g_{i=m}^*$) along each "calibrated" ground track now allows the use of (2) to compute the sea surface topographic profile along each ground track for other revs (other than $i = m$) as

$$\begin{aligned} \zeta_i &\cong \eta_i^* - g_{i=m}^* = G + (\zeta_i)_{\text{SHORT}} + (Z)_{\text{LONG}} - g_{i=m}^* \\ &\quad + (q_i)_{\text{SHORT}} \\ &= (\zeta_i)_{\text{SHORT}} + (Z)_{\text{LONG}} \\ &\quad + (q_i)_{\text{SHORT}} \quad g_{i=m}^* = G \end{aligned} \quad (4)$$

This surface topography represents the total surface topography (barotropic + baroclinic) and is estimated as the sum of the short wavelengths of the instantaneous topography ζ_i and the long wavelengths of the temporal mean topography Z along the selected ground track (plus some small residual orbit error).

3. ERROR ANALYSES OF GEOID PROFILE ESTIMATES

Random instantaneous errors or uncertainties will appear as errors in our alongtrack geoid profile estimate $\epsilon(g_{i=m}^*)$ as

$$\epsilon(g_{i=m}^*) = \{\epsilon^2(\eta_{i=m}^*) + \epsilon^2(D_{i=m}) + \epsilon^2(\zeta_{i=m}^{\text{BT}})\}^{1/2}$$

where $\zeta_{i=m}^{\text{BT}}$ is the simultaneous barotropic component of sea surface topography.

The direct appearance of these uncertainties is the major drawback in our instantaneous approach as opposed to estimates of the geoid profile based upon differences in temporal mean sea level and dynamic topography which will be addressed in a later paper. In this section we briefly examine likely errors in both η_i^* and D_i as summarized in Table 3.

TABLE 3. Root-Sum-Square (rss) Residual Error Budget for AXBT-Estimated Geoid Profiles

Error Source	rms Magnitude, cm
<i>Errors in Altimetric Sea Level</i>	
Range $\{\epsilon(R_i)\}$	6–10 [Lybanon and Crout, 1987]
Tides $\{\epsilon(\eta_i)\}$	1 [Lybanon and Crout, 1987]
EM bias $\{\epsilon(\eta_{\text{SWH}})_i\}$	2 [Lybanon and Crout, 1987]
Residual orbit error $\{(q_i)_{\text{SHORT}}\}$	8 (see section 4)
rss subtotal $\{\epsilon(\eta_i^*)\}$	10–13
<i>Errors in Estimating Total Topography</i>	
Regression $\{\epsilon(D_{i=m})\}$	7–8
Barotropic error $\{\epsilon(\zeta_{i=m}^{\text{BT}})\}$	10 [Hallock et al., 1989]
rss subtotal $\{\epsilon(\zeta_{i=m})\}$	12–13
Total rss	
Geoid profile error $\{\epsilon(g_{i=m}^*)\}$	16–18

Root-sum-square errors in Geosat ERM adjusted sea level ($\epsilon(\eta^*)$) arise as

$$\epsilon(\eta^*) = \{\epsilon^2(R) + \epsilon^2(\eta_i) + \epsilon^2(\eta_{\text{SWH}}) + q^2\}^{1/2}$$

where

- $\epsilon(R)$ error in altimeter range;
- $\epsilon(\eta_i)$ error in Schwiderski tide model;
- $\epsilon(\eta_{\text{SWH}})$ error in EM bias model;
- q orbit error.

Estimates of these four errors are provided in the following discussion.

Errors in satellite range ($\epsilon(R)$) arise from to unmodeled changes in the index of refraction at the 13.5 GHz operating frequency of Geosat's altimeter due to intervening water vapor and ionospheric free electrons. During solar minimum (i.e., 1986–1987), water vapor is likely the largest single source of error in range, while unmodeled ionospheric changes are small. Long-wavelength (i.e., defined as linear over the track segment) water vapor error in the simultaneous pass will be removed during the TB adjustment, as will as any other linear trend in the measured range. However, shorter-wavelength (i.e., higher-order) water vapor signatures often associated with the Gulf Stream front remain unmodeled and unobserved (since Geosat lacks a bore-sighted microwave radiometer for water vapor correction). From the error budget of G. H. Born (unpublished report, 1984) given by Lybanon and Crout [1987], we estimated the unmodeled water vapor error as approximately 6 cm rms. This is a global estimate. The error may well be somewhat higher in the Gulf Stream region. We ascribe an estimate of 6–10 cm rms for the total of both higher-order water vapor and ionospheric range error. However, the special sensor microwave imager (SSM/I) based studies of *Phoenix and Hawkins* [1990], while carried out in a different region, indicate that individual instantaneous passes of Geosat ERM altimetry (such as the simultaneous pass) can be contaminated with even larger episodic water vapor signatures.

Nonlinear errors in the Schwiderski tide model over the relatively short track segments used in this study are likely to be negligible [Schwiderski, 1980], while again, any error possessing a linear trend alongtrack will be implicitly removed during the TB adjustment (step 2).

We estimate the sum of the error due to surface wave

TABLE 4. Standard Deviations in Alongtrack Geoid Profiles

Ground Track	Standard Deviation, cm			
	July–April	AXBT–Lamont	AXBT–MM	MM–Lamont
A239	...	27.76	17.22	83.74
A240	15.69	24.53	36.72	92.96
A241	13.39	31.00	30.40	122.1
A242	...	32.00	43.68	164.5
A1	...	46.53	47.19	258.7
A2	...	81.39	91.01	232.1
A3	16.24	55.54	85.93	169.0
A4	...	49.73	97.74	123.8
A5	...	53.83	32.30	132.4
Mean	15.11	69.95	62.32	153.26
Differences		51.62	55.99	

MM, Marsh/Mader geoid heights.

skewness and residual EM bias as approximately 2 cm [see Lybanon and Crout, 1987].

Finally, as will be seen later in section 4, residual orbit determination error q contaminates the estimated geoid profiles with an alongtrack quadratic with an amplitude of approximately 8 cm.

Regression error $\epsilon(D_i)$ for either D15 or D17 is estimated to be somewhat larger than ± 7 cm. Error in representing the total topography by its dynamic (baroclinic) component is chiefly due to the presence of a barotropic component in the simultaneous topography (ξ_{i-m}^{BT}). Using IES/PG data from the initial (June 1985 to July 1986) REX deployment, Hallock *et al.* [1989] estimate the rms barotropic topographic variability as ± 10 cm. We accept this as the level of uncertainty due to the simultaneous barotropic surface topography ($\epsilon(\xi_{i-m}^{BT})$).

Table 4 presents standard deviations (i.e., mean differences removed) between various geoid height estimates including coincident transects through the Marsh/Mader [Marsh and Chang, 1978] and Lamont [Wessel and Watts, 1988] gravimetric geoids. Refer to Figures 4 and 5 for the location of transects.

Independent AXBT sections during both April 1987 and July 1987 were collected along three of these tracks (A240, A241, and A3; see Figure 4a). Note that the resulting two estimated geoid profiles are independent except that the same ensemble mean sea level H was used in the adjustment of the two simultaneous revs for orbit error. Ignoring, for the moment, seasonal steric height changes from April to July, differences in these independently estimated geoid profiles may be used as a measure of the total error of our procedure. The average standard deviation between the April and July estimates for these three ground tracks is about 15 cm (see Table 4), consistent with the Hallock *et al.* [1989] assessment of the amplitude of barotropic variability. Biases of entire profiles relative to each other appear as differences between the mean sea level value of each profile. The average profile-mean difference (July–April) between the AXBT/Geosat-estimated geoid profiles (for ground tracks A240, A241, and A3) is +10.5 cm, suggesting that seasonal steric height changes, which are expected to be of approximately

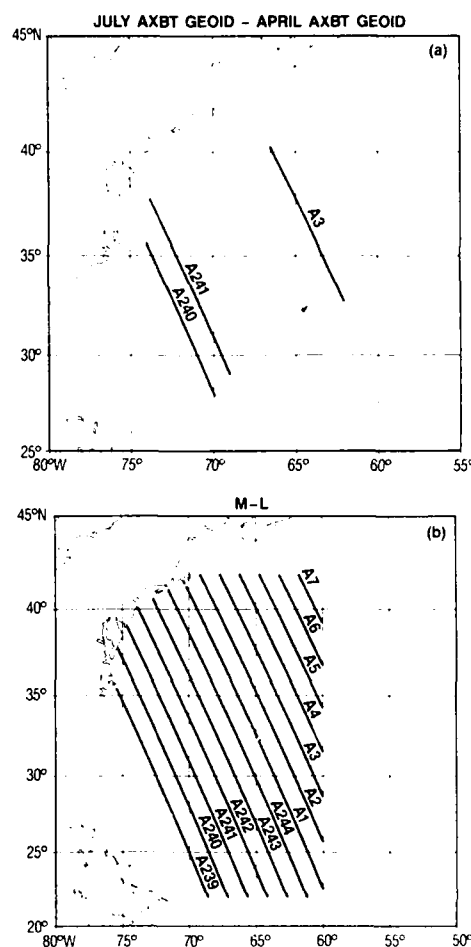


Fig. 4. Ground tracks used to compute geoid profile standard deviations listed in Table 4 for (a) repeats of AXBT/Geosat-derived geoid profiles and (b) transects through the Lamont and Marsh/Mader gravimetric geoids along Geosat-ERM ground tracks.

this magnitude and direction, are visible in our estimated geoid profiles.

The standard deviation (AXBT/Geosat–Lamont) along tracks shown in Figures 5a and 5c is 52 cm. The standard deviation (AXBT/Geosat–Marsh/Mader) along tracks shown in Figures 5b and 5d is 56 cm. The standard deviation between the two gravimetric geoids (Marsh/Mader–Lamont) along tracks shown in Figure 4b is even greater, at 153 cm. Given differences in reference levels between the two gravimetric geoids and the AXBT/Geosat-derived geoid profiles, as well as the arbitrary adjustment of the 3000-dbar level relative to the altimetric sea level in our estimation of the alongtrack geoid profiles, it is meaningless to compare profile mean differences between the various geoid profile estimates.

4. GEOSAT ERM ORBIT ERROR ANALYSIS

To compute instantaneous topography ζ_i from (4), we would like to justifiably neglect residual short wavelength orbit error in the reference pass (q_{i-m})_{SHORT}. The residual orbit error in the NAG ephemeris is very long wavelength (i.e., wavelength of the order of the orbit's circumference [see Haines *et al.*, 1989]). After the TB adjustment, the

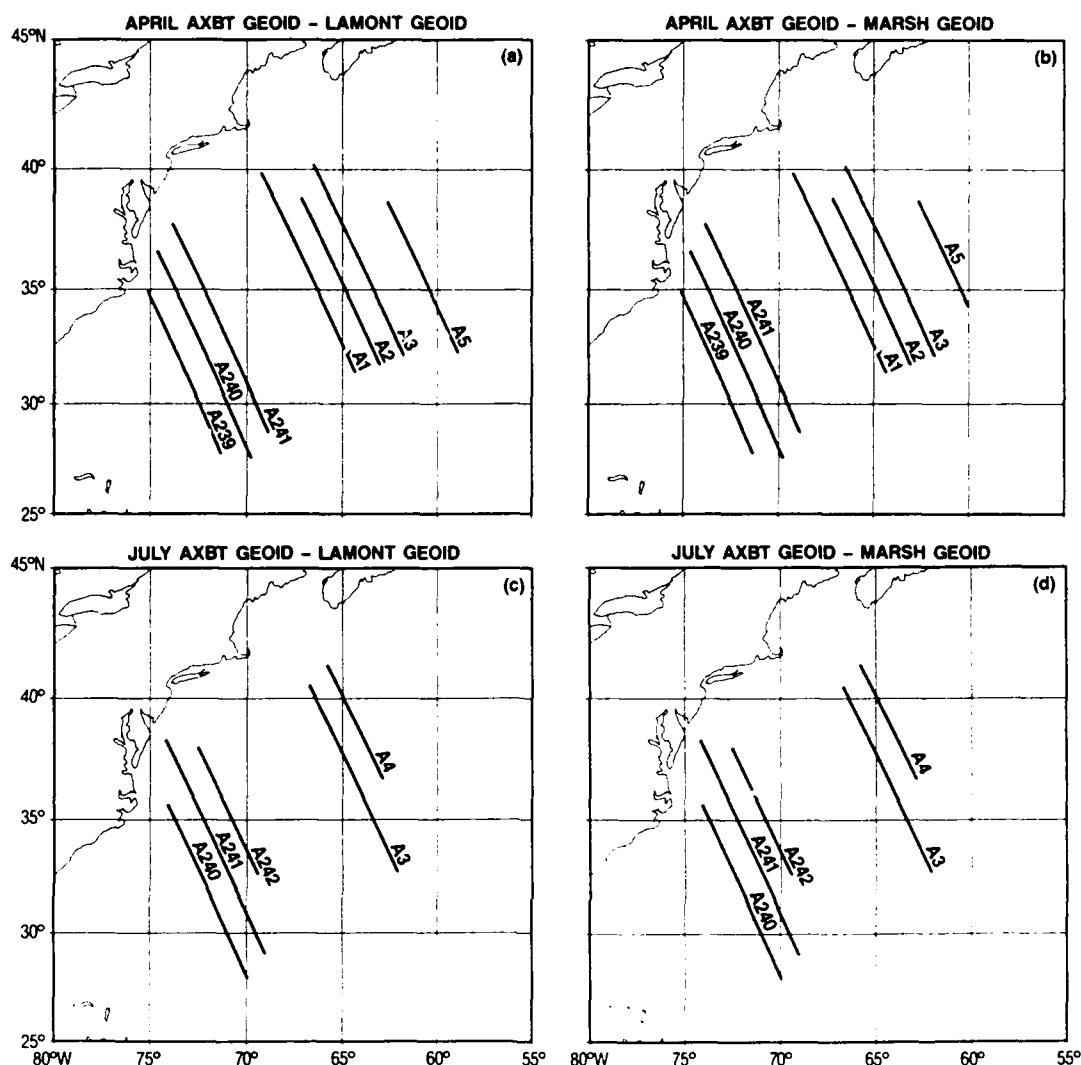


Fig. 5. Ground tracks used to compute geoid profile standard deviations listed in Table 4 for (a) differences between April 1987 AXBT/Geosat-derived geoid profiles and coincident transects through the Lamont gravimetric geoid, (b) differences between April 1987 AXBT/Geosat-derived geoid profiles and coincident transects through the Marsh/Mader gravimetric geoid, (c) differences between July 1987 AXBT/Geosat-derived geoid profiles and coincident transects through the Lamont gravimetric geoid, and (d) differences between July 1987 AXBT/Geosat-derived geoid profiles and coincident transects through the Marsh/Mader gravimetric geoid.

residual orbit error should be small (less than 10 cm; see discussion below). The extensive number of alongtrack repeat cycles used in this study allows for an assessment of this orbit error. The adjustment of the 3000-dbar level to an arbitrary zero relative to the altimetric reference ellipsoid precludes the interesting possibility of actually calibrating the orbit error by using the AXBT data.

Since the long-wavelength components of orbit error dominate fluctuations in the cumulative ensemble mean sea level, we may assess the magnitude of this error by examining the stationarity in the cumulative value of H from (1) as N increases. As an increasing number of repeated revs along a given ground track are accumulated, the cumulative rms of the biases ($\epsilon(B)$), defined here as

$$\epsilon(B) = \left[\sum_{i=1}^N (B_i^2/N) \right]^{1/2}$$

(where B_i is the bias of rev i , i.e., the constant term from the TB adjustment, and N is the total number of revs in the ensemble) approaches its stationary value. Again, B_i represents the mean track anomaly from an approximate 1-year average. Table 5 lists $\epsilon(B)$ for each of the 13 ascending ground tracks in the REX region. The cumulative rms of the biases ($\epsilon(B)$) may be used as an estimate of the amplitude of the long-wavelength orbit error inherent in the NAG ephemeris. Our a priori estimate of a NAG ephemeris orbit error of approximately 3 m rms includes both geographically correlated and uncorrelated orbit error. However, only geographically correlated orbit error is visible in our estimate of the alongtrack topography. Thus as can be seen in Table 5, an average $\epsilon(B)$ of approximately 166 cm is reasonable. The long-wavelength orbit error appears at shorter wavelengths as an artificial alongtrack gradient in sea level. This gradient is estimated using the polynomial expansion of a weighted sum of a sine and cosine error of roughly 40,000 km

TABLE 5. Ensemble Orbit Error Statistics for NW Atlantic Ascending Ground Tracks

Ground Track	$\epsilon(B)$, cm	q_{SHORT} , cm	No. of Repeats to Stationarity
A239	134	6.6	8
A240	199	9.8	10
A241	192	9.5	13
A242	108	5.3	7
A243	183	9.0	8
A244	160	7.9	10
A1	147	7.3	11
A2	161	7.9	13
A3	224	11.0	7
A4	193	9.5	9
A5	154	7.6	8
A6	141	7.0	14
A7	158	7.8	16
Mean	166	8.2	10.3

* Ground track for which there is no calibration-quality AXBT section.

wavelength (i.e., the orbital circumference):

$$q = A(B \sin \theta + C \cos \theta) \\ + A(C + B\theta + C\theta^2/2! + B\theta^3/3! + \dots)$$

where $\theta = 2\pi\Delta\xi/L$, L is orbital circumference, $\Delta\xi$ is ground track segment length, and A is estimated using $\epsilon(B)$.

Table 5 lists the estimated residual orbit error after removal of a tilt and bias (i.e., $q_{\text{SHORT}} = \theta^2/2!$) over ground track segments ($\Delta\xi$) of approximately 2000 km. Note that after removal of a tilt and bias, sea level measurements may still be contaminated by approximately 8 cm of residual quadratic orbit error over these segments. Of course, the temptation to remove increasingly higher order polynomials from sea level is abated by the knowledge that to do so will increasingly run the risk of inadvertent removal of actual oceanographic signal.

As the geographically uncorrelated component of orbit error is a once per rev error, it should be decorrelated over the 244-rev interval between consecutive repeats of a given ground track. Thus as the number of repeats of a given ground track increases, the cumulative rms bias or mean deviation (β), given by

$$\beta = [(\epsilon(B_i) - \epsilon(B_{i-1}))^2 / (i-1)]^{1/2}$$

should monotonically decrease like random noise as $1/(N-1)^{1/2}$. Note that as defined above, the rms mean deviation approaches zero at $i = N$. For example, Figure 6 illustrates for ground track A3 the time history of the decrease in β , slight decrease in q_{SHORT} (directly due to the decrease in β), and a least squares fit of a $1/(N-1)^{1/2}$ or random noise curve to β . Similar plots for each of the 13 ascending ground tracks in the REX region are characterized by a β with a sharper rate of decrease than the least squares fit of a random noise curve.

Equation (1) is used to establish a stationary mean sea level value within each bin (H). This accomplishes the damping of geographically uncorrelated orbit error. In addition, other time variable components of the measured sea level (e.g., variable surface topography or even unmodeled variable environmental corrections) are damped in the cu-

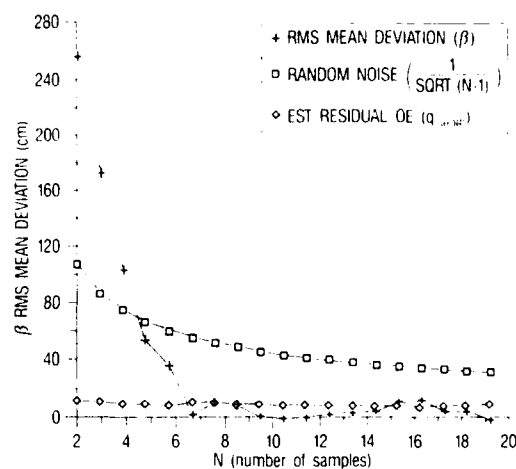


Fig. 6. Damping as a function of number of repeat revs (samples) of rms mean deviation in rev biases (β in section 4) (pluses), least squares fit to a random noise curve (squares), and estimated residual short wavelength orbit error q_{SHORT} (diamonds).

mulative estimate of H . Any temporally resolved oceanographic component of sea level (e.g., surface topography variability of frequencies lower than the sampling frequency of the collinear altimetry) will tend to damp faster than purely random noise (see appendix). In the region of the Gulf Stream, surface topographic temporal variance can be as large as several tens of centimeters (i.e., only somewhat smaller than the magnitude of the total geographically uncorrelated orbit error). As is shown in the appendix, collinear repeats which sample faster than the significant frequency of these surface topographic fluctuations, which chiefly arise from the propagation of Gulf Stream meanders, result in a cumulative ensemble mean sea level H that is uncontaminated by these fluctuations. Thus the rapid decrease in β suggests temporal resolution of a significant portion of the surface topographic variability by the repeat altimetry. Additionally, the time-incoherent components of surface topographic fluctuations will tend to result in the rapid stationarity of the cumulative ensemble mean H . For some ground tracks the decrease in β is not purely monotonic and is characterized by low-amplitude periodicity after longer (several months) averaging periods, suggesting either aliased or low frequency (i.e., time scales of months) temporally resolved periodicities in sea level. See Figure 7 for pertinent examples of both resolved and aliased topographic fluctuations.

An important parameter associated with ensemble averaging of collinear sea level measurements is provided by the number of repeat cycles necessary to reduce β to at least the level of q_{SHORT} (i.e., the number of repeat cycles necessary to reach stationarity; see appendix). Table 5 provides this information. Note that in most cases, approximately 10 repeat cycles (about 5–6 months) must be accumulated before the ensemble mean orbit error is reduced to the noise floor inherent in a TB-only adjustment (i.e., β is reduced to the level of q_{SHORT}). Beat frequencies between the collinear sampling frequency and topographic fluctuations associated with the regional oceanography make any global generalization of this result dangerous and lead to the damping undulations in H seen in Figure 7.

The ensemble statistics of Table 5 and the associated

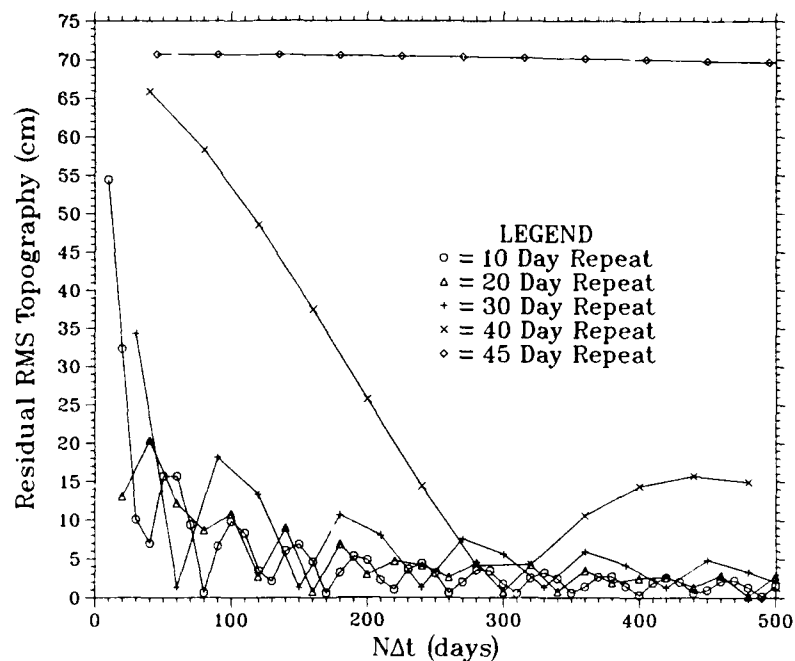


Fig. 7. Damping within an averaging bin of the residual rms surface topography associated with the propagation through the bin of a simulated 100-cm-amplitude, 330-km-wavelength, 8.4-cm/s eastward phase speed topographic fluctuation (identified by Halliwell and Mooers [1983] as the primary mode of Gulf Stream meanders). As described in the appendix, sampling is in discrete intervals (Δt). Damping curves for repeat periods of $\Delta t = 10, 20, 30, 40$, and 45 days are shown. The Geosat ERM has a repeat period of 17 days, thus resolving the major mode of Gulf Stream meander fluctuations. Severe aliasing of this 45-day meander mode occurs for repeat periods longer than 22.68 days.

curves of approach to stationarity (e.g., Figure 6) contain only geographically uncorrelated orbit error (by definition). However, geographically correlated orbit error, which is unaffected by the averaging process of (1), is a potentially major obstacle to the realization of two-dimensional maps of surface topography. Such maps depend upon the absence of corrugations or biases from one ground track to the next, regardless of whether these biases arise from geographically correlated or uncorrelated orbit error.

It is interesting to follow the propagation of both geographically correlated and uncorrelated orbit error through each step in our procedure for estimating geoid profiles from the combination of AXBT sections and simultaneous Geosat ERM altimetry. A summary of this propagation is provided by Table 6, where the algebraic rules of error propagation

TABLE 6. Step-by-Step Propagation of Geographically Correlated and Uncorrelated Orbit Error in the AXBT/Geosat Geoid Profile Estimation Procedure

Step	Description	Contains	
		Correlated OE	Uncorrelated OE
1	ensemble mean sea level	yes	no
2	TB adjusted instantaneous sea level	yes	yes (a little) -- no
3	AXBT regression topography	no	no
4	estimated geoid profile	yes	no
5	surface topography (computed as 2-4)	no	no

Rules of error propagation are as follows: yes + yes = yes; yes + no = yes; no + yes = yes; no + no = no.

are noted. In summary, note that the ensemble mean sea level (step 1 in section 2) contains only geographically correlated orbit error (by definition). Adjustment of each instantaneous sea level pass, based upon removal of the linear trend (tilt and bias) in the instantaneous sea level residual with respect to the ensemble mean, propagates correlated orbit error while introducing a very small amount of uncorrelated, higher-order orbit error (due to quadratic and higher-order orbit error not removed from the instantaneous pass). We assume that the dynamic topography based upon the AXBT sections contains little or no geographically correlated error. Hence the resulting estimated geoid profile, while contaminated by geographically correlated orbit error, is not subject to geographically uncorrelated orbit error.

Finally, since both the instantaneous adjusted pass (produced in step 2 of section 2) and the estimated geoid profile (produced in step 4 of section 2) contain geographically correlated orbit error, their difference (i.e., the surface topography) is contaminated by neither geographically correlated nor uncorrelated orbit error. This is an important characteristic of total alongtrack surface topography when computed using the AXBT/Geosat-estimated geoid profiles. In contrast, absolute surface topography computed using a gravimetrically derived geoid profile is contaminated by geographically correlated orbit error. While this contamination may not be particularly important for the computation of such quantities as alongtrack topographic variability, geographically correlated orbit error can introduce large track-to-track corrugations (in the cross-track direction) in the inferred surface topography. Unless the track-to-track biases are adequately removed, such corrugations would greatly limit (if not completely preclude) the use of altimeter data for objective input into regional numerical models of the

ocean circulation (one of the major REX objectives). Clearly, care must be taken in any altimetric attempt to estimate surface topography using a geoid which was constructed independently of that altimetry.

5. CONCLUSIONS

Our analysis of Geosat ERM collinear altimetry and simultaneous AXBT sections has resulted in the following conclusions.

1. Estimated alongtrack geoid profiles across the Gulf Stream appear to be repeatable to within 10–20 cm rms. This repeatability is consistent with an anticipated minimum error due to the presence of any instantaneous barotropic component in the surface topography of ± 10 cm. AXBT/Geosat-estimated geoid profiles typically agree better with available gravimetric geoids than the several available gravimetric geoids agree with each other.

2. The Geosat-ERM NAG ephemeris radial orbit error over the NW Atlantic has been assessed from the 1-year time series of repeated measurements of sea level. The rms magnitude of the geographically uncorrelated orbit error of approximately ± 1 –2 m is in agreement with earlier simulation studies designed to assess errors in the NAG ephemeris. Additionally, observed orbit error scales are consistent with the notion that this error occurs chiefly at a frequency of once per revolution.

A more desirable variation on this study would use deep air expendable CTDs (AXCTDs) in place of the AXBTs. The availability of reliable deep AXCTDs would allow geoid profile estimation in those regions where the dynamics are not clearly dominated by temperature (e.g., NE Pacific subarctic frontal region or the Greenland-Iceland-Norwegian Sea region). The marriage of satellite altimetry and complementary air-dropped expendable instrumentation anticipates a long and fruitful future.

APPENDIX: DEFINING MEAN TOPOGRAPHY WITH COLLINEAR GROUND TRACKS

In this appendix we examine the rate of approach to stationarity of the cumulative ensemble mean of a discretely sampled fluctuation. The unavoidably asymptotic sampling of surface topographic fluctuations by collinear altimetry will impact the nature of the approach of the ensemble mean topography to stationarity (e.g., Figure 6). In general, collinear repeat periods shorter than the significant time scale of the topographic fluctuations will resolve these fluctuations, and the resulting approach to stationarity by the ensemble mean will be accelerated. Alternately, collinear repeat periods longer than the significant time scale of the topographic fluctuations will alias the unresolved fluctuations and result in a topographic component of the ensemble mean sea level profile which does not monotonically approach stationarity. This appendix provides detailed discussion of this issue using as an example the cross-ground track propagation of a typical Gulf Stream meander across a fixed collinear Geosat ERM groundtrack.

Suppose that the observed sea surface topography within an averaging bin at time t (given by $\zeta(t)$) is equal to a fluctuating component plus a time invariant component:

$$\zeta(t) = \zeta'(x, \Psi, t) + \zeta_0 \quad (\text{A1})$$

We consider the time variable component ζ' at position x (the cross-track direction) to arise due to a topographic wave

(i.e., meander) of phase Ψ . The time invariant component ζ_0 represents the true temporal mean surface topography. We now examine the rate at which the running ensemble average estimate of the mean topography Z approaches ζ_0 .

Without any loss of generality we may consider the time variable component to be given by a series of sine waves, so that

$$\zeta'(x, \Psi, t) = \sum_{m=1}^M A_m \sin [k_m(x - x_0 - c_m t) + \Psi_m] \quad (\text{A2})$$

where we consider M wave components each of wave number k_m , phase speed c_m , and phase Ψ_m . Note that x represents the cross-track direction, where x_0 is some cross-track reference position defining the location of the ground track bin. The "exact" repeat Geosat ERM ground tracks are, in fact, allowed to wander over a cross-track envelope of up to ± 1 km. Thus x is not exactly equal to x_0 . We consider only that projection of wave components propagating transversely across track.

Flying in an exact repeat orbit, the altimeter samples the topography at point x in some regular time interval given by Δt , where $\Delta t = 17$ "days" for a 17-day repeat orbit (where a "day" equals 24 hours + the daily total precession rate of the satellite's orbit). Thus in the case of the Geosat ERM 17-day orbit, $\Delta t = 17 \times 86651$ s. From the collinear altimetry the time invariant component of the topography ζ_0 at point x on a collinear ground track is estimated by the ensemble temporal mean Z of the regularly sampled topography at point x , given by

$$Z = \overline{\zeta'(x, \Psi, t)} + \zeta_0 \quad (\text{A3})$$

where the overbar represents a running ensemble average. We are interested in the time rate at which the running ensemble mean of the fluctuating component ζ' approaches zero (i.e., the rate at which Z approaches ζ_0). For discrete sampling at intervals of Δt , this is given by the series

$$\begin{aligned} \overline{\zeta'(x, \Psi, N\Delta t)} &= \frac{1}{N+1} \sum_{n=0}^N \zeta'(x, \Psi, n\Delta t) \\ &= \frac{1}{N+1} \sum_{n=0}^N \left[\sum_{m=1}^M A_m \sin (k_m(x - x_0 - c_m n\Delta t) + \Psi_m) \right] \end{aligned} \quad (\text{A4a})$$

where N is the total number of samples in the ensemble of tracks.

We desire an analytic expression for $\overline{\zeta'}$ for the record length $N\Delta t$. First we expand the expression for $\overline{\zeta'}$ as

$$\begin{aligned} \overline{\zeta'(x, \Psi, N\Delta t)} &= \frac{1}{N+1} \left\{ \sum_{n=0}^N \left[\sum_{m=1}^M A_m \sin (k_m(x - x_0 - c_m n\Delta t) + \Psi_m) \cos (k_m c_m n\Delta t) \right] \right. \\ &\quad \left. + \sum_{n=0}^N \sum_{m=1}^M A_m \cos (k_m(x - x_0) + \Psi_m) \sin (k_m c_m n\Delta t) \right\} \end{aligned} \quad (\text{A5})$$

Recall the series formulae

$$\sum_{n=0}^N \sin n\alpha = \sum_{n=1}^N \sin n\alpha = \frac{\sin [\frac{1}{2}(N+1)\alpha] \sin (\frac{1}{2}N\alpha)}{\sin (\alpha/2)}$$

$$\sum_{n=0}^N \cos n\alpha = \frac{\sin [(N+\frac{1}{2})\alpha]}{2 \sin (\alpha/2)} + \frac{1}{2}$$

Thus (A5) may be rewritten as

$$\zeta'(x, \Psi, N\Delta t) = \frac{1}{N+1} \left\{ \sum_{m=1}^M A_m \sin [k_m(x-x_0) + \Psi_m] F_1(N, k_m c_m \Delta t) - \sum_{m=1}^M A_m \cos [k_m(x-x_0) + \Psi_m] F_2(N, k_m c_m \Delta t) \right\} \quad (A6)$$

where the functions F_1 and F_2 (for M orthogonal wave components) are defined as

$$F_1(N, k_m c_m \Delta t) = \frac{\sin [(N+\frac{1}{2})k_m c_m \Delta t]}{2 \sin (k_m c_m \Delta t/2)} + \frac{1}{2}$$

$$F_2(N, k_m c_m \Delta t) = \frac{\sin [\frac{1}{2}(N+1)k_m c_m \Delta t] \sin (\frac{1}{2}Nk_m c_m \Delta t)}{\sin (k_m c_m \Delta t/2)}$$

We now consider only one of the M components. Additionally, we shall at this point neglect the effects of very slight offsets in the ground track location and simply assume that $x = x_0$ (i.e., our analysis becomes strictly Eulerian). We are interested in the rate at which the power contained in the residual ζ' due to this monochromatic fluctuation approaches zero as a function of N . In general, the phase Ψ is arbitrary, since we will lack any independent realization of the topographic wave. Hence, we desire a phase-averaged rms measure of this residual power, which will be given by

$$\langle \zeta'^2 \rangle = \frac{1}{(N+1)^2} \{ P_1(N\Delta t) - P_2(N\Delta t) + P_3(N\Delta t) \} \quad (A7)$$

where

$$\langle \zeta'^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} [\zeta'(\Psi, N\Delta t)]^2 d\Psi$$

$$P_1(N\Delta t) = \frac{1}{2\pi} \int_0^{2\pi} F_1^2 A^2 \sin^2 \Psi d\Psi = \frac{F_1^2 A^2}{2}$$

$$P_2(N\Delta t) = \frac{1}{2\pi} \int_0^{2\pi} F_1 F_2 A^2 \sin \Psi \cos \Psi d\Psi = 0$$

$$P_3(N\Delta t) = \frac{1}{2\pi} \int_0^{2\pi} F_2^2 A^2 \cos^2 \Psi d\Psi = \frac{F_2^2 A^2}{2}$$

Thus (A7) simply becomes

$$\langle \zeta'^2 \rangle = \frac{A^2}{(N+1)^2} \left[\frac{F_1^2 + F_2^2}{2} \right] \quad (A8)$$

The square root of the residual power (i.e., residual surface topographic rms) defined by (A8) is depicted in Figure 7 for various altimeter repeat periods when sampling the most energetic scale of Gulf Stream meanders [see Halliwell and Mooers, 1983].

Finally, it is important to note that this fixed point ($x = x_0$) analysis made no attempt to take advantage of cross-track information on the phase of the fluctuation. In actual practice, if the total mission duration is longer than the time required for the topographic fluctuation to propagate from one track to the next adjacent track (which is certainly the case for Gulf Stream meanders as sampled by the 17-day repeat Geosat ERM orbit), then cross-track correlations in the topographic time series might be used to obtain a fit value for k , c , and Ψ . Spatial smoothing of the data will, to some extent, accomplish the same thing, although not nearly as precisely or eloquently.

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