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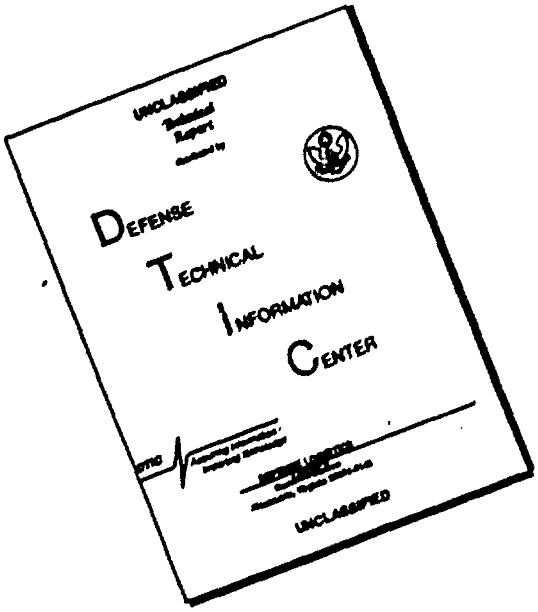

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DUALITY METHODS FOR INFINITE DIMENSIONAL OPTIMIZATION PROBLEMS

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This report includes a description of the progress and accomplishments achieved on our research program for the period June 15, 1988 to November 14, 1990 . Preliminary reports and publications of the research results are listed in section 6 . Papers presented at conferences and seminars are listed in Section 7.

Our proposal is in the general area of Nonlinear Optimization and the research issues focus on duality methods for infinite dimensional optimization problems and applications. The proposal covered the following areas:

1. Duality methods in convex optimization
2. Applications of generalized entropy
3. Semi-infinite programming
4. Nonsmooth Optimization
5. Stochastic programming

Results accomplished since this proposal has been submitted in October 1987, in the areas covered by the current grant are summarized below.

1. Duality methods in convex optimization

The results of the research in this area are described in the following papers [1], [2], [7]. In [1] we develop a general duality theory for minimizing a convex objective function over an infinite dimensional convex cone subject to a finite number of equality constraints. For a wide class of objective functions we are able to produce an explicit dual problem. This dual problem is typically finite dimensional and smooth. It can often be solved effectively and the infinite dimensional primal solution can be simply recovered. We apply our results to the problem of estimating a spectral density given finitely many correlations. The spectral estimation problem is formulated as a convex programming problem on L_p spaces with equality and non-negativity constraints. We demonstrate how classical spectral estimation results can be derived in this framework and how significant general results may be obtained on using generalized entropic functions.

Entropy optimization problems which seek to maximize the Shannon entropy functional or to minimize the Kullback-Liebler discrimination information measure between two probability measures over linear equality and inequality constraints arise in various fields of applications. Several interesting problems in engineering and social sciences include also in the constraint set quadratic inequality constraints. In [2] we derive duality results for these problems. Our approach is a direct application of Lagrangian duality and is extended to a more general class of entropy optimization problems with composite convex constraints. The results allow us to establish a simple and computationally tractible Lagrangian dual problem with only nonnegativity constraints.

Constraints qualifications in infinite dimensional programming problems are of fundamental importance to derive duality results. Many seemingly unrelated (CQ) have been recently proposed in the literature. In a joint venture with Dr. M. S. Gowda, we have investigated the relationships between old and new (CQ) in an effort to provide a unified framework to derive these (CQ). This work is summarized in [7]. Using Robinson's refinement of the duality result of Rockafeller, we demonstrate that the constraint qualification proposed by Rockafeller provides a systematic mechanism for comparing many constraints qualifications as well as to establish new results in different topological environments. Our results and examples demonstrate that these (CQ) seem to be the sharpest verifiable conditions that can be used in deriving duality results for the class of infinite dimensional optimization problems we have considered in this proposal.

2. Applications of generalized entropy

The results of our research in this area are summarized in [3], [4]. In [3] we provide a systematic survey of application of generalized entropy functional to a diversity of problems. These applications include: stochastic programming, Information theory, game theory, spectral estimation, characterization of means and characterization of probability measures.

In [4] we investigate the applicability of generalized entropy in the context of economics under uncertainty. Duality and extremal principles for certainty equivalent (in the sense of expected utility) and information measures are derived. We establish fundamental relations between the notion of divergence DIV (generalized entropy) of Csiszar and two objects used in the study of uncertainty, namely E (expectation), CE (certainty equivalent). Each member of the triple (E, CE, DIV) is the optimal value of an extremum problem involving the sum or difference of the other two. These relations provide a duality framework for economics under uncertainty and entropy, giving new concrete economic interpretations.

3. Semi-infinite linear programs

Most of the existing numerical *methods for* solving semi-infinite problems are based on discretization and approximation techniques. In our proposal we have suggested a new idea which does not attempt to destroy the continuum of the constraints by discretization. Our approach is essentially to look at a linear program in infinite dimension with a perturbed objective function. This allow us to derive a sequence of dual problems in finite dimension having the property of penalizing the constraints of a semi-infinite dual linear problem. This open the possibility of solving semi-infinite linear programs via unconstrained optimization techniques. A preliminary analysis is summarized in [11].

4. Nonsmooth Optimization

In the nature of research, new results not anticipated when writing the proposal, have lead to investigate ℓ_1-norm minimization problems and related non smooth problems and their corresponding duals. Results in this area are given in [5], [8], [10]. In [5], we proved that the solution of a least squares problem of an overdetermined system of linear equations is in the convex hull S of points, each of which is a solution of a square subsystem of the whole system. The result remain true for a weighted least squares solution, with a positive definite weighting matrix and for l_p-norm minimization problems, $1 < p < \infty$. The algorithmic implications of these results is yet to be explored in the context of l_p-norm problems and of linear programming, where the solution of a least squares problem is in the heart of Karmarkar's method. In [8], we develop a fairly general framework for smoothing nondifferentiable optimization problems. The methodology uses the concept of recession function to construct an approximate perturbed problem. A priori error bounds and corresponding perturbed duals are obtained. Another direction of research in this area was to apply our results to some problems in applied mechanics. This has been accomplished in a joint work with Drs. A. Ben-Tal and W. Yang. Our results are summarized in [6]. We consider there, the solution of a wide class of limit analysis problems (asymptotic behavior of elastic-plastic materials) that can be formulated as a pair of primal-dual optimization problems, the later being typically a l_1-norm optimization problem. To overcome the non differentiability of the objective function, we replace the original problem by an approximate smoothed one, which is controlled by a smoothing parameter. A new Combined Smoothing Successive Approximation (CSSA) method is then developed for solving this problem. The convergence of the CSSA algorithm is proved and some apriori error estimates are obtained. The performance of the above algorithm has some remarkable characteristics; in each of the numerical experiments a practically acceptable solution was obtained after only few iterations even from very bad starting points, and for a wide range of the smoothing parameter. In particular, the first iteration always produces a solution vector quite close to the final solution even with randomly generated initial solution. The subsequent iterations are less dramatic and are observed to be progressively slower. Extrapolation can then be used to accelerate convergence. To illustrate our results and give numerical evidence, two examples of truss and plate structures are solved.

Recently we studied and develop a general framework for regularizaton and approximation of nonsmooth optimization problems via proximal-like methods and augmented Lagrangians. Initial results are described in [13], [14].

5. Stochastic Programming

In [6], with J. Birge, new upper bounds are given for the expected value of a convex function. The bounds employ subgradient information and the conjugate function. In contrast to most other bounds, explicit moment information is not needed. These results are especially useful when (1) the integral is difficult to evaluate; (2) the moments are unknown or do not exist; (3) the integrand is unbounded over the support of the random vector. Examples illustrate the results. In particular, we consider integrals with logarithmic penalties and practical cases with unbounded moments. We also show how the bound may be particularly useful for integrating piecewise linear convex functions in multiple dimensions, such as the optimal value function of a stochastic linear program. In [9], the portfolio selection problem with one safe and n risky assets is analyzed via a new decision theoretic criterion, the Recourse Certainty Equivalent (RCE), introduced by the authors in 1986. Fundamental results in portfolio theory, previously studied under the expected Utility criterion (EU), such as separation theorems, comparative static analysis, and threshold values for inclusion or exclusion of risky assets in the optimal portfolio, are obtained. In contrast to the EU model, our results for the RCE maximizing investor do not impose restrictions on either the utility function or the underlying probability laws.

A list of articles, written or revised and in preparation under the grant follows.

6. Grant Supported Manuscripts

Published and accepted for publication

[1] "Spectral Estimation via Convex Programming", *Essays in Honor of Abraham Charnes*. Eds. A. Ben-Israel et al. (with A. Ben-Tal and J. M. Borwein). In Press.

[2] "Quadratically Constrained Entropy Functionals and Extensions ", *SIAM J. of Applied Mathematics*, **49**, 1989, pp.1845-1850.

[3] " On Φ-Divergence and its Applications", *Essays in Honor of Abraham Charnes*, Eds. A. Ben-Israel et al. In Press.

[4] "Certainty Equivalents and Information Measures: Duality and Extremal Principles", *J. of Mathematical Analysis and Applications* (with A. Ben-Tal and A. Ben-Israel). In Press.

[5] "Geometric Properties of the Least Square Solution of Linear Equations", *Linear Algebra and Applications*,**139**, 1990, pp.165-170 (with A. Ben-Tal).

[6] "Upper bounds on the expected value of a convex function using gradient and conjugate function information", *Mathematics of Operations Research*, **14**, 1989, pp. 745-759. (with J. Birge)

[7] " A Comparison of Constraint Qualifications in Infinite Dimensional Convex Programming ", *SIAM J. of Control and Optimization*, **28**, 1990, pp. 925-935. (with M. S. Gowda).

[8] " A smoothing technique for nondifferentiable optimization problems", *Optimization, Lecture Notes in Mathematics vol. 1405*, Ed. S. Dolecki, 1989, pp.(with A. Ben-Tal). 1-12.

[9] "Portfolio Theory for the recourse certainty equivalent maximizing investor", *Annals of Operations Research*, (with A. Ben-Tal). In Press.

[10] " A Combined Smoothing Successive Approximation Method for Limit Analysis Problems", *J. of Applied Math. and Optimization*, (with A. Ben-Tal and W. H. Yang). In Press.

[11] "Non Linear Perturbations in linear semi-infinite programming problems", *IEE CDC proceedings on Decision and Control*. In Press.

Submitted for publication

[12] "A primal-dual iterative algorithm for a maximum likelihood estimation problem", (with A. N. Iusem).

[13] "Entropic proximal mappings with applications to nonlinear programming".

[14] "Convergence analysis of a proximal-like minimization algorithm using Bregman functions".

7. Conferences, Meetings, Seminars

1. AMS-SIAM Summer Seminar on Computational Solution of Nonlinear Systems of Equations, July 18-29, 1988, Colorado State University, Ft. Collins Colorado. (Invited participant.)

2. RUTCOR, Rutgers Center for Operations Research, March 21, 1989, New Brunswick, N. J., Operations Research Seminar. " Nondifferentiable Optimization via Smoothing and Approximation". (Invited lecture)

3. SIAM Conference on Optimization, April 2-5, 1989, Boston, Mass. "Duality methods for Finite and Infinite Dimensional Optimization Problems". (Contributed paper.)

4. Mathematical Programming Workshop, April 28, 1989, The Johns Hopkins University, Baltimore, MD. "Nonsmooth Optimization". (Invited lecture)

5. Eleven Symposium on Mathematical Programming with Data Perturbations, May 25-26, 1989, Washington, D. C. " Smoothing and Approximation Methods in Nondifferentiable Optimization". (Invited talk)

6. Operation Research Society of Israel Meeting, May 1990. "Entropy-like proximal mappings and applications" (Invited talk).