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This report describes research conducted under the subject grant during the period 15 Nov 88 - 14 Nov 90. The principal areas described are nonsmooth analysis with applications to optimization, stochastic optimization, parametric design optimization, and force modeling using optimization techniques.

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Report FTR-0058

COMPUTATION AND THEORY IN NONLINEAR OPTIMIZATION

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11 January 1991

Final Technical Report for Period 15 November 1988–14 November 1990
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Prepared for:

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
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Summary.

During the period of this grant, substantial research progress was achieved in the areas of nonsmooth analysis with applications to optimization, stochastic optimization, parametric design optimization, and force modeling. Two Ph.D. students (Koohyun Park and Daniel Ralph) successfully defended their dissertations, and work was completed in areas including extension of classical embedding methods to nonsmooth functions, rank-1 approximations to the generalized Jacobian, piecewise-linear homeomorphisms, local analysis of nonsmooth functions, and parameter design optimization for complex systems with applications in manufacturing.

1. Research Objectives.

The research objectives of this program were:

1. To extend the theory base in nonsmooth analysis.

2. To apply that theory to develop algorithms that extend the range of optimization problems that can be solved.

3. To improve the understanding of convergence behavior in stochastic optimization.

4. To apply that understanding to the design and analysis of improved algorithms for solving optimization problems in important applications areas such as manufacturing.

2. Status of the Research.

Under this grant progress was made in four main areas: nonsmooth analysis and its applications; stochastic optimization; parametric design optimization; and force modeling using optimization. These activities will be described in turn.

The research program in nonsmooth analysis concentrated on extending the base of theoretical knowledge in the field and on applying that base to yield improved algorithmic methods for solving practical problems. Both components were considered essential to the program: theoretical analysis without any apparent application is of limited interest, while real advances in algorithms are unlikely without new knowledge upon which to base them.

The two main areas of nonsmooth analysis in which work was concentrated are local and global analysis of nonsmooth nonlinear functions. In local analysis, D. Ralph completed and successfully defended his Ph.D. dissertation on approximations to the generalized Jacobian; He also worked on a line of algorithmic application of local analysis, to develop for nonsmooth functions an analogue of the damped Newton method for smooth functions.

Koohyun Park completed his development of continuation methods for nonlinear programming, and reported his results in a thesis that he successfully defended in the summer of 1989.

Stephen M. Robinson continued work on local approximation of nonsmooth functions, particularly the development of implicit-function results and their application to numerical algorithms. He also established a basic characterization of when certain piecewise-linear maps, called *normal maps,* are homeomorphisms. Papers resulting from this work are reported in Section 3.

Robinson also worked on a method to decouple very large stochastic optimization problems so that they could be solved piece-by-piece using parallel methods. He succeeded in obtaining such a decomposition for the new "scenario analysis" method of stochastic optimization recently proposed by Rockafellar and Wets. A paper describing this work is reported in Section 3.

Work by Laura Morley resulted in progress toward the objective of optimizing parameter selection in the design of complex systems. This optimization has up to now been performed by experimental design techniques (the so-called "Taguchi methods"), but we have preliminary results indicating that it is feasible, and quite effective, to apply standard nonlinear optimization methods. Work in this area is continuing under the successor grant, AFOSR-91-0089.

Work by Robinson on force modeling using optimization produced two papers, reported in Section 3. The main result here was that it is possible to extend the so-called "eigenvalue weights," long used in combat modeling, to much more general situations. For example, models can be built in which non-killing systems have positive value. It was further shown that the eigenvalue weights are actually shadow prices of a mathematical programming problem.

3. Results from Research Activity.

The following dissertations and papers resulted from work performed under this grant.

a. Dissertations.

[D1] Koohyun Park, *Continuation Methods for Nonlinear Programming,* Ph.D. Dissertation, Department of Industrial Engineering, University of Wisconsin–Madison, 1989.

[D2] Daniel Ralph, *Rank-1 Support Functionals and the Rank-1 Generalized Jacobian. Piecewise Linear Homeomorphisms,* Ph.D. Dissertation, Computer Sciences Department, University of Wisconsin–Madison, 1990.

b. Papers.

[P1] S.M. Robinson, "Extended scenario analysis," accepted by *Annals of Operations Research.*

[P2] S.M. Robinson, "An implicit-function theorem for a class of nonsmooth functions," accepted by *Mathematics of Operations Research.*

[P3] K. Park, "New continuation methods for nonlinear programming by converting optimality conditions into nonlinear equations," *Technical Report 89-12,* Department of Industrial Engineering, University of Wisconsin–Madison, December 1989.

[P4] K. Park, "Stability and sensitivity analysis based multiple-path continuation method for nonlinear optimization," *Technical Report 89-13,* Department of Industrial Engineering, University of Wisconsin–Madison, December 1989.

[P5] K. Park, "Newton's method for nonsmooth equations and constrained optimization," *Technical Report 89-14,* Department of Industrial Engineering, University of Wisconsin–Madison, December 1989.

[P6] S.M. Robinson, "Mathematical foundations of nonsmooth embedding methods," *Mathematical Programming* **48** (1990) 221–229.

[P7] S.M. Robinson, "Normal maps induced by linear transformations," Preprint, March 1990, conditionally accepted by *Mathematics of Operations Research.*

[P8] S.M. Robinson, "Shadow prices for measures of effectiveness, I: Linear Model," Preprint, October 1990, submitted for publication.

[P9] S.M. Robinson, "Shadow prices for measures of effectiveness, II: General Model," Preprint, October 1990, submitted for publication.

4. Participating Professionals.

The following professional personnel participated in this research program during some part of the reporting period:

- Stephen M. Robinson, Professor of Industrial Engineering and Computer Sciences.
- Koohyun Park, Research Assistant.
- Daniel Ralph, Research Assistant.
- Chia-Ling Hwang, Research Assistant.
- Sen-Shung Chou, Research Assistant.
- Jun Ren, Research Assistant.
- Laura Morley, Research Assistant.
- M. Naceur Azaïez, Research Assistant.

5. Degrees Awarded.

Koohyun Park received the degree of Doctor of Philosophy in Industrial Engineering in August 1989.

Daniel Ralph received the degreee of Doctor of Philosophy in Computer Sciences in May 1990.

6. Interactions.

The following scientific activities received support from this grant.

a. Koohyun Park attended the joint National Meeting of the Operations Research Society of America, The Institute of Management Sciences, and the Canadian Operations Research Society, held in Vancouver, B.C., Canada in May 1989. He chaired a session at this meeting, and presented a lecture entitled "Continuation Methods for Nonlinear Programming."

b. Stephen M. Robinson attended the joint National Meeting of the Operations Research Society of America, The Institute of Management Sciences, and the Canadian Operations Research Society, held in Vancouver, B.C., Canada in May 1989. He presented an invited lecture entitled, "Nonsmooth Analysis and Variational Inequalities."

c. Prof. Dr. Klaus Ritter, Director of the Institute for Applied Mathematics and Statistics of the Technical University of München, West Germany, visited the University of Wisconsin–Madison in late September and early October of 1989. He conferred with the staff of this research project and presented a public lecture entitled, "Parallel Algorithms for Data Analysis in Maintenance and Quality Control Problems."

d. Daniel Ralph attended the Joint Mathematics Meetings in Louisville, KY, 17–20 January 1990. He presented an invited lecture entitled, "The Rank-1 Generalized Jacobian."

e. Prof. Alexander Ioffe of the Department of Mathematics, Technion–Israel Institute of Technology, visited the University of Wisconsin–Madison in September 1990. He conferred with the staff of this research project and presented a public lecture entitled. "Nonsmooth Analysis and Optimization Theory."

e. Stephen M. Robinson attended the Joint National Meeting of The Institute of Management Sciences and the Operations Research Society of America in Las Vegas, NV 7–9 May 1990. He presented an invited paper entitled, "Characterization of Regularity for Linear Generalized Equations."

In addition, Robinson participated in numerous other professional meetings and activities, with support from sources other than this grant.

7. Inventions and Patent Disclosures.

During the work under this grant, there were no inventions that appeared to have any patent possibilities. Other (non-patentable) discoveries are contained in the papers reported above.

8. Other Information.

Further information about any of the activities reported above, or other aspects of this research program, can be obtained from the principal investigator, Stephen M. Robinson, at the Department of Industrial Engineering, University of Wisconsin–Madison, 1513 University Avenue, Madison, WI 53706, telephone (608) 263-6862.