

1

AD-A233 431

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
Defence nationale



A RUDIMENTARY OVERVIEW OF THE CAPABILITIES AND PROBLEMS CONCERNING THE FINITE-ELEMENT METHOD

by

J.J.A. Klaasen

DTIC
APR 2 1991
D

PHOTOCOPY

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
REPORT NO.1067

Canada

February 1991
Ottawa

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
1067
1991

91 4 10 140

ABSTRACT

The finite-element method is very powerful and flexible to model complex geometries with computers, because the region of interest can be subdivided into finite elements accurately. The advantages and drawbacks of the finite-element method will be the focus of the rudimentary investigation presented in this report. Especially, the requirements in terms of computational effort and computer memory storage will be investigated with respect to Nuclear ElectroMagnetic Pulse (NEMP) research requirements, i.e., with configurations frequently found in NEMP research such as coupling and interaction studies, simulator design and sensor design. Such configurations are often three-dimensional and of intricate geometry.

It is found that with present day computer capability it is not yet possible to solve real-life three-dimensional geometries with the finite-element method, because of the memory requirements needed to store the resulting system of equations.

Two-dimensional geometries can at present be solved with the finite-element method, but the usefulness of two-dimensional geometries for NEMP research purposes is questionable.

RÉSUMÉ

Les avantages et inconvénients de la méthode des éléments finis sont présentés dans ce rapport. Plus particulièrement, les exigences en termes d'utilisation de mémoire et de temps d'ordinateur sont étudiées plus en détails. Cette étude est surtout orientée vers l'utilisation de la méthode appliquée à la recherches des effets des impulsions électromagnétiques (IEM), tels que l'étude des interactions électromagnétiques, de capteurs de champ électromagnétique et de simulateurs IEM. De tels problèmes sont généralement représentées par une géométrie tridimensionnelle très complexe.

Il a été trouvé qu'il n'est pas possible de résoudre des problèmes tridimensionnels complexes en utilisant la méthode des éléments finis; ces problèmes excèdent la capacité de mémoire des ordinateurs actuels.

Il est toutefois possible de résoudre des problèmes bidimensionnels par cette méthode; son utilité pour l'étude de problèmes IEM est par contre très limitée.

EXECUTIVE SUMMARY

At DREO, a number of well-known methods have been used to solve complicated electromagnetic scattering problems, which occur in Electromagnetic Pulse (EMP) analyses. We mention the Method of Moments and Transmission Line analyses. Most of these methods work in the frequency domain, and have certain disadvantages such as crude modelling capability and/or require prohibitively large computer resources and computer time. Therefore, the quest for better suited methods for EMP analyses continues.

More and more researchers employ the finite-element method to solve complicated scattering configurations. The finite-element method has unsurpassed modelling capabilities and looks promising for EMP research. The advantages and drawbacks of this relatively new method are presented in this report.

Table of Contents

	Page
Abstract/Résumé	iii
Executive summary	v
1 Introduction	1
2 The finite-element method	3
2.1 The finite-element formulation	3
2.2 Boundary conditions	5
2.3 Interior and exterior problems	5
2.4 Subdivision of the problem domain	6
3 Computer requirements for a time-domain finite-element method	7
4 Conclusions	8
References	9

1 INTRODUCTION

The finite-element method has been used in mechanical engineering for many years but was first applied to electromagnetics only 15 years ago. It is characterized by an intricate formulation, but the finite-element method is very powerful and flexible to model complex geometries with computers because the region of interest can be subdivided into finite elements accurately.

Probably the first book on the finite-element method applied to electromagnetics was written by Silvester [5] in 1980. Before that, articles on electromagnetic applications of the finite-element method have appeared in the literature since 1975. At first, the method was used to solve eddy-current problems and related problems. These applications allowed two-dimensional finite-element formulations based upon Maxwell's equations and neglected the displacement current (quasi-static solutions), thereby decreasing the complexity of the problem to be solved. The finite-element formulation was obtained using a variational principle, i.e., based on the Rayleigh-Ritz principle (Steele [3], p. 68).

Since 1984, researchers in electromagnetic disciplines other than power engineering have become more and more interested in the finite-element method because of its powerful properties. But many electromagnetic disciplines require that Maxwell's equations are solved without neglecting the displacement current. As examples we mention exploration geophysics (Stam [2]) and medical applications (hyperthermia). Modern finite-element formulations (in the time as well as in the frequency domain), which solve the full equations of Maxwell, are usually based on a Galerkin approach, i.e., the method of weighted residuals²⁾ with the same weighting and basis functions.

The advantages and drawbacks of the finite-element method will be the focus of the rudimentary investigation presented in this report. Especially, the requirements in terms of computer time and computer memory storage will be investigated with respect to NEMP research requirements, i.e., with configurations frequently found in NEMP research such as coupling and interaction studies, simulator and sensor design. Such configurations are often three-dimensional and of complicated geometry. One usually prefers time-domain formulations, because the large bandwidth of the NEMP spectrum requires that the configuration must be solved for many frequencies. Hence, time-domain type of formulations tends to require less computational time than frequency-domain formulations.

So, the principal intention of this report is to try to answer the question:

²⁾ The Method of Moments is a one-dimensional method of weighted residuals, and can be seen as a one-dimensional finite-element method. See Harrington [6].

Can the finite-element method be used for NEMP research purposes?

Since we have just mentioned what typical NEMP research configurations are, this question can be reformulated more universally as:

Can the finite-element method be used to solve complicated three-dimensional configurations?

Of course, the answer to this question is subject to the limitations the current state of computer technology offers.

2 THE FINITE-ELEMENT METHOD

As mentioned in the introduction, the finite-element method is capable of solving complicated geometries by subdividing the region of interest in finite elements. Compared with for example the finite-difference method, it is not restricted to a particular grid structure.

2.1 The finite-element formulation

For two-dimensional problems the finite element often used is a triangle or quadrilateral segment and in three-dimensional problems an isoparametric hexahedron. Fig. 2.1 depicts some of these finite elements.

With the weighted-residual approach, the unknown field quantities to be solved are projected on a basis of known basis functions, i.e., the field quantities are written in terms of known basis functions with coefficients to be determined later. These usually linear basis functions span over each finite element. The basis function pertaining to a node of a specific finite element has unity magnitude in that node-point only. Such a node is called the supporting node of the basis function. At every node other than the supporting node this basis function vanishes.

Finally, the finite-element formulation is obtained by weighting the residual of the pertinent differential-equation over each finite element to zero with a suitable weighting function. With the Galerkin-type approach the weighting function is the same as the basis function. The above described procedure yields a sparse³⁾ system of equations where the unknown coefficients, which result from the expansion of the unknown field quantities in basis functions, have to be solved for. In later sections it will be shown that storing and solving this system of equations is the main problem with the application of the finite-element problem.

³⁾ A matrix with few non-zero elements.

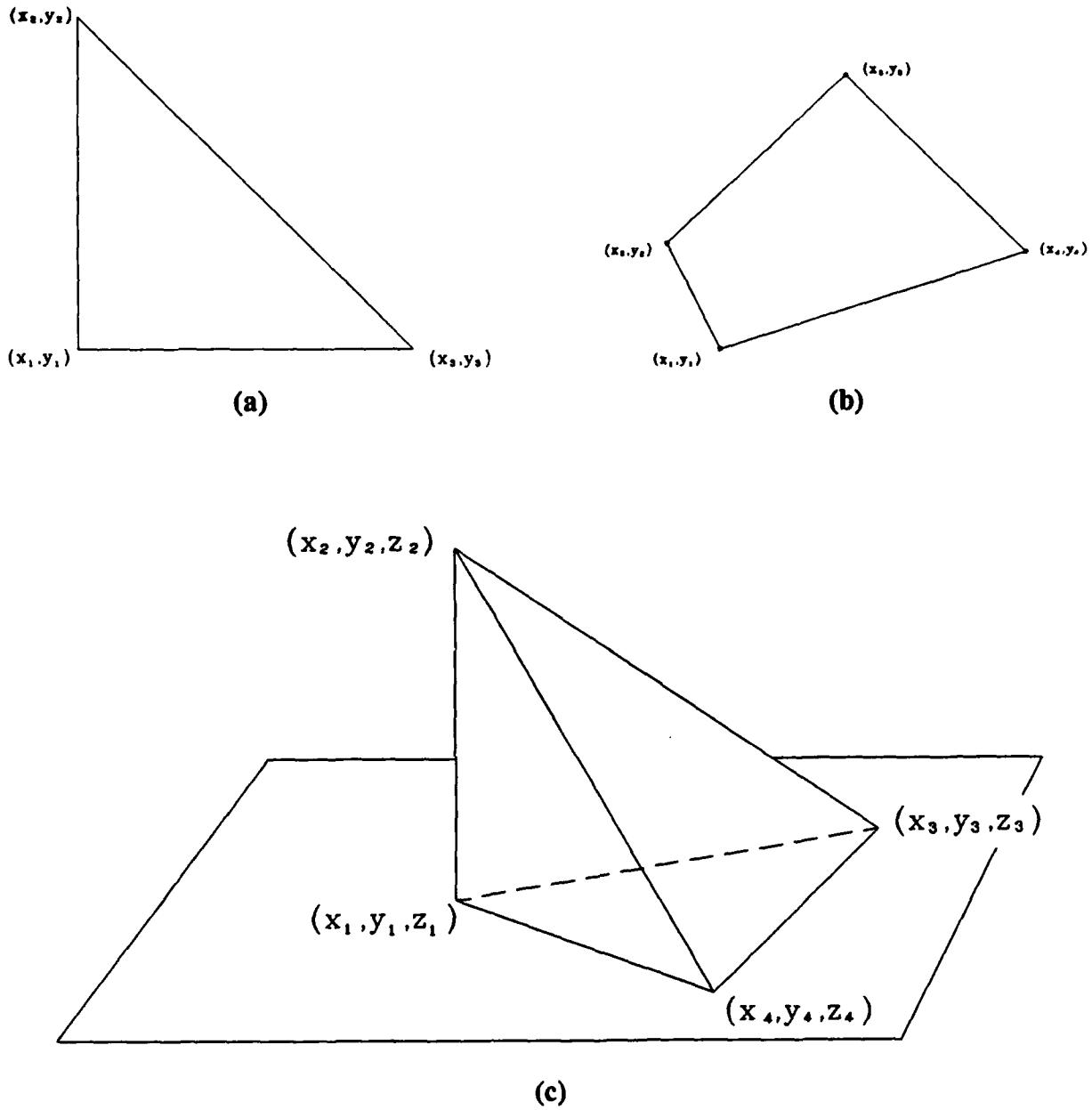


Fig. 2.1. Some often used finite elements.
 a) 2D triangle finite element
 b) 2D quadrilateral finite element
 c) 3D hexahedron finite element

2.2 Boundary conditions

At a boundary between finite elements with different electromagnetic properties, e.g., a boundary between free space and a scattering object or between two adjacent finite elements in a strongly inhomogeneous medium, the basis functions and finite elements have to be constructed so that the boundary conditions are satisfied—see pp. 34 - 38 Stratton [7]. The tangential parts of the electric field and the magnetic flux density must be enforced to be continuous across such adjacent finite elements and the normal parts may jump by a finite amount. In Mur [8] and [9], Mur describes finite elements and basis functions which preserve the boundary conditions. Mur calls his finite elements "boundary elements." Most finite elements used in the literature or in commercially available computer programs do not preserve the boundary conditions.

2.3 Interior and exterior problems

In general, the problem to be solved can be divided in two different kind of problems: an interior or exterior problem. An interior problem is a configuration where the region of interest is bounded, like waveguides, while with an exterior problem it is not. The latter is the more difficult one to solve because the unbounded region of interest has to be restricted in size as much as possible, because of available computer memory.

The truncation of the unbounded region of interest into a bounded problem domain which is sufficiently small in size that it can be stored in the computer's memory yields an artificial outer boundary. This boundary is also known as the numerical domain boundary. Appropriate boundary conditions have to be applied upon this boundary in such a way that the fields in the configuration are not influenced. When the sources generating the incident field are located within the problem domain, and as long as the fields have not yet reached the numerical domain boundary, zero-valued boundary conditions for the field quantities have to be applied. When the sources are located outside the problem domain, known prescribed boundary conditions have to be enforced upon the pertinent differential equations. Again as long as the fields inside the problem domain have not yet reached the numerical domain boundary. Usually the time window is then too small. To circumvent this, either the size of the problem domain has to be increased—thereby the number of finite elements usually becomes prohibitively large—or appropriate boundary conditions have to be applied which take into account the electromagnetic radiation through the numerical domain boundary.

Applying these appropriate boundary conditions efficiently is at present still a subject of research. It is known as the problem of "absorbing boundary conditions." This problem is not unique for the finite-element method; it also occurs with the finite-difference method.

2.4 Subdivision of the problem domain

The number of finite elements required to model the problem domain depends strongly on:

- the geometric complexity of the region of interest and its boundaries,
- the electromagnetic properties of the materials,
- the desired time resolution.

Obviously, the more complex the geometry of the configuration, the more finite elements are needed.

Costache (pp. 13 - 15, Costache [1]) found that for two-dimensional highly-lossy media, e.g., a copper or metal sheet, the number of finite elements required to model the sheet in the frequency domain is skin depth dependent. As the skin depth decreases, it becomes harder for the (linear) basis functions associated with the finite elements in the sheet to follow the field variations. For accurate results the size of the finite elements must be at least ten times the skin depth. For time-domain problems one expects that an analogous rule exists.

Another factor which determines the required number of finite elements is the desired time resolution Δt . Let h denote the size of the smallest finite element of the mesh. We then require that

$$\Delta t \leq \frac{h}{c_0\sqrt{3}}.$$

This condition is known as the Courant condition. So, an additional restraint for the time-domain case is that the size of the smallest finite element must be larger than the distance a wavefront propagates in $\sqrt{3} \Delta t$ seconds.

3 COMPUTER REQUIREMENTS FOR A TIME-DOMAIN FINITE-ELEMENT METHOD⁴⁾

The Maxwell equations allow a representation for the electric and magnetic fields, which reduces the six unknowns (for a three-dimensional problem) to four by introducing a vector potential \underline{A} (three unknowns) and a scalar potential ϕ (one additional unknown)—see pp. 23 - 34 Stratton [7]. Now, let the number of finite elements in which we subdivide the problem domain be denoted by N . For a three-dimensional problem the hexahedron finite-element, which has eight nodes, is often used. The number of unknowns is then $4 \times 8 \times N = 32 N$. The resulting system of equations to be solved is a $32 N \times 32 N$ system. To store this matrix equation with single precision (REAL *4 in FORTRAN) the minimum amount of computer memory required is $(4 \times 32 N)^2$ bytes = $16 N^2$ Kb. For frequency-domain solutions the necessary amount of memory is twice as large, because the matrices of the system of equations contain complex valued elements. This analysis is a worst-case estimate.

So, for a volume subdivided in only 64 finite elements the storage requirement is 64 Mb, which demonstrates that even for a modest number of finite elements the storage space becomes prohibitively large. Consequently, only very simple three-dimensional problems without much detail can be solved.

Although the finite-element method yields a sparse matrix, a sparse-matrix equation-solver which could reduce the computer memory storage requirements significantly seems not to exist. It can be concluded because most articles on the finite-element method give two-dimensional results. When these articles present three-dimensional results, which is not very often, the examples are always very simple (often highly symmetric, which allows reduction of the number of unknowns). Even for such simple configurations the required computer time is quite considerable, Mur [8] and [9].

Two-dimensional geometries require far less computer memory storage, but still in the order of N^2 .

⁴⁾ Most of the material presented in this Chapter is taken from Costache [1].

4 CONCLUSIONS

At present, some three-dimensional finite-element formulations are available. However, to be able to solve complicated three-dimensional electromagnetic scattering problems with the finite-element method, computers are required with memories and speeds which are at present not yet at hand. The problem lies with the enormous amount of memory required to store the resulting system of equations. Unless schemes are conceived to store this sparse matrix equation⁵⁾ more efficiently, the finite-element method is expected not to be useful for solving real-life three-dimensional electromagnetic problems in the near future.

The computation time to solve three dimensional problems is high. This is mainly caused by the time needed to solve the large system of equations. Iterative techniques have the preference above direct methods, because they are usually faster. It may be possible to formulate finite-element solutions which exploit the benefits of parallel processing, so that the computational time can be decreased significantly.

Two-dimensional problems can be solved with presently available computers, but are of limited use for NEMP research.

⁵⁾ The main matrix has only a few non-zero elements.

REFERENCES

- [1] *Costache, G., Ney, M.,*
A Study on the Penetration of Electromagnetic Fields in Cavities⁶⁾, Report University of Ottawa, August 1989, DREO Contract W7714-8-5560/01-SS.
- [2] *Stam, H.J.,*
A Time-Domain Finite-Element Method for the Computation of Three-Dimensional Acoustic Wave Fields in Inhomogeneous Fluids and Solids, Ph.D. Thesis, Et/EM 1990/12, Delft University of Technology, Faculty of Electrical Engineering, Laboratory of Electromagnetic Research.
- [3] *Steele, C.W.,*
Numerical Computation of Electric and Magnetic Fields, Van Nostrand Reinhold Company, New York, 1987.
- [4] *Lee, K.S.H.,*
EMP Interaction: Principles, Techniques, and Reference Data, Hemisphere Publishing Corporation, New York, 1986, pp. 69 - 88.
- [5] *Silvester, P., Chari, M.V.K.,*
Finite Elements in Electrical and Magnetic Field Problems, John Wiley & Sons, New York, 1980.
- [6] *Harrington, R.F.,*
Field Computations by Moment Methods, The Macmillan Company, New York, 1968.
- [7] *Stratton, J.A.,*
Electromagnetic Theory, McGraw-Hill Book Company, Inc., New York, 1941, pp. 23 - 34 and pp. 34 - 38.
- [8] *Mur, G., De Hoop, A.T.,*
"A Finite-Element Method For Computing Three-Dimensional Electromagnetic Fields in Inhomogeneous Media," IEEE Transactions on Magnetics, Vol. MAG-21, No. 6, November 1985, pp. 2188 - 2191.

⁶⁾ Although the title of this report does not show, this report studies penetration in cavities with the finite-element method.

- [9] *Mur, G., De Hoop, A.T.,*
"Optimum Choice of Finite Elements for Computing Three-Dimensional Electro-
magnetic Fields in Inhomogeneous Media," IEEE Transactions on Magnetics, Vol.
MAG-24, No. 1, January 1988, pp. 330 - 333.

SECURITY CLASSIFICATION OF FORM
(highest classification of Title, Abstract, Keywords)

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.) ELECTRONICS DIVISION/NUCLEAR EFFECTS SECTION DEFENCE RESEARCH ESTABLISHMENT OTTAWA OTTAWA, ONTARIO K1A 0Z4		2. SECURITY CLASSIFICATION (overall security classification of the document, including special warning terms if applicable) UNCLASSIFIED	
3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C or U) in parentheses after the title.) A RUDIMENTARY OVERVIEW OF THE CAPABILITIES AND PROBLEMS CONCERNING THE FINITE-ELEMENT METHOD (U)			
4. AUTHORS (Last name, first name, middle initial) J. J. A. KLAASEN			
5. DATE OF PUBLICATION (month and year of publication of document) NOVEMBER 1990	6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.) 12	6b. NO. OF REFS (total cited in document) 9	
7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.) DREO REPORT BY A VISITING NATO FELLOW			
8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.) DEPARTMENT OF NATIONAL DEFENCE DEFENCE RESEARCH ESTABLISHMENT OTTAWA OTTAWA, ONTARIO K1A 0Z4			
9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant) 041LT	9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)		
10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.) DREO REPORT 1067	10b. OTHER DOCUMENT NOS. (Any other numbers which may be assigned this document either by the originator or by the sponsor)		
11. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification) <input checked="" type="checkbox"/> Unlimited distribution <input type="checkbox"/> Distribution limited to defence departments and defence contractors; further distribution only as approved <input type="checkbox"/> Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved <input type="checkbox"/> Distribution limited to government departments and agencies; further distribution only as approved <input type="checkbox"/> Distribution limited to defence departments; further distribution only as approved <input type="checkbox"/> Other (please specify):			
12. DOCUMENT ANNOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)			

UNCLASSIFIED

SECURITY CLASSIFICATION OF FORM

13. **ABSTRACT** (a brief and factual summary of the document ; it may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual).

The finite-element method is very powerful and flexible to model complex geometries with computers, because the region of interest can be subdivided into finite elements accurately. The advantages and drawbacks of the finite-element method will be the focus of the rudimentary investigation presented in this report. Especially, the requirements in terms of computational effort and computer memory storage will be investigated with respect to NEMP research requirements, i.e., with configurations frequently found in NEMP research such as coupling and interaction studies, simulator design and sensor design. Such configurations are often three dimensional and of intricate geometry.

It is found that with present day computer capability it is not yet possible to solve real-life three-dimensional geometries with the finite-element method, because of the memory requirements to store the resulting system of equations.

Two-dimensional geometries can at present be solved with the finite-element method, but the usefulness of two-dimensional geometries for NEMP research purposes is questionable.

14. **KEYWORDS, DESCRIPTORS or IDENTIFIERS** (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

FINITE-ELEMENT METHOD
NUCLEAR ELECTROMAGNETIC PULSE
NUMERICAL ELECTROMAGNETICS