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TECHNICAL REPORT ARCCB-TR-91013

**CORRELATION BETWEEN MACHINE TWIST
ANGLE AND UNIT TWIST ANGLE IN CALCULATING
SHEAR STRESSES FOR ELASTIC AND
PLASTIC STRAINS IN TORSION**

ROBERT R. FUJCAK

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**US ARMY ARMAMENT RESEARCH,
DEVELOPMENT AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
BENÉT LABORATORIES
WATERVLIET, N.Y. 12189-4050**



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INTRODUCTION

In certain applications such as spring testing and twist testing of solid or hollow shafts, the torsion test has been valuable. However, it has not been as widely accepted and utilized as the tension test. Therefore, it has not been standardized to the same extent as the tension test. Material specifications rarely require torsion tests, therefore, the data are less common than tensile data (refs 1-3).

Torsion-testing equipment consists of a twisting head, with a chuck for gripping the specimen and for applying the twisting moment to the specimen, and a fixed end. Specimen deformation may be determined by unit twist angle over a specified gage length, but this method is limited to a small angular range, 4 to 5 degrees/inch gage length. Another measurement that may be determined during testing is the machine twist of the chuck applying the twist moment. This report shows the correlation between the machine twist angle and the limited unit twist angle and how they both can be used to calculate shear stress, even in the case of large plastic strains.

ELASTIC TORSION EQUATIONS

Figure 1 is a schematic of a torsional moment, M_T , on a cylindrical bar. The twisting moment is opposed by shear stresses in the bar cross section. The shear stress, τ , is zero at the center of the bar and increases linearly with the radius, r . The following is an equation of the twisting moment:

$$M_T = \int_{r=0}^{r=a} \tau r dA = \frac{\tau}{r} \int_0^a r^2 dA \quad (1)$$

However, $\int r^2 dA$ is J , the polar moment of inertia of the area with respect to the axis of the bar. Therefore,

$$M_T = \frac{\tau J}{r}$$

or

$$\tau = \frac{M_T r}{J} \quad (2)$$

where τ = shear stress, psi

M_T = torsional moment, in.-lb

r = radial distance measured from center of bar, in.

J = polar moment of inertia, in.⁴

Since the shear stress is a maximum at the outside surface of the bar, for a solid cylindrical specimen where $J = \pi D^4/32$, the maximum shear stress is

$$\tau_{\max} = \frac{M_T D/2}{\pi D^4/32} = \frac{16M_T}{\pi D^3} \quad (3)$$

The angle of twist, θ , is usually expressed in radians. If L is the test length of the specimen, from Figure 1 it can be seen that the shear strain, γ , is given by

$$\gamma = \tan \phi = \frac{r\theta}{L} \quad (4)$$

where ϕ = the torsional angular displacement across the gage length, L . This angle, γ , remains constant over any gage length, so twist angle θ is directly related to the gage length.

PLASTIC TORSION EQUATIONS

Once the torsional yield strength is surpassed, the shear stress over the cross section is no longer a linear function of the distance from the axis, and Eqs. (2) and (3) do not apply. Nadai (ref 4) presented a method to calculate

the shear stress in the plastic range if the torque-twist curve is known. To simplify the analysis, the angle of twist per unit length may be defined as θ' , where $\theta' = \theta/L$. Referring to Eq. (4), the shear strain is

$$\gamma = r\theta' \quad (5)$$

Equation (1), for the resisting torque in a cross section of the bar, can be expressed as follows:

$$M_T = 2\pi \int_0^a \tau r^2 dr \quad (6)$$

Now the shear stress is related to the shear strain by the stress-strain curve in shear. The shear stress τ can be related as a function of the shear strain γ , where $\tau = f(\gamma)$.

Introducing this equation into Eq. (6) and changing the variable from r to γ by means of Eq. (5) gives the following equation:

$$M_T = 2\pi \int_0^{\gamma_a} f(\gamma) \frac{(\gamma^2)}{(\theta')^2} \frac{d\gamma}{\theta'} \quad (7)$$

$$M_T(\theta')^3 = 2\pi \int_0^{\gamma_a} f(\gamma) \gamma^2 d\gamma$$

where $\gamma_a = a\theta'$ and $d\gamma = a d\theta'$. Differentiating Eq. (7) with respect to θ'

$$\frac{d}{d\theta'} (M_T \theta'^3) = 2\pi a [f(a\theta') a^2 (\theta')^2] = 2\pi a^3 (\theta')^2 f(a\theta')$$

However, the maximum value of shear stress in the bar at the outer fiber is $\tau_a = f(a\theta')$. Therefore, the equation becomes

$$\frac{d(M_T \theta'^3)}{d\theta'} = 2\pi a^3 (\theta')^2 \tau_a$$

$$3M_T (\theta')^2 + (\theta')^3 \frac{dM_T}{d\theta'} = 2\pi a^3 (\theta')^2 \tau_a$$

Rearranged, the equation becomes

$$\tau_a = \frac{1}{2\pi a^3} \left[\theta' \frac{dM_T}{d\theta'} + 3M_T \right] \quad (8)$$

The shear stress may be calculated with Eq. (8) if a torque-twist curve is available. Figure 2 shows a schematic of how this is done. Equation (8) can be rewritten in terms of geometry shown in Figure 2 as follows:

$$\tau_a = \frac{1}{2\pi a^3} (BC + 3CD) \quad (9)$$

It can also be noticed from Figure 2 that at the maximum value of torque the derivative $dM_T/d\theta' = 0$. Therefore, the ultimate torsional shear strength or modulus of rupture can be expressed by the following equation:

$$\tau_u = \frac{3M_{\max}}{2\pi a^3} \quad (10)$$

If Eq. (9) is used at or before the elastic limit, then $M_T = BC = CD$, and since $a = D/2$, Eqs. (3) and (9) are equivalent up to this point. Thus, Eq. (9) may be used throughout the entire elastic-plastic range.

EXPERIMENTAL PROCEDURE

Figure 3 shows the torsional test specimen designed for use in an Instron Model No. 1323 Triaxial Test Machine, capable of static and fatigue testing in combinations of 100,000 lb tension-compression, 50,000 in.-lb torsion, and 50,000 psi pressure. In our case, only the static torsion capability was used.

The specimen was mounted in the twist chucks with instrumentation monitoring applied torque, unit twist angle, and machine twist angle at the chuck position. Figure 4 shows a reproduction of the original plot. The lower curve represents the unit twist angle from the biaxial extensometer, which had a

maximum capacity of only about 5 degrees twist. So, this extensometer was removed before 5 degrees twist to prevent gage damage. Also, the upper curve represents the applied machine twist angle, but at over 40 degrees twist up to a value close to the maximum twist moment. The maximum twist moment also appears on the curve, but without any twist angle data, because the plasticity exceeded the machine's angle measurement capacity.

DATA ANALYSIS

Data points were taken from the curves in Figure 4, and Figure 5 represents a schematic of both machine angle and unit twist angle versus machine torque. Since calculation of the shear stresses along the torque-twist curve depends on derivatives, a cubic spline technique (ref 5) was used. The raw data and calculated values appear in Table I. Data for unit twist and machine twist angles were taken at equivalent torque values up to the limit of the unit twist values. After that, only machine twist values were recorded.

The first column in Table I is untitled and just identifies the data points as "pt 0," "pt 1," etc., taken from Figure 4. The next column entitled "Mach" contains values for the machine twist angle in total degrees. The column entitled "Unit" contains values for the unit twist angle in degrees/inch and is limited to less than 5 degrees/inch. The column entitled "M(T)" contains values of machine torque in in.-lb from minimum to maximum torque.

The column entitled "dy/dx" contains values of the derivatives calculated by the cubic spline approximation for the unit twist curve, and the column entitled "(dy/dx)M" contains values for the machine twist curve. "Tau-a" refers to the values of the shear stresses calculated from Eq. (9) using the unit twist derivative values. "Tau-M" refers to the values of the shear stresses using the machine twist derivative values.

TABLE I. TORQUE DATA FOR 4150H SPEC AA-51

	Mach	Unit	M(T)	(dy/dx)M	Tau-a	dy/dx	Tau-M
pt 0	0.00	0.00	0	4,626.80	0.00	16,868.75	0.00
pt 1	1.75	0.48	8,097	3,963.30	41,237.68	16,868.75	39,759.16
pt 2	3.00	0.85	12,218	2,894.50	57,381.58	9,898.10	57,725.50
pt 3	4.25	1.20	15,369	2,377.00	71,198.65	8,176.91	71,567.84
pt 4	5.55	1.59	18,278	1,869.50	83,192.97	6,607.30	83,027.60
pt 5	7.10	2.01	20,702	1,664.40	92,283.91	5,161.00	94,121.99
pt 6	8.45	2.53	23,126	1,616.70	101,669.05	4,139.40	105,728.68
pt 7	10.00	3.06	25,066	1,082.80	108,406.27	3,249.70	109,531.71
pt 8	11.85	3.73	27,005	945.80	115,405.34	2,580.20	117,421.63
pt 9	14.25	4.55	28,944	752.40	122,810.64	2,115.00	124,209.23
pt 10	16.25		30,399	655.40			129,675.95
pt 11	20.20		32,338	397.80			133,753.25
pt 12	25.00		34,035	291.40			139,279.67
pt 13	29.75		35,102	140.00			139,382.81
pt 14	34.70		35,732	68.00			139,490.52
pt 15	40.00		36,023	37.50			139,507.58
pt 16	60.00		36,556	0.00			139,633.63

By using Eq. (9) from the torque-twist diagram for unit twist and also machine twist angle, the torsion stress may be calculated either way. Figure 6 shows a comparison of torsional stress values calculated from both twist angles. Although the twist angles from each method are different with respect to twist moment, their corresponding derivative values yield comparable torsional stress values.

Figure 7 shows that the torsional stress calculated from unit twist is very closely related to the stress calculated from machine twist. Therefore, the machine twist angle may be used to calculate torsional stress with reasonable accuracy, using the limited unit twist angle to calculate torsional strain.

Figure 8 shows the static torsion plot calculated from the machine twist angle. Since the ratio between machine twist and unit twist is roughly 4 to 1, the machine twist data beyond about 16 degrees, or beyond the unit twist limit of about 4 degrees/inch, is the only means by which the torsional stress may be calculated up to maximum value.

CONCLUSIONS

1. For a solid torsion bar of 4150H steel, the ratio between the unit twist angle and the machine twist angle remains basically the same, even when the twist shows plastic deformation. For the configuration used herein, the ratio between machine angle and unit twist angle, θ/θ' , is approximately 3.2. For every degree of unit twist, the machine twist is about 3.2 degrees.

2. Torsional stresses may be calculated by using the slopes from a torque-twist curve, whether the twist is unit twist per inch gage length or machine twist measured at the twisting head.

3. Since torsional stress may be calculated throughout the elastic-plastic range with the plastic equation, either twist may be used to calculate torsional stress up to the capability limit of the unit twist gage, but beyond that point the only method available to calculate torsional stress is the machine twist angle.

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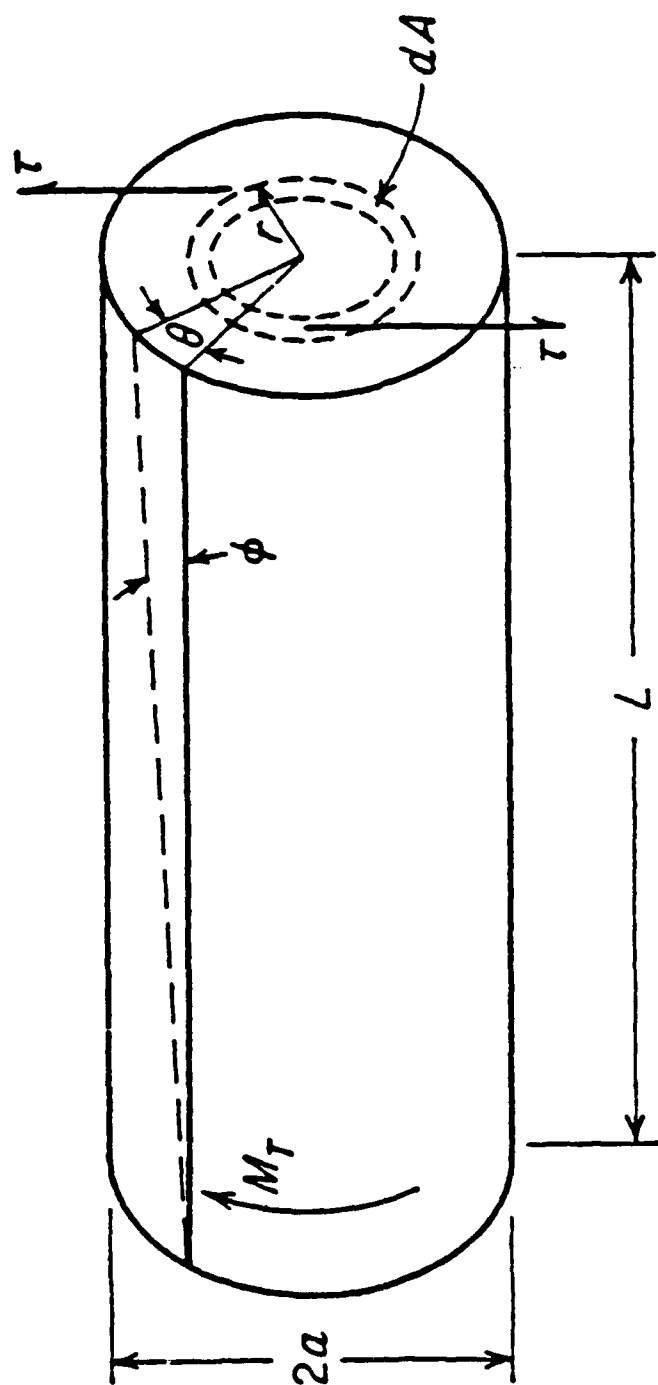
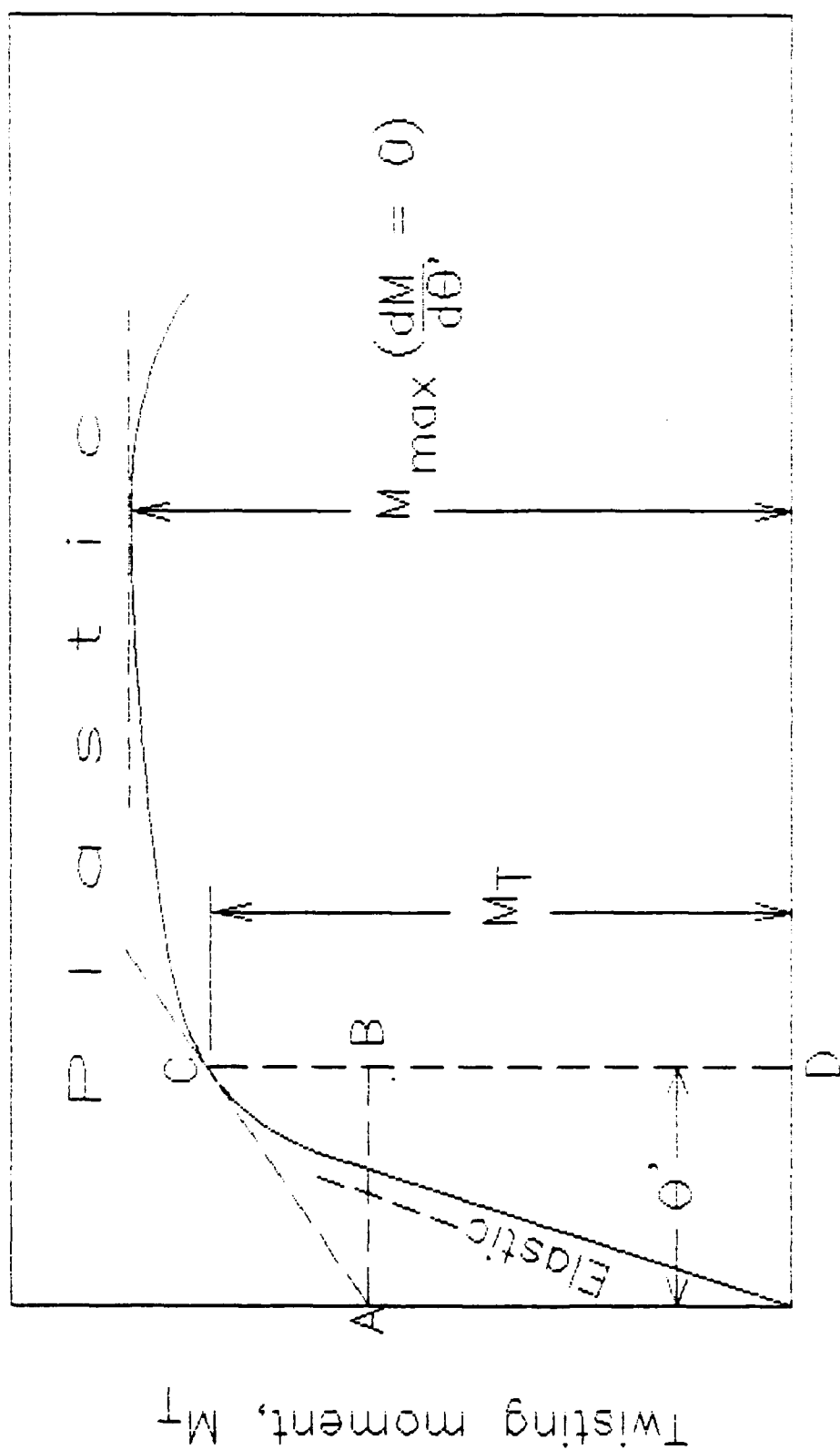


Figure 1. Torsion of a solid cylindrical bar.



Angle of twist per unit length, θ'

Figure 2. Method of calculating shear stress from torsion-twist curve.

TORSION TEST SPECIMEN

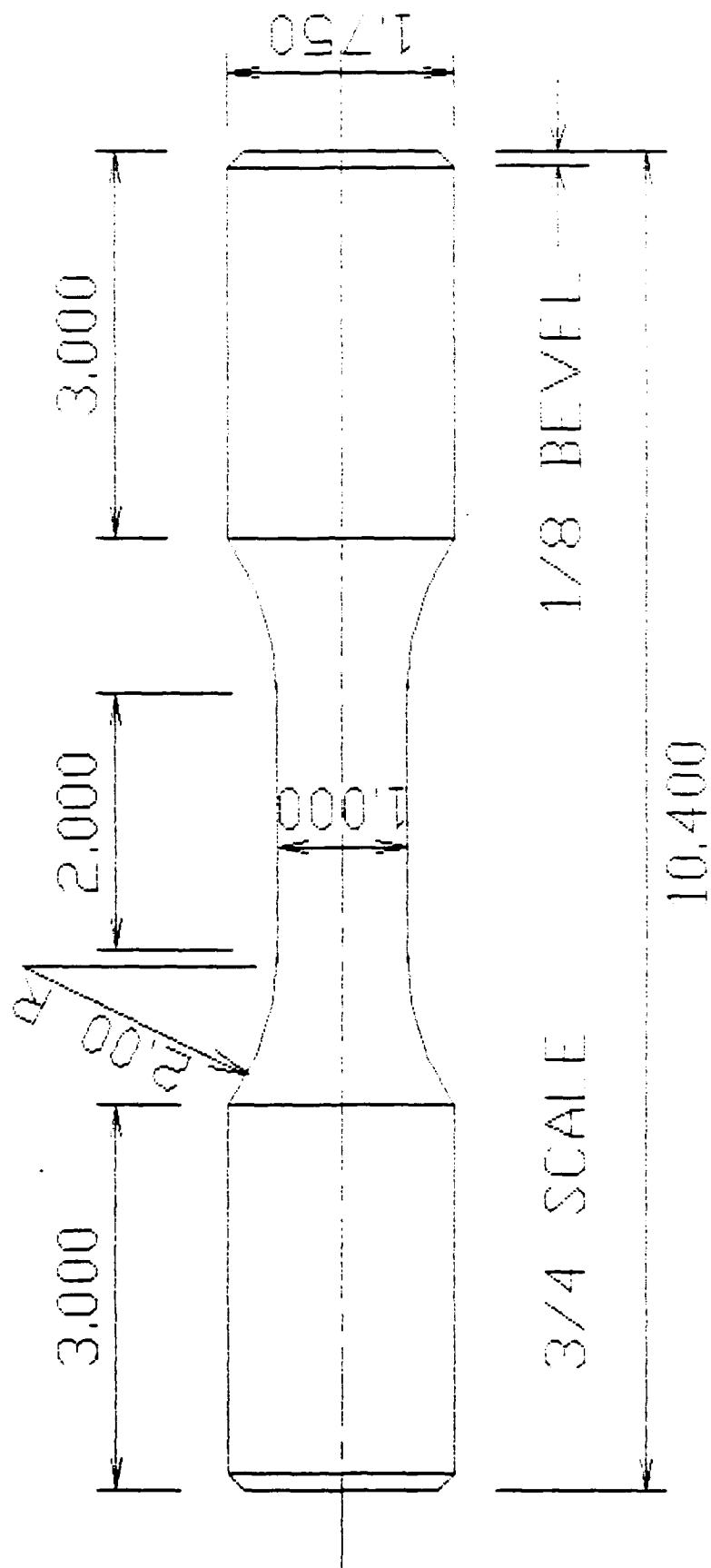
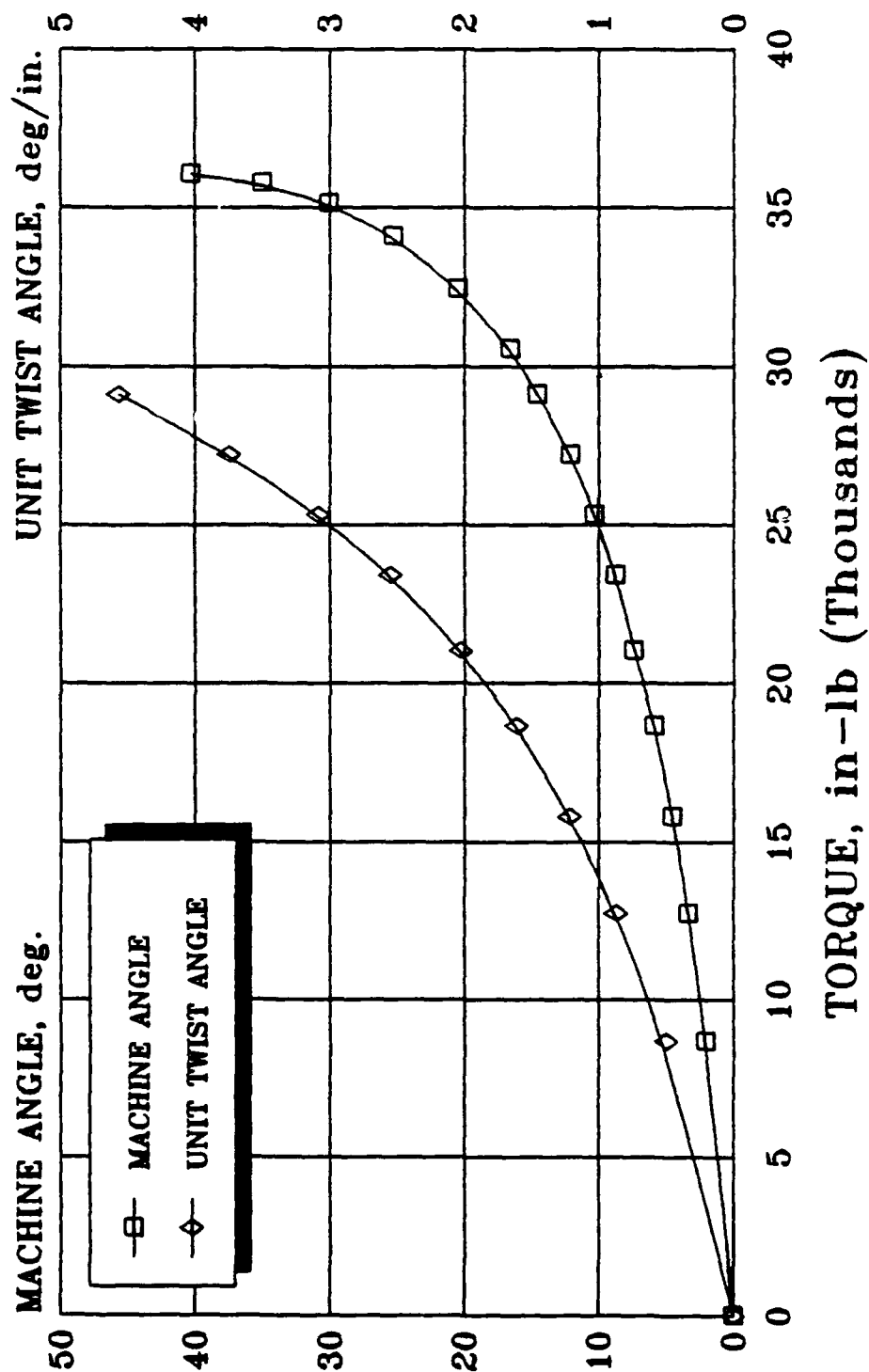


Figure 3. Torsion test specimen.



TWIST ANGLES VS. TORQUE

MACHINE ANGLE AND UNIT TWIST ANGLE

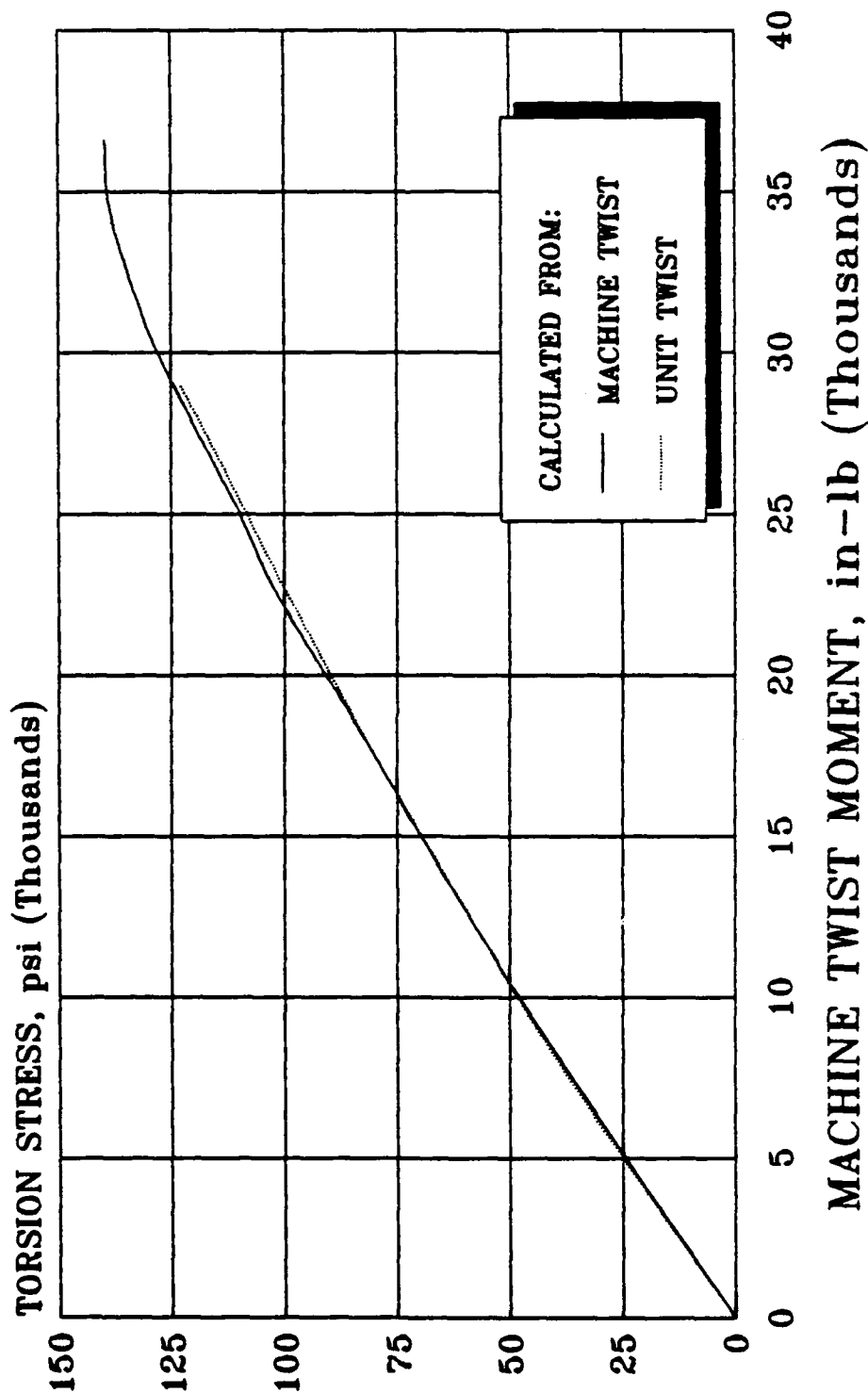


DATA FROM TORSION CURVE FOR SPEC. AA-51

Figure 5. Schematic of torsion-twist curve.

TORSION STRESS vs. MOMENT

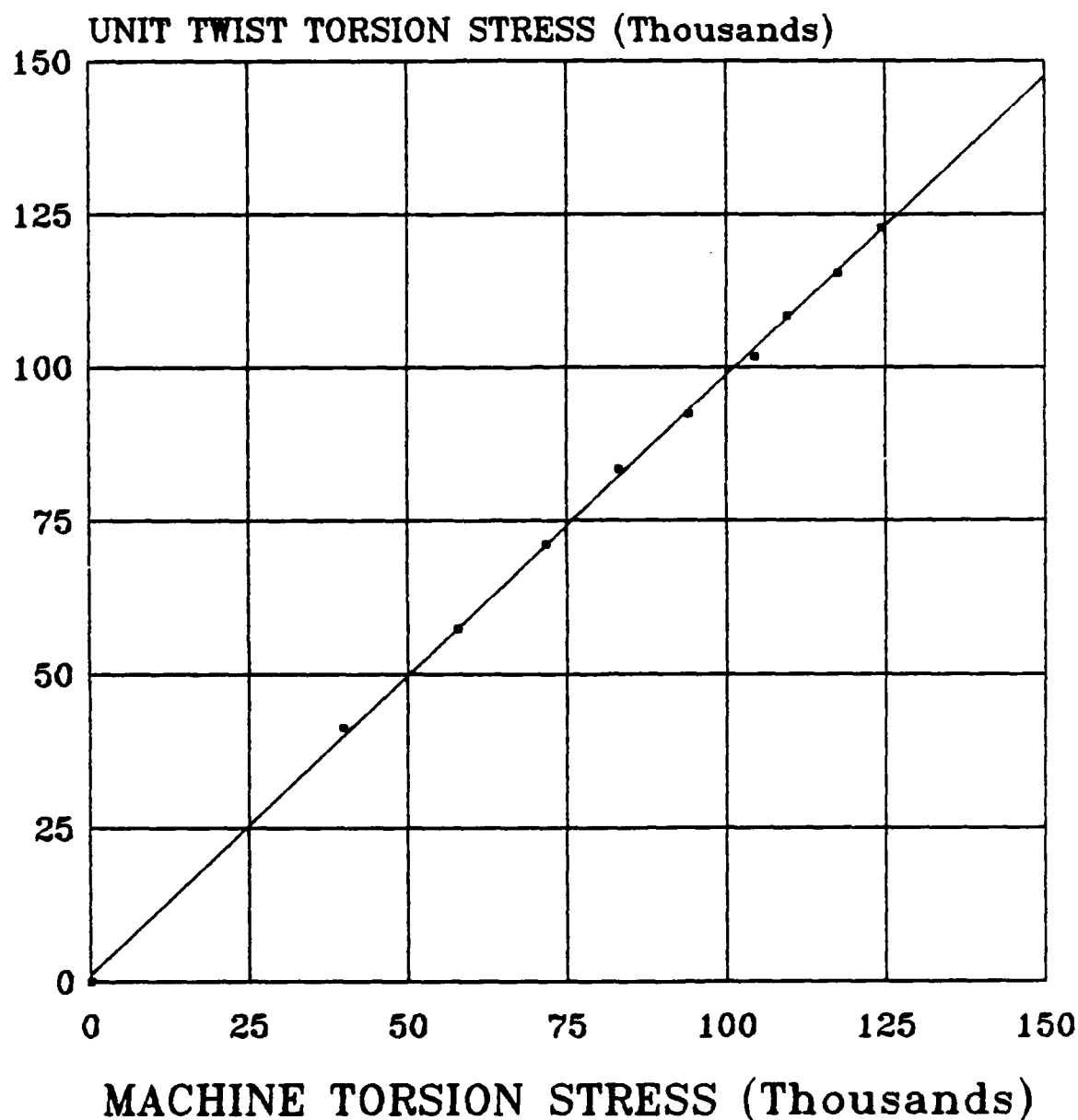
UNIT TWIST AND MACHINE TWIST



FOR 4150H SPEC NO. AA-51

Figure 6. Comparison of calculated torsional stress values.

TAU(T) vs. TAU(M) FOR 4150H SPECIMEN AA-51



CALCULATED FROM UNIT TWIST (TAU(T))
AND MACHINE TWIST (TAU(M))

Figure 7. Ratio between unit twist angle and machine twist angle.

STATIC TORSION PLOT TORSION STRESS VS. TWIST ANGLE FOR 4150H SPEC NO. AA-51

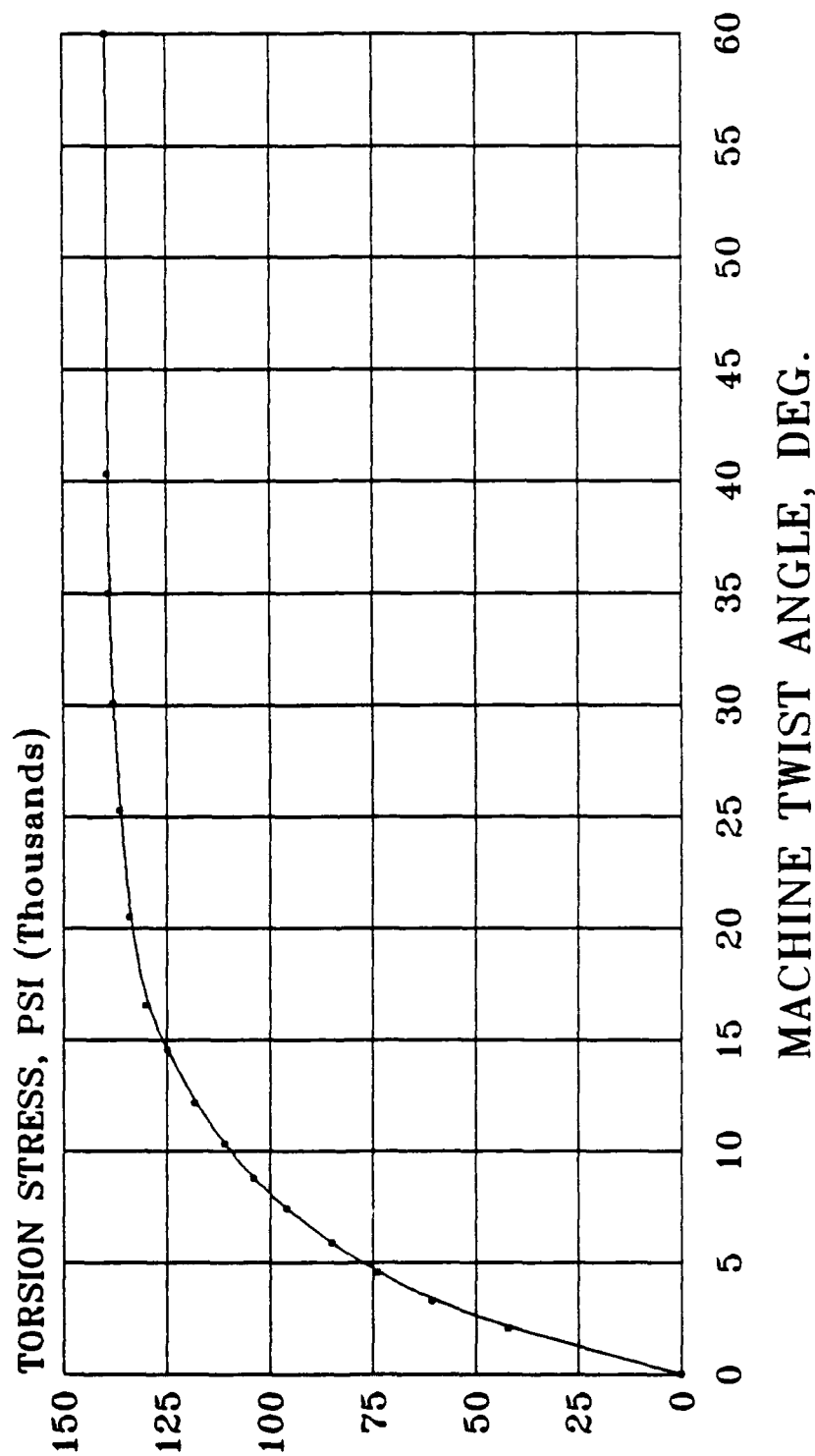


Figure 8. Static torsion plot calculated from machine twist angle data.

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