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AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Flight Mechanics Technical Memorandum 432

A FORTRAN PROGRAM FOR SPECTRAL ANALYSIS USING THE FAST FOURIER TRANSFORM

by

L.D. MacLaren and S.D. Hill

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SUMMARY

The analysis of random data is frequently carried out 'on-line' using a hardware based Fourier analyser. Capabilities similar to those provided in the 'on-line' analysers have been developed in a Fortran computer program that can be used to analyse a digital time-dependent signal at the post-test stage. An overview of the calculation methods used within this software is presented.

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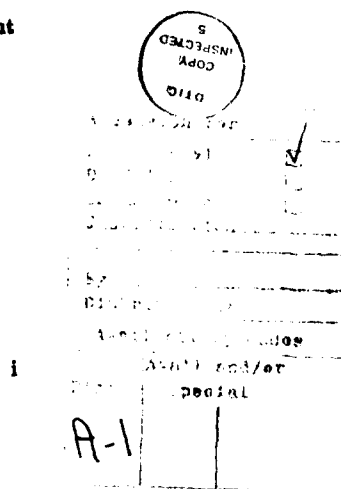
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Notation

c	Modified boxcar function
f	Frequency
f_c	Nyquist frequency
G	Power spectral density
H	Transfer function
h	Sampling time interval
l	Number of adjacent frequencies used in frequency smoothing
N	Number of discrete points used in each FFT computation
q	Number of ensembles used in the averaging process
R	Correlation function
sf	Scale factor
T	Signal length
t	Time
U	Fourier transform of u
u	Boxcar function
X, Y	Fourier transforms of x and y
x, y	Input signals
γ	Coherence function
θ	Argument of cross power spectral density function
τ	Time lag
ϕ	Phase of transfer function
Ψ	Mean square value

Superscripts

$\sim$	Raw estimate
$\frown$	Smoothed estimate
$'$	Modified function
$*$	Complex conjugate

1 Introduction

The development of the Fast Fourier Transform algorithm has extended the procedures available for the analysis of random data. These procedures can be carried out on-line using a hardware based machine, or the data can be processed using computer software.

The Wavetek 804A machine is an example of a digital Fourier analysis system which can process unsteady signals 'real time'. To provide post-test analysis facilities with similar capabilities, a PC-based computer program has been developed. The computer program SPEC which has been written in Fortran 77 uses a Fast Fourier Transform routine as the basis for all subsequent calculations and can be linked to the public domain program GNUPLOT for graphical output.

This report introduces the user to basic spectral analysis theory and highlights some aspects of data sampling and interpretation that need to be considered when dealing with discrete Fourier transforms. A more detailed description of spectral analysis is given by Bendat and Piersol (Reference [1]). This text forms the basis for much of the theory presented in this report.

An outline of the computer program SPEC is also given with descriptions of both input and output.

2 The Fourier Integral and Discrete Fourier Transform

In this report, the equations used in the analysis program are presented. The basic properties and classification of random data, its measurement and analysis are described in Reference [1]. Consideration of the data characteristics should be made prior to the selection of the analysis techniques.

In the analysis of transient data, the Fourier Integral is used to determine a frequency spectrum from any arbitrary time dependent signal. The infinite range Fourier Integral is given by (Ref. [1]):

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \qquad (1)$$

Realistically, the signal of interest is defined over a finite time interval $[0, T]$. The infinite range integral can be used within this time frame but now becomes a function of the signal length T, as well as the frequency f. $X(f, T)$ is the finite Fourier transform of $x(t)$.

$$X(f, T) = \int_{0}^{T} x(t)e^{-j2\pi ft}dt \qquad (2)$$

The analogue signal $x(t)$ can be sampled at N discrete time intervals. If we define the sampling interval as h such that $(N-1)h = T$ and h is chosen to produce a sufficiently high cutoff frequency, the above finite range integral can be expressed as a summation. For arbitrary f,

$$X(f, T) = h \sum_{n=0}^{N-1} x_n \exp\left[-j2\pi fnh\right] \qquad (3)$$

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If the frequency spectrum is to be defined at N discrete frequencies (i.e. $f = k/Nh$), then the Discrete Fourier Transform (DFT) is,

$$\begin{aligned}
X(f_k, T) &= h \sum_{n=0}^{N-1} x_n \exp\left[-j\frac{2\pi kn}{N}\right] \\
&= h X_k
\end{aligned}$$

(4)

where

$$X_k = \sum_{n=0}^{N-1} x_n \exp\left[-j\frac{2\pi kn}{N}\right]$$

(5)

The computation of the Fourier components X_k at the discrete frequency values is usually carried out using Fast Fourier Transform (FFT) techniques which are discussed in Section 3.

Upon calculation of the Fourier components, it is possible to return to the original input signal x_n by performing an Inverse Discrete Fourier Transform (IDFT). The IDFT equation is:

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \exp\left[+j\frac{2\pi kn}{N}\right]$$

(6)

Because the equations for DFT's and IDFT's are very similar, any algorithm used to compute a DFT can also be used to calculate its inverse by simply substituting the discrete frequencies for the input signal, changing the sign of the complex argument from negative to positive and finally dividing the results by N.

3 The Fast Fourier Transform

The Fast Fourier Transform (FFT) is an algorithm that enables the spectrum of an unsteady signal to be calculated using significantly less computational effort than the standard Fourier series procedure. The basis of the FFT is to split an N point transform into two $N/2$ point transforms. The derivation of the FFT algorithm from the DFT equation (5) is as follows:

$$\begin{aligned}
X_k &= \sum_{n=0}^{N-1} x_n \exp\left[-j\frac{2\pi kn}{N}\right] \\
&= \sum_{n=0}^{\frac{N}{2}-1} \left\{ x_{2n} \exp\left[-j\frac{4\pi kn}{N}\right] + x_{2n+1} \exp\left[-j\frac{2\pi k(2n+1)}{N}\right] \right\} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x_{2n} \exp\left[-j\frac{4\pi kn}{N}\right] + \exp\left[-j\frac{2\pi k}{N}\right] \sum_{n=0}^{\frac{N}{2}-1} x_{2n+1} \exp\left[-j\frac{4\pi kn}{N}\right]
\end{aligned}$$

(7)

This can be re-written as

$$X_k = A_N^k + B_N^k W_N^k$$

(8)

where

$$A_N^k = \sum_{n=0}^{\frac{N}{2}-1} x_{2n} \exp\left[-j\frac{4\pi kn}{N}\right]$$

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$$B_N^k = \sum_{n=0}^{\frac{N}{2}-1} x_{2n+1} \exp\left[-j\frac{4\pi kn}{N}\right]$$

$$W_N^k = \exp\left[-j\frac{2\pi k}{N}\right]$$

As an example, consider a case where the input signal consists of eight discrete points. The eight point transform is split into two four point transforms which are in turn split into four two point transforms. The equation below shows this splitting process with the selected data points identified beneath each individual summation

For $N = 8$ and $x = \{x_n : x_0, x_1, x_2, ...x_7\}$,

$$
\begin{aligned}
X_k &= \underset{\{x_0, x_2, x_4, x_6\}}{A_8^k} + \underset{\{x_1, x_3, x_5, x_7\}}{B_8^k W_8^k} \\[2ex]
&= \left[\underset{\{x_0, x_4\}}{A_4^k} + \underset{\{x_2, x_6\}}{B_4^k W_4^k}\right] + \left[\underset{\{x_1, x_5\}}{A_4^k} + \underset{\{x_3, x_7\}}{B_4^k W_4^k}\right] W_8^k \\[2ex]
&= \left[\left(\underset{\{x_0\}}{A_2^k} + \underset{\{x_4\}}{B_2^k W_2^k}\right) + \left(\underset{\{x_2\}}{A_2^k} + \underset{\{x_6\}}{B_2^k W_2^k}\right) W_4^k\right] \\
&\quad + \left[\left(\underset{\{x_1\}}{A_2^k} + \underset{\{x_5\}}{B_2^k W_2^k}\right) + \left(\underset{\{x_3\}}{A_2^k} + \underset{\{x_7\}}{B_2^k W_2^k}\right) W_4^k\right] W_8^k
\end{aligned}
$$

The FFT process can also be depicted diagramatically as shown for the eight point transform in Figure 1 (Ref. [2]). The order in which the raw data on the left appears, is determined using a 'bit reversal' procedure. That is, the address of the n th point is found by taking the mirror image of the binary form of n. For example, point 3 (011) of the 8 point transform shown is relocated to position 6 (110). The dashed lines indicate that the quantity be multiplied by a complex coefficient W_N^k whereas the dotted lines indicate a multiplication by unity. The junction of two lines at the dots indicate that the two quantities should be added.

When calculating the Fourier Transform of an N point signal using the DFT approach, it involves a total of N^2 multiplications each followed by addition. For all FFT applications, N is selected to be a power of 2 (i.e. $N = 2^p$ where p is a positive integer), and for this arrangement, the FFT algorithm reduces the number of computations to $N \log_2 N$ (Ref. [2]). As p increases, the advantage of the FFT over the DFT becomes apparent as indicated in Table 1.

Table 1: Computational requirements for Fourier Transform calculations

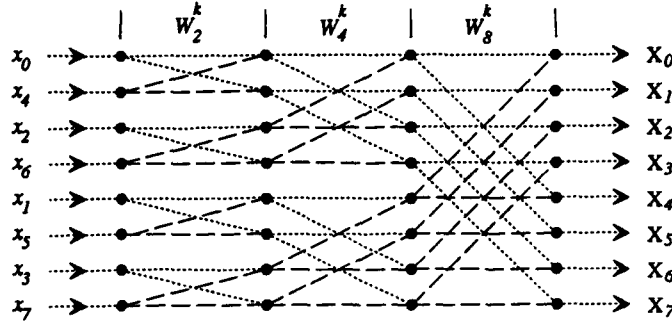| p	2	3	4	5	6	7	8	9	10
N	4	8	16	32	64	128	256	512	1024
N^2	16	64	256	1024	4096	16384	65536	262144	1048576
$N \log_2 N$	8	24	64	160	384	896	2048	4608	10240
$N^2/N \log_2 N$	2.0	2.7	4.0	6.4	10.7	18.3	32.0	56.9	102.4

Figure 1: Illustration of the FFT process for an $N = 8$ transform

4 Spectral Density

The Power Spectral Density (PSD) function can be defined as the rate of change of the mean square value of a given signal with respect to frequency (Ref. [3]). The mean square value Ψ_x^2 of an infinitely long signal $x(t)$ is as follows:

$$\Psi_x^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T x^2(t)dt \qquad (9)$$

In practice, signal analysis is confined to a finite length time history $x(t, T)$ where

$$
\begin{aligned}
x(t, T) &= x(t) \quad 0 \le t \le T \\
&= 0 \quad\ otherwise
\end{aligned}
$$

Therefore, by defining $x(t, T)$ between $\pm\infty$, the mean square value can be re-written as

$$\Psi_x^2 = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} x^2(t, T)dt \qquad (10)$$

By making use of Parseval's theorem, which states that if $X(f)$ is the Fourier transform of $x(t)$ then

$$\int_{-\infty}^{\infty} x(t)^2 dt = \int_{-\infty}^{\infty} |X(f)|^2\, df \qquad (11)$$

and since $X(f, T)$ is the Fourier transform of $x(t, T)$, the mean square value can be defined as a function of the frequency spectrum.

$$
\begin{aligned}
\Psi_x^2 &= \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |X(f, T)|^2\, df \\
&= 2 \lim_{T \to \infty} \frac{1}{T} \int_0^{\infty} |X(f, T)|^2\, df
\end{aligned}
\qquad (12)
$$

4

The PSD function $G_x(f)$ is the rate of change of Ψ_x^2 with respect to frequency, therefore

$$G_x(f) = \frac{d\Psi_x^2}{df} = 2 \lim_{T \to \infty} \frac{1}{T} E\left[|X(f,T)|^2\right] \tag{13}$$

As 'E' denotes the expected value in the above equation, an estimate of the power spectral density function can be expressed as

$$\tilde{G}_x(f) = \frac{2}{T} |X(f,T)|^2 \tag{14}$$

with the tilde ($\sim$) indicating that the function is an estimate only. From equation (4) it is apparent that at discrete frequencies, $X(f_k, T) = hX_k$. The power spectral density function is therefore related to the power spectrum, $|X_k|^2$ as follows.

$$\tilde{G}_x(f_k) = \frac{2h^2}{T} |X_k|^2 = \frac{2h}{N} |X_k|^2 \tag{15}$$

For a second signal $y(t)$, another PSD estimate can be calculated using the above equations. To investigate the relationship between $x(t)$ and $y(t)$, there is a need to compute the Cross Spectral Density (CSD) function. The definition of CSD is presented below as a function of the Fourier components of each signal.

$$\begin{aligned} \tilde{G}_{xy}(f_k) &= \frac{2}{T} [X^*(f_k,T)Y(f_k,T)] \\ &= \frac{2h}{N} [X_k^* Y_k] \end{aligned} \tag{16}$$

Here, the asterisk (*) indicates the complex conjugate.

Discrete frequency spectra estimates are subject to two main errors. These are discussed in Reference [3] and are summarised as follows.

1. Statistical error due to finite data length, which can be improved by averaging the results over a number of samples.

2. Bias error due to the finite bandwidth filter used to separate the various frequency components of a signal. Bias errors are large when the PSD changes value rapidly as the frequency is varied.

5 Data Manipulation

5.1 Sampling Considerations

A finite length signal of duration T should be sampled at N equally spaced time intervals h. The parameters N and h determine the range and resolution of the final frequency spectrum.

5.1.1 Time Interval, h

The sampling interval h is selected in order to obtain the required frequency range. If it is known that a signal contains no frequencies greater than a value f_c, then the signal

can be completely described if it is sampled at a rate greater than $2f_c$. f_c is known as the Nyquist frequency and is equal to half of the sampling frequency. The frequency spectrum as calculated by Fast Fourier Transform techniques, produces a mirror image of itself about this point.

$$f_c = \frac{f_{sample}}{2} = \frac{1}{2h} \tag{17}$$

If the sampling rate is selected to be too low, superposition of the high and low frequency components in the original data occurs. Figure 2 shows how a high frequency signal is indistinguishable from a lower frequency if the sampling interval is too large. This is called 'aliasing'.
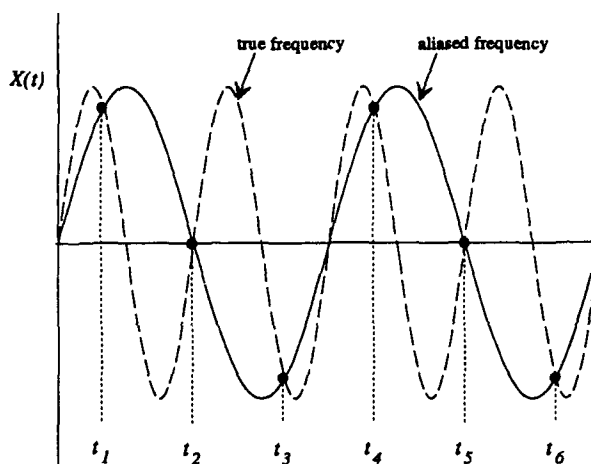

Figure 2: The aliasing effect in the time domain

For any frequency f within the range $0 \leq f \leq f_c$, the higher frequencies which are aliased to it are

$$(2f_c \pm f),(4f_c \pm f),.....(2nf_c \pm f),.....$$

Figure 3 illustrates the aliasing effect on the frequency spectrum. The components that lie beyond the Nyquist frequency are folded back about this point to corrupt the true frequency spectrum.

The aliasing phenomenon can be avoided by either

1. filtering the raw analogue signal before digitising to remove frequencies higher than the chosen Nyquist frequency (i.e. low pass filtering),

2. Selecting h small enough to ensure that no frequencies with significant power occur beyond the Nyquist frequency.
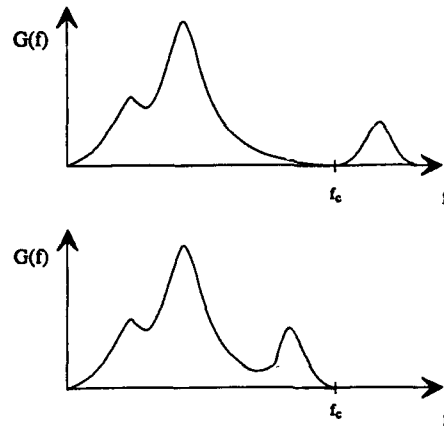
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Figure 3: The aliasing effect in the frequency domain

5.1.2 Number of points, N

After h has been determined, N must be chosen such that Nh is not greater than the available signal length T. N also controls the resolution of the output spectrum. The number of discrete frequency points within the range $(0, f_c)$ is $N/2 + 1$. N must be selected large enough to enable sudden changes (i.e. spikes) in the frequency spectrum to be easily identified, and to avoid this information becoming lost or smeared into neighbouring frequencies. The number of points must also be selected to maintain a good compromise between required output resolution and computational effort.

5.2 Centring

Data that has a non-zero mean value will result in power at a frequency of 0 Hz. If this is large then it may adversely affect the results at other frequencies. The time history can be modified to centre the signal on a zero mean value to remove this effect.

$$x'_n = x_n - \frac{1}{N} \sum_{k=0}^{N-1} x_k \tag{18}$$

5.3 Windowing

Because it is impractical to compute the Fourier transform of an infinitely long signal, it is necessary to select a finite length signal from the available data. This is accomplished by multiplying the infinite signal by a *boxcar* function, $u(t)$ defined as follows.

$$
\begin{aligned}
u(t) \quad &= 0 \quad t < -T/2 \\
&= 1 \quad -T/2 \leq t \leq T/2 \\
&= 0 \quad t > T/2
\end{aligned} \tag{19}
$$

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By multiplying the infinite signal by the boxcar function $u(t)$ in the time domain, the estimate of the power spectral density function $\check{G}_x(f)$ has been affected by the transform of the boxcar function $U(f)$ in the frequency domain.

The Fourier transform of $u(t)$ is

$$U(f) = T\left(\frac{\sin \pi fT}{\pi fT}\right) \tag{20}$$

and is called a *window function* (Reference. [1]).

The estimate $\check{G}_x(f)$ is the convolution of the true $G_x(f)$ with the window function $U(f)$. The main lobe of the window function spreads the power at discrete frequencies over the sample bandwidth and the smaller side lobes cause *leakage* to occur .

To reduce the effect of leakage, it is necessary to modify the weighting function in the time domain such that in the frequency domain, the width of the main lobe is increased and the magnitude of the side lobes is decreased.

One common technique for weighting the data is by using a cosine distributed taper on the ends of the signal. This ensures that there are no discontinuities of amplitude and/or slope between the start and finish of each sample period (Reference [3]). The Fourier transform is based on the assumption that the data is periodic.

If the total number of digitised points in the signal sample is N and the number of points to be tapered at each end is m, then the weighting function c_n for $n = 0$ to $N - 1$ is given by

$$c_n = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{\pi n}{m}\right)\right) & : n \leq m \\ 1 & : m < n < N - m \\ \frac{1}{2}\left(1 - \cos\left(\frac{\pi(m-n)}{m}\right)\right) & : n \geq N - m \end{cases} \tag{21}$$

This function is known as the 'Tukey' window (Ref. [4]). When the number of points at each end is equal to half of the total number of points, the Tukey window becomes commonly known as the 'Hanning' window. Figure 4. illustrates the difference between the Hanning window function and the boxcar window function in the frequency domain.

The inclusion of a taper function will reduce the variance of the tapered data with respect to the original data. This in turn reduces the magnitude of the FFT and the subsequent PSDs. A correction scale factor should be applied to the resulting PSD if a taper function has been used in order to compensate for the reduced variance. Parseval's theorem suggests that this factor can be determined by integrating the square of the window function (i.e. eq.22) in the time domain.

$$\begin{aligned} sf &= \int_{-\infty}^{\infty} [c(t)]^2 \, dt \\ &= 4/(4 - 5\frac{m}{N}) \end{aligned} \tag{22}$$

Reference [3] indicates that for a full Hanning window (i.e. $m/N = 0.5$), the above scale factor should be $8/3$ and for a rectangular window (i.e. $m/N = 0$) this factor should be 1.
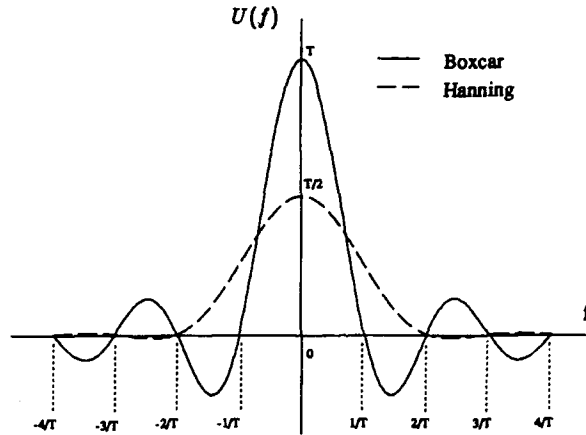
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Figure 4: Comparison of Boxcar and Hanning window functions.

5.4 Smoothing of results

With real data, the spectral density estimates will contain both random and bias errors. These are discussed in Reference [1]. To improve the accuracy of the estimates, it is usual to average the results using one of the following techniques. The smoothed result is indicated by the use of a hat ($\hat{}$) which replaces the tilde ($\tilde{}$) used to indicate a raw estimate.

The time history (if sampled for a long enough period) can be split up into q separate segments. The power spectra for each of these segments should be calculated independently and the average spectrum determined according to the following equation.

$$\hat{G}_k = \frac{1}{q} \left[\tilde{G}_{k,1} + \tilde{G}_{k,2} + ... + \tilde{G}_{k,q} \right] \qquad k = 0, 1, 2, ...N/2 - 1 \qquad (23)$$

As the number of averages increases, the results assymptote to the true spectrum, but there is a practical restriction on the number of averages that can be used due to the total length of available data. In order to increase the number of averages, it is useful to employ an 'overlapping' technique where the trailing portion of data in the current segment can be used as the initial portion for the next segment. An overlap of more than 50 % is deemed to be unnecessary, as each new data block then becomes too similar to the last, and therefore continued averaging would yield no real benefit.

6   Auto-correlation Functions

The auto-correlation function describes the relationship between a signal value at a particular time to that at a different time. For a sample time history $x(t)$, an estimate of the auto-correlation between signal values at times t and $t + \tau$ can be made by taking the product of the two signals and averaging over the time T. This process is represented

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by the following equation.

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt \qquad (24)$$

$R_x(\tau)$ is always a real valued even function (symmetric about $\tau = 0$) with a maximum at $\tau = 0$, and may be either positive or negative or zero at all other values of τ.

Because the auto-correlation function can also be defined as the inverse Fourier transform of the power spectrum, it can be efficiently obtained using FFT techniques.

$$R_x(\tau_n) = \frac{1}{N} \sum_{k=0}^{N-1} \widehat{G}_x(f_k) \exp \left[+j \frac{2\pi k n}{N} \right] \qquad (25)$$

7   Cross-correlation Functions

In a similar manner to the auto-correlation function, the cross-correlation function describes the relationship in the time domain between two different signals. For a pair of signals $x(t)$ and $y(t)$, the cross-correlation function is defined as:

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt \qquad (26)$$

$R_{xy}(\tau)$ is a real function that can be positive or negative or zero but unlike autocorrelation functions, it does not necessarily have a maximum at $\tau = 0$ nor does it have to be an even function.

Again by using FFT procedures, the cross-correlation function can be calculated as the inverse Fourier transform of the cross-spectrum.

$$R_{xy}(\tau_n) = \frac{1}{N} \sum_{k=0}^{N-1} \widehat{G}_{xy}(f_k) \exp \left[+j \frac{2\pi k n}{N} \right] \qquad (27)$$

Averaging of a number of ensembles is required to provide estimates with satisfactory accuracy.

Cross-correlation plots can be used in the measurement of time delays. As the signal output is displaced in time relative to the input, the cross-correlation plot will peak at a time that is equal to the time difference. It should be noted that the two signals must be digitised with exactly the same starting time. Any misalignment in the $t = 0$ point will result in a phase angle error in the cross spectral density estimate, which will in turn cause errors in the cross-correlation and in any other function that requires CSD estimates such as coherence.

8   Coherence

When dealing with two signals $x(t)$ and $y(t)$, an indication of the relation between the signals can be obtained using a coherence function $\gamma_{xy}^2(f)$. This function varies between 0 and 1 over the specified frequency range. When γ_{xy}^2 is equal to 1, the signals are said to be fully coherent (i.e. there is a linear relationship between them), whereas if

γ_{xy}^2 equals 0 the signals are incoherent or totally uncorrelated. The coherence function relates the cross-spectrum to both of the power spectra.

$$\hat{\gamma}_{xy}^2(f_k) = \frac{\left|\hat{G}_{xy}(f_k)\right|^2}{\hat{G}_x(f_k)\hat{G}_y(f_k)} \tag{28}$$

It should be noted that the above equation uses the hat (^) notation indicating that each of the spectra should be averaged estimates. The use of raw spectra results in a coherence function that is everywhere equal to unity, since:

$$\tilde{\gamma}_{xy}^2(f_k) = \frac{\left|\tilde{G}_{xy}(f_k)\right|^2}{\tilde{G}_x(f_k)\tilde{G}_y(f_k)} = \frac{\frac{2}{T}[X^*Y]^* \frac{2}{T}[X^*Y]}{\frac{2}{T}[X^*X]\frac{2}{T}[Y^*Y]} = 1 \tag{29}$$

Smoothing of frequency spectra is discussed in Section 5.4.

9 Transfer Functions

If we consider a system where a signal $y(t)$ is a response to an input $x(t)$, the relationship between them is the frequency response function $H(f)$. For a single-input, single-output system, $H(f)$ (a complex function of frequency which contains both gain and phase information) can be best calculated using the averaged cross power spectrum and the input power spectrum.

$$\hat{H}(f) = \frac{\hat{G}_{xy}(f)}{\hat{G}_x(f)} \tag{30}$$

The frequency response function gain is the the the modulus from the above equation.

$$\left|\hat{H}(f)\right| = \frac{\left|\hat{G}_{xy}(f)\right|}{\hat{G}_x(f)} \tag{31}$$

and the phase angle is the argument of $\hat{G}_{xy}$.

$$\hat{\phi}(f) = arg\left(\hat{G}_{xy}(f)\right) = \hat{\theta}_{xy}(f) \tag{32}$$

10 Program SPEC

SPEC (truncated from Spectral Analysis) is written in standard Fortran 77 except for a one line system call to run the graphics routine GNUPLOT. If this routine is unavailable or unsuitable for the computer in use, then the line should be commented out and the program recompiled.

The subroutines used by SPEC are located in a library file called SPECLIB which must be linked to SPEC after compilation. The library includes a routine to remove the mean value from a signal (see Section 5.2) and also a routine to apply a cosine windowing function to the raw data (see Section 5.3). The most important routine within this library is the FFT routine itself which was obtained from Reference [5].

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10.1 Input File SPEC.IN

SPEC requires the creation of an input file. This file contains all of the parameters required to set up the calculation and is a simple 10 record file with only one number on each record. A sample of such an input file is given below. The *comments* within the file are quite acceptable as SPEC will only read the first number on each line.

```
1         **  GNUPLOT plotting to be used ( 1 yes, 0 no)
1         **  Channel number
3         **  Channel number ( 0 if only one channel used)
2000.0    **  Sampling rate (Hz)
200.0     **  Maximum output frequency
30        **  No. of averages
10        **  Exponent for no. of samples/average (2^n1)
50        **  Percentage of points in window clip
50        **  Percentage overlap
0.09844   **  Engineering calibration for channel 1
2.32515   **  Engineering calibration for channel 3
```

The purpose of each record is as follows.

1. Record number 1 is a switch to turn the optional GNUPLOT plotting routine on or off.

2. Record numbers 2 and 3 select the data files to be used in the analysis. Here, channels 1 and 3 have been chosen to represent signals x and y respectively and these data files must be called CHAN01.DAT and CHAN03.DAT. If record number 2 is a zero then the analysis will be for a single channel only and no cross signal calculations will be attempted.

3. Record number 4 is the rate at which the data under consideration was digitised from the analogue signal.

4. Record number 5 is the maximum output frequency required for the calculation which cannot be greater than the maximum Nyquist cutoff frequency (half of the sampling rate). If it is less than this value then the effective sampling rate will be reduced by reading less frequently than the sampling rate to satisfy the relation given in Section 5.1.1.

5. Record number 6 is the number of segments required to be averaged. The program will adjust this record to the maximum possible if there is not enough data available.

6. Record number 7 is the base 2 logarithm of the number of points required for one segment. For the above example, the number of points is $2^{10} = 1024$.

7. Record number 8 is the percentage of the total number of points required at each end of the data segment for the cosine window clip. For the above example, the number of points is $0.5 \times 2^{10} = 512$.

8. Record number 9 is the percentage overlap of raw data from one segment to the next.

9. Records 10 and 11 are any calibration constants that need to be applied to the raw signal. If the raw signals are in volts and they need to be converted to engineering units, then these constants should be specified in units/volt.

10.2   Analysis Format

Assuming that the input file SPEC.IN is available and is in the correct form, the spectral analysis program will read all parameters within the file to initialise the calculation. A check will first be made on the accessibility of the requested data files and warning flags will be output to the screen if these files cannot be opened. If the input information is correct, a summary of the set-up parameters is provided. This allows the user to double check that the analysis has been specified correctly.

The calculation proceeds with the reading of the first segment of data from either one or two data files. The data segment is passed to a centering subroutine (see Section 5.2) and then the cosine windowing function is applied (Section 5.3). The data is adjusted according to the calibration constant and finally the resulting real number is converted to a complex number with the imaginary component equal to zero, as required by the Fast Fourier Transform algorithm. The results from the FFT routine X_k, Y_k are in the form of complex numbers which are saved in an array while the next data segment is read and processed in a similar manner. The following parameters are calculated and summed over the total number of data segments.

$$X_k, \ Y_k, \ |X_k|, \ |Y_k|, \ |X_k|^2, \ |Y_k|^2, \ X_k Y_k, \ |X_k Y_k|$$

These are used in the final calculation of PSD and coherence, etc. After the requested number of averages is reached or the total available data has been read, the running totals above are converted into averages by dividing by the number of segments used in the calculation. A summary of the final calculation setup is then provided followed by a table of output options.

10.3   Output

Selection of one of the analysis options, will create a data file into which the results are to be stored. The name of the data file is a combination of the type of analysis requested (the first three characters in the file name) and also the channel numbers used in the calculation. For example, a request for a PSD output on channel 1 would have results stored in file PSD01.OUT. Similarly, a request for coherence between channels 1 and 3 creates a file COH01_03.OUT. The ouput files possible will have names using the following codes:

1. DAT - Raw data from the last segment analysed.

2. FFT - The average Fourier transform over the time history.

3. INV - The inverse Fourier transform of the average FFT.

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4. PSD - The average Power Spectral Density. This option also calculates a variance estimate by integrating the PSD function.

5. AUT - The auto-correlation function obtained from the inverse Fourier transform of the average PSD.

6. CSD - The average Cross Spectral Density.

7. CCO - The cross-correlation function obtained from the inverse Fourier transform of the average CSD (in complex form).

8. COH - The coherence function.

9. FRF - The magnitude of the frequency response function.

10. PHA - The phase angle of the frequency response function.

If the graphics routine GNUPLOT has been made available, the requested output will automatically be plotted on the screen for instant appraisal. Further analysis can be requested until the EXIT option is used. All data files created by SPEC are saved for later use upon exit from the program.

An example of SPEC output is presented in Appendix A. This includes a comparison with results calculated on a Wavetek 804A Fourier analyser.

11 Conclusion

The development of a PC based computer program (SPEC) provides a capability to perform post test spectral analysis on digitised time-dependent signals.

The mathematical definitions have been included to ensure that the basis for the calculations used in the computer program is clearly documented. For further information regarding the application of the techniques, reference should be made to standard texts such as Ref. [1]. Some general guidelines on the preparation of measured data are included.

Acknowledgements

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References

[1] Bendat, J. S. and Piersol, A. G., 'Random Data: Analysis and Measurement Procedures', Wiley-Interscience 1971

[2] De Lorenzo, L. J. and Gottleib, P., 'A High Speed Serial FFT Processor', IREE Proceedings, November 1974

[3] Williams, D. A., 'The Analysis of Random Data', AGARD Flight Test Instrumentation Series, Volume 14, AGARD-AG-160-VOL.14, 1981

[4] Geckinli, N. C. and Yavuz, D., 'Discrete Fourier Transformation and Its Applications to Power Spectra Estimation', Elsevier 1983

[5] Cooley, J. W., Lewis, P. A. W. and Welch, P. D., 'The Fourier Transform and Its Applications', IEEE Transactions on Education, Vol 12, No. 1, March 1969

Appendix A - Example SPEC Output

The output presented in this appendix has been calculated using signals from an accelerometer and an unsteady pressure transducer obtained during testing in the ARL $9' \times 7'$ Low Speed Wind Tunnel. Both transducers were mounted on the vertical stabiliser of a 1/9th scale model aircraft which was subjected to aerodynamic buffet.

The analysis was completed under the following conditions:

1. The accelerometer and pressure transducer were on channels 1 and 2 respectively.

2. The sampling rate was 2000 Hz.

3. Maximum output frequency was set to 1000 Hz (of which 0 – 500 Hz has been plotted in the following figures).

4. 30 block averages were used with each block containing 1024 points and overlapping by 50 %.

5. The data was unfiltered at the time of recording and smoothed using a full Hanning window.

A typical block of data from these transducers is given in Figure A1.

Figure A2 shows the correlation functions as calculated by SPEC. It can be construed from Parseval's theorem that the value of autocorrelation functions at $\tau = 0$ should be equal to the variance of raw signal. Program SPEC provides the user with a variance estimate by integrating the Power Spectral Density function. The two signals chosen for this analysis produced variances of $1376g^2$ and $108000Pa^2$ which compare quite well with Figure A2 at $\tau = 0$.

The Wavetek804A Fourier analyser is used as a basis for comparison with SPEC output. As this machine does not have a built in facility for the calculation of correlation functions, a direct comparison with a hardware based Fourier analyser could not be made. However, for the parameters presented in the frequency domain, its output is useful.

For the frequency based functions such as PSD, coherence etc., a direct comparison between Wavetek and SPEC results is graphically presented in Figures A3-A7. This indicates that the software-based method is correct in determining both the frequency information within a random signal and the level of energy at each discrete frequency.
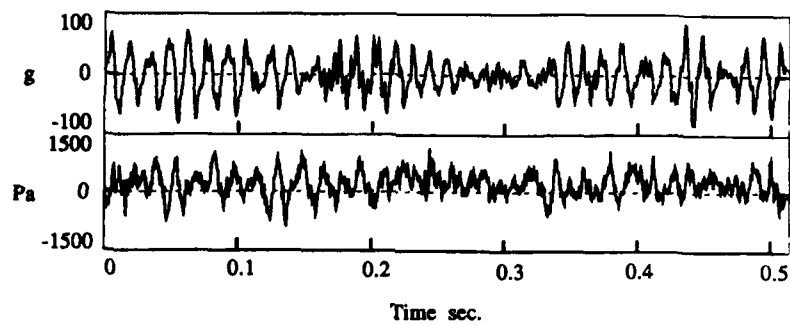
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FIGURE A1. Typical data blocks for accelerometer and pressure transducer
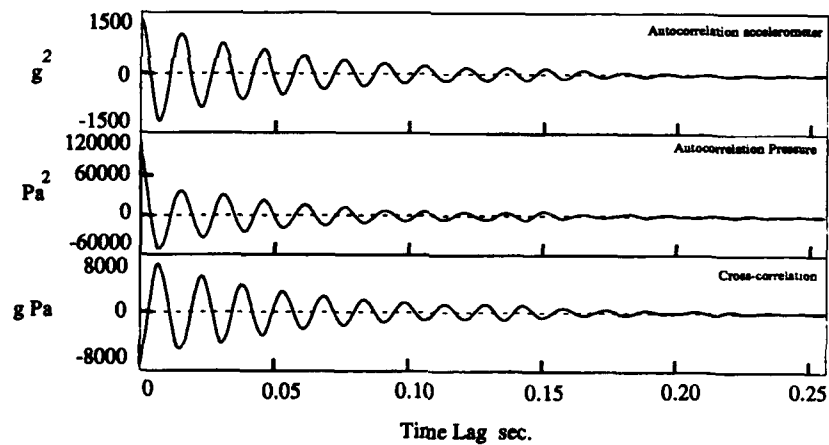Sampling rate = 2000 Hz, N = 1024

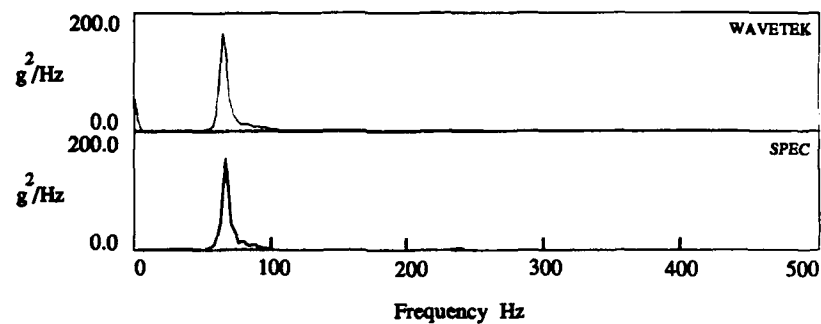FIGURE A2. Correlation functions calculated using program SPEC

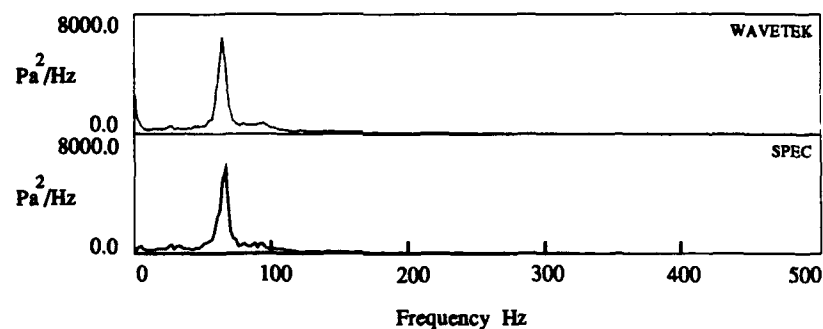FIGURE A3. Power Spectral Density of Accelerometer Signal

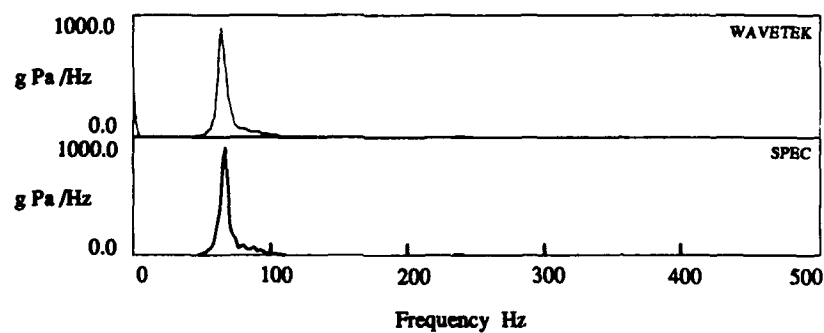FIGURE A4. Power Spectral Density of Pressure Transducer Signal

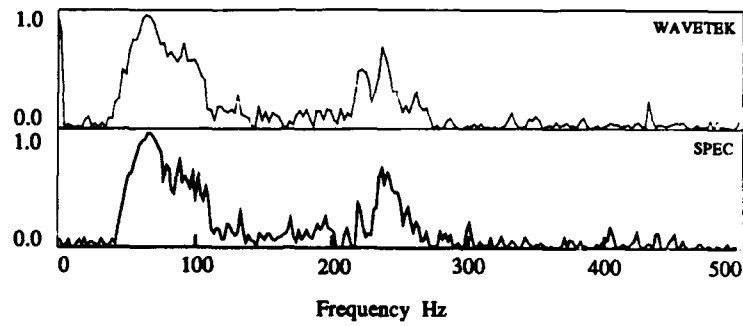FIGURE A5. Cross Power Spectral Density of Above Signals
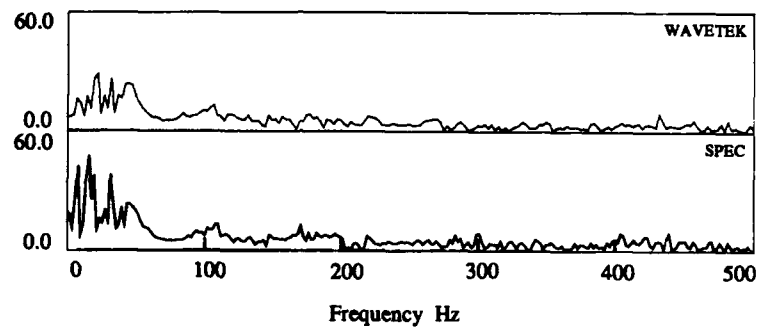
FIGURE A6. Coherence Function

FIGURE A7. Modulus of Transfer Function

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16. ABSTRACT

The analysis of random data is frequently carried out 'on-line' using a hardware based Fourier analyser. Capabilities similar to those provided in the 'on-line' analysers have been developed in a Fortran computer program that can be used to analyse a digital time-dependent signal a. the post-test stage. An overview of the calculation methods used within this software is also presented.

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