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A GOODNESS-OF-FIT TEST FOR A FAMILY OF TWO PARAMETER WEIBULLS WITH KNOWN SHAPE USING MINIMUM DISTANCE ESTIMATION OF PARAMETERS

THESIS

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of the Air Force Institute of Technology
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John S. Crown, B.S.
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Of course I would be remiss if I forgot to express my gratitude to my parents, whose undying support and encouragement gave me the strength and determination to complete this project.

John S. Crown

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Abstract

This research is to produce a modified Anderson-Darling goodness-of-fit test for the Weibull distribution when the location parameter is found by minimum distance estimation, the shape parameter is assumed known, and the MLE is used for the scale parameter. The critical values for the Anderson-Darling test are generated via Monte Carlo simulation when both the Anderson-Darling and Cramer-Von Mises distance statistics are minimized. These critical values are then used for a power study in testing whether a set of observations follows a Weibull distribution when the scale and location parameters are unspecified and are estimated from the sample while the shape is assumed known. The Monte Carlo simulation used 5000 repetitions for sample sizes of 5, 8, 12, 15, 16, 20 and 25 with the Weibull shape parameter of .5(.5)4.0. The power study is made for the same sample sizes as above with the hypothesized Weibull shape parameter of 1.0 and 3.5 against ten alternate hypothesized distributions. For small sample sizes, improvement can be seen over tests which use MLEs for the location and scale parameters. However, for larger sample sizes, more than 20, the power is similar to other goodness-of-fit tests for the Weibull. In most cases, minimizing the Anderson-Darling distance statistic to estimate the Weibull location parameter had more power than minimizing the Cramer-Von Mises distance statistic.

A GOODNESS-OF-FIT TEST FOR A FAMILY OF TWO PARAMETER WEIBULLS WITH KNOWN SHAPE USING MINIMUM DISTANCE ESTIMATION OF PARAMETERS

I. Introduction

1.1 Background

The Air Force depends on technology to support its mission. Considering the increasing cost of today's technology, it is imperative to purchase the best weapon systems available. Part of the decision between alternative weapon systems includes system reliability (the probability the system will work, as expected, for a specified duration of time). Computing system reliability is usually achieved through statistical analysis of test data where items have been placed on test and failure times are recorded. With the price of new weapon systems restricting the number of test items, it is becoming more important to be able to make decisions based on a small number of data points.

To make decisions about a set of failure data, statisticians try to estimate the probability density function (pdf) which the data most closely matches. The data is ordinarily hypothesized to come from a classical distribution, such as the exponential, normal, or beta, and the parameters of this distribution are estimated. Through the use of a goodness-of-fit test this hypothesis can be rejected or hopefully accepted. The "goodness" refers to how well the distribution corresponds to the sample data.

For a hypothesized distribution, there are many goodness-of-fit tests to choose from. When deciding which test to use, the statistician will pick the test with the highest power. Power is the probability of rejecting the null hypothesis when in fact this hypothesis is false (10:472). The higher the power of a test, the lower the chance of accepting a distribution when it is false. If the test allows the acceptance of the hypothesized distribution, reliability characteristics, such as expected time to failure and shelf life, can be computed from the distribution and its parameters. These characteristics can be used in the overall decision to purchase and build the weapon system.

The Weibull distribution, developed by W. Weibull in 1939, is used extensively in reliability models of mechanical and electrical components. It is also useful in the goodness-of-fit arena since it can represent an infinite number of shapes, including the exponential. Thus, improved tests for the Weibull distribution can prove useful in many ways.

1.2 Objective

The proposed research will be to develop a new goodness-of-fit test for the Weibull distribution where the location parameter is obtained by minimum distance estimation. Maximum likelihood estimation of the scale parameter will be based on this improved estimate of location and the assumed known shape parameter.

1.3 Sub-objectives

1) Generate random numbers from different classical distribution functions. This will be necessary in generating data to develop the new test.

2) Find the minimum distance estimator for parameters.

3) Generate tables of critical values of the new test statistic.

4) Perform a power study for the new test against many other classical distributions.

1.4 Support Requirements

This research will require a large amount of AFIT computer resources. Most of the computer routines will be in PASCAL.

II. Literature Review

2.1 Introduction

This research project hinges on whether a goodness-of-fit test using minimum distance estimation for the location parameter of the Weibull distribution will be of use. Thus, it is important to locate evidence to determine if minimum distance has advantages over other estimation techniques. To complete this task it will be necessary to review the literature in the areas of parameter estimation and goodness-of-fit test statistics.

2.2 Parameter Estimation

In the statistical area of hypothesis testing it is rare to know anything about the parameters of a distribution being tested. The parameters must be estimated from observed data before proceeding with any test. Two methods of parameter estimation will be discussed here.

2.2.1 Maximum Likelihood. Many studies on estimating parameters of a statistical distribution can be found in the literature. The most common and accepted method of parameter estimation is the maximum likelihood method. This method selects values as estimates that maximize the likelihood of the observed sample, where the likelihood function is the joint density function (10:419). An iterative algorithm to compute the maximum likelihood estimates (MLEs) for the Weibull distribution is presented by Harter and Moore (7:639-643). A modification to Harter and Moore's method described by Gallagher (6:575-580) converges on the MLE in less iterations, thus less computer time. The existence and uniqueness of the scale and location parameter MLEs are proven by Rockette et al. (13:246-249) whenever the shape parameter is known and is greater than or equal to one.

An important property of the MLEs for the location and scale parameters is that they are invariant (2:251-253). An estimator is invariant if, when the sample data is transformed by $aX_i + b$, then the estimator computed from this new transformed data is equal to $a\theta + b$, where θ is the original estimate value. For example, suppose the MLE for the location parameter is 10, and when each sample data element is multiplied by 15 the new MLE for location is 150, the MLE is said to be invariant. This property allows the estimation of one set of location and scale parameters to be generalized to any location and scale. This research will use a location parameter of 10 and a scale parameter of 1. The shape parameter and sample sizes will be varied in the Monte Carlo simulations.

The MLEs are the standard to which other estimation techniques are compared. Sinha and Sloan (15:364-369) compare maximum likelihood estimators (MLEs) to Bayes estimators of the parameters of the three-parameter Weibull distribution. According to Sinha and Sloan, "The Bayes technique is based on asymptotic theory and hence, performs best with larger sample sizes" (15:364-369). Indeed, for small sample sizes, it is possible to produce negative variance estimates for the Bayes estimators. These negative variance estimates were shown in a table to turn positive somewhere between the sample sizes of 40 and 100. This result would deter the use of Bayes estimates for small sample sizes. For larger sample sizes, greater than 100, Sinha and Sloan show the Bayes estimators to have smaller variances than the MLEs. Small variance is a favorable quality in estimators.

A similar comparison of Bayes estimators and MLEs is given by Smith and Naylor (16:358-369). This study uses two sets of data to show the Bayes estimators are superior to the MLEs. The data sets used in the study have sample sizes of 63 and 46 elements. Smith and Naylor also show the Bayes estimators have smaller variances than the MLEs.

2.2.2 Minimum Distance. Another parameter estimation procedure is minimum distance. Wolfowitz published two papers in the 1950's to introduce this method and showed minimum distance (MD) estimators to be consistent (18:75). A consistent estimator converges stochastically to its parameter value with probability one. Sahler outlined conditions for the existence and consistency of MD estimates (14:85). Knusel examined the robustness of the MD method in 1969 and showed that MD estimators have similar robust properties as the maximum likelihood estimators. Robustness refers to an estimating procedure that is good for a broad class of underlying models but which is not necessarily the best estimating procedure for any one of the models. Woodward et al. show MD estimators to be more robust than the MLE in a study of the mixture of two normal components (20:598). The term robust is used here in the sense that the MD estimator is less sensitive to symmetric departures from the underlying assumption of normality of component distributions. Parr and Schucany performed more studies in robustness of the MD method and used MD estimators for the location parameter of symmetric distributions (12:616). Parr and Schucany also noted that MD estimates were invariant, another property in common with maximum likelihood estimators.

The minimum distance estimation method involves minimizing the distance between the cumulative distribution function (CDF) values of the sample data and the empirical distribution function (EDF). The CDF values come from a parameterized family of theoretical distribution functions. In this thesis, the CDF will be the Weibull distribution with estimated location and scale parameters, but known shape parameter. The EDF for n ordered data points $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ is a step function given by:

$$EDF(x) = \begin{cases} 0, & x < x_{(1)} \\ \frac{i}{n}, & x_{(i)} \leq x < x_{(i+1)}, \qquad i = 1, \ldots, (n-1) \\ 1, & x \geq x_{(n)} \end{cases} \tag{2.1}$$

Distance is measured by a goodness-of-fit statistic, which quantifies the difference between the CDF and the EDF. Adjustments are made to the parameter being estimated until the distance is minimized, resulting in a final estimate of the parameter. The proposed new goodness-of-fit test will use a minimum distance estimator for the location parameter and an MLE for the scale parameter.

Gallagher and Moore have recently shown that minimum distance estimation on the location parameter and maximum likelihood on the shape and scale parameters of the Weibull distribution is preferred over MLE of all three parameters (6:575-580). Some criteria that lead to this preference were normalized mean square error of estimators, goodness-of-fit statistics, integrated absolute difference between CDFs, integrated squared difference between CDFs, percentage of times better than MLE, and robustness of estimators. Several estimators were compared and the one selected to be used in this thesis outperformed the rest in most cases.

2.3 Goodness-of-Fit Statistics

A goodness-of-fit test uses a test statistic to measure how closely data "fits" a hypothesized distribution. The first step of a test is to make a hypothesis (guess) of the distribution the data resembles. Next, the parameters of the distribution must be estimated from the data, unless they are known *a priori*. Once the parameters are estimated, the CDF and EDF values can be computed for each of the ordered data points in the sample. The goodness-of-fit statistic is then computed. The goodness-of-fit statistic is a measure of the distance between the CDF and the EDF. The statistic is compared to tables constructed specifically for that statistic to make a decision about the hypothesized distribution. If the statistic's value is too large, the two distributions are too far apart; hence, the distribution being tested is rejected. Woodruff and Moore specify that modified goodness-of-fit tests, where the parameters have been estimated, must not use tables built based on known parameters (19:115).

Although there are many goodness-of-fit statistics available, this research will only consider two: the Cramer-von Mises and the Anderson-Darling. Parr and Schucany define the weighted Cramer-von Mises distance as follows:

$$W_{\psi}^2(K,F) = \int_{-\infty}^{\infty} (K(x) - F(x))^2 \psi(F(x)) dF(x) \tag{2.2}$$

where $K(x)$ is the EDF,

$F(x)$ is the CDF,

and $\psi(F(x))$ is a weighting function (12:616).

This distance represents the area discrepency between the EDF and the CDF. When the weighting function is a constant one, $\psi(F(x)) = 1$, the equation above becomes the Cramer-von Mises statistic. The computational formula for this statistic was presented by Stephens (17:731).

$$CVM = W^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left(z_i - \frac{2i-1}{2n} \right)^2 \tag{2.3}$$

where $z_i = F(x_i)$ for $i = 1, 2, \ldots, n$.

Clarke gives a comparison of a minimum distance estimator and the maximum likelihood estimator (MLE) of the proportion parameter in a mixture of two normal distributions (5:275-281). By minimizing a Cramer-von Mises type distance with an empirical weight function, Clarke devises a parameter estimate which is shown to be superior to the MLE.

The Anderson-Darling statistic is a special case of the Cramer-von Mises distance when the weight function is given as follows:

$$\frac{1}{F(x)(1 - F(x))} \tag{2.4}$$

where $0 \le F(x) \le 1$.

This causes more emphasis to be placed on the tail discrepancy of the distributions. The equation for the Anderson-Darling statistic is as follows: (1:765)

$$AD = A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\ln(z_i) + \ln(1 - z_{n+1-i})] \qquad (2.5)$$

where $z_i = F(x_i)$ for $i = 1, 2, \ldots, n$.

Minimum distance estimation of the location parameter for the Weibull distribution, using the Anderson-Darling statistic, has been shown to outperform maximum likelihood estimation by Miller (11:495).

Other uses of the goodness-of-fit statistic and minimum distance estimation are presented by Hobbs, Moore, and James. In their study, several new estimators were developed for the three- parameter gamma distribution using minimum distance estimators for the location parameter with the remaining parameters determined by MLEs. All these new estimators were compared to the MLEs. The three distance statistics used were Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling. First the three parameters were estimated by the MLEs, then the location parameter was estimated again by minimizing these three distance statistics. Finally, using the new location parameter, new estimates of the other two parameters were made using the MLEs. The results show the Anderson-Darling statistic was the best choice of minimum distance statistics to estimate the location parameter in all sample sizes. Hobbs', Moore and James results also show that the minimum distance estimation gave better estimates than the maximum likelihood estimation (8:237-240).

Woodruff and Moore developed a modified goodness-of-fit test for the Weibull distribution with unknown location and scale and known shape parameters (19:2465). Maximum likelihood estimation was used for the location and scale parameter estimates. Depending on the value of the shape parameter, the most powerful tests were obtained by using either the Anderson-Darling or Cramer-von Mises statistic. This test can be used for comparison of the power of the new test.

To conclude, there has been much literature compiled in the areas of minimum distance estimation and goodness-of-fit test statistics. There is much evidence that a new goodness-of-fit test using the improved technique of minimum distance estimation will be more powerful than existing tests. Minimum distance estimators have many properties of the maximum likelihood estimators and seem to outperform them in most cases. The Cramer-von Mises and Anderson-Darling test statistics have been shown to be most powerful, so these two statistics will be used as the distance measures to minimize. To reiterate, the proposed research will develop a new goodness-of-fit test for the Weibull distribution using minimum distance estimation techniques to determine the location parameter, and maximum likelihood techniques to estimate the scale parameter.

III. Methodology

3.1 Introduction

The Weibull distribution function is the basis for this research effort. The cumulative distribution function and the probability density function are below.

3.1.1 Weibull Cumulative Distribution Function (CDF)

$$W(x;\beta,\delta,\theta) = 1 - \exp\left[-\left(\frac{x-\delta}{\theta}\right)^{\beta}\right], \qquad \delta \leq x \tag{3.1}$$

where $\theta > 0$ is the scale parameter,

$\beta > 0$ is the shape parameter,

δ is the location parameter

3.1.2 Weibull Probability Density Function (PDF)

$$w(x;\beta,\delta,\theta) = \frac{\beta}{\theta}\left[\frac{x-\delta}{\theta}\right]^{\beta-1} \exp\left[-\left(\frac{x-\delta}{\theta}\right)^{\beta}\right], \qquad \delta \leq x \tag{3.2}$$

The Weibull function can take on many looks based on a variety of shape values. including the exponential. This is shown in figure 3.1 on page 3-2. The shape value of 1.0 is an exponential distribution. A shape value of 2.0 looks like an F distribution, while a shape value of 3.0 to 4.0 looks like a normal distribution.

This thesis presents a Monte Carlo method of generating the critical values for the new goodness-of-fit test. A flow diagram of this procedure will be presented on page 3-6.

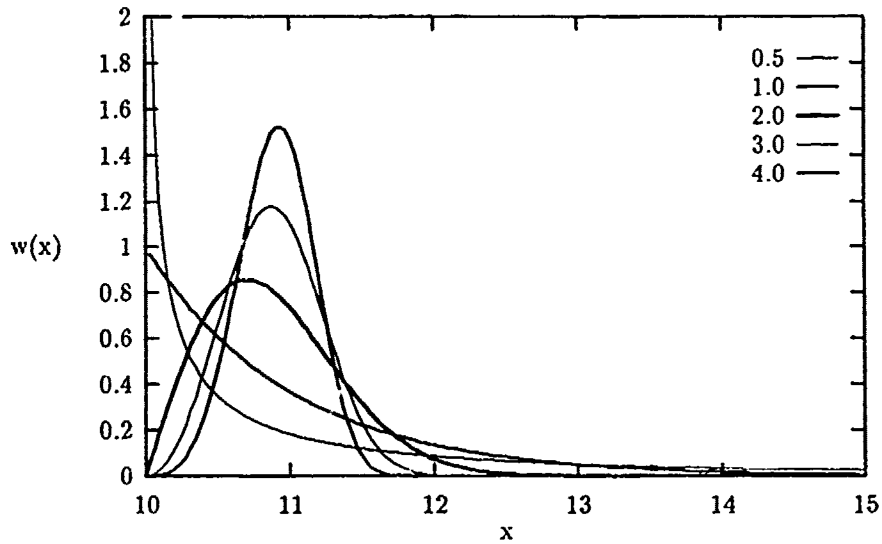

Figure 3.1. Weibull PDF for Different Shape Values

3.2 Random Deviate Generation

The first step in generating the critical values for the goodness-of-fit test is to generate random samples from the Weibull distribution with $\theta = 1$, $\delta = 10$, and the shape parameter (β) varies from $\frac{1}{2}$ to 4 by an increment of .5. That large value of the location parameter keeps the random samples far enough away from zero to ensure the location parameter estimates are always positive. This would provide a goodness-of-fit test which will be invariant. Sample sizes of 5, 8, 12, 15, 16, 20 and 25 are drawn 5000 times from each of the above distributions. To provide these samples, a computer program was written using the cumulative distribution function method to generate random variates from the Weibull distribution.

3.3 Parameter Estimation

For each sample, the location and scale parameters are initially estimated using the maximum likelihood estimators (MLEs). The shape parameter is assumed to be known throughout the procedure. The minimum distance procedure introduced by

Wolfowitz is used to improve the initial location estimate. This is achieved by shifting the location estimate in a linear fashion until the goodness-of-fit distance statistics, Anderson-Darling (A-D) and Cramer-Von Mises (C-VM), are minimized. The value of the location estimate which minimizes this search produces the closest fit of the sample data to the Weibull distribution. This new location estimate, along with the known shape parameter value, is used in the MLE formula to compute the new scale parameter estimate. The linear search is accomplished with modifications to an existing program from similar research (6:575-580) using a golden-section search routine. Caution should be shown in using the existing program since it does not allow negative values for the location estimate.

3.3.1 Golden-Section Search. The golden-section search routine is an iterative linear search method that will find the minimum of a unimodal function over a given interval. The basis of this procedure is to hook the function minimum in some interval, then to collapse the interval around this minimum until the length of the interval is within a small error tolerance level. What makes the golden-section search more efficient than other similar search routines is the way the interval is reduced. Suppose l_k and r_k are the ieft and right endpoints, respectively, of the kth iteration. Then two more points x_k^1 and x_k^2, where $x_k^1 < x_k^2$, are placed symmetrically inside the interval $[l_k, r_k]$ by the equations:

$$x_k^1 = l_k + (1 - \alpha)(r_k - l_k) \tag{3.3}$$

$$x_k^2 = l_k + \alpha(r_k - l_k) \tag{3.4}$$

where $\alpha = 0.618$, $\frac{1}{\alpha} = \frac{2}{-1+\sqrt{5}} \Leftarrow$ the golden ratio

Once the intial interval is broken into three sections by these points, the unimodal function values at the interior two points are used to reduce the interval. The section of the interval outside the interior point with the higher function value will be removed leaving the function minimum inside the other two sections. This interior point, with the higher function value, will become the endpoint of the interval, while the other interior point remains an interior point for the next iteration. This means that only one new interior point must be computed along with its function value, resulting in an efficient algorithm. The steps of the procedure once the initial four points are computed are as follows:

3.3.1.1 Step 1: If $r_k - l_k < \epsilon$ stop; the optimal point is in $[l_k, r_k]$. If $f(x_k^1) > f(x_k^2)$, go to step 2. If $f(x_k^1) \leq f(x_k^2)$, go to step 3.

3.3.1.2 Step 2: $l_{k+1} = x_k^1$ and $r_{k+1} = r_k$; $x_{k+1}^1 = x_k^2$ and $x_{k+1}^2 = l_{k+1} + \alpha(r_{k+1} - l_{k+1})$; evaluate $f(x_{k+1}^2)$ and go to step 4.

3.3.1.3 Step 3: $l_{k+1} = l_k$ and $r_{k+1} = x_k^2$; $x_{k+1}^2 = x_k^1$ and $x_{k+1}^1 = l_{k+1} + (1-\alpha)(r_{k+1} - l_{k+1})$; evaluate $f(x_{k+1}^1)$ and go to setp 4.

3.3.1.4 Step 4: $k = k + 1$; go to step 1.

3.4 Critical Values

Once the parameters have been estimated, the critical values of the new test statistic are found. Using the parameter estimates in the Weibull CDF with the sample data, the A-D test statistic is computed for each of the 5000 samples. These 5000 values are ranked and tables are made of the .01, .05, .10, .15, and .20 testing level critical values for all sample sizes. The A-D test statistic is found for both of the MD estimators, the A-D and the C-VM, to determine which has the most power.

One method of computing the critical values is to simply pull off the ranked number corresponding to the percentile wanted. For example, out of 5000 ranked values, the 4950th would correspond to the 99th percentile. This procedure infers that the range of the statistic is fixed between the 1st and 5000th ranked values, which is a false representation.

This thesis uses a slightly more complicated method of computing the critical values involving linear interpolation. This method plots the 5000 test statistics on the horizontal axis versus some plotting position on the vertical axis. For the plotting position used in this thesis, the median rank approximation shown in Kapur and Lamberson (9:300) is as follows:

$$y_i = \frac{i - .3}{n + .4} \tag{3.5}$$

The plotting position values on the vertical axis create a scale between zero and one which represent percentiles. The 80th, 85th, 90th, 95th, and 99th percentiles are calculated by linear interpolation between the two plotted points whose vertical axis values surround the respective percentile value. For example, the 85th pecentile is calculated by finding the median rank value on the vertical axis just below .85, say i, and the value just above .85, given by i+1. Thus the 85th percentile is found by interpolating between the ith and the (i+1)st entry in the horizontal axis. This percentile will represent the .15 level of significance for hypothesis testing.

The 5000 values of the plotting position with n=5000 and i=1,2,...,5000 are entered in the vertical array in position 2 to 5001. The interval [0,1] is completed by entering zero and one into position 1 and 5002, respectively. The 5000 ranked test statistics are placed into the horizontal array into positions 2 and 5001. The first and last elements of the horizontal array are found by linear extrapolation. The first is found using the second and third elements (first and second order statistics) with a minimum value of zero (non-negative restriction). The last is found using the 5000th and 5001st element with no maximum restriction. Once the arrays are complete the [0,1] interval is piecewise continuous and the desired percentiles are computed.

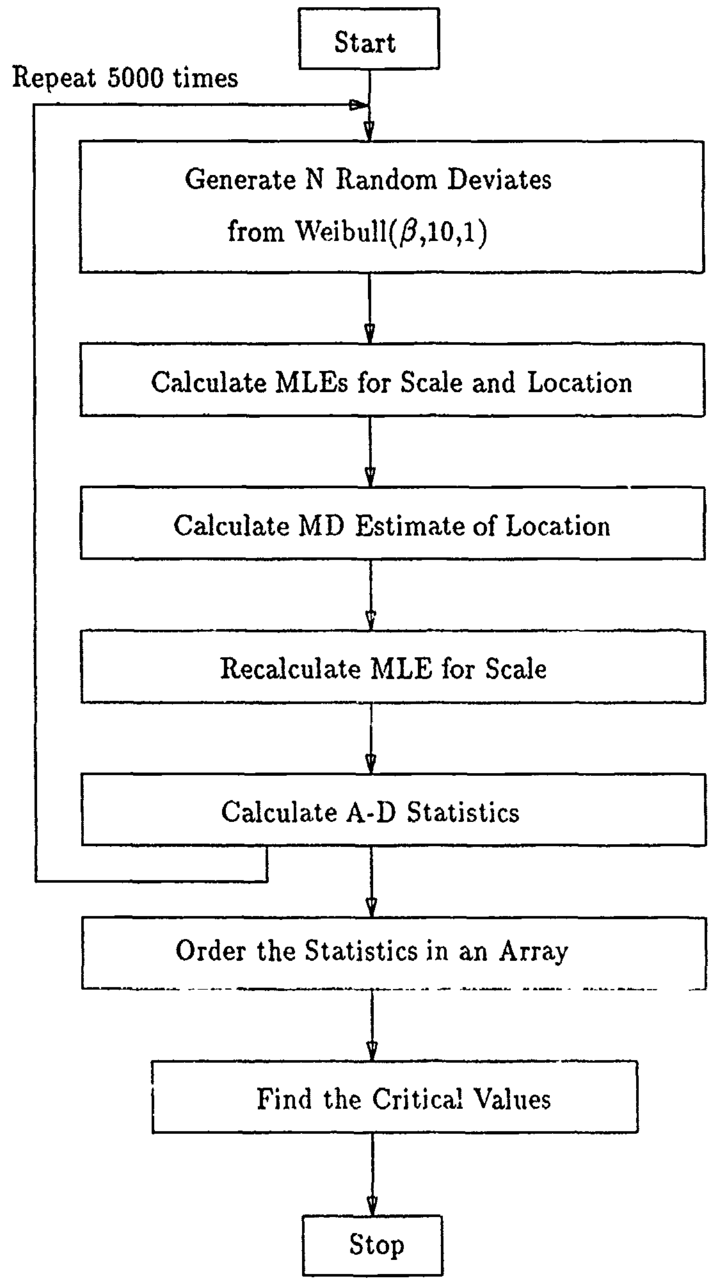

Figure 3.2. Generation of Critical Values

3.5 Power Study

Once the tables of critical values have been made, a power study is completed by using a random number generating program to produce random samples from the following distributions:

1. Weibull with shape = 1.0, location = 10.0 and scale = 1.0
2. Weibull with shape = 2.0, location = 10.0 and scale = 1.0
3. Weibull with shape = 3.5, location = 10.0 and scale = 1.0
4. Gamma with shape = 1.0, location = 10.0 and scale = 0.2
5. Gamma with shape = 2.0, location = 10.0 and scale = 0.2
6. Gamma with shape = 3.0, location = 10.0 and scale = 0.2
7. Normal with mean = 15.0 and variance = 2.0
8. Uniform on interval [10,15]
9. Beta with p = 2 and q = 2 on [10,11]
10. Beta with p = 2 and q = 3 on [10,11]

Graphs of the gamma distributions and the beta distributions appear in figures 3.3 and 3.4, respectively. A random sample is generated from one of the above distributions and is then tested with the null hypothesis that the data is from a Weibull distribution with known shape and estimated location and scale parameters. The alternative hypothesis is that the data is in fact from the distribution used in generating the sample. The A-D test statistic is obtained using distance estimation for the location parameter and MLE for the scale parameter, and this statistic is compared to the critical values from the new tables. If the test statistic is larger than the critical value, the data is determined to be too far from the hypothesized Weibull distribution, and the null hypothesis is rejected.

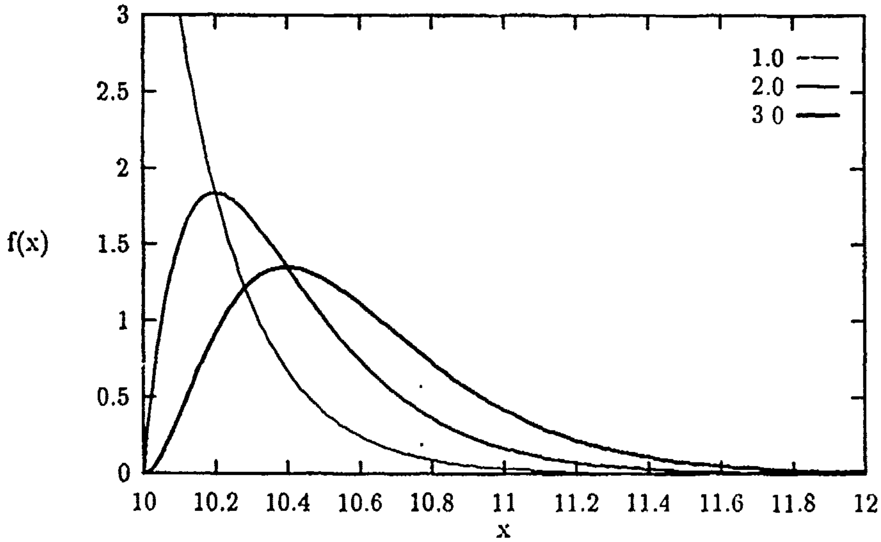

Figure 3.3. Gamma PDF for Different Shape Values

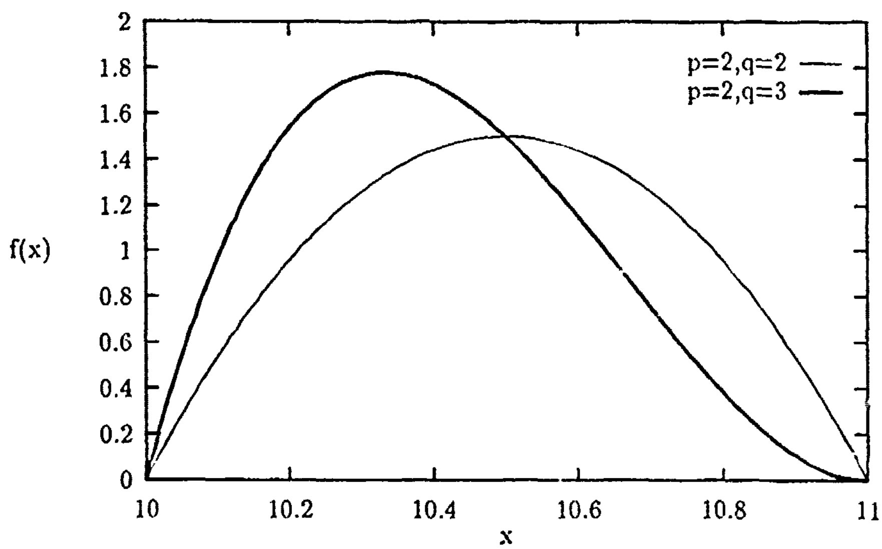

Figure 3.4. Beta PDF for Different Parameter Values

This test is done for sample sizes of 5, 8, 12, 15, 16, 20 and 25 with 5000 of each case. The power of the test for a distribution is then computed by dividing the number of rejections by the total number of trials (5000). This power can be compared to the power of other known goodness-of-fit tests and should show an improvement. This method is shown graphically on the following page and the results of the power study are shown in the next chapter.

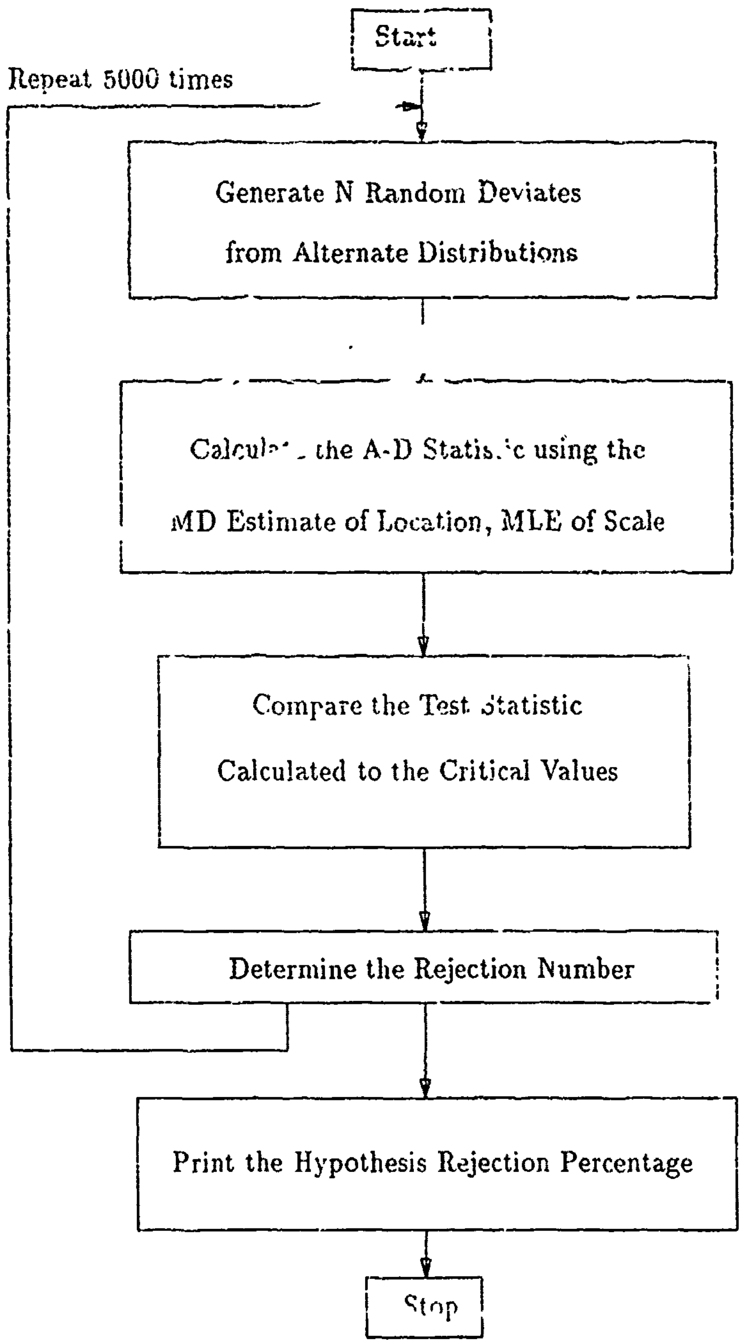

Figure 3.5. Power Study

IV. Results

4.1 Introduction

This chapter presents the results of the research including the tables of critical values of the new test and the power studies.

4.2 Critical Values

Both the Anderson-Darling and Cramer-Von Mises distance statistics were used as the minimum distance estimator of the location parameter, and the resulting critical values of an Anderson-Darling test statistic were obtained using the linear interpolation method discussed in the previous chapter. The critical values for sample sizes of 5, 8, 12, 15, 16, 20 and 25 and shape parameter of 0.5(.5)4.0 are shown for both distance estimators. In the first set of tables, the Anderson-Darling distance was minimized. The trend for these critical values is interesting. With shape parameter equal to 0.5, 1.0 and 1.5, the critical values are monotonically increasing for almost every level of significance as the sample size increases from 5 to 25. When the shape parameter equals 2.0, the critical values are relatively constant for each significance level as the sample size increases. Once the shape parameter equals 2.5 or better, the critical values are monotonically decreasing as the sample size increases.

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.534051	0.607026	0.820025	1.036443	1.213872
8	0.614272	0.664982	0.731374	0.841802	1.162243
12	0.596347	0.654188	0.739516	0.899161	1.328168
15	0.597870	0.666509	0.756392	0.926976	1.330167
16	0.599857	0.670554	0.781886	0.968018	1.358705
20	0.618552	0.693435	0.788167	0.976673	1.429640
25	0.642861	0.719960	0.821111	0.984538	1.458325

Table 4.1. Critical values for shape = 0.5 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.466557	0.510328	0.567311	0.667923	0.901375
8	0.516715	0.572171	0.646776	0.7[illegible]0109	1.069050
12	0.541523	0.594925	0.668569	0.802379	1.129110
15	0.540875	0.599271	0.676492	0.802037	1.130876
16	0.541330	0.602171	0.683986	0.823289	1.154745
20	0.543625	0.607112	0.691740	0.835489	1.169701
25	0.564979	0.617836	0.701454	0.842974	1.195907

Table 4.2. Critical values for shape = 1.0 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.503905	0.552334	0.610183	0.713866	0.917968
8	0.536365	0.589516	0.660859	0.779269	1.042963
12	0.549963	0.604351	0.678464	0.799135	1.115089
15	0.550019	0.605383	0.688189	0.801943	1.094021
16	0.546901	0.605420	0.679028	0.821214	1.130356
20	0.550156	0.604839	0.682869	0.825576	1.086475
25	0.556496	0.612490	0.694096	0.817816	1.118761

Table 4.3. Critical values for shape = 1.5 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.555533	0.601215	0.660493	0.756353	0.962344
8	0.573819	0.622119	0.688422	0.786025	1.035773
12	0.564881	0.620026	0.685421	0.804985	1.068284
15	0.560963	0.615073	0.689270	0.802330	1.071177
16	0.558839	0.613181	0.685358	0.808226	1.115043
20	0.555708	0.609571	0.684567	0.812978	1.065246
25	0.557890	0.609158	0.687513	0.809534	1.071019

Table 4.4. Critical values for shape = 2.0 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.652263	0.699009	0.766547	0.911380	1.210330
8	0.637913	0.680989	0.744459	0.848517	1.081090
12	0.590492	0.643969	0.711170	0.822863	1.111514
15	0.574905	0.629193	0.699603	0.805880	1.059591
16	0.574553	0.619276	0.686357	0.817216	1.120905
20	0.555046	0.608867	0.681335	0.812909	1.076719
25	0.551553	0.600628	0.670653	0.790761	1.062129

Table 4.5. Critical values for shape = 2.5 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.861117	0.951203	1.026658	1.114341	1.333277
8	0.706825	0.758969	0.829725	0.960148	1.219758
12	0.618356	0.671919	0.748272	0.853347	1.135138
15	0.576304	0.634377	0.702875	0.813199	1.051891
16	0.567829	0.618995	0.685710	0.816015	1.116338
20	0.546148	0.600488	0.672285	0.797350	1.075683
25	0.541523	0.588886	0.666653	0.781134	1.037753

Table 4.6. Critical values for shape = 3.0 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	1.042698	1.091778	1.165292	1.290300	1.648458
8	0.747824	0.830331	0.924592	1.051246	1.337583
12	0.605114	0.670964	0.755143	0.876348	1.177469
15	0.562710	0.628160	0.698871	0.812235	1.068987
16	0.558354	0.609675	0.679308	0.812555	1.097881
20	0.535269	0.586810	0.662448	0.779746	1.056165
25	0.533402	0.580727	0.652551	0.769762	1.023432

Table 4.7. Critical values for shape = 3.5 for A-D distance

Anderson-Darling Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	1.199158	1.270814	1.376768	1.576195	1.901552
8	0.711753	0.815421	0.950338	1.121645	1.424366
12	0.587628	0.656820	0.743248	0.881114	1.179224
15	0.546244	0.615950	0.692655	0.807316	1.058057
16	0.548205	0.603818	0.675277	0.809559	1.087799
20	0.528212	0.578695	0.650561	0.762568	1.035120
25	0.529384	0.577706	0.651832	0.763390	1.028816

Table 4.8. Critical values for shape = 4.0 for A-D distance

The following tables of critical values were constructed by minimizing the Cramer-Von Mises distance statistic in the estimation of the Weibull location parameter. The actual critical values are still based on an Anderson-Darling test statistic. For shape parameter equal to 0.5 and 1.0, the values tend to increase as the sample size increases for each level of significance. At a shape equal to 1.5, the values are fairly constant as the sample size increases. For shape parameter equal to 2.0 and larger, the values are monotonically decreasing as the sample size increases for each level of significance. These critical values are slightly larger than the values using the Anderson-Darling distance statistic for smaller values of shape. However, when the shape parameter increases to 2.5, where the Weibull PDF appears mound shaped, the critical values begin to be less than the critical values from when the Anderson-Darling distance was minimized. This may be due to the increased sensitivity the Anderson-Darling statistic places on the tails of a distribution, and for symmetric distributions, the Cramer-Von Mises statistic seems to fit the sample data better.

Both distances produce A-D critical values that are generally smaller than those found when MLEs are used for the location and scale parameters of the Weibull (4:2470). This would suggest that the minimum distance estimator produces a better fit to the data.

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.965486	0.998768	1.039347	1.114850	1.296970
8	0.674697	0.721906	0.794348	0.956306	2.391288
12	0.666256	0.766539	0.985229	1.501673	2.292211
15	0.775618	1.022719	1.152037	1.321945	2.690094
16	0.778373	0.994853	1.096769	1.324162	2.618572
20	0.825711	0.919646	1.057656	1.854281	3.418259
25	0.810191	0.940940	1.316671	1.601477	2.875148

Table 4.9. Critical values for shape = 0.5 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.542990	0.738448	1.023448	1.122639	1.327904
8	0.616389	0.675847	0.755433	0.892519	2.330585
12	0.588321	0.650347	0.757689	1.045055	1.732435
15	0.605844	0.676801	0.800474	1.167642	1.972804
16	0.601291	0.686925	0.834646	1.136493	2.420991
20	0.626597	0.726251	0.870821	1.078427	2.045386
25	0.666003	0.743799	0.857493	1.101560	1.821127

Table 4.10. Critical values for shape = 1.0 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.619172	0.727991	0.992829	1.085388	1.296969
8	0.648949	0.711729	0.800799	0.970381	2.026298
12	0.616561	0.689801	0.794308	0.995101	1.681581
15	0.618371	0.690594	0.798964	1.046778	1.619372
16	0.611952	0.679501	0.804540	1.052238	1.584326
20	0.609062	0.687071	0.789086	0.969580	1.520080
25	0.614664	0.695683	0.797370	0.974221	1.424828

Table 4.11. Critical values for shape = 1.5 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.821194	0.965702	1.035227	1.135092	1.352597
8	0.669553	0.728181	0.803138	0.963331	1.306347
12	0.614077	0.677099	0.754498	0.910881	1.296074
15	0.599767	0.655612	0.741584	0.887657	1.267243
16	0.594161	0.650002	0.738407	0.897428	1.282969
20	0.581169	0.639461	0.718920	0.855779	1.241834
25	0.575536	0.631678	0.720798	0.854290	1.192441

Table 4.12. Critical values for shape = 2.0 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.950715	1.010451	1.077548	1.238630	1.779887
8	0.693967	0.748623	0.826515	0.977111	1.329779
12	0.612581	0.669015	0.744432	0.873301	1.201634
15	0.590743	0.644539	0.716680	0.838352	1.151605
16	0.582513	0.629862	0.709518	0.850830	1.196046
20	0.559817	0.614693	0.693314	0.834221	1.151676
25	0.554977	0.606766	0.683063	0.807105	1.080129

Table 4.13. Critical values for shape = 2.5 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	0.993611	1.011896	1.125477	1.344089	1.931437
8	0.730110	0.785420	0.863328	1.007794	1.368944
12	0.620297	0.674001	0.718487	0.864871	1.191455
15	0.577006	0.634555	0.706111	0.816307	1.089967
16	0.569706	0.619530	0.693463	0.824516	1.157864
20	0.513748	0.595005	0.674024	0.806164	1.124944
25	0.539265	0.589540	0.666551	0.787912	1.023957

Table 4.14. Critical values for shape = 3.0 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	1.050638	1.102468	1.199908	1.445662	2.049980
8	0.743112	0.823153	0.913536	1.049059	1.412587
12	0.602938	0.670812	0.753125	0.870835	1.200510
15	0.563493	0.620364	0.692680	0.808454	1.077812
16	0.558580	0.609075	0.679240	0.806696	1.104273
20	0.531122	0.582526	0.658279	0.781475	1.078567
25	0.531532	0.577235	0.655035	0.774304	1.006227

Table 4.15. Critical values for shape = 3.5 for C-VM distance

Cramer-Von Mises Distance Minimized					
	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
5	1.173730	1.225865	1.300520	1.515403	2.130315
8	0.708673	0.808945	0.940283	1.096419	1.473698
12	0.580940	0.650908	0.737226	0.875456	1.176087
15	0.546469	0.611492	0.686612	0.802979	1.076161
16	0.547523	0.601431	0.672484	0.801461	1.097661
20	0.524349	0.574334	0.644397	0.761153	1.045939
25	0.526483	0.573224	0.645495	0.762069	1.027702

Table 4.16. Critical values for shape = 4.0 for C-VM distance

4.3 Power Study

In this section the results of the power study are shown. For example, in a given power test, the data was randomly drawn from a distribution, say normal(15,2), and was tested to be Weibull with shape equal to 3.5. The location and scale parameters were estimated, the Anderson-Darling test statistic was computed and compared to the critical value of this new test. If the test statistic was greater than the critical value, the hypothesized Weibull distribution was rejected and the alternate hypothesized normal distribution was accepted. The number of rejections divided by the total number of test repetitions is the power of the test.

The tables show the hypothesized Weibull distribution with shape equal to either 1.0 or 3.5 and level of significance of 0.05 and 0.01. First the tables computed by minimizing the Anderson-Darling distance statistic are presented. Following these tables are the tables computed by minimizing the Cramer-Von Mises test statistic. After these tables are shown, a table is presented with the Weibull shape of 2.0 and level of significance of 0.05 minimizing only the Anderson-Darling distance. A listing, with several graphs, of the alternate distributions used in the power study was presented in the previous chapter. Based on the shapes of the chosen alternate distribution, we would expect the following observed results. A Weibull with shape of 1.0 is an exponential, as is the gamma distribution with shape of 1.0. With this in mind, the power against the Weibull with shape of 1.0 should be close to the power against the gamma with shape of 1.0. Likewise, the Weibull with shape of 3.5 is similar to the normal distribution, with the exception that the normal is not anchored on the left side like the Weibull which might result in slight differences in power. Also, if the power of this test against an alternate distribution is small, it is important to realize that both the hypothesized distribution and the alternate distribution would do a good job of fitting the data. Thus, either distribution can be used to represent the population from which the data was obtained. Of course, when the hypothesized distribution and the alternate distribution are both Weibull with the same shape value, the power should be the same as the level of significance. This prior intuition of the similarities in the alternate distributions was verified by the results in this thesis effort.

When these power quantities are compared to power quantities using MLEs, there is a mixture of results depending on the value of shape and sample size. It is known that as the sample size increases, both the MD estimator and the MLE approach the true value of the parameter (consistent). For many small sample sizes the power is increased by the MD estimator over the MLE power study presented by Bush, Woodruff and Moore for shape values of 1.0(4:2471-2472). But for shape equal to 3.5 the MLE has higher power at small sample sizes against most of the alternate distributions tested. It appears that as the Weibull shape parameter increases the density becomes more peaked. This spike in the distribution might enable the MLE estimator to become more effective. This would increase the power of the MLE based goodness-of-fit test. The test developed in this research has good power at low sample sizes and lower values of the shape parameter. Generally, the power is better when the Anderson-Darling distance is minimized. The power tables are as follows.

Anderson-Darling Distance Minimized						
	Alternate Distribution					
n	Weibull (1,10,1)	Weibull (2,10,1)	Weibull (3.5,10,1)	Gamma (.2,1)	Gamma (.2,2)	Gamma (.2,3)
5	.0462	.0850	.1598	.0878	.1126	.1118
8	.0522	.2228	.4274	.0594	.1144	.1580
12	.0474	.2586	.5184	.0482	.0948	.1468
15	.0468	.3290	.6516	.0532	.1208	.1992
16	.0460	.3442	.6674	.0554	.1210	.1958
20	.0510	.4630	.8142	.0556	.1526	.2614
25	.0574	.6076	.9222	.0556	.2030	.3638

Table 4.17. Power Study for alpha = .05, Ho: shape = 1.0 for A-D

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.2002	.1402	.1660	.1774
8	.4018	.2662	.3792	.3946
12	.5522	.3996	.4538	.4734
15	.6804	.5184	.5940	.6224
16	.7038	.5266	.6216	.6528
20	.8190	.6740	.7660	.7922
25	.9254	.8134	.8784	.9134

Table 4.18. Power Study for alpha = .05, Ho: shape = 1.0 for A-D (Cont.)

Anderson-Darling Distance Minimized						
	Alternate Distribution					
n	Weibull (1,10,1)	Weibull (2,10,1)	Weibull (3.5,10,1)	Gamma (.2,1)	Gamma (.2,2)	Gamma (.2,3)
5	.0088	.0214	.0666	.0694	.0942	.0926
8	.0084	.0492	.1382	.0110	.0188	.0280
12	.0078	.0632	.2216	.0074	.0186	.0332
15	.0092	.1172	.3622	.0088	.0302	.0566
16	.0084	.1268	.3900	.0136	.0326	.0640
20	.0090	.2130	.5850	.0128	.0460	.0924
25	.0112	.3122	.7604	.0094	.0550	.1368

Table 4.19. Power Study for alpha = .01, Ho: shape = 1.0 for A-D

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.1618	.0908	.0852	.1004
8	.1362	.0798	.1108	.1102
12	.2500	.1402	.1602	.1786
15	.4060	.2330	.2862	.3152
16	.4358	.2478	.3274	.3512
20	.6044	.3882	.4848	.5240
25	.7870	.5382	.6388	.7188

Table 4.20. Power Study for alpha = .01, Ho: shape = 1.0 for A-D (Cont.)

Anderson-Darling Distance Minimized						
	Alternate Distribution					
n	Weibull (1,10,1)	Weibull (2,10,1)	Weibull (3.5,10,1)	Gamma (.2,1)	Gamma (.2,2)	Gamma (.2,3)
5	.0160	.0398	.0518	.0866	.0378	.0298
8	.1062	.0324	.0514	.2212	.0738	.0454
12	.3852	.0544	.0452	.4584	.2036	.1374
15	.5792	.0978	.0502	.6226	.3290	.2352
16	.6276	.1000	.0468	.6566	.3670	.2394
20	.7768	.1456	.0536	.7878	.4924	.3334
25	.8912	.1830	.0550	.8904	.6178	.4424

Table 4.21. Power Study for alpha = .05. Ho: shape = 3.5 for A-D

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0442	.0636	.0564	.0638
8	.0542	.0512	.0486	.0572
12	.0532	.0712	.0418	.0430
15	.0590	.1236	.0528	.0502
16	.0562	.1182	.0524	.0444
20	.0644	.1818	.0638	.0576
25	.0672	.2534	.0780	.0748

Table 4.22. Power Study for alpha = .05, Ho: shape = 3.5 for A-D (Cont.)

Anderson-Darling Distance Minimized						
	Alternate Distribution					
n	Weibull (1,10,1)	Weibull (2,10,1)	Weibull (3.5,10,1)	Gamma (.2,1)	Gamma (.2,2)	Gamma (.2,3)
5	.0028	.0070	.0082	.0518	.0108	.0034
8	.0348	.0058	.0110	.1532	.0288	.0096
12	.1934	.0114	.0074	.3084	.0906	.0472
15	.3814	.0320	.0104	.4522	.1758	.1148
16	.4004	.0242	.0100	.4750	.1864	.1078
20	.5838	.0456	.0122	.6232	.2900	.1672
25	.7574	.0674	.0126	.7728	.4152	.2646

Table 1.23. Power Study for alpha = .01, Ho: shape = 3.5 for A-D

	Alternate Distribution			
n	Normal(15,2)	Uniform(10.15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0078	.0128	.0128	.0112
8	.0090	.0102	.0104	.0128
12	.0094	.0102	.0072	.0074
15	.0142	.0316	.0104	.0100
16	.0120	.0194	.0080	.0102
20	.0152	.0486	.0130	.0124
25	.0198	.0858	.0200	.0172

Table 4.24. Power Study for alpha = .01, Ho: shape = 3.5 for A-D (Cont.)

Cramer-Von Mises Distance Minimized						
	Alternate Distribution					
n	Weibull K = 1.0	Weibull K = 2.0	Weibull K = 3.5	Gamma K = 1.0	Gamma K = 2.0	Gamma K = 3.0
5	.0452	.1286	.2000	.0498	.0788	.0928
8	.0464	.1782	.3390	.0512	.0924	.1272
12	.0532	.2076	.3916	.0490	.1002	.1394
15	.0502	.2844	.5042	.0566	.1372	.1868
16	.0502	.3212	.5590	.0600	.1432	.2056
20	.0566	.4232	.7366	.0584	.1624	.2514
25	.0614	.4900	.8524	.0578	.1876	.2914

Table 4.25. Power Study for alpha = .05, Ho: shape = 1.0 for C-VM

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.1990	.1770	.1974	.1986
8	.3524	.2786	.3036	.3238
12	.4214	.3376	.3532	.3644
15	.5196	.4374	.4558	.4782
16	.5852	.4798	.5242	.5396
20	.7444	.6508	.6842	.7060
25	.8682	.7526	.7894	.8352

Table 4.26. Power Study for alpha = .05, Ho: shape = 1.0 for C-VM (Cont.)

Cramer-Von Mises Distance Minimized						
	Alternate Distribution					
n	Weibull K = 1.0	Weibull K = 2.0	Weibull K = 3.5	Gamma K = 1.0	Gamma K = 2.0	Gamma K = 3.0
5	.0090	.0292	.0538	.0128	.0198	.0202
8	.0078	.0256	.0328	.0102	.0132	.0192
12	.0134	.0560	.1192	.0102	.0206	.0316
15	.0084	.0448	.0912	.0094	.0184	.0228
16	.0092	.0380	.0596	.0084	.0200	.0226
20	.0102	.0930	.1938	.0078	.0328	.0490
25	.0136	.1600	.4680	.0154	.0502	.0796

Table 4.27. Power Study for alpha = .01, Ho: shape = 1.0 for C-VM

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0594	.0530	.0504	.0512
8	.0326	.0412	.0376	.0358
12	.1292	.1374	.1194	.1254
15	.1002	.1128	.0844	.0888
16	.0650	.0972	.0708	.0720
20	.2224	.2204	.1774	.1910
25	.5082	.4058	.3850	.4186

Table 4.28. Power Study for alpha = .01, Ho: shape = 1.0 for C-VM (Cont.)

Cramer-Von Mises Distance Minimized						
	Alternate Distribution					
n	Weibull K = 1.0	Weibull K = 2.0	Weibull K = 3.5	Gamma K = 1.0	Gamma K = 2.0	Gamma K = 3.0
5	.0144	.0392	.0478	.0718	.0310	.0292
8	.0964	.0296	.0536	.2188	.0702	.0424
12	.3688	.0506	.0474	.4466	.1930	.1292
15	.5590	.0922	.0508	.6068	.3140	.2222
16	.6082	.0926	.0498	.6416	.3520	.2280
20	.7540	.1308	.0522	.7694	.4716	.3166
25	.8784	.1630	.0536	.8760	.5934	.4180

Table 4.29. Power Study for alpha = .05, Ho: shape = 3.5 for C-VM

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0432	.0572	.0526	.0536
8	.0520	.0534	.0508	.0586
12	.0538	.0752	.0444	.0476
15	.0604	.1248	.0552	.0522
16	.0586	.1238	.0566	.0506
20	.0618	.1780	.0654	.0586
25	.0644	.2500	.0746	.0718

Table 4.30. Power Study for alpha = .05, Ho: shape = 3.5 for C-VM (Cont.)

Cramer-Von Mises Distance Minimized						
	Alternate Distribution					
n	Weibull K = 1.0	Weibull K = 2.0	Weibull K = 3.5	Gamma K = 1.0	Gamma K = 2.0	Gamma K = 3.0
5	.0030	.0070	.0096	.0320	.0086	.0044
8	.0252	.0066	.0130	.1410	.0252	.0072
12	.1684	.0096	.0072	.2984	.0786	.0408
15	.3530	.0272	.0118	.4358	.1634	.1046
16	.3752	.0214	.0096	.4596	.1736	.1010
20	.5464	.0360	.0114	.5908	.2588	.1504
25	.7494	.0644	.0146	.7622	.4082	.2626

Table 4.31. Power Study for alpha = .01, Ho: shape = 3.5 for C-VM

	Alternate Distribution			
n	Normal(15.2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0066	.0144	.0120	.0106
8	.0072	.0112	.0094	.0130
12	.0092	.0102	.0074	.0084
15	.0144	.0334	.0108	.0102
16	.0118	.0228	.0098	.0116
20	.0126	.0486	.0116	.0122
25	.0208	.0954	.0208	.0232

Table 4.32. Power Study for alpha = .01, Ho: shape = 3.5 for C-VM (Cont.)

Finally, a power table is shown for the null hypothesis of a Weibull with shape equal to 2.0. This shows the power of the test against a null hypothesis of a distribution with shape similar to an F distribution. The level of significance is 0.05 and the distance used for the location parameter was the A-D distance. Good power is achieved for an alternate of Weibull with shape equal to 1.0 and it appears to have good power against the normal distribution and the Weibull with shape of 3.5.

Anderson-Darling Distance Minimized						
	Alternate Distribution					
n	Weibull (1,10,1)	Weibull (2,10,1)	Weibull (3.5,10,1)	Gamma (.2,1)	Gamma (.2,2)	Gamma (.2,3)
5	.0824	.0472	.0672	.0808	.0568	.0460
8	.1732	.0536	.0992	.1770	.0828	.0706
12	.2762	.0508	.1062	.2930	.1202	.0814
15	.3770	.0584	.1216	.3804	.1516	.1004
16	.3964	.0536	.1266	.4184	.1632	.0992
20	.5078	.0510	.1588	.5192	.1964	.1090
25	.6410	.0498	.2062	.6392	.2458	.1324

Table 4.33. Power Study for alpha = .05, Ho: shape = 2.0 for A-D

	Alternate Distribution			
n	Normal(15,2)	Uniform(10,15)	Beta(p=2,q=2)	Beta(p=2,q=3)
5	.0724	.0716	.0634	.0624
8	.1056	.1134	.0926	.0942
12	.1252	.1388	.1050	.1016
15	.1494	.1862	.1254	.1282
16	.1542	.1784	.1374	.1336
20	.1954	.2282	.1414	.1604
25	.2588	.3040	.1780	.2024

Table 4.34. Power Study for alpha = .05, Ho: shape = 2.0 for A-D (Cont.)

The following figure illustrates the greatest power difference obtained with the selected alternative distributions between the MD estimator and the MLE. The test was a null hypothesized Weibull distribution with shape of 1.0 versus the alternative uniform on [10,15] at a significance level of 0.01.

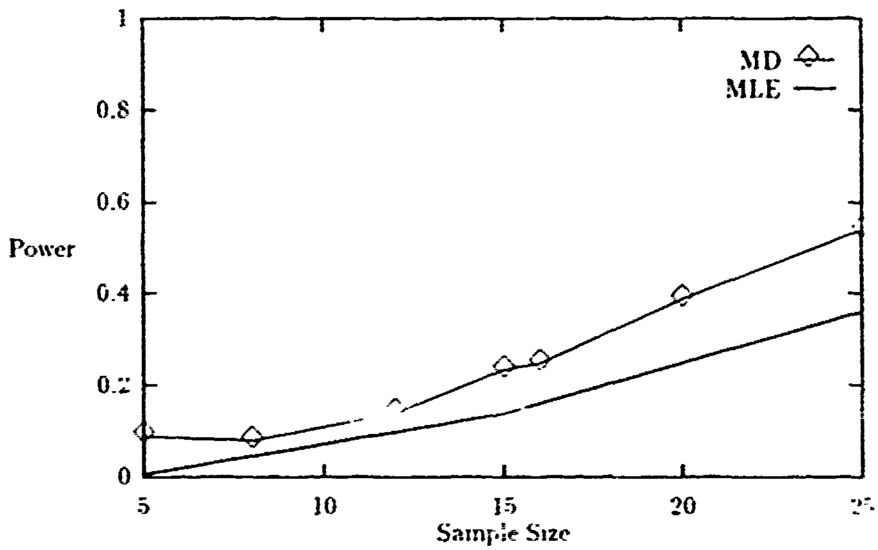

Figure 4.1. Power Comparison of MD to MLE for Uniform

4.4 Verification and Validation

Verification of the computer code was accomplished by an extensive line by line check to compare the code to the actual routines, such as the routines for generating random deviates from the different distributions found in Banks and Carson (3:294-300). The random number generator used was the linear congruential method from the IMSL routines. The seed was the tested and recommended seed used in the same IMSL generator.

Validation of the computer code was accomplished by matching the output to hand calculations of the data involving 10 repetitions instead of the 5000 used in the Monte Carlo simulation. The computer calculations of the linear interpolation method to compute the critical values were confirmed in this manner.

V. Conclusions and Recommendations

5.1 Conclusions

The Anderson-Darling critical values for the Weibull distribution presented in this thesis are valid. Ir , power study, a true null hypothesis achieved the expected level of significance.

The conclusions based on the power study presented in chapter IV are applicable to the ten alternate distributions. Good power is achieved for small sample sizes and some shape values for certain alternate distributions. Improvement over MLE based goodness-of-fit tests for the Weibull is seen for almost all cases of the uniform distribution. Also for small values of the shape parameter and small sample sizes, the new test performs well compared to the MLE based test for most of the alternate distributions tested. However, for sample sizes of 20 or more, the new test shows similar results to the MLE based test. In general, the Anderson-Darling distance statistic is more powerful than the Cramer-Von Mises distance statistic for estimating the location parameter.

5.2 Recommendations for Further Research

' ' th all Monte Carlo studies, there can be increased accuracy if the number of repetitions is increased. This thesis used 5000 repetitions and might show more consistent results if 10,000 repetitions were used. Possibly a more powerful goodness-of-fit test would result where both the location and scale parameters are found by minimum distance estimation by some iterative technique. Finally, since the minimum distance goodness-of-fit test performs well at small sample sizes, further computer runs could be made for the sample sizes less than 12 that were not included in this research to complete the tables of critical values.

Appendix A. *Computer Code*

```
program thesis(input, output, out);
   const
      error     = 0.0001;
      tolerance = 0.0001;
      repetitions = 5000;

   type
      data = array[0..30] of real;    (* generated random variables       *)
                                      (* position 0 is the number of rvs. *)
      para = array[1..3] of real;     (* array of the Weibull paramters   *)
                                      (* in order location, scale, shape  *)
      gofstat = (AD, CVM);         (* Anderson-Darling or Cramer VonMises *)

      teststat = array[0..5001] of real; (* Array of GOF Statistics used  *)
                                         (* to pick off critical values   *)
      critv = array[0..5] of real;       (* array of critical values      *)


   var
      out    :text;          (* out is the output file                          *)
      dataset:data;          (* generated random numbers                        *)
      TRU,                   (* True parameters used to generate the data       *)
      MLE,                   (* Maximum Likelihood Estimates                    *)
      FOSL,                  (* First Order Stat for Location MLE for others    *)
      MDLAD,                 (* Min Dist on Location using Anderson-Darling     *)
      MDLCVM  :para;         (* Min Dist on Location using Cramer-Von Mises     *)

      which   : gofstat;     (* Which goodness of fit statistic AD or CVM       *)
      power,
      mlegof,                (* mle goodness of fit statistic                   *)
      gofvalue: real;        (* each estimates goodness of fit statistic        *)
      seed,                  (* seed for uniform random number generator        *)
      i,j,k   : integer;
      trueloc : boolean;     (* true if location is assumed known for MLE       *)
      ADCRIT,
      CVMCRIT : teststat;    (* Sorted GOF statistics for crit values           *)
      NRrejAD,               (* Number of rejections in power study             *)
      NRrejCVM,
```

```
      cvad,
      cvcvm   : critv;      (* Critical values for A-D and CV-M                  *)

function uniform(var seed:integer): real;
(*  Generates a uniform random number *)
(*  Introduction to Simulation by Payne(1982) page 310 *)
   const
      a = 62605;          (* multiplier  *)
      c = 113218009;      (* increment   *)
      m = 536870912;      (* modulus     *)

   begin
      seed := (a * seed + c) mod m;
      if seed = 0 then
         seed := 1;
      uniform := seed / m;
   end; (* function uniform *)

function gamma(m:integer): real;
(* This function generates a gamma random variate      *)
(* using the method described in Law and Kelton (1991) *)
(* shape = m                                           *)
   var
      i : integer;
      temp, temp2 : real;

   begin
      temp2 := 1.0;
      for i := 1 to m do
         begin
            temp  := uniform(seed);
            temp2 := temp2 * temp;
         end;
      gamma := (-1/m)*ln(temp2);
   end; (* gamma *)

function cvmgof(x:data;param:para): real;
(*  This function returns the Cramer Von-Mises Goodness of Fit    *)
(*  Statitic for the three parameter Weibull.                     *)
(*  Formulas published in Woodruff, Moore, and Dunne (1983)       *)
(*  Data must be ORDERED !                                        *)
   var
```

```
      cum    : data;             (* cummulative distribution *)
      i,num : integer;
      sum,
      temp  : real;
   begin
      num := trunc(x[0]);
      for i := 1 to num do
         begin
            if x[i] <= param[1] then
               cum[i] := 0
            else
               begin
                  temp := -1*exp(param[3] * ln((x[i]-param[1])/param[2]));
                  cum[i] := 1 - exp(temp);
               end;
         end; (* for *)

      sum := 0;
      for i := 1 to num do
         begin
            temp := cum[i] - (2*i - 1)/(2*num);
            sum := sum + temp*temp;
         end; (* for *)

      cvmgof := (1/(12*num)) + sum;
   end; (* function cvmgof *)


function adgof(x:data;param:para): real;
(*  This function returns the Anderson-Darling Goodness of Fit     *)
(*  Statitic for the three parameter Weibull.                      *)
(*  Formulas published in Woodruff, Moore, and Dunne (1983)        *)
(*  Data must be ordered!                                          *)
   var
      cum : data;          (* cummulative distribution                   *)
      i,num : integer;     (* num is number of data values               *)
      sum,
      temp : real;
   begin
      num := trunc(x[0]);

      for i := 1 to num do
```

```
        begin
           if x[i] <= param[1] then
              cum[i] := 0.001
           else
              begin
                 temp := -1*exp(param[3] * ln((x[i]-param[1])/param[2]));
                 cum[i] := 1 - exp(temp);
                 if cum[i] < 0.001 then
                    cum[i] := 0.001;
                 if cum[i] > 0.99999 then
                    cum[i] := 0.99999;
              end;
         end; (* for *)

      sum := 0;
      for i := 1 to num do
         begin
            temp := (2*i -1)*(ln(cum[i]) + ln(1-cum[num+1-i]));
            sum := sum + temp;
         end; (* for *)

      adgof := -1*num - (sum/num);
   end; (* function adgof *)


function gof(x:data; pars:para; which:gofstat):real;
   begin
      if which = AD then    (* Anderson-Darling Goodness of Fit Statistic *)
         gof := adgof(x, pars)
      else if which = CVM then
         gof := cvmgof(x, pars);
   end;



procedure MLEest(x:data; ctrue:boolean; var location, scale, shape:real);
   (****************************************************************)
   (*  Maximum-Likelihood estimation of three parameters weibull   *)
   (*  Iterative technique developed by H. Leon Harter and Albert *)
   (*  H. Moore and published in Technometrics (Nov 1965).         *)
   (*  Formulas (for two parameter) from ATC Notes page 235 were   *)
```

```
(*  adjusted.  Also see Miller's 1980 thesis. Shape Known!      *)
(***************************************************************)
var
   r,                      (* number of data points less than location *)
   num    : integer;       (* number of data points                     *)
   lastlocation : real;

function scaleest:real;  (* part of MLE procedure  *)
(* scaleest estimatate the scale parameter.  The scale is determined by   *)
(* the location and shape paramters.  Formula in Miller's thesis. (1980)  *)
   var
      temp : real;
      i    : integer;
   begin
      temp := 0;
      for i := (r+1) to num do
         temp := temp + exp(ln(x[i]-location)*shape);
      scaleest := exp(ln(temp/(num-r))/shape);
   end; (* function scaleest *)

function locationest(c:real):real;  (* part of MLE procedure *)
(* locationest estimates the location.  The iterative technique was used *)
(* by Harter and Moore (1965).  Their equation 3.5 is simplified in that *)
(* only complete samples are allowed.  Values for location are tried and *)
(* then adjusted to until the equation is equal to zero.                 *)
   var
      upper,            (* upper limit on c                  *)
      lower,            (* lower limit on c                  *)
      value,            (* value of partial derivative of L *)
      lowerval,         (* value at lower limit              *)
      upperval,         (* value at upper limit              *)
      k  : real;        (* shape parameter                   *)
      i  : integer;     (* c is estimate of location         *)

   function partial(c:real):real;(* part of locationest under MLE proc. *)
   (* returns the derivative by c of ln(max likelihood function)      *)
   (* With the correct estimate of c the equation will be zero.       *)
   (* Harter and Moore (1965) found that with shape <= 1 the partial *)
   (* is monotone.  The resulting estimate is either 0 or the first  *)
   (* order statistic.  The function is positive with too low a c     *)
   (* and negative with too high a c.                                 *)
        var
```

```
      sumx,          (* sum of 1/(x[i]-c)  (k is shape)  *)
      sumxck,        (* sum of (x[i]-c) to the kth power *)
      sumxck1:real;  (* sum of (x[i]-c) to the k-1 power *)
   begin
      sumx := 0;
      sumxck := 0;
      sumxck1 := 0;

      i := 1;
      r := 0;
      while x[i] - c <= 0.0001 do    (* censors data from below *)
        begin                        (* assumes data is ordered *)
           r := r + 1;
           i := i + 1;
        end;

      for i := (r+1) to num do
         begin
            sumx := sumx + 1/(x[i]-c);
            sumxck := sumxck + exp(k*ln(x[i]-c));
            sumxck1 := sumxck1 + exp((k-1)*ln(x[i]-c));
         end;
      partial := (1-k)*sumx+(num*k*sumxck1/sumxck);
   end; (* function partial *)

begin (* function locationest *)
   k := shape;
   value := partial(c);  (* c is last estimate of location           *)

   if value > 0 then      (* bound c between a lower and higher value *)
      begin               (* the lower value is pos and higher is neg *)
         lower := c;
         lowerval := value;
         if (c + 1.0) < x[1] then  (* try to get a small interval *)
            begin
               upper := c + 1.0;
               upperval := partial(upper);
               if upperval > 0 then
                  begin
                     upper := x[1];
                     upperval := partial(upper);
                  end;
```

```
            end;
      end

   else if value < 0 then
      begin
         upper := c;
         upperval := value;
         if (c - 1.0) < x[1] then  (* try to get a small interval *)
            begin
               lower := c - 1.0;
               lowerval := partial(lower);
               if lowerval < 0 then
                  begin
                     lower := 0;
                     lowerval := partial(lower);
                  end;
            end;
      end

   else if value = 0 then     (* if is zero then quit *)
      begin
         upper := c;
         lower := upper;      (* prevents entering loop below *)
      end;

   if upperval = 0 then
      begin
         c := upper;
         lower := upper;
      end;

   if lowerval = 0 then
      begin
         c := lower;
         upper := lower;
      end;

   if (upperval*lowerval) > 0 then (* the function is monotone  *)
      begin                        (* and c is an end point     *)
         upper := x[1];
         upperval := partial(upper);  (* check entire interval  *)
         lower := 0;                  (* to see if its monotone *)
```

```
      lowerval := partial(lower);

      if (lowerval > 0) and (upperval > 0) then
         begin
            lower := x[1];
            c := x[1];
            upper := x[1];
         end;

      if (lowerval < 0) and (upperval < 0) then
         begin
            c := 0;
            lower := 0;
            upper := 0;
         end;
   end; (* if monotone function *)

while ((upper-lower)>tolerance) and (abs(value) > tolerance) do
   begin
      c := (upper + lower)/2;   (* binary search for zero *)
      value := partial(c);

      if value > 0 then
         begin
            lower := c;
            lowerval := value;
         end;
      if value < 0 then
         begin
            upper := c;
            upperval := value;
         end;
   end; (* while *)

i := 1;
r := 0;
while x[i] <= c do      (* censors data from below *)
   begin                (* r is used in shapeest   *)
      r := r + 1;
      i := i + 1;
   end;
```

```
        locationest := c;
     end; (* function locationest *)


  begin (* procedure MLEest *)
    num  := trunc(x[0]); (* the number of data points          *)

    if ctrue then (* location is known -- from GoldenSearch routine *)
      begin
        i := 1;
        r := 0;
        while x[i] <= location do  (* censors data from below *)
          begin
            r := r + 1;
            i := i + 1;
          end;
      end
    else (* location is unknown *)
      begin
        location := x[1]/2;  (* forces location below first data point *)
        r := 0;
        location := locationest(location);
      end;
    scale := scaleest;
  end; (* of procedure MLEest *)

procedure GoldenSearch(x:data; which:gofstat; var pars:para);
   (* starting at "a" searches in "direction" until the function stops   *)
   (* decreasing.  Then begins a golden search on the last two           *)
   (* intervals just prior to the function increasing.                  *)
   var
      a,b,               (* current right and left endpoints   *)
      ab,                (* midpoint between a and b           *)
      left,right,        (* golden search midpoints            *)
      fa,fab,fb,
      fleft,fright,      (* function value at current points   *)
      step,              (* line search interval lenth         *)
      r,                 (* sets golden search interval width  *)
      bound: real;       (* golden search iteration error bound*)

   begin
```

```
     step := pars[1]/20;              (* line interval step size       *)
     if step = 0 then
step := .1;
     r := 0.618034;                   (* golden search multiplier      *)

     a := pars[1];
     fa := gof(x,pars,which);         (* current objective value       *)
     fb := fa + 1;                    (* initiate loop                 *)

     while (fb - fa) > error do       (* loop determines direction to  *)
        begin                         (* decrease the function or if   *)
           b := a + step;             (* current point is the minimum  *)
           pars[1] := b;
           fb := gof(x,pars,which);

           if fb > fa then             (* try the other direction   *)
              begin
                 step := -1 * step;
                 b := a + step;
                 pars[1] := b;
                 fb := gof(x,pars,which);
              end;
           step := step/4;
        end;

     if fb > fa then             (* the original point was the minimum *)
           pars[1] := a
     else
        begin                    (* line search to find interval with minimum *)
           ab  := a;                  (* initialize search  *)
           fab := fa;

           repeat                (* line search checks every step to find     *)
              a   := ab;         (* where the function starts to increase     *)
              fa  := fab;
              ab  := b;
              fab := fb;
              b   := b + step;
              pars[1] := b;
              fb  := gof(x,pars,which);
           until (fb >= fab);
```

```
left  := b - r*(b-a);    (*** GOLDEN SEARCH begins ***)
right := a + r*(b-a);
bound := 2 * abs(step);

pars[1] := left;
fleft := gof(x,pars,which);
pars[1] := right;
fright:= gof(x,pars,which);

while abs(fb-fa) > error do
   begin
      if fleft < fright then          (* delete right interval *)
         begin
            b := right;
            fb := fright;
            right := left;
            fright := fleft;
            left := b - r*(b-a);
            pars[1] := left;
            fleft := gof(x,pars,which);
         end; (* if *)

      if fright <= fleft then         (* delete left interval *)
         begin
            a := left;
            fa := fleft;
            left := right;
            fleft := fright;
            right := a + r*(b-a);
            pars[1] := right;
            fright := gof(x,pars,which);
         end; (* if *)
      bound := r*bound;
   end; (* of whil. *)

 if fleft < fright then
   begin
      if fa < fleft then
         pars[1] := a
      else
         pars[1] := left
   end
```

```
         else
            begin
               if fb < fright then
                  pars[1] := b
               else
                  pars[1] := right;
            end;
      end; (* of else (from long time ago) *)

      trueloc := true;
      MLEest(x,trueloc,pars[1],pars[2],pars[3]);
      (* re estimate scale using min dist estimate of location *)
   end; (* of procedure goldensearch *)

procedure findcrit(x:teststat; var cvpass:critv);
   var
      mr      : teststat;  (* Median Rank plotting position    *)
                           (* used in bootstrap technique.     *)
      m, b,                (* slope, intercept,                *)
      alpha   : real;      (* 1-alpha for critical value       *)
      i, num  : integer;

   function cv(x, mr:teststat; alpha:real):real;
      var
         m,
         b : real;
         i : integer;

      begin (* function cv *)
         for i := 0 to num do
            begin
               if (mr[i] < alpha) and (mr[i+1] > alpha) then
                  begin
                     m := (mr[i+1] - mr[i])/(x[i+1] - x[i]);
                     b := mr[i] - m*x[i];
                     cv := (alpha - b)/m;
                  end

               else if mr[i] = alpha then
                  cv := x[i];
            end;
      end; (* function cv *)
```

```
   begin  (* procedure findcrit *)
      num := repetitions;
      for i := 1 to num do
         begin
            mr[i] := (i - 0.3)/(num + 0.4);
         end;

      mr[0] := 0;
      mr[num + 1] := 1;
      m := (mr[2] - mr[1])/(x[2] - x[1]);
      b := mr[1] - m*x[1];
      x[0] := -b/m;

      if x[0] < 0 then
         x[0] := 0;

      m := (mr[num]-mr[num-1])/(x[num]-x[num-1]);
      b := mr[num] - m*x[num];
      x[num + 1] := (1.0 - b)/m;

      writeln(out,'  alpha    critical value');

      for i := 1 to 5 do
         begin
            alpha := 0.75 + 0.05*i;
            if alpha > 0.96 then
               alpha := 0.99;
            cvpass[i] := cv(x,mr,alpha);
            writeln(out,alpha:8:4,cvpass[i]:10:6);
         end;
   end; (* procedure findcrit *)

procedure bubble(var critsort:teststat);

   var
      number,
      i,j : integer;
      temp : real;
   begin
      number := repetitions;  (* number of elements to sort *)
      for j := (number-1) downto 1 do  (* bubble sort of data values *)
```

```
      for i.= 1 to j do              (* Wirth (1976) pages 65-66   *)
        begin
          if critsort[i] > critsort[i+1] then
            begin
              temp := critsort[i];
              critsort[i] := critsort[i+1];
              critsort[i+1] := temp;
            end;
        end; (* for i *)
  end; (* bubble *)

procedure datasort(var dataset:data);

  var
    number,
    i,j : integer;
    temp : real;
  begin
    number := trunc(dataset[0]);  (* number of elements to sort *)
    for j := (number-1) downto 1 do  (* bubble sort of data values *)
      for i:= 1 to j do              (* Wirth (1976) pages 65-66   *)
        begin
          if dataset[i] > dataset[i+1] then
            begin
              temp := dataset[i];
              dataset[i] := dataset[i+1];
              dataset[i+1] := temp;
            end;
        end; (* for i *)
  end; (* datasort *)

procedure Weibull(TR:para; var dataset:data);
(* Weibull generates a data set of Weibull random variables *)
(* Uses the inverse transform technique. Banks and Carson    *)
(* (1984) pages 294-300.                                     *)
(* DATA MUST BE ORDERED FOR OTHER PROCEDURES                 *)
  var
    number,
    i : integer;
    temp : real;
  begin
    number := trunc(dataset[0]);  (* number of random variables (<30) *)
```

```
(* writeln(out,'        SEED = ',seed:10); *)
      for i := 1 to number do
         begin
            temp := uniform(seed);
            dataset[i] := TR[2]*exp((1/TR[3])*ln(-ln(temp))) + TR[1];
         end;

      datasort(dataset);

   end; (* Weibull *)

procedure Normal(var dataset:data);
(* Normal generates a data set of Normal random variables   *)
(* with mean = 15 and std dev = 2.  Uses the Box Muller     *)
(* technique.  Banks and Carson (1984) page 316.            *)
(* DATA MUST BE ORDERED FOR OTHER PROCEDURES                *)
   var
      number,
      i,j : integer;
      temp2,
      temp : real;
   begin
      number := trunc(dataset[0]);  (* number of random variables (<30) *)
(* writeln(out,'        SEED = ',seed:10); *)
      j := trunc((number+1)/2);
      for i := 1 to j do
         begin
            temp := uniform(seed);
            temp2 := uniform(seed);
            temp := sqrt(-2*ln(temp));
            temp2 := temp2 * 6.2831853;
            dataset[i] := 15 + 2*temp*cos(temp2);
            dataset[i+j] := 15 + 2*temp*sin(temp2);
         end;

      datasort(dataset);

   end; (* Normal *)

procedure Erlang(m:integer; var dataset:data);
(* Erlang generates a data set of Gamma random variables    *)
(* Uses the inverse transform technique.                    *)
```

```
(* Banks and Carson (1984) page 317 and 295.  m = shape       *)
(* DATA MUST BE ORDERED FOR OTHER PROCEDURES                    *)
   var
      number,
      i,j : integer;
      temp2,
      temp : real;
   begin
      number := trunc(dataset[0]);  (* number of random variables (<30) *)
(* writeln(out,'       SEED = ',seed:10); *)
      for i := 1 to number do
         begin
            temp2 := 1.0;
            for j := 1 to m do
               begin
                  temp := uniform(seed);
                  temp2 := temp2 * temp;
               end;
            (* k = m and theta = 1/5 Banks and Carson pg 146 and 317 *)
            (* location = 10.0 *)
            dataset[i] := (-5/m)*ln(temp2) + 10.0;
         end;

      datasort(dataset);

   end; (* Erlang *)

procedure Beta(p,q:integer; var dataset:data);
   var
      i, number : integer;
      x1,x2     : real;
   begin
      number := trunc(dataset[0]);
      for i := 1 to number do
         begin
            x1 :=      (p);
            x2 := Gamma(q);
            dataset[i] := x1/(x1+x2) + 10.0;
         end;

      datasort(dataset);
```

```
   end; (* Beta *)

procedure Unif(var dataset:data);
(* method to generate uniform numbers between 10 and 15 *)
   var
      number,
      i : integer;
      temp : real;
   begin
      number := trunc(dataset[0]);
      for i := 1 to number do
         begin
            temp := uniform(seed);
            dataset[i] := 5.0 * temp + 10.0;
         end;

      datasort(dataset);

   end; (* Unif *)

procedure Getdata(k:integer; var dataset:data);
   var
      TR  : para;            (* Array of true parameter values *)
   begin
      if k = 1 then
         begin
            TR[1] := 10.0;
            TR[2] :=  1.0;
            TR[3] :=  1.0;
            Weibull(TR,dataset);
         end
      else if k = 2 then
         begin
            TR[1] := 10.0;
            TR[2] :=  1.0;
            TR[3] :=  2.0;
            Weibull(TR,dataset);
         end
      else if k = 3 then
         begin
            TR[1] := 10.0;
            TR[2] :=  1.0;
```

```
          TR[3] :=  3.5;
          Weibull(TR,dataset);
        end
     else if k = 4 then
        begin
          Erlang(1,dataset);
        end
     else if k = 5 then
        begin
          Erlang(2,dataset);
        end
     else if k = 6 then
        begin
          Erlang(3,dataset);
        end
     else if k = 7 then
        begin
          Normal(dataset);
        end
     else if k = 8 then
        begin
          Unif(dataset);
        end
     else if k = 9 then
        begin
          Beta(2,2,dataset);
        end
     else if k = 10 then
        begin
          Beta(2,3,dataset);
        end
  end; (* procedure Getdata *)

begin (* thesis *)
  rewrite(out);
  seed := 7774755;


  TRU[1] := 10.0;    (* location    *)
  TRU[2] :=  1.0;    (* scale       *)
  TRU[3] :=  1.0;    (* shape       *)
  MLE[3] := TRU[3];
```

```
   dataset[0] := 25;  (* sample size *)

writeln(out,'NUMBER OF REPETITIONS IS ',repetitions:6);
writeln(out,'TRUE PARAMETERS ARE ');
writeln(out,'    Location = ',TRU[1]:6:3);
writeln(out,'    Scale    = ',TRU[2]:6:3);
writeln(out,'    Shape    = ',TRU[3]:6:3);
writeln(out,'SAMPLE SIZE = ',dataset[0]);
writeln(out,' ');

   for j := 1 to repetitions do (* Loop for Monte Carlo Simulations  *)
      begin
         Weibull(TRU,dataset); (* generate data set   *)

         if j <= 3 then                   (* writes first three data sets *)
            begin
               writeln(out,' ');
               for i := 1 to trunc(dataset[0]) do
                  writeln(out,'data ',dataset[i]:10:4);
            end;

         trueloc := false;  (* calculates MLE estimates of parameters       *)
         MLEest(dataset,trueloc,MLE[1],MLE[2],MLE[3]);

         (**************************************************************)
         (***   CALCULATE MINIMUM DISTANCE ESTIMATES                 ***)
         (**************************************************************)
         for i := 1 to 3 do
            begin
               MDLAD[i]  := MLE[i]; (* Starting points are MLE values *)
               MDLCVM[i] := MLE[i];
            end;

         which := AD;
         GoldenSearch(dataset,which,MDLAD);

         which := CVM;
         GoldenSearch(dataset,which,MDLCVM);

      (*****************************************************************)
      (***             ANDERSON-DARLING EVALUATION                   ***)
      (*****************************************************************)
```

```
      gofvalue := adgof(dataset,MDLAD);
      ADCRIT[j]  := gofvalue;

      gofvalue := adgof(dataset,MDLCVM);
      CVMCRIT[j] := gofvalue;

   end; (* loop for monte carlo repetitions *)

bubble(ADCRIT);
bubble(CVMCRIT);

writeln(out,'Anderson-Darling:');
findcrit(ADCRIT,cvad);

writeln(out,'Cramer-Von Mises:');
findcrit(CVMCRIT,cvcvm);

(**************************************************************)
(***                     POWER STUDY                        ***)
(**************************************************************)

for k := 1 to 10 do       (* main loop for 10 power studies *)
   begin
      for i := 1 to 5 do
         begin
            NRrejAD[i] := 0;
            NRrejCVM[i] := 0;
         end;
      for j := 1 to repetitions do
         begin
            Getdata(k,dataset);
            trueloc := false;
            MLEest(dataset,trueloc,MLE[1],MLE[2],MLE[3]);

            for i := 1 to 3 do
               begin
                  MDLAD[i]  := MLE[i]; (* Starting points *)
                  MDLCVM[i] := MLE[i];
               end;

            which := AD;
```

```
            GoldenSearch(dataset,which,MDLAD);
            which := CVM;
            GoldenSearch(dataset,which,MDLCVM);

            gofvalue := adgof(dataset,MDLAD);
            for i := 1 to 5 do
               begin
                  if gofvalue > cvad[i] then
                     NRrejAD[i] := NRrejAD[i] + 1;
               end;

            gofvalue := adgof(dataset,MDLCVM);
            for i := 1 to 5 do
               begin
                  if gofvalue > cvcvm[i] then
                     NRrejCVM[i] := NRrejCVM[i] + 1;
               end;

         end; (* for j loop *)
      writeln(out,'Anderson-Darling Distance');
      for i := 1 to 5 do
         begin
            power := NRrejAD[i]/repetitions;
            writeln(out,'i = ',i,' power =  ',power:8:6);
         end;
      writeln(out,'Cramer-Von Mises Distance');
      for i := 1 to 5 do
         begin
            power := NRrejCVM[i]/repetitions;
            writeln(out,'i = ',i,' power =  ',power:8:6);
         end;

   end; (* for k loop *)

 close(out);

end. (* thesis *)
```

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13. ABSTRACT (*Maximum 200 words*)
This research is to produce a modified Anderson-Darling goodness-of-fit test for the Weibull distribution when the location parameter is found by minimum distance estimation, the shape parameter is assumed known, and the MLE is used for the scale parameter. The critical values for the Anderson-Darling test are generated via Monte Carlo simulation when both the Anderson-Darling and Cramer-Von Mises distance statistics are minimized. These critical values are then used for a power study. The Monte Carlo simulation used 5000 repetitions for sample sizes of 5, 8, 12, 15, 16, 20 and 25 with the Weibull shape parameter of .5(.5)4.0. The power study is made for the same sample sizes as above with the hypothesized Weibull shape parameter of 1.0 and 3.5 against ten alternate hypothesized distributions. For small sample sizes, improvement can be seen over tests which use MLEs for the location and scale parameters. However, for larger sample sizes, more than 20, the power is similar to other goodness-of-fit tests for the Weibull. In most cases, minimizing the Anderson-Darling distance statistic to estimate the Weibull location parameter had more power than minimizing the Cramer-Von Mises distance statistic.

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