ARO 24456.5-EG

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Response and Stability Analysis of Helicopter Rotor Blades Using Computerized Symbolic Manipulation

Final Report

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May 1991

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U.S. Army Research Office

Grant DAAL03-87-K-0116

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
May 91

3. REPORT TYPE AND DATES COVERED
Final 20 Sep 87 - 19 Mar 91

4. TITLE AND SUBTITLE
Response and Stability Analysis of Helicopter Rotor Blades Using Computerized Symbolic Manipulation

5. FUNDING NUMBERS
ARO Grant Number
DAAL03-87-K-0116

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
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Aeronautical Engineering & Mechanics
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Troy, NY 12180-3590

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 24476.5-EG

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The importance of nonlinearities, and of certain approximations, in the analysis of beams and rotor blades has been addressed in detail. For this, special care was taken in formulating the differential equations of motion of rotor blades in order to be able to identify sources of problematic approximations. In addition, the response of a rotor blade was analyzed by a methodology using an essentially exact solution for the equilibrium state of the blade and by subsequently determining the "essentially exact" eigenfunctions and eigenvalues associated with the perturbed motion of the blade. The results obtained could serve as a gauge for validating approximate results such as those obtained by a Galerkin procedure. A perturbation methodology was also used to analyze the response of a blade that is either in hover or in forward flight.

14. SUBJECT TERMS
rotor blades, stability

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

STATEMENT OF THE PROBLEM

Although most of the work on helicopter rotor blades presented in the literature to date is based on a set of differential equations of motion that include only quadratic nonlinearities (obtained by expanding the full nonlinear equations about the undeformed state of the blade), it has been recently determined that cubic nonlinearities can play a significant role in the equilibrium and response of a rotor blade in hover. This is to be expected since equations where only quadratic nonlinearities are retained do not reduce to the proper nonlinear equations of motion when the blade is not rotating.

One of the objectives of this work was to provide a thorough understanding of the importance and treatment of nonlinearities in the response and stability of helicopter rotor blades, and to pinpoint which approximations caused by "ordering schemes" lead to troblesome results. Another objective was to refine a perturbation methodology, valid for the cases of hover and forward flight, that was initiated by the principal investigator. The analysis of the response of rotor blades by such a perturbation methodology provides an alternative to the numerically-intensive Floquet theory. It also provides, with the aid of symbolic computation, closed form approximations for the various levels of approximation of the response, and discloses a number of important features of the response for different flight conditions. In addition, the effect of truncating higher order nonlinearities on the equilibrium state and eigenvalues associated with perturbed motions of beams and rotor blades was assessed in detail.

SUMMARY OF MOST IMPORTANT RESULTS

It is now well known that the nonlinear differential equations of an initially straight non-rotating beam have cubic nonlinearities that are due to both curvature and inertia effects. These nonlinearities can play a significant role in the response of the beam to external loads. Although most of the work on helicopter rotor blades presented in the literature to date is based on a set of differential equations of motion that include only quadratic nonlinearities, it has been recently determined that cubic nonlinearities can play a significant role in the equilibrium and response of a rotor blade in hover. This is to be expected since equations where only quadratic nonlinearities are retained do not reduce to the proper nonlinear equations of motion when the blade is not rotating.

There are a number of analyses of rotor blade dynamics in the helicopter industry. The computer programs for those analyses predict different eigenvalues associated with the perturbed motion of the blade. Also, in some of the published work available in the open literature, there have been a number of questions regarding the proper treatment and influence of nonlinear terms, especially quadratic and cubic order terms, in the dynamics of a rotor blade. These problems were addressed in this research work and the effect of truncating higher order nonlinearities on the equilibrium state and eigenvalues associated with perturbed motions of a non-rotating beam and of a rotor blade were assessed in detail.

In our work, special care was taken in formulating the differential equations of motion for a blade in order to be able to identify which commonly used approximations lead to invalid results. We have been able to show [1] that approximating all the diagonal elements of transformation matrices by unity while retaining nonlinear terms off the diagonal is one of such erroneous approximations. This approximation alone renders the subsequent nonlinear equations invalid for analyzing the dynamics of beams or rotor blades. Equations obtained when such approximation is used lead to results that depend on Euler angle sequence, which is an incorrect result even on physical grounds alone. In developing this work the analysis of a cantilever beam with a tip mass proved to be invaluable. Complete details of this work were reported in [1]. The work was done using the facilities of our Symbolic Computation Laboratory. We made use of MACSYMA to develop and to analyze the equations of motion and to generate error free extensive Fortran code which was needed to generate all the graphical results presented in [1]. The effect of analyzing the dynamics of a beam by replacing the full nonlinear differential equations of motion by a set of nonlinear equations expanded about the undeformed state of the beam, when such state is not an equilibrium, was studied in detail [2]. In addition, the use of a transfer matrix methodology, instead of Galerkin's method, was also explored in [2]. Based on the results and conclusions presented in [2], the differential equations of motion for a rotor blade were reformulated and the blade's response in hover was analyzed with such methodology. For this, an essentially exact equilibrium solution for the blade was obtained by numerically integrating its full nonlinear differential equations of motion. For the case of hover, considered in [3], the differential equations governing the equilibrium state of the blade are a set of nonlinear ordinary differential equations. Standard IMSL routines were used to solve the two-point boundary value problem represented by such equations. When the equilibrium state is perturbed, and the resulting equations are linearized for infinitesimally small perturbations, the solution to the linearized equations can be written as $\underline{E}(s)\, e^{rt}$ where s denotes distance measured along the blade and t denotes time; $\underline{E}(s)$ is a column of state variables that were properly chosen so that the perturbed differential equations are written as a set of first order equations that can be integrated numerically [3]. The resulting differential equations for $\underline{E}(s)$ contain the complex quantity r as a parameter. Those equations were integrated in the same manner described in [2] to determine the eigenvalues r. Plots showing how the equilibrium solution and the real and imaginary parts of r depend on a number of important parameters for the system were presented in [3]. The results obtained were compared with those generated by using a Galerkin procedure with non-rotating beam modes and significant differences were noted for several cases. Such results could serve as a gauge for validating the approximate results obtained with Galerkin's procedure.

It should be noted that by avoiding Galerkin's procedure, such as done in the work presented in [2,3], the calculation of a large number of Galerkin coefficients was avoided and the use of ordering schemes was not necessary. Also, the eigenvalues associated with the perturbed motion were calculated without the need for iterating on the number of trial modes, such as non-rotating beam modes, required by the Galerkin procedure. Another advantage of the the work presented in [2,3] is that the solution obtained for the equilibrium state of the rotor blade was, except for small round-off errors inherent in any numerical scheme, essentially exact since the full nonlinear differential equations were used in the numerical integration. Such solution could also have been obtained by other numerical methods, such as finite elements, without major difficulties. Additional details of part of the work reported in [1] are included in [4,5].

Another objective of this work was to refine the development of a perturbation methodology valid for the cases of hover and forward flight. In contrast with Floquet theory, a perturbation analysis provides significant insight into the physics of the problem. In a perturbation analysis one wants to render the first order differential equations linear and to make the analysis valid for any flight condition.

The perturbation methodology is based on the method of multiple time scales, and is valid both for the cases of hover and of forward flight. In order to make the perturbation analysis valid for both of these cases, the advance ratio cannot be used as the perturbation parameter. Instead, a small arbitrary "bookkeeping parameter" that is introduced artificially, and that linearizes the differential equations of motion, was used. The significant amount of "algebra" associated with the perturbation analysis was done with computerized symbolic manipulation using MACSYMA.

It should be noted that perturbation analyses have been used by a few other researchers in helicopter dynamics. However, the analyses that have been presented in the literature to date deal separately with the cases of hover and of forward flight. For each case a different small perturbation parameter was used, such as the collective pitch angle for the case of hover and the advance ratio for the case of forward flight. However, none of these parameters render the equations of motion linear in the limit as they approach zero. Thus such parameters are not ideally suited for a perturbation analysis in the presence of nonlinearities.

In order to make our perturbation analysis valid for both cases of hover and of forward flight, and also to make it handle nonlinearities, a small arbitrary "bookkeeping parameter" is introduced in the analysis. As in other research work published by the principal investigator, such parameter is used to keep track of orders of smallness from a perturbation point of view. It should be noted that ordering here does not have the same meaning as that of ordering schemes that have been used elsewhere to eliminate complicated terms in the analysis.

The details of the perturbation analysis, including the effects of nonlinear terms up to $O(\varepsilon^3)$, are described in [6]. An alternative perturbation analysis based on a set of differential equations used for a Floquet stability analysis was also performed. The latter analysis is simpler than the full nonlinear perturbation analysis. It was found that the two analyses produce essentially the same results for advance ratios up to about 0.3, which is in the range of practical interest for most applications. The perturbation analysis also discloses the resonance conditions in the system, clearly points to them as the cause for instabilities, and allows one to concentrate the search for unstable regions in the system's parameter space near those lines defined by the resonance conditions.

The refined perturbation methodology developed in this work consists in *first determining* a particular solution to the full nonlinear differential equations of motion. This was done numerically. The particular solution is either an equilibrium solution, for the case of hover, or a periodic solution that satisfies some imposed conditions. The latter is the "trim solution" for the case of forward flight. By transferring the periodic coefficient and the damping terms from the $O(\varepsilon)$ to the next order approximation, solvability conditions (i.e., conditions for elimination of secular terms) were obtained at both the $O(\varepsilon^2)$ and at the $O(\varepsilon^3)$ approximations. To obtain the solvability conditions one has to distinguish between several resonant conditions disclosed by the perturbation analysis for the $O(\varepsilon^2)$ and $O(\varepsilon^3)$ differential equations. For the case of a hinged spring-restrained rigid blade, it was found that the perturbation analysis discloses the existence of two important resonance conditions, namely $\omega_\zeta = \omega_\eta$ and $\omega_\zeta = 1 + \omega_\eta$. Here, ω_ζ and ω_η denote, respectively, the lead-lag and the flap natural frequencies. The solvability conditions for the motion "near" these resonance conditions were obtained and, from these, the stability of the motion was then determined. Since the $O(\varepsilon^n)$, $n>1$, differential equations obtained from the perturbation analysis are equations with constant coefficients, stability determination using such equations is readily done using the Routh-Hurwtiz criterion.

LIST OF PUBLICATIONS AND REPORTS

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6. Crespo da Silva, M.R.M. and Cho, Y., Stability Analysis of Helicopter Rotor Blades by a Perturbation Technique". In Appendix III of RPI Proposal Number 413(02R)056(20F) submitted to ARO on March 27, 1990, pp. 53-63.

Other Related Publication:

Hassenpflug, W.C. and Crespo da Silva, M.R.M., "Dynamics of Rotor Blades in Curved Steady Maneuver". Proc. Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Duke University, (1990).

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Mr. Cho is still working on his Ph.D. thesis.

The following graduate students also worked on this grant on a temporary basis:

Mr. Clifford Zaretzky
Mr. H.D. Kim
Mr. Walter Hassenpflug

Mr. Hassenpflug is expected to receive his M.S. degree in August 1991. He is continuing for the Ph.D degree.