AD-A238 757

...ATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	06|06|91	Final Report 1 Jun 88 - 31 May 91

4. TITLE AND SUBTITLE

Systems of Nonlinear Conservation Laws

5. FUNDING NUMBERS

DAAL03-88-K-0080

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

ARO 24904.5-MA

11. SUPPLEMENTARY NOTES

The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This description of results from the project covers research in two areas:

1. Plastic flow in two and three dimensions.

2. Hyperbolic conservation laws.

14. SUBJECT TERMS

Shock waves, plasticity, conservation laws

15. NUMBER OF PAGES

5

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500
Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std Z39-18

SYSTEMS OF NONLINEAR CONSERVATION LAWS

FINAL REPORT

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U.S. ARMY RESEARCH OFFICE

GRANT NUMBER DAAL03-88-K-0080

NORTH CAROLINA STATE UNIVERSITY

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Research Results

This description of results from the project covers research in two areas:

1. Plastic flow in two and three dimensions.

2. Hyperbolic conservation laws.

1. Plastic flow. This part of the project was carried out in collaboration with David Schaeffer of Duke University. Our research focused on the issue of loss of stability and well posedness in the equations of motion of granular materials. The partial differential equations are derived from conservation of mass and momentum, augmented by constitutive laws that relate the dependent variables algebraically. The starting point was the motion in two dimensions of a rigid-plastic material, with the constitutive laws coming from critical state soil mechanics. We established a criterion for stability in [8], and related this to the quasidynamic equations in [9], showing broadly speaking that the full equations are linearly stable if and only if the quasidynamic equations are. This is important because the quasidynamic equations are much more convenient for computation, and our results show that, with precisely formulated conditions, the simpler equations capture the stability properties of the original system. A third paper in preparation analyzes corresponding issues for flow in three dimensions.

We recently looked at yield vertex constitutive equations. These differ markedly from those of critical state soil mechanics, in that the equations become fully nonlinear rather than merely quasilinear. We found that the equations lose hyperbolicity, a phenomenon mathematically associated with a loss of well-posedness, and physically associated with localization, leading to the failure of the material through the formation of shear bands. A notable success of the analysis in [10] is the correct prediction of the preferred orientation of the shear band in an experiment in which a granular material is sheared between parallel plates.

2. Hyperbolic conservation laws. The classification [A] of 2x2 systems of hyperbolic conservation laws with quadratic nonlinearities identifies four different types of equations. The Riemann problem was solved in detail in [B] for three of the four types. The fourth type of equation, Case I, is the most significant for applications to models of multiphase flow in oil reservoirs, as discussed in [A]. This case involves undercompressive shocks, which are physical shock waves closely associated with systems that change type.

Using ideas from dynamical systems to understand the role of undercompressive shocks, the Riemann problem was solved for Case I equations in [1]. This paper exploits the special properties of quadratic nonlinearities. In particular the solution of the Riemann problem is not stable to perturbation of the nonlinearity by higher order terms. This problem was solved using equilibrium bifurcation theory augmented by a characterization of heteroclinic orbits using Melnikov's integral. Preliminary results were announced in conference proceedings [2,3], and written up in detail in [4].

A detailed study of change of type for model equations of three phase flow in porous media is given in [6]. These equations typically have small elliptic regions that may be somewhat accentuated by the inclusion of gravity effects, as noted in numerical results with specific models. Special classes of equation are now known to lose strict hyperbolicity while remaining hyperbolic. Corners of the physical domain correspond to umbilic points, at which the characterisitic speeds coincide, but they are degenerate, leading us to an analysis of the higher order terms.

There are classes of nonstrictly hyperbolic systems for which the characteristic speeds are real, but coincide along a curve. In [7], we give a local analysis and classification of such equations, and show how new types of shocks, known as *singular shocks*, are a crucial part of solving Riemann problems. We give a detailed asymptotic analysis of corresponding solutions of the regularized, parabolic, equations. These solutions blow up at a single point as the dissipation approaches zero.

Glimm's method was implemented for the full initial boundary value problem describing the motion of an elastic string stretched between two fixed points [11]. The numerical results are strongly indicative of the presence of a periodic solution. This is reinforced by the discovery of two exact solutions that describe the motion of sections of the string. The analytic solutions can be combined to form a periodic function that resembles the numerical solution.

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PERSONNEL SUPPORTED

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