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## REPORT DOCUMENTA



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| 1a. REPORT SECURITY CLASSIFICATION<br><b>UNCLASSIFIED</b>                                                                                                                           |       |                                                                                                         | 1b. RESTRICTIVE MARKINGS                                                                           |                                                                                            |                                       |
| 2a. SECURITY CLASSIFICATION AUTHORITY<br><b>NA</b>                                                                                                                                  |       |                                                                                                         | 3. DISTRIBUTION / AVAILABILITY OF REPORT<br>Approved for Public Release;<br>Distribution Unlimited |                                                                                            |                                       |
| 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE<br><b>NA</b>                                                                                                                            |       |                                                                                                         | 5. MONITORING ORGANIZATION REPORT NUMBER(S)<br><b>AFOSR-TR-90 0985</b>                             |                                                                                            |                                       |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S)<br><b>STANFORD UNIVERSITY</b>                                                                                                           |       |                                                                                                         | 7a. NAME OF MONITORING ORGANIZATION<br><b>AFOSR/NM</b>                                             |                                                                                            |                                       |
| 6a. NAME OF PERFORMING ORGANIZATION<br><b>STANFORD UNIVERSITY</b>                                                                                                                   |       | 6b. OFFICE SYMBOL<br>(if applicable)                                                                    |                                                                                                    | 7b. ADDRESS (City, State, and ZIP Code)<br><b>Bldg. 410<br/>Bolling AFB, DC 20332-6448</b> |                                       |
| 6c. ADDRESS (City, State, and ZIP Code)<br><b>Department of Electrical Engr.<br/>Durand 117<br/>Stanford, CA 94305</b>                                                              |       | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br><b>AFOSR 88-0327A<br/>(Contract or Grant Number)</b> |                                                                                                    | 10. SOURCE OF FUNDING NUMBERS                                                              |                                       |
| 8a. NAME OF FUNDING / SPONSORING ORGANIZATION<br><b>AFOSR</b>                                                                                                                       |       | 8b. OFFICE SYMBOL<br>(if applicable)<br><b>NM</b>                                                       |                                                                                                    | PROGRAM<br>ELEMENT NO.<br><b>61102F</b>                                                    | TASK<br>NO.<br><b>2304</b>            |
| 8c. ADDRESS (City, State, and ZIP Code)<br><b>Bldg. 410<br/>Bolling AFB, DC 20332-6448</b>                                                                                          |       | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br><b>AFOSR 88-0327A<br/>(Contract or Grant Number)</b> |                                                                                                    | TASK<br>NO.<br><b>A6</b>                                                                   | WORK UNIT<br>ACCESSION NO.            |
| 11. TITLE (Include Security Classification)<br><b>Fast Algorithms for Structured Matrices with Arbitrary Rank Profile</b>                                                           |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 12. PERSONAL AUTHOR(S)<br><b>Debajyoti Pal</b>                                                                                                                                      |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 13a. TYPE OF REPORT<br><b>Thesis</b>                                                                                                                                                |       | 13b. TIME COVERED<br>FROM <b>1989</b> TO <b>1990</b>                                                    |                                                                                                    | 14. DATE OF REPORT (Year, Month, Day)<br><b>1990, May</b>                                  |                                       |
| 15. PAGE COUNT<br><b>211</b>                                                                                                                                                        |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 16. SUPPLEMENTARY NOTATION                                                                                                                                                          |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 17. COSATI CODES                                                                                                                                                                    |       |                                                                                                         | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)                  |                                                                                            |                                       |
| FIELD                                                                                                                                                                               | GROUP | SUB-GROUP                                                                                               | triangular factorization, computation of rank profile<br>solution to linear equations, inversion   |                                                                                            |                                       |
|                                                                                                                                                                                     |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 19. ABSTRACT (Continue on reverse if necessary and identify by block number)<br><br><b>See Back</b>                                                                                 |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 20. DISTRIBUTION / AVAILABILITY OF ABSTRACT<br><input type="checkbox"/> UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 21. ABSTRACT SECURITY CLASSIFICATION<br><b>UNCLASSIFIED</b>                                                                                                                         |       |                                                                                                         |                                                                                                    |                                                                                            |                                       |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL<br><b>Jon A. Sjogren</b>                                                                                                                        |       |                                                                                                         | 22b. TELEPHONE (Include Area Code)<br><b>202/ 767-4939</b>                                         |                                                                                            | 22c. OFFICE SYMBOL<br><b>AFOSR/NM</b> |

91-05958



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Triangular factorization, solution to linear equations, inversion, computation of rank profile and inertia (in the Hermitian case) etc. of general  $n \times n$  matrices require  $O(n^3)$  operations. For certain structured matrices including Toeplitz and Hankel matrices the computational complexity is known to be  $O(n^2)$  or better. These structured matrices often arise in a wide variety of areas including Signal processing, Systems theory and Communications. Fast (i.e.  $O(n^2)$ ) algorithms for these structured matrices have been actively studied for over twenty five years. However almost all the authors have assumed that the underlying matrices are strongly regular i.e. every principal submatrix is nonsingular. Although some fast algorithms have recently been developed for certain problems involving some of these structured matrices which may have one or more zero minors, several other problems remain unresolved; also a unified approach towards these problems is lacking. In this dissertation, we obtain several new results through a unified approach to the problems mentioned earlier.

First we derive fast (i.e.  $O(n^2)$ ) procedures for computing a "modified" triangular factorization which is a *LDU* factorization where *L* is lower triangular (resp. *U* is upper triangular) with unit diagonal entries and *D* is a block diagonal matrix with possibly varying block sizes. Only strongly nonsingular matrices will always have a purely diagonal nonsingular *D* matrix. For the matrices we study the diagonal blocks are also structured: in particular they are Hankel (resp. Quasi-Toeplitz) for Hankel (resp. Toeplitz) and Quasi-Hankel (resp. Quasi-Toeplitz) matrices.

A particular application of our result is a fast method of computing rank profile and inertia, leading to alternative proofs of certain results due to Iohvidov (1974) on rank profile and inertia of Hankel and Toeplitz matrices.

Next using the results on modified triangular factorization we extended the Schur complement based approach of Chun [Chu89] for inversion of strongly regular Toeplitz and Hankel matrices to Hermitian Toeplitz, Quasi-Toeplitz, Hankel, and Quasi-Hankel matrices with arbitrary rank profile.

We apply our procedures for computing inertia to derive  $O(n^2)$  procedures for computing root distribution of an  $n$ -th order polynomial with respect to the unit circle and the imaginary axis. In particular it has been possible to derive the first general recursive procedure for determining the root-distribution of a polynomial with respect to the unit circle: the classical Schur-Cohn test for this problem fails in the presence of certain "singularities," which correspond to a Quasi-Toeplitz Bezoutian matrix being non-strongly-regular.

