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M. M. Weiner

Influence of Non-Homogeneous Earth on the Performance of High-Frequency Receiving Arrays with Electrically-Small Ground Planes

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M. M. Weiner

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Department Approval: R. G. M. Bush

MITRE Project Approval: Edward A. Palo



ABSTRACT

The performance of ground-based high-frequency (HF) antenna arrays is reduced when the array elements have electrically-small ground planes. Performance degradations include: (1) a decrease in directive gain near the horizon (caused by earth multipath), (2) a decrease in radiation efficiency and an increase in internal noise (caused by ground losses), (3) an array RMS phase error (caused by exterior currents on element feed cables), and (4) an array rms phase error and beam pointing errors (caused by non-uniform Fresnel reflection RMS from a non-homogeneous earth).

This paper models the degradation described in (4) above. Numerical results are presented for cases of randomly-distributed and systematically-distributed earth non-homogeneities where one-half of vertically-polarized array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. The maximum rms phase errors, for the cases examined, are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum beam pointing errors are 0 and 0.3 beamwidths for randomly-distributed and systematically-distributed non-homogeneities, respectively.



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The least square's algorithm given by equation (A-3) in the appendix was brought to the author's attention by J. D. R. Kramer.

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SECTION 1

INTRODUCTION

High-frequency (HF) over-the-horizon (OTH) radars generally employ transmitting and receiving antenna arrays whose elements are in close proximity to earth, but with electrically-large ground planes to mitigate the influence of earth on antenna performance. However, in the design of advanced OTH radar systems with sparse receiving arrays comprising hundreds or thousands of elements pseudo-randomly distributed over a large area, electrically-large metallic ground planes are prohibitively expensive to construct or to maintain for so many elements. One alternative approach is to employ salt-water ground planes to achieve an electrically-large ground plane. However, suitable salt-water sites are not readily available and have their own unique problems. A second alternative approach is to use elements with electrically-small ground planes and to accept the reduction in antenna performance caused by a greater susceptibility to the influence of the earth. This second alternative approach is considered in this paper.

The performance of ground-based HF antenna arrays is reduced when the array elements have electrically-small ground planes. Performance degradations include (1) a decrease in directive gain near the horizon (caused by earth multipath), (2) a decrease in radiation efficiency and an increase in internal noise (caused by ground losses), (3) an array RMS phase error (caused by field-induced exterior currents on element feed cables), and (4) an array RMS phase error and beam pointing errors (caused by non-uniform Fresnel reflection from a non-homogeneous earth).

This paper models the array RMS phase error and beam pointing errors caused by non-uniform Fresnel reflection from a non-homogeneous earth. Numerical results are presented for cases of randomly-distributed and systematically-distributed earth non-homogeneities, where one-half of vertically-polarized array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. It is found, for the cases examined, that the maximum expected values of

RMS phase error are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum expected values of the beam-pointing error are 0 and 0.3 beamwidths from randomly-distributed and systematically-distributed non-homogeneities, respectively.

The performance degradation, caused by non-uniform Fresnel reflection by a non-homogeneous earth is modeled in section 2. Numerical results are presented in section 3. The summary and conclusions are given in section 4.

SECTION 2

MODEL

Consider a sparse HF receiving array of m vertically-polarized elements with ground planes, of radius a , pseudo-randomly distributed on flat earth over a circular area of radius $r_A \gg a$. The elements, in close proximity to earth, are identical except that the earth below each element may vary from element to element. The element length ℓ and ground plane radius a are assumed to be electrically-small ($\ell \ll \lambda_o$, $a \ll \lambda_o$ where λ_o is the RF wavelength in free space). The midpoint of each element is at a height h above the earth.

Consider now a plane wave incident from the true direction (θ, Φ) where θ is the elevation angle of incident with respect to zenith (the z axis) and Φ is the azimuthal angle with respect to the x axis. The electric field at the midpoint of the k -th element, in the absence of mutual coupling among elements, is the sum of the fields from a direct ray incident from the direction (θ, Φ) and an indirect ("multipath") ray reflected at a point P_k at a horizontal distance $h \tan \theta$ from the element local origin O_k (see figure 1). We assume that the ground plane is sufficiently small so that the indirect ray is reflected from the earth rather than from the ground plane. Accordingly, the radius of the ground plane satisfies the condition $a < h \tan \theta$. (At angles of incidence $\theta < 60^\circ$ this condition is more stringent than the condition $a \ll \lambda_o$.)

The radiation pattern of the k th element is then approximately identical to that of a vertically-polarized Hertzian dipole at height h above flat earth (see figure 1).

The relative permittivity $\epsilon_k^* / \epsilon_o$ of the earth at the k th element is given by

$$\epsilon_k^* / \epsilon_o = \epsilon_{rk} - j 60 \lambda_o \sigma_k \quad (2-1)$$

where

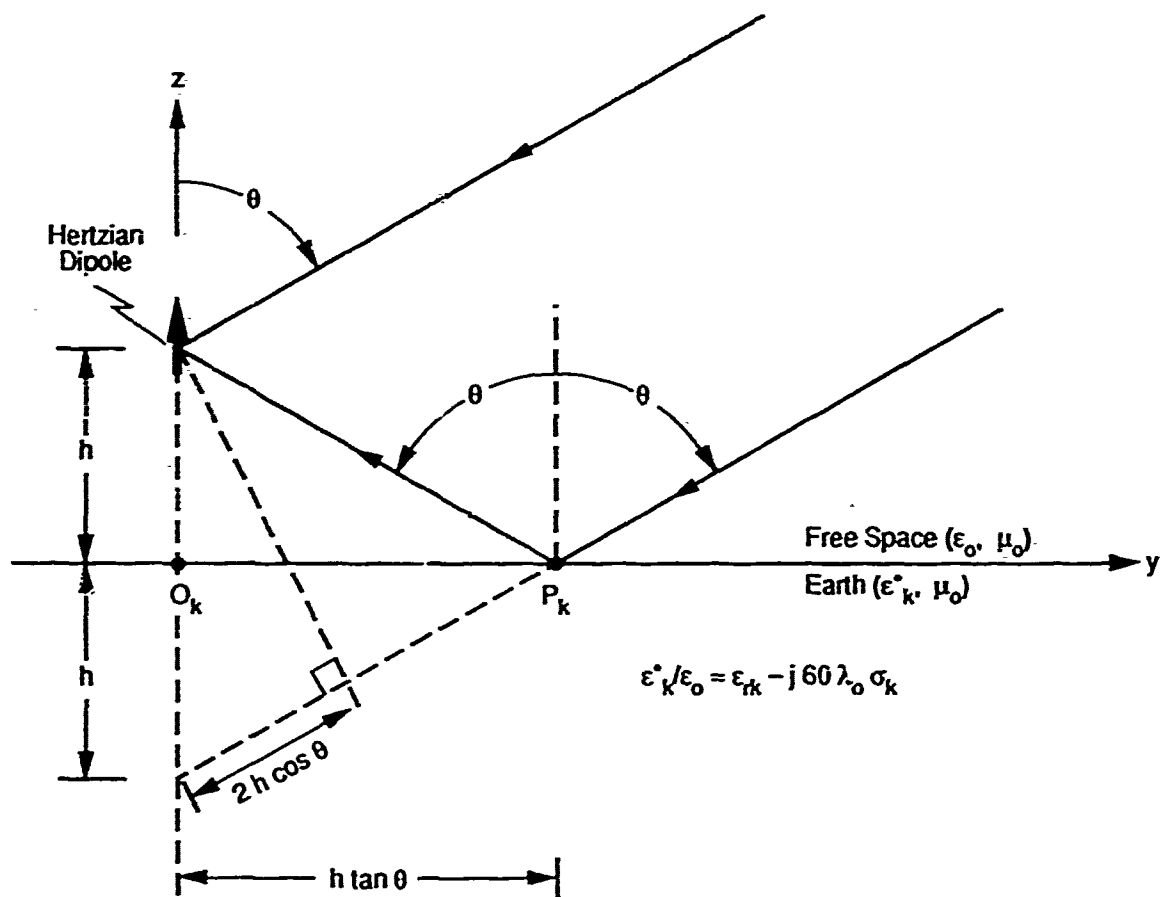


Figure 1. The k th Element Approximated by a Vertically-Polarized Hertzian Dipole Above Flat Earth

ϵ_{rk} = dielectric constant of the earth at the kth element (numeric)

σ_k = conductivity of the earth at the kth element (S/m)

λ_o = RF wavelength in free space (m)

For homogeneous earth,

$$\epsilon^* / \epsilon_o = \epsilon_r - j 60 \lambda_o \sigma \quad (2-2)$$

The ground constants ϵ_r and σ , loss tangent, and penetration depth δ , for the International Radio Consultative Committee (CCIR) 527-1 classifications of homogeneous earth, are summarized in table 1.

The field $E_k(\theta, \phi)$ at the kth element, after elimination of the true phase advance of the direct field at each element relative to that at the origin, is given by

$$E_k(\theta) = A \sin \theta \left\{ 1 + R_{k,v} \exp[-j(2\pi / \lambda_o) 2h \cos \theta] \right\} \quad 0 \leq \theta \leq \pi / 2 \text{ rad} \quad (2-3)$$

where

A = constant (V/m)

θ = angle of incidence (measured from zenith) of the direct ray (degrees)

$R_{k,v} = |R_{k,v}| \exp[j(\text{angle } R_{k,v})]$ = Fresnel reflection coefficient of the earth at the kth element for a vertically-polarized direct wave incident from the direction (θ, ϕ) .

Equation (2-3) is valid for computing the indirect field in air relative to the direct field in air. The single ray (plane wave) Fresnel reflection coefficient model of equation (2-3) is not

Table 1. Permittivity, Loss Tangent, and Penetration Depth of CCIR-527-1
Classifications of Earth

Cases	CONSTANTS		LOSS TANGENT			PENETRATION DEPTH		
	ϵ_r	σ (S/m)	$\alpha(\omega\epsilon_r\epsilon_0) = (60\lambda)(\sigma/\epsilon_r)$			δ (m)		
			Frequency (MHz) [Wavelength (m)]			Frequency (MHz) [Wavelength (m)]		
			3	15	30	3	15	30
(1) Perfect Ground	1.0	∞	[99.93]	[19.986]	[9.993]	[99.93]	[19.986]	[9.993]
(2) Sea Water (average Salinity 20°C)	70.0	5.0	4.282×10^2	8.425×10^1	4.283×10^1	0	0	0
(3) Fresh Water	80.0	3.0×10^{-2}	2.251×10^0	4.497×10^{-1}	2.248×10^{-1}	1.3×10^{-1}	5.8×10^{-2}	4.1×10^{-2}
(4) Wet Ground	30.0	1.0×10^{-2}	1.999×10^0	3.997×10^{-1}	1.999×10^{-1}	2.1×10^0	1.6×10^0	1.6×10^0
(5) Medium Dry Ground	15.0	1.0×10^{-3}	3.997×10^{-1}	7.995×10^{-2}	3.997×10^{-2}	3.7×10^0	3.0×10^0	2.9×10^0
(6) Very Dry Ground	3.0	1.0×10^{-4}	1.999×10^{-1}	3.997×10^{-2}	1.999×10^{-2}	2.1×10^1	2.1×10^1	2.1×10^1
(7) Pure Water, 20°C	80.0	1.8×10^{-6} 5.0×10^{-4} 1.7×10^{-3}	1.350×10^{-4}	7.495×10^{-3}	-	9.2×10^1	9.4×10^2	2.7×10^1
(8) Ice (fresh water, -1°C)	3.0	6.0×10^{-5} 9.0×10^{-5} 1.0×10^{-4}	1.199×10^{-1}	3.597×10^{-2}	1.999×10^{-2}	1.5×10^2	1.0×10^2	9.2×10^1
(9) Ice (fresh water, -10°C)	3.0	1.8×10^{-5} 2.7×10^{-5} 3.5×10^{-5}	3.597×10^{-2}	1.079×10^{-2}	-	5.1×10^2	3.4×10^2	2.6×10^2
(10) Average Land (TCI)	10.0	5.0×10^{-3}	2.998×10^0	5.996×10^{-1}	2.998×10^{-1}	4.8×10^0	1.6×10^0	3.4×10^0
(11) Free Space	1.0	0	0	0	0	∞	∞	∞

valid for calculating the amplitude A of the direct wave because the model does not include the substantial surface wave near-field losses in the earth.

The Fresnel reflection $R_{k,v}$ for parallel (vertical) polarization (the E-field is parallel to the plane of incidence) is given by [1]

$$R_{k,v} = \frac{(\epsilon_k^* / \epsilon_o) \cos \theta - [(\epsilon_k^* / \epsilon_o) - \sin^2 \theta]^{1/2}}{(\epsilon_k^* / \epsilon_o) \cos \theta + [(\epsilon_k^* / \epsilon_o) - \sin^2 \theta]^{1/2}} \quad (2-4)$$

where ϵ_k^* is given by equation (2-1). The Fresnel reflection coefficient $R_{k,v}$ is a function of k and θ . For homogeneous earth, equation (2-4) reduces to

$$R_{k,v} = R_v = \frac{(\epsilon^* / \epsilon_o) \cos \theta - [(\epsilon^* / \epsilon_o) - \sin^2 \theta]^{1/2}}{(\epsilon^* / \epsilon_o) \cos \theta + [(\epsilon^* / \epsilon_o) - \sin^2 \theta]^{1/2}}, \text{ homogeneous earth} \quad (2-5)$$

where $\epsilon_k^* / \epsilon_o$ is given by equation (2-2). For homogeneous earth, R_v is a function only of the angle of incidence θ and the relative permittivity ϵ^* / ϵ_o . The Fresnel reflection coefficient R_v , computed by MITRE program MODIFIED IMAGES, is given in tables 2 through 11 at 6, 15, and 30 MHz for the ground constants specified in table 1. The coefficient R_v is tabulated as a function of the grazing angle ψ (with respect to zenith). Please note that $\psi = (\pi/2) - \theta$ (rad) or $\psi = 90 - \theta$ (degree).

The numeric directive gain $d(\theta)$ of a vertically-polarized Hertzian dipole at a height h above flat homogeneous lossy earth is given by reference 2 as

$$d(\theta) = \begin{cases} \frac{2|E(\theta)|^2}{\int_0^{\pi/2} |E(\theta)|^2 \sin \theta d\theta}, & 0 \leq \theta \leq \pi/2 \\ \text{lossy earth} & \\ 0 & , -\pi/2 \leq \theta \leq 0 \end{cases} \quad (2-6)$$

where $E(\theta)$ is given by equation (2-3) (with the subscript k suppressed). The integration in the elevation plane is restricted to the upper hemisphere because, for a lossy earth, there is no far-field radiation in the lower hemisphere. The directive gain $D(\theta) = 10 \log_{10} d(\theta)$ (dBi), computed by MITRE program MODIFIED IMAGES, is tabulated in tables 2 through 11 for the case $h = 0$.

The field $E_k(\theta)$, given by equation (2-3), may be rewritten as

$$E_k/A \sin \theta = (1 + |R_{k,v}| \cos \beta_k) + j |R_{k,v}| \sin \beta_k \quad (2-7)$$

where

β_k = phase delay of the indirect array with respect to the direct ray

$$= \begin{cases} \beta_k \text{ (rad)} = \text{angle } R_{k,v} \text{ (rad)} - (2\pi/\lambda_o) 2h \cos \theta \\ \beta_k \text{ (deg)} = \text{angle } R_{k,v} \text{ (rad)} - (180/\pi) (2\pi/\lambda_o) 2h \cos \theta \end{cases} \quad (2-8)$$

The argument α_k , of the complex quantity $E_k/A \sin \theta$, is given by

$$\alpha_k(\theta) = \arctan [|R_{k,v}| \sin \beta_k / (1 + |R_{k,v}| \cos \beta_k)] \quad (2-9)$$

For homogeneous earth, $\alpha_k \equiv \alpha$ and $\beta_k = \beta$. The phase delay β and argument α are tabulated in tables 12, 13, and 14 for homogeneous very dry ground, medium dry ground,

Table 2. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Perfect Ground

Grazing ang. deg	Perfect Ground (Relative Dielectric Constant = 1.0, Conductivity = INFINITY S/m)				Constant = 1.0, Conductivity = INFINITY S/m				FREQUENCY = 30.0 MHz.			
	FREQUENCY = 6.0 MHz.				FREQUENCY = 15.0 MHz.				FREQUENCY = 30.0 MHz.			
	Fresnel Reflect Coef	Directive gain	[Rv]	ang Rv(deg)	Fresnel Reflect Coef	Directive gain	[Rv]	ang Rv(deg)	Fresnel Reflect Coef	Directive gain	[Rv]	ang Rv(deg)
0.00	1.00000	4.7712	1.00000	0.00000	1.00000	4.7712	1.00000	0.00000	1.00000	4.7712	1.00000	0.00000
0.50	1.00000	4.7709	1.00000	0.00000	1.00000	4.7709	1.00000	0.00000	1.00000	4.7709	1.00000	0.00000
1.00	1.00000	4.7699	1.00000	0.00000	1.00000	4.7699	1.00000	0.00000	1.00000	4.7699	1.00000	0.00000
1.50	1.00000	4.7682	1.00000	0.00000	1.00000	4.7682	1.00000	0.00000	1.00000	4.7682	1.00000	0.00000
2.00	1.00000	4.7659	1.00000	0.00000	1.00000	4.7659	1.00000	0.00000	1.00000	4.7659	1.00000	0.00000
2.50	1.00000	4.7629	1.00000	0.00000	1.00000	4.7629	1.00000	0.00000	1.00000	4.7629	1.00000	0.00000
3.00	1.00000	4.7593	1.00000	0.00000	1.00000	4.7593	1.00000	0.00000	1.00000	4.7593	1.00000	0.00000
3.50	1.00000	4.7550	1.00000	0.00000	1.00000	4.7550	1.00000	0.00000	1.00000	4.7550	1.00000	0.00000
4.00	1.00000	4.7500	1.00000	0.00000	1.00000	4.7500	1.00000	0.00000	1.00000	4.7500	1.00000	0.00000
4.50	1.00000	4.7431	1.00000	0.00000	1.00000	4.7431	1.00000	0.00000	1.00000	4.7431	1.00000	0.00000
5.00	1.00000	4.7381	1.00000	0.00000	1.00000	4.7381	1.00000	0.00000	1.00000	4.7381	1.00000	0.00000
6.00	1.00000	4.7235	1.00000	0.00000	1.00000	4.7235	1.00000	0.00000	1.00000	4.7235	1.00000	0.00000
7.00	1.00000	4.7062	1.00000	0.00000	1.00000	4.7062	1.00000	0.00000	1.00000	4.7062	1.00000	0.00000
8.00	1.00000	4.6863	1.00000	0.00000	1.00000	4.6863	1.00000	0.00000	1.00000	4.6863	1.00000	0.00000
9.00	1.00000	4.6636	1.00000	0.00000	1.00000	4.6636	1.00000	0.00000	1.00000	4.6636	1.00000	0.00000
10.00	1.00000	4.6382	1.00000	0.00000	1.00000	4.6382	1.00000	0.00000	1.00000	4.6382	1.00000	0.00000
12.00	1.00000	4.5993	1.00000	0.00000	1.00000	4.5993	1.00000	0.00000	1.00000	4.5993	1.00000	0.00000
14.00	1.00000	4.5420	1.00000	0.00000	1.00000	4.5420	1.00000	0.00000	1.00000	4.5420	1.00000	0.00000
16.00	1.00000	4.4781	1.00000	0.00000	1.00000	4.4781	1.00000	0.00000	1.00000	4.4781	1.00000	0.00000
18.00	1.00000	4.4082	1.00000	0.00000	1.00000	4.4082	1.00000	0.00000	1.00000	4.4082	1.00000	0.00000
20.00	1.00000	4.3353	1.00000	0.00000	1.00000	4.3353	1.00000	0.00000	1.00000	4.3353	1.00000	0.00000
22.00	1.00000	4.2609	1.00000	0.00000	1.00000	4.2609	1.00000	0.00000	1.00000	4.2609	1.00000	0.00000
24.00	1.00000	4.1845	1.00000	0.00000	1.00000	4.1845	1.00000	0.00000	1.00000	4.1845	1.00000	0.00000
26.00	1.00000	3.9858	1.00000	0.00000	1.00000	3.9858	1.00000	0.00000	1.00000	3.9858	1.00000	0.00000
28.00	1.00000	3.8444	1.00000	0.00000	1.00000	3.8444	1.00000	0.00000	1.00000	3.8444	1.00000	0.00000
30.00	1.00000	3.6899	1.00000	0.00000	1.00000	3.6899	1.00000	0.00000	1.00000	3.6899	1.00000	0.00000
32.00	1.00000	3.5218	1.00000	0.00000	1.00000	3.5218	1.00000	0.00000	1.00000	3.5218	1.00000	0.00000
34.00	1.00000	3.3536	1.00000	0.00000	1.00000	3.3536	1.00000	0.00000	1.00000	3.3536	1.00000	0.00000
36.00	1.00000	3.1847	1.00000	0.00000	1.00000	3.1847	1.00000	0.00000	1.00000	3.1847	1.00000	0.00000
38.00	1.00000	2.9304	1.00000	0.00000	1.00000	2.9304	1.00000	0.00000	1.00000	2.9304	1.00000	0.00000
40.00	1.00000	2.7019	1.00000	0.00000	1.00000	2.7019	1.00000	0.00000	1.00000	2.7019	1.00000	0.00000
42.00	1.00000	2.4563	1.00000	0.00000	1.00000	2.4563	1.00000	0.00000	1.00000	2.4563	1.00000	0.00000
44.00	1.00000	2.1927	1.00000	0.00000	1.00000	2.1927	1.00000	0.00000	1.00000	2.1927	1.00000	0.00000
46.00	1.00000	1.9099	1.00000	0.00000	1.00000	1.9099	1.00000	0.00000	1.00000	1.9099	1.00000	0.00000
48.00	1.00000	1.6066	1.00000	0.00000	1.00000	1.6066	1.00000	0.00000	1.00000	1.6066	1.00000	0.00000
50.00	1.00000	1.2814	1.00000	0.00000	1.00000	1.2814	1.00000	0.00000	1.00000	1.2814	1.00000	0.00000
52.00	1.00000	0.9326	1.00000	0.00000	1.00000	0.9326	1.00000	0.00000	1.00000	0.9326	1.00000	0.00000
54.00	1.00000	0.5581	1.00000	0.00000	1.00000	0.5581	1.00000	0.00000	1.00000	0.5581	1.00000	0.00000
56.00	1.00000	0.1556	1.00000	0.00000	1.00000	0.1556	1.00000	0.00000	1.00000	0.1556	1.00000	0.00000
58.00	1.00000	-0.2776	1.00000	0.00000	1.00000	-0.2776	1.00000	0.00000	1.00000	-0.2776	1.00000	0.00000
60.00	1.00000	-0.7446	1.00000	0.00000	1.00000	-0.7446	1.00000	0.00000	1.00000	-0.7446	1.00000	0.00000
62.00	1.00000	-1.2494	1.00000	0.00000	1.00000	-1.2494	1.00000	0.00000	1.00000	-1.2494	1.00000	0.00000
64.00	1.00000	-1.7966	1.00000	0.00000	1.00000	-1.7966	1.00000	0.00000	1.00000	-1.7966	1.00000	0.00000
66.00	1.00000	-2.3919	1.00000	0.00000	1.00000	-2.3919	1.00000	0.00000	1.00000	-2.3919	1.00000	0.00000
68.00	1.00000	-3.0425	1.00000	0.00000	1.00000	-3.0425	1.00000	0.00000	1.00000	-3.0425	1.00000	0.00000
70.00	1.00000	-3.7573	1.00000	0.00000	1.00000	-3.7573	1.00000	0.00000	1.00000	-3.7573	1.00000	0.00000
72.00	1.00000	-4.5478	1.00000	0.00000	1.00000	-4.5478	1.00000	0.00000	1.00000	-4.5478	1.00000	0.00000
74.00	1.00000	-5.4291	1.00000	0.00000	1.00000	-5.4291	1.00000	0.00000	1.00000	-5.4291	1.00000	0.00000
76.00	1.00000	-6.4220	1.00000	0.00000	1.00000	-6.4220	1.00000	0.00000	1.00000	-6.4220	1.00000	0.00000
78.00	1.00000	-7.5553	1.00000	0.00000	1.00000	-7.5553	1.00000	0.00000	1.00000	-7.5553	1.00000	0.00000
80.00	1.00000	-8.8712	1.00000	0.00000	1.00000	-8.8712	1.00000	0.00000	1.00000	-8.8712	1.00000	0.00000
82.00	1.00000	-10.4354	1.00000	0.00000	1.00000	-10.4354	1.00000	0.00000	1.00000	-10.4354	1.00000	0.00000
84.00	1.00000	-12.3577	1.00000	0.00000	1.00000	-12.3577	1.00000	0.00000	1.00000	-12.3577	1.00000	0.00000
86.00	1.00000	-14.8441	1.00000	0.00000	1.00000	-14.8441	1.00000	0.00000	1.00000	-14.8441	1.00000	0.00000
88.00	1.00000	-18.3571	1.00000	0.00000	1.00000	-18.3571	1.00000	0.00000	1.00000	-18.3571	1.00000	0.00000
90.00	1.00000	-24.3724	1.00000	0.00000	1.00000	-24.3724	1.00000	0.00000	1.00000	-24.3724	1.00000	0.00000
		-142.4169	1.00000	0.00000		-142.4169	1.00000	0.00000		-142.4169	1.00000	0.00000

Table 3. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Sea Water

CASE (2): Sea Water (av salinity 20 ‰ C)		FREQUENCY = 6.0 Mhz.		FREQUENCY = 15.0 Mhz.		FREQUENCY = 30.0 Mhz.		FREQUENCY = 5.000000 S/m	
Grazing ang. deg	PSI(deg)	Fresnel Reflect Coef		Directive gain		Fresnel Reflect Coef		Directive gain	
		IRV	ang RV(deg)	D(dBi)		IRV	ang RV(deg)	D(dBi)	
0.00	0.00	1.00000	180.00000	-999.9999		1.00000	180.00000	-999.9999	
0.50	0.4143	0.4143	-84.60905	0.0552		0.43255	-119.74591	1.9779	
1.00	0.54337	0.54337	-40.19803	2.3618		0.43633	-66.46782	1.1092	
1.50	0.56235	0.56235	-25.98895	3.2328		0.44345	-42.49292	2.3677	
2.00	0.72220	0.72220	-19.24878	3.6853		0.52775	-31.06702	3.0450	
2.50	0.76935	0.76935	-15.30605	3.9605		0.60595	-24.51376	3.4653	
3.00	0.80304	0.80304	-12.71306	4.1441		0.66472	-20.26586	3.7498	
3.50	0.82823	0.82823	-10.67583	4.2744		0.70315	-17.28586	3.9541	
4.00	0.84775	0.84775	-9.50509	4.3708		0.73455	-15.07728	4.1070	
4.50	0.86198	0.86198	-8.32721	4.4519		0.76082	-12.01869	4.3178	
5.00	0.87598	0.87598	-7.15013	4.5193		0.78362	-9.59944	4.4527	
5.50	0.88953	0.88953	-6.32299	4.5737		0.80359	-8.56400	4.5456	
6.00	0.90279	0.90279	-5.42309	4.6230		0.82047	-7.66246	4.6032	
6.50	0.91584	0.91584	-4.71795	4.6680		0.83532	-6.86844	4.6483	
7.00	0.92829	0.92829	-4.17955	4.7093		0.84881	-6.16681	4.6881	
7.50	0.94016	0.94016	-3.70375	4.7475		0.86111	-5.55994	4.7221	
8.00	0.95158	0.95158	-3.27619	4.7833		0.87255	-5.00789	4.7521	
8.50	0.96253	0.96253	-2.89330	4.8169		0.88311	-4.50039	4.7783	
9.00	0.97300	0.97300	-2.55325	4.8480		0.89286	-4.03239	4.8004	
9.50	0.98308	0.98308	-2.25325	4.8775		0.90184	-3.60699	4.8194	
10.00	0.99275	0.99275	-2.00000	4.9053		0.91000	-3.22277	4.8359	
10.50	0.99999	0.99999	-1.79220	4.9308		0.91744	-2.88018	4.8491	
11.00	0.99999	0.99999	-1.62220	4.9538		0.92421	-2.57743	4.8599	
11.50	0.99999	0.99999	-1.48220	4.9745		0.93049	-2.30270	4.8684	
12.00	0.99999	0.99999	-1.36220	4.9930		0.93620	-2.05219	4.8750	
12.50	0.99999	0.99999	-1.26220	5.0095		0.94149	-1.82461	4.8800	
13.00	0.99999	0.99999	-1.18220	5.0243		0.94629	-1.61496	4.8836	
13.50	0.99999	0.99999	-1.11220	5.0378		0.95059	-1.42192	4.8860	
14.00	0.99999	0.99999	-1.05220	5.0503		0.95438	-1.24444	4.8874	
14.50	0.99999	0.99999	-1.00000	5.0619		0.95767	-1.08199	4.8880	
15.00	0.99999	0.99999	-0.95767	5.0727		0.96049	-0.94500	4.8880	
15.50	0.99999	0.99999	-0.92421	5.0827		0.96286	-0.83195	4.8874	
16.00	0.99999	0.99999	-0.89999	5.0920		0.96481	-0.73438	4.8860	
16.50	0.99999	0.99999	-0.88220	5.0999		0.96633	-0.65000	4.8840	
17.00	0.99999	0.99999	-0.86165	5.1065		0.96756	-0.57844	4.8819	
17.50	0.99999	0.99999	-0.83783	5.1119		0.96849	-0.51744	4.8797	
18.00	0.99999	0.99999	-0.81588	5.1169		0.96909	-0.46444	4.8774	
18.50	0.99999	0.99999	-0.79619	5.1215		0.96938	-0.41844	4.8750	
19.00	0.99999	0.99999	-0.77833	5.1257		0.96938	-0.37844	4.8727	
19.50	0.99999	0.99999	-0.76217	5.1295		0.96909	-0.34444	4.8704	
20.00	0.99999	0.99999	-0.74756	5.1329		0.96849	-0.31544	4.8680	
20.50	0.99999	0.99999	-0.73438	5.1360		0.96756	-0.29144	4.8656	
21.00	0.99999	0.99999	-0.72253	5.1389		0.96633	-0.27144	4.8632	
21.50	0.99999	0.99999	-0.71190	5.1415		0.96481	-0.25444	4.8608	
22.00	0.99999	0.99999	-0.70242	5.1437		0.96308	-0.24044	4.8584	
22.50	0.99999	0.99999	-0.69403	5.1457		0.96111	-0.22844	4.8560	
23.00	0.99999	0.99999	-0.68666	5.1475		0.95899	-0.21844	4.8536	
23.50	0.99999	0.99999	-0.68027	5.1491		0.95666	-0.21044	4.8512	
24.00	0.99999	0.99999	-0.67481	5.1505		0.95421	-0.20444	4.8488	
24.50	0.99999	0.99999	-0.66935	5.1519		0.95166	-0.20044	4.8464	
25.00	0.99999	0.99999	-0.66389	5.1533		0.94909	-0.19744	4.8440	
25.50	0.99999	0.99999	-0.65843	5.1547		0.94649	-0.19544	4.8416	
26.00	0.99999	0.99999	-0.65297	5.1561		0.94381	-0.19444	4.8392	
26.50	0.99999	0.99999	-0.64751	5.1575		0.94111	-0.19444	4.8368	
27.00	0.99999	0.99999	-0.64205	5.1589		0.93844	-0.19544	4.8344	
27.50	0.99999	0.99999	-0.63659	5.1603		0.93577	-0.19744	4.8320	
28.00	0.99999	0.99999	-0.63113	5.1617		0.93311	-0.19944	4.8296	
28.50	0.99999	0.99999	-0.62567	5.1631		0.93049	-0.20244	4.8272	
29.00	0.99999	0.99999	-0.62021	5.1645		0.92786	-0.20644	4.8248	
29.50	0.99999	0.99999	-0.61475	5.1659		0.92521	-0.21144	4.8224	
30.00	0.99999	0.99999	-0.60929	5.1673		0.92255	-0.21744	4.8199	
30.50	0.99999	0.99999	-0.60383	5.1687		0.91989	-0.22444	4.8175	
31.00	0.99999	0.99999	-0.59837	5.1701		0.91722	-0.23244	4.8151	
31.50	0.99999	0.99999	-0.59291	5.1715		0.91455	-0.24144	4.8127	
32.00	0.99999	0.99999	-0.58745	5.1729		0.91188	-0.25144	4.8103	
32.50	0.99999	0.99999	-0.58199	5.1743		0.90922	-0.26244	4.8079	
33.00	0.99999	0.99999	-0.57653	5.1757		0.90655	-0.27444	4.8055	
33.50	0.99999	0.99999	-0.57107	5.1771		0.90389	-0.28744	4.8031	
34.00	0.99999	0.99999	-0.56561	5.1785		0.90122	-0.30144	4.8007	
34.50	0.99999	0.99999	-0.56015	5.1799		0.89855	-0.31644	4.7983	
35.00	0.99999	0.99999	-0.55469	5.1813		0.89589	-0.33244	4.7959	
35.50	0.99999	0.99999	-0.54923	5.1827		0.89322	-0.34944	4.7935	
36.00	0.99999	0.99999	-0.54377	5.1841		0.89055	-0.36744	4.7911	
36.50	0.99999	0.99999	-0.53831	5.1855		0.88789	-0.38644	4.7887	
37.00	0.99999	0.99999	-0.53285	5.1869		0.88522	-0.40644	4.7863	
37.50	0.99999	0.99999	-0.52739	5.1883		0.88255	-0.42744	4.7839	
38.00	0.99999	0.99999	-0.52193	5.1897		0.87989	-0.44944	4.7815	
38.50	0.99999	0.99999	-0.51647	5.1911		0.87722	-0.47244	4.7791	
39.00	0.99999	0.99999	-0.51101	5.1925		0.87455	-0.49644	4.7767	
39.50	0.99999	0.99999	-0.50555	5.1939		0.87188	-0.52144	4.7743	
40.00	0.99999	0.99999	-0.50009	5.1953		0.86922	-0.54744	4.7719	
40.50	0.99999	0.99999	-0.49463	5.1967		0.86655	-0.57444	4.7695	
41.00	0.99999	0.99999	-0.48917	5.1981		0.86389	-0.60244	4.7671	
41.50	0.99999	0.99999	-0.48371	5.1995		0.86122	-0.63144	4.7647	
42.00	0.99999	0.99999	-0.47825	5.2009		0.85855	-0.66144	4.7623	
42.50	0.99999	0.99999	-0.47279	5.2023		0.85589	-0.69244	4.7599	
43.00	0.99999	0.99999	-0.46733	5.2037		0.85322	-0.72444	4.7575	
43.50	0.99999	0.99999	-0.46187	5.2051		0.85055	-0.75744	4.7551	
44.00	0.99999	0.99999	-0.45641	5.2065		0.84789	-0.79144	4.7527	
44.50	0.99999	0.99999	-0.45095	5.2079		0.84522	-0.82644	4.7503	
45.00	0.99999	0.99999	-0.44549	5.2093		0.84255	-0.86244	4.7479	
45.50	0.99999	0.99999	-0.44003	5.2107		0.83989	-0.90044	4.7455	
46.00	0.99999	0.99999	-0.43457	5.2121		0.83722	-0.93944	4.7431	
46.50	0.99999	0.99999	-0.42911	5.2135		0.83455	-0.98044	4.7407	
47.00	0.99999	0.99999	-0.42365	5.2149		0.83189	-1.02244	4.7383	
47.50	0.99999	0.99999	-0.41819	5.2163		0.82922	-1.06644	4.7359	
48.00	0.99999	0.99999	-0.41273	5.2177		0.82655	-1.11244	4.7335	
48.50	0.99999	0.99999	-0.40727	5.2191		0.82389	-1.16044	4.7311	
49.00	0.99999	0.99999	-0.40181	5.2205		0.82122	-1.21044	4.7287	
49.50	0.99999	0.99999	-0.39635	5.2219		0.81855	-1.26244	4.7263	
50.00	0.99999	0.99999	-0.39089	5.2233		0.81589	-1.31644	4.7239	
50.50	0.99999	0.99999	-0.38543	5.2247		0.81322	-1.37244	4.7215	
51.00	0.99999	0.99999	-0.37997	5.2261		0.81055	-1.43044	4.7191	
51.50	0.99999	0.99999	-0.37451	5.2275		0.80789	-1.49044	4.7167	
52.00	0.99999	0.99999	-0.36905	5.2289		0.80522	-1.55244	4.7143	
52.50	0.99999	0.99999	-0.36359	5.2303		0.80255	-1.61644	4.7119	
53.00	0.99999	0.99999	-0.35813	5.2317		0.80000	-1.68244	4.7095	
53.50	0.99999	0.99999	-0.35267	5.2331		0.79744	-1.75044	4.7071	
54.00	0.99999	0.99999	-0.34721	5.2345		0.79489	-1.82044	4.7047	
54.50	0.99999	0.99999	-0.34175	5.2359		0.79233	-1.89244	4.7023	
55.00	0.99999	0.99999	-0.33629	5.2373		0.78977	-1.96644	4.7000	
55.50	0.99999	0.99999	-0.33083	5.2387		0.78722	-2.04244	4.6976	
56.00	0.99999	0.99999	-0						

Table 4. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Fresh Water

CASE(3):		Fresh Water FREQUENCY = 6.0 MHz.				Dielectric Constant = 80.0, Conductivity = 0.030000 S/m				FREQUENCY = 15.0 MHz.				FREQUENCY = 30.0 MHz.			
Grazing ang. deg	PSI(deg)	Fresnel Reflect Coef		Directive gain		Fresnel Reflect Coef		Directive gain		Fresnel Reflect Coef		Directive gain		Fresnel Reflect Coef		Directive gain	
		Rv	ang Rv(deg)	D(dBi)		Rv	ang Rv(deg)	D(dBi)		Rv	ang Rv(deg)	D(dBi)		Rv	ang Rv(deg)	D(dBi)	
0.00	0.00	1.00000	180.00000	-999.99999		1.00000	180.00000	-999.99999		1.00000	180.00000	-999.99999		1.00000	180.00000	-999.99999	
0.50	0.83894	0.83894	-175.49135	-13.8869		0.85125	-178.03345	-14.8065		0.85354	-179.00006	-14.9947		0.85354	-179.00006	-14.9947	
1.00	0.70309	0.70309	-170.78516	-8.5548		0.72332	-175.99016	-9.4131		0.72713	-177.96196	-9.5900		0.72713	-177.96196	-9.5900	
1.50	0.58017	0.58017	-165.66431	-5.6783		0.61235	-173.78303	-6.4785		0.61698	-176.84261	-6.6446		0.61698	-176.84261	-6.6446	
2.00	0.49104	0.49104	-159.87224	-3.7860		0.51541	-171.30666	-4.5318		0.52021	-175.58774	-4.6876		0.52021	-175.58774	-4.6876	
2.50	0.40955	0.40955	-153.09621	-2.4194		0.43031	-168.41162	-3.1146		0.43461	-174.12224	-3.2607		0.43461	-174.12224	-3.2607	
3.00	0.34241	0.34241	-144.96724	-1.3761		0.35841	-164.88354	-2.0242		0.35845	-172.33318	-2.1612		0.35845	-172.33318	-2.1612	
3.50	0.28915	0.28915	-135.11427	-0.5493		0.28957	-160.38875	-1.1536		0.29041	-170.03693	-1.2822		0.29041	-170.03693	-1.2822	
4.00	0.24989	0.24989	-123.33838	0.1240		0.23217	-154.38623	-0.4398		0.23251	-166.90907	-0.5605		0.23251	-166.90907	-0.5605	
4.50	0.21363	0.21363	-109.87959	1.1563		0.14395	-133.80182	0.6830		0.12723	-154.89259	0.5588		0.12723	-154.89259	0.5588	
5.00	0.18377	0.18377	-95.87959	1.9109		0.10523	-104.41348	1.4823		0.05995	-114.20327	1.3887		0.05995	-114.20327	1.3887	
5.50	0.15960	0.15960	-82.81333	2.4851		0.07561	-86.40618	2.1111		0.07561	-146.19757	2.0287		0.07561	-146.19757	2.0287	
6.00	0.13928	0.13928	-70.96504	2.9536		0.05400	-77.59242	2.6085		0.05400	-24.50036	2.5360		0.05400	-24.50036	2.5360	
6.50	0.12049	0.12049	-60.51323	3.2832		0.04193	-67.61649	3.0100		0.04193	-16.79543	2.9463		0.04193	-16.79543	2.9463	
7.00	0.10374	0.10374	-51.35164	3.5053		0.03408	-59.02290	3.3389		0.03408	-12.94742	3.2831		0.03408	-12.94742	3.2831	
7.50	0.08942	0.08942	-43.39118	4.0219		0.02858	-51.35164	3.8385		0.02858	-9.08301	3.7963		0.02858	-9.08301	3.7963	
8.00	0.78085	0.78085	-36.59118	4.3212		0.02425	-44.02251	4.1892		0.02425	-7.11487	4.1580		0.02425	-7.11487	4.1580	
8.50	0.64014	0.64014	-28.74165	4.5356		0.02051	-38.02251	4.4360		0.02051	-5.90497	4.4141		0.02051	-5.90497	4.4141	
9.00	0.53495	0.53495	-23.74165	4.7780		0.01751	-33.02251	4.6061		0.01751	-4.7824	4.5922		0.01751	-4.7824	4.5922	
9.50	0.46152	0.46152	-19.69151	4.6603		0.01544	-29.02251	4.7168		0.01544	-1.01156	4.7096		0.01544	-1.01156	4.7096	
10.00	0.40818	0.40818	-16.69151	4.7778		0.01395	-26.02251	4.7955		0.01395	-0.64546	4.7881		0.01395	-0.64546	4.7881	
10.50	0.36333	0.36333	-14.06998	4.7780		0.01293	-23.02251	4.8021		0.01293	-0.57693	4.8057		0.01293	-0.57693	4.8057	
11.00	0.32745	0.32745	-11.86998	4.7403		0.01237	-20.02251	4.7999		0.01237	-0.34817	4.7980		0.01237	-0.34817	4.7980	
11.50	0.29810	0.29810	-10.01432	4.6863		0.01193	-17.02251	4.7470		0.01193	-0.62763	4.7500		0.01193	-0.62763	4.7500	
12.00	0.27460	0.27460	-8.31297	4.6002		0.01166	-15.02251	4.6761		0.01166	-0.64147	4.6916		0.01166	-0.64147	4.6916	
12.50	0.25650	0.25650	-7.1638	4.4897		0.01141	-13.02251	4.5791		0.01141	-0.21794	4.5977		0.01141	-0.21794	4.5977	
13.00	0.24219	0.24219	-6.20329	4.3560		0.01111	-11.02251	4.4573		0.01111	-0.24321	4.5788		0.01111	-0.24321	4.5788	
13.50	0.23099	0.23099	-5.45144	4.1996		0.01086	-9.02251	4.3117		0.01086	-0.21488	4.5785		0.01086	-0.21488	4.5785	
14.00	0.22177	0.22177	-4.84144	4.0209		0.01062	-7.02251	3.9504		0.01062	-0.70025	4.5785		0.01062	-0.70025	4.5785	
14.50	0.21404	0.21404	-4.32169	3.8199		0.01039	-5.02251	3.7347		0.01039	-0.71418	4.5785		0.01039	-0.71418	4.5785	
15.00	0.20759	0.20759	-3.90999	3.5962		0.01014	-3.02251	3.4950		0.01014	-0.72790	4.5785		0.01014	-0.72790	4.5785	
15.50	0.20219	0.20219	-3.59144	3.3494		0.00989	-1.02251	3.2307		0.00989	-0.73595	4.5785		0.00989	-0.73595	4.5785	
16.00	0.19774	0.19774	-3.34944	3.0786		0.00964	0.97767	2.9407		0.00964	-0.74894	4.5785		0.00964	-0.74894	4.5785	
16.50	0.19419	0.19419	-3.14144	2.7827		0.00939	2.97767	2.6238		0.00939	-0.75601	4.5785		0.00939	-0.75601	4.5785	
17.00	0.19144	0.19144	-2.94144	2.4604		0.00914	4.97767	2.2783		0.00914	-0.76155	4.5785		0.00914	-0.76155	4.5785	
17.50	0.18944	0.18944	-2.74144	2.1100		0.00889	6.97767	1.9022		0.00889	-0.76660	4.5785		0.00889	-0.76660	4.5785	
18.00	0.18744	0.18744	-2.54144	1.7294		0.00864	8.97767	1.4930		0.00864	-0.76951	4.5785		0.00864	-0.76951	4.5785	
18.50	0.18719	0.18719	-2.34144	1.3161		0.00839	10.97767	1.0477		0.00839	-0.77119	4.5785		0.00839	-0.77119	4.5785	
19.00	0.18744	0.18744	-2.14144	0.8671		0.00814	12.97767	0.5626		0.00814	-0.77324	4.5785		0.00814	-0.77324	4.5785	
19.50	0.18819	0.18819	-1.94144	0.3786		0.00789	14.97767	0.0332		0.00789	-0.77537	4.5785		0.00789	-0.77537	4.5785	
20.00	0.18944	0.18944	-1.74144	-0.1538		0.00764	16.97767	-0.5460		0.00764	-0.77719	4.5785		0.00764	-0.77719	4.5785	
20.50	0.19144	0.19144	-1.54144	-0.7358		0.00739	18.97767	-1.1807		0.00739	-0.77900	4.5785		0.00739	-0.77900	4.5785	
21.00	0.19419	0.19419	-1.34144	-1.3744		0.00714	20.97767	-1.8339		0.00714	-0.78085	4.5785		0.00714	-0.78085	4.5785	
21.50	0.19774	0.19774	-1.14144	-2.0783		0.00689	22.97767	-2.5036		0.00689	-0.78270	4.5785		0.00689	-0.78270	4.5785	
22.00	0.20219	0.20219	-0.94144	-2.8592		0.00664	24.97767	-3.2007		0.00664	-0.78454	4.5785		0.00664	-0.78454	4.5785	
22.50	0.20759	0.20759	-0.74144	-3.7320		0.00639	26.97767	-3.9321		0.00639	-0.78638	4.5785		0.00639	-0.78638	4.5785	
23.00	0.21404	0.21404	-0.54144	-4.7174		0.00614	28.97767	-4.6985		0.00614	-0.78821	4.5785		0.00614	-0.78821	4.5785	
23.50	0.22177	0.22177	-0.34144	-5.8441		0.00589	30.97767	-5.5028		0.00589	-0.78999	4.5785		0.00589	-0.78999	4.5785	
24.00	0.23099	0.23099	0.05856	-8.6409		0.00564	32.97767	-6.3519		0.00564	-0.79174	4.5785		0.00564	-0.79174	4.5785	
24.50	0.24219	0.24219	0.25856	-10.3216		0.00539	34.97767	-7.2421		0.00539	-0.79358	4.5785		0.00539	-0.79358	4.5785	
25.00	0.25650	0.25650	0.45856	-12.1660		0.00514	36.97767	-8.1791		0.00514	-0.79541	4.5785		0.00514	-0.79541	4.5785	
25.50	0.27460	0.27460	0.65856	-14.2212		0.00489	38.97767	-9.1671		0.00489	-0.79724	4.5785		0.00489	-0.79724	4.5785	
26.00	0.29810	0.29810	0.85856	-16.5216		0.00464	40.97767	-10.2007		0.00464	-0.79907	4.5785		0.00464	-0.79907	4.5785	
26.50	0.32745	0.32745	1.05856	-19.0216		0.00439	42.97767	-11.2751		0.00439	-0.80089	4.5785		0.00439	-0.80089	4.5785	
27.00	0.36333	0.36333	1.25856	-21.7216		0.00414	44.97767	-12.3951		0.00414	-0.80272	4.5785		0.00414	-0.80272	4.5785	
27.50	0.40818	0.40818	1.45856	-24.6216		0.00389	46.97767	-13.5651		0.00389	-0.80454	4.5785		0.00389	-0.80454	4.5785	
28.00	0.46152	0.46152	1.65856	-27.7216		0.00364	48.97767	-14.7901		0.00364	-0.80637	4.5785		0.00364	-0.80637	4.5785	
28.50	0.53495	0.53495	1.85856	-31.0216		0.00339	50.97767	-16.0651		0.00339	-0.80820	4.5785		0.00339	-0.80820	4.5785	
29.00	0.64014	0.64014	2.05856	-34.5216		0.00313											

Table 5. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Wet Ground

CASE (4): Wet Ground (Relative Dielectric Constant = 30.0, Conductivity = 0.010000 S/m)		FREQUENCY = 6.0 MHz.		FREQUENCY = 15.0 MHz.		FREQUENCY = 30.0 MHz.	
Grazing ang. deg	psi(deg)	Fresnel Reflect Coef [Rv]	ang Rv(deg)	Directive gain	Fresnel Reflect Coef [Rv]	ang Rv(deg)	Directive gain
0.00	0.00	1.00000	180.00000	-999.9999	1.00000	180.00000	-999.9999
0.50	0.89913	-177.53148	-177.53148	-17.0270	0.90372	-178.94031	-17.0270
1.00	0.80820	-175.02328	-175.02328	-11.4200	0.81997	-176.86513	-11.4200
1.50	0.72605	-172.43419	-172.43419	-8.3347	0.74170	-175.75836	-8.3347
2.00	0.65170	-169.71970	-169.71970	-6.2364	0.67002	-175.60228	-6.2364
2.50	0.58435	-166.83031	-166.83031	-4.6839	0.60417	-174.37682	-4.6839
3.00	0.52334	-163.70963	-163.70963	-3.4721	0.54353	-173.05827	-3.4721
3.50	0.46815	-160.29240	-160.29240	-2.4921	0.48756	-171.61778	-2.4921
4.00	0.41835	-156.50290	-156.50290	-1.6792	0.43581	-170.01923	-1.6792
4.50	0.37389	-147.44798	-147.44798	-0.4018	0.34352	-166.14746	-0.4018
5.00	0.33389	-135.76707	-135.76707	0.5602	0.26445	-160.85295	0.5602
5.50	0.29911	-120.91679	-120.91679	1.3121	0.19754	-153.01488	1.3121
6.00	0.26972	-103.51066	-103.51066	1.9153	0.14340	-140.41916	1.9153
6.50	0.24508	-85.89762	-85.89762	2.4085	0.10591	-119.41214	2.4085
7.00	0.22536	-70.66362	-70.66362	2.8178	0.08252	-89.66296	2.8178
7.50	0.20936	-58.87074	-58.87074	3.1513	0.06287	-73.24990	3.1513
8.00	0.19572	-49.86417	-49.86417	3.4307	0.04622	-58.48850	3.4307
8.50	0.18393	-42.96202	-42.96202	3.6638	0.03259	-46.80100	3.6638
9.00	0.17357	-37.77112	-37.77112	3.8560	0.02166	-38.13327	3.8560
9.50	0.16431	-33.82700	-33.82700	4.0073	0.01320	-31.27667	4.0073
10.00	0.15600	-30.72000	-30.72000	4.1322	0.00715	-25.90581	4.1322
				4.2346	0.00415	-21.41214	4.2346
				4.3178	0.00252	-18.49762	4.3178
				4.3853	0.00156	-15.81214	4.3853
				4.4413	0.00093	-13.48850	4.4413
				4.4878	0.00053	-11.47472	4.4878
				4.5259	0.00031	-9.71970	4.5259
				4.5573	0.00019	-8.27470	4.5573
				4.5831	0.00011	-7.08668	4.5831
				4.6043	0.00006	-6.11268	4.6043
				4.6218	0.00003	-5.39146	4.6218
				4.6363	0.00002	-4.78126	4.6363
				4.6488	0.00001	-4.26617	4.6488
				4.6593	0.00000	-3.83914	4.6593
				4.6681	0.00000	-3.48126	4.6681
				4.6753	0.00000	-3.18126	4.6753
				4.6813	0.00000	-2.93126	4.6813
				4.6861	0.00000	-2.72617	4.6861
				4.6899	0.00000	-2.55126	4.6899
				4.6928	0.00000	-2.40126	4.6928
				4.6948	0.00000	-2.27161	4.6948
				4.6960	0.00000	-2.16126	4.6960
				4.6973	0.00000	-2.06617	4.6973
				4.6987	0.00000	-1.98126	4.6987
				4.6999	0.00000	-1.90617	4.6999
				4.7011	0.00000	-1.84126	4.7011
				4.7022	0.00000	-1.78617	4.7022
				4.7033	0.00000	-1.74126	4.7033
				4.7043	0.00000	-1.70617	4.7043
				4.7053	0.00000	-1.68126	4.7053
				4.7063	0.00000	-1.66617	4.7063
				4.7073	0.00000	-1.66126	4.7073
				4.7083	0.00000	-1.66617	4.7083
				4.7093	0.00000	-1.68126	4.7093
				4.7103	0.00000	-1.70617	4.7103
				4.7113	0.00000	-1.74126	4.7113
				4.7123	0.00000	-1.78617	4.7123
				4.7133	0.00000	-1.84126	4.7133
				4.7143	0.00000	-1.90617	4.7143
				4.7153	0.00000	-1.98126	4.7153
				4.7163	0.00000	-2.06617	4.7163
				4.7173	0.00000	-2.16126	4.7173
				4.7183	0.00000	-2.27161	4.7183
				4.7193	0.00000	-2.40126	4.7193
				4.7203	0.00000	-2.55126	4.7203
				4.7213	0.00000	-2.72617	4.7213
				4.7223	0.00000	-2.93126	4.7223
				4.7233	0.00000	-3.18126	4.7233
				4.7243	0.00000	-3.48126	4.7243
				4.7253	0.00000	-3.83914	4.7253
				4.7263	0.00000	-4.26617	4.7263
				4.7273	0.00000	-4.78126	4.7273
				4.7283	0.00000	-5.39146	4.7283
				4.7293	0.00000	-6.11268	4.7293
				4.7303	0.00000	-6.93126	4.7303
				4.7313	0.00000	-7.86617	4.7313
				4.7323	0.00000	-8.93126	4.7323
				4.7333	0.00000	-10.14126	4.7333
				4.7343	0.00000	-11.50617	4.7343
				4.7353	0.00000	-13.04126	4.7353
				4.7363	0.00000	-14.76617	4.7363
				4.7373	0.00000	-16.69126	4.7373
				4.7383	0.00000	-18.83617	4.7383
				4.7393	0.00000	-21.21126	4.7393
				4.7403	0.00000	-23.83617	4.7403
				4.7413	0.00000	-26.71126	4.7413
				4.7423	0.00000	-30.84617	4.7423
				4.7433	0.00000	-36.24126	4.7433
				4.7443	0.00000	-43.01617	4.7443
				4.7453	0.00000	-51.29126	4.7453
				4.7463	0.00000	-61.16617	4.7463
				4.7473	0.00000	-72.74126	4.7473
				4.7483	0.00000	-86.16617	4.7483
				4.7493	0.00000	-101.54126	4.7493
				4.7503	0.00000	-119.86617	4.7503
				4.7513	0.00000	-140.24126	4.7513
				4.7523	0.00000	-162.66617	4.7523
				4.7533	0.00000	-187.14126	4.7533
				4.7543	0.00000	-213.66617	4.7543
				4.7553	0.00000	-242.24126	4.7553
				4.7563	0.00000	-272.86617	4.7563
				4.7573	0.00000	-305.54126	4.7573
				4.7583	0.00000	-340.26617	4.7583
				4.7593	0.00000	-377.04126	4.7593
				4.7603	0.00000	-415.86617	4.7603
				4.7613	0.00000	-456.74126	4.7613
				4.7623	0.00000	-500.66617	4.7623
				4.7633	0.00000	-547.64126	4.7633
				4.7643	0.00000	-597.66617	4.7643
				4.7653	0.00000	-650.64126	4.7653
				4.7663	0.00000	-706.56617	4.7663
				4.7673	0.00000	-765.44126	4.7673
				4.7683	0.00000	-827.26617	4.7683
				4.7693	0.00000	-892.04126	4.7693
				4.7703	0.00000	-959.76617	4.7703
				4.7713	0.00000	-1030.44126	4.7713
				4.7723	0.00000	-1104.06617	4.7723
				4.7733	0.00000	-1180.64126	4.7733
				4.7743	0.00000	-1260.16617	4.7743
				4.7753	0.00000	-1342.64126	4.7753
				4.7763	0.00000	-1428.06617	4.7763
				4.7773	0.00000	-1516.34126	4.7773
				4.7783	0.00000	-1608.46617	4.7783
				4.7793	0.00000	-1704.44126	4.7793
				4.7803	0.00000	-1804.26617	4.7803
				4.7813	0.00000	-1907.94126	4.7813
				4.7823	0.00000	-2015.46617	4.7823
				4.7833	0.00000	-2126.74126	4.7833
				4.7843	0.00000	-2241.76617	4.7843
				4.7853	0.00000	-2360.44126	4.7853
				4.7863	0.00000	-2482.76617	4.7863
				4.7873	0.00000	-2608.74126	4.7873
				4.7883	0.00000	-2738.36617	4.7883
				4.7893	0.00000	-2871.54126	4.7893
				4.7903	0.00000	-3008.26617	4.7903
				4.7913	0.00000	-3148.54126	4.7913
				4.7923	0.00000	-3292.36617	4.7923
				4.7933	0.00000	-3439.64126	4.7933
				4.7943	0.00000	-3590.36617	4.7943
				4.7953	0.00000	-3744.54126	4.7953
				4.7963	0.00000	-3902.16617	4.7963
				4.7973	0.00000	-4063.24126	4.7973
				4.7983	0.00000	-4227.76617	4.7983
				4.7993	0.00000	-4395.74126	4.7993
				4.8003	0.00000	-4567.16617	4.8003

Table 6. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Medium Dry Ground

CASE 5): Medium Dry Ground (Relative Dielectric Constant = 15.0, Conductivity = 0.001000 S/m)		FREQUENCY = 15.0 MHz.		FREQUENCY = 30.0 MHz.	
Grazing angle, deg	PSI(deg)	Fresnel Reflect Coef [Rv]	Directive gain D(dBi)	Fresnel Reflect Coef [Rv]	Directive gain D(dBi)
0.00	0.00	1.00000	-999.9999	1.00000	-999.9999
0.50	0.3273	-179.62875	-19.6288	0.93239	-179.52548
1.00	0.86812	-179.52478	-13.5006	0.86920	-179.52548
1.50	0.80935	-178.87526	-10.6617	0.81003	-179.77422
2.00	0.75387	-178.48729	-8.3680	0.75450	-179.6937
2.50	0.70135	-178.08771	-6.7675	0.70230	-179.61615
3.00	0.65210	-177.67305	-5.4447	0.65314	-179.53296
3.50	0.60566	-177.23953	-4.3601	0.60676	-179.44598
4.00	0.56181	-176.78376	-3.4485	0.56294	-179.35435
4.50	0.52058	-176.30633	-2.6111	0.52020	-179.25292
5.00	0.48158	-175.79328	-1.9099	0.48252	-179.15292
5.50	0.44485	-175.24728	-1.3265	0.44378	-179.05403
6.00	0.41058	-174.67288	-0.8655	0.40952	-178.95803
6.50	0.37835	-174.07183	-0.5124	0.37378	-178.86399
7.00	0.34781	-173.44025	-0.3674	0.34378	-178.77177
7.50	0.31852	-172.78236	-0.2804	0.31444	-178.68066
8.00	0.29021	-172.10016	-0.2227	0.28606	-178.59066
8.50	0.26252	-171.40236	-0.1804	0.25927	-178.50188
9.00	0.23521	-170.68916	-0.1485	0.23368	-178.42293
9.50	0.20821	-169.96016	-0.1227	0.20867	-178.34493
10.00	0.18152	-169.21516	-0.1004	0.18367	-178.26693
10.50	0.15521	-168.45516	-0.0804	0.15867	-178.18893
11.00	0.12921	-167.68016	-0.0627	0.13367	-178.11093
11.50	0.10352	-166.89016	-0.0474	0.10867	-178.03293
12.00	0.07821	-166.08516	-0.0344	0.08367	-177.95493
12.50	0.05321	-165.26516	-0.0234	0.05867	-177.87693
13.00	0.02852	-164.43016	-0.0144	0.03367	-177.79893
13.50	0.00421	-163.58016	-0.0074	0.00867	-177.72093
14.00	0.00000	-162.71516	-0.0024	0.00367	-177.64293
14.50	0.00000	-161.83516	-0.0004	0.00067	-177.56493
15.00	0.00000	-160.94016	-0.0000	0.00000	-177.48693
15.50	0.00000	-160.03016	-0.0000	0.00000	-177.40893
16.00	0.00000	-159.10516	-0.0000	0.00000	-177.33093
16.50	0.00000	-158.16516	-0.0000	0.00000	-177.25293
17.00	0.00000	-157.21016	-0.0000	0.00000	-177.17493
17.50	0.00000	-156.24016	-0.0000	0.00000	-177.09693
18.00	0.00000	-155.25516	-0.0000	0.00000	-177.01893
18.50	0.00000	-154.25516	-0.0000	0.00000	-176.94093
19.00	0.00000	-153.24016	-0.0000	0.00000	-176.86293
19.50	0.00000	-152.21016	-0.0000	0.00000	-176.78493
20.00	0.00000	-151.16516	-0.0000	0.00000	-176.70693
20.50	0.00000	-150.10516	-0.0000	0.00000	-176.62893
21.00	0.00000	-149.03016	-0.0000	0.00000	-176.55093
21.50	0.00000	-147.94016	-0.0000	0.00000	-176.47293
22.00	0.00000	-146.83516	-0.0000	0.00000	-176.39493
22.50	0.00000	-145.71516	-0.0000	0.00000	-176.31693
23.00	0.00000	-144.58016	-0.0000	0.00000	-176.23893
23.50	0.00000	-143.43016	-0.0000	0.00000	-176.16093
24.00	0.00000	-142.26516	-0.0000	0.00000	-176.08293
24.50	0.00000	-141.08516	-0.0000	0.00000	-176.00493
25.00	0.00000	-139.89016	-0.0000	0.00000	-175.92693
25.50	0.00000	-138.68016	-0.0000	0.00000	-175.84893
26.00	0.00000	-137.45516	-0.0000	0.00000	-175.77093
26.50	0.00000	-136.21516	-0.0000	0.00000	-175.69293
27.00	0.00000	-134.96016	-0.0000	0.00000	-175.61493
27.50	0.00000	-133.69016	-0.0000	0.00000	-175.53693
28.00	0.00000	-132.40516	-0.0000	0.00000	-175.45893
28.50	0.00000	-131.10516	-0.0000	0.00000	-175.38093
29.00	0.00000	-129.79016	-0.0000	0.00000	-175.30293
29.50	0.00000	-128.46016	-0.0000	0.00000	-175.22493
30.00	0.00000	-127.11516	-0.0000	0.00000	-175.14693
30.50	0.00000	-125.75516	-0.0000	0.00000	-175.06893
31.00	0.00000	-124.38016	-0.0000	0.00000	-174.99093
31.50	0.00000	-122.99016	-0.0000	0.00000	-174.91293
32.00	0.00000	-121.58516	-0.0000	0.00000	-174.83493
32.50	0.00000	-120.16516	-0.0000	0.00000	-174.75693
33.00	0.00000	-118.73016	-0.0000	0.00000	-174.67893
33.50	0.00000	-117.28016	-0.0000	0.00000	-174.60093
34.00	0.00000	-115.81516	-0.0000	0.00000	-174.52293
34.50	0.00000	-114.33516	-0.0000	0.00000	-174.44493
35.00	0.00000	-112.84016	-0.0000	0.00000	-174.36693
35.50	0.00000	-111.33016	-0.0000	0.00000	-174.28893
36.00	0.00000	-109.80516	-0.0000	0.00000	-174.21093
36.50	0.00000	-108.26516	-0.0000	0.00000	-174.13293
37.00	0.00000	-106.71016	-0.0000	0.00000	-174.05493
37.50	0.00000	-105.14016	-0.0000	0.00000	-173.97693
38.00	0.00000	-103.55516	-0.0000	0.00000	-173.89893
38.50	0.00000	-101.95516	-0.0000	0.00000	-173.82093
39.00	0.00000	-100.34016	-0.0000	0.00000	-173.74293
39.50	0.00000	-98.71016	-0.0000	0.00000	-173.66493
40.00	0.00000	-97.06516	-0.0000	0.00000	-173.58693
40.50	0.00000	-95.40516	-0.0000	0.00000	-173.50893
41.00	0.00000	-93.73016	-0.0000	0.00000	-173.43093
41.50	0.00000	-92.04016	-0.0000	0.00000	-173.35293
42.00	0.00000	-90.33516	-0.0000	0.00000	-173.27493
42.50	0.00000	-88.61516	-0.0000	0.00000	-173.19693
43.00	0.00000	-86.88016	-0.0000	0.00000	-173.11893
43.50	0.00000	-85.13016	-0.0000	0.00000	-173.04093
44.00	0.00000	-83.36516	-0.0000	0.00000	-172.96293
44.50	0.00000	-81.58516	-0.0000	0.00000	-172.88493
45.00	0.00000	-79.79016	-0.0000	0.00000	-172.80693
45.50	0.00000	-77.98016	-0.0000	0.00000	-172.72893
46.00	0.00000	-76.15516	-0.0000	0.00000	-172.65093
46.50	0.00000	-74.31516	-0.0000	0.00000	-172.57293
47.00	0.00000	-72.46016	-0.0000	0.00000	-172.49493
47.50	0.00000	-70.59016	-0.0000	0.00000	-172.41693
48.00	0.00000	-68.70516	-0.0000	0.00000	-172.33893
48.50	0.00000	-66.80516	-0.0000	0.00000	-172.26093
49.00	0.00000	-64.89016	-0.0000	0.00000	-172.18293
49.50	0.00000	-62.96016	-0.0000	0.00000	-172.10493
50.00	0.00000	-61.01516	-0.0000	0.00000	-172.02693
50.50	0.00000	-59.05516	-0.0000	0.00000	-171.94893
51.00	0.00000	-57.08016	-0.0000	0.00000	-171.87093
51.50	0.00000	-55.09016	-0.0000	0.00000	-171.79293
52.00	0.00000	-53.08516	-0.0000	0.00000	-171.71493
52.50	0.00000	-51.06516	-0.0000	0.00000	-171.63693
53.00	0.00000	-49.03016	-0.0000	0.00000	-171.55893
53.50	0.00000	-46.98016	-0.0000	0.00000	-171.48093
54.00	0.00000	-44.91516	-0.0000	0.00000	-171.40293
54.50	0.00000	-42.83516	-0.0000	0.00000	-171.32493
55.00	0.00000	-40.74016	-0.0000	0.00000	-171.24693
55.50	0.00000	-38.63016	-0.0000	0.00000	-171.16893
56.00	0.00000	-36.50516	-0.0000	0.00000	-171.09093
56.50	0.00000	-34.36516	-0.0000	0.00000	-171.01293
57.00	0.00000	-32.21016	-0.0000	0.00000	-170.93493
57.50	0.00000	-30.04016	-0.0000	0.00000	-170.85693
58.00	0.00000	-27.85516	-0.0000	0.00000	-170.77893
58.50	0.00000	-25.65516	-0.0000	0.00000	-170.70093
59.00	0.00000	-23.44016	-0.0000	0.00000	-170.62293
59.50	0.00000	-21.21016	-0.0000	0.00000	-170.54493
60.00	0.00000	-18.96516	-0.0000	0.00000	-170.46693
60.50	0.00000	-16.70516	-0.0000	0.00000	-170.38893
61.00	0.00000	-14.43016	-0.0000	0.00000	-170.31093
61.50	0.00000	-12.14016	-0.0000	0.00000	-170.23293
62.00	0.00000	-9.83516	-0.0000	0.00000	-170.15493
62.50	0.00000	-7.51516	-0.0000	0.00000	-170.07693
63.00	0.00000	-5.18016	-0.0000	0.00000	-170.00000
63.50	0.00000	-2.83016	-0.0000	0.00000	-170.00000
64.00	0.00000	-0.46516	-0.0000	0.00000	-170.00000
64.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
65.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
65.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
66.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
66.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
67.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
67.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
68.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
68.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
69.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
69.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
70.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
70.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
71.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
71.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
72.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
72.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
73.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
73.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
74.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
74.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
75.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
75.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
76.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
76.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
77.00	0.00000	0.00000	-0.0000	0.00000	-170.00000
77.50	0.00000	0.00000	-0.0000	0.00000	-170.00000
78.00	0.00000	0.00000	-0.0000	0.0000	

Table 7. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Very Dry Ground

Very Dry Ground (Relative Dielectric Constant = 3.0, Conductivity = 0.000100 S/m)		FREQUENCY = 15.0 MHz.		FREQUENCY = 30.0 MHz.	
Grazing ang. deg	PS(deg)	Fresnel Reflect Coef [Rv]	Directive gain D(dBi)	Fresnel Reflect Coef [Rv]	Directive gain D(dBi)
0.00	0.00000	1.00000	-999.9999	1.00000	-999.9999
0.50	-179.94652	0.96366	-22.4733	0.96366	-22.4647
1.00	-179.89288	0.92861	-16.6108	0.92861	-16.6023
1.50	-179.83899	0.89489	-13.2454	0.89489	-13.2371
2.00	-179.78468	0.86229	-10.5016	0.86218	-10.4934
2.50	-179.72981	0.83083	-9.1168	0.83069	-9.1088
3.00	-179.67427	0.80043	-7.6853	0.80028	-7.6774
3.50	-179.61789	0.77106	-6.4970	0.77089	-6.4893
4.00	-179.56052	0.74267	-5.4866	0.74248	-5.4790
4.50	-179.50214	0.71528	-4.6336	0.71508	-4.6262
5.00	-179.44214	0.68866	-3.8436	0.68844	-3.8362
5.50	-179.38178	0.66308	-3.1178	0.66283	-3.1108
6.00	-179.32054	0.63846	-2.4506	0.63763	-2.4436
6.50	-179.25854	0.61478	-1.8412	0.61353	-1.8342
7.00	-179.19584	0.59206	-1.2807	0.59038	-1.2737
7.50	-179.13244	0.57034	-0.7691	0.56823	-0.7621
8.00	-179.06834	0.54956	-0.2964	0.54696	-0.2894
8.50	-179.00354	0.52966	0.1474	0.52666	0.1404
9.00	-178.93804	0.51066	0.5807	0.50766	0.5737
9.50	-178.87184	0.49256	1.0074	0.48956	1.0004
10.00	-178.80504	0.47536	1.4287	0.47236	1.4217
10.50	-178.73764	0.45906	1.8447	0.45606	1.8377
11.00	-178.66964	0.44366	2.2557	0.44066	2.2487
11.50	-178.60104	0.42906	2.6617	0.42606	2.6547
12.00	-178.53184	0.41526	3.0627	0.41226	3.0557
12.50	-178.46204	0.40226	3.4587	0.39926	3.4517
13.00	-178.39164	0.38996	3.8507	0.38696	3.8437
13.50	-178.32064	0.37836	4.2387	0.37536	4.2317
14.00	-178.24904	0.36746	4.6227	0.36446	4.6157
14.50	-178.17684	0.35726	5.0027	0.35426	5.0000
15.00	-178.10404	0.34776	5.3787	0.34476	5.3717
15.50	-178.03064	0.33896	5.7507	0.33596	5.7437
16.00	-177.95664	0.33076	6.1187	0.32776	6.1117
16.50	-177.88204	0.32316	6.4827	0.32016	6.4757
17.00	-177.80684	0.31616	6.8427	0.31316	6.8357
17.50	-177.73104	0.30976	7.1987	0.30676	7.1917
18.00	-177.65464	0.30396	7.5507	0.30096	7.5437
18.50	-177.57764	0.29876	7.8987	0.29576	7.8917
19.00	-177.49994	0.29416	8.2427	0.29116	8.2357
19.50	-177.42154	0.28996	8.5827	0.28696	8.5757
20.00	-177.34254	0.28626	8.9187	0.28326	8.9117
20.50	-177.26294	0.28306	9.2507	0.28006	9.2437
21.00	-177.18274	0.28026	9.5787	0.27726	9.5717
21.50	-177.10194	0.27786	9.9027	0.27486	9.8957
22.00	-177.02054	0.27576	10.2227	0.27276	10.2157
22.50	-176.93854	0.27396	10.5387	0.27096	10.5317
23.00	-176.85594	0.27236	10.8507	0.26936	10.8437
23.50	-176.77274	0.27096	11.1587	0.26796	11.1517
24.00	-176.68894	0.26976	11.4627	0.26676	11.4557
24.50	-176.60454	0.26876	11.7627	0.26576	11.7557
25.00	-176.51954	0.26796	12.0587	0.26496	12.0517
25.50	-176.43394	0.26736	12.3507	0.26436	12.3437
26.00	-176.34774	0.26696	12.6387	0.26396	12.6317
26.50	-176.26094	0.26676	12.9227	0.26376	12.9157
27.00	-176.17354	0.26676	13.2027	0.26376	13.1957
27.50	-176.08554	0.26696	13.4787	0.26396	13.4717
28.00	-176.00000	0.26736	13.7507	0.26436	13.7437
28.50	-175.91294	0.26796	14.0187	0.26496	14.0117
29.00	-175.82434	0.26876	14.2827	0.26576	14.2757
29.50	-175.73424	0.26976	14.5427	0.26676	14.5357
30.00	-175.64264	0.27096	14.7987	0.26796	14.7917
30.50	-175.54954	0.27236	15.0507	0.26936	15.0437
31.00	-175.45494	0.27396	15.2987	0.27096	15.2917
31.50	-175.35884	0.27576	15.5427	0.27276	15.5357
32.00	-175.26124	0.27786	15.7827	0.27486	15.7757
32.50	-175.16214	0.28026	16.0187	0.27726	16.0117
33.00	-175.06154	0.28296	16.2507	0.27996	16.2437
33.50	-174.95944	0.28596	16.4787	0.28296	16.4717
34.00	-174.85584	0.28926	16.7027	0.28626	16.6957
34.50	-174.75074	0.29276	16.9227	0.28976	16.9157
35.00	-174.64414	0.29646	17.1387	0.29346	17.1317
35.50	-174.53604	0.30036	17.3507	0.29736	17.3437
36.00	-174.42644	0.30446	17.5587	0.30146	17.5517
36.50	-174.31534	0.30876	17.7627	0.30576	17.7557
37.00	-174.20274	0.31326	17.9627	0.31026	17.9557
37.50	-174.08864	0.31796	18.1587	0.31496	18.1517
38.00	-173.97304	0.32286	18.3507	0.31986	18.3437
38.50	-173.85594	0.32796	18.5387	0.32496	18.5317
39.00	-173.73734	0.33326	18.7227	0.33026	18.7157
39.50	-173.61724	0.33876	18.9027	0.33576	18.8957
40.00	-173.49564	0.34446	19.0787	0.34146	19.0717
40.50	-173.37254	0.35036	19.2507	0.34736	19.2437
41.00	-173.24794	0.35646	19.4187	0.35346	19.4117
41.50	-173.12184	0.36276	19.5827	0.35976	19.5757
42.00	-172.99424	0.36926	19.7427	0.36626	19.7357
42.50	-172.86514	0.37596	19.8987	0.37296	19.8917
43.00	-172.73454	0.38286	20.0507	0.37986	20.0437
43.50	-172.60244	0.38996	20.1987	0.38696	20.1917
44.00	-172.46884	0.39726	20.3427	0.39426	20.3357
44.50	-172.33374	0.40476	20.4827	0.40176	20.4757
45.00	-172.19714	0.41246	20.6187	0.40946	20.6117
45.50	-172.05904	0.42036	20.7507	0.41736	20.7437
46.00	-171.91944	0.42846	20.8787	0.42546	20.8717
46.50	-171.77834	0.43676	21.0027	0.43376	21.0000
47.00	-171.63574	0.44526	21.1227	0.44226	21.1200
47.50	-171.49164	0.45396	21.2387	0.45096	21.2350
48.00	-171.34604	0.46286	21.3507	0.45986	21.3450
48.50	-171.19894	0.47196	21.4587	0.46896	21.4510
49.00	-171.05034	0.48126	21.5627	0.47826	21.5550
49.50	-170.90024	0.49076	21.6627	0.48776	21.6550
50.00	-170.74864	0.50046	21.7587	0.49746	21.7510

Table 8. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Pure Water, 20°C

CASE (7): Pure Water, 20 deg C		FREQUENCY = 6.0 MHz		FREQUENCY = 15.0 MHz		FREQUENCY = 30.0 MHz	
Grating		COND = 0.000007 S/m		COND = 0.000500 S/m		COND = 0.001700 S/m	
ang. deg	PSI(deg)	[Rv]	ang Rv(deg)	Directive gain	[Rv]	ang Rv(deg)	Directive gain
0.00	0.00	1.00000	180.00000	-999.99999	1.00000	180.00000	-999.99999
0.50	0.50	0.95435	-179.99883	-15.0628	0.85435	-179.94301	-15.0626
1.00	1.00	0.72848	-179.99762	-9.6541	0.72848	-179.88383	-9.6539
1.50	1.50	0.61863	-179.99631	-6.7048	0.61863	-179.82038	-6.7046
2.00	2.00	0.52193	-179.99483	-4.7442	0.52193	-179.74884	-4.7440
2.50	2.50	0.43616	-179.99310	-3.3139	0.43616	-179.66524	-3.3137
3.00	3.00	0.35957	-179.99101	-2.2112	0.35957	-179.56335	-2.2110
3.50	3.50	0.29077	-179.98831	-1.3291	0.29077	-179.43227	-1.3290
4.00	4.00	0.22862	-179.98460	-0.6045	0.22862	-179.25249	-0.6044
4.50	4.50	0.17081	-179.96973	0.5199	0.17083	-178.52922	0.5200
5.00	5.00	0.12082	-179.96973	1.3544	0.12083	-177.52922	1.3545
5.50	5.50	0.08052	-179.87846	1.9984	0.08056	-174.11409	1.9985
6.00	6.00	0.04617	-0.08025	2.5092	0.04621	-3.89019	2.5093
6.50	6.50	0.11210	-0.03271	2.9227	0.11216	-1.58797	2.9228
7.00	7.00	0.16937	-0.02130	3.2624	0.16942	-1.03415	3.2625
7.50	7.50	0.21956	-0.01610	3.5624	0.21960	-0.78182	3.5625
8.00	8.00	0.26334	-0.01112	3.7806	0.26337	-0.53936	3.7806
8.50	8.50	0.30041	-0.00655	4.1484	0.30044	-0.28228	4.1484
9.00	9.00	0.43524	-0.00116	4.4059	0.43527	-0.34765	4.4060
9.50	9.50	0.47066	-0.00814	4.5868	0.47068	-0.29839	4.5869
10.00	10.00	0.50934	-0.00341	4.7068	0.50936	-0.26255	4.7068
10.50	10.50	0.54220	-0.00484	4.7775	0.54222	-0.23520	4.7775
11.00	11.00	0.57055	-0.00440	4.8069	0.57057	-0.21360	4.8069
11.50	11.50	0.59521	-0.00404	4.8009	0.59523	-0.19609	4.8009
12.00	12.00	0.61684	-0.00374	4.7633	0.61685	-0.18160	4.7633
12.50	12.50	0.63592	-0.00349	4.6971	0.63594	-0.16942	4.6971
13.00	13.00	0.65285	-0.00327	4.6044	0.65287	-0.15904	4.6044
13.50	13.50	0.66795	-0.00309	4.4865	0.66797	-0.15010	4.4864
14.00	14.00	0.68148	-0.00293	4.3443	0.68149	-0.14334	4.3443
14.50	14.50	0.69363	-0.00279	4.1783	0.69365	-0.13553	4.1783
15.00	15.00	0.70460	-0.00267	3.9888	0.70461	-0.12954	3.9888
15.50	15.50	0.71452	-0.00256	3.7756	0.71453	-0.12422	3.7756
16.00	16.00	0.72350	-0.00246	3.5383	0.72352	-0.11949	3.5382
16.50	16.50	0.73167	-0.00237	3.2760	0.73168	-0.11527	3.2760
17.00	17.00	0.73909	-0.00230	2.9880	0.73911	-0.11148	2.9880
17.50	17.50	0.74585	-0.00223	2.6728	0.74587	-0.10807	2.6728
18.00	18.00	0.75201	-0.00216	2.3289	0.75203	-0.10500	2.3289
18.50	18.50	0.75763	-0.00211	1.9542	0.75764	-0.10223	1.9542
19.00	19.00	0.76274	-0.00205	1.5463	0.76276	-0.09974	1.5463
19.50	19.50	0.76740	-0.00201	1.1022	0.76741	-0.09748	1.1022
20.00	20.00	0.77164	-0.00197	0.6182	0.77165	-0.09544	0.6182
20.50	20.50	0.77549	-0.00193	0.0898	0.77550	-0.09361	0.0898
21.00	21.00	0.77897	-0.00189	-0.4884	0.77899	-0.09195	-0.4885
21.50	21.50	0.78212	-0.00186	-1.1237	0.78213	-0.09047	-1.1237
22.00	22.00	0.78495	-0.00184	-1.8246	0.78496	-0.08914	-1.8247
22.50	22.50	0.78749	-0.00181	-2.6028	0.78749	-0.08796	-2.6029
23.00	23.00	0.78973	-0.00179	-3.4733	0.78974	-0.08692	-3.4733
23.50	23.50	0.79171	-0.00177	-4.4566	0.79172	-0.08600	-4.4566
24.00	24.00	0.79343	-0.00175	-5.5815	0.79344	-0.08520	-5.5816
24.50	24.50	0.79490	-0.00174	-6.8903	0.79491	-0.08453	-6.8904
25.00	25.00	0.79613	-0.00173	-8.4485	0.79614	-0.08396	-8.4486
25.50	25.50	0.79713	-0.00172	-10.3660	0.79714	-0.08350	-10.3661
26.00	26.00	0.79790	-0.00171	-12.6487	0.79791	-0.08315	-12.6488
26.50	26.50	0.79844	-0.00171	-16.3591	0.79845	-0.08290	-16.3591
27.00	27.00	0.79877	-0.00170	-22.3728	0.79878	-0.08275	-22.3728
27.50	27.50	0.79888	-0.00170	-140.4163	0.79889	-0.08272	-140.4163

Table 9. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Ice (Fresh Water, -1°C)

CASE (B):		ICE (Fresh water, -1.0 deg C)				(Relative Dielectric Constant = 3.0. Conductivity is defined below)			
Grazing ang. deg		FREQUENCY = 6.0 MHz.		FREQUENCY = 15.0 MHz.		FREQUENCY = 30.0 MHz.		FREQUENCY = 30.0 MHz.	
Psi(deg)		COND = 0.000070 S/m		COND = 0.00090 S/m		COND = 0.000100 S/m		COND = 0.000100 S/m	
		Fresnel Reflect Coef		Fresnel Reflect Coef		Fresnel Reflect Coef		Fresnel Reflect Coef	
		Directive gain		Directive gain		Directive gain		Directive gain	
		[Rv]		[Rv]		[Rv]		[Rv]	
		and Rv(deg)		and Rv(deg)		and Rv(deg)		and Rv(deg)	
0.00	1.00000	180.00000	-999.99999	1.00000	180.00000	-999.99999	1.00000	180.00000	-999.99999
0.50	0.96367	-179.96271	-22.4688	0.96365	-179.98088	-22.4655	0.96385	-179.98940	-22.4647
1.00	0.92864	-179.92532	-16.6063	0.92861	-179.96172	-16.6031	0.92861	-179.97874	-16.6023
1.50	0.89485	-179.88776	-13.2411	0.89481	-179.94246	-13.2379	0.89480	-179.96805	-13.2371
2.00	0.86224	-179.84990	-10.8973	0.86219	-179.92303	-10.8942	0.86218	-179.95728	-10.8934
2.50	0.83076	-179.81165	-9.1126	0.83070	-179.90343	-9.1096	0.83069	-179.94640	-9.1088
3.00	0.80035	-179.77292	-7.6811	0.80029	-179.88356	-7.6781	0.80028	-179.93536	-7.6774
3.50	0.77097	-179.73360	-6.4930	0.77090	-179.86340	-6.4900	0.77089	-179.92416	-6.4893
4.00	0.74257	-179.69359	-5.5826	0.74250	-179.84286	-5.4797	0.74248	-179.91277	-5.4790
4.50	0.71514	-179.65294	-4.8682	0.71506	-179.82195	-4.8689	0.71504	-179.89928	-4.8682
5.00	0.68854	-179.61104	-4.3097	0.68846	-179.80054	-4.3069	0.68844	-179.88628	-4.3062
5.50	0.66279	-179.56834	-3.8997	0.66270	-179.77866	-3.8969	0.66267	-179.87271	-3.8962
6.00	0.63795	-179.52434	-3.5906	0.63786	-179.75636	-3.5878	0.63783	-179.86458	-3.5871
6.50	0.61395	-179.47934	-3.3590	0.61385	-179.73366	-3.3562	0.61382	-179.85181	-3.3555
7.00	0.59081	-179.43345	-3.1845	0.59071	-179.71052	-3.1817	0.59068	-179.83841	-3.1810
7.50	0.56845	-179.38618	-3.0545	0.56834	-179.68698	-3.0517	0.56831	-179.82501	-3.0510
8.00	0.54671	-179.33792	-2.9645	0.54658	-179.66340	-2.9617	0.54655	-179.81169	-2.9610
8.50	0.52557	-179.28823	-2.9045	0.52546	-179.63872	-2.9017	0.52543	-179.79829	-2.9010
9.00	0.50491	-179.23754	-2.8645	0.50480	-179.61400	-2.8617	0.50477	-179.78489	-2.8610
9.50	0.48471	-179.18529	-2.8445	0.48460	-179.58980	-2.8417	0.48457	-179.77149	-2.8410
10.00	0.46495	-179.13199	-2.8345	0.46484	-179.56510	-2.8317	0.46481	-179.75809	-2.8310
10.50	0.44561	-179.07714	-2.8345	0.44550	-179.54000	-2.8289	0.44547	-179.74469	-2.8282
11.00	0.42667	-179.02129	-2.8445	0.42656	-179.51450	-2.8289	0.42653	-179.73129	-2.8282
11.50	0.40813	-178.96394	-2.8645	0.40802	-179.48860	-2.8289	0.40799	-179.71789	-2.8282
12.00	0.39000	-178.90559	-2.8945	0.38989	-179.46230	-2.8289	0.38986	-179.70449	-2.8282
12.50	0.37226	-178.84674	-2.9345	0.37215	-179.43560	-2.8289	0.37212	-179.69109	-2.8282
13.00	0.35491	-178.78679	-2.9845	0.35480	-179.40850	-2.8289	0.35477	-179.67769	-2.8282
13.50	0.33795	-178.72594	-3.0545	0.33786	-179.38100	-2.8289	0.33783	-179.66429	-2.8282
14.00	0.32149	-178.66459	-3.1445	0.32138	-179.35310	-2.8289	0.32135	-179.65089	-2.8282
14.50	0.30553	-178.60214	-3.2445	0.30542	-179.32480	-2.8289	0.30539	-179.63749	-2.8282
15.00	0.29007	-178.53879	-3.3545	0.29000	-179.29610	-2.8289	0.28997	-179.62409	-2.8282
15.50	0.27511	-178.47494	-3.4745	0.27500	-179.26700	-2.8289	0.27497	-179.61069	-2.8282
16.00	0.26065	-178.41059	-3.5945	0.26054	-179.23750	-2.8289	0.26051	-179.59729	-2.8282
16.50	0.24669	-178.34574	-3.7245	0.24658	-179.20760	-2.8289	0.24655	-179.58389	-2.8282
17.00	0.23323	-178.28039	-3.8645	0.23312	-179.17730	-2.8289	0.23309	-179.57049	-2.8282
17.50	0.22027	-178.21454	-4.0145	0.22016	-179.14660	-2.8289	0.22013	-179.55709	-2.8282
18.00	0.20781	-178.14819	-4.1745	0.20770	-179.11550	-2.8289	0.20767	-179.54369	-2.8282
18.50	0.19585	-178.08134	-4.3445	0.19574	-179.08400	-2.8289	0.19571	-179.53029	-2.8282
19.00	0.18439	-178.01409	-4.5245	0.18428	-179.05210	-2.8289	0.18425	-179.51689	-2.8282
19.50	0.17343	-177.94634	-4.7145	0.17332	-179.02000	-2.8289	0.17329	-179.50349	-2.8282
20.00	0.16297	-177.87809	-4.9145	0.16286	-178.98760	-2.8289	0.16283	-179.49009	-2.8282
20.50	0.15291	-177.80934	-5.1245	0.15280	-178.95490	-2.8289	0.15277	-179.47669	-2.8282
21.00	0.14335	-177.74009	-5.3445	0.14324	-178.92190	-2.8289	0.14321	-179.46329	-2.8282
21.50	0.13429	-177.67034	-5.5745	0.13418	-178.88860	-2.8289	0.13415	-179.44989	-2.8282
22.00	0.12573	-177.60059	-5.8145	0.12562	-178.85510	-2.8289	0.12559	-179.43649	-2.8282
22.50	0.11767	-177.53084	-6.0645	0.11756	-178.82130	-2.8289	0.11753	-179.42309	-2.8282
23.00	0.11011	-177.46059	-6.3245	0.11000	-178.78720	-2.8289	0.10997	-179.40969	-2.8282
23.50	0.10305	-177.39034	-6.5945	0.10294	-178.75280	-2.8289	0.10291	-179.39629	-2.8282
24.00	0.09649	-177.32009	-6.8745	0.09638	-178.71810	-2.8289	0.09635	-179.38289	-2.8282
24.50	0.09043	-177.24974	-7.1645	0.09032	-178.68310	-2.8289	0.09029	-179.36949	-2.8282
25.00	0.08487	-177.17949	-7.4645	0.08476	-178.64780	-2.8289	0.08473	-179.35609	-2.8282
25.50	0.07981	-177.10924	-7.7745	0.07970	-178.61220	-2.8289	0.07967	-179.34269	-2.8282
26.00	0.07525	-177.03899	-8.0845	0.07514	-178.57630	-2.8289	0.07511	-179.32929	-2.8282
26.50	0.07119	-176.96874	-8.4045	0.07108	-178.54010	-2.8289	0.07105	-179.31589	-2.8282
27.00	0.06763	-176.89849	-8.7245	0.06752	-178.50360	-2.8289	0.06749	-179.30249	-2.8282
27.50	0.06457	-176.82824	-9.0545	0.06446	-178.46680	-2.8289	0.06443	-179.28909	-2.8282
28.00	0.06201	-176.75799	-9.3945	0.06190	-178.42970	-2.8289	0.06187	-179.27569	-2.8282
28.50	0.05995	-176.68774	-9.7445	0.05984	-178.39230	-2.8289	0.05981	-179.26229	-2.8282
29.00	0.05839	-176.61749	-10.0945	0.05828	-178.35470	-2.8289	0.05825	-179.24889	-2.8282
29.50	0.05733	-176.54724	-10.4545	0.05722	-178.31680	-2.8289	0.05719	-179.23549	-2.8282
30.00	0.05677	-176.47699	-10.8245	0.05666	-178.27860	-2.8289	0.05663	-179.22209	-2.8282
30.50	0.05715	-176.40674	-11.1945	0.05704	-178.24010	-2.8289	0.05701	-179.20869	-2.8282
31.00	0.05809	-176.33649	-11.5745	0.05798	-178.20130	-2.8289	0.05795	-179.19529	-2.8282
31.50	0.05953	-176.26624	-11.9645	0.05942	-178.16230	-2.8289	0.05939	-179.18189	-2.8282
32.00	0.06147	-176.19599	-12.3645	0.06136	-178.12300	-2.8289	0.06133	-179.16849	-2.8282
32.50	0.06391	-176.12574	-12.7745	0.06380	-178.08340	-2.8289	0.06377	-179.15509	-2.8282
33.00	0.06685	-176.05549	-13.1945	0.06674	-178.04360	-2.8289	0.06671	-179.14169	-2.8282
33.50	0.07029	-175.98524	-13.6245	0.07018	-178.00360	-2.8289	0.07015	-179.12829	-2.8282
34.00	0.07423	-175.91499	-14.0645	0.07412	-177.96330	-2.8289	0.07409	-179.11489	-2.8282
34.50	0.07867	-175.84474	-14.5145	0.07856	-177.92280	-2.8289	0.07853	-179.10149	-2.8282
35.00	0.08361	-175.77449	-14.9745	0.08350	-177.88210	-2.8289	0.08347	-179.08809	-2.8282
35.50	0.08905	-175.70424	-15.4445	0.08894	-177.84120	-2.8289	0.08891	-179.07469	-2.8282
36.00	0.09500	-175.63399	-15.9245	0.09489	-177.80000	-2.8289	0.09486	-179.06129	-2.8282
36.50	0.10144	-175.56374	-16.4145	0.10133	-177.75880	-2.8289	0.10130	-179.04789	-2.8282
37.00	0.10838	-175.49349	-16.9145	0.10827	-177.71730	-2.8289	0.10824	-179.03449	-2.8282
37.50	0.11582	-175.42324	-17.4245	0.11571	-177.67560	-2.8289	0.11568	-179.02109	-2.8282
38.00	0.12376	-175.35299	-17.9445	0.12365	-177.63370	-2.8289	0.12362	-179.00769	-2.8282
38.50	0.13220	-175.28274	-18.4745	0.13209	-177.59160	-2.8289	0.13206	-178.99429	-2.8282
39.00	0.14114	-175.21249	-19.0145	0.14103	-177.54930	-2.8289	0.14100	-178.98089	-2.8282
39.50	0.15058	-175.14224	-19.5645	0.15047	-177.50680	-2.8289	0.15044	-178.96749	-2.8282
40.00	0.16052	-175.07199	-20.1245	0.16041	-177.46410	-2.8289	0.16038	-178.95409	-2.8282
40.50	0.17096	-175.00174	-20.6945	0.17085	-177.42120	-2.8289	0.17082	-178.94069	-2.8282
41.00	0.18190	-174.93149	-21.2745	0.18179	-177.37810	-2.8289	0.18176	-178.92729	-2.8282
41.50	0.19334	-174.86124	-21.8645	0.19323	-177.33480	-2.8289	0.19320	-178.91389	-2.8282
42.00	0.20528	-174.79099	-22.4645	0.20517	-177.29130	-2.8289	0.20514	-178.90049	-2.82

Table 10. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Ice (Fresh Water, -10°C)

CASE(9): ICE (fresh water, -10.0 deg C)		Relative Dielectric Constant = 3.0. Conductivity is defined below)			
		FREQUENCY = 15.0 Mhz.		FREQUENCY = 30.0 Mhz.	
		COND = 0.00020 S/m		COND = 0.00035 S/m	
Grazing ang. deg	PST(deg)	[Rv]	ang Rv(deg)	[Rv]	ang Rv(deg)
0.00		1.00000	180.00000	1.00000	180.00000
0.50		0.96365	-179.98940	0.96365	-179.99629
1.00		0.92861	-179.97874	0.92861	-179.99257
1.50		0.89480	-179.96805	0.89480	-179.98883
2.00		0.86218	-179.95728	0.86218	-179.98505
2.50		0.83069	-179.94640	0.83069	-179.98125
3.00		0.80028	-179.93536	0.80027	-179.97739
3.50		0.77089	-179.92416	0.77088	-179.97346
4.00		0.74248	-179.91277	0.74247	-179.96948
5.00		0.68844	-179.88928	0.68843	-179.96490
6.00		0.63783	-179.86458	0.63782	-179.95962
7.00		0.59038	-179.83841	0.59037	-179.95345
8.00		0.54583	-179.81039	0.54582	-179.94645
9.00		0.50396	-179.78020	0.50395	-179.93865
10.00		0.46455	-179.74736	0.46454	-179.93010
12.00		0.39243	-179.67151	0.39241	-179.91159
14.00		0.32817	-179.57721	0.32816	-179.88506
		0.27073	-179.45573	0.27072	-179.85205
		0.21923	-179.29282	0.21921	-179.81252
		0.17289	-179.06290	0.17288	-179.75204
		0.13112	-178.71512	0.13112	-179.55029
		0.09338	-178.39066	0.09338	-179.34578
		0.05921	-178.35573	0.05920	-179.93414
		0.02823	-178.49508	0.02820	-177.69432
		0.00180	-89.89668	0.00117	-89.93313
		0.02572	-4.11866	0.02568	-2.67239
		0.04907	-2.21075	0.04905	-1.43338
		0.07040	-1.57657	0.07038	-1.02304
		0.08989	-1.26164	0.08987	-0.81784
		0.10771	-1.07448	0.10769	-0.69650
		0.12401	-0.95119	0.12400	-0.61657
		0.13894	-0.86438	0.13892	-0.56029
		0.15261	-0.80033	0.15259	-0.51878
		0.16513	-0.75145	0.16511	-0.48709
		0.17659	-0.71314	0.17658	-0.46226
		0.18709	-0.68252	0.18708	-0.44241
		0.19670	-0.65764	0.19669	-0.42628
		0.20548	-0.63716	0.20547	-0.41300
		0.21350	-0.62012	0.21349	-0.40196
		0.22081	-0.60584	0.22080	-0.39270
		0.22746	-0.59378	0.22745	-0.38488
		0.23349	-0.58355	0.23348	-0.37825
		0.23894	-0.57484	0.23893	-0.37261
		0.24384	-0.56741	0.24383	-0.36779
		0.24823	-0.56108	0.24821	-0.36369
		0.25212	-0.55569	0.25211	-0.36019
		0.25555	-0.55112	0.25553	-0.35723
		0.25853	-0.54727	0.25851	-0.35474
		0.26108	-0.54407	0.26106	-0.35267
		0.26321	-0.54146	0.26319	-0.35097
		0.26494	-0.53938	0.26492	-0.34963
		0.26627	-0.53781	0.26626	-0.34860
		0.26729	-0.53670	0.26721	-0.34785
		0.26779	-0.53604	0.26777	-0.34746
		0.26798	-0.53582	0.26796	-0.34732
		0.26799	-138.7999	0.26799	-138.8000

Table 11. Fresnel Reflection Coefficient for Parallel (Vertical) Polarization, Average Land (TCI)

Grazing ang. deg psi(deg)	Average Land (TCI) (Relative Dielectric Constant = 10.0, Conductivity = 0.005000 S/m)				FREQUENCY = 10.0 MHz.				FREQUENCY = 30.0 MHz.			
	Fresnel Reflect Coef		Directive gain	Directive gain	Fresnel Reflect Coef		Directive gain	Directive gain	Fresnel Reflect Coef		Directive gain	Directive gain
	[Rv]	ang Rv(deg)			[Rv]	ang Rv(deg)			[Rv]	ang Rv(deg)		
0.00	1.00000	180.00000	-999.9999	0.00000	1.00000	180.00000	-999.9999	0.00000	1.00000	180.00000	-999.9999	0.00000
0.50	0.93505	-179.05515	-15.4723	0.04173	0.93505	-179.05515	-15.4723	0.04173	0.93505	-179.05515	-15.4723	0.04173
1.00	0.87429	-176.09825	-12.7329	0.08879	0.87429	-176.09825	-12.7329	0.08879	0.87429	-176.09825	-12.7329	0.08879
1.50	0.81743	-174.11749	-10.4802	0.13492	0.81743	-174.11749	-10.4802	0.13492	0.81743	-174.11749	-10.4802	0.13492
2.00	0.76421	-172.10227	-8.2507	0.17980	0.76421	-172.10227	-8.2507	0.17980	0.76421	-172.10227	-8.2507	0.17980
2.50	0.71440	-170.03292	-6.5860	0.22308	0.71440	-170.03292	-6.5860	0.22308	0.71440	-170.03292	-6.5860	0.22308
3.00	0.66777	-167.90511	-5.2614	0.26484	0.66777	-167.90511	-5.2614	0.26484	0.66777	-167.90511	-5.2614	0.26484
3.50	0.62414	-165.69302	-4.1702	0.30454	0.62414	-165.69302	-4.1702	0.30454	0.62414	-165.69302	-4.1702	0.30454
4.00	0.58333	-163.40370	-3.2659	0.34163	0.58333	-163.40370	-3.2659	0.34163	0.58333	-163.40370	-3.2659	0.34163
4.50	0.54526	-161.03757	-2.5008	0.37654	0.54526	-161.03757	-2.5008	0.37654	0.54526	-161.03757	-2.5008	0.37654
5.00	0.51001	-158.59544	-1.8608	0.40980	0.51001	-158.59544	-1.8608	0.40980	0.51001	-158.59544	-1.8608	0.40980
5.50	0.47764	-156.07824	-1.3307	0.44163	0.47764	-156.07824	-1.3307	0.44163	0.47764	-156.07824	-1.3307	0.44163
6.00	0.44807	-153.48625	-0.8959	0.47229	0.44807	-153.48625	-0.8959	0.47229	0.44807	-153.48625	-0.8959	0.47229
6.50	0.42129	-150.82044	-0.5408	0.50200	0.42129	-150.82044	-0.5408	0.50200	0.42129	-150.82044	-0.5408	0.50200
7.00	0.39724	-148.08224	-0.3307	0.53000	0.39724	-148.08224	-0.3307	0.53000	0.39724	-148.08224	-0.3307	0.53000
7.50	0.37587	-145.27244	-0.1644	0.55654	0.37587	-145.27244	-0.1644	0.55654	0.37587	-145.27244	-0.1644	0.55654
8.00	0.35700	-142.39224	-0.0307	0.58163	0.35700	-142.39224	-0.0307	0.58163	0.35700	-142.39224	-0.0307	0.58163
8.50	0.34054	-139.44224	0.0644	0.60544	0.34054	-139.44224	0.0644	0.60544	0.34054	-139.44224	0.0644	0.60544
9.00	0.32629	-136.42224	0.1800	0.62800	0.32629	-136.42224	0.1800	0.62800	0.32629	-136.42224	0.1800	0.62800
9.50	0.31400	-133.34224	0.2900	0.64944	0.31400	-133.34224	0.2900	0.64944	0.31400	-133.34224	0.2900	0.64944
10.00	0.30354	-130.20224	0.3900	0.66980	0.30354	-130.20224	0.3900	0.66980	0.30354	-130.20224	0.3900	0.66980
10.50	0.29464	-127.00224	0.4800	0.68900	0.29464	-127.00224	0.4800	0.68900	0.29464	-127.00224	0.4800	0.68900
11.00	0.28724	-123.84224	0.5600	0.70720	0.28724	-123.84224	0.5600	0.70720	0.28724	-123.84224	0.5600	0.70720
11.50	0.28129	-120.72224	0.6300	0.72444	0.28129	-120.72224	0.6300	0.72444	0.28129	-120.72224	0.6300	0.72444
12.00	0.27654	-117.64224	0.6900	0.74080	0.27654	-117.64224	0.6900	0.74080	0.27654	-117.64224	0.6900	0.74080
12.50	0.27280	-114.60224	0.7400	0.75644	0.27280	-114.60224	0.7400	0.75644	0.27280	-114.60224	0.7400	0.75644
13.00	0.26994	-111.60224	0.7800	0.77144	0.26994	-111.60224	0.7800	0.77144	0.26994	-111.60224	0.7800	0.77144
13.50	0.26780	-108.64224	0.8100	0.78580	0.26780	-108.64224	0.8100	0.78580	0.26780	-108.64224	0.8100	0.78580
14.00	0.26629	-105.72224	0.8300	0.79980	0.26629	-105.72224	0.8300	0.79980	0.26629	-105.72224	0.8300	0.79980
14.50	0.26534	-102.84224	0.8400	0.81344	0.26534	-102.84224	0.8400	0.81344	0.26534	-102.84224	0.8400	0.81344
15.00	0.26494	-100.00224	0.8500	0.82680	0.26494	-100.00224	0.8500	0.82680	0.26494	-100.00224	0.8500	0.82680
15.50	0.26494	-97.20224	0.8600	0.83980	0.26494	-97.20224	0.8600	0.83980	0.26494	-97.20224	0.8600	0.83980
16.00	0.26534	-94.44224	0.8700	0.85244	0.26534	-94.44224	0.8700	0.85244	0.26534	-94.44224	0.8700	0.85244
16.50	0.26629	-91.72224	0.8800	0.86480	0.26629	-91.72224	0.8800	0.86480	0.26629	-91.72224	0.8800	0.86480
17.00	0.26764	-89.04224	0.8900	0.87680	0.26764	-89.04224	0.8900	0.87680	0.26764	-89.04224	0.8900	0.87680
17.50	0.26934	-86.40224	0.9000	0.88844	0.26934	-86.40224	0.9000	0.88844	0.26934	-86.40224	0.9000	0.88844
18.00	0.27134	-83.80224	0.9100	0.89980	0.27134	-83.80224	0.9100	0.89980	0.27134	-83.80224	0.9100	0.89980
18.50	0.27364	-81.24224	0.9200	0.91080	0.27364	-81.24224	0.9200	0.91080	0.27364	-81.24224	0.9200	0.91080
19.00	0.27624	-78.72224	0.9300	0.92144	0.27624	-78.72224	0.9300	0.92144	0.27624	-78.72224	0.9300	0.92144
19.50	0.27900	-76.24224	0.9400	0.93180	0.27900	-76.24224	0.9400	0.93180	0.27900	-76.24224	0.9400	0.93180
20.00	0.28194	-73.80224	0.9500	0.94180	0.28194	-73.80224	0.9500	0.94180	0.28194	-73.80224	0.9500	0.94180
20.50	0.28500	-71.40224	0.9600	0.95144	0.28500	-71.40224	0.9600	0.95144	0.28500	-71.40224	0.9600	0.95144
21.00	0.28814	-69.04224	0.9700	0.96080	0.28814	-69.04224	0.9700	0.96080	0.28814	-69.04224	0.9700	0.96080
21.50	0.29134	-66.72224	0.9800	0.96980	0.29134	-66.72224	0.9800	0.96980	0.29134	-66.72224	0.9800	0.96980
22.00	0.29464	-64.44224	0.9900	0.97844	0.29464	-64.44224	0.9900	0.97844	0.29464	-64.44224	0.9900	0.97844
22.50	0.29800	-62.20224	1.0000	0.98680	0.29800	-62.20224	1.0000	0.98680	0.29800	-62.20224	1.0000	0.98680
23.00	0.30144	-60.00224	1.0100	0.99480	0.30144	-60.00224	1.0100	0.99480	0.30144	-60.00224	1.0100	0.99480
23.50	0.30494	-57.84224	1.0200	1.00244	0.30494	-57.84224	1.0200	1.00244	0.30494	-57.84224	1.0200	1.00244
24.00	0.30854	-55.72224	1.0300	1.00980	0.30854	-55.72224	1.0300	1.00980	0.30854	-55.72224	1.0300	1.00980
24.50	0.31224	-53.64224	1.0400	1.01680	0.31224	-53.64224	1.0400	1.01680	0.31224	-53.64224	1.0400	1.01680
25.00	0.31600	-51.60224	1.0500	1.02344	0.31600	-51.60224	1.0500	1.02344	0.31600	-51.60224	1.0500	1.02344
25.50	0.31984	-49.60224	1.0600	1.02980	0.31984	-49.60224	1.0600	1.02980	0.31984	-49.60224	1.0600	1.02980
26.00	0.32374	-47.64224	1.0700	1.03580	0.32374	-47.64224	1.0700	1.03580	0.32374	-47.64224	1.0700	1.03580
26.50	0.32774	-45.72224	1.0800	1.04144	0.32774	-45.72224	1.0800	1.04144	0.32774	-45.72224	1.0800	1.04144
27.00	0.33184	-43.84224	1.0900	1.04680	0.33184	-43.84224	1.0900	1.04680	0.33184	-43.84224	1.0900	1.04680
27.50	0.33600	-42.00224	1.1000	1.05180	0.33600	-42.00224	1.1000	1.05180	0.33600	-42.00224	1.1000	1.05180
28.00	0.34024	-40.20224	1.1100	1.05644	0.34024	-40.20224	1.1100	1.05644	0.34024	-40.20224	1.1100	1.05644
28.50	0.34454	-38.44224	1.1200	1.06080	0.34454	-38.44224	1.1200	1.06080	0.34454	-38.44224	1.1200	1.06080
29.00	0.34894	-36.72224	1.1300	1.06480	0.34894	-36.72224	1.1300	1.06480	0.34894	-36.72224	1.1300	1.06480
29.50	0.35344	-35.04224	1.1400	1.06844	0.35344	-35.04224	1.1400	1.06844	0.35344	-35.04224	1.1400	1.06844
30.00	0.35800	-33.40224	1.1500	1.07180	0.35800	-33.40224	1.1500	1.07180	0.35800	-33.40224	1.1500	1.07180
30.50	0.36264	-31.80224	1.1600	1.07500	0.36264	-31.80224	1.1600	1.07500	0.36264	-31.80224	1.1600	1.07500
31.00	0.36734	-30.24224	1.1700	1.07790	0.36734	-30.24224	1.1700	1.07790	0.36734	-30.24224	1.1700	1.07790
31.50	0.37200	-28.72224	1.1800	1.08060	0.37200	-28.72224	1.1800	1.08060	0.37200	-28.72224	1.1800	1.08060
32.00	0.37674	-27.24224	1.1900	1.08300	0.37674	-27.24224	1.1900	1.08300	0.37674	-27.24224	1.1900	1.08300
32.50	0.38154	-25.80224	1.2000	1.08520	0.38154	-25.80224	1.2000	1.08520	0.38154	-25.80224	1.2000	1.08520
33.00	0.38634	-24.40224	1.2100	1.08720	0.38634	-24.40224	1.2100	1.08720	0.38634	-24.40224	1.2100	1.08720
33.50	0.39114	-23.04224	1.2200	1.08900	0.39114	-23.04224	1.2200	1.08900	0.39114	-23.04224	1.2200	1.0

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Table 12. Argument α . Very Dry Ground ($\epsilon_r = 3.0, \sigma = 1.0 \times 10^{-4} \text{ S/m}$)

6 MHz ($\lambda_0 = 50\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v ^2 \cos \beta$		α (rad)	
			h/λ_0		h/λ_0		h/λ_0		h/λ_0	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.689	-3.132	-3.132	-3.191	-0.007	0.034	0.311	0.312	-0.023	0.109
10	0.465	-3.119	-3.119	-3.157	-0.011	0.007	0.535	0.535	-0.021	0.109
16	0.271	-3.094	-3.094	-3.281	-0.013	0.038	0.729	0.732	-0.018	0.052
20	0.173	-3.060	-3.060	-3.292	-0.014	0.026	0.828	0.829	-0.017	0.031
26	0.061	-2.850	-2.880	-3.177	-0.016	-0.002	0.942	0.939	-0.017	-0.002
30	0.017	-1.554	-1.554	-1.893	-0.017	-0.016	1.000	0.995	-0.017	-0.016

30 MHz ($\lambda_0 = 10\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v ^2 \cos \beta$		α (rad)	
			h/λ_0		h/λ_0		h/λ_0		h/λ_0	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.689	-3.140	-3.140	-3.436	-0.001	0.200	0.311	0.341	-0.003	0.530
10	0.465	-3.137	-3.137	-3.726	-0.002	0.257	0.535	0.612	-0.004	0.398
16	0.271	-3.132	-3.132	-4.067	-0.003	0.216	0.729	0.837	-0.004	0.253
20	0.173	-3.125	-3.125	-4.285	-0.003	0.157	0.827	0.928	-0.004	0.168
26	0.059	-3.088	-3.088	-4.575	-0.003	0.058	0.941	0.992	-0.003	0.058
30	0.003	-1.567	-1.567	-3.263	-0.003	0.000	0.997	0.997	-0.003	0.000

β (rad) = angle R_v (rad) - $(2\pi/\lambda_0) 2 h \sin \psi$

$\alpha = \arctan [|R_v| \sin \beta / (1 + |R_v| \cos \beta)]$

Table 13. Argument α . Medium Dry Ground ($\epsilon_r = 15.0$, $\sigma = 1.0 \times 10^{-4} \text{ S/m}$)

6 MHz ($\lambda_o = 50\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v \cos \beta$		α (rad)	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	π	π	π	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$	$-\pi/2$
5	0.481	-3.068	-3.068	-3.127	-0.035	-0.007	0.520	-0.067	-0.013	-0.013
10	0.182	-2.893	-2.893	-3.011	-0.045	-0.024	0.824	-0.055	-0.029	-0.029
16	0.070	-0.717	-0.717	-0.904	-0.046	-0.055	1.053	-0.044	-0.053	-0.053
20	0.165	-0.277	-0.277	-0.509	-0.045	-0.080	1.159	-0.039	-0.070	-0.070
26	0.279	-0.155	-0.155	-0.452	-0.043	-0.126	1.276	-0.034	-0.100	-0.100
30	0.338	-0.124	-0.124	-0.463	-0.042	-0.151	1.335	-0.031	-0.113	-0.113

30 MHz ($\lambda_o = 10\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v \cos \beta$		α (rad)	
			h/λ_o		h/λ_o		h/λ_o		h/λ_o	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	π	π	π	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$	$-\pi/2$
5	0.482	-3.127	-3.127	-3.423	-0.026	-0.134	0.518	-0.050	0.245	0.245
10	0.180	-3.091	-3.091	-3.680	-0.010	-0.092	0.820	-0.012	0.108	0.108
16	0.050	-0.189	-0.189	-1.124	-0.000	-0.045	1.049	0.000	0.044	0.044
20	0.155	-0.059	-0.059	-3.452	-0.000	-0.047	1.155	0.000	0.055	0.055
26	0.272	-0.032	-0.032	-1.519	-0.000	-0.272	1.272	0.000	-0.268	-0.268
30	0.331	-0.026	-0.026	-1.727	-0.000	-0.327	1.331	0.000	-0.332	-0.332

 β (rad) = angle R_v (rad) - $(2\pi/\lambda_o) 2 h \sin \psi$
 $\alpha = \arctan [R_v \sin \beta / (1 + R_v \cos \beta)]$

Table 14. Argument α . Wet Ground ($\epsilon_r = 30$, $\sigma = 1.0 \times 10^{-4} \text{ S/m}$)6 MHz ($\lambda_0 = 50\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v \cos \beta$		α (rad) $= \arctan [(1)/(2)]$	
			h/λ_0		h/λ_0		h/λ_0		h/λ_0	
			0	0.054	0	0.054	0	0.054	0	0.054
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.334	-2.573	-2.573	-2.632	-0.180	-0.163	0.719	0.708	-0.245	-0.226
10	0.205	-1.233	-1.233	-1.351	-0.193	-0.200	1.068	1.045	-0.179	-0.189
16	0.347	-0.539	-0.539	-0.726	-0.178	-0.230	1.298	1.259	-0.136	-0.181
20	0.429	-0.396	-0.396	-0.628	-0.165	-0.252	1.396	1.347	-0.118	-0.185
26	0.519	-0.290	-0.290	-0.587	-0.148	-0.287	1.497	1.432	-0.099	-0.198
30	0.564	-0.248	-0.248	-0.587	-0.138	-0.312	1.547	1.470	-0.089	-0.209

30 MHz ($\lambda_0 = 10\text{m}$)

Grazing Angle, ψ (deg)	$ R_v $	Angle R_v (rad)	β (rad)		$ R_v \sin \beta$		$1 + R_v \cos \beta$		α (rad) $= \arctan [(1)/(2)]$	
			h/λ_0		h/λ_0		h/λ_0		h/λ_0	
			0	0.270	0	0.270	0	0.270	0	0.270
0	1.000	$-\pi$	$-\pi$	$-\pi$	0.000	0.000	0.000	0.000	$-\pi/2$	$-\pi/2$
5	0.346	-3.019	-3.019	-3.315	-0.042	-0.060	0.659	0.659	-0.064	-0.091
10	0.049	-1.822	-1.822	-2.411	-0.047	-0.033	1.988	0.964	-0.048	-0.034
16	0.220	-0.209	-0.209	-1.144	-0.046	-0.200	1.215	1.091	-0.038	-0.181
20	0.318	-0.136	-0.136	-1.296	-0.043	-0.306	1.315	1.086	-0.033	-0.275
26	0.424	-0.093	-0.093	-1.580	-0.039	-0.424	1.422	0.996	-0.027	-0.402
30	0.476	-0.079	-0.079	-1.775	-0.003	-0.466	1.475	0.903	-0.002	-0.476

 β (rad) = angle R_v (rad) - $(2\pi/\lambda_0) 2h \sin \psi$ $\alpha = \arctan [R_v \sin \beta / (1 + R_v \cos \beta)]$

and wet ground, respectively. Numerical values are given at 6 MHz for $h/\lambda_0 = 0, 0.054$ and at 30 MHz for $\lambda_0 = 0, 0.270$. The normalized heights $h/\lambda_0 = 0, 0.054$ and 0.270 correspond to the mid-point at 6 MHz and 30 MHz, respectively, of a vertical monopole of length $\ell = 5.4$ m.

The mean argument $\bar{\alpha}(\theta)$, averaged over the m elements of the array for a plane wave incident from the direction θ , is given by

$$\bar{\alpha}(\theta) = (1/m) \sum_{k=1}^m \alpha_k(\theta) \quad (2-10)$$

The variability of the argument α_k from element to element causes an array RMS phase error α_{rms} and beam pointing errors B_θ, B_ϕ . The RMS phase error, at the diffraction focus of the array [3], is given by

$$\alpha_{rms}(\theta) = \left\{ \left(\sum_{k=1}^m (1/m) [\alpha_k(\theta) - \bar{\alpha}(\theta)]^2 \right) - \alpha_B^2(\theta) \right\}^{1/2} \quad (2-11)$$

where $\alpha_B(\theta)$ is the RMS phase error caused by a linear phase shift resulting from beam pointing errors. The rms phase error $\alpha_B(\theta)$ is given by equation (A-13) in the appendix as

$$\alpha_B = \left\{ \frac{(1/m) \left[\sum_{k=1}^m (x_k / r_A) (\alpha_k - \bar{\alpha}) \right]^2}{\sum_{k=1}^m (x_k / r_A)^2} + \frac{(1/m) \left[\sum_{k=1}^m (y_k / r_A) (\alpha_k - \bar{\alpha}) \right]^2}{\sum_{k=1}^m (y_k / r_A)^2} \right\}^{1/2} \quad (2-12)$$

are uncorrelated

where

x_k, y_k = projections of the kth element location onto the x and y axes, respectively, whose origin is at the center of the array.

The non-homogeneous earth causes beam pointing errors B_θ and B_ϕ , in the elevation and azimuthal directions, respectively. The beam pointing errors, for a plane wave incident from the true direction (θ, ϕ) are given by equations (A-20) and (A-21) as

$$B_\theta = \theta_{\text{apparent}} - \theta = -(1 / \cos \theta)[(\hat{u} - u) \cos \phi + (\hat{v} - v) \sin \phi], B_\theta \ll 1 \text{ rad} \quad (2-13)$$

$$B_\phi = \phi_{\text{apparent}} - \phi = -(1 / \sin \theta)[(\hat{u} - u) \sin \phi + (\hat{v} - v) \cos \phi], B_\phi \ll 1 \text{ rad} \quad (2-14)$$

where $(\theta_{\text{apparent}}, \phi_{\text{apparent}})$ is the apparent beam direction when the beam has a true direction (θ, ϕ) . The quantities $(\hat{u} - u)$ and $(\hat{v} - v)$ are given by equations (A-9) and (A-10), respectively, as

$$\hat{u} - u = (\lambda_o / 2\pi r_A) \sum_{k=1}^m (x_k / r_A)(\alpha_k - \bar{\alpha}) / \sum_{k=1}^m (x_k / r_A)^2 \quad (2-15)$$

$$\hat{v} - v = (\lambda_o / 2\pi r_A) \sum_{k=1}^m (y_k / r_A)(\alpha_k - \bar{\alpha}) / \sum_{k=1}^m (y_k / r_A)^2 \quad (2-16)$$

where x_k, y_k are assumed to be uncorrelated.

The parameters $\alpha_B, B_\theta, B_\phi, \hat{u} - u$, and $\hat{v} - v$ are zero for the cases: (1) homogeneous earth ($\alpha_k = \bar{\alpha}$) and (2) non-homogeneous earth with randomly-distributed non-homogeneities and elements $(\sum x_k (\alpha_k - \bar{\alpha}) = \sum y_k (\alpha_k - \bar{\alpha}) = 0)$. However, for non-homogeneous earth with systematically-distributed non-homogeneities or elements $(\sum x_k (\alpha_k - \bar{\alpha}) \neq 0, \sum y_k (\alpha_k - \bar{\alpha}) \neq 0)$, then $\alpha_B, B_\theta, B_\phi, \hat{u} - u$ and $\hat{v} - v$ are non-zero.

SECTION 3

NUMERICAL RESULTS

Numerical results for the RMS phase error α_{rms} and boresight errors B_θ, B_ϕ , are given in this section for cases where a fraction of the elements are located in proximity to earth of permittivity ϵ_I^* and the remaining fraction are in proximity to earth of permittivity ϵ_{II}^* . This type of non-homogeneous earth is designated as "two-level" non-homogeneous earth. Accordingly, the parameters $\epsilon_k^*, \gamma_k, \beta_k$, and $R_{k,v}$ at the k th element are given by

$$\epsilon_k^*, \alpha_k, \beta_k, R_{k,v} = \begin{cases} \epsilon_I^*, \alpha_I, R_{I,v} & \text{at } p \text{ elements,} \\ \text{two-level non-homogeneous earth} \\ \epsilon_{II}^*, \alpha_{II}, R_{II,v} & \text{at } m-p \text{ elements} \end{cases} \quad (3-1)$$

Furthermore, it is assumed that the elements are randomly distributed over the circular area of radius r_A and that projections x_k and y_k of the k th element onto the x and y axes, respectively, are uncorrelated.

3.1 Randomly-Distributed Two-Level Non-Homogeneous Earth

Consider first the case the two permittivities ϵ_I^* and ϵ_{II}^* are randomly distributed with occurrence frequencies p/m for ϵ_I^* and $1 - (p/m)$ for ϵ_{II}^* . Consequently,

$$\sum_{k=1}^m x_k \alpha_k = \sum_{k=1}^m y_k \alpha_k = 0, \text{ randomly distributed non-homogenities and elements} \quad (3-2)$$

For this case, equations (2-10) through (2-16) reduce to

$$\bar{\alpha}(\theta) = (p/m) \alpha_I(\theta) + [1 - (p/m)] \alpha_{II}(\theta),$$

two-level non-homogeneous earth (3-3)

$$\alpha_{rms}(\theta) = \left\{ (p/m) [\alpha_I(\theta) - \bar{\alpha}(\theta)]^2 + [1 - (p/m)] [\alpha_{II}(\theta) - \bar{\alpha}(\theta)]^2 \right\}^{1/2},$$

two-level non-homogeneous earth, randomly-
distributed elements and non-homogenities (3-4)

$$\alpha_B = B_\theta = B_\phi = \hat{u} - u = \hat{v} - v = 0,$$

randomly distributed elements and earth
non-homogenities (3-5)

Numerical values of the RMS phase error $\alpha_{rms}(\theta)$ are given in tables 15 through 17 for three compositions of two-level randomly-distributed non-homogeneous earth for $p/m = 0.5$ and elevation angles $60 \leq \theta \leq 90$ degrees. Table 15 is for very dry ground/medium dry ground. Table 16 is for medium dry ground/wet ground. Table 17 is for very dry ground/wet ground. Numerical values are given at 6 MHz for normalized height $h/\lambda_0 = 0, 0.054$ and at 30 MHz for $h/\lambda_0 = 0, 0.270$.

The RMS phase errors of tables 15 through 17 are summarized in table 18. The largest phase errors occur for very dry ground/wet ground. At 6 MHz, the maximum RMS phase error is 10 degrees and occurs for $\theta = 85$ degrees, $h/\lambda_0 = 0, 0.054$. At 30 MHz, the maximum RMS phase error is 18 degrees and occurs for $\theta = 85$ degrees, $h/\lambda_0 = 0, 0.270$. The RMS phase error generally increases with increasing values of h/λ_0 (for modest value of h/λ_0). The dependence of RMS phase error upon θ is generally non-monotonic.

3.2 SYSTEMATICALLY-DISTRIBUTED TWO-LEVEL NON-HOMOGENEOUS EARTH

Consider now the case where the two-permittivities ϵ_I^* and ϵ_{II}^* are not randomly distributed over the array area of radius r_A . In particular consider the case where the left half of the array of $p = m/2$ elements is over earth with a permittivity ϵ_I^* and the right half of the array of $m - p = m/2$ elements is over earth with a permittivity ϵ_{II}^* . Accordingly, $p/m = 0.5$ and

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Table 15. RMS Phase Error α_{rms} (Very Dry Ground/Medium Dry Ground, $p/m = 0.5$, Randomly-Distributed Non-Homogeneities)

6 MHz ($\lambda_o = 50m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Very Dry Ground		Medium Dry Ground							
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
0	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
5	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
10	-0.023	0.109	-0.067	-0.013	-0.045	0.048	0.022	0.065	1.26	3.72
16	-0.021	0.109	-0.055	-0.029	-0.038	0.040	0.017	0.069	0.97	3.95
20	-0.018	0.052	-0.044	-0.053	-0.040	-0.001	0.016	0.053	0.92	3.04
26	-0.017	0.031	-0.039	-0.070	-0.028	-0.020	0.011	0.051	0.63	2.92
30	0.017	-0.002	-0.034	-0.100	-0.026	-0.051	0.009	0.049	0.52	2.81
30	0.017	0.016	-0.031	-0.113	-0.024	-0.065	0.007	0.049	0.40	2.81

30 MHz ($\lambda_o = 10m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Very Dry Ground		Medium Dry Ground							
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
0	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
5	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
10	-0.003	0.530	-0.050	0.245	-0.027	0.388	0.023	0.143	1.32	8.19
16	-0.004	0.398	-0.012	0.108	-0.008	0.253	0.004	0.145	0.23	8.31
20	-0.004	0.253	0.000	0.044	-0.002	0.149	0.002	0.105	0.11	6.02
26	-0.004	0.168	0.000	0.055	-0.002	0.112	0.002	0.057	0.11	3.27
30	-0.003	0.058	0.000	-0.268	-0.002	-0.105	0.002	0.163	0.11	9.33
30	-0.003	0.000	0.000	-0.332	-0.002	-0.166	0.002	0.166	0.11	9.51

$$\bar{\alpha} = 0.5 (\alpha_I + \alpha_{II})$$

$$\bar{\alpha}_{rms} = \{0.5 [(\alpha_I - \bar{\alpha})^2 + (\alpha_{II} - \bar{\alpha})^2]\}^{1/2}$$

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Table 16. RMS Phase Error α_{rms} (Medium Dry Ground/Wet Ground, $p/m = 0.5$, Randomly-Distributed Non-Homogeneities)

6 MHz ($\lambda_o = 50m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Medium Dry Ground		Wet Ground							
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
0	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
5	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
10	-0.067	-0.013	-0.245	-0.226	-0.156	-0.120	0.089	0.168	5.10	9.64
16	-0.055	-0.029	-0.179	-0.189	-0.117	-0.109	0.062	0.080	3.52	4.58
20	-0.044	-0.053	-0.136	-0.181	-0.090	-0.117	0.046	0.064	2.63	3.67
26	-0.039	-0.070	-0.118	-0.185	-0.079	-0.128	0.040	0.057	2.29	3.27
30	-0.034	-0.100	-0.099	-0.198	-0.067	-0.149	0.033	0.052	1.89	2.98
	-0.031	-0.113	-0.089	-0.209	-0.060	-0.161	0.029	0.051	1.66	2.92

30 MHz ($\lambda_o = 10m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Medium Dry Ground		Wet Ground							
	h/λ_o		h/λ_o		h/λ_o		h/λ_o		h/λ_o	
0	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
5	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
10	-0.050	0.245	-0.064	-0.091	-0.057	0.077	0.007	0.168	0.40	9.62
16	-0.012	0.108	-0.048	-0.034	-0.030	0.037	0.018	0.071	1.03	4.07
20	0.000	0.044	-0.038	-0.181	-0.019	-0.069	0.019	0.113	1.09	6.47
26	0.000	0.055	-0.033	-0.275	-0.017	-0.110	0.017	0.165	0.97	9.45
30	0.000	-0.268	-0.027	-0.402	-0.014	-0.335	0.017	0.067	0.80	3.83
	0.000	-0.332	-0.002	-0.476	-0.001	-0.404	0.001	0.072	0.06	4.13

$$\bar{\alpha} = 0.5 (\alpha_I + \alpha_{II})$$

$$\alpha_{rms} = \{0.5 [(\alpha_I - \bar{\alpha})^2 + (\alpha_{II} - \bar{\alpha})^2]\}^{1/2}$$

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Table 17. RMS Phase Error α_{rms} (Very Dry Ground/Wet Ground, $p/m = 0.5$, Randomly-Distributed Non-Homogeneities)

6 MHz ($\lambda_o = 50m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Very Dry Ground		Wet Ground		h/λ_o		h/λ_o		h/λ_o	
	0	0.054	0	0.054	0	0.054	0	0.054	0	0.054
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	-0.023	0.109	-0.245	-0.226	-0.134	-0.059	0.111	0.168	6.36	9.62
10	-0.021	0.109	-0.179	-0.189	-0.100	-0.040	0.080	0.149	4.58	8.54
16	-0.018	0.052	-0.136	-0.181	-0.077	-0.065	0.059	0.117	3.38	6.70
20	-0.017	0.031	-0.118	-0.185	-0.068	-0.077	0.051	0.108	2.92	6.19
26	-0.017	-0.002	-0.099	-0.198	-0.058	-0.100	0.041	0.098	2.35	5.61
30	-0.017	-0.016	-0.089	-0.209	-0.053	-0.113	0.036	0.097	2.06	5.56

30 MHz ($\lambda_o = 10m$)

Grazing Angle, ψ (deg)	α_I (rad)		α_{II} (rad)		$\bar{\alpha}$ (rad)		α_{rms} (rad)		α_{rms} (deg)	
	Very Dry Ground		Wet Ground		h/λ_o		h/λ_o		h/λ_o	
	0	0.270	0	0.270	0	0.270	0	0.270	0	0.270
0	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$	0	0	0	0
5	-0.003	0.530	-0.064	-0.091	-0.034	0.220	0.031	0.311	1.78	17.8
10	-0.004	0.398	-0.048	-0.034	-0.026	0.182	0.022	0.216	1.26	12.4
16	-0.004	0.253	-0.038	-0.181	-0.021	0.036	0.017	0.217	0.97	12.4
20	-0.004	0.168	-0.033	-0.275	-0.019	-0.054	0.015	0.222	0.86	12.7
26	-0.003	0.058	-0.027	-0.402	-0.015	-0.172	0.012	0.230	0.69	13.2
30	-0.003	0.000	-0.002	-0.476	-0.003	-0.238	0.001	0.238	0.57	13.6

$$\bar{\alpha} = 0.5 (\alpha_I + \alpha_{II})$$

$$\bar{\alpha}_{rms} = [0.5 [(\alpha_I - \bar{\alpha})^2 + (\alpha_{II} - \bar{\alpha})^2]]^{1/2}$$

Table 18. Summary of RMS Phase Errors ($p/m = 0.5$, Randomly-Distributed Two-Level Non-Homogeneous Earth)(a) 6 MHz ($\lambda_o = 50m$)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.4	2.8	1.7	2.9	2.1	5.6
64	0.5	2.8	1.9	3.0	2.3	5.6
70	0.6	2.9	2.3	3.3	2.9	6.2
74	0.9	3.0	2.6	3.7	3.4	6.7
80	1.0	4.0	3.5	4.6	4.6	8.5
85	1.3	3.7	5.1	9.6	6.4	9.6
90	0	0	0	0	0	0

(b) 30 MHz ($\lambda_o = 10m$)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	0.1	9.5	0.1	4.1	0.6	13.6
64	0.1	9.3	0.8	3.8	0.7	13.2
70	0.1	3.3	1.0	9.5	0.9	12.7
74	0.1	6.0	1.1	6.5	1.0	12.4
80	0.2	8.3	1.0	4.1	1.3	12.4
85	1.3	8.2	0.4	9.6	1.8	17.8
90	0	0	0	0	0	0

$$\epsilon_{k,v}, \alpha_k, \beta_k, R_{k,v} = \begin{cases} \epsilon_I, \alpha_I, \beta_I, R_{I,v} & \text{at } (m/2) \text{ elements, } -r_A \leq x_k \leq 0 \\ \epsilon_{II}, \alpha_{II,v}, \beta_{II}, R_{II,v} & \text{at } (m/2) \text{ elements, } 0 \leq x_k \leq r_A \end{cases} \quad (3-6)$$

If the probability densities of the spacing of element location projections on the x and y axes were specified, then the cumulative probabilities, of the RMS phase error and beam pointing errors exceeding specified values, could be determined [4]. In this study, only the expected value of the RMS phase error and beam pointing errors are determined.

The expected value $\langle s \rangle$ of the spacing between element location projections x_k, x_{k+1} on the y axis, is given by

$$\langle s \rangle = r_A / (m-1) \quad (3-7)$$

For $m \gg 1$

$$\langle s \rangle / r_A \ll \langle |x_k| \rangle / r_A, \langle |y_k| \rangle / r_A \ll 1 \quad (3-8)$$

Denoting the ordered projections of x_k on the negative and positive x axes as x_q then

$$\langle (x_k)_{ordered} \rangle = \langle x_q \rangle = \begin{cases} -q \langle s \rangle; q = 1, 2, \dots, m/2; & -r_A \leq x_q < 0 \\ q \langle s \rangle; q = 1, 2, \dots, m/2; & 0 \leq x_q \leq r_A \end{cases} \quad m \gg 1 \quad (3-9)$$

For α_k and x_k given by eqs. (3-6) and (3-9), respectively,

$$\sum_{k=1}^m (x_k / r_A) (\alpha_k - \bar{\alpha}) = \frac{1}{m-1} \left[(\alpha_I - \bar{\alpha}) \sum_{q=1}^{m/2} -q + (\alpha_{II} - \bar{\alpha}) \sum_{q=1}^{m/2} q \right]$$

$$= \frac{1}{m-1}(\alpha_{II} - \alpha_I) \frac{(m/2)[(m/2)+1]}{2} = \frac{m(m+2)}{q(m-1)}(\alpha_{II} - \alpha_I) \quad (3-10)$$

$$\sum_{k=1}^m (y_k / r_A)(\alpha_k - \bar{\alpha}) = 0; \quad y_k \text{ and } x_k \text{ are randomly distributed} \quad (3-11)$$

$$\begin{aligned} \sum_{k=1}^m (x_k / r_A)^2 &= \sum_{k=1}^m (y_k / r_A)^2 = 2 \sum_{q=1}^{m/2} (x_q / r_A)^2 \\ &= \frac{2}{(m-1)^2} \sum_{q=1}^{m/2} q^2 = \frac{2}{(m-1)^2} \frac{(m/2)[(m/2)+1](m+1)}{6} \\ &= \frac{m(m+1)(m+2)}{12(m-1)^2} \end{aligned} \quad (3-12)$$

Substituting equations (3-10) through (3-12) into equations (2-10) through (2-16),

$$\bar{\alpha}(\theta) = (\alpha_I + \alpha_{II}) / 2 \quad (3-13)$$

$$\alpha_{rms}(\theta) = \{0.5[(\alpha_I - \bar{\alpha})^2 + (\alpha_{II} - \bar{\alpha})^2] - \alpha_B^2\}^{1/2} \quad (3-14)$$

$$\begin{aligned} \alpha_B &= \left\{ \frac{1}{m} \left[\frac{m(m+2)(\alpha_{II} - \alpha_I)}{8(m-1)} \right]^2 \frac{12(m-1)^2}{m(m+1)(m+2)} \right\}^{1/2} \\ &= \left[\frac{3}{16} \frac{(m+2)(\alpha_{II} - \alpha_I)^2}{(m+1)} \right]^{1/2} \\ &= [(3/16)(m+2)/(m+1)]^{1/2} |\alpha_{II} - \alpha_I| \end{aligned} \quad (3-15)$$

$$\hat{u} - u = \frac{\lambda_o}{2\pi r_A} \frac{m(m+2)(\alpha_{II} - \alpha_I)}{8(m-1)} \frac{12(m-1)^2}{m(m+1)(m+2)}$$

$$= \frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_H - \alpha_I)}{(m+1)} \quad (3-16)$$

$$\hat{v} - v = 0 \quad (3-17)$$

$$\begin{aligned} B_\theta &= -\frac{1}{\cos\theta} (\hat{u} - u) \cos\phi = -\frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_H - \alpha_I)}{(m+1)} \\ &\approx -(BW)_{\theta, 3dB} \frac{3}{2\pi} \frac{(m-1)(\alpha_H - \alpha_I) \cos\phi}{(m+1)}, B_\theta \ll 1 \text{ rad} \end{aligned} \quad (3-18)$$

where

$$\begin{aligned} (BW)_{\theta, 3dB} &= \text{elevation beamwidth of the array main beam at its half-power points} \\ &= (1.02 \lambda_o / 2r_A \cos\theta) \text{ rad} \end{aligned} \quad (3-19)$$

$$\begin{aligned} B_\phi &= \frac{1}{\sin\theta} (\hat{u} - u) \sin\phi = \frac{\sin\phi}{\sin\theta} \frac{\lambda_o}{r_A} \frac{3}{4\pi} \frac{(m-1)(\alpha_H - \alpha_I)}{(m+1)} \\ &= (BW)_{\phi, 3dB} \frac{3}{2\pi} \frac{(m-1)(\alpha_H - \alpha_I) \sin\phi}{(m+1) \sin\theta}, B_\phi \ll 1 \text{ rad} \end{aligned} \quad (3-20)$$

where

$$\begin{aligned} (BW)_{\phi, 3dB} &= \text{azimuth beamwidth of the array main beam at its half-power points} \\ &= (1.02 \lambda_o / 2r_A) \text{ rad} \end{aligned} \quad (3-21)$$

Numerical values of the RMS phase error α_{rms} and beam pointing errors B_θ , B_ϕ are given in tables 19 through 21 for systematically-distributed non-homogeneous very dry ground/medium dry ground, medium dry ground/wet ground, and very dry ground/wet ground, respectively. The RMS phase errors are approximately 50 percent less than those

Table 19. RMS Phase Error and Beam Pointing Errors (Very Dry Ground/Medium Dry Ground, $p/m = 0.5$, Earth Non-Homogeneities Systematically Distributed so that each Half of Array is over Different Earth, $m = 96$ Elements)

6 MHz ($\lambda_o = 50m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\phi/(BW)_{\phi,3}$ dB		$B_\phi/(BW)_{\phi,3}$ dB
	0	0.054	0	0.054	0	0.054	0	0.054	0
0	0	0	0	0	0	0	0	0	0
5	-0.044	-0.122	0.008	0.023	0.020	0.061	0.021	0.057	-0.021
10	-0.034	-0.138	0.006	0.026	0.016	0.064	0.016	0.065	-0.016
16	-0.026	-0.105	0.005	0.020	0.015	0.049	0.012	0.049	-0.012
20	-0.022	-0.101	0.004	0.019	0.010	0.047	0.010	0.047	-0.011
26	-0.017	-0.098	0.003	0.019	0.008	0.045	0.008	0.046	-0.009
30	-0.014	-0.097	0.003	0.018	0.006	0.046	0.007	0.045	-0.008

30 MHz ($\lambda_o = 10m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\phi/(BW)_{\phi,3}$ dB		$B_\phi/(BW)_{\phi,3}$ dB
	0	0.270	0	0.270	0	0.270	0	0.270	0
0	0	0	0	0	0	0	0	0	0
5	-0.047	-0.285	0.009	0.054	0.021	0.132	0.022	0.133	-0.022
10	-0.008	-0.290	0.002	0.055	0.003	0.134	0.004	0.136	-0.044
16	0.004	-0.209	0.001	0.040	0.002	0.097	-0.002	0.098	0.002
20	0.004	-0.113	0.001	0.021	0.002	0.053	-0.002	0.053	0.002
26	0.003	-0.326	0.001	0.062	0.002	0.151	-0.001	0.152	0.001
30	0.003	-0.332	0.001	0.063	0.002	0.154	-0.001	0.155	0.001

$$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2} \alpha_{II} - \alpha_I, \alpha_{RMS} = (\alpha_{RMS}^2 \alpha_B^2 = 0 - \alpha_B^2)^{1/2}$$

$$\frac{B_\phi}{(BW)_{\phi,3} \text{ dB}} = \frac{-3}{2\pi} \frac{m-1}{m+1} (\alpha_{II} - \alpha_I) \cos \phi, \frac{B_\phi}{(BW)_{\phi,3} \text{ dB}} = \frac{3}{2\pi} \frac{m-1}{m+1} \sin \phi (\alpha_{II} - \alpha_I)$$

Table 20. RMS Phase Error and Beam Pointing Errors (Medium Dry Ground/Wet Ground,
 $p/m = 0.5$, Earth Non-Homogeneities Systematically Distributed so that each
 Half of Array is over Different Earth, $m = 96$ Elements)

6 MHz ($\lambda_o = 50m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\theta/(BW)_{\theta,3}$ dB		$B_\theta/(BW)_{\theta,3}$ dB
	0	0.054	0	0.054	0	0.054	0	0.054	0
0	0	0	0	0	0	0	0	0	0
5	-0.178	-0.213	0.077	0.093	0.045	0.140	0.083	0.100	-0.083
10	-0.124	-0.160	0.054	0.070	0.030	0.039	0.058	0.075	-0.059
16	-0.092	-0.128	0.040	0.056	0.023	0.031	0.043	0.060	-0.045
20	-0.079	-0.115	0.034	0.050	0.021	0.027	0.037	0.054	-0.039
26	-0.065	-0.098	0.028	0.043	0.021	0.029	0.030	0.046	-0.033
30	-0.058	-0.096	0.025	0.042	0.015	0.029	0.027	0.045	-0.031

30 MHz ($\lambda_o = 10m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\theta/(BW)_{\theta,3}$ dB		$B_\theta/(BW)_{\theta,3}$ dB
	0	0.270	0	0.270	0	0.270	0	0.270	0
0	0	0	0	0	0	0	0	0	0
5	-0.014	-0.336	0.006	0.146	0.004	0.083	0.007	0.157	-0.007
10	-0.036	-0.142	0.016	0.062	0.008	0.035	0.017	0.066	-0.017
16	-0.038	-0.225	0.017	0.098	0.008	0.056	0.018	0.105	-0.018
20	-0.033	-0.330	0.014	0.144	0.010	0.081	0.015	0.154	-0.016
26	-0.027	-0.134	0.012	0.058	0.007	0.034	0.013	0.063	-0.014
30	-0.002	-0.144	0.001	0.063	0	0.035	0.001	0.067	-0.001

$$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2} \alpha_{II} - \alpha_I, \alpha_{RMS} = (\alpha_{RMS}^2, \alpha_B = 0 - \alpha_B^2)^{1/2}$$

$$\frac{B_\theta}{(BW)_{\theta,3} \text{ dB}} = \frac{-3}{2\pi} \frac{m-1}{m+1} (\alpha_{II} - \alpha_I) \cos \phi, \frac{B_\theta}{(BW)_{\theta,3} \text{ dB}} = \frac{3}{2\pi} \frac{m-1}{m+1} \sin \phi (\alpha_{II} - \alpha_I)$$

Table 21. RMS Phase Error and Beam Pointing Errors (Very Dry Ground/Wet Ground, $p/m = 0.5$, Earth Non-Homogeneities Systematically Distributed so that each Half of Array is over Different Earth, $m = 96$ Elements)

6 MHz ($\lambda_o = 50m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\phi / (BW)_{\phi, 3}$ dB		$B_\phi / (BW)_{\phi, 3}$ dB
	0	0.054	0	0.054	0	0.054	0	0.054	h/λ_o
0	0	0	0	0	0	0	0	0	0
5	-0.222	-0.335	0.097	0.146	0.054	0.083	0.104	0.157	-0.104
10	-0.158	-0.298	0.069	0.130	0.040	0.073	0.074	0.139	-0.075
16	-0.118	-0.233	0.051	0.101	0.030	0.059	0.055	0.109	-0.057
20	-0.101	-0.216	0.044	0.094	0.026	0.053	0.047	0.101	-0.050
26	-0.082	-0.196	0.036	0.085	0.020	0.050	0.038	0.092	-0.042
30	-0.072	-0.193	0.031	0.084	0.018	0.049	0.034	0.090	-0.039
30 MHz ($\lambda_o = 10m$)									
Grazing Angle, ψ (deg)	$\alpha_{II} - \alpha_I$ (rad)		Pointing RMS Phase Error α_B (rad)		RMS Phase Error α_{RMS} (rad)		$\phi = 0$ rad		$\phi = \pi/2$ rad
	h/λ_o		h/λ_o		h/λ_o		$B_\phi / (BW)_{\phi, 3}$ dB		$B_\phi / (BW)_{\phi, 3}$ dB
	0	0.270	0	0.270	0	0.270	0	0.270	h/λ_o
0	0	0	0	0	0	0	0	0	0
5	-0.061	-0.621	0.027	0.270	0.015	0.154	0.029	0.290	-0.029
10	-0.044	-0.432	0.019	0.188	0.011	0.106	0.021	0.202	-0.021
16	-0.034	-0.434	0.015	0.189	0.008	0.107	0.016	0.202	-0.017
20	-0.029	-0.443	0.013	0.193	0.007	0.110	0.014	0.207	-0.015
26	-0.024	0.460	0.010	0.200	0.007	0.114	0.011	-0.215	-0.012
30	-0.001	0.476	0.000	0.207	0.001	0.117	0.000	-0.223	0.000

$$\alpha_B = [(3/16)(m+2)/(m+1)]^{1/2} \alpha_{II} - \alpha_I, \alpha_{rms} = (\alpha_{rms}^2 - \alpha_B^2)^{1/2}$$

$$\frac{B_\phi}{(BW)_{\phi, 3} \text{ dB}} = \frac{-3}{2\pi} \frac{m-1}{m+1} (\alpha_{II} - \alpha_I) \cos \phi, \frac{B_\phi}{(BW)_{\phi, 3} \text{ dB}} = \frac{3}{2\pi} \frac{m-1}{m+1} \frac{\sin \phi}{\sin \theta} (\alpha_{II} - \alpha_I)$$

given in tables 15 through 17 for randomly-distributed non-homogeneous earth. The elevation and azimuthal beam pointing errors are approximately one-tenth the 3 dB beamwidths of the array main beam.

The expected values of the RMS phase error and elevation beam pointing error of tables 19 through 21 are summarized in tables 22 and 23, respectively. The largest RMS phase error is 9 deg. The largest beam pointing error is 0.3 of a beamwidth. The maximum errors occur for $\theta = 85$ deg, 30 MHz, $h/\lambda = 0.270$, and very dry ground/wet ground.

Table 22. Summary of RMS Phase Errors ($p/m = 0.5$, Two-Level Non-Homogeneous Earth Systematically Distributed so that each Half of Array is over Different Earth)

(a) 6 MHz ($\lambda_o = 50m$)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.3	2.6	0.9	1.7	1.0	4.8
64	0.5	2.6	1.2	1.7	1.1	4.2
70	0.6	2.7	1.2	1.5	1.5	3.4
74	0.9	2.8	1.3	1.8	1.7	3.0
80	0.9	3.7	1.7	2.2	2.3	2.9
85	1.1	3.5	2.6	8.0	3.1	2.8
90	0	0	0	0	0	0

(b) 30 MHz ($\lambda_o = 10m$)

Angle of Incidence, θ (deg)	RMS Phase Error, α_{RMS} (deg)					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	0.1	8.8	0.3	4.8	0.9	6.7
64	0.1	8.7	0.5	2.0	0.6	6.5
70	0.1	3.0	0.5	3.2	0.5	6.3
74	0.1	5.6	0.6	4.6	0.4	6.3
80	0.2	7.7	0.4	1.9	0.4	6.1
85	1.2	7.5	0	2.0	0.1	8.8
90	0	0	0	0	0	0

Table 23. Summary of Elevation Beam Pointing Errors ($p/m = 0.5$, Two-Level Non-Homogeneous Earth Systematically Distributed so that each Half of Array is over Different Earth, $\phi = 0^\circ$)

(a) 6 MHz ($\lambda_o = 50m$)

Angle of Incidence, θ (deg)	Elevation Beam Pointing Error, $B_\theta/(BW)_{\theta,3}$ dB					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$	$h/\lambda_o = 0$	$h/\lambda_o = 0.054$
60	0.01	0.05	0.03	0.05	0.03	0.16
64	0.01	0.05	0.03	0.05	0.04	0.14
70	0.01	0.05	0.04	0.05	0.05	0.11
74	0.01	0.05	0.04	0.06	0.06	0.10
80	0.02	0.07	0.06	0.08	0.07	0.09
85	0.02	0.06	0.08	0.10	0.10	0.09
90	0	0	0	0	0	0

(b) 30 MHz ($\lambda_o = 10m$)

Angle of Incidence, θ (deg)	Elevation Beam Pointing Error, $B_\theta/(BW)_{\theta,3}$ dB					
	Very Dry Ground/ Medium Dry Ground		Medium Dry Ground/ Wet Ground		Very Dry Ground/ Wet Ground	
	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$	$h/\lambda_o = 0$	$h/\lambda_o = 0.270$
60	-0.001	0.16	0.001	0.07	0.00	-0.22
64	-0.001	0.15	0.01	0.06	0.01	-0.22
70	-0.002	0.05	0.02	0.02	0.01	0.21
74	-0.002	0.10	0.02	0.11	0.02	0.20
80	0.004	0.14	0.02	0.07	0.02	0.20
85	0.02	0.13	0.01	0.16	0.03	0.29
90	0	0	0	0	0	0

SECTION 4

SUMMARY AND CONCLUSIONS

The electric field, at each element of a ground-based HF receiving array, is the sum of a direct field and an indirect (multipath) field. For elements with sufficiently-small ground planes, the indirect field is reflected not from the ground plane but from the earth in proximity to that element. The indirect field, relative to the direct field, is the product of the earth Fresnel reflection coefficient and a path-length phase delay that is proportional to the height of the element above the earth. If the earth beneath the array is not homogeneous, then the argument of the total electric field at each element (after allowance for the true phase advance of the direct field at each element) is not uniform from element to element. The non-uniform argument causes an array rms phase error. The non-uniform argument also causes beam pointing errors when the non-homogeneous earth is systematically-distributed.

The earth Fresnel reflection coefficients and the arguments of the total field, for a vertically-polarized Hertzian dipole at height h above the earth, are tabulated in this paper for CCIR 527-1 classifications of earth and $h/\lambda_0 = 0, 0.054$, and 0.270 . The normalized heights $h/\lambda_0 = 0.054$ and 0.270 correspond to the midpoint at 6 MHz and 30 MHz, respectively, of a 5.4 m length vertical monopole. The RMS phase errors and beam pointing errors are modeled for arbitrary distributions of non-homogeneous earth.

Numerical results are presented for randomly-distributed and systematically-distributed earth non-homogeneities for cases where one-half of the array elements are located in proximity to one type of earth and the remaining half are located in proximity to a second type of earth. The combinations of earth types that are considered are very dry ground/medium dry ground, medium dry ground/wet ground, and very dry ground/wet ground.

The RMS phase error is an increasing monotonic function of h/λ_0 (for modest values of h/λ_0). The RMS phase error is generally a non-monotonic function of the angle of incidence θ .

The maximum expected values of the RMS phase errors at the best diffraction focus of the array for the cases examined, are 18 degrees and 9 degrees for randomly-distributed and systematically-distributed non-homogeneities, respectively. The RMS phase error is less for systematically-distributed non-homogeneities because the linear phase error caused by beam pointing errors has been subtracted from the RMS phase error. The maximum expected values of the beam pointing error (in elevation and in azimuth) are 0 and 0.3 beamwidths for randomly-distributed and systematically-distributed non-homogeneities, respectively. The maximum RMS phase errors and beam pointing errors occur for very dry ground/wet ground, $\theta = 85$ degrees, $h/\lambda_0 = 0.270$.

The above numerical results suggest that the influence of non-homogeneous earth is appreciable, but not significant, on the performance of HF receiving arrays with electrically-small ground planes that do not require very low sidelobes.

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APPENDIX

BEAM POINTING ERRORS CAUSED BY A NON-HOMOGENEOUS EARTH

Consider a plane wave that is incident from the true direction (θ, ϕ) at the k th element of a large array of m randomly distributed elements within a circular area of radius r_A . The elements are in proximity to non-homogeneous earth whose non-homogeneities are not necessarily randomly distributed over the circular area. A systematic distribution of non-homogeneities causes beam pointing errors B_θ, B_ϕ that in turn cause a linear phase shift. The beam pointing errors reduce the tracking accuracy of the array. The linear phase shift causes a mean phase-squared error α_B^2 which should be subtracted from the array mean phase-squared error computed at the diffraction focus of the array. Expressions for the beam pointing errors B_θ, B_ϕ and RMS phase error α_B are derived in this appendix.

The true phase advance $\delta_{true, k}$, of the direct field at the k th element, relative to that of an element at the center of the array, is given by reference 1.

$$\delta_{true, k} = (2\pi / \lambda_o)(x_k \sin \theta \cos \phi + y_k \sin \theta \sin \phi) = (2\pi / \lambda_o)(x_k u + y_k v) \quad (A-1)$$

where

x_k, y_k = coordinates of the k th element along the x and y axes, respectively,
with the origin at the center of the array.

$$u = \sin \theta \cos \phi$$

$$v = \sin \theta \sin \phi$$

The measured phase delay δ_k at the k th element, in the presence of an indirect field reflected from the earth, is given by

$$\delta_k = \delta_{true,k} + (\alpha_k - \bar{\alpha}) \quad (A-2)$$

where α_k , $\bar{\alpha}$ are given by equations (2-9) and (2-10), respectively.

The estimates $\hat{u} = \sin \theta \cos \phi$, $\hat{v} = \sin \theta \sin \phi$ may be determined from the least squares algorithm given by

$$\text{minimize with respect to } \hat{u}, \hat{v}: \sum_{k=1}^m [\delta_k - (2\pi / \lambda_o)(x_k \hat{u} + y_k \hat{v})]^2 \quad (A-3)$$

The minimization in equation (A-3) is executed by

$$\frac{\partial}{\partial \hat{u}} \sum_{k=1}^m [\delta_k - (2\pi / \lambda_o)(x_k \hat{u} + y_k \hat{v})]^2 = 0 \quad (A-4)$$

$$\frac{\partial}{\partial \hat{v}} \sum_{k=1}^m [\delta_k - (2\pi / \lambda_o)(x_k \hat{u} + y_k \hat{v})]^2 = 0 \quad (A-5)$$

Equations (A-4) and (A-5) reduce to

$$\sum_{k=1}^m [\delta_k x_k - (2\pi / \lambda_o) x_k^2 \hat{u} - (2\pi / \lambda_o) x_k y_k \hat{v}] = 0 \quad (A-6)$$

$$\sum_{k=1}^m [\delta_k y_k - (2\pi / \lambda_o) x_k y_k \hat{u} - (2\pi / \lambda_o) y_k^2 \hat{v}] = 0 \quad (A-7)$$

The projections x_k and y_k are uncorrelated (because the element locations are randomly distributed). Accordingly,

$$\sum_{k=1}^m x_k y_k \approx 0, \text{ elements randomly distributed} \quad (\text{A-8})$$

Substituting equation (A-8) into equations (A-6) and (A-7),

$$\begin{aligned} \hat{u} &= \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^m x_k \delta_k}{\sum_{k=1}^m x_k^2} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^m x_k [(2\pi / \lambda_o) x_k u + (\alpha_k - \bar{\alpha})]}{\sum_{k=1}^m x_k^2} \\ &= u + \frac{\lambda_o}{2\pi r_A} \frac{\sum_{k=1}^m (x_k / r_A) (\alpha_k - \bar{\alpha})}{\sum_{k=1}^m (x_k / r_A)^2}; x_k, y_k \text{ are uncorrelated} \end{aligned} \quad (\text{A-9})$$

$$\begin{aligned} \hat{v} &= \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^m y_k \delta_k}{\sum_{k=1}^m y_k^2} = \frac{\lambda_o}{2\pi} \frac{\sum_{k=1}^m y_k [(2\pi / \lambda_o) y_k v + (\alpha_k - \bar{\alpha})]}{\sum_{k=1}^m y_k^2} \\ &= v + \frac{\lambda_o}{2\pi r_A} \frac{\sum_{k=1}^m (y_k / r_A) (\alpha_k - \bar{\alpha})}{\sum_{k=1}^m (y_k / r_A)^2}; x_k, y_k \text{ are uncorrelated} \end{aligned} \quad (\text{A-10})$$

It should be noted that $\hat{u} = u$, $\hat{v} = v$ for the cases of homogeneous earth ($\alpha_k = \bar{\alpha}$) and non-homogeneous earth with randomly-distributed elements and non-homogeneities ($\sum x_k (\alpha_k - \bar{\alpha}) = \sum y_k (\alpha_k - \bar{\alpha}) = 0$). However, for non-homogeneous earth with systematically-distributed elements or non-homogeneities ($\sum x_k (\alpha_k - \bar{\alpha}) = \sum y_k (\alpha_k - \bar{\alpha}) \neq 0$),

then $\hat{u} \neq u$, $\hat{v} \neq v$.

The RMS phase error α_B contributed by beam pointing errors is given by

$$\alpha_B = (1/m) \sum_{k=1}^m [(2\pi r_A / \lambda_o)^2 \{(x_k / r_A)(\hat{u} - u) + (y_k / r_A)(\hat{v} - v)\}^2] \quad (A-11)$$

where $\hat{u} = u$, $\hat{v} = v$ are given by equations (A-9) and (A-10), respectively.

Since x_k and y_k have been assumed to be uncorrelated, equation (A-11) reduces to

$$\alpha_B = (2\pi r_A / \lambda_o) [(1/m) (\hat{u} - u)^2 \sum_{k=1}^m (x_k / r_A)^2 + (1/m) (\hat{v} - v)^2 \sum_{k=1}^m (y_k / r_A)^2]^{1/2},$$

x_k, y_k are uncorrelated (A-12)

Substituting equations (A-9) and (A-10) into equation (A-12),

$$\alpha_B = \left\{ \frac{(1/m) [\sum_{k=1}^m (x_k / r_A) (\alpha_k - \bar{\alpha})]^2}{\sum_{k=1}^m (x_k / r_A)^2} + \frac{(1/m) [\sum_{k=1}^m (y_k / r_A) (\alpha_k - \bar{\alpha})]^2}{\sum_{k=1}^m (y_k / r_A)^2} \right\}^{1/2}, \quad (A-13)$$

x_k, y_k are uncorrelated

The beam pointing errors B_θ , B_ϕ in the elevation and azimuthal directions, respectively, are defined as

$$B_\theta = \theta_{\text{apparent}} - \theta \quad (A-14)$$

$$B_\phi = \phi_{\text{apparent}} - \phi \quad (A-15)$$

where $(\theta_{\text{apparent}}, \phi_{\text{apparent}})$ is the apparent beam direction when the beam has a true direction (θ, ϕ) . The beam pointing errors B_θ, B_ϕ are related to $\hat{u}, \hat{v}$ by

$$\begin{aligned}\hat{u} &= \widehat{\sin \theta \cos \phi} = \widehat{\sin(\theta_{\text{apparent}} - B_\theta) \cos(\phi_{\text{apparent}} - B_\phi)} \\ &= \text{estimate} [(\sin \theta_{\text{apparent}} \cos B_\theta - \sin B_\theta \cos \theta_{\text{apparent}})(\cos \phi + B_\phi \sin \phi \\ &\quad + \sin \phi_{\text{apparent}} \sin B_\phi)] = (\sin \theta - B_\theta \cos \theta)(\cos \phi - B_\phi \sin \phi) \\ &\approx \sin \theta \cos \phi - B_\theta \cos \theta \cos \phi + B_\phi \sin \theta \sin \phi, \quad B_\theta \ll 1 \text{ rad}, B_\phi \ll 1 \text{ rad} \quad (\text{A-16})\end{aligned}$$

$$\begin{aligned}\hat{v} &= \widehat{\sin \theta \sin \phi} = \widehat{\sin(\theta_{\text{apparent}} - B_\theta) \sin(\phi_{\text{apparent}} - B_\phi)} \\ &= \text{estimate} [(\sin \theta_{\text{apparent}} \cos B_\theta - \sin B_\theta \cos \theta_{\text{apparent}})(\sin \phi_{\text{apparent}} \cos B_\phi \\ &\quad - \cos \phi_{\text{apparent}} \sin B_\phi)] = (\sin \theta - B_\theta \cos \theta)(\sin \phi - B_\phi \cos \phi) \\ &\approx \sin \theta \sin \phi - B_\theta \cos \theta \cos \phi - B_\phi \sin \theta \cos \phi, \quad B_\theta \ll 1 \text{ rad}, B_\phi \ll 1 \text{ rad} \quad (\text{A-17})\end{aligned}$$

Rearranging terms,

$$\hat{u} - u = -(\cos \theta \cos \phi) B_\theta + (\sin \theta \sin \phi) B_\phi \quad (\text{A-18})$$

$$\hat{v} - v = -(\cos \theta \sin \phi) B_\theta + (\sin \theta \cos \phi) B_\phi \quad (\text{A-19})$$

$$\text{Denoting } \Delta = \begin{vmatrix} -\cos \theta \cos \phi & \sin \theta \sin \phi \\ -\cos \theta \sin \phi & -\sin \theta \cos \phi \end{vmatrix} = \cos \theta \sin \theta,$$

$$B_\theta = \frac{1}{\Delta} \begin{vmatrix} \hat{u} - u & \sin \theta \sin \phi \\ \hat{v} - v & -\sin \theta \cos \phi \end{vmatrix}$$

$$\begin{aligned}
&= (1 / \Delta)[(\hat{u} - u)(-\sin \theta \cos \phi) - (\hat{v} - v)(\sin \theta \sin \phi)] \\
&= -(1 / \cos \theta)[(\hat{u} - u) \cos \phi + (\hat{v} - v) \sin \phi], \quad B_\theta \ll 1 \text{ rad}
\end{aligned} \tag{A-20}$$

$$\begin{aligned}
B_\phi &= \frac{1}{\Delta} \begin{vmatrix} -\cos \theta \cos \phi & \hat{u} - u \\ -\cos \theta \sin \phi & \hat{v} - v \end{vmatrix} \\
&= (1 / \Delta)[(\hat{v} - v)(-\cos \theta \cos \phi) - (\hat{u} - u)(\cos \theta \sin \phi)] \\
&= (1 / \sin \theta)[(\hat{u} - u) \sin \phi + (\hat{v} - v) \cos \phi], \quad B_\phi \ll 1 \text{ rad}
\end{aligned} \tag{A-21}$$

where $\hat{u} - u$, $\hat{v} - v$ are given by equations (A-9) and (A-10), respectively.

REFERENCES TO APPENDIX

1. R. S. Elliot, 1981, *Antenna Theory and Design*, Prentice-Hill, Englewood, Cliffs, N.J., p. 114.