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INTRODUCTION

Colloquium 127, *Reference Systems*, was proposed by the U.S. Naval Observatory during the XX General Assembly in Baltimore. The proposal was accepted by the IAU Executive Committee and the dates, 14-20 October 1990, and the venue, Virginia Beach, were given in the First Announcement mailed in February 1990.

This Colloquium was somewhat unique in that its primary purpose was to serve as a meeting opportunity for the IAU Working Group On Reference Systems (WGRS). In conjunction with this pivotal animus however, the meeting also provided an excellent forum for the presentation and discussion of matters essential to the concerns of the WGRS by individuals whether members of the WGRS or not. Thus both invited oral and poster papers were presented on topics ranging from theoretical relativistic considerations to new observational programs and results. The oral contributions are printed here in the order in which they were presented at the meeting. The poster papers are given in alphabetical order by the first author's name.

The Colloquium was characterized by the inclusion of several discussion periods. Some of these were closed sessions of the WGRS, but most of the discussions were open to all participants. The final recommendations of the WGRS evolved from these discussions. The starting points for this evolution are contained in the opening papers given by the four leaders of the sub-groups of the WGRS, namely, B. Guinot, J. Kovalevsky, T. Fukushima and D. McCarthy. It is interesting to compare these papers with the final form of the recommendations as they appear in the Appendix of these Proceedings.

Recordings were made of the exchanges which took place after the presentations of the oral papers, and the original intention was to publish this material as is usually done. However, due to the high level of interest and the associated disinclination to curtail discussion, the length of the transcription was overwhelming. Straightforward question and answer exchanges were rare, with long commentaries being more the rule. Thus, to keep these Proceedings to a reasonable size, it would have been necessary to severely edit the commentary. The first editor made a few attempts at such a procedure, but it soon became clear that such editing, without detailed discussions with the speakers, was not likely to faithfully reproduce the ideas expressed. Therefore, with some reluctance, it was decided to forego printing this material.

In general, it is fair to say that the meeting has been deemed a success by the vast majority of the attendees. Certainly from the point of view of the SOC and the WGRS it may be said that the objectives of the Colloquium were well met. It is equally true that the LOC did an admirable job, and the thanks and appreciation expressed by so many are repeated here. Thanks are also due those who refereed papers both at Virginia Beach and subsequently.

Attention is drawn to the final Recommendations of the WGRS which, together with a copy of the letter forwarding them to the IAU, appear as an Appendix to this volume.

James A. Hughes
Chairman, SOC, WGRS

ACKNOWLEDGEMENTS

Attendance at the Colloquium was greatly increased and many miscellaneous expenses were covered due to the generous financial support of the International Astronomical Union, the National Aeronautics and Space Administration and the Office of Naval Technology. It is a pleasure to acknowledge this support which contributed so much to the success of the Colloquium.

Thanks are also due to the U.S. Naval Observatory for hosting the meeting, and in particular for providing administrative and logistic support. Not only the institution, but many of its professional and clerical staff, rendered assistance where and when needed. Such contributions addressed many of the critical needs of the Colloquium and of its participants.

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PART 1. ORAL PAPERS

REPORT OF THE SUB-GROUP ON TIME

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1. Introduction

The report of the Sub-Group on Time (SGT) of the IAU Working Group on Reference Systems (WGRS) is divided as follows.

Sections 2 to 4 recall the scope of the work of the SGT and briefly describe its activity.

Section 5 summarizes the discussions on several topics which form the background of the draft recommendations and notes of Section 6.

2. Scope of the work

This has been defined in a letter of 1 June 1989 by the chairman of the WGRS, Dr J.A. Hughes, : "Define time- scales and time-like arguments and their interrelationships within the framework of General Relativity. Provision must be made for projecting these time scales into the past. The consequences of adopting any time-scale, when viewed in the context of a relativistic space-time continuum, must be carefully considered. The need for theoretically acceptable transformations meeting the accuracy requirements for all the diverse applications must be considered. The practical difficulty of defining and determining appropriate units of time as well as the precise transfer of time must also be addressed with reference to protocols already established."

3. Historical background

The atomic definition of the unit of time, in 1967, and the availability of atomic time scales since 1955, with accuracy requiring a relativistic treatment, generated new problems in dynamical astronomy. These problems were considered by a Working Group on Units and Time-Scales, created in 1970, chaired by G.A. Wilkins. The activity of this SG led to IAU Recommendation 5 (1976) on "Time-scales for dynamical theories and ephemerides", defining the time-scales which were designated in 1979 as the Terrestrial Dynamical Time (TDT) and the Barycentric Dynamical Time (TDB).

The 1976/79 recommendations raised much controversy (Guinot and Seidelmann, 1988) for reasons which are essentially:
-the lack of a correct definition in a relativistic framework encompassing space and time,
-the ambiguity between the ideal form of atomic time and the time-like argument of dynamical theories.

4. Development of the work of the SGT

The list of members of the SGT is given in Annex I. The necessity of receiving a wider range of opinions and advices, especially from experts in relativity, appeared in the course of the discussions. Annex I also gives the list of persons who were consulted. In the following, all these persons are usually designated by the initials given in Annex I.

The development of the activity of the SG on Coordinate Frames and Origins (SGFO) made clear the need of a strong interaction between the SGFO and the SGT. This led to frequent contacts with J. Kovalevsky, leader of the SGFO, and to common draft recommendations, as will be seen later. The work was performed almost entirely by correspondence, but I had the privilege of fruitful meetings with J. Kovalevsky and T. Damour.

The documents I received from my correspondents have been circulated. In the following abbreviated presentation, it is not possible to refer fully to the opinions which have been expressed : I rather concentrate on controversial topics. In the subtle problems which were encountered, opinions may diverge, even if based on correct scientific reasoning. In such cases, I followed the majority, at least when it is well marked.

5. Background of draft recommendations and notes

5.1. SPACE-TIME (DRAFT RECOMMENDATIONS G1 AND G2)

The necessity of a global treatment of space-time coordinates was unanimously recognized: "We must resolve the complete question of space and time transformations" (I quote L). In such a treatment there appears "the necessity of defining and using on an equal footing several systems of space-time coordinates, especially the systems centered at the barycenter of the solar system and of the Earth" (D). Brumberg and Kopejkin (1990) give a comprehensive study of "a set of reference systems mutually superimposing and covering altogether the whole of space-time" and advocate the use of non-rotating systems with the harmonic condition on the metric tensor; their work offers a complete and self-consistent set of coordinate transformations for the four space-time coordinates.

However, the choice of the coordinate conditions is still a domain of theoretical researches (Damour et al., 1990) and no particular solution can be recommended officially, at present.

It is in the nature of our work on recommended references and constants that the IAU recommendations often cannot meet the requirements of the most advanced studies : the freedom of innovating is essential. We were guided, J. Kovalevsky and myself, by the necessity of offering a sound basis which is a good and accepted approximation of the most advanced theories and which can be improved without bringing drastic changes when it it is desirable and possible.

These considerations led to the draft of the Conceptual Recommendation G1, common to the SGFO and SGT, initially written by D, then slightly modified by K and Gu.

I would like to stress that G1 was made quite general and covers the cases where, for example, selenocentric or planetocentric coordinate systems are convenient. In the case of the system centered at the barycenter of the solar system ("barycentric system"), the effect of external bodies is presently negligible, but this is nevertheless covered by the general wording of the recommendation.

The Constraint Recommendation G2 fixes the state of rotation of the space coordinate grids by a constraint on barycentric coordinates, while the time coordinates are defined by a geocentric constraint. This was imposed by the physical measurement methods and did not meet any objection. Several correspondents observed that the state of rotation of the geocentric grid is left undetermined: this aspect of the barycentric/geocentric transformation cannot be solved unambiguously at the level of the metric in G1.

Both G1 and G2 exclude scaling factors of the units of time and length in the coordinate transformations, using the metric coefficients of G1. This important issue is discussed in 5.2.

5.2 THE PROBLEM OF SCALING THE UNITS

We can reasonably assume that the physical units of the International System of Units (SI) are given by ideal standards on their world line, because it ensures the universality of physical measurements and constants in local experiments, devoid of any conversion factors. It is well known that this point of view leads to secular divergence between the coordinate times in various coordinate systems and also between coordinate times and proper time, if we assume that far from the space origin of the coordinate systems, the metrics tends to be Minkowskian, as in G1.

In Recommendation 5 (1976), secular divergence between the ideal form of TAI (TDT + const.), the geocentric and barycentric coordinate times were cancelled by retaining only the periodic terms in the transformations. This is equivalent to the introduction of "scaling factors" in the unit of time, and, therefore in the unit of length, assuming a constant value of the velocity of light. As far as I remember, this decision was taken without controversy. However, difficulties appeared later.

(a) A practical problem is that the distinction between secular and periodic terms is not clear when long periods are considered. The distinction is even not possible in case of numerical integration, the separation then depending upon the averaging time.

(b) A more fundamental problem is the consequence of scaling factors on other units, constants and values of physical quantities (Fukushima et al., 1986).

It turned out, in our present discussions, that the dissatisfaction with the introduction of scaling factors was latent and crystallized in the form of a majority in favor of making them equal to unity.

Since it is an important change with respect to the 1976 Recommendation, with consequences for other sub-groups, especially on astronomical constants, I will discuss it further, for the benefit of those who did not participate in the work of the SGT, by summarizing the most characteristic opinions I received.

I first quote D. "This choice of normalization [scaling factors equal to 1] ensures the smooth merging of these relativistic frames with all the standard approximate description of the relativistic gravitational field... This normalization has the great advantage that it leads to simple formulas for extracting the physical 'gravitational mass' GM, expressed in SI units, from measured properties of the motions... If one uses a different normalization of e.g. time scales, this leads not only to an appreciable complication of the theoretical relativistic description, but, moreover, this can lead to real physical mistakes, as it implies that the various GM's that one can read off from the metric coefficients, or equations of the motion, for various systems differ from the physically well defined SI-measured GM by some 'redshift factors'... In conclusion, I think that the spirit of universality that motivated the definition of the International System of Units, has, to-day, the consequence that one should abandon the IAU recommendations of linking in a 'regional' way the graduation units of both Earth-based and Solar-system-based time scales to the SI second, and should prefer a more 'universal' (and theoretically preferred) way of linking them by requiring the graduation units to tend asymptotically in space, in each reference system when one neglects external influences, to the physical units : meter and second."

Murray writes : "You are right in quoting my opinion that we should avoid conventions which are adopted for pratical reasons, but which obscure principles. My reaction to the statement in Question (h) [The SGT recommends that the convention requiring that in the relation TDB-TDT only periodic terms are kept be abandoned] is, therefore, approval in principle; this should be accompanied by a corresponding statement that the unit of TDT differs from the SI second by the geopotential factor. However, I realized that to change now might cause difficulties for some people; therefore, before a decision is made, a careful survey of the likely consequences for the existing software used by observers and theoreticians should be carried out."

Fukushima has first expressed some fears that the user be confused by secular differences among time-scales. Then he expressed his agreement in keeping secular parts in the relations. "This is because this option makes the new system of astronomical constants <u>simpler</u>". He agrees with the "opinion that unnecessary conventions should be avoided, <u>as long as</u> the ordinary users will not be confused".

At this stage, I would like to recall that the secular divergence between TAI and a barycentric coordinate time, without rescaling the units, is $1{,}55 \times 10^{-8}T$, and amounts to 49s in a century. It is of the same order as the difference between TAI and UTC (if the UTC system is maintained under the present form). These differences are sufficiently small to avoid any risk of confusion in the day number in a reasonable future. Anyway, draft recommendation T2 states that the apparent geocentric ephemerides shoud use a time scale without rate offset with respect to TAI and UTC : this should avoid confusion for the 'ordinary user'.

Other persons have expressed their agreement with the proposal of setting to 1 the scaling factors : B (and Kopejkin), Gr, K, Se, and myself, for similar reasons as above. In contrast, A and X disagree.

Xu writes that he "rather disapprove, it is inconvenient to treat old observations".

Aoki recalls that the Kepler's third law holds in the isotropic form of the Schwartzschild coordinates when the mean motion n of an orbiting test particle is expressed with the proper time along the world line of the particle (for details refer to Murray, 1983, Chap. I).But using the coordinate time at the barycenter, we have

$$n = (1 - L)\, k \quad \mathrm{rad}/D_B, \tag{1}$$

where k is the Gaussian gravitational constant, D_B is a day of 86400 coordinate seconds of the barycentric frame and L is given by

$$L = 3GM / 2c^2A \approx 1{,}48 \times 10^{-8}, \tag{2}$$

A, being the astronomical unit, all quantities being expressed in SI units.

We can make L = 0 in (1) by rescaling the unit of time, as it was done by the IAU Recommendation 5 (1976) on time scales. But the consequence is that L appears in the units of time and length, and in most of the quantities expressed with these units (in particular the GM's).

I would like to point out that accepting to rescale the units of time and length is not limited to the geocentric and barycentric frames, but could be extended to other frames centered on planets, barycenter of multiple stars, etc. This would be extremely confusing.

The draft recommendations of Sub-Groups of the WGRS are a first step toward a correct and coherent treatment of the reference systems in relativistic theories. I am aware that much remains to be done, but, at least, we must start on a sound basis. We have to accept all the consequences of relativistic theories. I am personally opposed to unnecessary conventions adopted on purely practical grounds. In the present case, I even fail to see the practical advantages of the scaling factors.

These considerations led to define :
(a) a Terrestrial Time TT (draft T2), which is an ideal form of TAI (with possible constant time offset, see discussion below),
(b) a Geocentyric Coordinate Time TCG, with a constant frequency offset with respect to TT, due to the fact that the unit of TT and TAI is the second of SI as obtained on the geoid (draft T1),
(c) a Barycentric Coordinate Time TCB, keeping all the terms of the TCB - TCG conversion, including a mean frequency offset (draft T1).

All my correspondents estimated that the relativistic definition of TAI, given in a CCDS declaration in 1980, is sufficient for the time being (see Huang et al.,1989).

5.3. ORIGINS OF TT, TCG, TCB

In a previous draft recommendation TB, I suggested that TT have the same reading as TAI on 1977 January 1. With such an origin, it would have been possible, in many cases, to use TAI (available) instead of TT as time argument of ephemerides. This proposal has been unanimously rejected. The enclosed draft recommendations retain the historical time offset of 32,184s, so that TT is equivalent to TDT of the 1976/79 recommendations. Possible realizations of TT are, using the notation of note (h) of draft T2:

until July 1955 TT(TEi) = TEi,
since July 1955 TT(TAI) = TAI + 32,184s.

5.4. THE TCB - TCG RELATIONSHIP

The TCB - TCG relationship is a full 4-dimensional transformation of coordinates. Most of my correspondents expressed the wish that a conventional development of the 'geocentric' part be given, but no clear preference has been expressed between the use of numerical integration and analytical formulas. Therefore two possibilities are offered in note (c) of T1.

5.5. TIME-LIKE ARGUMENT OF THEORIES

An important problem is raised by Se: "Presumably we can have a theoretical version of TCB and TCG, which is the time-like argument for a planetary theory or numerical integration, before being fit to observations. Would that be designated as TCB(Theo)?". According to T1 and T2, TT, TCG and TCB are definitely ideal forms of atomic time. I reproduce here my answer to Se.

"Let us take the example of TCB. My proposal is that TCB be the ideal form of TAI, transformed in the barycentric frame, in an ideal way. Thus TCB is an ideal form of quantum time. The chain of transformations is :

TAI realized atomic time,
TT ideal form of TAI,
TCB resulting from tranformation topocentric to barycentric of TT.

These two steps involve uncertainties :

TAI - TT , physical defects of time standards,

TT - TCB , approximation of the theory and of the
required numerical constants.

For these reasons, in note (e) of recommendation T1, I suggest designating a realization of TCB by TCB(xxx), where xxx states the source of the realized time scale and the theory.

Now, the time-like argument of theories, *t*, is something different which might not be easily reconciled with TCB, especially if there is some fundamental divergence between dynamical time and quantum time. But in the latter case, there should be a relation between *t* and TCB.

As I see this problem, the ideal TCB should be approximated from two sides :
-by the best possible realization from terrestrial clocks (to which observations are ultimately referred),
-by the most adequate relationship with the time-like argument of theories. This time argument could be called TCB(theor), as you propose, but I would prefer some freedom for its designation; the letter *t* seems convenient."

5.6. Designation of time-scales

As said previously, TT is equivalent to TDT of 1976/79. I nevertheless suggest abandoning the letter "D" which is confusing because it suggests that TDT is obtained from a dynamical theory, as was Ephemeris Time. For TCG and TCB, I propose notations which seem to be more explicit.

5.7. Coordinated Universal Time

Fukushima and Zhu stress the inconvenience of the leap seconds of UTC. The definition of UTC is primarily a matter to be considered by CCIR, IMO (International Maritime Organization), ICAO (International Civil Aviation Organization). Our only possible action could be, I believe, to ask for a reevaluation of the usefulness of the UTC system. I leave this question open for discussion.

6. Draft recommendations and notes

6.1. Recommendations G1 and G2 common to SGFO and SGT

Conceptual recommendation G1

.... considering

that it is necessary to define in the framework of the General Relativity Theory several systems of space-time coordinates,

recommends that

the four space-time coordinates ($x^0 \equiv ct$, x^1, x^2, x^3) be selected in such a way that in each coordinate system centered at the barycenter of an ensemble of masses exerting the main action, the interval ds^2 be expressed at the minimum degree of approximation by

$$ds^2 = -\left(1 - \frac{2U}{c^2}\right)(dx^0)^2 + \left(1 + \frac{2U}{c^2}\right)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right]$$

where c is the velocity of light and U the sum of the gravitational potentials of the above mentioned ensemble of masses and of a potential generated by the external bodies, the latter vanishing at the barycenter.

Constraint recommendation G2

.... considering

(a) the necessity to define a barycentric coordinate system centered on the barycenter of the solar system and a geocentric coordinate system, centered on the barycenter of the Earth,

(b) the desirability that the coordinate systems be linked to the best physically realized references in space and time,

(c) that the use of the International System of Units (SI) should be extended to outer space, without introduction of scaling factors depending on the coordinate system under consideration,

recommends that

1. the state of rotation of the space coordinate grid centered at the solar system barycenter be such that the coordinates of a set of distant extragalactic objects present no global rotation,

2. the time coordinates be derived from the geocentric coordinate time realized by atomic clocks operating in conformity with the definition of the second,

3. the physical basic units of the space-time be the second of SI for the proper time and that it be connected to the meter of SI for proper length by the value of the velocity of light $c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$.

6.2. Notes on Recommendations G1 and G2

Although G1 and G2 are common recommendations of SGFO and SGT, no attempt has been made to unify the notes. The following notes are drafted for the SGT.

(a) It does not seem possible, at the present stage of our knowledge, to agree on a particular form of the metric beyond approximation given in Recommendation G1. Recommendation G1 allows the possibility of using all forms of metric accepting this approximation.

(b) Recommendations G1 and G2 exclude the use of scaling factors for the units of length and time in the transformations of coordinates. These points are considered more specifically in recommendations T1 and T2.

(c) Recommendation G2 fixes the state of rotation of the grid of barycentric coordinates by a kinematical constraint. It is recognized that G2, 1 cannot be rigorously fulfilled, either for fundamental reasons such as a possible incompatibility of reference systems dynamically and kinematically defined, or for practical reasons such as the uncertainties of the realization of the kinematical reference frame. Constraint G2, 1 must be realized in so far as possible.

(d) Recommendation G2 does not fix the state of rotation of the geocentric grid because it cannot be done unambiguously at the considered level of approximation. [These matters are considered by the SGFO].

(e) While the state of rotation of the coordinate grids is fixed by a constraint on barycentric coordinates, the time coordinates are defined by a geocentric constraint. These hybrid constraints are imposed by the need to realize the reference frames, in space and time, without degrading data obtained from measurements and from standards.

6.3. RECOMMENDATIONS AND NOTES OF THE SGT

Recommendation T1

The ....

considering

- the desirability of standardization of the units and origins of coordinate times used in astronomy,

- the importance of coordinate systems having their origins at the center of mass of the Earth and at the center of mass of the solar system,

recommends that

1. the unitary interval of coordinate times of all coordinate systems centered at the barycenter of material systems tend asymptotically to the proper SI second, far from the spatial origins of these coordinate systems,

2. the reading of these coordinate times be 1977 January 1, 0 h 0 m 32,184 s on 1977 January 1, 0 h 0 m 0 s TAI (MJD = 43 144.0..., TAI), at the geocenter,

3. coordinate times in non-rotating reference systems having their spatial origins respectively at the geocenter and at the solar system barycenter, and established in conformity with the above recommendations, be designated as Geocentric Coordinate Time (TCG) and Barycentric Coordinate Time (TCB).

Notes on Recommendation T1

(a) Recommendation T1 recognizes that the space-time cannot be covered with a single reference system, because a good choice of coordinate system may significantly facilitate the treatment of the problem at hand and elucidate the meaning of the relevant physical events. In "recommends 1", it must be understood that, far from the space origin, the potential of the material system to which the coordinate system pertains becomes negligible, while the potential of external bodies manifests itself only by tidal terms which vanish at the space origin. In the domain common for two coordinate systems "recommends 1" implies that the tensor transformation law, applied to the metric tensor, is valid without re-scaling the unit of time. Therefore, the various coordinate times under consideration exhibit secular variations. Recommendation 5 (1976) of IAU Commissions 4, 8 and 31, completed by Recommendation 5 (1979) of IAU Commissions 4, 19 and 31, stated that the Terrestrial Dynamical Time (TDT) and the Barycentric Dynamical Time (TDB) should differ only by periodic variations. This requirement has now been cancelled.

(b) According to Recommendations G1 and G2, the absence of re-scaling of the unit of time, implies the absence of re-scaling of the unit of length, in conformity with its definition by the 17th Conférence Générale des Poids et Mesures (1983). The astronomical constants are thus expressed in SI units, without conversion factors depending on the coordinate systems to which they belong.

(c) The relation TCB-TCG involves a full 4-dimensional transformation. For observers on the surface of the Earth, the terms depending on their terrestrial coordinates are diurnal, with a maximum amplitude of 2,1 μs. The numerical expression of TCB - TCG can be evaluated from the positions and velocities of the solar system bodies obtained by numerical integration, using the formula by Moyer [Moyer, T.D., 1981, Celest. Mechanics, 23, 33-68]. Another possibility is to use the analytical formula by Hirayama et al. [Proc. IUGG Symposia, Vancouver, 1987], with the secular part in conformity with the IAU System of Astronomical Constants. The secular term is approximately, in seconds

$$[\mathrm{TCB} - \mathrm{TCG}]_{\mathrm{secular}} = 1{,}4808 \times 10^{-8} \times (\mathrm{MJD} - 43144{,}0) \times 86400$$

the MJD being reckoned in TAI.

(d) The origin of the coordinate times has been arbitrarily set so that they all coincide with the Terrestrial Time TT of Recommendation T2, at the geocenter, on 1977 January 1, 0 h 0 m 0 s TAI. See note (c) of Recommendation T2.

(e) When realizations of TCB and TCG are needed, it is suggested that these realizations be designated by expressions such as TCB(xxx), where xxx states the source of the realized time scale (TAI, for example) and the theory used for the transformation into considered time.

Recommendation T2

The ....

considering

- that the time scales used for dating events observed from the surface of the Earth and for terrestrial metrology should have a unitary interval close to the SI second, as realized by terrestrial time standards,

- the definition of the International Atomic Time, TAI, approved by the 14th General Conference of Weights and Measures (1971) and completed by a declaration of the 9th session of the Comité Consultatif pour la Définition de la Seconde (1980),

recommends that

1. the time reference for apparent geocentric ephemerides be the Terrestrial Time TT,

2. TT be a coordinate time in a non-rotating geocentric coordinate system, its unitary interval being chosen so that it agrees with the SI second on the geoid,

3. at instant 1977 January 1, 0 h TAI exactly, TT have the reading 1977 January 1, 0 h 0 m 32,184 s.

Notes on Recommendation T2

(a) The basis of the measurement of time on the Earth is International Atomic Time, TAI, which is made available by the dissemination of corrections to be added to the readings of national time scales and clocks. The time scale TAI has been defined by the 59th session of the Comité International des Poids et Mesures (1970) and approved by the 14th Conférence Générale des Poids et Mesures (1971) as a realized time scale. As the errors in the realization of TAI are not always negligible, it has been found necessary to define an ideal form of TAI now designated "Terrestrial Time", TT.

(b) In order to define TT without ambiguity, it would be necessary to define the coordinate system precisely, by the metric form, to which it belongs. However, ambiguities can be tolerated if they generate frequency errors much smaller than the uncertainties of the frequency of the best standards. It is at present (1990) sufficient to consider that the reference system does not rotate, in a broad sense (i.e. with respect to the average direction of distant bodies such as quasars), and to use the metric of the first post-Newtonian approximation of the General Relativity theory.

(c) For ensuring an approximate continuity of the time argument of ephemerides, previously the Ephemeris Time, a time offset between TT and TAI is introduced, so that TT - TAI = 32,184 s on 1977 January 1, 0 h TAI. This date corresponds to the implementation of a steering process of the TAI frequency, so that the TAI unitary scale interval remain in close agreement with the best realizations of the SI second on the geoid. TT can be considered as equivalent to TDT as defined by the IAU Recommendations 5 (1976) and 5 (1979) of Commissions 4, 19 and 31.

(d) The divergence between TAI and TT is a consequence of physical defects of atomic time standards. In the interval 1977-1990 in addition to the constant offset of 32,184 s, the deviation remained probably within the approximate limits of ± 10 μs. It is expected to increase more slowly in the future, as a consequence of the progress of atomic time standards. In many cases, especially for the publication of ephemerides, this deviation is negligible. In such cases, it can be stated that the argument of the ephemerides is TAI + 32,184 s.

(e) The Terrestrial Time differs from TCG of Recommendation T1 uniquely by a scaling factor:

$$TCG - TT = 6{,}969 \times 10^{-10} \times (MJD - 43144{,}0) \times 86400 \text{ in seconds}$$

These two time scales are distinguished by different names to avoid scaling errors.

(f) The time interval unit of TT is the SI second on the geoid(coordinate second). The usual multiples such as the TT day of 86400 TT seconds, the TT julian century of 36525 TT days can be used, providing that the reference to TT be clearly indicated. The corresponding time interval units of TAI are in agreement with the TT units within the uncertainties of the primary atomic time standards (for example, within $\pm 2 \times 10^{-14}$ in 1990, on yearly average).

(g) The markers of the TT scale can follow any date system based on the TT second, for example the usual calendar date or the Modified Julian Date, providing that the reference to TT be clearly indicated.

(h) It is suggested that realizations of TT be designated by TT(xxx) where xxx is an identifyer. In most cases, a convenient approximation is

$$TT(TAI) = TAI + 32{,}184 \text{ s.}$$

But in some applications it may be advantageous to use other realizations; for example, the BIPM has issued time scales such as TT(BIPM90).

7. Acknowledgement

I consider this report as a collective work. We must thank all those who participated in the preparatory discussions, who are mentioned in the text and appear in Annex I

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Annex I

Working Group on Reference Systems

Sub-Group on Time

1. Members of the Sub-Group

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PRELIMINARY REPORT OF THE WORK OF THE SUBGROUP ON COORDINATE FRAMES AND ORIGINS

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1. PARTICIPANTS IN THE WORK OF THE SUBGROUP

The membership of the subgroup on coordinate frames and origins included the following : V.K. Abalakin, S. Aoki, F. Arias, C. Boucher, N. Capitaine, K. Johnston, J. Kovalevsky (chairman), C. Ma, I.I. Mueller, C.A. Murray, H. Schwan, C.A. Smith, C. de Vegt and R. Wielen.

In addition, the following colleagues have contributed to the work : V.A. Brumberg, T. Damour, J. Dickey, M. Feissel, T. Fukushima, B. Guinot, T. Huang, M. Standish, J.G. Williams and B.X. Xu.

2. SCOPE OF THE STUDY

The subgroup on coordinate frames and origins of the IAU Working group on reference systems was formed by J.A. Hughes as a follow up of the resolution C1 adopted by the IAU General Assembly in Baltimore. In particular, its work was to propose a practical realization of this resolution and, in particular of its section 3 :

"The International Astronomical Union should adopt a celestial reference based upon a consistent set of coordinates for a sufficient number of suitable extragalactic objects when the required observational data have been successfully obtained and appropriately analyzed. This reference frame should be based upon a common, simultaneous discussion of the observations using agreed upon conventions. This reference frame is likely to be based, initially at least, exclusively upon radio astrometry, and transformations between this reference frame and the conventional celestial and terrestrial reference systems as well as the dynamical frame should be defined. The reference frame should be updated as required".

To this, J.A. Hughes added the task of considering the origin of the celestial reference frame, having in mind that this problem is, "a separable problem which can be addressed somewhat independently. For example, the origins of the coordinate frames used for celestial coordinates and the metering of the Earth's rotation need not, in principle, be identical".

In practice, the problem of the terrestrial reference frame was not considered because I judged that this is essentially a problem to be addressed first by IUGG. This has been confirmed by C. Boucher who will make a proposal to this body. The presence of geodesists on the subgroup was intended to ensure that the proposals for a celestial reference frame and origins would not be incompatible with what could be the terrestrial reference frame in the future.

During the course of the work of the subgroup, I also decided not to discuss the origin of the Earth's rotation reckoning, because it was linked to the definition we would adopt for the celestial reference frame. I underestimated the difficulty of reaching a consensus on this first problem, but it can now be taken up at this stage.

3. TERMINOLOGY

In this presentation, the following terminology will be used :

i) **Ideal reference system** : Theoretical principle on which the final reference frame is based.
Example 1 : the equations of motion of a set of celestial bodies should have no Coriolis or linear acceleration terms when written in the ideal reference system.
Example 2 : the ensemble of very distant bodies has no global rotation in the ideal reference system.

ii) **Reference system** : It identifies the physical system on which the ideal reference system definition is applied. The solar system together with the physical laws governing it (general relativity or Newtonian mechanics) corresponds to the first example above. For the second example, a certain number of quasars form the system with a recipe on how the "non-rotation" is obtained..

iii) **Conventional reference system** : In addition to the statements 1 and 2, parameters describing the physical system are assigned (and are therefore conventional).
Example : masses and initial conditions of motions in the first case; they are given in the system of fundamental constants. In the case of extragalactic objects, a list of such objects will be given. The definition of the coordinate axes must also be given.

iv) **Conventional reference frame or, simply, reference frame** : It is a set of fiducial points with their coordinates that materialize the conventional reference system. The origin and axes of coordinates may either be materialized by, or simply inferred from the coordinates of the fiducial points. The coordinates of a point is obtained by interpolating the coordinates of fiducial points.

4. BACKGROUND OF THE REFERENCE FRAME DEFINITION

Before presenting the report on the work of the subgroup, I believe that it is appropriate to analyze somewhat deeply the meaning of the IAU resolution and its consequences. To do this, let us first examine the original meaning and the evolution of the notion of celestial reference frames and systems.

The objective of a celestial reference frame is to provide a means of assigning, in a unique way, coordinates of a celestial body, whether observed by an instrument, or derived from some theory. This can be - and has been - done by different methods. Let us examine them.

4.1. Definition of a system of coordinates

The most direct answer to the problem is to construct a system of rectangular or spherical coordinates in space. This is possible since there exists an ensemble of material bodies - Sun, planets and satellites - whose motions can be described by a theory referred to some cartesian coordinate system. The observed coordinates of these bodies are moving markers of the coordinate system. It is then sufficient to define an origin and a scale. The vernal equinox at a given date, and the mean equator at the same date, are a possible choice together with the astronomical unit of distance. Together with an absolute time, we obtain a self consistent definition of a dynamical reference system in the Newtonian sense.

Unfortunately, this not sufficient. We have no practical way of assigning the coordinates to α Centauri from a direct comparison of its position with the observed positions of one or several planets. This has to be done by a very complex procedure applied to a number of stars, so that one obtains a *fundamental catalogue* of stars, like the FK4. It is this catalogue that is used to determine the position of other bodies by relative astrometry. At this point, the user forgets completely the definition of the dynamical system, and only uses the frame represented by the catalogue.

4.2. The case of the FK5

As pointed out by H. Schwan and R. Wielen, and in opposition to a common opinion (that I must confess to having shared), the FK5 system is not a dynamical system, but is essentially based on some determination of galactic rotation. Of course, this was ultimately linked to the position of the equinox point, in particular via a new constant of precession. But this is only an *ad'hoc, a posteriori* link.

So, although FK5 frame looks very much like the FK4 frame, the FK5 system is dramatically different. It is a kinematic reference system, based upon a certain model of galactic rotation. This choice was made because the observations of bodies in the solar system did not permit a sufficiently accurate definition of fixed points and a sufficiently accurate procedure for accessing these fixed coordinate axes.

4.3. Accessibility of a position in a fixed coordinate system

To obtain the coordinates of a body in the present reference system one requires a number of conventional parameters such as, the constant of precession, a theory of nutation, etc... that are to be applied in order to obtain its position in fixed coordinate system, chosen as being the position for J.2000.0 of the instantaneous mean equator and equinox. Any error in these conventional values introduces a spurious additional apparent motion of objects. This remark is particularly important when comparing the positions of quasars observed at several times and reduced to a given epoch by the present IAU conventional precession and nutation. This is equivalent, in the best cases, to a rotation of the system, more likely a rotation plus a deformation, the latter being due to systematic, position dependent errors in the proper motions.

The IAU resolution proposes a very elegant and easy solution for most of these problems. Let us, for clarity of exposition, assume that with an extragalactic objects are observable in the same wavelengths as the body with an unknown position. Since these objects are fixed, they materialize at *any moment* the reference system, and one will have directly the apparent position of the body in the coordinate system, without any use of precession or nutation correction. The reference system is directly accessed. This is a major simplification and corresponds exactly to what mathematically one means by a system of coordinates.

If the unknown body is observable only in visible light, one would interpolate its observation between observations of stars in a catalogue that is constructed with respect to the extragalactic frame defined beforehand. The positions of stars will be given at any epoch using a position at some date and proper motions. They will constitute an intermediary catalogue that may be continuously updated and improved, but it will be possible at any moment, and in any direction of the sky to link them with the actual extragalactic reference, since they will always be available as are presently available FK5 stars.

4.4. Four dimensional reference system

One of the drawbacks of the present FK5 reference system is that it is two dimensional, giving two angular parameters of position. In this sense it is not complete and it has to be completed by dimensional units. This is done by introducing a decoupled system of astronomical constants. This is no longer acceptable, and one should think of four dimensional reference system since it is now impossible to ignore General Relativity. However, a complete consistency in the four dimensions would have the drawbacks described in section 4.1. Therefore in some ways, one has to accept a certain decoupling. The proposal made later in this paper reduces this decoupling to a minimum since it imposes only SI units for time and length.

5. DEVELOPMENT OF THE WORK

The work of the subgroup was done mostly by correspondence, although I had the privilege of direct discussions with about half of the members. In addition, I took the advice of several persons who were not members of the subgroup. After my first introductory letter sent on July 27, 1989, I prepared three others, taking into account the letters received previously. These answers posed, in the beginning, more questions and put forward more difficulties than they solved, so that it was not possible to test a first definition before April of this year. The present report is essentially based on the proposals included in my fourth letter and on the answers received thereafter. However, before entering into this presentation, it is useful to briefly describe some of the problems raised in the first part of the work.

5.1. Structure of the reference system

From answers to my first letter, it appeared that several members did not accept the IAU resolution and insisted upon either keeping the present situation, or having a dynamical definition of the reference system based upon the dynamics of the solar system.

Concerning the second point of view, my challenge to formalize a strictly dynamical definition was not taken up, and nothing was proposed until an incomplete attempt by S. Aoki was received. It will be presented and discussed during the meeting. However, dynamical aspects have indeed to be included in the definition and this was implemented later.

The conservative point of view corresponds to a concern that the reference system should be accessible to all observers and in particular, to optical astrometry. This objection however is waived by the first section of the resolution C1 of the IAU which reads :

"In order to avoid a confusing proliferation of reference frames, the FK5 should be retained as the IAU reference at optical wavelengths for the present and immediate future".

It is clear that this will remain true until a satisfactory extension of the radio frame to optically observable objects is achieved. Taking this into account, the opposition to a "primary radio-frame" lost part of its supporters. For this reason, in the following parts of the work I considered this problem as settled. Another point of doubt is the possibility of reconciling the variety of catalogues of radio sources into a single system. Later results and some presentations at this colloquium should dispel the fears.

But the problems that were encountered should not hide the fact that the majority of the members of the subgroup do agree with the necessity of shifting quickly to a definition of a celestial reference frame based upon a system represented, as far as directions are concerned, by extragalactic objects.

5.2. Relativistic aspects

Although all agreed that the definition of a celestial reference system should be expressed in a form compatible with General Relativity, there was almost no input from the members on how to express this. I am grateful to T. Damour who finally suggested the general structure of the wording which will be presented here. It has the advantage of being consistent with the definitions of time scales and of being sufficiently concise so as to permit subsequent additions to the expression for ds^2 if required.

5.3. Non-rotation of an extragalactic reference frame

Some members of the subgroup expressed fears that the coordinate system derived from the positions of extragalactic radio sources such as quasars would have a rotation. Let me quote a couple of sentences from a letter by R. Wielen: *"It is an observed fact (and not only a plausible assumption) that the universe does not rotate within the very small errors of measurements. A stringent upper limit for the rotation of the universe is much beyond our present needs in astrometry".* I believe that this point should not bring any difficulty. However a definition depending on the coordinates of two sources would not fulfill the non-rotation condition because of the possible instability of the sources, and one needs a large number of sources to stabilize the global behaviour. Actually, even if a rotation of the Universe is discovered, it will be orders of magnitude smaller than the rotation of the FK5 system realized by the present IAU conventional constant of precession.

5.4. Presentation of the provisional conclusions

In my last letter to the members of the subgroup, dated July 17, I made a proposal that now has to be discussed by the Colloquium. It consists of two general draft resolutions that provide the basis not only of the celestial reference system, but also of the time scales: hence they are common to our subgroup and to the subgroup on time scales. Two other draft recommendations are intended to describe how, from these conceptual recommendations, one could achieve the construction of a reference frame.

6. GENERAL RECOMMENDATION

6.1. Conceptual recommendation (G1)

.... considering

that it is necessary to define in the framework of the General Relativity Theory several systems of space-time coordinates,

recommends

the four space-time coordinates ($x^0 = ct, x^1, x^2, x^3$) be selected in such a way that in each coordinate system centered at the barycenter of an ensemble of masses exerting the main action, the interval ds^2 be expressed at the minimum degree of approximation by

$$ds^2 = -(1-\frac{2U}{c^2})(dx^0)^2 + (1+\frac{2U}{c^2})\left[(dx^1)^2 + (dx^2)^2 + dx^3)^2\right] \qquad (1)$$

where c is the velocity of light and U the sum of the gravitational potentials of the above mentioned ensemble of masses and of a potential generated by the external bodies, the latter vanishing at the barycenter.

This recommendation explicitely introduces General Relativity as the theoretical background for the definition. Only two terms, common to several representations of the field are kept, neglecting those terms that are different but are presently beyond the reach of observations. This permits the use of any of the possible more elaborate representation compatible with the given terms and still remain in conformity with the definition. It assumes the relevant PPN parameters to be equal to their values in General Relativity proper and leaves the possibility to add higher order terms. Finally it may apply to any frame origin (geocentric, barycentric, etc...). All members of the subgroup agreed with this principle which actually also introduces dynamics in the definitions.

6.2. Constraint recommendation (G2)

... considering

a) the necessity to define a barycentric coordinate system centered on the barycenter of the solar system and a geocentric coordinate system, centered on the barycenter of the Earth,

b) the desirability that the coordinate systems be linked to the best physically realized references in space and time,

c) that the use of the International System of Units (SI) should be extended to outer space, without introduction of scaling factors depending on the coordinate system under consideration,

recommends that

1. *the state of rotation of the space coordinate grid centered at the solar system barycenter be such that the coordinates of a set of distant extragalactic objects present no global rotation,*
2. *the time coordinates be derived from the geocentric coordinate time realized by ato mic clocks operating in conformity with the definition of the second,*
3. *the physical basic units of the space-time be the second of SI for the proper time and that it be connected to the meter of SI for proper length by the value of the velocity of light c=299 792 458 ms*$^{-1}$.

This recommendation is the one that actually defines the characteristics of the reference systems. It fixes the state of rotation of the system (but its realization will have, of course, a certain amount of arbitrariness). To clarify the first point, one might state that the set should be large enough (not only two objects). The second part of the recommendation is to be discussed by the subgroup on time. It is to be noticed that these two recommendations exclude the use of scaling factors for the units of length and time.

T. Fukushima proposes to add that there should be no secular rotation among the space grids for coordinate systems to be used. I am not sure that this is compatible with 1 as applied directly to barycentric and geocentric frames. This is not true for a more general expression of the ds^2. If it is true for the present case, it does add anything.

A. Murray suggests that this recommendation should include that the reference frame is defined by the ephemeris which have been derived from the relativistic equations of motion. I do not agree with this. The definition as given here plus a set of positions of quasars as implied by (1) is sufficient. The proposed addition would introduce internal inconsistencies.

Most of the correspondents agreed with this wording. S. Aoki however, presented a drastically different proposal. It will not be discussed here since it is described elsewhere in this volume. A fear was expressed by J. Dickey and M. Standish who state that the introduction of a new unit of barycentric coordinate time that would not have the presently agreed scaling factor would introduce large corrections to the present ephemerides amounting to 2".0 per century for the Earth's orbit and 27".0 per year in the case of the Moon. They think that this would cause serious mistakes among users. However, not to reduce this scaling factor to one, would dramatically change the whole philosophy of the proposal and introduce inconsistency in the realization of the ideal system as proposed in G1.

7. CONVENTIONAL REFERENCE SYSTEM

Recommendations G1 and G2 set up the working rules for further developments in the construction of usable reference frames. The first step is to model the structure chosen to define the reference system. This means constructing the conventional reference system. Draft recommendation R1 proposes a route to do so. It reads :

.... considering
- the desirability to implement a unique conventional celestial barycentric reference system based upon the observed positions of extragalactic radio-sources,
noting
- the existence of tentative reference frames constructed by IERS and other Institutes,
recommends
- that intercomparisons of these frames be extensively made in order to assess their accuracy and systematic differences,
- that IERS, in consultation will all the Institutes constructing catalogues of extragalactic radio-sources, establish a list of candidates for primary sources defining the new conventional reference system, together with a list of substitute sources that may later be added to or replace some of the primary sources,
requests
- that such a list be presented to the 1994 IAU General Assembly for a decision on the definition of the new conventional reference frame,
- that onwards, the objects in this list be systematically observed by all VLBI astrometric programes.

The object of this recommendation is to request that the work on the implementation of the next barycentric reference frame consistent with G1 and G2 be started. It actually addresses the choice of the "set of distant extragalactic objects" that are mentionned in G2, first point.

This point has encountered one objection: the fact that IERS is requested to establish the list of objects. A strong request is that this should be the responsibility of an *ad'hoc* IAU Working Group. I would agree to this, provided however that it is shared with IERS and that the final compilation be made by IERS. It is not correct to say that IERS celestial reference frame is constructed for the rotation of the Earth. It has now more than 200 objects, only 10% of which are used for Earth's rotation. It is part of the tasks of IERS as defined by its bylaws to include the construction and the maintenance of a celestial reference frame and it has a sub-bureau that is devoted to this job. But I admit that further inputs from astronomers having various interests in the choice would be valuable.

A more fundamental objection was made by C. Ma: IERS only combines the positions in existing catalogues and does not go into the actual raw data, so that the full strength of the underlying data is not used. This second approach is to be prefered to a simple concatenation. As a consequence, the work might have to be performed elsewhere in addition to the catalogue comparisons made by IERS which are as such very important to evaluate random and systematic errors of the catalogues.

Let me remark at this stage that at least one voice regretted that we do not adopt the existing IERS celestial reference frame.

An improvement of the wording has been requested by C.A. Murray: one should say more explicitely what is meant by "based upon" in the considering. I have no proposal at this point. It means that the catalogue that materializes the reference system is a set of coordinates of extragalactic radio-sources (just as the FK5 catalogue materializes the FK5 system).

8. COORDINATE FRAME AND ORIGIN

Once one has a catalogue of positions of extragalactic objects, one may apply to them an arbitrary rotation. Draft recommendation R2 makes a proposal and reads as follows :

... considering
- *that the new conventional celestial barycentric reference frame should be as close as possible to the existing FK5 reference frame as refered to J.2000.0,*
- *that is should be accessible to astrometry in visual wavelengths as well as in radio wavelengths,*

recommends
- *that the positions of the extragalactic sources given in the catalogue representing the reference frame be computed for the epoch J.2000.0 using the best available values of precession and nutation,*
- *that a great effort be deployed in intercomparing reference frames of all types between them and particularly with FK5 and extragalactic reference frames,*
- *that all types of observing programs be undertaken or continued in order to link to a catalogue of extragalactic sources positions the best catalogues of star positions, in particular FK5 and the HIPPARCOS catalogues with the accuracy of these catalogues,*
- *that the Ox axis of the spherical coordinates of the new conventional celestial reference frames be as close as possible of the FK5, equinox J.2000.0 and that the principal plane be as close as possible to the mean equator at epoch J.2000.0.*

This wording has brought a number of comments. The rationale for it was that one would like to avoid as much as possible a jump in positions and proper motions of stars and ensure the smoothest possible transition from the present situation. Clearly, it is not possible to have an alignment between FK5 and the new system other that at epoch. The proposed epoch is J.2000.0. The open questions are :

1. Should we use a conventional value of the precession to extrapolate the FK5 equinox or the continuously measured? The problem was not discussed throughly and should be an issue for the Colloquium: should one use as the origin at J.2000.0 the equinox and the mean equator as observed in year 2000 or extrapolate the FK5 value with the present IAU system of constants? The second solution is the easiest to implement, but the risk is that we shall not be at the actual mean equinox of date. I do not, feel very strongly in favour of my proposal in the recommendation, and most, but not all of the members of the subgroup that replied prefer using a conventional precession. An additional point is proposed by B. Guinot :

   "-That the new conventional celestial reference frame be not rotated to fit determination of the position of the mean pole of rotation and mean equinox for epoch

J.2000.0".

This proposal implies in practice that we use a conventional value of precession. However, analysing the reasons of each side, one possibly may find a common ground.

i) On one side (and this was the point I wanted to make in the proposal R2), it is only by using the best *observed* values of nutation and precession that observation of relative positions of radio-sources (extragalactic or radio-stars) can be compared and extrapolated to a reference date.

ii) On the other side, if one does this, the positions obtained will correspond to a position of the ecliptic and mean equator that is not the one of the FK5 system. In order to satisfy the last condition, on should then apply a rotation to the result obtained by (i) so that it corresponds to the FK5 mean pole and equinox at J.2000.0. It is to be noted that this double procedure is not equivalent to the procedure that consists to use systematically FK5 precession and nutation.

9. TIMING AND TRANSITION PERIOD

It is desirable that resolutions R1 and R2 are fulfilled by 1994. This would permit linking HIPPARCOS to the extragalactic frame as well as to FK5 before it is published. Whether this would be sufficient to be ready with a stellar catalogue for the year 2000 is difficult now to say, and whether would it be better than the present FK5 is still questionable. For this reason, I support the proposal made by H. Schwan which reads :

"As long as the relation between the optical and radio systems is not sufficiently accurately determined, the FK5 shall be considered as an additional realization of the celestial reference system in optical wavelengths".

Actually, it is simply a rephrasing of the first part of resolution C1 of the IAU (1988) quoted in 2.

10. CONCLUSION

The proposals made in this presentation are not perfect. Some improvements have been proposed that should be discussed during this Colloquium. There is also (S. Aoki) a completely different counter proposal that destroys most of the features of the proposed construction and in many ways reverts to the original situation. Science is advancing very quickly, and if IAU does not follow the trend, we shall arrive at an uncontrolable situation in which various interest groups will use different references without coordination. Actually, with the present considerable increase in the precision of observation, this temptation is already great for people working in VLBI and possibly also in space probes. It is time to take these improvements into consideration and the draft recommendations presented here are a tentative effort to meet their needs while they still remain acceptable to most classical astronomers. Then, of course, different realizations of the proposed system may be used for different usages or techniques, but all one of them, the conventional reference frame adopted, should be the standard to which other frames are to be identified as such as possible.

But this proposal has a more revolutionnary aspect which was perceived by some persons and perhaps made them more cautious. Reference frames have been until now based at least partially on the rotation and space motion of the Earth. The proposal removes all connection with the Earth. The Earth's motions become physical quantities that must be observed and studied for themselves but they lose the basic character of being the reference. There was a kind of geocentrism that our proposal destroys. Similarly, the meter and the second become truly universal by means of relativistic transformations and will no longer be terrestrial units to which others are referred using rescaling. I believe that by adopting the ideas developed here, we would make a great step forward towards universality by throwing away the remainders of geocentrism.

ACTIVITY REPORT OF THE IAU WORKING GROUP ON REFERENCE SYSTEMS SUB-GROUP ON ASTRONOMICAL CONSTANTS

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1. Introduction

The IAU Working Group on Reference Systems (WGRS) Sub-Group on Astronomical Constants (SGAC) was established in June, 1989, as a consequence of resolutions adopted by Commissions 4, 7, 8 and 24 at the IAU General Assembly at Baltimore in 1988. The given missions of this sub-group were stated clearly by J. Hughes, the chairman of the WGRS, as:

> "Provide numerical values for the primary constants and specify the relationships between these and other, secondary constants within the framework of general relativity. This task will involve the documentation of the constants themselves as well as of the procedures and algorithms associated with their use. Recognition must be given to the fact that approaches which are specific to various techniques exist. The group must recommend the best estimates which can meet the varied requirements of astronomy. The apparent dichotomy between adopting fixed values for various quantities on the one hand, and the need for current, highly accurate values on the other hand, must be addressed by the group. Indeed, the crafting of effective procedures for incorporating new determinations into the values assigned to the constants, and the setting up of a mechanism for disseminating information regarding new determinations as an interim measure, are important tasks for this group."

This is a report of the activity of the SGAC prior to IAU Colloquium No. 127. Section 2 summarizes the questionnaire prepared in the course of discussion. In Section 3, the discussions on major issues are introduced. The drafts of Recommendations from the SGAC prepared before the Colloquium are shown in Section 4. Note that these drafts were very different from the final form as indicated in this Proceeding. Sections 5 and 6 deal with the current status of astronomical constants used widely now and the best estimates of some major constants which are available now, respectively. I believe these will serve as a guideline for current astronomical constants.

2. Summary of Activity

After the formation of SGAC, we have discussed the matters of astronomical constants and units through the exchange of letters and about a dozen circulars. Among them, a questionnaire on astronomical constants was sent to all members of WGRS including the SGAC and some other specialists at the beginning of 1990. It contained 22 questions with some possible selections. The essence of the questions are as follows;

1) Should the system of astronomical constants be limited within the solar system?
2) What constants beyond the solar system should be included?
3) What systems of units beyond the solar system are needed?
4) Should standard procedures be included in the scope of the SGAC?
5) How many systems of units should be prepared when taking general relativity into account?
6) If we accept the IAU Recommendation 5 (1976) on time-like arguments, how should we construct the systems of units consistent with it?
7) What combination of defining constants is most appropriate?
8) What expression is most suitable for a primary constant defining the AU?

9) Should the mean angular velocity of the Earth be included as a primary constant?
10) Should the masses of minor planets and natural satellites except for the Moon be discussed?
11) What changes are needed in the classification of constants?
12) Should accuracies be indicated also?
13) What policy should we take on compatibility with the existing systems of constants in other fields?
14) What new constants should be added?
15) What constants should be excluded from the present system?
16) What mechanism to update the system should be adopted?
17) Is the form of reciprocal masses reasonable?
18) What auxiliary units of length are needed?
19) What auxiliary units of time are needed?
20) Should units of angle be specified?
21) Is there any other question?
22) To whom should we send this questionnaire?

From 22 members and consultants, answers were obtained. They are too long (47 pages) to be quoted here.

3. Discussed Topics

3.1. RELATIVISTIC EFFECTS ON UNITS

As stated in the paper of Fukushima *et al.* (1986), the present IAU convention on time-like arguments to have no secular difference among them will force one to use different sets of units in different coordinate systems within the framework of general relativity; the terrestrial meter and second in the geocentric coordinate system on the one hand, and the barycentric meter and second in the barycentric coordinate system on the other hand, for example. Furthermore both of these units differ from SI units. It was acknowledged that there are two options to solve this problem. The one is to keep the present convention and to introduce different systems of units. In this case, a [illegible]ng factor should be introduced to connect different systems of units. The other is to abandon the present convention and to use only one system of units, SI. In any case, it was noted that the numerical values of constants determined by using the TDB-based observation should be examined carefully.

Ref.: Fukushima *et al.*: 1986, *Celestial Mechanics*, **36**, 215.

3.2. CHOICE OF DEFINING CONSTANTS

Kubo argued that

> "The defining constants should be the constants such that no inconsistency will be caused by assigning them whatever numerical values. The Gaussian constant k defines the length of AU. However, we should remark that k never fixes the length of AU. If the physical magnitude of GM_{Sun} changes, the length of AU will change even if k is kept the same. The length of AU should be unchanged even the mass of actual Sun or G will change, shouldn't it? To define AU by means of k is just the same as to define a day by means of the mean solar day, isn't it? If so, we must define AU by the light time for unit distance τ_A and we should adopt τ_A as a defining constant."

Some members supported his idea, however many others preferred to keep the present style, namely k and c as defining constants.

3.3. DISCRIMINATION OF CONSTANTS AND QUANTITIES

The answers to the questionnaire indicated the tendency to extend the coverage of the system of astronomical constants to include, for example, the transformation matrix between FK5 and a galactic reference frame, the mean rotational angular velocity of the Earth, *GM* of many natural satellites and so on. However, it is clear that the degree of accuracy for estimated numerical values differs very much depending on the nature of the determination. Some constants have more than 10 well-determined digits while others are estimated with 50% accuracy. Also it was argued that there are two contradictory requirements on constants; 1) to seek the latest and most accurate values and to update them as frequently as possible, and 2) to keep them as standards for long-term references. To solve this dilemma, Sinclair proposed to discriminate primary and secondary constants and to call the latter as "best estimates". This proposal was welcomed by many members, however, to draw a line between them will require a deep consideration as Seidelmann stressed. Also many people felt that to extend the coverage is beyond the mission of the SGAC even if it is desirable.

3.4. NEED FOR STANDARD PROCEDURES

Almost all replies stressed the importance of providing standard procedures in fundamental astronomy. In other words, they required a kind of IAU version of the IERS Standards. The IERS Standards do not cover the whole of fundamental astronomy so that another standards, say IAU Standard Procedures, are required. Also possible media for their distribution were discussed; to utilize E-mail systems or to provide them in a machine-readable form. However, some members argued that to prepare them is far beyond the mission of the SGAC. So it was proposed to establish a special working group for their establishment.

3.5. UPDATE MECHANISM

The update mechanism of the geodetic system of constants and their best estimates in the IAG was discussed and almost all members and consultants agreed to introduce a similar kind of mechanism into the IAU. Namely, to keep a system of constants for long-term references and to update a list of best estimates for other specified 'quantities' at every General Assembly.

4. Proposed Recommendations

Note that the proposed Recommendations listed in the following are different from the final ones. They are quoted here just to show the change of opinions in the course of discussions in our sub-group.

Recommendation C1: *Separation of constants and quantities in astronomy and the establishment of a permanent working group to maintain the list of astronomical quantities*

recognizing the importance in astronomy to discriminate the well-established constants and the quantities whose estimates will be improved frequently,

recommends that the former constants and the units remain as a system of astronomical units and constants which should be used as numerical standards to produce long-term references, and should be unchanged unless the adopted values of constants deviate greatly from their latest estimates or the structure of system becomes inadequate due to the increased knowledge,
that the estimates of latter quantities shall be presented and updated at every General Assembly from now on, and
that a permanent working group named the Working Group on Astronomical Quantities (WGAQ) shall be established for the update and improvement of the list of such estimates.

Recommendation C2: *IAU (1991) System of Astronomical Units and Constants*

recommends that the attached list of units and constants Annex I (dropped from this report) shall be adopted as the "IAU (1991) System of Astronomical Units and Constants", which should be used as the numerical standards to produce long-term references in astronomy.

Recommendation C3: *IAU (1991) Estimates of Astronomical Quantities* *recommends* that the attached list of estimates of quantities Annex II (dropped from this report) shall be referred as the "IAU (1991) Estimates of Astronomical Quantities", which can be used as numerical standards in astronomy.

Recommendation C4: Establishment of electronic accesses to the IAU (1991) System of Astronomical Units and Constants and the IAU (1991) Estimates of Astronomical Quantities

noting that the recommended lists of constants and estimates of quantities contain a great deal of digits in their expressions,

recognizing the necessity to avoid mistakes caused by copying these digits by hands,

recommends establishing electronic accesses to these lists.

Recommendation C5: Establishment of a working group to prepare the IAU Standards of Procedures

noting that the MERIT Standards and the IERS Standards have contributed significantly in the progress of astronomy and geodesy,
that these standards of procedures do not cover the whole of basic astronomy, and

recognizing that it is not sufficient for the establishment of references to prepare only the numerical standards, namely a system of astronomical constants and a list of estimates of astronomical quantities, unless the standard procedures for using these numerical values are also given,

recommends establishing a working group named the Working Group on Standards of Procedures (WGSP) to prepare a draft report on standards of procedures needed in astronomy, which
1) should have a maximum degree of compatibility with the IERS Standards,
2) should include the implementations of procedures in the form of tested software as often as possible, and
3) should be made available not only in the form of literature but also in the form which computers can read easily,

at least six months before the next General Assembly.

5. Current Systems of Astronomical Constants

There are three sets of astronomical constants currently used as self-consistent systems; the IAU1976 System, DE200 System and IERS Standards (1989). Note that both *the Connaisance des Temps* (French Ephemeris) and *the Japanese Ephemeris* were obtained by fitting their positional values to those of DE200 but by using the IAU1976 System. While *the Astronomical Almanac* and other national ephemerides are based on the DE200 both in positional values and in the system of constants. Some constants are the same for these three systems;

k	=	0.01720209895	$\mathrm{AU}^{3/2}\mathrm{day}^{-1}M_{\mathrm{Sun}}^{-1/2}$,
c	=	299792458	m/s,
p	=	5029.0966	"/jc,
$M_{\mathrm{Sun}}/M_{\mathrm{Mercury}}$	=	6023600,	
$M_{\mathrm{Sun}}/M_{\mathrm{Venus}}$	=	408523.5,	
$M_{\mathrm{Sun}}/M_{\mathrm{Mars}}$	=	3098710,	
$M_{\mathrm{Sun}}/M_{\mathrm{Neptune}}$	=	19314,	

where square brackets denote a derived constant. Constants whose values are different among three systems are shown in the table below where the units for G, GM_{Earth} and GM_{Sun} are $10^{-11}\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$,

$10^{+14}m^3/s^2$ and $10^{+20}m^3/s^2$, respectively. In the table, dashes mean that the value of corresponding constant is not defined explicitly in the literature nor able to be computed from other defined values in the system.

Constants	IAU1976	DE200	IERS1989
τ_A/s	499.004782	[499.00478370]	499.00478370
AU/m	[149597870]	149597870.66	[149597870.66]
G	6.672	—	6.67259
$a_{e\mathrm{Earth}}$/m	6378140	—	6378136
$J_{2\mathrm{Earth}}$	0.00108263	—	0.001082626
GM_{Earth}	3.986005	[3.98600448]	3.98600448
f_{Earth}	[1/298.257]	—	[1/298.257]
$M_{\mathrm{Sun}}/(M_{\mathrm{Earth}}+M_{\mathrm{Moon}})$	[328900.5]	328900.55	[328900.55]
$M_{\mathrm{Sun}}/M_{\mathrm{Earth}}$	[332946.0]	[332946.038]	DE200
$M_{\mathrm{Moon}}/M_{\mathrm{Earth}}$	0.01230002	[0.012300034]	0.012300034
$M_{\mathrm{Earth}}/M_{\mathrm{Moon}}$	[81.30068]	81.300587	[81.300588]
ε_0	23° 26' 21".448	[23° 26' 21".4119]	DE200
$M_{\mathrm{Sun}}/M_{\mathrm{Jupiter}}$	1047.355	1047.350	DE200
$M_{\mathrm{Sun}}/M_{\mathrm{Saturn}}$	3498.5	3498.0	DE200
$M_{\mathrm{Sun}}/M_{\mathrm{Uranus}}$	22869	22960	DE200
$M_{\mathrm{Sun}}/M_{\mathrm{Pluto}}$	3000000	130000000	DE200
π_{Sun}	[8".794148]	—	[8".794141]
GM_{Sun}	[1.32712438]	[1.327124399]	[1.32712440]

Ref.: (IAU1976) Duncombe *et al.*: 1977, *Transactions of the IAU*, **XVIB**, 56.
(DE200) Standish: 1990, *Astron. and Astrophys.*, **223**, 252.
(IERS1989) McCarthy *et al.*: 1989, *IERS Tech. Note*, No.3, 1.

This table clearly shows that the numerical standards in the IERS Standards (1989) were mostly based on the DE200 System though its classification of primary and derived constants differs from that of the DE200 System sometimes. Note that the listed values of f_{Earth} are different with each other though they seem to be the same. This is because f_{Earth} is analytically derived from the values of $a_{e\mathrm{Earth}}$, $J_{2\mathrm{Earth}}$, and GM_{Earth} plus the mean angular velocity of the Earth rotation ω_{Earth}, and the adopted values of first three are different in the IAU1976 System and in the IERS Standards (1989). The adopted values of ω_{Earth} are the same in these two systems and is

$$\omega_{\mathrm{Earth}} = 7.292115 \times 10^{-5} \text{ radian/s}$$

The readers can consult with the explanation of the Geodetic Reference System 1980 (GRS1980) by Moritz for the further information on the derivation procedures of geodetic constants such as f_{Earth}.

Ref.: (GRS1980) Moritz: 1988, *Bulletin Géodésique*, **62**, No. 3, 348.

6. Current Best Estimates

The followings are the best estimates of astronomical constants whose references are available at present, *i.e.* October, 1990. We must note that some of these are not final determinations and research to update these values is still on going. Also note that these estimates were all derived by using TDB as a time-like argument in the solar system barycentric coordinate system. Thus, if we adopt the way to allow secular differences among the time-like arguments, the numerical value of some constants shall be changed, especially for those with more than 8 significant digits.

WARNING: Much care should be taken in the use of following estimated values in conjunction with ephemerides or star catalogs which are based on the existing systems of astronomical constants such as described in the preceding section!

6.1. LIGHT TIME FOR AU

Best estimates of τ_A and/or AU/m have been obtained as a solve-for parameter in creating planetary ephemerides. The following is a table of their values determined in creating JPL's DE series.

System/Ephemerides	AU/m	τ_A/s
IAU1976	[149597870]	499.004782
DE96	149597871411	[499.006708]
DE102	0683	[3779]
DE108	0705	[3853]
DE111	0652	[3676]
DE118, DE200	0660	[3703]
DE125, DE201	0614	[3549]
DE130, DE202	0609	[3533]

Ref.: (DE96-DE118, DE200) Standish: 1990, *Astron. and Astrophys.*, 223, 252.
(DE125, 130; DE201, 202) Standish: 1990, private communication.

Standish reported that the standard deviation for AU/m is ±50 for the DE118 or DE200 value because AU/m would change by that amount as a result of improvement of asteroid modelling. He also said that the DE130/DE202 was not a final version based on the latest observations including Voyager encounters with outer planets. Therefore, we do not recommend any value of these as a best estimate at present.

6.2. PRECESSION CONSTANT

According to McCarthy, there have been the following estimates of a correction to the precession constant in IAU1976 System, where the unit is mas/Julian year = 0".1/jc.

−2.39	(± 0.13)	Herring *et al.*: 1986, *Jour. Geophys. Res.*, 91, 4745.
−5.00	(± 1.10)	Herring: 1988, *BIH Annual Report for 1987*, D-106. However, Herring recommended the correction −3.00.
−1.80	(± 0.13)	Sovers and Edwards: 1988, *BIH Annual Report for 1987*, D-109.
−2.05	(± 0.15)	Steppe *et al.*: 1989, *IERS Tech. Note*, No.2.
−3.76	(± 0.47)	Zhu *et al.*: 1989, *Astron. Jour.* 99, 1024.
−2.53	(± 0.24)	McCarthy and Luzum: 1990, to be submitted to *Astron. Jour.*
−2.22	(± 0.14)	Whipple: 1990, private communication.
−2.7	(± 0.4)	Williams *et al.*: 1990, Submitted to *Astron. and Astrophys. Let.* This paper gives also an estimate of general precession as 5028".82/jc.

We take the unweighted mean of these except for the value −5.00. The resulting correction becomes −2.493 (± 0.634). After rounding the last digit, we have a best estimate of the precession constant as

p = 5028.847 (± 0.063) "/jc

However, we should remark that this value is not compatible with the published national ephemerides, the existing star catalogs such as FK5, nor the IAU (1976) Precession theory.

6.3 OBLIQUITY OF THE ECLIPTIC

According to Standish, best estimates of the obliquity of ecliptic have been obtained dynamically from the analysis of motion of ecliptic realized in the planetary ephemerides. In this sense, it is not a primary constant but a kind of derived constant although its derivation procedure is complicated. The following is a table of their values determined from JPL's DE series and an analytical planetary theory VSOP82.

System/Ephemerides	ε_0
IAU1976	23° 26' 21".448
DE96	.327
DE102	.412
DE108	.310
DE111	.412
DE118, DE200	.412
VSOP82	.409
DE125, 130; DE201, 202	.411

Ref.: (DE96-DE118, DE200) Standish: 1990, *Astron. and Astrophys.*, 223, 252.
(VSOP82) Bretagnon: 1982, *Astron. and Astrophys.*, 114, 278.
(DE125, 130; DE201, 202) Standish: 1990, private communication.

Judging from the trend of convergence seen in this list, we recommend its best (derived) estimate as

$$\varepsilon_0 = 23° 26' 21''.411 \ (\pm 0''.002).$$

However, we should remark that this value is not compatible with the published national ephemerides, the existing star catalogs such as FK5, nor the IAU (1976) Precession theory. Also note that one should use the corresponding value of obliquity when one uses a certain planetary ephemeris.

6.4 EARTH-MOON MASS RATIO

Best estimates of the Earth-Moon mass ratio have been obtained as a solve-for parameter in creating planetary ephemerides. The following is a table of their values determined and/or adopted in JPL's DE series. This value may be better treated as a derived constant to be derived from the two primary constants GM_{Earth} and $M_{\mathrm{Sun}}/M_{\mathrm{Earth+Moon}}$ with the use of τ_A or AU/m.

System/Ephemerides	$M_{\mathrm{Earth}}/M_{\mathrm{Moon}}$	$\mu = M_{\mathrm{Moon}}/M_{\mathrm{Earth}}$
IAU1976	[81.300681]	0.01230002
DE96, 102	81.3007	[0.0123000171]
DE108	49	[489]
DE111, 118-130; DE200-202	587	[342]

Ref.: (DE96-DE118, DE200) Standish, 1990: *Astron. and Astrophys.*, 223, 252.
(DE125, 130; DE201, 202) Standish, 1990: private communication.

6.5 *GM* OF PLANETARY SYSTEMS

According to Standish, the values in the table below are the current best estimates of GM of the planets, where square bracket denotes the derived value using the DE200 estimate of AU/m = 149597870660. Note that these GM_{Planet} values include the contribution of satellites.

Planet	M_{Sun}/M_{Planet}		$GM_{Planet}/(km^3/s^2)$	
Mercury	6023600.	(± 250)		
Venus	[408523.71	(± 0.06)]	324858.60	(± 0.05)
Earth+Moon	328900.55			
Mars	3098708.	(± 9)		
Jupiter	[1047.3486	(± 0.0008)]	126712767.	(±100)
Saturn	3497.898	(± 0.018)		
Uranus	22902.94	(± 0.04)		
Neptune	[19412.240	(± 0.057)]	6836534.	(± 20)
Pluto+Charon	1.350	(± 0.006) $\times 10^{+8}$		

Ref.: (Mercury) Anderson *et al.*: 1987, *Icarus*, 71, 337.
This is the same as those in IAU1976, DE200 and IERS1989 systems.

(Venus) Sjogren *et al.*: 1990, *Geophys. Res. Lett.*, 17, 1485.

(Earth+Moon) Standish: 1987, "Ephemerides DE130/LE130 & DE202/LE202", *JPL Interoffice Memo.*, 314.6-891.
This is the same as that in DE200.

(Mars) Null: 1969, *Astron. Jour.*, 72, 1292.

(Jupiter) Campbell and Synnott: 1985, *Astron. Jour.*, 90, 364.

(Saturn) Campbell and Anderson: 1989, *Astron. Jour.*, 97, 1485.

(Uranus) Anderson *et al.*: 1987, *Jour. Geophys. Res.*, 92, 14877.

(Neptune) Tyler *et al.*: 1989, *Science*, 246, 1466.
This paper gives a value for the reciprocal ratio to GM_{Sun}, however, the conversion was done using an old value for the speed of light. A correct value is shown here.

(Pluto+Charon) Derived from Tholen and Buie: 1988, *Astron. Jour.*, 96, 1977.
The value and uncertainty are derived from the cited values of 19640 (± 320) km for the semi-major axis and 6.387230 (± 0.000021) days for the period of the orbital motion of Pluto+Charon.

Acknowledgment

This report is a product of the cooperative work of not only members of the SGAC but also other members of the WGRS and some distinguished experts, whose names are listed in the Appendix. Here I would like to thank all those who returned answers to the questionnaire, participated in the discussion, sent fruitful comments and information, and prepared some parts of this report.

Appendix. Members and Consultants of SG on Astronomical Constants

1. MEMBERS

M. Bursa,	Astronomical Institute, Praha, Czechoslovakia.
J. Campbell,	Geodätisches Institut der Univ. Bonn, F.R. Germany.
J. Chapront,	Bureau des Longitudes, Paris, France.
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P.K. Seidelmann,	U.S. Naval Observatory, Washington, D.C., U.S.A.
A.T. Sinclair,	Royal Greenwich Observatory, Cambridge, U.K.
E.M. Standish, Jr.,	Jet Propulsion Laboratory, Pasadena, U.S.A.
Ch. Veillet,	C.E.R.G.A., Grasse, France.

2. CONSULTANTS

S. Aoki,	National Astronomical Observatory, Tokyo, Japan.
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Z.-G. Cheng,	Shanghai Observatory, Shanghai, P.R. China.
V. Dehant,	Observatoire Royal de Belgique, Bruxelles, Belgique.
E. Groten,	Institut für Phys. Geodäsie, Darmstadt, F.R. Germany.
B. Guinot,	B.I.P.M., Sèvres Cedex, France.
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RELATIVISTIC HIERARCHY OF REFERENCE SYSTEMS AND TIME SCALES

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ABSTRACT. Relativistic hierarchy of reference systems (RS) developed in recent years by different authors is examined in detail. Metric expressions and transformation relations for solar system barycentric RS (BRS), heliocentric RS (HRS), Earth-Moon local RS (LRS), geocentric RS (GRS), topocentric RS (TRS) and Earth satellite RS (SRS) may be obtained explicitly in harmonic coordinates of GRT. The time coordinate of any RS involves the corresponding time scale. Particular attention is given to the closed form representation of GRS avoiding expansions in powers of the geocentric coordinates. GRS has been constructed in both versions of dynamically non-rotating GRS (DGRS) or kinematically non-rotating GRS (KGRS). DGRS and KGRS differ in their space axes orientation by the amount of the geodesic precession. Similarly, taking into account the motion of the Sun around the center of the Galaxy one should distinguish between dynamically non-rotating BRS (DBRS) and kinematically non-rotating BRS (KBRS) differing in their space axes orientation by the amount of the galactic precession. Reduction to the galactic time and the galactic space axes may be needed in the nearest future.

1. INTRODUCTION

Presently, any theory of astronomical reference systems and time scales adequate to the precision of modern observations may be developed only within the GRT framework. However, the preliminary discussion by the IAU Working Group on Reference Systems of the corresponding relativistic formulations suggested in (Brumberg and Kopejkin, 1989a, 1990) has shown that many astronomers are not ready to apprehend these formulations and regard them as "too technical". The difficulties of apprehension increase when considering not only one approach to develop relativistic reference systems but a whole set of the alternative approaches based, for instance, on PPN formalism (Will, 1981), generalization of Fermi normal coordinates (Ashby and Bertotti, 1986; Fukushima, 1988), tetrad formalism (Soffel, 1989), etc. The aim of the present paper is to elucidate the key statements of relativistic formulations and to simplify as much as possible their "technical" aspects not diminishing the level of necessary experimental precision and mathematical accuracy. The paper is based on the technique to construct the hierarchy of reference systems in harmonic coordinates exposed in (Brumberg and Kopejkin, 1989a, 1990).

2. REFERENCE SYSTEM IN GRT

A reference (coordinate) system in GRT is given by the symmetric quadratic form ds^2 determining the metric of the four dimensional pseudo-Riemannian space of events of GRT

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad , \; x^0 = ct \quad , \; \mu, \nu = 0,1,2,3 \quad , \qquad (1)$$

c being the velocity of light. The quantity t is called the coordinate time of the system. The spatial coordinates of the system are designated by x^i (i=1,2,3). Einstein summation rule over repeated greek or latin index is used everywhere. In absence of the gravitating masses the reference system may be chosen so that the components of the metric tensor $g_{\mu\nu}$ (the gravitation potentials) take the values $\eta_{\mu\nu}$ with

$$\eta_{oo} = 1 \; , \; \eta_{oi} = 0 \; , \; \eta_{ij} = -\delta_{ij} , \quad i, j = 1,2,3 \qquad (2)$$

(pseudo-Euclidean or Minkowskian or Galilean metric). These values determine the inertial reference system of special relativity theory. In presence of the gravitating masses the components $g_{\mu\nu}$ determined by the Einstein field equations are represented by expansions in powers of v/c (v being the characteristic velocity of bodies)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad , \qquad (3)$$

$$h_{00} = h^{(2)}_{00} + h^{(4)}_{00} + \ldots \quad , \qquad (4)$$

$$h_{0i} = h^{(1)}_{0i} + h^{(3)}_{0i} + \ldots \quad , \qquad (5)$$

$$h_{ij} = h^{(2)}_{ij} + \ldots \quad . \qquad (6)$$

The upper index in parentheses indicates the order of smallness with respect to v/c. The quantity $h^{(1)}_{0i}$ is caused not by the gravitating masses but by the motion of the reference system. Two cases are typical. The first one is related with a system in rotation. If a system is originated by rotation of the spatial axes of the inertial system and the angular velocity components on the moving axes x^i are designated by ω^i then the metric (1) will be characterized by the occurrence of the term

$$h^{(1)}_{0i} = -c^{-1}\varepsilon_{ijk}\omega^j x^k = -c^{-1}(\omega \times x)^i \quad . \qquad (7)$$

ε_{ijk} is the three dimensional fully antisymmetric Levi-Civita symbol (ε_{123}= +1). ω and x are the triplets of components ω^i and x^i respectively. In the second case a reference system is resulted from formal application to the inertial system of three dimensional Galileo transformation characterized by the translatory velocity v^i. Then the metric (1) will contain a typical term

$$h^{(1)}_{0i} = - c^{-1} v^i \quad . \qquad (8)$$

In what follows we shall be interested only in reference systems transforming in absence of the gravitating masses into inertial systems of special relativity theory. Metric (1) of such systems cannot contain the terms (7) or (8) and expansion (5) begins only with the third order terms $h^{(3)}_{0i}$.

No matter where to stop in expansions (4)-(6) one obtains different accuracies for astrometric problems (based on the equations of light propagation) and for celestial mechanics problems (based on the equations of motion of celestial bodies). In fact, in neglecting $h_{\mu\nu}$ at all, metric (1) yields the Newtonian equations of light propagation. Retaining only $h^{(2)}_{00}$ enables one to derive the Newtonian equations of motion of celestial bodies. This permits also to take into account the post-Newtonian terms in the problem of clock synchronization and time scale relations. Considering $h^{(2)}_{00}$ and $h^{(2)}_{ij}$ results in the post-Newtonian equations of light propagation in the static field (ignoring the motion of the gravitating masses). In taking into account $h^{(2)}_{00}$, $h^{(2)}_{ij}$ and $h^{(3)}_{0i}$ one yields the complete post-Newtonian equations of light propagation (considering the motion of the masses). Moreover, the terms $h^{(3)}_{0i}$ are crucial to describe the relativistic rotation of the spatial axes of the reference system. Finally, including $h^{(2)}_{00}$, $h^{(2)}_{ij}$, $h^{(3)}_{0i}$ and $h^{(4)}_{00}$ one obtains the post-Newtonian equations of motion.

Nowadays, one may meet suggestion to define relativistic reference systems by fixing only $h^{(2)}_{00}$ and $h^{(2)}_{ij}$. This is sufficient for present day astrometry. But this is quite inadequate for modern celestial mechanics using the post-Newtonian equations of motion of the solar system bodies and will be soon insufficient for high-precision astrometry (millisecond pulsar timing, POINTS project, etc.) demanding the complete post-Newtonian equations of light propagation. It seems to us that one should define a reference system with some excess of accuracy and therefore it is reasonable to fix all the terms indicated in expansions (4)-(6).

The dots in (4)-(6) mean the terms of more high order, in particular, the terms responsible for gravitational radiation of the system of bodies. For ephemeris astronomy problems these terms may be ignored as yet.

3. BRS IN HARMONIC COORDINATES AND IN PPN FORMALISM

Dynamically non-rotating barycentric reference system for the solar system (DBRS) may be represented in the post-Newtonian approximation of GRT in arbitrary quasi-Galilean coordinates in the form

$$h_{00}^{(2)} = -2c^{-2}U \quad , \quad h_{ij}^{(2)} = -2c^{-2}U\delta_{ij} + a_{i,j} + a_{j,i} \quad , \tag{9}$$

$$h_{0i}^{(3)} = 4c^{-3}U^{i} + a_{0,i} + a_{i,0} \quad . \tag{10}$$

$$h_{00}^{(4)} = 2c^{-4}(U^{2} - W) + 2a_{0,0} + 2c^{-2}U_{,k}a_{k} + 2c^{-2}\sum_{A} \frac{\partial U}{\partial x_{A}^{k}}(\dot{a}_{k})_{A} \tag{11}$$

with

$$W = \frac{3}{2}\Phi_{1} - \Phi_{2} + \Phi_{3} + 3\Phi_{4} + \frac{\partial^{2}\chi}{\partial t^{2}} \quad . \tag{12}$$

Newtonian potential U, vector-potential U^{i} and complementary potentials Φ_1, Φ_2, Φ_3, Φ_4 and χ satisfy Poisson equations

$$U_{,kk} = -4\pi G\rho \;,\; U^{i}_{,kk} = -4\pi G\rho v^{i} \;,\; \Phi_{1,kk} = -4\pi G\rho v^{2} \;,$$
$$\Phi_{2,kk} = -4\pi G\rho U \;,\; \Phi_{3,kk} = -4\pi G\rho\Pi \;,\; \Phi_{4,kk} = -4\pi Gp,\; \chi_{,kk} = U \;. \tag{13}$$

Comma with subsequent index denotes the derivative with respect to the corresponding variable. ρ is the conserved density, v^{i} is the velocity of the matter, p is the pressure and Π is the internal energy. G is the gravitational constant. a_0 and a_i are four arbitrary functions (a_i are of the second order of smallness, a_0 is of the third order). $x_A^k(t)$ are the spatial coordinates of body A, $(a_k)_A$ means the regular part of a_k in substituting $x^k = x_A^k$. It is of importance to note that function a_0 entering in the Lagrangian in form of total derivative has no influence on the post-Newtonian equations of motion of bodies.

Functions a_0 and a_i are specified by the coordinate conditions represented by four differential or algebraic relations for the metric tensor components and their first derivatives. These functions determine the type of the coordinates employed. One uses rather often the harmonic coordinate conditions. Their main mathematical advantage is the existence of explicit mathematical formulation

$$[(-g)^{1/2}g^{\mu\nu}]_{,\nu} = 0 \quad , \quad g = det(g_{\mu\nu}) \quad , \quad g^{\mu\lambda}g_{\nu\lambda} = \delta^{\mu}_{\nu} \quad . \tag{14}$$

If the tilde denotes harmonic coordinates then the relationship of the arbitrary coordinates used above with the harmonic ones is of the form

$$\tilde{x}^{0} = x^{0} + a_{0} \;,\; \tilde{x}^{k} = x^{k} - a_{k} \quad . \tag{15}$$

This means that metric (1)-(6), (9)-(11) becomes harmonic with $a_0 = a_i = 0$.

Potential χ in the point mass approximation has the form

$$\chi = \frac{1}{2}\sum_{A} GM_{A}r_{A} \quad , \quad r_{A} = (r_{A}^{k}r_{A}^{k})^{1/2}, \quad r_{A}^{k} = x^{k} - x_{A}^{k} \tag{16}$$

so that

$$\frac{\partial^{2}\chi}{\partial t^{2}} = -\frac{1}{2}\sum_{A} GM_{A}\{\frac{1}{r_{A}}[(n_{A}^{k}v_{A}^{k})^{2} - v_{A}^{2}] + n_{A}^{k}a_{A}^{k}\} \quad , \quad n_{A}^{k} = r_{A}^{k}/r_{A} \;, \tag{17}$$

v_A^k and a_A^k being velocity and acceleration of the center of mass of body A. In Newtonian approximation

$$a_A^k = \sum_{B \neq A} GM_B \frac{x_B^k - x_A^k}{r_{AB}^3} \quad , \quad r_{AB} = [(x_B^k - x_A^k)(x_B^k - x_A^k)]^{1/2} \quad .$$

At infinity (but let us remember that BRS is not valid for too large distances from the solar system) all potentials except for χ vanish and coefficients (9)-(11) will contain only one non-zero term

$$h_{\infty}^{(4)} = c^{-4} \sum_A GM_A n_A^k a_A^k \quad . \tag{18}$$

Hence, ar infinity the spatial part of the BRS metric in harmonic coordinates takes the Euclidean form but there remains in g_{∞} relativistic term (18).

Along with harmonic representation the BRS metric in PPN formalism coordinates is also widely used (Misner et al., 1973; Will, 1981). The PPN system is characterized by its spatial isotropy (as well as the harmonic representation) and its Galilean form at infinity. This involves the choice of the coordinate functions as follows:

$$a_i = 0 \quad , \quad a_o = c^{-3} \frac{\partial \chi}{\partial t} \quad . \tag{19}$$

In so doing, function (17) disappears from (11) and in the right-hand side of (10) there will be an additive term

$$a_{o,i} = c^{-3} \frac{\partial^2 \chi}{\partial t \, \partial x^i} = - \frac{1}{2} c^{-3} \sum_A \frac{GM_A}{r_A} v_A^k (\delta_{ik} - n_A^i n_A^k) \quad .$$

It is easy to see that at infinity all coefficients (9)-(11) for the BRS metric in PPN coordinates vanish resulting in Galilean form for the BRS metric.

Hence, the spatial harmonic and PPN coordinates are the same

$$\hat{x}^i = x^i \tag{20}$$

whereas the time coordinates are related by the equation

$$\hat{t} = t + c^{-4} \frac{\partial \chi}{\partial t} \quad , \quad \frac{\partial \chi}{\partial t} = - \frac{1}{2} \sum_A GM_A n_A^k v_A^k \quad . \tag{21}$$

Using physical terminology one says that PPN and harmonic coordinate systems belong to one and the same reference frame differing only by the time coordinates.

Relations (20) and (21) enable one to conclude that PPN formulation and harmonic representation of BRS are practically equivalent within the present accuracy (v^2/c^2). Only in dealing with the higher order effects (v^4/c^4 as in discussing POINTS observations) this difference might be taken into account. As pointed above, the advantage of harmonic coordinates is due to their explicit mathematical formulation. In transforming to any other reference system (geocentric, topocentric, etc.) it is easy to impose the harmonic conditions on new metric coefficients and to ensure therewith the harmonic form of the coordinate transformation. On the contrary, the PPN formulation has been developed only for BRS and, moreover, only in the post-Newtonian approximation. The transformation within the PPN framework to other systems has not been elaborated so far. There is no definite PPN procedure to take into account the higher order terms. With respect to these two aspects the harmonic representation is more advantageous.

Let us make one technical remark useful for comparison with formulations of (Misner et al., 1973; Will, 1981). One often uses the density ρ^* satisfying the relation

$$(-g)^{1/2}\rho^* dx^0 = \rho ds$$

or

$$\rho = \rho^*[1 + c^{-2}(\frac{1}{2} v^2 + 3U)] \quad .$$

Therefore,

$$U = U^* + c^{-2}(\frac{1}{2}\Phi_1 + 3\Phi_2)$$

where U^* is the same Newtonian potential as U but expressed with the aid of ρ^* (all other potentials are the same within the adopted accuracy). Hence, in virtue of (3), (9), (11) and (19) coefficient g_{00} of the PPN formalism BRS metric takes the form

$$g_{00} = 1 - 2c^{-2}U^* + c^{-4}(2U^2 - 4\Phi_1 - 4\Phi_2 - 2\Phi_3 - 6\Phi_4) \quad .$$

In (Misner et al., 1973; Will, 1981) ρ (designated by ρ^*) is used in the equations of motion and ρ^* (designated by ρ) is used in the field metric. Taking into account the difference in designations and signature we obtain the expressions of these papers (with the GRT values for the PPN parameters). We think it reasonable to use one and the same conserved density ρ both for the field metric and the equations of motion.

4. HIERARCHY OF ASTRONOMICAL REFERENCE SYSTEMS

For astronomical purposes it is suitable to have the hierarchy of relativistic reference systems (RS) as follows:

$$\begin{tikzcd} \mathrm{HRS} & \mathrm{LRS} \arrow[d] & \mathrm{SRS} \\ \mathrm{BRS} \arrow[u] \arrow[ur] \arrow[r] & \mathrm{GRS} \arrow[r] \arrow[ur] & \mathrm{TRS} \arrow[u] \end{tikzcd}$$

with BRS (solar system barycentric RS), HRS (heliocentric RS), LRS (local Earth-Moon barycentric RS), GRS (geocentric RS), TRS (topocentric RS) and SRS (satellite RS). Such an hierarchy (except for HRS and LRS which may be constructed by analogy) has been constructed in (Brumberg and Kopejkin, 1989a,b) conforming with three principles:

1) harmonic coordinates are used for each system;

2) the principle of equivalence is met in each system, i.e. the influence of the external masses is described only by tidal terms (in particular, the terms like (8) are absent);

3) each system is dynamically non-rotating (there are no terms like (7) or similar third order terms in $h^{(3)}_{01}$).

For BRS there are no external bodies (in ignoring the influence of the Galaxy) and all the solar system bodies taken into account are considered as internal ones. For LRS the internal bodies are the Earth and the Moon whereas all remaining bodies are considered as external ones. Both HRS and GRS have one internal body, the Sun or the Earth respectively. For TRS and SRS all the bodies are external.

The matching procedure of two reference systems used in (Brumberg and

Kopejkin, 1989a,b) starts from the known metric coefficients of one system and permits to determine 1) metric coefficients of the second system, 2) transformation formulae relating two metrics and 3) the equations of motion of the origin of the second system with respect to the first system.

Obviously, a similar hierarchy may be constructed using alternative approaches mentioned above. But the use of one and the same type of coordinate conditions for all systems in combination with the matching procedure has definite advantages, for instance:

1) the finite transformation formulae for spatial coordinates of two systems (quadratic functions in contrast to the power series of the alternative approaches);

2) unambiguous determination of the required functions;

3) no difficulties in considering figure characteristics and proper rotation of the bodies.

So far, in all papers indicated above the GRS metric has been constructed in form of the series in powers of the geocentric spatial coordinates. It is to be noted that to derive the GRS post-Newtonian rigorous (avoiding expansions in powers of geocentric coordinates) equations of motion of Earth distant satellites it is sufficient to apply to the known rigorous BRS equations the finite transformation formulae from BRS to GRS. Just in the same manner one can get the post-Newtonian HRS equations of motion of the planets. It is not evident how to solve these two problems with the alternative approaches based on power series expansions. For astrometric purposes (reduction of observations) GRS within the level of $h^{(2)}_{00}$, $h^{(2)}_{ij}$ and $h^{(3)}_{0i}$ is quite adequate. As shown in the next section, the technique of (Brumberg and Kopejkin, 1989a,b) permits easily to construct GRS within this level of accuracy in the closed form. Needless to say that the same is valid for any other reference system.

Let us add that each system entering in the hierarchy under consideration may be subjected to the rigid body spatial rotation. The resulting system (RS^+) is not harmonic anymore. Its metric will contain a term like (7). Such a system is used for solving astrometric problems.

5. CLOSED FORM OF GRS METRIC (DGRS AND KGRS)

Transformation from DBRS determined by relations (1)-(6) and (9)-(11) with $a_0 = a_1 = 0$ to GRS defined by

$$ds^2 = \hat{g}_{\mu\nu} dw^\mu dw^\nu \quad , \quad w^0 = cu \qquad (22)$$

and having its origin in the center of mass of the Earth E is given by the formulae generalizing the Lorentz transformation of special relativity theory (Brumberg and Kopejkin, 1989a,b)

$$u = t - c^{-2}[S(t) + v^k_B r^k_B] + c^{-4}[B(t,r_E) - \frac{1}{2} v^2_B v^k_B r^k_B] + \dots , \qquad (23)$$

$$w^i = r^i_B + c^{-2}\{[\frac{1}{2} v^i_B v^k_B + qF^{ik}(t) + D^{ik}(t)]r^k_B + D^{ijk}(t) r^j_B r^k_B\} + \dots (24)$$

In transformation (24) of the spatial coordinates functions $D^{ik} = D^{ki}$

and $D^{ijk} = D^{ikj}$ determine the relativistic contraction of length. Antisymmetric function $F^{ik} = -F^{ki}$ acts as the angular velocity of rotation of the spatial axes. Indeed, introducing the triplet $F = (F^1, F^2, F^3)$ with

$$F^i = \frac{1}{2} \varepsilon_{ijk} F^{jk} \tag{25}$$

one has

$$F^{ik} r_B^k = r_B \times F \tag{26}$$

resulting in rotation of the spatial axes. q is the scalar parameter to be defined below. The inverse transformation is of the form

$$t = u + c^{-2}[S(t) + v_B^k(t) w^k] - c^{-4}[B(u, w) + v_B^i(qF^{ik} + D^{ik}) w^k + v_B^i D^{ijk} w^j w^k] + \ldots \quad , \tag{27}$$

$$x^i = x_B^i(t) + w^i - c^{-2}[(\frac{1}{2} v_k^i v_B^k + qF^{ik} + D^{ik}) w^k + D^{ijk} w^j w^k] + \ldots \tag{28}$$

Substituting the derivatives of t and x^i with respect to u and w^k into the tensor transformation of metric matching

$$\hat{g}_{\alpha\beta}(u, w) = g_{\mu\nu}(t, x) \frac{\partial x^\mu}{\partial w^\alpha} \frac{\partial x^\nu}{\partial w^\beta} \tag{29}$$

one gets

$$\hat{g}_{oo} = 1 + c^{-2}(- 2U + 2\dot{S} - v_B^2 + 2a_B^k w^k) \quad , \tag{30}$$

$$\hat{g}_{oi} = c^{-3}[4U^i - 4Uv_B^i - B_{,i} + v_B^i(\dot{S} - \frac{1}{2} v_B^2) + (\frac{3}{2} v_B^i a_B^k + \frac{1}{2} v_B^k a_B^i + q\dot{F}^{ik} + \dot{D}^{ik}) w^k + \dot{D}^{ijk} w^j w^k] \quad , \tag{31}$$

$$\hat{g}_{ij} = - \delta_{ij} - 2c^{-2}[U\delta_{ij} - D^{ij} - (D^{ijk} + D^{jik}) w^k] \quad . \tag{32}$$

Function B may be presented in the form

$$B(u, w) = B^{(o)}(u) + B_i^{(1)} w^i + B_{ij}^{(2)} w^i w^j + B^{(3)}(u, w) \quad . \tag{33}$$

Function $B^{(o)}$ cannot be determined within the adopted accuracy of the matching procedure. Its expression may be found in (Kopejkin, 1989a,b; Brumberg and Kopejkin, 1990). Functions S, $B_i^{(1)}$, $B_{ij}^{(2)}$, F^{ik}, D^{ik}, D^{ijk} and a_B^i are determined on the basis of the conditions resulted from the tidal form of the external mass action. Separating Newtonian potential U and vector-potential U^i into the parts U_B and U_B^i due to the Earth alone and the parts $\bar{U}_B$ and $\bar{U}_B^i$ caused by the external masses one has

$$U = U_B + \bar{U}_B \quad , \quad U^i = U_B^i + \bar{U}_B^i \quad , \tag{34}$$

$$\dot{S} = \frac{1}{2} v_B^2 + \bar{U}_B(x_B) \quad , \tag{35}$$

$$D^{ik} = \delta_{ik} \bar{U}(x_B) \quad , \quad D^{ijk} = \frac{1}{2} (\delta_{ij} a_B^k + \delta_{ik} a_B^j - \delta_{jk} a_B^i), \tag{36}$$

$$a_B^i = \bar{U}_{B,i}(x_B) - Q_i \quad , \quad Q_i = - \frac{1}{2} M_B^{-1} I_B^{jk} \bar{U}_{B,ijk}(x_B) \quad , \tag{37}$$

$$B_i^{(1)} = 4\bar{U}_B^i(x_B) - 3v_B^i \bar{U}_B(x_B) \quad , \tag{38}$$

$$B^{(2)}_{ik} = \frac{1}{2}\,\dot{D}^{ik} + \bar{U}^{i}_{E,k}(x_E) + \bar{U}^{k}_{E,i}(x_E) - (v^{i}_{E}Q_k + v^{k}_{E}Q_i) - \frac{1}{2}\,(v^{i}_{E}a^{k}_{E} + v^{k}_{E}a^{i}_{E}) \quad , \tag{39}$$

$$\dot{F}^{ik} = -\,2[\bar{U}^{i}_{E,k}(x_E) - \bar{U}^{k}_{E,i}(x_E)] + 2(v^{i}_{E}Q_k - v^{k}_{E}Q_i) + \frac{3}{2}\,(v^{i}_{E}a^{k}_{E} - v^{k}_{E}a^{i}_{E}) \quad . \tag{40}$$

In distinction to other quantities functions S, F^{ik} and $B^{(3)}$ are determined by differential relations. Function S yields (in the post-Newtonian approximation) the difference of BRS and GRS coordinate time scales in the geocenter ($r^{k}_{E} = x^{k} - x^{k}_{E} = 0$). Function F^{ik} describes in its main part the geodesic precession. Equations (37) represent the BRS Newtonian equations of motion of the Earth (at the next step of approximation the matching procedure results in the post-Newtonian equations). Function Q_i is the correction for the non-geodesic motion of the Earth due to the interaction of the Earth quadrupole moment and the external masses. One has therewith

$$M_E = \oint_{(E)} \rho d^3x \quad , \qquad I^{ik}_{E} = \oint_{(E)} \rho r^{i}_{E} r^{k}_{E} d^3x \quad .$$

In consequence, the closed form GRS metric for approximation (30)-(32) will be

$$\hat{g}_{oo} = 1 - 2c^{-2}[\hat{U}_E + Q_k w^k + \bar{U}_E(x_E+w) - \bar{U}_E(x_E) - \bar{U}_{E,k}(x_E)w^k] + \ldots, \tag{41}$$

$$\hat{g}_{oi} = c^{-3}\{4\hat{U}^{i}_{E} + (q-1)\dot{F}^{ik}w^k + 4[\bar{U}^{i}_{E}(x_E+w) - \bar{U}^{i}_{E}(x_E) - \bar{U}^{i}_{E,k}(x_E)w^k] - 4v^{i}_{E}[\bar{U}_E(x_E+w) - \bar{U}_E(x_E) - \bar{U}_{E,k}(x_E)w^k] - B^{(3)}_{,i} + \dot{D}^{ijk}w^j w^k\} + \ldots, \tag{42}$$

$$\hat{g}_{ij} = -\,\delta_{ij} - 2c^{-2}[\hat{U}_E + Q_k w^k + \bar{U}_E(x_E+w) - \bar{U}_E(x_E) - \bar{U}_{E,k}(x_E)w^k]\delta_{ij} + \ldots \tag{43}$$

with

$$\hat{U}_E = U_E \quad , \qquad \hat{U}^{i}_{E} = U^{i}_{E} - v^{i}_{E}U_E \quad . \tag{44}$$

Function $B^{(3)}$ is determined by the harmonic conditions for the GRS metric. These conditions within the adopted accuracy result in the equations (with $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$)

$$\hat{h}_{oo,i} - \hat{h}_{kk,i} + 2\hat{h}_{ik,k} = 0 \quad , \tag{45}$$

$$\hat{h}_{oo,o} + \hat{h}_{kk,o} - 2\hat{h}_{ok,k} = 0 \quad . \tag{46}$$

Equation (45) is satisfied identically in virtue of the structure of expressions (41) and (43). Equation (46) involves the equation

$$c\hat{U}_{E,o} + \hat{U}^{k}_{E,k} = 0 \tag{47}$$

satisfied in virtue of the equation of continuity and the Poisson equation determining function $B^{(3)}$

$$B^{(3)}_{,kk} = 4[\bar{U}^k_{E,k}(x_E+w) - \bar{U}^k_{E,k}(x_E)] - 4v^k_E[\bar{U}_{E,k}(x_E+w) - \bar{U}_{E,k}(x_E)] + $$
$$+ 4[c\bar{U}_{E,0}(x_E+w) - \dot{\bar{U}}_E(x_E)] - \dot{a}^k_E w^k \quad . \qquad (48)$$

Function $B^{(3)}$ and its derivatives of the first and second order with respect to the spatial coordinates should vanish with $w = 0$. Using explicit expressions of scalar and vector potentials it is easy to verify the identity

$$\dot{\bar{U}}^k_{E,k}(x_E+w) - v^k_E\bar{U}_{E,k}(x_E+w) + c\bar{U}_{E,0}(x_E+w) = 0 \qquad (49)$$

(relation (46) is imposed on expressions (41)-(43) and the substitution $x = x_E + w$ is performed before differentiating). Hence, equation (48) takes the simple form

$$B^{(3)}_{,kk} = - \dot{a}^k_E w^k \quad . \qquad (50)$$

As a particular solution of this equation one may choose, for example

$$B^{(3)} = - \frac{1}{10}\dot{a}^i w^i w^j w^j \qquad (51)$$

so that

$$B^{(3)}_{,i} = - \frac{1}{5}\dot{a}^k_E w^k w^i - \frac{1}{10}\dot{a}^i_E w^k w^k \quad . \qquad (52)$$

Construction of the GRS metric in the closed form is completed by substituting (52) into (42).

Numerical parameter q plays an important role. Value $q = 1$ corresponds to dynamically non-rotating GRS (DGRS). In this case there are no terms due to geodesic precession in $\hat{g}_{0i}$. But the DGRS spatial axes rotate with respect to the BRS spatial axes as seen from transformation (24). Value $q = 0$ corresponds to kinematically non-rotating GRS (KGRS). In this case the spatial coordinate transformation (24) between BRS and GRS does not involve terms due to geodesic precession. Such terms occur in $\hat{g}_{0i}$. In investigating Earth satellite motion it is suitable to use DGRS (Brumberg and Kopejkin, 1989b). For reduction of observations both systems might be useful.

Let us add a technical remark. Function (51) differs from the function used in (Kopejkin, 1988; Brumberg and Kopejkin, 1989a,b). Function $B^{(3)}$ has been constructed in these papers in form of the series in powers of the geocentric coordinates starting with the terms

$$B^{(3)} = B_{ijk} w^i w^j w^k + \dots , \qquad (53)$$

$$B_{ijk} = \frac{2}{9}[\bar{U}^i_{E,jk}(x_E) + \bar{U}^j_{E,ki}(x_E) + \bar{U}^k_{E,ij}(x_E)] - \frac{2}{9}[v^i_E\bar{U}_{E,jk}(x_E) + $$
$$+ v^j_E\bar{U}_{E,ki}(x_E) + v^k_E\bar{U}_{E,ij}(x_E)] + \frac{4}{45}[\delta_{jk}\dot{\bar{U}}_{E,i}(x_E) + \delta_{ki}\dot{\bar{U}}_{E,j}(x_E) + $$
$$+ \delta_{ij}\dot{\bar{U}}_{E,k}(x_E)] - \frac{1}{30}(\delta_{jk}\dot{a}^i_E + \delta_{ki}\dot{a}^j_E + \delta_{ij}\dot{a}^k_E) \quad . \qquad (54)$$

It is easy to see that $B_{ikk} = - \frac{1}{6}\dot{a}^i_E$ and equation (50) is satisfied. Such a choice of B^{0i} provides the fulfillment of some relations of symmetry in the expansion of g_{0i}, in particular, the condition $C_{jm} = C_{mj}$ in quadratic terms

$$g_{0i} = 4c^{-3}[\dot{L}_i + \frac{1}{4}(q-1)\dot{F}^{ik}w^k + \varepsilon_{ijk}C_{jm}w^k w^m - \frac{3}{10}\dot{Q}_k w^k w^i + \\ + \frac{1}{10}\dot{Q}_i w^k w^k + \ldots] \quad . \tag{55}$$

6. CONSIDERATION OF THE INFLUENCE OF THE GALAXY

In ignoring the influence of the Galaxy BRS described by expressions (9)-(11) (with $a_0 = a_1 = 0$) is non-rotating system both dynamically and kinematically. But in not so distant future consideration of the influence of the Galaxy may become necessary. Ignoring all local irregularities and considering the mass M of the Galaxy as being concentrated at its center one may relate galactic time T and galactic spatial coordinates X^i with BRS time-space coordinates t and x^i just in the same way as BRS-GRS transformation

$$t = T - c^{-2}[S_G(T) + V^k_B R^k_B] + \ldots, \quad R^k_B = X^k - X^k_B(T) \quad , \tag{56}$$

$$x^i = R^i_B + c^{-2}\{[\frac{1}{2} V^i_B V^k_B + q_G F^{ik}_G(T) + D^{ik}_G(T)]R^k_B + D^{ijk}_G(T) R^j_B R^k_B\} + \\ + \ldots \tag{57}$$

with $X^k_B(T)$ and $V^k_B(T) = dX^k_B(T)/dT$ being galactic coordinates and velocity components of the solar system barycenter B. q_G is a constant leading to dynamically ($q_G = 1$) or kinematically ($q_G = 0$) non-rotating BRS respectively (DBRS or KBRS). Assuming the galactic circular motion of the solar system barycenter with radius $X_B = (X^k_B X^k_B)^{1/2}$ and mean motion $N = 2\pi/P = (GM)^{1/2}/X_B^{3/2}$ one has $V^2_B = GM/X_B$ and

$$\dot{S}_G = \frac{3}{2}\frac{GM}{X_B} \quad , \quad \dot{F}^{ik}_G = \frac{3}{2}\frac{GM}{X^3_B}(X^i_B V^k_B - X^k_B V^i_B) \quad , \tag{58}$$

$$D^{ik}_G = \delta_{ik}\frac{GM}{X_B} \, , \quad D^{ijk}_G = \frac{1}{2}\frac{GM}{X^3_B}(\delta_{jk}X^i_B - \delta_{ij}X^k_B - \delta_{ik}X^j_B) \quad . \tag{59}$$

Function F^{ik}_G is responsible for the galactic precession caused by the motion of the Sun around the center of the Galaxy (quite similar to the geodesic precession due to the heliocentric motion of the Earth). Introducing as in (25) triplet F_G one has

$$\dot{F}_G = \frac{3}{2}\frac{GM}{X^3_B}(X_B \times V_B) = \frac{3}{2} N \frac{GM}{X_B} k \quad , \tag{60}$$

k being a unit vector normal to the plane of the orbit of the Sun. In vector notation the transformation (57) is rewritten in the form

$$x = R_B + c^{-2}[\ \frac{1}{2}(V_B R_B)V_B + \frac{GM}{X_B} R_B + q_G(R_B \times F_G) + \frac{1}{2}\frac{GM}{X_B^3}(R_B^2)\ X_B -$$

$$- \frac{GM}{X_B^3}(X_B R_B)\ R_B] + \ldots \quad . \qquad (61)$$

Using numerical values $M = 1.6 \cdot 10^{11}\ M_\odot$, $X_B = 2.5 \cdot 10^{22}$ cm, $P = 2.2 \cdot 10^8$ years = $6.6 \cdot 10^{15}$ s, $c^{-2}GM_\odot = 1.5$ km, $c = 3 \cdot 10^{10}$ cm/s one finds that the DBRS spatial axes rotate with respect to the galactic axes with angular velocity $|c^{-2}\dot{F}_G| = 0.85'' \cdot 10^{-6}$ per century. The term $c^{-2}V_B^k R_B^k$ in the time transformation (56) applied to the Earth represents an annual periodic term with the amplitude 0.4 s. The coefficient in the secular term of this transformation is $c^{-2}\dot{S}_G = 1.44 \cdot 10^{-6}$.

Component g_{0i} of BRS metric (9)-(11) with $a_0 = a_i = 0$ is represented now in the form

$$h^{(3)}_{0i} = c^{-3}[4U^i + (q_G - 1)\dot{F}_G^{ik}\ x^k] \quad . \qquad (62)$$

Let us repeat once again that with $q_G = 1$ we have DBRS whose spatial axes rotate with respect to the galactic axes. With $q_G = 0$ we have KBRS whose spatial axes do not rotate with respect to the galactic axes. But the KBRS equations of motion of the solar system bodies contain Coriolis terms. For celestial mechanics purposes DBRS is preferable. For astrometric purposes both systems are useful.

7. TIME SCALES

The problem of time scales is to be solved finally by the IAU recommendations. But irrespective of definitions given by these recommendations the problem is based actually on the relations between the coordinate times of BRS (t), GRS (u) and TRS (τ) (in future the galactic time scale T may be needed). Relation between t and u is given in the form (23) (presently, $O(c^{-4})$ terms therewith may be omitted). Function S determined by (35) is presented in the form

$$S(t) = S^* t + S_p(t) \qquad (63)$$

where S^* is a constant and $S_p(t)$ includes both periodic terms as well as non-periodic terms due to the secular evolution of the planetary orbits. Therefore, relation (23) is rewritten in the form

$$u = (1 - c^{-2}S^*)t - c^{-2}[S_p(t) + v_B^k r_E^k] + \ldots \quad . \qquad (64)$$

Just in the same manner the relation between u and τ is

$$\tau = u - c^{-2}[V(u) + v_T^k(w^k - w_T^k)] + \ldots \qquad (65)$$

with w_T^k and v_T^k being the GRS coordinates and velocity components of the TRS origin. Function $V(u)$ is determined by the differential relation

$$\frac{dV}{du} = \frac{1}{2}\, v_T^2 + \hat{U}_E(w_T) + Q_k w_T^k + \bar{U}_E(x_E+w_T) - \bar{U}_E(x_E) - \bar{U}_{E,k}(x_E) w_T^k \quad .(66)$$

Its solution is represented in the form

$$V(u) = V^* u + V_p(u) \tag{67}$$

with V^* being a constant one and the same for all possible TRS and hence independent both of time and coordinates of the TRS origin. Function $V_p(u)$ includes corrections for the height of the ground station above the surface of geoid, for lunar and solar tidal influence and for the geophysical factors (deviation of the Earth rotation from the rigid body rotation). Substitution of (67) into (65) yields

$$\tau = (1 - c^{-2} V^*)\, u - c^{-2}[V_p(u) + v_T^k(w^k - w_T^k)] \quad . \tag{68}$$

Relations (64) and (68) are crucial for the relativistic theory of time scales (Brumberg and Kopejkin, 1990).

8. CONCLUSION

The aim of this paper is to elucidate some key questions of relativistic theory of reference systems avoiding technical aspects as much as possible. The main results of the paper may be formulated as follows:

1. BRS metrics in the PPN formalism coordinates and in the harmonic representation are practically equivalent. Harmonic coordinates have advantage of being used for constructing the hierarchy of astronomical reference systems.

2. To elaborate the definitions of time scales it is presently sufficient to retain in the BRS metric only $O(c^{-2})$ terms in g_{00} (Newtonian potential). For relativistic reduction of observations in the static field (ignoring the motion of the masses) the $O(c^{-2})$ terms in g_{00} and g_{ij} are sufficient. For relativistic reduction of observations taking into account the motion of the masses and for rigorous formulation of dynamically or kinematically non-rotating reference system (DRS or KRS) the terms $O(c^{-3})$ in g_{0i} should be added. For unambiguous formulation of the post-Newtonian equations of motion of celestial bodies the terms $O(c^{-4})$ in g_{00} should be also taken into account.

3. Each system entering in the hierarchy of relativistic reference systems may be considered in version of dynamically non-rotating system (DRS) or kinematically non-rotating system (KRS). For celestial mechanics problems it is suitable to use DRS implying the absence of the Coriolis terms in the equations of motion. For astrometric problems both types are of importance.

4. The metric of each system may be presented in the closed form (as illustrated by the GRS metric taking into account only $O(c^{-2})$ and $O(c^{-3})$ terms).

Technical details of constructing reference systems and their relationships discussed here may be found in (Kopejkin, 1988, 1989a,b; Brumberg and Kopejkin, 1989a,b, 1990; Voinov, 1990; Brumberg, 1991).

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RELATIVISTIC CELESTIAL MECHANICS AND REFERENCE FRAMES

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ABSTRACT. A new formalism for treating the relativistic celestial mechanics of systems of N, arbitrarily composed and shaped, weakly self-gravitating, rotating, deformable bodies is presented. This formalism is aimed at yielding a complete description, at the first post-Newtonian approximation level, of (i) the global dynamics of such N-body systems ("external problem"), (ii) the local gravitational structure of each body ("internal problem"), and, (iii) the way the external and the internal problems fit together ("theory of reference systems").

1. Introduction

The problem of describing the dynamics of N gravitationally interacting extended bodies, called "celestial mechanics", has been thoroughly investigated (see e.g. Tisserand, 1960) in the framework of Newton's theory of gravity. Very shortly after the discovery of Einstein's theory of gravity, Einstein (1915), Droste (1916), De Sitter (1916) and Lorentz and Droste (1917) devised an approximation method (called "post-Newtonian") which allowed them to compare General Relativity with Newton's theory of gravity, and to predict several "relativistic effects" in celestial mechanics, such as the relativistic advance of the perihelion of planets, and the relativistic precession of the Moon's orbit. This post-Newtonian approach to relativistic celestial mechanics was subsequently developed (and completed) by many authors, notably by Fock (1959), Papapetrou (1951), Chandrasekhar and colleagues

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(1965, 1969, 1970), Caporali (1981), Grishchuk and Kopejkin (1986) and others (for a review of the development of the problem of motion in General Relativity see e.g. Damour, 1987).

However, in order to match the high precision which is already achieved by means of space techniques such as Satellite Laser Ranging (SLR), Lunar Laser Ranging (LLR) or Very Long Baseline Interferometry (VLBI), one needs a correpondingly accurate relativistic theory of celestial mechanics able to describe both the global gravitational dynamics of a system of N extended bodies, the local gravitational structure of each, arbitrarily composed and shaped, rotating deformable body, and the way each of these N local structures meshes into the global one. The traditional post-Newtonian approach to relativistic celestial mechanics using only one global coordinate system $x^\mu = (ct, x, y, z) \equiv (ct, x^i)$, $i = 1, 2, 3$, to describe an N-body system, fails in this task, for both conceptual and technical reasons; concepts like "center of mass", "multipole moments", "mass-centered frames" are used while they are ill defined. Usually such "mass-centered frames" with spatial coordinates X^i, e.g. given by

$$X^i = x^i - z^i(t), \tag{1}$$

where z^i denotes the global coordinates of the "center of mass" of the body under consideration are not dynamically useful in the sense that they do not efface the external gravitational field down to tidal effects but, instead, introduce into the description of the internal dynamics of the bodies many external "relativistic" effects proportional to the square of the orbital velocity or the external gravitational potential. This is because the external (global) description of each body contains many "apparent deformations" (Lorentz contractions etc.) which are not intrinsic to the body itself.

In recent years, several authors have tried to remedy some of the defects of the traditional post-Newtonian approach to the N-body problem. For instance, Martin et al. (1985) and Hellings (1986) have tried, in an essentially heuristic manner, to explicitly take into account the main apparent deformations due to the use of an external coordinate representation. More recently a notable progress in the theory of such local relativistic frames (at the post-Newtonian approximation, relevant to systems of N weakly self-gravitating bodies) has been achieved by Brumberg and Kopejkin (1988a,b) (Kopejkin, 1988; Brumberg, 1990) in a series of publications (see also Voinov, 1988). Their approach combines the usual post-Newtonian-type expansions with the multipole expansion formalisms for internally generated (Thorne, 1980; Blanchet and Damour, 1986, 1989) and externally generated (Thorne and Hartle, 1985), gravitational fields, and with asymptotic matching techniques (D'Eath, 1975, Damour, 1983). We believe, however, that the approach by Brumberg and Kopejkin has several drawbacks: ad hoc assumptions about the structure of various expansions (as e.g. in the coordinate transformation between global and local coordinates) are made, which are only partially justified by some later consistency checks; the scheme is confined to a particular model for the matter (isentropic perfect fluid) and rigidly restricts itself to considering only some special (harmonic) coordinate system; moreover, their approach is basically incomplete in that it neither describes the full multipole moment structure of the bodies with post-Newtonian accuracy, nor gets (translational or rotational) equations of motion with full post-Newtonian accuracy.

2. A New Approach Towards Relativistic Celestial Mechanics

We have introduced (Damour, Soffel and Xu, 1990a) a new formalism for treating the relativistic celestial mechanics of systems of N, arbitrarily composed and shaped, weakly self-gravitating, rotating, deformable bodies. This formalism yields a complete description, at the first post-Newtonian level, of the global dynamics of such N-body systems ("external problem"), the local gravitational structure of each body ("internal problem"), and the way they fit together ("relativistic theory of reference systems"). This new scheme successfully overcomes, in our opinion, the problems encountered by previous approaches (notably the one of Brumberg and Kopejkin): only very general assumptions are made for the structure of the formalism which is developed in a constructive way by proving a number of theorems; the structure of the stress-energy tensor of the matter is left completely open; the scheme is formulated in a certain "gauge-invariant" way which leaves a convenient flexibility in the choice of the time gauge (at the order $\delta t = O(c^{-4})$); the scheme describes with full post-Newtonian accuracy the gravitational structure of each body by means of a set of multipole moments which are linked in an operational way to what can be observed in the local gravitational environment of each body; finally, the scheme succeeds in getting translational and rotational equations of motion with full post-Newtonian accuracy, and inclusion of all multipole moments, for the N-body system. Our approach does not use any asymptotic matching technique but takes advantage of two different recent progresses in the first post-Newtonian approximation method· (i) linearization of Einstein's field equations by means of certain "exponential parametrization" of the metric tensor (introduced by Blanchet and Damour (1989), and Blanchet, Damour and Schäfer (1990)), and (ii) definition, by Blanchet and Damour (1989), of a set of post-Newtonian multipole moments of an isolated body given as compact support integrals of the stress-energy tensor of the matter. A third basic element of the present approach is our way of restricting (without fixing completely) the coordinate freedom inherent to the theory of General Relativity. We do that not by imposing one of the two *differential* coordinate conditions generally used in the post-Newtonian literature (namely "harmonic gauge" versus "standard post-Newtonian gauge") but by imposing, in all coordinate systems, some *algebraic* conditions on the metric coefficients, which can be written as $(i, j = 1, 2, 3)$

$$g_{00}\, g_{ij} = -\delta_{ij} + O(1/c^4). \qquad (2)$$

This condition can be described by saying that the spatial coordinates are "conformally cartesian" or "isotropic". This condition is compatible with both usual choices but is, at once, more flexible (for the time gauge) and more rigid (for the space gauge) than either one of them. It plays an important technical role in freezing down the coordinate freedom to a level which is nearly the usual freedom in Newtonian celestial mechanics (arbitrary choice of a time-dependent spatial origin and of a time-dependent rotation matrix).

3. Theory Of Reference Systems

For our problem of N gravitationally interacting extended bodies we employ $N+1$ coordinate charts (reference systems): one "global" chart with coordinates $x^\mu = (ct, x^i)$ and N "local" charts with coordinates $X^\alpha = (cT, X^a)$. Each one of the local charts is defined in the vicinity of some body, and comoving with it. For most practical purposes in the solar system two of these coordinate system will be sufficient: one global "barycentric" system (ct, x^i) and one local "geocentric" system (cT, X^a). The global, barycentric coordinate time t is also named TCB, whereas the geocentric coordinate time T also is called TCG.

In each of these reference systems we use an exponential representation for the metric tensor of the form

$$g_{00} = -e^{-2w/c^2} + O(6) \tag{3a}$$

$$g_{0i} = -\frac{4}{c^3} w_i + O(5) \tag{3b}$$

$$g_{ij} = +e^{+2w/c^2} \gamma_{ij} + O(4) \tag{3c}$$

where $O(n) \equiv O(c^{-n})$ indicates the post-Newtonian order of magnitude. In this expression w is a generalization of the usual Newtonian potential and w^i is some gravitational vector potential arising because of "magnetic type gravity". One finds that the Riemann curvature tensor of γ_{ij} is of order $O(4)$, i.e. to post-Newtonian (PN) order the space metric is conformally flat. Hence, there exists a preferred class of spatial coordinates, where

$$\gamma_{ij} = \delta_{ij} + O(4), \tag{4}$$

or, equivalently, where condition (2) holds. In the formulation of our framework we systematically use such preferred spatially isotropic coordinates. Then, for each reference system, the information in the metric tensor is fully contained in the scalar field w and the vector field w_i. The Einstein field equations remarkably become *linear* in terms of these variables:

$$\Delta w + \frac{3}{c^2}\partial_t^2 w + \frac{4}{c^2}\partial_{ti}^2 w_i = -4\pi G\sigma + O(4) \tag{5a}$$

$$\Delta w_i - \partial_{ij}^2 w_j - \partial_{ti}^2 w = -4\pi G\sigma^i + O(2), \tag{5b}$$

where

$$\sigma = \frac{T^{00} + T^{ss}}{c^2} \tag{6a}$$

$$\sigma^i = \frac{T^{0i}}{c}. \tag{6b}$$

Though, for each reference system, our spatial coordinates are fixed (modulo a choice of origin and rigid rotation) by our spatial isotropy condition, we do not fix completely our time coordinate, but keep a certain flexibility linked to a gauge invariance of the 1PN field equations: if $w_\mu \equiv (w, w_i)$ is a solution of eqs.(5) with some given source terms $\sigma^\mu \equiv (\sigma, \sigma^i)$ so is $w'_\mu = (w', w'_i)$ (modulo PN error terms) with

$$w' = w - \frac{1}{c^2}\partial_t \lambda, \tag{7a}$$

$$w'_i = w_i + \frac{1}{4}\partial_i \lambda, \tag{7b}$$

where $\lambda(x^\mu)$ is an arbitrary (differentiable) function. This gauge invariance corresponds to a shift of the time variable according to

$$\delta t = \frac{1}{c^4}\lambda(t, \boldsymbol{x}), \tag{8}$$

which affects none of the physical quantities at the 1PN level. This especially applies to the problem of time scales: the relation between barycentric coordinate time (t = TCB) and geocentric coordinate time (T = TCG) is not affected by transformation (8) to post-Newtonian order. Note, that our gauge freedom encompasses both the choice of the "harmonic gauge" as well as of the "standard post-Newtonian gauge". E.g., for the harmonic gauge the solution of the field equations reads

$$w = GI_{-1}[\sigma] + \frac{G}{2c^2}\partial_t^2 I_1[\sigma] + O(4) \tag{9a}$$

$$w_i = GI_{-1}[\sigma^i] + O(2), \tag{9b}$$

where

$$I_\alpha[f](t, \boldsymbol{x}) \equiv \int d^3x' |\boldsymbol{x} - \boldsymbol{x}'|^\alpha f(t, \boldsymbol{x}'). \tag{10}$$

Similarly to what is done in Maxwell's theory of electromagnetism, we can introduce gauge-invariant (gravito-electric and gravito-magnetic) fields e and b by

$$\boldsymbol{e} \equiv \nabla w + \frac{4}{c^2}\partial_t \boldsymbol{w} \tag{11a}$$

$$\boldsymbol{b} \equiv -4\nabla \times \boldsymbol{w}, \tag{11b}$$

satisfying (in each system) "Maxwell-like" equations of the form

$$\nabla \cdot \boldsymbol{b} = 0, \tag{12a}$$

$$\nabla \times \boldsymbol{e} = -\frac{1}{c^2}\,\partial_t \boldsymbol{b}, \tag{12b}$$

$$\nabla \cdot \boldsymbol{e} = -\frac{3}{c^2}\,\partial_t^2 w - 4\pi G\sigma + O(4), \tag{12c}$$

$$\nabla \times \boldsymbol{b} = 4\,\partial_t \boldsymbol{e} - 16\pi G\boldsymbol{\sigma} + O(2). \tag{12d}$$

For the coordinate transformation between each of the local coordinates X^α and the global coordinates x^μ we start with the completely general ansatz

$$x^\mu = f^\mu(X^\alpha) = z^\mu(X^0) + e^\mu_a(X^0)X^a + \xi^\mu(X^0, X^a). \tag{13}$$

Here, $a = 1, 2, 3$ labels the spatial coordinates in the local system, $z^\mu(X^0)$ $(X^a = 0)$ describes the "global" motion of some "central worldline" of the body under consideration (which will later be chosen as the worldline of the "center of mass" of the body) and ξ^μ is assumed to be at least quadratic in X^a. Now, our PN assumptions plus spatially isotropic coordinates essentially determine $f^\mu(X^\alpha)$ completely, modulo the choices of some arbitrary central worldline and of some (slowly varying) rotation matrix R^j_a in $e^i_a(T)$ (see eq.(23) below). E.g., one finds uniquely

$$\xi^i(T, \boldsymbol{X}) = \frac{1}{c^2} e^i_a(T)\left[\frac{1}{2}A_a \boldsymbol{X}^2 - X^a(\boldsymbol{A}\boldsymbol{X})\right] + O(4), \tag{14}$$

where

$$A_a \equiv f_{\mu\nu}\, e^\mu_a \frac{d^2 z^\nu}{dT^2} \tag{15}$$

(the 4-acceleration of the central worldline projected into the corresponding local system). Here, $f_{\mu\nu}$ denotes the usual flat Minkowski metric in Cartesian coordinates ($f_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$).

Because of the linearity of the field equations, in each local system, we can uniquely split the metric potentials† $W_\alpha \equiv (W, W_a)$ into "self-" and "external-part"

$$W_\alpha = \overset{+}{W}_\alpha + \overline{W}_\alpha. \tag{16}$$

Here, the self-part ($W^+{}_\alpha$) describes the gravitational influence of the central body itself, while the external-part describes the action of all the *other* bodies of the system (plus *inertial* terms).

† We use capital letters for local quantities.

As central results of our scheme we find the following transformation rules of potentials between some local and the global system to PN order:

$$w = \left(1 + \frac{2}{c^2} V^a V^a\right) W + \frac{4}{c^2} V^a W_a + \frac{c^2}{2} \ln \left[A_0^0 A_0^0 - A_a^0 A_a^0\right] \tag{17a}$$

$$w^i = R_a^i W^a + v^i W + \frac{c^3}{4} \left[A_0^0 A_0^i - A_a^0 A_a^i\right], \tag{17b}$$

where

$$A_\alpha^\mu \equiv \frac{\partial x^\mu}{\partial X^\alpha} \tag{18}$$

and v^i is the velocity of the "central point" in the global system ($V^a = R_i^a v^i$). Hence, not only the field equations are linear, but also the various $w \leftrightarrow W$ relationships! Writing this affine transformation in the form

$$w^\mu(x) = \mathcal{A}_\alpha^\mu(T) W^\alpha(X) + \mathcal{B}^\mu(X), \tag{19}$$

we find the transformation of the self-parts to take the simple form

$$\overset{+}{w}{}^\mu(x) = \mathcal{A}_\alpha^\mu(T) \overset{+}{W}{}^\alpha(X), \tag{20}$$

a remarkable result indeed. Hence, in contrast to previous works on relativistic reference systems, we obtain the various transformation laws in closed, i.e., non-expanded form (we do *not* use a matched asymptotic expansion technique like, e.g, Brumberg and Kopejkin), which has in fact many advantages for practical applications.

We introduce the following (BD) mass (M_L) and current moments (S_L) of the central body defined by (L is a multi spatial index, $L \equiv a_1 \dots a_l$)

$$\begin{aligned} M_L = \int d^3X \hat{X}_L \Sigma + \frac{1}{2(2l+3)c^2} \frac{d^2}{dt^2} \left[\int d^3X \hat{X}_L \boldsymbol{X}^2 \Sigma\right] \\ - \frac{4(2l+1)}{(l+1)(2l+3)c^2} \frac{d}{dt} \left[d^3X \hat{X}_{aL} \Sigma_a\right], \end{aligned} \tag{21a}$$

$$S_L = \int d^3X \epsilon^{ab<c_l} \hat{X}^{L-1>a} \Sigma^b, \tag{21b}$$

where all quantities are considered in the local system of the considered body, where the caret and the bracket <> indicates that the symmetric and trace-free (STF) part should be taken (see e.g. Thorne, 1980) and the integration extends over the support of the body under consideration. These BD-moments are called "physical" by us because the self-part of the local gravitational potentials of the considered body can be expanded in terms of these multipole moments (modulo an irrelevant gauge transformation). See Blanchet and Damour (1989) for the other physical meanings of these moments when considering isolated systems.

Not only are the self-potentials $W^{+}{}_{\alpha}$ expanded in terms of (STF) moments, but similarly the external-potentials $\overline{W}_{\alpha}$ or the corresponding $\overline{E}$ and $\overline{B}$-fields are "skeletonized" by defining (for each local system) two corresponding (gravito-electric and gravito-magnetic) sets of post-Newtonian *tidal moments*:

$$(l \geq 1) \quad G_L \equiv \left[\partial_{<L-1}\overline{E}_{a_l>}(T,X)\right]_{X^a=0}, \tag{22a}$$

$$(l \geq 1) \quad H_L \equiv \left[\partial_{<L-1}\overline{B}_{a_l>}(T,X)\right]_{X^a=0}. \tag{22b}$$

Using the various expansions of the self-potentials and the transformation laws we can get the external tidal moments explicitly as functions of the intrinsic moments M_L, S_L of all the *other* bodies, plus some inertial contributions.

If we require (as we may) the quantities e^{μ}_{α} ($e^{\mu}_{0} \equiv c^{-1}dz^{\mu}(T)/dT$) to represent an orthonormal tetrad with respect to the "external metric" defined by $\overline{w}_{\mu}$, our theory of reference systems is completely specified up to the choice of:

- the time gauge
- the central worldlines, $z^i(T)$

and a slowly time dependent rotation matrix $R^j_a(T)$ appearing in

$$e^i_a(T) = \left(1 - \frac{1}{c^2}\overline{w}|_{X^a=0}\right)\left(\delta^{ij} + \frac{1}{2c^2}v^i v^j\right) R^j_a(T). \tag{23}$$

4. A User's Guide To Reference Systems

For practical purposes let us summarize our results for relativistic reference frames for the problem of barycentric and geocentric coordinate systems. The global, barycentric coordinate system was denoted by (ct, x^i), the local geocentric one by (cT, X^a). The barycentric (geocentric) coordinate time t (T) is also called TCB (TCG). The relation between the barycentric and the geocentric system is written in the form (13). For many practical applications presently one can neglect the quadratic and higher order terms ξ^{μ} in (13); then the relation between these coordinate systems is simply given by:

$$ct = z^0(T) + e^0_a(T)X^a \tag{24a}$$

$$x^i = z^i(T) + e^i_a(T)X^a. \tag{24b}$$

For the relation between $t = \text{TCB}$ and $T = \text{TCG}$ one finds the result:

$$\left.\frac{dt}{dT}\right|_{X^a=0} \equiv \left.\frac{d(\text{TCB})}{d(\text{TCG})}\right|_{X^a=0} = e_0^0 = 1 + \frac{1}{c^2}\left(\overline{w}(z_\oplus) + \frac{1}{2}v_\oplus^2\right). \tag{25}$$

Here, $X^a = 0$ refers to the geocenter, $\boldsymbol{v}_\oplus$ is the velocity of the geocenter in the barycentric system and $\overline{w}$ can be replaced by the *external* Newtonian potential U_{ext} taken at the geocenter. For the linear term in the time transformation $e_a^0(T)$ in eq.(24a) one finds to sufficient approximation

$$e_a^0 \simeq R_a^j v_\oplus^j. \tag{26}$$

In this last formula the matrix R_a^j relates the spatial barycentric coordinates with the spatial geocentric ones, which not necessarily point into the same direction. Using these results the TCB-TCG relation can be written in the form

$$\text{TCB} - \text{TCG} = c^{-2}\int_{t_0}^{t}\left(\frac{1}{2}v_\oplus^2 + U_{\text{ext}}(z_\oplus)\right)dt + \boldsymbol{v}_\oplus \cdot \boldsymbol{X}. \tag{27}$$

Here, $\boldsymbol{v}_\oplus \cdot \boldsymbol{X} = v_\oplus^j R_a^j X^a \simeq v_\oplus^j(x^j - z_\oplus^j)$.

To post-Newtonian accuracy the spatial coordinates x^i and X^a are related by

$$x^i = z_\oplus^i(T) + e_a^i X^a + O(X^2), \tag{28}$$

where $z_\oplus^i(T)$ describes the motion of the geocenter and the coefficients e_a^i are determined by eq.(23) which we may write in the form

$$e_a^i(T) = \left(1 - \frac{1}{c^2}U_{\text{ext}}(z_\oplus)\right)\left(\delta^{ij} + \frac{1}{2c^2}v_\oplus^i v_\oplus^j\right)R_a^j(T). \tag{29}$$

The matrix $R_a^j(T)$ finally determines the precise relation between the barycentric and the geocentric spatial coordinates. Two choices for $R_a^j(T)$ are preferred, leading to geocentric coordinates which are

- fixed star oriented (kinematically non-rotating)
- or locally inertial (dynamically non-rotating).

In the first case of kinematically non-rotating geocentric coordinates we might take

$$R_a^j(T) = \delta_{ij}.$$

In this case the direction of geocentric spatial coordinates are given by the corresponding directions of barycentric coordinates, i.e. practically by some catalogue of extragalactic sources. If such kinematically non-rotating geocentric coordinates are chosen then additional (time dependent) inertial forces, mainly due to the geodesic precession (of inertial axes) have to be taken into account in any dynamical equation (e.g. for an artificial satellite). The geodesic precession in this case has to be considered also in the precession matrix as well as in the nutation series (because the geodesic precession has an annual term proportional to the eccentricity of the Earth's orbit).

On the other hand, if dynamically non-rotating, locally inertial geocentric coordinates are chosen the geodesic precession is contained in the $R^j_a(T)$ matrix and not in the precession (nutation) matrix. This choice, however, has the disadvantage, that the local geocentric spatial coordinate lines precess w.r.t. the barycentric ones.

5. Equations Of Motion

In our approach, global equations of motion are derived by combining the local energy-momentum balance equations

$$T^{\beta}_{\alpha;\beta} = 0 \tag{30}$$

with conditions chosen to relate the central worldline of a body with the corresponding energy-momentum distribution. We find that a theorem of the following form holds in each local frame

Theorem. *The energy-momentum conservation equations (24) in each local frame imply constraints on the time-evolution of the three lowest BD multipole moments of the form:*

$$\frac{dM}{dT} = \frac{1}{c^2}\mathcal{E}^{1\mathrm{PN}}(\overset{(p)}{M}_L, \overset{(p')}{G}_{L'}) + O(4), \tag{31a}$$

$$\frac{d^2 M_a}{dT^2} = \sum_{l\geq 0}\frac{1}{l!}M_L G_{aL} + \frac{1}{c^2}\mathcal{F}_a^{(1\mathrm{PN})}(\overset{(p)}{M}_L, \overset{(q)}{S}_L; \overset{(p')}{G}_{L'}, \overset{(q')}{H}_{L'}) + O(4), \tag{31b}$$

$$\frac{dS_a}{dT} = \sum_{l\geq 0}\frac{1}{l!}\epsilon_{abc}M_{bL}G_{cL} + \frac{1}{c^2}\mathcal{G}_a^{(1\mathrm{PN})}(\overset{(p)}{M}_L, \overset{(q)}{S}_L; \overset{(p')}{G}_{L'}, \overset{(q')}{H}_{L'}) + O(2/4), \tag{31c}$$

where

$$\overset{(p)}{M} \equiv \frac{d^p}{dT^p}M \qquad \text{etc.}$$

and all the right-hand sides of eqs.(31) are bilinear in the BD multipole moments and in the above-introduced tidal moments, and their time derivatives.

More explicitly, the right-hand sides of eqs.(31) consist of an infinite series of terms, each having the form

$$\overset{(p)}{M}\overset{(q)}{G},\ \overset{(p)}{M}\overset{(q)}{H},\ \overset{(p)}{S}\overset{(q)}{G},\quad \text{or}\quad \overset{(p)}{S}\overset{(q)}{H}.$$

The special notation $O(2/4)$ in eq.(31c) means that, when one is working strictly within the 1PN approximation, it is sufficient to know S_a to Newtonian accuracy and therefore the explicitly written Newtonian torque is enough. However, it is possible to define a local spin vector for body A (differing from the Newtonian spin moment (21b) by $O(c^{-2})$ additional terms) whose time evolution is given, modulo $O(4)$, by an equation of the form (31c).

To relate the central worldline of a body with the corresponding energy-momentum distribution, we choose each central worldline to coincide with the BD center of mass of the considered body, i.e., we require $M_a = 0$. From eq.(31b) this then implies

$$0 = \frac{d^2 M_a}{dT^2} = \sum_{l \geq 0} \frac{1}{l!} M_L G_{La} \quad + (c^{-2} - \text{terms}). \tag{32}$$

Since one can prove that

$$G_a = -\frac{d^2 z^a(t)}{dt^2} + \overline{w}_{,a}|_{X^a=0} \quad + (c^{-2} - \text{terms}), \tag{33}$$

we see that the "local equation of motion" (32) can be rewritten in the looked for global form for the equation of translational motion:

$$\frac{d^2 z^a(t)}{dt^2} = \overline{w}_{,a}|_{X^a=0} \quad + \sum_{l \geq 2} \frac{1}{l!} M_L G_{La}/M + (c^{-2} - \text{terms}), \tag{34}$$

where we have derived the complete PN expression on the right hand side of eq.(34) for arbitrary mass- and current-moments of the individual bodies (Damour, Soffel and Xu, 1990b). We have explicitly verified (Damour, Soffel and Xu, 1990a) that in the monopole limit without spins ("spherical, non-rotating" bodies) one recovers the usual Lorentz - Droste - Einstein - Infeld - Hoffmann equations of motion used for modern numerical ephemeris programs (such as the DE programs from JPL). Work for the PN-spin motion is still in progress.

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HIPPARCOS: ITS LINK TO AN EXTRAGALACTIC REFERENCE FRAME

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ABSTRACT. This paper reviews the links currently envisaged for a mission of duration $3\frac{1}{2}$ years, with estimates of the precision of each link.

1 Introduction

The Catalogue of star coordinates to be produced by HIPPARCOS will be highly rigid and homogeneous, but will have arbitrary zero-points because no observations are being made by which the mean equator and the dynamical equinox could be located directly. In the first instance the Catalogue, comprising about 120,000 stars, will be aligned to FK5 by the two Data Analysis Consortia as an off-line task, working independently. Then, if data from the so-called 'Super-high Priority Link' (Section 2) are ready in time, the alignment will be further refined; if not, the FK5-aligned Catalogue will be published as it stands, and further refinement by the methods of Sections 2, 3 and 4 carried out subsequently. The ultimate purpose is to establish a conventional celestial barycentric reference frame, as close as possible to the existing FK5 equator and equinox referred to J2000.0, and accessible to astrometry at visual and radio frequencies, in compliance with the recommendation of the IAU WG on Reference Systems made at this Colloquium.

The VLBI Reference Frame due to be presented to the IAU General Assembly in 1994 will be made up from a number of tentative frames combined in the manner now carried out by IERS. At present, the right ascension scales of these frames are purely relative scales, and alignment to the FK5 right ascension scale is done through optical astrometry of some tens of quasars (Feissel, 1990. The practice of using one object alone, such as Algol or 3C273B, has largely been discontinued). The FK5 frame is known to contain regional warps (Morrison et al., 1990), so that HIPPARCOS will give a better realization of the FK5 system in RA than the VLBI frames. In RA the link will thus be in the sense of linking VLBI to HIPPARCOS-FK5. Thereafter the RA zero-point will be a non-rotating vector locked into the extragalactic frame. The question of whether this frame does or does not possess a cosmic rotation, is not one that will be considered here.

The accuracies of the HIPPARCOS positions and proper motions for the $3\frac{1}{2}$ year mission are estimated at $\pm 0''.002$ and $\pm 0''.002 yr^{-1}$ respectively. It would be desirable if the rms error of the rotation matrix R and its time derivative R' applied to the HIPPARCOS-FK5 frame to link to VLBI, be not greater than one or two tenths of these quantities. The rms error in R specifies the accuracy with which the optical and radio frames have been unified at the specified epoch, while that of R' gives limits to their relative rotation.

2 The Super-high Priority Link

Two methods have been selected: (i) Hubble Space Telescope and (ii) Radio Stars.

2.1. HUBBLE SPACE TELESCOPE

The intention is to measure the separation, and its rate of change with time, between pairs of quasars and HIPPARCOS stars, both situated within the same 'pickle' of the Fine Guidance Sensor. Unfortunately so many difficulties have arisen that the schedule of Hubble observations for this program has had to be suspended for the present (Jefferys et al., 1990).

2.2. RADIO STARS

Many radio stars are sufficiently bright to be well within the range of accurate measurement by HIPPARCOS (the limit is about 11^m, but the accuracy improves with increasing brightness). Radio stars are not extragalactic, so that measurements must be repeated over suitable timescales to obtain their proper motions. In addition, their radio fluxes are highly variable and usually very faint (below a few mJy at 5 GHz), but many are amenable to phase-reference relative to VLBI reference frame quasars situated within a few degrees on the sky. Because of their extreme variablity, VLBI observations can accumulate only rather slowly. Some radio stars such as Algol display finite structure so that a series of measurements is needed in order to model the offset between the optical and radio emitting regions at the epoch of each HIPPARCOS observation (Lestrade et al., 1990). But the selection has been weighted in favour of RS CVn binaries which are more compact: many are resolved only at the $0''.002$ level.

The programs to be described are drawn from only a small fraction of those stars that have confirmed radio emission and are included in the HIPPARCOS Input Catalogue (there are 186 in the Super-high Priority Link alone) but any new programs commencing only now would probably not be completed in time for that link. An inventory of radio stars has been compiled by Walter (1990).

2.2.1. *NRL/USNO/CSIRO.* This program consists of 54 stars north of $\delta = -26°$ with plans for a southern extension, giving altogether about 100 stars (De Vegt et al., 1990).

2.2.2. *JPL.* Eleven stars[1] north of $\delta = 0°$ (Lestrade et al., 1990).

2.2.3. *MERLIN (Multi-Element Radio Linked Interferometer Network, Jodrell Bank).* Twelve stars[2] have been accepted for measurement (Anderson et al., 1989).

[1] LS161 303; Algol; UX Ari; HR1099; FK Com; HR5110; σCrB; Cyg X1; AR Lac; SZ Psc; II Peg.

[2] HD1061; UX Ari; HD26337; b Per; 54 Cam; BD+52°1579; κ Dra; 55 Boo; HD341475; BD+43°3571;

2.2.4. *Accuracy of Link.* If at the end of the mission 50 radio stars have been measured by HIPPARCOS and VLBI, all to the same accuracy of $0''.002$ in position and $0''002yr^{-1}$ in proper motion, in both optical and radio, the rms errors in R and R' will be:
$\sigma_R = 0''.0005$ and $\sigma_{R'} = 0''.0007yr^{-1}$ (Froeschlé and Kovalevsky, 1982).

3 Ground-based Optical Astrometry of Quasars

The brightness of the optical counterparts of the VLBI quasars runs from about 15^m to fainter, and are well below the range of HIPPARCOS[3]. The resolution needed to relate the radio and optical feature in a quasar requires an optical telescope of aperture at least 2m, which means in practice that the field size is limited to a diameter of order 1°, and this is not sufficient to include enough HIPPARCOS stars for a link (on average there are fewer than 3 stars per □°). So the step from HIPPARCOS to quasar must be bridged by one or two intermediate steps provided by wide field astrographs. These also reduce the difficulties encountered in the photography over such large brightness ranges. The precision of a single object is about 15 times rms worse than the HIPPARCOS and VLBI precision, so to obtain accuracies in R and R' that are comparable to those anticipated for the Super-high Priority Link, it is necessary to increase both the time base and the number of quasars. For example, a 5-year program of 400 quasars would yield:

$$\sigma_R = 0''.0019 \text{ and } \sigma_{R'} = 0''.0006yr^{-1}.$$

No programs have yet been offered involving ground-based interferometry or CCDs used in the manner of the Hubble pickle: undoubtedly such methods will play important roles in the future, but are unlikely to be ready in time for the present mission.

3.1. LARGE TELESCOPE PROGRAMS

3.1.1. *NRL/HAMBURG/NASA/GSFC/CSIRO/USNO.* This 5-year program was commenced in 1987, and is described by Russell et al. (1990). The aim is for 400 sources with optical counterparts suitable for astrometry, distributed over the whole sky.

3.1.2. *CONFOR.* A Russian program intended for completion by 1992. It is described by Kumkova et al. (1990). There are about 200 sources with suitable optical counterparts, mostly north of $\delta = -26°$.

3.2. FICTITIOUS PROPER MOTIONS OF QUASARS

A program described by Brosche et al. (1990), based on plates some as old as 90 years, with

Cyg OB2 No 5; V509 Cas.

[3]3C273B was measured on 4th. July 1990, but the signal-to-noise ratio was insufficient for the measurements to play any significant role in the link.

about 10 quasars. A preliminary reduction for four quasars[4] gives for the three components of the rotational link:

$$\sigma(\omega_1) = 0''.0010 yr^{-1}; \sigma(\omega_2) = 0''.0010 yr^{-1}; \sigma(\omega_3) = 0''.0014 yr^{-1}.$$

4 Absolute Proper Motion Surveys

Provision has been made in the HIPPARCOS Input Catalogue for three surveys of proper motion of stars measured relative to galaxies, as follows:

4.1. LICK NPM

This survey contains about 300,000 stars in the brightness range B $= 9^m$ to 18^m north of $\delta = -23°$ (Klemola, 1990). The rms error of a single proper motion is $0''.005 yr^{-1}$. There are about 19,000 stars in common with the HIPPARCOS Input Catalogue, of which 14,100 are from AGK3 and the remainder, all south of $\delta = -2°$, from SAO.

4.2. YALE-SAN JUAN SPM

This is similar to the Lick Survey of which it is a southern extension, but because the measuring phase had not been entered until after the HIPPARCOS Input Catalogue had reached an advanced stage, it was possible to make a more link-oriented selection of stars. Selection was based mainly on the SIMBAD Database, and there are about 12,000 stars in common with the HIPPARCOS Input Catalogue (Van Altena et al., 1990).

Combining both the NPM and the SPM Surveys into one single all-sky survey, the formal rms error for R', ignoring possible zonal errors, is $0''.00007 yr^{-1}$.

4.3. TAUTENBURG

This program uses the Tautenburg Schmidt Telescope and the ASCORECORD, APM (Cambridge) and MAMA (Paris) measuring machines. The present status is 15 link fields with at least 250 HIPPARCOS stars, giving an anticipated $\sigma_{R'} = 0''.004 yr^{-1}$. Preliminary details have been published by Dick et al. (1987).

5 Appendix

The members of the HIPPARCOS Input Catalogue Consortium WG on the Extragalactic Link are:

A.N. Argue (Cambridge, Chairman); P. Brosche (Bonn); C. De Vegt (Hamburg); R.L. Duncombe (Austin); P.D. Hemenway (Austin); A.R. Klemola (Lick); J. Kovalevsky (CERGA); J.-F. Lestrade (Paris); C.A. Murray (c/o RGO); G. Ruben (Potsdam); H.G. Walter (Hei-

[4] 3C273B; OQ208; 3C371; 3C309.3.

delberg); G.L. White (c/o CSIRO).

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CURRENT STATUS OF THE ASTROMETRIC CAPABILITIES OF THE HUBBLE SPACE TELESCOPE FINE GUIDANCE SENSORS

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ABSTRACT. The Fine Guidance Sensors (FGSs) are the instrument of choice for most astrometric measurements with the Hubble Space Telescope (HST). The observed amount of spherical aberration in the Ritchey Chretien optical system does not affect positional measurements with perfectly aligned FGSs because they are interferometers. The FGSs combine wavefronts from points in the exit pupil with other points which are at the same radial distance from the optical axis. Asymmetric aberrations such as coma and astigmatism do affect the measured positions. The current knowledge of the HST wavefront error, the FGS operation and the implications for milliarcsecond relative astrometry are discussed. It is still planned to use the HST to tie the HIPPARCOS and VLBI Reference Frames together at the few milliarcsecond level.

1. Introduction

On the morning of 24 April 1990, STS-31 carried the Hubble Space Telescope into space. It was placed into orbit the following day with an inclination of 28.5 degrees, perigee of 331 nautical miles and apogee of 332 nautical miles; all very close to nominal.

Of the six scientific instruments which comprise the instrumental package of the Hubble Space Telescope (HST), only the Fine Guidance Sensors (FGSs) will be discussed. Three Fine Guidance Sensors are associated with the HST. These interferometric devices, as part of the Pointing Control System, fix the orientation of HST with respect to known stars, or may be used for astrometric measurements. The location of the three FGS fields of view in the focal plane of HST are shown in figure 1. The design aspects of the Fine Guidance Sensors as well as their operational and observational modes are completely described in references 7 and 8 given in the bibliography.

Figure 2 shows the fields of view of the three FGSs with respect to the optical axis.

The FGSs use Koester's prisms to combine two images of the exit pupil which are reversed with respect to each other. Two photomultiplier tubes measure the light on either side of the interference pattern; call these outputs A and B. A Fine Error Signal (FES) is produced by measuring (A-B)/(A+B). A hypothetical plot of the FES as a function of the position offset from the interferometer "null" position is given in figure 2. The null position is the center of the pattern where the transfer function passes through zero.

This paper presents the results of consistant sets of transfer functions observed under a variety of conditions. Affects of the observed vehicle jitter, the telescope optics problems, and possible internal FGS problems are discussed.

2. The Observations

We have obtained transfer scans in all FGSs, but with limited data. We have not obtained a consistent set of data (different magnitude stars, jittered and non-jittered conditions, same secondary mirror position, etc.) in POS mode with full aperture in any FGS and in POS mode with 2/3 aperture in any FGS except FGS3. However, fine lock has been achieved for guiding in all three FGSs on bright stars (above 13th magnitude).

The HST ground system is slow to respond to new input. From the time an observation is first formulated, written, submitted, approved and scheduled by the Space Telescope Science Institute, at least 4 to 6 weeks will elapse before the observation is integrated into the timeline and actually performed.

2.1. THE DATA

A series of Transfer scans were obtained on sets of stars during two programs. The first program was a series of secondary mirror position changes to characterize the change of image structure in the Planetary Camera and the Faint Object Camera frames as a function of focal position. At each mirror position, WFPC and FOC images were taken (except when guide stars were lost due to vehicle jitter) and transfer scans were performed with the FGSs on a 9th , a 12th, and a 15th magnitude star. Each star was scanned five times, providing reproducibility checks. Scans were performed with both clear filter and 2/3 apertures. These scans provided the first consistent set of data on the behavior of the three FGSs. All data were taken while guiding in Coarse Track.

The second series of positions was obtained in an attempt to determine the optical field angle distortions (OFAD) within each pickle. The observations were made in NGC188 where the relative positions are known from ground based observations at the 0.01 arcsecond level of accuracy (rms). Ultimately, the OFAD will be determined by a priori measurements and a full plate overlap solution at the 0.002 arcsecond level, but the initial characterization of the OFAD is done with respect to the ground-based measurements to get first-order values for the OFAD coefficients.

2.2. REVIEW OF THE DATA--SYMPTOMS, PROBLEMS, AND POSSIBLE CAUSES

Three areas that contribute to the FGS problems may be identified: Vehicle jitter, OTA (primary and/or secondary mirror) misfigurement and misalignment, and possible internal FGS problems.

2.2.1 *Vehicle Jitter.* When the Telescope is guided in Coarse Track, the sum of the PMT counts from the four quadrants of the 5 arcsecond square field stop (IFOV) are used to determine the centroid position of the guide stars, as the stop is "nutated" around the star. The position is updated 4 times per second while using all of the data from the previous full second; therefore, the Coarse Track information can be used to estimate how much the vehicle line of sight is moving as a function of time. The thermal variation of the solar arrays as the telescope moves from orbit day into orbit night causes the pointing of the telescope to "jitter". The peak-to-peak excursions after the day/night transition are on the order of .2 arcseconds while the "quiet time" peak-to-peak excursions are on the order of .050 arcseconds. A dominant frequency of 0.1 Hz can be clearly seen with 0.6 Hz also present. A digital filtering algorithm is being put into the Pointing Control System Flight Software to attempt to remove the 0.1 Hz jitter. If the 0.6 Hz jitter is driven by the 0.1 Hz, then the 0.6Hz jitter will be reduced also. We hope the jitter will be significantly reduced, since we believe it to be perturbing the transfer scan data (and hence varying position measurements) with the additional possibility of contributing to the loss of Fine Lock on guide stars fainter than 13th magnitude.

Consecutive scans of the same star with the same FGS show large variations in the character of the Transfer Functions. Dr. Gerald Nurre has simulated the effect of jitter (input stimulus) at 0.1 Hz and 0.6 Hz on an ideal transfer function, while varying the phase of the input stimulus with respect to the phase of the transfer scan. The variation of the signature in some of his simulations show induced characteristics which are strikingly similar to the observations from one scan to another.

2.2.2 *OTA Misalignments/Possible Internal FGS Problems*

i. Figure 3 shows transfer scans for FGS2, 1, and 3 in x and y on the same star with the same aperture (2/3). The ordinate is the normalized Fine Error Signal. The separation between the tick marks is either 0.1 or 0.2 unitless numbers. The expected theoretical amplitude for a perfect FGS is between 0.7 and 0.8. The amplitudes of FGS3 are close to nominal, whereas the amplitudes of the other two are significantly reduced. The abscissa is in arcseconds where the total data plotted is about one or about two arcseconds depending on the individual plots. The width of the ideal transfer function from the positive peak to the negative peak is about 40 milliarcseconds and can be used to estimate the scales of the individual graphs. The plots have been auto-scaled and are reproduced from the online data extraction system. The different FGS scans were observed at different times. The transfer scans taken so far are predominantly taken in the center of the pickle. Clear differences between the transfer scans indicate that FGS to FGS differences exist. The x coordinate in FGS2 corresponds to the y coordinates in FGSs 1 and 3, while the y coordinate in FGS2 corresponds to the x coordinates in FGSs 1 and 3. If a star is double, then the component separations which show up in x and/or y in FGS2 will show up in the reversed coordinates in the other two FGSs. Therefore, if a scan which appears as a "W" were due to a double star in the y component of FGS2, then it would show up as a "W" in x in FGS1 and FGS3. While structure exists in the odd FGSs, it is unlikely to be due to a double star because of the different nature of the transfer functions. 12th magnitude stars were used for this comparison because comprehensive consistant data on brighter stars do not yet exist.

ii. Focus Shift. A comparison has been made between transfer function scans of the same star at two slightly different focus settings of the secondary mirror. The position difference between the two settings was 10 microns along the optical axis. Within the noise, no difference between the two scans can be determined. However, the focus position difference is so slight that nothing can be said about the change in the characteristic of the transfer function for widely separated focus settings. Still, the result indicates that the system is stable after a mirror move.

iii. Magnitude Comparison. A comparison has been made between scans with the same FGS on a 12th magnitude and a 15th magnitude star. Except for the noise, the characteristics appear to be the same. However, the difficulty of "picking out" the transfer function from the noise in the fainter star's data is evident.

The effect of averaging within the FGS on real data has been demonstrated. Successive tests indicate the data with binning at possible FGS averaging times. With good data (note these data were taken in FGS3 with 2/3 aperture-the best conditions we have at the moment) we should be able to detect and lock onto stars as faint as 15th with the system as it presently exists.

iv. Clear vs 2/3 Aperture Comparison. Comparisons have been made between scans using a full aperture and a 2/3 aperture stop. A 2/3 aperture observation produces a reasonably "clean" S-Curve while the full aperture produces a degraded signal/noise ratio, or poor modulation in the S-Curve.

v. Change in tilt and decenter of the secondary mirror.

The initial transfer functions observed showed the following characteristic: The transfer function in FGS 3 yields best modulation in the signal. The signal peak is almost to the expected amplitude (about .7 on a normalized scale) for a fully functional FGS. The transfer functions in the other FGSs show greatly reduced modulation (around 0.2), FGS2 being far worse than FGS1 or 3. The poor performance was ascribed to off-axis coma and astigmatism. A mirror correction was made to correct for the inferred astigmatism and coma. The amplitudes are now more consistant with each other, ranging from 0.4 to 0.6 under different conditions for the different FGSs. The amplitude on FGS3 was in fact slightly reduced after the mirror moves.

2.2.3. *Summary of the Current Status of the FGSs.* The HST is being shaken by solar array thermal shock as it passes through the earth's terminator with sporadic thermal aftershock occurring during orbit day. The thermal gradients are causing "vehicle jitter" resulting in disturbances around the 0.1 Hz and 0.6 Hz modes, with amplitudes up to 0.2 arcseconds peak-to-peak initially through the terminator. The jitter is making the use of Fine Lock difficult-to-impossible on fainter stars, and making the interpretation of the transfer function scans in the FGSs difficult. However, engineering solutions to the jitter problem are being sought. The first implementation is due around the middle of October.

Compared with an ideal, stationary transfer function of a point source (single unresolved star), the transfer functions obtained vary from nearly ideal to very complex and convoluted. Comparison between transfer functions near day/night transitions and during quiescent periods demonstrate that the jitter is perturbing the signature of the transfer functions, but that in some cases (FGS2), non-ideal transfer functions with multiple values exist, which are not influenced by jitter. We do not have enough information yet to determine whether the problem is an optical alignment/mirror figure problem in the main telescope or possible internal FGS problems. The solution to determining the nature of the problems we have observed is to make carefully controlled observations which will isolate one or another of these possibilities.

3. Recommendations

The recommendation is to establish a controlled experiment by observing the same star at the same focal position "across FGS boundaries", i.e., to obtain transfer functions of the star at nearly the same position in the field of view for each FGS. The entire test will be performed twice for "repeatability" while providing some protection against failed guide star acquisitions. The transfer scans will be performed throughout each pickle to determine intra FGS changes in the characteristics of the transfer functions.

The result of these and other considerations is the "9 points of light" test, to be performed around the end of November. Observations of a star will be made at nine points within each FGS. See figure 4. The points have been chosen to be sensitive to radial and transverse variations within each FGS and to interFGS variations due to the primary and secondary mirror optical distortions. 5 scans with both the clear and 2/3 pupil filters will be performed at each position through each aperture. Assuming that at least 50 percent of the data are good, we then may have sufficient information to determine the source of the corrupted transfer signature, and/or whether they have either internal FGS dependence or OTA field dependence.

The second test to be performed in the November timeframe, will demonstrate the capability of the FGSs to perform astrometric measurements. This test was originally designed as a series of tests to check the engineering capabilities of the instrument, such as fundamental capabilities of each FGS to lock onto stars in Fine Lock with full aperture, utilization of different filters, determine whether or not each FGS is capable of measuring the position of a 17th magnitude star, and whether or not the FGSs are capable of performing meaningful transfer scans.

Our current plan is to perform these tests, to take whatever corrective action is indicated, and to repeat the tests until they are successful.

4. Conclusion

If the jitter fix is successful, and we are able to characterize the FGS/OTA problems, we eventually should be able to reach our original goal of 17th magnitude with a few milliarcseconds (rms) relative pointing accuracy.

We hope that the successful combination of these tests will indicate that the FGSs are capable of performing milliarcsecond astrometry and that we will know which FGS to choose as the Astrometer. The tests should indicate adjustments that must be made before we can reach astrometric level performance. Only after successful completion of these tests can we then proceed to the Science Verification portion of our program, where milliarcsecond calibrations are performed.

5. Acknowledgements

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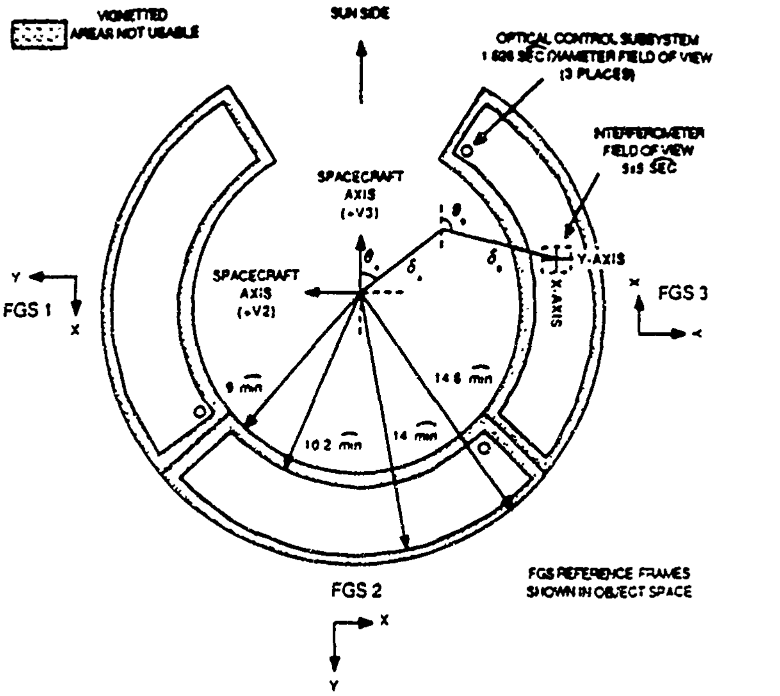
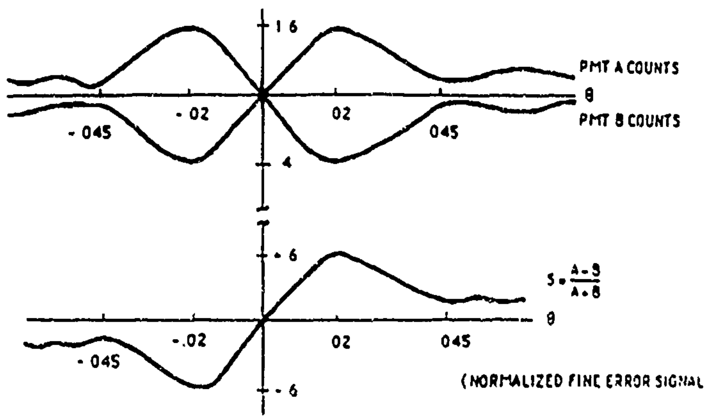


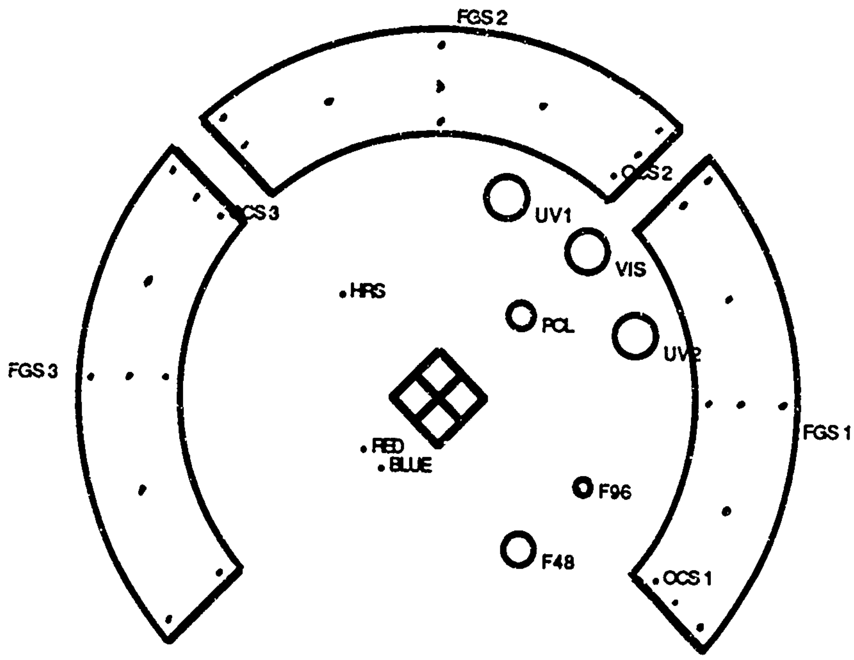
FGS FIELD OF VIEW PROJECTED
ONTO THE CELESTIAL SPHERE

FIGURE 1



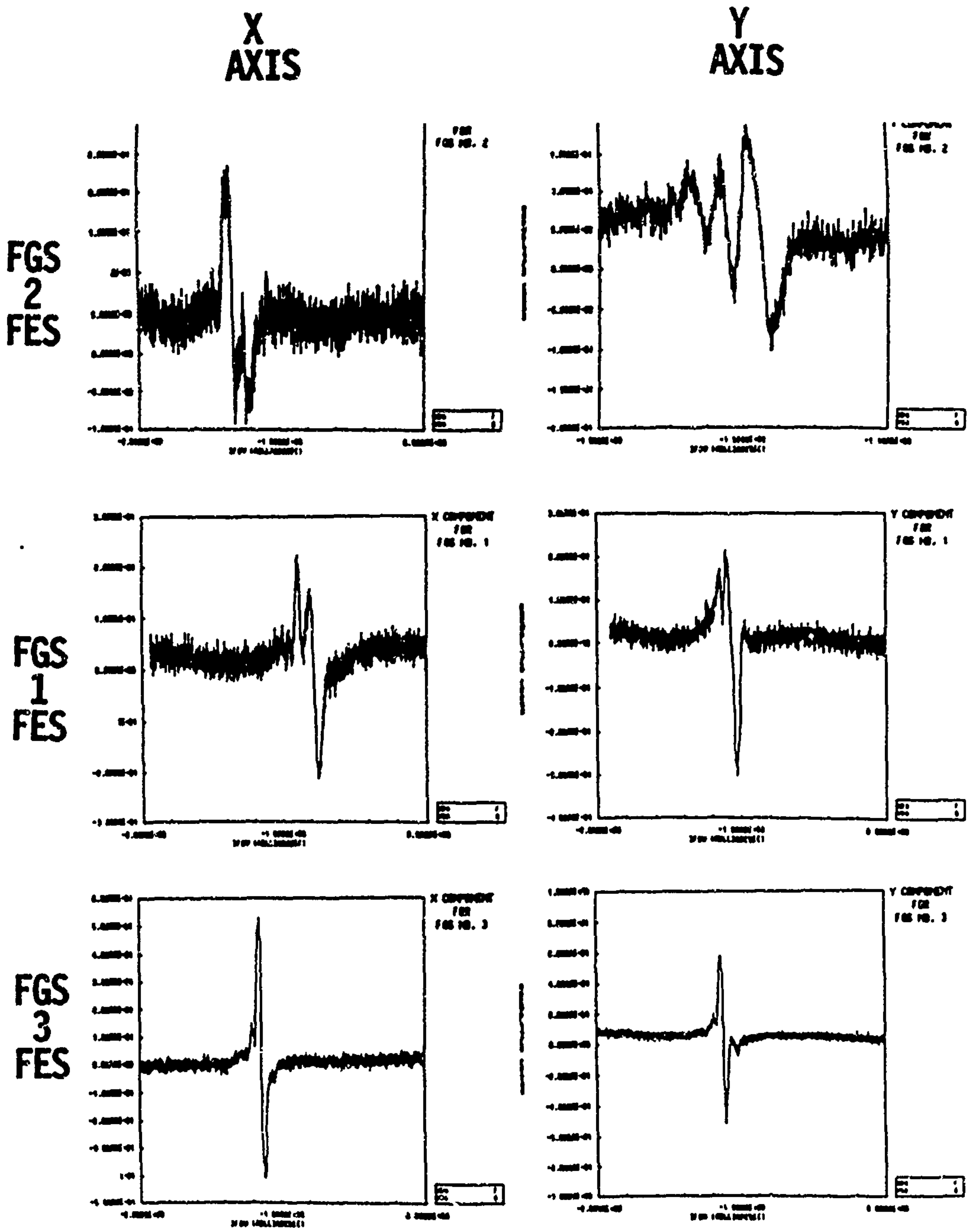
CONSTRUCTION OF A
HYPOTHETICAL S-CURVE

FIGURE 2



LOCATIONS OF THE
9 POIINTS OF LIGHT
OBSERVATIONS

FIGURE 4



X AND Y TRANSFER FUNCTIONS
OBSERVED IN FGSs 2, 1, AND 3
(top to bottom)
Normalized Fine Error Signal vs Positional Offset

FIGURE 3

PROCEDURE FOR VLBI ESTIMATES OF EARTH ROTATION PARAMETERS REFERRED TO THE NONROTATING ORIGIN

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ABSTRACT. This paper investigates the practical use of the nonrotating origin (NRO) (Guinot 1979) for estimating the Earth Rotation Parameters from VLBI data, which is based on the rotational transformation between the geocentric celestial and terrestrial frames as previously derived by Capitaine (1990). Numerical checks of consistency show that the transformation referred to the NRO is equivalent to the classical one referred to the equinox and considering the complete "equation of the equinoxes" (Aoki & Kinoshita 1983). The paper contains the expressions for the partial derivatives of the VLBI geometric delay to be used for the adjustment of the pole coordinates, UT1 and deficiencies in the two celestial coordinates of the Celestial Ephemeris Pole (CEP) in the multiparameters fits to VLBI data. The use of the NRO is shown to simplify the estimates of these parameters and to free the estimated UT1 parameter from the model for precession and nutation.

1. INTRODUCTION

The classical procedure for estimating the Earth Rotation Parameters (ERP) from VLBI observables refers, due to historical reasons, to the equinox of date. This leads to a coordinate transformation from the Terrestrial Reference System (TRS) to the Celestial Reference System (CRS) in which the concepts of precession, nutation and the celestial Earth's angle of rotation are mixed.

As VLBI observations are nearly not sensitive to the position of the ecliptic (and therefore of the equinox), but only to the position of the equator, the use of a coordinate transformation from the TRS to the CRS based both on the nonrotating origin (NRO) (Guinot 1979) and on the two celestial coordinates (Capitaine 1990) of the Celestial Ephemeris Pole (CEP) should be more convenient for deriving the ERP from VLBI observations.

The purpose of this paper is to investigate the practical use of this proposed transformation in the computation of the geometric delay for VLBI estimates of the ERP; it contains numerical checks of consistency between this transformation and the classical one and gives the expressions of the partial derivatives of the geometric delay with respect to the fundamental parameters.

2. DEFINITION, USE AND POSITIONING OF THE NRO

2.1. Definition

Let (Oxyz) be the instantaneous system based on the instantaneous equator, its corresponding pole P and the NRO; the NRO has been defined by Guinot (1979) by the kinematical condition that when P moves in the CRS, the system (Oxyz) has no component of instantaneous rotation along the equator with respect to the CRS (Figure 1).

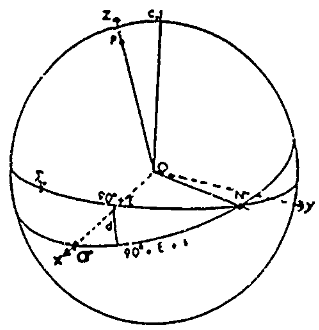

Figure 1: Kinematical definition of the NRO

The kinematical condition defining the NRO corresponds to a necessary concept to describe any motion of rotation along the moving equator. It allows to define a NRO in the CRS, denoted by σ, and also a NRO denoted by ϖ in the TRS, as an exact definition of the "instantaneous origin of the longitudes" (i.e. instantaneous prime meridian).

2.2. The use of the NRO for the representation of the Earth Rotation

The "stellar angle", $\theta = \widehat{\varpi O \sigma}$, gives the "specific Earth angle of rotation", such that $\dot{\theta} = \omega_z$, and UT1 should therefore be conceptually defined as an angle proportional to θ (Guinot 1979).

2.3. Positioning of the NRO

The positioning of the NRO can be easily derived from the origin Σ_0 on the fixed equator of the CRS by the use of the quantity s. A similar quantity s' is necessary for positioning ϖ in the TRS (Figure 2).

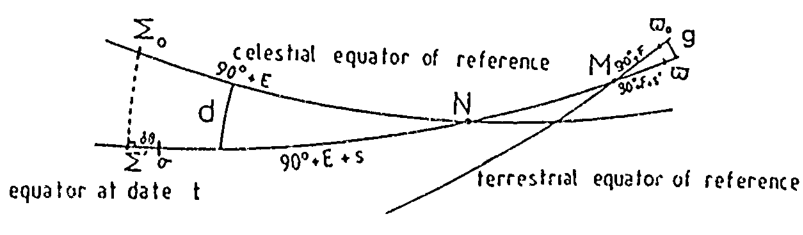

Figure 2: The positioning of the NRO

The quantity s can be written as (Guinot 1979): $s = \int_{t_0}^{t} (\cos d - 1)\, \dot{E}\, dt$, which provides the position of σ on the moving equator as soon as the celestial motion of the CEP is known between the epoch t_0 and the date t. Its expression as a function of time has been shown to be nearly not sensitive to the model of the pole trajectory (Capitaine *et al.* 1986).

3. THE COORDINATE TRANSFORMATION FROM THE TRS TO THE CRS TO BE USED IN THE VLBI EARTH ROTATION PARAMETERS ESTIMATES

Any procedure for parameter estimation from VLBI observables requires the calculation of the "geometric delay" : $\tau = -\vec{B}.\vec{K}/c$ (where $\vec{B}$ is the baseline vector, $\vec{K}$ is the unit vector pointing in the direction of the observed source and c is the velocity of light).

One step in such a calculation is to apply to the baseline vector in the TRS, the rotational transformation Q of coordinate frames from the TRS to the CRS:

$$[CRS] = Q\,[TRS],$$

Q being composed of several separate rotations.

3.1. The classical transformation

In the classical procedure Q is written as: $Q=Q_1.Q_2.Q_3$, such that, if $R_i(\eta)$ represents, as usual, the rotation matrix of angle η around the i-axis :

1) $Q_1 = R_3(\zeta_A).R_2(-\theta_A).R_3(z_A).R_1(-\varepsilon_A).R_3(\Delta\psi).R_1(\varepsilon_A+\Delta\varepsilon)$,
ε_A being the mean obliquity of the ecliptic at date t, and z_A, ζ_A, θ_A, $\Delta\varepsilon$, $\Delta\psi$ the usual precession and nutation quantities in right ascension, obliquity and ecliptic longitude respectively, referred to the mean ecliptic of epoch (or of date),

2) $Q_2 = R_3(-GST)$, GST being Greenwich True Sidereal Time at date t, including both the effect of Earth rotation and the precession and nutation in right ascension,

3) $Q_3 = R_1(y_p).R_2(x_p)$,
x_p and y_p being the "pole coordinates" of the CEP in the TRS,

3.2. The use of the NRO in the transformation from the TRS to the CRS

The use of the NRO allows us to separate Q into three independant rotation matrices, such that: $Q=Q_1.Q_2.Q_3$, each of the matrix Q_i corresponding to one component of the Rotation of the Earth around its center of mass:

1) $Q_1 = R_3(-E).\ R_2(-d).\ R_3(E).\ R_3(s)$,
for those rotations arising from the celestial motion of the CEP (see Fig 2), including the rotation s which takes into account the displacement of σ on the instantaneous equator due to the celestial motion of the CEP,

2) $Q_2 = R_3(-\theta)$ for the rotation of the Earth around the axis of the CEP,

3) $Q_3 = R_3(-s').R_3(-F).\ R_2(g).\ R_3(F) = R_3(-s'+x_py_p/2).R_1(y_p).R_2(x_p)$,
for those rotations arising from the terrestrial motion of the CEP (see Fig 2), including the rotation s' (Capitaine 1990) which takes into account the displacement of ϖ on the instantaneous equator due to polar motion.

4. NUMERICAL CHECKS OF CONSISTENCY OF THE COORDINATE TRANSFORMATIONS

The coordinate transformation from the TRS to the CRS has been applied to vectors in the terrestrial frame such that: ($r = 1$, $\varphi = 0°$, λ = 0h, 6h, 12h; $\varphi = 45°$, λ = 0h, 6h, 12h), from t = 1900.0 to t = 2100.0, every 0.1 century.

4.1. Numerical expressions to be used for the parameters referred to the NRO

In order to check the consistency of the coordinate transformation from the TRS to the CRS referred to the NRO with the classical one referred to the equinox, it is necessary to have consistent numerical expressions for the parameters used in the two transformations.

(1) Numerical relationship between θ and UT1

We have used the following relationship, which has been given by Capitaine *et al.* (1986) in order to be consistent with the conventional relationship between GMST and UT1 (Aoki *et al.* 1982):

$$\theta = 2\pi\,\{0.779\,057\,273\,264 + 1.002\,737\,811\,911\,354\,T_u\}, \quad (1)$$

T_u being the number of days elapsed since 2000 January 1, 12h UT1.

(2) Numerical expression for the celestial pole coordinates of the CEP

We have used the developments as functions of time of the coordinates X=sindcosE and Y=sindsinE of the CEP in the CRS as given by Capitaine (1990) with a consistency of 5×10^{-5}" after a century with the conventional developments for precession and nutation.

Each development, including both effects of precession and nutation, is the sum of a polynomial form of t, of periodic terms corresponding to the nutations and of pseudo-periodic terms arising from the cross terms between general precession and luni-solar nutations. It can be expressed as:

$$X(t) = X(t_0)+2004.310\,9''t-0.426\,65''t^2-0.198\,656''t^3+0.000\,014\,0''t^4+\Sigma_i(a_{io}+a_{i1}t)\sin(\omega_i t-\phi_i)$$
$$-0.000\,35''\sin 2\Omega+\Sigma_j a'_{j1}t\cos(\omega_j t-\phi_j)+0.002\,04''t^2\sin\Omega+0.000\,16''t^2\sin 2\odot+0.000\,06''t^2\cos\Omega \quad (2)$$

$$Y(t) = Y(t_0)-22.409\,92''t^2+0.001\,836''t^3+0.001\,113\,0''t^4+\Sigma_i(b_{io}+b_{i1}t)\cos(\omega_i t-\phi_i)$$
$$+0.000\,13''\cos 2\Omega+\Sigma_j b'_{j1}t\sin(\omega_j t-\phi_j)-0.002\,31''t^2\cos\Omega-0.000\,14''t^2\cos 2\odot,$$

where $(a_{io}, b_{io})_{i=1,106}$ are the coefficients in longitude$\times\sin\varepsilon_o$ and obliquity of arguments $(\omega_i t-\phi_i)_{i=1,106}$ of the 1980 IAU nutation, and $a_{i1}, b_{i1}, a'_{j1}, b'_j$ are quantities lower than 5×10^{-5}", except for a few terms (5 for index i and 18 for index j).

(3) Numerical expression for the quantity s

We have used the numerical expression of s as derived, by the relation: $s=-\delta\theta-XY/2$, from the numerical values of X and Y and from the following numerical development of $\delta\theta$, with an accuracy of 5×10^{-5}" after a century (Capitaine 1990):

$$\delta\theta = -0.003\,85''t+0.072\,59''t^3+0.002\,65''\sin\Omega+0.000\,06''\sin 2\Omega$$
$$-0.000\,74''t^2\sin\Omega-0.000\,06''t^2\sin 2\odot, \quad (3)$$

which is equivalent to the expression of s previously given (Capitaine *et al.* 1986).

4.2. Internal checks for the transformation referred to the NRO

Three different forms of Q_1 have been tested, using the previous numerical developments (1), (2) (3) and a polar motion equal to zero (i.e. $x_p=y_p=s'=0$) :

(i): Q_1 as defined as a product of rotation matrixes, with: $E=\arctan(Y/X)$, $d=\arcsin\sqrt{X^2+Y^2}$

$$Q_1=R_3(-E).\,R_2(-d).\,R_3(E).\,R_3(s), \quad (4)$$

(ii): Q_1 as given directly as a function of X and Y and s:

$$Q_1 = \begin{pmatrix} 1-aX^2 & -aXY & X \\ -aXY & 1-aY^2 & Y \\ -X & -Y & 1-a(X^2+Y^2) \end{pmatrix} . R_3(s) \quad (5)$$

with $a = 1/(1+\cos d) = 1/2 + 1/8\,(X^2+Y^2) + ...,$

(iii): Q_1 as given with a uracy better than 10^{-7}" after a century (Capitaine 1990), as a function of X, Y and $\delta\theta$, ıetrical form of the matrix transformation Q_1):

$$Q_1 = \begin{pmatrix} 1-aX^2 & -2aXY+a^2X^3Y & X \\ 0 & 1-aY^2 & Y \\ -X(1+aY^2) & -Y(1-aX^2) & 1-a(X^2+Y^2) \end{pmatrix} . R_3(-\delta\theta) \quad (6)$$

The numerical checks of the transformation applied to the terrestrial vectors show the identity of the transformation (i) and (ii) and show moreover that (ii) and (iii) are in all cases equivalent with an accuracy better than 10^{-8}".

4.3. Numerical checks of consistency between the transformation referred to the NRO and the classical transformation

The proposed coordinate transformation using the most simple form (4) for Q_1 has been compared with the classical transformation for the terrestrial vectors as considered in the previous section, for the same period of time, assuming as previously, a polar motion equal to zero (i.e. $x_p=y_p=s'=0$). The numerical developments as given by (1), (2), (3) have been used for the transformation referred to the NRO, whereas for the classical transformation, $\zeta_A, z_A, \theta_A, \varepsilon_A$ are the precession parameters as given by Lieske *et al.* (1977), and $\Delta\psi, \Delta\varepsilon$ are the parameters as given by the IAU 1980 theory of nutation.

Greenwich Sidereal Time, GST, has been derived from the expression of Greenwich Mean Sidereal Time, $GMST_{ohUT1}$, as given by Aoki *et al*.(1982):

$$GMST_{ohUT1} = 24\,110.548\,41s + 8\,640\,184^s.812\,866\,T'_u + 0.^s0931\,04\,T'^2_u - 6.^s2\times10^{-6}\,T'^3_u, \quad (7)$$

with $T'_u = d_u/36525$, d_u being the number of days elapsed since 2000 January 1, 12h UT1, taking on values ±0.5, ±1.5, ..., and from the periodic terms of the so-called "equation of the equinoxes", in two different forms:

(i) as: $GST = GMST + \Delta\psi\cos\varepsilon_A + 0.002\,65" \sin\Omega + 0.000\,06"\sin 2\Omega,$ (8)
corresponding to the periodic terms as given by Woolard (1953), or by Aoki and Kinoshita (1983) for the "equation of the equinoxes" in a "wider sense",

(ii) as: $GST = GMST + \Delta\psi\cos\varepsilon_A,$ (9)
corresponding to the "equation of the equinoxes" limited, as it is the general case, to the nutation in right ascension.

It should be noted that the expression (8) of the equation of the equinoxes can only be obtained by using implicitely the concept of the NRO in order to express the accumulated precession and nutation on the moving equator between the epoch and the date.

The numerical tests of consistency of the coordinate transformation referred to the NRO with the classical transformation referred to the equinox gives the following results for the differences in the equatorial coordinates α and δ of the terrestrial vector in the CRS:

(i) with the complete expression (8) for the equation of the equinoxes:

from 1900.0 to 2000.0: $0.4 \times 10^{-6''} \leq \delta\alpha \leq 1 \times 10^{-4''}$

$0.4 \times 10^{-6''} \leq \delta\delta \leq 1 \times 10^{-4''}$,

except for a very few cases for which the difference reachs $1.4 \times 10^{-4''}$

from 2000.0 to 2100.0: $0.1 \times 10^{-6''} \leq \delta\alpha \leq 0.9 \times 10^{-4''}$

$0.5 \times 10^{-6''} \leq \delta\delta \leq 0.9 \times 10^{-4''}$.

In most cases, from 1900.0 to 2100.0, the differences $\delta\alpha$, $\delta\delta$ are lower than $5\times10^{-5''}$, which is the order of the consistency of the used developments (1), (2), (3) with the conventional developments for precession, nutation, and Greenwich Sidereal Time (7) and (8).

(ii) with the incomplete expression (9) for the equation of the equinoxes, nearly identical results are obtained for $\delta\delta$, which is not sensitive to a rotation around the axis of the CEP, but periodic variations appear in the differences in right ascension, with the period of Ω and an amplitude of the order of 2 mas (see Table 1).

1900.0: $-2.2 \times 10^{-3''}$	1920.0: $-2.0 \times 10^{-3''}$	1940.0: $-1.1 \times 10^{-3''}$
1960.0: $-0.5 \times 10^{-5''}$	1980.0: $+1.2 \times 10^{-3''}$	2000.0: $+2.2 \times 10^{-3''}$

Table 1: examples of the differences in right ascension of u.. Gx terrestrial axis in the CRS resulting from the use of the incomplete expression (9) for the "equartion of the equinoxes"

Complementary calculations show, for example, that the difference is minimum in 1988 and 1997 and maximum in 1992 and 2002.

Such numerical checks show the consistency of the coordinate transformation referred to the NRO and using the numerical developments (1) for the stellar angle, (2) for the coordinates of the CEP in the CRS and (3) for the positioning of the NRO with the classical transformation referred to the equinox and using the conventional models for precession and nutation, Greenwich Mean Sidereal Time, as well as the complete expression (8) for the "equation of the equinoxes". Such numerical checks show moreover that the use of the incomplete equation of the equinoxes in the expression of Greenwich Sidereal Time at the date of the observation, as it is presently the case in the reduction of VLBI data, leads to a spurious periodic rotation around the axis of the CEP with an amplitude of a few mas. Such a rotation is probably mainly absorbed in the estimated UT1, which includes therefore a periodic error of this amplitude.

This results show the advantage of using the NRO as an explicit origin fo reckoning the Earth's rotation in order to derive an accurate UT1 parameter from the observations.

5. PROCEDURE FOR ERP ESTIMATES FROM THE VLBI OBSERVABLES

5.1. General case of the partial derivatives of the geometric delay with respect to the ERP

The ERP affect the expression of the VLBI geometric delay only through the orientation of the Earth as a whole. Therefore, if τ is the geometric delay in the geocentric frame, we have, for each of the ERP (Sovers and Fanselow 1987), K_i, L_k being the components of $\vec{K}$ and $\vec{L}$ in the CRS and the TRS respectively and Q_{ik} being the elements of the matrix Q:

$$\frac{\partial\tau}{\partial\eta} = \frac{K_i}{c}\frac{(\partial Q_{ik})}{\partial\eta} L_k,$$

The corrections to the ERP can therefore be estimated by a least squares fit among the VLBI observed delays, using the expression for the partial derivatives of the rotational matrix transformation Q with respect to each of the parameter x_p, y_p, UT1, X and Y.

In the proposed transformation referred to the NRO, the coordinates X and Y of the CEP in the CRS appear only in the matrix Q_1, UT1 appears only in the matrix Q_2 and the coordinates x_p and y_p of the CEP in the TRS appear only in the matrix Q_3. Such a form of the rotational transformation Q simplifies the calculation of its partial derivatives with respect to the ERP as compared to the classical case in which the precession and nutation parameters appear in a complicate way both in Q_1 and Q_2.

The rotation matrix Q_2 being independent of the celestial pole coordinates X and Y, the UT1 parameter as estimated from VLBI observables using this transformation would be free from the errors on the model for precession and nutation. This represent an improvement as compared to the classical method in which the estimated UT1 is dependent, due to the used relationship between GST and UT1, on the precession and nutation model in right ascension.

The partial derivatives with respect to UT1 and the pole coordinates are, such that, k being the conversion factor between the stellar angle, θ, and UT1: $\dfrac{\partial Q}{\partial \mathrm{UT1}} = \dfrac{1}{k} Q_1 . \dfrac{\partial R_3(\theta)}{\partial \theta} . Q_3$,

$$\frac{\partial Q}{\partial x_p} = Q_1.Q_2.R_3(-s'+x_p y_p/2).R_1(y_p).\frac{\partial R_2(x_p)}{\partial x_p}, \quad \frac{\partial Q}{\partial y_p} = Q_1.Q_2.R_3(s'+x_p y_p/2).\frac{\partial R_1(y_p)}{\partial y_p}.R_2(x_p),$$

the derivatives $\partial R_2(x_p)/\partial x_p$, $\partial R_1(y_p)/\partial y_p$, $\partial R_3(\theta)/\partial\theta$ being easily obtained as the derivative of a rotation matrix of angle η with respect to η.

The partial derivatives with respect to the coordinates of the CEP in the CRS are such that:

$$\frac{\partial Q}{\partial X} = \frac{\partial Q_1}{\partial X}.Q_2.Q_3, \quad \frac{\partial Q}{\partial Y} = \frac{\partial Q_1}{\partial Y}.Q_2.Q_3 \ .$$

5.2. Expression of the partial derivatives with respect to the celestial coordinates of the CEP

For computing the partial derivatives of Q_1 with respect to X and Y, the forms (5) or (6) are the most convenient ones, as, firstly X and Y appear explicitly, and secondly, the partial derivatives of δθ or s with respect to X and Y can be neglected.

Such partial derivatives can be written as:

$$\frac{\partial Q_1}{\partial X} = \begin{pmatrix} -X-\frac{X^3}{2}-\frac{XY^2}{4} & -Y+\frac{Y^3}{4} & 1 \\ 0 & -\frac{Y^2}{4}X & 0 \\ -1-\frac{Y^2}{2} & XY+\frac{X^3Y}{2} & -X-\frac{X}{2}(X^2+Y^2) \end{pmatrix} .R_3(\delta\theta) \qquad (10)$$

$$\frac{\partial Q_1}{\partial Y} = \begin{pmatrix} -\frac{X^2Y}{4}(1+X^2+Y^2) & -X-\frac{3XY^2}{4} & 0 \\ 0 & -Y-\frac{Y^3}{2}-Y\frac{X^2}{4} & 1 \\ -XY-\frac{YX^3}{4} & -1+\frac{X^2}{2}+\frac{X^4}{8}+\frac{3X^2Y^2}{8} & -Y-\frac{Y}{2}(X^2+Y^2) \end{pmatrix} .R_3(\delta\theta) \qquad (11)$$

5.3. Interpretation of the estimated quantities dX and dY

The corrections dX and dY which can be estimated from VLBI observables, using the partial derivatives (10) and (11) of the rotational matrix Q_1, correspond to the deficiencies in the conventional model for the coordinates X and Y of the CEP in the CRS. They can be written as:

$dX = \xi_0 + \delta X$, $dY = \eta_0 + \delta Y$, where ξ_0, η_0 are for the constant offset between the pole of the CRS and the pole of the epoch of the model and δX, δY for the deficiencies in the models (including both precession and nutation) at the date of the observation. Using the developments of X and Y as given by Capitaine (1990), the deficiencies δX, δY in the models can be written, with an accuracy better than 5×10^{-5}" after a century, as: $\delta X = d\psi_A \sin\varepsilon_0 + d\Delta\psi \sin\varepsilon_0$, $\delta Y = d\varepsilon_A + d\Delta\varepsilon$, where $d\psi_A$ and $d\varepsilon_A$ are the errors on the model for precession and $d\Delta\psi$, $d\Delta\varepsilon$, the errors on the model for nutation.

dX and dY are the quantities to which the VLBI observations of radio-sources are actually sensitive, as they provide the position of the instantaneous equator with respect to the equator of the CRS. Such quantities are presently derived from VLBI observables on the form $d\psi \sin\varepsilon_0$ and $d\varepsilon$ by using a more complicated procedure involving separately precession and nutation of which the effects are in fact not separable using VLBI observations.

This shows the advantage of using X and Y as the two fundamental parameters for the celestial motion of the equator instead of the large number of the precession and nutation parameters generally considered.

6. CONCLUSION

The coordinate transformation from the TRS to the CRS using both the NRO and the coordinates of the CEP in the CRS has been shown to be numerically consistent with the classical one (when the complete equation of the equinoxes is used) with an accuracy better than 0.1mas. Such a representation, which clarifies the involved concepts, has been shown to simplify the estimates of the Earth Rotation Parameters from VLBI observables and to free the estimated UT1 parameter from the precession and nutation model.

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THE NECESSARY PROCEDURES TO REACH AN AGREEABLE REFERENCE FRAME -COUNTER-PROPOSAL TO THE CIRCULAR LETTER N°4 OF KOVALEVSKY-

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ABSTRACT The procedures are summarized in order to obtain a reference system which is available at the present moment and in the near future for the astronomical purposes. The target of the present discussions is the IAU General Assembly to be held in the summer of year 1991, in order to find out an agreeable result there. Discussions beyond this are postponed to a further occasion.

Introduction

Reviewing the voluminous Circular Letter n°4 of Kovalevsky to Subgroup on Coordinate Frame and Origin (SgCFO), dated July 17, 1990, on the very delicate problems, I feel nevertheless some difficulties to accept these proposals as they are. I have here made the modifications and / or counter-proposals and comments, respectively, as follows:
section 1. corresponds to the original section 2.1 (G1); section 2. to section 2.2 (G2); section 3. to section 3.1 (R1); section 4. to section 3.2 (R2); and section 5. to section 4. The reason(s) why I have made the counter-proposals may be found in the comments, which are arranged as the second subsection of each section. Each subsection is further divided into several subsubsections in some cases.

In the following, I have used the abbreviations: A for Aoki, and K for Kovalevsky.

1. Recommendation on the Definition of the Reference Frame to be Aimed (G1)

1.1. PROPOSAL

I have proposed the expression of the line element in the form that it gives the time-like argument instead of the original form which give the space-like argument. The concrete expression was given in a letter to K, September 20, 1990.

1.2. COMMENTS

1.2.1 *Conceptual / Conventional Definition.* To my opinion, the discrimination between the conceptual and conventional definitions is not clear (For some details, connecting to the CEP, see A 1990, Appendix B) . Therefore, I have chosen the wordings "definition to be aimed" and "definition to be realized or realizable" instead. For I don't prefer, in particular, the usage of "conventional" in the meaning of "which should be avoided".

1.2.2 *Reference Frame / System.* The usage of "reference frame" and "reference system" by Kovalevsky seems not to be recommendable. See A 1990, Appendix A. Also see the arguments by Eichhorn 1990.

1.2.3 *Form of the Line Element.* The original expression of the line element is given to be applicable for the barycentric system of the solar system, as well as in order to cover the local systems, such as the geocentric system. However, according to my opinion, the latter case is derivable from the former, and it is not necessary to give it. Instead, I think it is necessary to give the expression of the potential function in a more explicit form, in order to avoid a possible confusion.

2. Constraint Recommendation (G2)

2.1. PROPOSAL

...

considering
a) the necessity to define a barycentric coordinate system centered at the barycenter of the solar system, a geocentric coordinate system centered at the barycenter of the earth, and the terrestrial coordinate system,
b) the desirability that the coordinate system be linked to the best physically realizable references in time and space, and
c) that the theory of general relativity is appropriate to describe the reference frames,

recommends

1. that it is urgent to research whether the coordinate system without rotation considering only the masses within the solar system conforms with that of the whole universe or not,
2. that the geocentric coordinate system is constructed so as to be tangential [in other words, in such that its time coordinate is selected to be the proper time at the geocenter (which is the origin of the coordinate system) given by the line element of the basic coordinate system for which the mass of the earth is omitted and its spatial coordinates to be perpendicular to its time coordinate at the origin], but of which the so-called *geodesic or relativistic precession* is discarded, in order to be a natural coordinate system.
3. that the relation between the geocentric coordinate system and the terrestrial coordinate system should be accounted for the rotation of the earth and for the attracting force arising from the mass distribution within the earth, and

4. that in order to be parallel in average between the time reading of the coordinate time in the barycentric coordinate system of the solar system and that realized on the (timely averaged) geoid of the terrestrial coordinate system, the units of time arguments are selected to be

$$[s_T] = (1 - 1.55050 \times 10^{-8})[s_B], \tag{1}$$

where $[s_T]$ is the time unit of the terrestrial coordinate system, identified with the SI Unit, and $[s_B]$ is the time unit of the barycentric coordinate system.

2.2. COMMENTS

2.2.1 *Natural Coordinate System.* It is not yet sufficiently established that the quasar system links to the non-rotating system in the dynamical sense. For detail, see A 1990, Appendix A (in which I have discussed that the so-called "ideally quasi-inertial system" should be replaced by the natural coordinate system), and section 6.2 (in which I have discussed the (supposedly apparent) motion of 3C273B).

I think it is necessary to add the last clause in the item 2. of the Recommendation beginning with "but." See a discussion on the geodesic precession in A 1990, Appendix A.

2.2.2 *Barycentric, Geocentric, and Terrestrial System.* I think it is necessary to give the relations among the barycentric, geocentric, and terrestrial systems in a clearer and more concrete expression than the original text.

2.2.3 *Scaling of the Time Argument.* I still have the opposite opinion against to abandon the present convention of scaling the time argument.

The main reason of my opposition is that the existing planetary ephemerides in terms of TDB are to be neither excluded nor re-calculated, and for the sake of continuity it is necessary to have these ephemerides also in the future. The similar opposisions are expressed by people of JPL, such as Lieske, Dickey, Williams, and Standish.

The numerical value given in eq.(1) comes from Fukushima et al. 1986.

3. Recommendation on Reference Frames to be Aimed (R1)

3.1. PROPOSAL

...

considering
a) that the reference frame should be still defined to be referred to the mean equator and equinox of the fundamental epoch of J2000.0 (say),
b) that the catalogue to be realized should include objects being well coordinated and emitting any electromagnetic wave available for the astronomical purposes, and

c) that this catalogue should be an extension of the presently existing FK5,

recommends

1. that an international working group be set up, in order to select the appropriate candidate objects for constructing the reference coordinate system, consisting of members of Commissions 4, 8, 19, 24, and 31, and other pertinent experts, with consultation of Commissions 5, 33, 40, and all the pertinent (or relevant) institutes;
2. and that its subgroup collect all the observational data of their positions and principal characteristics available and compile the positions, in particular, with respect to the reference frame defined above, as early as possible, so that the new fundamental catalogue serves as a continuation and/or an extension of the FK5.

3.2. COMMENTS

3.2.1 *On the Definition of Reference Frame.* I have a strong objection to *define* the reference frame [the reference system, in Kovalevsky and Mueller(1981)'s sense] by a set of quasar system. The reason is that we do not know whether the quasar system represents the ideal system which is considered in Recommendation G1. (See A 1990, sections 2 and 4). Wielen comments " If we would *define* the coordinate system at J2000.0 implicitly by a set of quasar positions, how should we improve this list by further observations?")

De Vegt argues that VLBI uses objects [quasars] which stem from a class displaying most variable astrophysical properties and which in addition are poorly understood; and further argues that this could have a serious impact on the longterm usability of this object class (change of spatial structure and emission strength). Murray comments " We know from experience that even in the best fundamental catalogues there are individual and systematic regional errors. How can we be certain that VLBI catalogues do not have analogous errors, albeit on a small scale?"

The problem depends on the accuracy obtained or obtainable. When we have the more accuracy, we have to take the more care of it. I have a common opinion that the reference frame should be defined by the mean equator and equinox at some epoch, say J2000.0. We should not change this conventional principle as was stated by Murray.

3.2.2 *A Working Group.* As was discussed by de Vegt in his letter to K. on 23.05[*sic*].90, the IERS group has interest only on the quasar positions. However, the objects, to be coordinated, are not restricted only to the radio sources. Therefore, the IERS to which the original proposal refers has been replaced by an international working group, according to his suggestion.

4. Recommendation on the (Conventional) Reference System to be Realized (R2)

4.1. PROPOSAL

...

considering
a) that the new conventional celestial barycentric reference system to be realized should be as closely as possible to the existing FK5 reference system as referred to J2000.0, and
b) that it should be accessible to astrometry[-rist] in visual wavelengths as well as in radio wavelengths,

recommends

1. that the position of the extragalactic sources given in the catalogue representing the reference system be computed for the epoch J2000.0 using the presently adopted value of precession and nutation,
2. that a great effort be developed in intercomparison of reference systems of all types between them and particularly with FK5 and extragalactic reference systems,
3. that all types of observing programs be undertaken or continued in order to link to a catalogue of extragalactic source positions to the best catalogue of star positions, in particular FK5 and HIPPARCOS catalogues with the accuracy of these catalogues, and
4. that the Ox axis of the spherical coordinates of the conventional celestial reference system be as close as possible to that of the FK5, equinox J2000.0, and that the principal plane be as close as possible to the mean equator at epoch J2000.0.

4.2. COMMENTS

4.2.1 *Designation of the Fundamental Epoch.* J2000.0 is employed here for the designation of fundamental epoch, in accordance with the IAU recommendation 1976, instead of J.2000 of the original text.

4.2.2 *Reference System / Frame.* Exchanges between "reference system" and "reference frame" are taken place. As for the reason, see section 1.2.2.

4.2.3 *What is the Best Value?* I have changed "the best available values of precession and nutation", in item 1. of the original text. The reason is as follows:

(i) It is very hard to accept the statement which gives an allowance to use the best value for the precession constant instead of the presently adopted constant, since the meaning of the "best" will depend on each researcher's judgment. In fact, we cannot separate the precession from nutation, even using the very accurate VLBI observational data presently available (For detail, see A 1990, section 4.3.)

(ii) Moreover, it is unusual in the history of the IAU to give such an allowance without specifying the value to be used of an important astronomical constant. If this would take place, we would not be able, afterwards, or at least it would be very difficult to follow up what value some person actually used.

(iii) Schwan in his letter to K, 30 August, 1990, stressed that "We should therefore make the extrapolation to J2000 with the IAU standard values because a frequent change in the constants is very unconvenient[in-] and even dangerous if different observations are to be compared (see also my [Schwan's] letter of 28 May 1990)." De Vegt, in his letter to K, 23.05[sic].90, gave a similar opinion.

4.2.4 *Reference System to Which the Motion is Described.* There was some confusion and misunderstanding on the "motionless." In fact, in the past, there were some quasar catalogues without mentioning their mean observation dates explicitly, according to Walter 1990, probably from a reason because the quasar system is "motionless." They were, I think, coming from a bad practice. We don't know exactly the precession constant; therefore, the reduced position for each obserevation epoch using an adopted precession constant is not necessarily fixed. After assemling such data, we can separate the precession constant statistically in such a way that the precession refers to the mean position of ensemble of the observed objects. This convention has been applied since the days of Bessel. It is very important to notice that the method does not depend on whether the objects do move or not. In other words, it is not pre-requested that the ensemble is motionless or not. It is still valid for quasar system, albeit the degree of accuracy is much higher than the stellar case. Yet, I am afraid that there is a misunderstanding on this point. It is important to recall that we should always mention something to which "the motion" is referred. In fact, Smith asked "but relative to what [the motions of quasars are measured]?" It is a tautology and trivial, as is easily understood, to say that the quasar system is motionless with respect to the quasar system.

4.2.5 *Dynamical Reference System.* There is another kind of reference system which is called dynamical. This reference is based on the dynamical solution of the members of the solar system. Such a system should be connected with the statistically obtained quasar system. Presently the link is not so tight because of the lower accuracies in the optical observations of the sun and the planets than radio range observations. However, I have a hope for a time when sufficient observational materials of the so-called millisecond pulsars are collected, since the analysis of these materials is expected to be able to afford the orbital motion of the earth with respect to the radio source system. See, for more detail, A 1990, section 3.

4.2.6 *The necessary Steps.* It is not yet established whether the apparent differences in orientation among various VLBI catalogues at a level of milli-arcsecond (mas) are really due to the nutation offsets depending on the selected reference days, respectively, (A 1990, section 4.3) or due to the other systematic errors involved in the reduction analysis. In fact, during a discussion with me when Ma visited Japan, he recognized that a presently compiled catalogue, such as Ma et al. 1990, depending on the reference day, is a catalogue with tentative character. Therefore, the steps to be taken are as follows:

(i) To establish observationally the offsets of the nutational coefficients at least for the short periods (equal to or less than one year), by excluding any contamination caused by the other effects.
The subgroup on nutation has decided not to adopt new nutation series for the time being since the theoretical consistency is not yet obtained. But we may obtain observationally a tentative nutation series for the components including the so-called out-of-phase components [See, e.g., Herring et al. 1986] at least with periods of or less than a year.

(ii) To obtain theoretically the real causes of the offsets on the well-established internal constitution of the earth.
The terrestrial reference coordinate system should be referred to the so-called Tisserand mean axes (see Munk and MacDonald 1960, p. 10f.), which is defined in such that the total angular momentum of the earth referred to this coordinate system is zero. However, this is not yet accomplished, because the observation sites are scanty on the earth's surface, and because we have to use a modeling of the motion of crustal plates, e.g., such as given by Mister and Jordan 1978.
We do not know the modeling of plates, on which the observatories reside separately, is altogether correct, because of the scanty number of the observatories. The present practice may lead us thus to a divergence in the definition of reference coordinate systems among the different networks depending on the different choice of the VLBI observation sites.

(iii) Upon completing the above steps, we may have the quasar positions with respect to the *adopted reference system*, not on the basis of day by day but of year by year.
It is not recommendable to analyze the quasar positions using a reference day discussed by Ma et al. 1990 [Also see Sovers 1990], because the quasar catalogue reduced to a fundamental epoch such as J2000.0 depends on the reference day, since the adopted values are not so accurate that they do not represent the orientation of the earth in space.
This means that the reference coordinate systems, to which such catalogues are referred, may be their own tentative reference systems but do not represent the unique reference system. According to Ma's opinion, the quasar positions are sufficient if they are expressed in a relative sense. This may be true as far as we consider only the quasar observations, but this does not mean that such a treatment produces the reference coordinate system to which the every astrometric observations should be referred.

(iv) Then we have the separation of the precession from the nutation if the observational materials are accumulated at least for a score of years or more.
Note that the present situation, on the contrary, is not sufficient. Note that it has a merit to separate precession from nutation, because the precessional motion is considered as if the earth is solid, while the amplitude ratio of the deformable earth to the solid earth for a nutation component depends on its period [See A 1988, section 5.] Thus the theoretical nutation amplitude can be easily treated, if each component of the nutation is separated from the precession.

It is not yet accomplished so far, however, to separate the precession from the nutation in the VLBI observations at the mas level because sufficient data are not yet available only from the decade observations. It needs at least 18.6 years to separate fully the precession from the nutation component associated with the nodal motion of the moon, in such a degree that this is comparable with the highest order of magnitude for the accuracy presently obtained.

(v) And then we can test whether the quasar system is connected to the dynamical system expressed in the relativistic theory discussed in Recommendation G1.

As was discussed in A 1990, Appendix A, we do not know so far that the extragalactic system represented by quasar system is rotation-free or not, because this is given only kinematically. We should have a test by the dynamical system, namely, by the natural coordinate system. [I still oppose the expression "ideally quasi-inertial", because this is very misleading.] We know that the optically observed data are not sufficiently accurate, but we may expect radio observations of millisecond pulsars and/or artificial proves traveling through the solar system with much accuracy within a decade or so [as was discussed in a letter by Williams to K, February 1]. This does not mean, however, that the terrestrial optical observations are obsolete, because the optical observations have their own long history and the separation of precession from the proper motion system of the galactic objects has been made using the optical observations. We have to do, therefore, many works such as coordinating radio sources within the FK5 system before we could construct the reference frame.

Note that the precession of order 0."1 / century is not yet known even from the VLBI observations. Moreover, the quasar observations are not all the observations in the field of astrometry. We should take also care of the optical observations hitherto obtained and to be observed in future. Also we should keep in mind that the FK5, e.g., has much more objects in number than those of the quasar system so that we can coordinate easily the other objects referred to the former catalogue. Without such a catalogue, we cannot compare the positions directly with quasar system because the quasars generally have only their very weak optical counterparts so as to be observable for comparison.

(vi) Finally when all these steps would have been well done, then we could know whether the quasar system is really rotating or not.

I have heard, however, from Ma that it would take quite a lot time if we want to compile and to combine all the material such as of Crustal Dynamical Project (CDP), International Radio Interferometry Survey (IRIS) and Deep Space Network (DSN) obtained from the different networks and different purposes, although the techniques used in the VLBI observations are almost the same. What we should do right now is, however, to reduce the quasar positions to J2000.0 by using the adopted precession and the adopted nutation series *mutatis mutandis* [with alternations if necessary] for the short periods given by the above analysis. The reduced positions thus obtained can be at least free from the uncertainty of the nutation series of period one year or less, and could represent a common catalogue.

Note, however, that the reference frame is dependent on the adopted precession and adopted nutation series at the highest degree of accuracy, because we do not know the exact precession nor nutation series beyond the observation accuracy of its date. [See, for detail, A 1988, section 6.] This implies that the reduced positions to a fundamental epoch are not necessarily constant but are subject to a proper motion referred to the adopted precession constant. This situation could not be avoided at any historical dates and will not be able to be altered for the future, as long as we observe celestial objects from the surface of the earth, because the precession and the nutation are not given *a priori* but they should be given from the observations depending on the orientation of the earth.
It should not be considered it as a degrade to use the precession and nutation. Also one should not be reluctant by the degrade (uncertainty) limited by the adopted precession and nutation series, but one should determine them by observations themselves, because the terrestrial observations anyhow depend on the precession and nutation.

Unless these steps are taken, I think we cannot have a unique or the unified celestial reference system proposed up to the mas level acceptable among those who have interest in the present issue. Make haste slowly!

5. Recommendation on the Relation between the Tentative (Conventional) Terrestrial and Celestial Reference Coordinate Systems

5.1. PROPOSAL

...

considering
the present situation of the realized celestial reference coordinated systems,

recommends

that the relation between the (spatial) terrestrial reference system mentioned in Recommendation G2 and the presently realized (spatial) celestial reference system be given by

$$[\mathrm{TRS}] = W R_3(\dot{\phi}) N P [\mathrm{CRS}]. \tag{2}$$

The mathematical symbols employed here denote as follows: [TRS] and [CRS] are the vector representations of the spatial terrestrial and celestial coordinate systems (in three dimension), respectively;
W, N and P are the wobble, nutation and precession matrices given, respectively, by

$$W = R_2(-x_\mathrm{p}) R_1(-y_\mathrm{p}), \tag{3}$$
$$N = R_1(-\epsilon_A - \Delta\epsilon) R_3(-\Delta\psi) R_1(\epsilon_A), \tag{4}$$

and

$$P = R_3(-z_A)R_2(\theta_A)R_A(-\zeta_A), \tag{5}$$

where x_p and y_p are the terrestrial polar coordinates in the left hand coordinate system usually employed, $\Delta\epsilon$ and $\Delta\psi$ are the nutation series given in Seidelmann 1982, and $\epsilon_A, z_A, \theta_A$,and ζ_A are the components of the precession quantities given by Lieske et al. 1977, together with the rotation matrix R_1 being defined as

$$R_1(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{pmatrix} \tag{6}$$

for any dummy angle x, and the other matrices being obtained by cyclic permutations of the coordinates; and $\hat{\phi}$ is the true Greenwhich Sidereal Time defined by

$$\hat{\phi} = \mathrm{GMST} + (\Delta q)_{\mathrm{per}}, \tag{7}$$

where GMST is the Greenwich Mean Sidereal Time, which has a relation with the UT1, given in Aoki et al. 1982 and endorsed by the 18th GA of IAU (Resolution C5),and $(\Delta q)_{\mathrm{per}}$ is the equation of equinoxes in wider sense and given by

$$(\Delta q)_{\mathrm{per}} = \Delta\psi \cos\epsilon + 0.''00264 \sin \Omega_{☾} + 0.''000063 \sin 2\Omega_{☾}, \tag{8}$$

with $\Omega_{☾}$ being the mean longitude of the lunar node.

5.2. COMMENTS

5.2.1 *Where Should the So-called Non-Rotating Origin (NRO) be Discussed?* Objection against the transfer of discussion introducing the so-called NRO to the IERS. It is quite curious and ironical that Capitaine and myself have a common opinion that it is the task of the subgroup, as was discussed by Capitaine, notwithstanding our opinions on the problem itself may be quite different from each other.

5.2.2 *Demerits of NRO.* According to my opinion, the so-called NRO has demerits as follows:

(i) This Origin is only locally inertial (namely as far as the rotation of the earth is concerned) but moves with respect to space even to the right ascension direction, against its naming, and is very misleading: For example, we can easily know that after a complete revolution of the precessional motion (26,000 years), the right ascension (RA) of the NRO increases by an amount 360° cos ϵ, while an object at the equinox at the beginning will recover the same RA or increase 360° after the same period, so that the NRO moves 360°(cos $\epsilon - 1$) with respect to space in this period. In order to make clear this fact, I prefer the departure point (or the local inertial direction) on the moving equator, to the NRO, as the naming, according as the usage of celestial mechanics. In case of the nutational motion of 18.6 years, this amount is estimated to be −0.72 mas per period, which is not negligible for accurate determination of the sources. Besides, we have the periodic motion given by the last two terms of the right hand side of eq.(8). See also A 1989.

This phenomenon can be explained mathematically from the fact: a direction on a tangential plane to a sphere of the unit length cannot keep its original direction when the plane contacts without sliding and local rotation around the contact point and returns to the the starting point of contact. The change or difference of the direction during this route is equal to the area described by the normal of the tangential plane on the unit sphere. In other words, the tangential plane is only local and does not represent the direction in the original space.

(ii) The difference between usual RA and the instantaneous ascension (IA, which is the RA direction coordinate referred to NRO, according to Guinot[1979]'s naming, although I dislike this naming), is nothing but the RA of the NRO and is completely calculatable, if one adopts a precession constant. In this meaning, the choice of two ascensions does not make any difference mathematically and has no merit. [See Aoki, S. and H. Kinoshita 1983.]

(iii) Moreover, the fixing the coordinates at the beginning and the continual prolongating the position of the NRO by integration, which CGS (1986) intend, has a demerit, because we cannot have any recovering procedure when once we have committed errors which would be serious at later times, however they might be slight at the standards of the beginning, if we do not have a room for adjustment procedures such as the equinox correction. To abandon such a degree of freedom for adjustment procedures is, I think, to loose the relation to the fiducial point at the beginning.

5.2.3 *Equinox Correction.* Incidentally, it should be noted that the neglection of the equinox correction ΔE (or the correction to the fiducial point) already takes place in the formulation given in IERS Annual Report, for the year 1988. Correctly, the fourth line of equation (3) should read

$$\Delta d\psi = A2/\sin\epsilon = -(A3 - \Delta E)/\cos\epsilon$$

(with the signs taken in ERRATUM), instead of $\Delta d\psi = A2/\sin\epsilon = -A3/\cos\epsilon$, since the difference A3 in orientation can be interpreted as to include generally the difference of RA of the fiducial points in comparing catalogues not only the difference in the offsets implicitly taken in precession and nutation. A similar error is found in Arias' letter to K, October 26, 1989. If this or similar erroneous formulations are widely spread, the influence is very serious.

5.2.4 *Current System.* Anyhow, my recommendation stated above is a confirmation of the currently adopted or to be adopted with a slight but important modification [see section 5.2.5] from my stand-point.

Now, the expressions adopted here are taken from those given in Aoki, S and H. Kinoshita 1983, and A 1988, except for the notations.

5.2.5 *Amelioration.* An important amelioration of the relation between the Greenwich True and the Greenwich Mean Sidereal Time adopted here is an introduction of "the equation of equinoxes in wider sense" which differs from "the equation of equinoxes = $\Delta\psi\cos\epsilon$" currently adopted, by the amount coming from the last two terms of the right hand side of eq.(8). The terms with similar numerical coefficients were discussed by Woolard (1953), but the astronomical circle has not yet formally adopted these terms.

As for the light deflection and aberration due to the relativistic effect through the gravitational field of the solar system, which is not discussed here, see IERS (or Merit) Standards.

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THE ORGANIZATION OF SPACE: FRAMES, SYSTEMS AND STANDARDS

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ABSTRACT. The term "frame of reference" or "reference frame" has long been established in physics. The recent emergence of the use of this very term in astronomy to denote "a catalogue of the adopted coordinates of reference points that serves to define, or realize, a particular coordinate frame" is misleading because "a catalogue of adopted coordinates" must necessarily consists of estimates which cannot uniquely define a coordinate system.

Physical events happen in space-time and their complete description requires the assignment of spatial coordinates and epochs. This self-evident fact has led physicists and astronomers to consider certain relevant concepts and to agree on names for them. It can be argued that astronomy and physics are two branches of the same science; their concerns intersect often, perhaps more often than those of any other two sciences, and one would therefore expect that astronomers and physicists call the same entities by the same names. It will lead to confusion if a particular name, preempted in one of the disciplines for one concept is used in the other for a different concept, especially since contemporary physicists frequently work on astronomical problems and vice versa. Such a practice discourages graduate students from choosing astronomy and especially astrometry as a field of specialization.

At the beginning of this century the advent of the theory of relativity opened a discussion of space-time and the various systems used to assign coordinates to points in space-time. The term "inertial frame of reference" emerged for those spaces in which Newton's first postulate and third law of motion are valid. This is the set of spaces (with certain properties) in which certain events take place and this does not imply that these spaces must be organized by any one particular coordinate system. It is, in fact, well known that there is no particular inertial system of coordinates which is privileged, but that all coordinate systems whose axes do not rotate with respect to any of the others but whose origins move uniformly with respect to each other can be used to organize the same inertial frame of reference. The understanding of the community of physical scientists is well expressed by C. A. Murray (1989): "...a 'reference frame' is a physical entity, independent of its numerical realization, just as a vector or tensor is a physical entity independent of any triad (or tetrad) by which it is described. Thus we have reference frames defined by the solar system, the stars in our galaxy and distant matter such as quasars. Nearer home, we have the terrestrial reference frame and also local inertial frames of different observers. These are in accordance with the usual terminology of physics in which 'frames' do not necessarily imply specific coordinates..." This passage states how the term "frame" or "frame of reference" has been used in physics for almost a century, and this is how it should be

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interpreted in astronomy, especially since many who are now astronomers received their formal training as physicists. Unfortunately, a movement has recently emerged to use this term in a different sense (cf. Kovalevsky and Mueller 1989).

In order to operate within a particular reference frame, it must be *organized*, which means that a system of coordinates must have been defined which is in a known (not necessarily fixed) relationship with the frame and allows one to assign coordinates, that is numbers which characterize the locations of specific points within the reference frame. In order to minimize the necessity for frequent revisions, this assignment should preferably be conceptual (Eichhorn 1983), that is, based on some physical (or geometric) concept which dictates and determines unambiguously the choice of the coordinate axes. These axes determine what has been called a "coordinate system" or a "system". The coordinate systems in kinematic astronomy are examples: The H (horizon) system at a particular point of the Earth's surface has its z-axis pointing in the direction opposite to that of the vector of gravitation at this point, and the y-axis points in the direction of the vector $\omega \times \mathbf{k}$, where $\mathbf{k}$ is a vector along the z-axis and ω is a vector in the direction of the Earth's axis.[2] The Q_t-system, the true equatorial system as we understand it at this time, is defined purely conventionally: Let n_E be a vector normal to the ecliptic - a prime example for a conventionally defined entity - pointing toward its northpole, and n_Q a vector pointing toward the CEP, that is, the conventional celestial ephemeris pole. The x-axis of Q_t is parallel to $n_Q \times n_E$. Note that the nonrigidity of the Earth would (or could) render the ω which belongs to a particular volume element of the Earth different from one place on the Earth to the next (even though ever so slightly and at this time immeasurably little) if ω were in the direction of the axis of any volume element's instantaneous rotation. This potential dependence of ω on the location of the observer is avoided by the introduction of the CEP, whose direction, by definition, cannot depend on the observer's position on the Earth's surface.

The term "system" has been used in astronomy for rigorously defined constructs which organize (the space-like components of) frames of reference. These definitions may also involve the fixing of the time dependence of the system with respect to the frame; note, for example, that the time dependence of Q_t with respect to an inertial frame is fixed by the theories of precession and of nutation, including the numerical parameters in these theories.

Once such a system has been defined, it becomes possible to estimate the coordinates of points[3] with respect to the system itself and thus with respect to the frame which the system organizes. If this is done for a representative sample of points, such as the stars in the FK5, we obtain what astronomers in the past have - somewhat loosely - also called a system, or perhaps more specifically a "system of standards" or a "reference system". The most accurate and self-explanatory description of such an entity would be "Estimated System" (Owen 1990). Even though the statement is trivial that no system of standards (or estimated system) can consist of the actual coordinates of the points in the system it represents, astronomers are making statements such as "catalogue X is 'on the system' of the FK5". The rather recent suggestion by Kovalevsky and Mueller (1989) to use the term "frame of reference" or "reference frame" for such sets of coordinate estimates has been followed by quite a number of investigators.

This paper suggests that we abandon this practice for two reasons: one merely historical but the other more substantive. We have already stated the historical reason: the term "frame of reference", which emerged in the connotation suggested by Kovalevsky and Mueller only recently in fundamental astronomy, has long been preempted for a different concept in physics, a

[2]After the adoption of the CEP (celestial ephemeris pole), this is no longer conceptually defined, because the CEP itself is conventionally defined. A conceptual definition would require that ω itself be conceptually defined, either as the direction of the instantaneous axis of rotation of the observer's element of the Earth's surface or the (physically uniquely defined) principal axis of inertia.

[3]Without restricting generality, we can restrict our considerations to points rather than to extended bodies which may, after all, themselves be regarded as sets of points.

very closely related discipline; its parallel and simultaneous use for a totally different concept in astronomy cannot but lead to confusion and will discourage some young scientists at the beginning of their career from entering a field where such confusion reigns.

The second, substantive reason is that there cannot be an unambiguous or rigorous relationship between a system of standards and its target system, that is, the coordinate system it is intended to represent. This was pointed out by Eichhorn (1982). The values which constitute a (system of) standard(s) are the results of measurements and therefore affected by unknown errors. These errors are not only accidental (having their origin in the random errors of the relevant measurements) but unavoidably also systematic. The reason for this is that the process of estimating the values which constitute the (system of) standard(s) also involved the estimation of certain parameters, each of which was used in the calculation of at least some groups of standard values. Only parameter estimates and not the parameters themselves are available for the calculation of the values which make up the standard. These estimates will therefore deviate systematically from those quantities which they are intended to estimate as a consequence of the (unknown and unknowable) differences between the actual values of the parameters and their estimates. This unavoidable parameter variance (cf. Eichhorn and Williams 1963) is present even in the absence of noticeable systematic trends in the adjustment residuals (Eichhorn and Cole 1985). It is thus impossible to establish empirically a bias-free set of estimates of the coordinates of any sets of objects with respect to any given system; all we can hope to achieve is to establish a standard which contains estimates whose systematic and random errors are small, at least below whatever upper limit is imposed by the nature of the task toward which they are to be used.

Let it be further emphasized that the use of a standard as reference for the computation of coordinates in the system, from measurements on objects whose coordinates are not part of the standard, cannot yield coordinates of the nonstandard objects whose systematic errors against the target system are identical to those of the standard in the domain covered by standard as well as nonstandard values. The reason (cf. Eichhorn 1982) is again the parameter variance: the differences between the actual values of the parameters involved in the reduction and their estimates will cause systematic differences of the secondary standard values against the primary reference standard. This means, for example, that there is no finite operation by which one could reduce an independent catalogue "to the system of the FK5". The end/product of any operation undertaken toward this purpose will always be a set of position estimates whose systematic errors are different from those of the FK5 (cf. Cole 1988). Even worse: different investigators, making different judgements about reduction models and reduction methods will, from the same set of measurements on the same objects, arrive at different sets of estimates for what are physically the same values, all claiming to have achieved a reduction onto the FK5 system. There will obviously be systematic differences as well between the results of these different reductions. Yet how can sets of estimates, all intended to estimate the identical target quantities, be "on the same system" if there are systematic differences between them?

The incongruities inherent in the statement that a set of estimated positions can define a coordinate system is further illustrated by the following considerations: both polar coordinates of any object and one of the polar coordinates of a second object will fix the coordinate system to which these objects' coordinates are referred. Any combination of three coordinates will fix the system, and one can determine as many different systems found as combinations of coordinates can be put together, because all estimated coordinates are affected by unknown errors, random as well as systematic. One would have to agree on an unambiguous procedure to get "the" system by somehow averaging all these slightly different systems. One can see that the possible ambiguities of the mode of averaging and the questions concerning systematic errors make the *definition* of coordinate system by a set of coordinate estimates with respect to it unpracticable.

In view of the fact that each realization of a system is of necessity of a stochastic nature, we propose any of the terms "standard", "standard system", "realized system" or "estimated system" instead of the misleading term "frame of reference", which connotes solidity and firmness, properties which are conspicuously absent from these standards.

"Quasi-inertial" is another term suggested by Kovalevsky and Mueller which deserves some comment. What they mean by this phrase has been known among relativists as a "local inertial system" and, as in the case of "frame", there is no conspicuous reason why long established terminology ought to be changed. One hears and reads, however, the term "quasi-inertial", now with a different meaning, also applied to entities such as the FK5. This is because the FK5 represents the best effort to date by the community of astrometrists to approximate, empirically, an inertial system in terms of star positions. A much more precise and accurate standard for the realization of an inertial reference frame is available in terms of the ephemerides of selected bodies in the solar system, cf. Williams and Standish (1989). For reasons already mentioned above, the positions in the FK5 (and all its not yet constructed successors and substitutes) show random and systematic differences against a true inertial system. The prefix "quasi-", when used in the literature, always had a clear and well defined meaning (cf. "quasiperiodic function"). We ought to respect this tradition and avoid the term "quasi-inertial" altogether. Instead, we should use the established terms "local inertial system" or "local inertial reference frame" when we mean these entities and characterize the property of, e.g., the FK5, the dynamical standard DE 200 or the contemplated galactic reference standard as "nearly inertial", which seems to convey the nature of these standards more clearly than the ambiguous "quasi-inertial".

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PERMANENT MILLIARCSECOND LINK OF CELESTIAL AND TERRESTRIAL REFERENCE SYSTEMS

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ABSTRACT. The celestial reference system and the terrestial reference system of the International Earth Rotation Service (IERS) are realized on the basis of observation programs in Very Long Baseline radio Interferometry and laser ranging to the Moon and artificial satellites. The celestial frame is materialized by the equatorial coordinates of radio sources observed in VLBI; the terrestrial frame is materialized by the cartesian coordinates of the terrestrial sites monitored by the three techniques. Series of the Earth Orientation Parameters are derived from the same observations. These series provide a permanent link between the celestial system and the terrestrial system at the level of 0.001".

The global adjustment in which the reference systems are defined and realized is described, and the metrological properties of the frames and of the derived EOP are evaluated.

1. Introduction

In 1985, when IAU decided to create the new International Earth Rotation Service and endorsed the recommendations of the MERIT Working Group (Wilkins and Mueller, 1986), it was recognized that the accurate determination of the Earth's orientation makes it necessary that the appropriate terrestrial and celestial frames be monitored in a consistent fashion with the series of Earth Orientation Parameters (EOP).

The operation of IERS is based on observation and analysis by space geodesy techniques: VLBI, Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR); the colocation of some of the stations in common terrestrial sites allows the setting up of a terrestrial reference frame by combining into a common system the coordinates estimated from the various techniques of observation. The celestial reference frame is established on the extragalactic compact radio sources observed in the VLBI programs participating to IERS. The EOP are those angles which describe the orientation of the Celestial Ephemeris Pole in the terrestrial system and in the celestial system (x, y; $d\psi$, $d\varepsilon$), and the orientation angle of the Earth around this axis (UT1-TA1), as a function of time. The time series of the EOP thus provides a permanent tie between the IERS Terrestrial Reference Frame and the IERS Celestial Reference Frame. The accuracy of this link can be evaluated by considering various aspects: the stability of the two realized reference systems, the stability of the EOP evaluations, and their consistency with the reference frames.

2. The observations and their analysis

The observations are organized in networks, which have some sites in common. Table 1 summarizes the contribution of the various techniques to the three objectives of IERS. Although GPS observations are planned to be incorportated in IERS only after 1991, their expected performances are listed for reference.

Table 1. Participation of the various techniques to IERS objectives. Numbers of fiducial points are given in the case of reference frames; in the EOP columns, the number of * reflects a compound of the precision, accuracy and density of measurements.

Observed objects	Technique	Terrestrial frame stations	Celestial frame sources	Earth Orientation celest. pole	terrest. pole	UT
Extragalactic radio sources	VLBI	3-40	50-250	***	***	***
Lunar reflectors	LLR	3-5	(Lun.eph)	**		**
GPS	radio signals from satellites	20-200	(Sat.eph)		**	*
Satellite reflectors	SLR	40-100	(Sat.eph)		***	**

Analysis centres specialized in the various techniques analyse the observations and submit to the Central Bureau of IERS sets of results comprising a terrestrial frame (set of station coordinates) and the corresponding time series of the EOP over several years. In the case of VLBI, the celestial frames are also submitted, under the form of sets of estimated equatorial coordinates of extragalactic radio sources. A selection of these ensembles of results is combined in order to derive realizations of the IERS terrestrial and celestial systems and time series of EOP, following the concepts developped by Boucher *et al.* (1988).

The mutual consistency of the frames and EOP series was set originally at the end of the first year of operation of IERS, and the results were published in the Annual Report for 1988. The system was initialized under the form of the terrestrial and the celestial system. The series of EOP was attached to them by deriving, from the rotation angles of individual frames with respect to the IERS ones, biases for the corresponding time series of EOP (see Section 5); the series could then be combined into a homogeneous one. This global analysis is made possible by the high degree of consistency in the astronomical and geophysical modelling used by the various analysis centres which participate in it, in particular through the reference to the IERS Standards (McCarthy, 1989).

3. The IERS Terrestrial Reference System

The principles on which the IERS Terrestrial Reference System is established and maintained are described by Boucher and Altamimi (1990a).

It is geocentric, the centre of mass being defined for the whole Earth, including oceans and atmosphere. Only observations which can be modelled by dynamical techniques (presently SLR and LLR for IERS) can determine the center of mass. The VLBI results can be referred to a geocentric system by adopting for a station its geocentric position at a reference epoch as provided from external information.

The unit of length is the metre (SI). The scale is that of a local Earth frame, in the sense of a relativistic theory of gravitation. It is obtained by appropriate relativistic modelling, particularly for VLBI and LLR which are usually modelled in a barycentric frame.

The directions of axes are consistent with those of the previous conventional terrestrial reference frame (CIO/BIH pole and BIH origin of longitudes) within the uncertainty of the latter.

As new site coordinates become available, as new observing techniques provide terrestrial frames, or as improved modelling is developped, new realizations of the *frame* are constructed. To insure the constancy of the direction of axes relative to the Earth's crust, appropriate treatment for the internal consistency of the site positions and motions is applied. Then the axes of the new frame are rotated to those of the preceding realization. Through this procedure, the frame is constantly improved and extended, while all successive realizations give access to the same conventional *system*. Checks on the successive versions indicate that the constancy of axes directions is insured within ± 0.0005".

The terrestrial reference system is made available by the site coordinates (119 in the 1990 realization, see Boucher and Altamimi, 1990b), or by the transformation parameters of individual terrestrial frames relative to the IERS ones. As an example, the IERS Annual Report for 1989 gives this information for 18 different terrestrial frames. The system is used as a common reference in worldwide applications, such as time transfer by satellite links or the connection of tide gauges for the study of mean sea level.

4. The IERS Celestial Reference System

The definition of the IERS celestial reference system and the maintenance process of the associated frame are similar to those of the terrestrial system and frame.

The IERS Celestial Reference Frame (ICRF) is established at the Central Bureau of IERS on the basis of extragalactic reference frames obtained by the IERS Analysis Centres for VLBI (in 1990: GSFC, JPL, NGS, USNO). These centres use consistent standards in order to make their frames barycentric, and they impose the condition that the sources do not move relative to one another. Appropriate procedures are applied so that the source coordinates are unaffected by the inaccuracy of the precession-nutation model, except for arbitrary offsets in the reference poles. The use of the standard precession-nutation model in computing the coordinates at J2000.0 introduces a small offset of the pole of the frame relative to the real position of the mean pole. The various ways in which the right ascension origin is fixed also introduce small offsets. The above mentionned offsets are modeled in the Central Bureau combination as three rotation angles for each individual frame relative to the IERS one; they are in general smaller than three milliarcseconds.

As new information on source positions becomes available, new realizations of the ICRF are introduced, insuring that a new realization has globally no rotation with respect to the previous one. The maintenance process includes the selection of *primary sources* on which the no rotation condition is applied; this procedure aims at improving the source coordinates and at extending the list of sources (primary and others), while keeping the initial direction of the axes. The 1990 version (IERS Annual Report for 1989) includes a total of 228 radio sources, among which 51 are primary; 113 sources have position uncertainties under 0.001" (vs. 76 in the previous realization) and the rotation angles relative to the previous realization are insignificant at the level of 0.0001".

Details on the IERS celestial reference system and frame are given by Arias and Feissel (1990a, 1990b). A tentative evaluation of the consistency of the IERS celestial system with the FK5 is given in the Annex. We conclude that the directions of axes of the IERS celestial system are in agreement with those of the FK5 within the uncertainty of the latter.

5. The Earth Orientation parameters

Comparing pairs of terrestrial frames or pairs of celestial frames, one can predict the biases between the respective EOP series (Δx, Δy, $\Delta UT1$, $\Delta d\psi$, $\Delta d\varepsilon$). Let R_1, R_2, and R_3 be the rotation angles between two terrestrial frames, reckoned respectively around the Ox axis (in the direction of the prime meridian), the Oy axis (longitude 90° E) and the Oz axis (polar axis); let A_1, A_2, and A_3 be the rotation angles between two celestial frames (see definition in Annex.) The predicted biases in EOP's are as follows (Zhu and Mueller, 1983).

$$\Delta x = R_2, \qquad \Delta y = R_1,$$
$$\Delta UT1 = A_3 - R_3,$$
$$\Delta d\psi = A_2/\sin\varepsilon, \;\; \Delta d\varepsilon = -A_1. \tag{1}$$

Table 2 gives the average discrepancies between the predicted and observed differences between time series, for VLBI and SLR. The discrepancies are in the range 0.0002" - 0.002".

Table 2. Average discrepancies between reference frames relative rotations and relative biases in EOP series. The uncertainties listed refer to the individual estimates of the discrepancies. Unit: 0.001".

Techniques	Earth Orientation Parameters				
	x	y	UT1	$d\psi\sin\varepsilon$	$d\varepsilon$
VLBI-VLBI	0.7 ±0.6	1.5 ±0.6	0.3 ±0.6	0.4 ±0.2	0.2 ±0.2
SLR-SLR	0.2 ±0.6	0.7 ±0.6			
VLBI-SLR	1.8 ±1.1	1.5 ±1.0			

The comparisons of Table 2 are derived at a common epoch (1988.0). Provided that two series of the pole coordinates and UT1 are referred to the same site motion model, the difference of two series of EOP should not change with time. A check of the relative linear drifts indicates that the unexplained relative drifts in any of the five EOP are generally insignificant (absolute value lower than 0.0001"/year), with some exceptions up to the level of 0.0005"/year.

The two above evaluations set an upper limit at the level of 0.001" to the *accuracy* of the best Earth orientation results. The *precision* of the time series, *i.e.*, the stability of their measurement errors, can be estimated from comparisons two by two with the help of the Allan variance. In an earlier study (Feissel, 1990), we showed that the individual series have a white noise error spectrum for sampling times of five days (0.001") through one year (0.0001"). The IERS series of EOP, which is obtained by combining several of these series, is expected to have also a white noise error spectrum, at a somewhat better level.

Summary

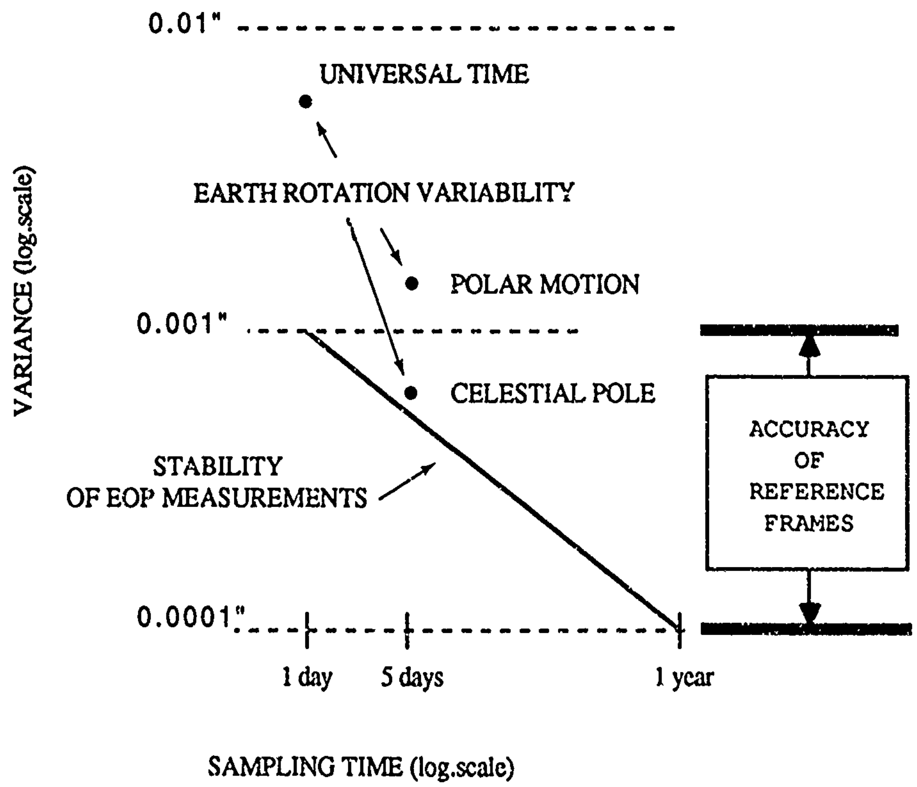
The IERS terrestrial and celestial systems are realized by sets of fiducial points which coordinates have a precision better than 0.001" and an accuracy of 0.001-2"; their axes are maintained within ±0.0005".

The time series of the EOP which relate them have a measurement stability of 0.001" (one day interval) to 0.0001" (one year); their link to the reference system is known within 0.001".

Figure 1 shows these uncertainties, together with an estimation of the natural unstability of the Earth's rotation. UT1-TA1 has a stability of about ±0.006" at one day interval, with a very sharp increase for longer time intervals; the polar motion spectrum starts at 0.0015" for a five day interval, with a relatively rapid rise for longer time intervals; the celestial pole coordinates have a flicker noise spectrum, at the level of 0.0006".

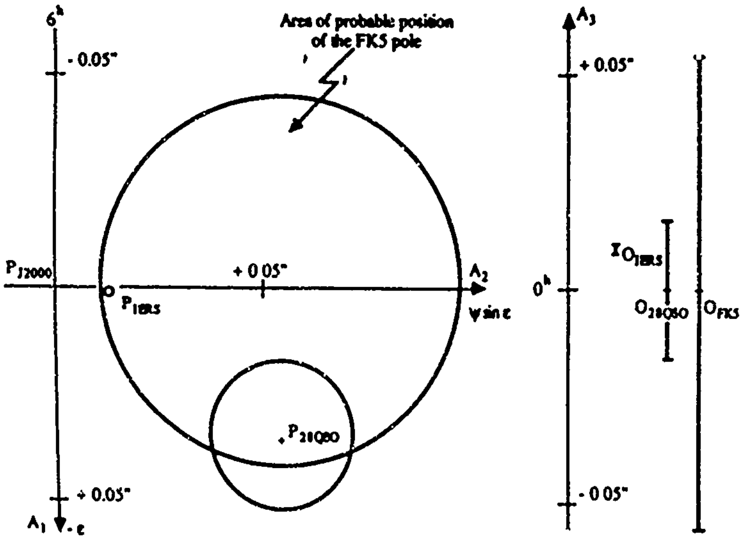
In conclusion, the observations collected and treated in the operation of IERS give a permanent tie at the level of 0.001" between its terrestrial system and its celestial system. The realization of the reference systems as well as their ties are unaffected by the geophysical noise in the Earth's rotation.

Figure 1. Spectra of the Earth's rotation irregularities and of their measurement.



Annex. Relationship between the FK5 and IERS axes.

The figure represents estimated positions of the FK5 and IERS polar axes relative to the mean celestial pole at J2000.0, and estimated relative positioning of the FK5 and IERS right ascension origins. The estimated parameters are rotation angles A_1, A_2, and A_3 around three axes in the directions $\alpha = 0^h, \delta = 0$; $\alpha = 6^h, \delta = 0$; $\delta = 90°$. A_1, A_2 are plotted in a plane tangent to the celestial sphere at the celestial pole; A_3 is plotted in a plane tangent to the celestial sphere in the vicinity of the equinox.



Pole. The coordinate axes are centered at the mean celestial pole at J2000.0. P_{IERS} is an estimation of the IERS polar axis derived from VLBI analyses (e.g. Steppe *et al.*, 1990); the various VLBI estimations agree within 0.001" (radius of the circle around P_{IERS}). The area of probable position of the FK5 pole is obtained by first considering that the systematic part is dominated by a correction of -0.25"/cy to the precession constant imbedded in the FK5 System, and second by adopting Fricke's (1982) estimation of the accuracy of the FK5 equator (± 0.02"), and Schwan's (1988) estimation of the limit of the residual rotation (± 0.07"/cy), taking the epochs of observations from Fricke et al., (1988). If one assumes that the error in the precession rate is absorbed by the proper motions of stars, the circle has to be centered at P_{J2000}, keeping the same radius. P_{28QSO} and its error circle are derived from a comparison of the coordinates of 28 quasars in the FK5 System (Ma *et al.*, 1990) with their coordinates in the IERS Celestial Reference System.

Origin of right ascensions. O_{28QSO}, corresponding to the origin of the right ascensions of 28 quasars in the FK5 System given by Ma *et al.* (1990), is set arbitrarily as the origin. The position of O_{IERS} is derived from a least squares adjustment between the 28 quasars FK5 right ascensions and the IERS ones; the error bar on O_{28QSO} is derived from this adjustment. The error bar on O_{IERS} results from an estimation of the stability of the IERS origin of right ascensions.

O_{FK5} is set at the same position as O_{28QSO}; the error bar is derived from the quadratic sum of the accuracies given by Fricke (± 0.045") and Schwan (± 0.07"/cy), considering a mean epoch of 1955 for the proper motions in right ascension.

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REALIZATION OF THE PRIMARY TERRESTRIAL REFERENCE FRAME

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ABSTRACT. The PTRF is based on 43 sites with 64 SSC collocation points with the optimum geographic distribution, which were selected from all stations of the ITRF89 according to the criterion of the minimum value of the errors of 7 parameters of transformation. The ITRF89 was computed by the IERS Terrestrial Frame Section in Institut Geographique National - IGN and contains 192 VLBI and SLR stations (points) with 119 collocation ones. The PTRF has been compared with the ITRF89. The errors of the 7 parameters of transformation between the PTRF and 18 individual SSC as well as the mean square errors of station coordinates are of the same order as those for the ITRF89. The transformation parameters between the ITRF89 and the PTRF are negligible and their errors are of the order of 3 mm.

1. Introduction

Geodetic and geodynamic investigations on the level of subcentimeter accuracy are the goals of the next decade. A well defined and stable terrestrial reference frame is needed for these purposes (Mueller et al., 1989).

The IERS Terrestrial Reference Frames ITRF88, ITRF89 (Boucher, 1988; Boucher et al. 1989; IERS, 1989-1990) are the best combine solutions developed from previous BIH Terrestrial Systems - BTS 1984 - BTS 1987 (BIH; 1986-1989; Boucher et. al., 1984, 1985, 1986, 1988). The ITRF89 is defined by 192 VLBI, SLR and LLR stations which coordinates were computed as a combination of 18 Sets of Station Coordinates - SSC (IERS, 1990).

The accuracy of the transformation parameters between the individual SSC and the ITRF89 is now of the order of 1 to 10 cm, and depends on the number, the distribution and the accuracy of collocation stations in each SSC. The accuracy of the ITRF89, defined by the accuracy of 7 parameters of transformation depends mostly on distribution of the SSC collocation stations. The ITRF89 stations are located mostly in Western Europe and North America. The most number of such stations, which are not located homogeneously all over the Earth do not improve the accuracy of the ITRF89. A terrestrial reference frame can be defined with the same accuracy by smaller number of stations with accurate coordinates and optimum geographic distribution (Kosek et al., 1990). In the paper the choice of the optimum number and geographic distribution of stations taken from the ITRF89 set of stations was done and new terrestrial reference frame named the Primary Terrestrial Reference Frame - PTRF was determined. The PTRF consisted of 64 SSC collocation points with a good distribution and accuracy of their coordinates is presented as a combination of 18 Sets of Station Coordinates. Such system can be used in geodynamic investigations and for other global geodetic activities. The ITRF with larger number of stations with coordinates in the PTRF system can be used for all other purposes.

2. Method of analysis

The sites of the PTRF with the optimum geographic distribution were selected from the ITRF89 stations according to the criterion of the minimum value of the errors of 7 parameters of transformations obtained by the least-squares adjustment.

The observational equation for the least-squares adjustment written for the $i-th$ station is given by the following formula:

$$\begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix} = \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} - \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} - \begin{pmatrix} D & -R3 & R2 \\ R3 & D & -R1 \\ -R2 & R1 & D \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \tag{1}$$

where: X_k, Y_k, Z_k are disturbed by the white noise station coordinates X_i, Y_i, Z_i,
T_1, T_2, T_3 are the translation parameters,
D is the excess to 1 of the scale factor,
$R1$, $R2$, $R3$ are the small Euler rotation angles.

The equation (1) have been weighted by dividing it by the mean square error of a station computed from the mean square errors of station coordinates given in the ITRF89 solution:

$$m_i = \sqrt{(m_{x_i}^2 + m_{y_i}^2 + m_{z_i}^2)}, \tag{2}$$

The errors of 7 parameters of transformation are given by the following formula:

$$m_j = m_o\sqrt{(A^T A)_{jj}^{-1}} \quad ,\text{for } j = 1,2,\ldots,7 \tag{3}$$

A is the $t \times l$ matrix of the observational equation coefficients with $t = 7$ and $l = 3N$,
N is the number of stations.

In this equation the errors of the parameters of transformation depend only on the values of the diagonal elements of the matrix $(A^T A)^{-1}$ since the value of m_o is constant for the same number of stations. These diagonal elements depend on the stations distribution as well as on the m_i errors.

First the points of the highest accuracy of coordinates have been chosen in each site. Next the stations of not good distribution and accuracy have been eliminated sequentially according to the criterion of the minimum value of the errors of 7 parameters of transformation. These errors are minimum when the trace of the variance covariance matrix has a minimum value ($tr(A^T A)^{-1} = min$). In order to choose the optimum distribution of $N-1$ stations from the set of N the $\binom{N}{N-1}$ combinations were analysed. From N number of stations a number of $N-1$ such stations were selected, for which the value of $tr(A^T A)^{-1} = min$. It enables elimination of one station and after that the number of $N-2$ stations from $N-1$ stations can be selected. All collocation stations of VLBI and SLR or LLR instruments have been included in the computation of the PTRF in order to get better tie of the VLBI to SLR systems, though for some of them like METSAHOVI, FLAGSTAFF, PASADENA, YUMA, PLATTEVILLE and SAN DIEGO the trace matrix analysis show that they ought to be eliminated.

Additionally points like: GOLDSTONE (S009, S014, S019), FORT DAVIS (M006), CANBERRA (S003) and MADRID (S001, S010) were also included to the computation in order to get the solution for the transformation parameters for each SSC for which at least 3 stations are necessary. All the SSC collocation stations of the Southern hemisphere are important for the computation of the PTRF, because of the small number of stations in this hemisphere. The improvement of the accuracy of their coordinates is very important for the better definition of the Terrestrial Reference Frame and the PTRF.

Finally, the PTRF consisting of 43 sites with 64 SSC collocation points was computed (Fig. 1). Using the least squares adjustment and the IGN program for ITRF computation (Boucher et al., 1988, 1989) the coordinates X_i, Y_i, Z_i, of the PTRF as well as the transformation parameters: T_1, T_2, T_3, D, $R1$, $R2$, $R3$, between this frame and the individual SSC were obtained (Tab. 1.) on the base of formula (1). The scale and the origin of the SSC-CSR89L02 have been adopted for definition of the ITRF88, ITRF89 and the PTRF systems and they were held fixed in the adjustment of the PTRF. The orientation of the PTRF axes is adopted to be the same as in the ITRF88 and ITRF89 systems and differs about fixed rotation angles from the SSC-CSR89L02 system. The transformation parameters from the PTRF to the individual SSC's are in a very good agreement with the transformation parameters from ITRF89 to the same individual SSC's (Tab. 1.,2.). The differences are smaller than their RMS errors except for such SSC like: GAOUA90L01 (T_2) and LPAC90L01 (T_2, D). The errors of these transformation parameters are of the same order as for the ITRF89. The errors of the coordinates of the PTRF stations (Tab. 3.) and the $\sigma_o = 1.35cm$ are of the same order as for the ITRF89 ($\sigma_o = 1.34cm$).

The transformation parameters from the PTRF to ITRF89 reference frames are negligible (Tab. 1.) and their RMS are of the order of 3 mm.

3. Conclusions

The PTRF consisting of 64 SSC collocation points in 43 sets carefully chosen defines the terrestrial system as well as the ITRF89 consisting of 119 SSC collocation points. About half a number of SSC collocation points due to not homogeneous distribution and lower coordinate accuracy have no influence on the accuracy of determination of the ITRF89. More number of the eliminated collocation station are located in Europe and North America. The PTRF defines well an ITRF for global geodynamic investigations. The ITRF with larger number of stations is useful for other geodetic activities.

4. Acknowledgements

The computation was made during the visit of Mr. W. Kosek at the Institut Geographique National due to kind invitation of Dr. C. Boucher. Mr. W. Kosek thanks Dr. Z. Altamimi for his kind assistance in the use of the IGN software for ITRF determination.

A "PRIMARY" TERRESTRIAL REFERENCE FRAME

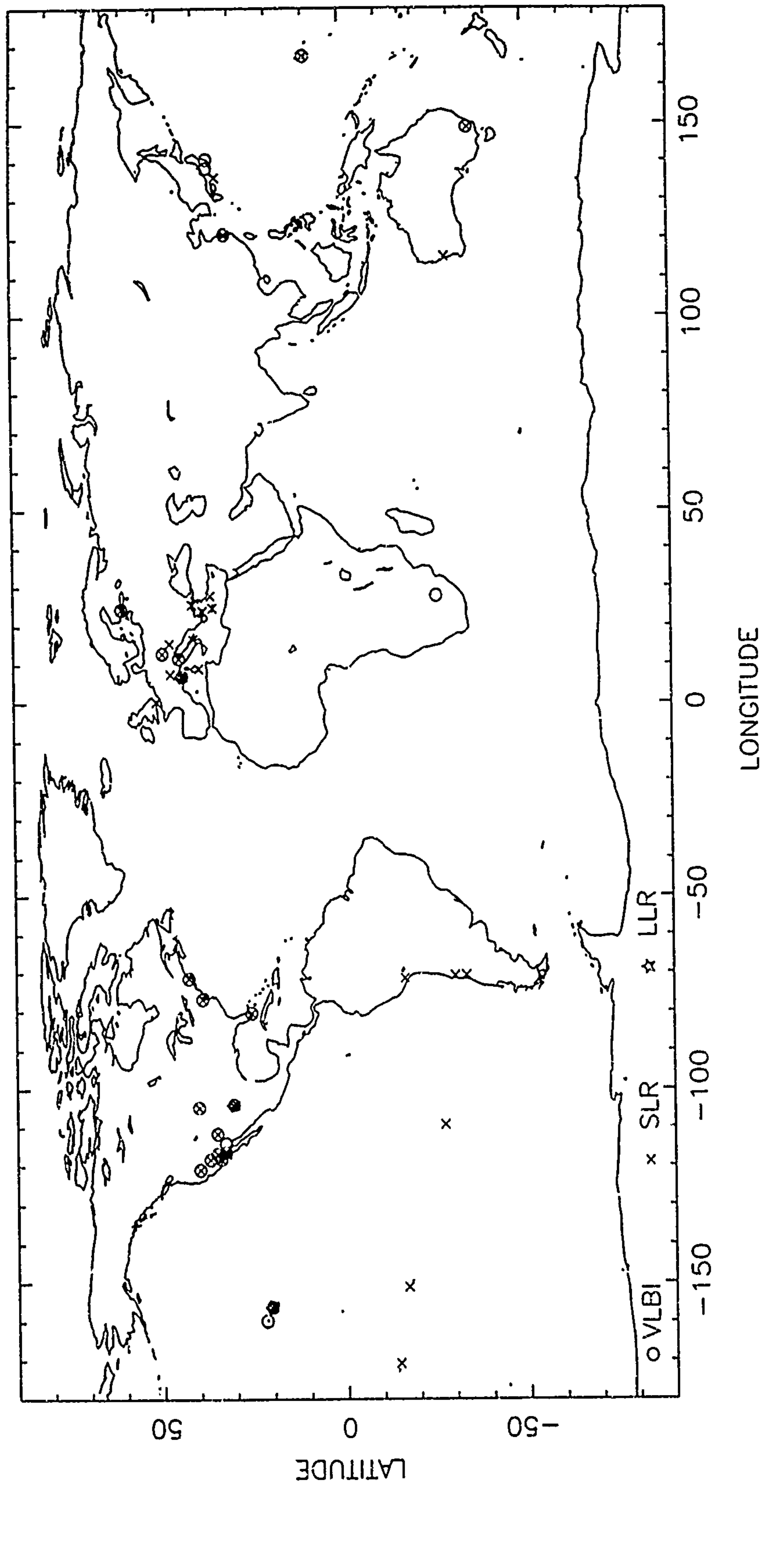

Table 1. Transformation parameters from the Primary Terrestrial Reference Frame to individual SSC systems and to the ITRF89 at epoch 1988.0.

SSC	T1	T2	T3	D	R1	R2	R3	Coll. stations	
		[cm]		10^{-8}		[mas]		chosen	elimin.
GSFC 90R02	161.4	-86.0	52.0	-0.7	2.1	1.5	0.2	21	15
	1.1	1.2	1.1	0.2	0.5	0.4	0.4		
NGS 90R01	-0.2	-4.7	5.1	-0.3	-3.8	10.5	-0.2	14	12
	1.2	1.2	1.2	0.2	0.5	0.5	0.4		
USNO 90R01	-8.5	-16.5	3.5	-1.8	-0.4	2.2	0.5	4	4
	2.0	2.4	2.3	0.3	1.1	0.8	0.6		
SO 88R01	162.8	-107.3	36.9	-0.5	-7.4	7.0	-2.0	3	2
	7.8	4.7	6.4	0.5	2.5	3.4	1.4		
NAOMZ 89R01	-10.1	13.3	-4.2	1.3	-4.1	9.1	-1.4	3	3
	9.0	8.6	7.5	1.2	3.3	3.5	2.5		
JPL 90R02	-0.2	-4.6	9.1	-3.1	2.3	1.6	0.1	3	2
	2.7	2.9	2.8	0.4	1.4	1.2	1.5		
CSR 89L02	0.0	0.0	0.0	0.0	-3.4	4.3	-15.9	46	37
	-	-	-	-	-	-	-		
GSFC 89L01	-2.0	-0.7	1.7	0.1	-0.7	5.2	-7.5	40	35
	1.1	1.1	1.0	0.2	0.4	0.4	0.4		
DGFII 89L03	0.4	-0.3	4.6	-0.2	-286.1	-46.0	3.4	18	4
	1.5	1.5	1.4	0.2	0.6	0.6	0.5		
DUT 90LO1	-0.4	-1.2	-5.9	-1.1	0.1	1.7	-4.3	22	9
	1.3	1.3	1.3	0.2	0.5	0.5	0.5		
ZIPE 90L01	-1.8	0.5	-8.0	0.1	0.5	0.8	0.6	29	9
	1.3	1.3	1.2	0.2	0.5	0.5	0.4		
S0 90L01	-0.5	0.5	1.1	-0.6	-3.7	5.2	-14.3	30	12
	1.3	1.3	1.2	0.2	0.5	0.5	0.4		
GAOUA 90L01	-2.0	0.4	-0.6	-1.0	1.1	2.6	-0.8	26	14
	1.4	1.3	1.3	0.2	0.5	0.5	0.4		
LPAC 90L01	0.8	1.9	8.9	-0.6	2.2	-0.1	-4.2	20	7
	1.7	1.6	1.6	0.2	0.6	0.7	0.6		
UTXMO 90M01	-2.4	-0.7	34.3	-4.2	-0.8	0.2	-3.1	6	0
	5.9	7.7	5.4	1.0	2.4	1.7	3.3		
JPL 90M01	-9.8	-1.8	0.4	-2.0	1.5	3.2	-1.4	6	0
	4.5	5.0	4.7	0.6	2.3	1.7	1.3		
CERGA 89M01	-13.2	3.6	-4.7	-5.3	0.6	3.5	-11.5	5	0
	4.6	5.3	4.8	0.6	2.5	1.7	1.3		
S0 86M01	-17.2	21.8	6.8	-1.8	16.0	6.9	-6.1	4	0
	9.1	10.5	9.5	1.2	5.0	3.5	2.6		
ITRF89	-0.1	-0.6	-0.1	0.0	-0.1	0.0	0.0	64	55
	0.3	0.3	0.3	0.1	0.1	0.1	0.1		

Table 2. Differences between transformation parameters from the PTRF to the individual SSC systems and from the ITRF89 to the individual SSC systems.

SSC	T1	T2	T3	D	R1	R2	R3
		[cm]		10^{-8}		[mas]	
GSFC 90R02	0.2	1.0	0.1	0.1	0.3	-0.2	0.0
NGS 90R01	0.0	0.6	0.3	0.0	0.1	-0.1	0.2
USNO 90R01	-0.6	0.5	-1.8	0.0	-0.4	0.3	0.4
SO 88R01	1.6	-0.5	-0.2	0.1	-0.2	-0.5	-0.4
NAOMZ 89R01	-2.3	1.5	1.2	0.1	-0.2	1.1	0.1
JPL 90R02	1.5	-0.8	2.2	0.5	0.1	-0.2	0.2
CSR 89L02	0.0	0.0	0.0	0.0	0.0	0.0	0.0
GSFC 89L01	-1.0	0.3	-0.7	0.1	0.3	-0.1	0.2
DGFII 89L03	0.4	0.7	0.2	0.0	0.2	0.0	0.0
DUT 90LO1	0.1	1.1	0.2	0.0	0.1	0.0	0.1
ZIPE 90L01	1.3	0.1	0.6	0.0	0.2	-0.2	-0.1
S0 90L01	0.1	1.2	-0.6	0.2	-0.2	0.2	-0.1
GAOUA 90L01	1.3	2.2	1.1	0.2	0.3	-0.6	0.0
LPAC 90L01	-0.7	2.8	0.0	-0.4	-0.1	0.6	0.5
UTXMO 90M01	0.2	1.3	0.1	0.1	0.1	-0.1	0.0
JPL 90M01	0.2	1.0	0.2	0.0	0.1	-0.1	0.0
CERGA 89M01	0.2	1.0	0.1	0.0	0.1	-0.1	0.0
S0 86M01	0.2	1.0	0.1	0.1	0.1	-0.1	0.1

Table 3. The station coordinates of the Primary Terrestrial Reference Frame.

	STATION	X	m_x	Y	m_y	Z	m_z
				[m]			
S010	CANBERRA	-4460935.076	.022	2682765.771	.021	-3674381.711	.021
S010	MADRID-R	4849336.797	.019	-360488.893	.019	4114748.621	.020
S014	GOLDSTON	-2351129.050	.015	-4655477.095	.015	3660956.884	.015
M102	WASHINGT	1130686.678	.011	-4831353.025	.011	3994110.875	.011
M002	MAUI I	-5465998.456	.012	-2404408.544	.012	2242228.403	.013
S001	JOHANNES	5085442.829	.024	2668263.391	.025	-2768697.273	.024
S002	WESTFORD	1492404.925	.012	-4457266.488	.012	4296881.692	.013
S003	FORT DAV	-1324210.866	.010	-5332023.144	.011	3232118.375	.011
S001	KASHIMA	-3997892.273	.020	3276581.312	.019	3724118.288	.022
S001	KAUAI	-5543845.987	.014	-2054564.165	.013	2387813.766	.013
S003	KWAJALEI	-6143536.502	.017	1363997.208	.017	1034707.353	.019
S001	BOLOGNE	4461370.198	.016	919596.751	.017	4449559.046	.018
M013	GOLDSTON	-2356494.019	.011	-4646607.672	.012	3668426.599	.012
S009	GOLDSTON	-2356170.906	.011	-4646755.885	.011	3668470.584	.011
M003	MONUMENT	-2386289.312	.011	-4802346.566	.012	3444883.962	.012
M004	OWENS VA	-2410421.134	.013	-4477800.428	.013	3838690.309	.013
M002	PLATTEVI	-1240708.017	.013	-4720454.337	.013	4094481.633	.013
S001	RICHMOND	961258.174	.013	-5674090.035	.013	2740533.732	.014
S009	SHANGHAI	-2831686.666	.018	4675733.885	.018	3275327.806	.020
S004	WETTZELL	4075540.093	.014	931735.168	.014	4801629.244	.014

Table 3. cont.

	STATION	X	m_x	Y [m]	m_y	Z	m_z
M001	YUMA	-2196777.811	.018	-4887337.065	.018	3448425.231	.018
S001	MADRID-R	4849092.729	.067	-360180.596	.068	4115108.964	.070
M003	SAN DIEG	-2428826.596	.012	-4799754.325	.015	3417267.044	.015
M006	FORT DAV	-1330020.936	.010	-5328401.851	.012	3236480.796	.011
M001	FORT DAV	-1330125.263	.010	-5328526.640	.011	3236150.244	.011
M001	YARRAGAD	-2389006.532	.020	5043329.288	.019	-3078525.383	.020
M001	WESTFORD	1492453.790	.012	-4457278.752	.012	4296815.886	.013
M001	KWAJALEI	-6143447.268	.018	1364700.185	.017	1034163.112	.019
M001	SAMOA	-6100045.855	.029	-996203.144	.029	-1568976.327	.029
M002	EASTER I	-1884984.374	.021	-5357608.169	.020	-2892853.406	.020
M105	WASHINGT	1130719.790	.011	-4831350.573	.011	3994106.477	.011
M002	QUINCY	-2517234.690	.013	-4198556.250	.013	4076569.654	.013
M001	MONUMENT	-2386277.917	.011	-4802354.367	.012	3444881.438	.012
M001	PLATTEVI	-1240678.122	.013	-4720463.373	.013	4094480.608	.013
M001	OWENS VA	-2410422.357	.013	-4477802.689	.013	3838686.692	.013
M002	GOLDSTON	-2350861.547	.015	-4655546.275	.015	3660997.827	.015
M003	HUAHINE	-5345865.399	.025	-2958246.717	.025	-1824623.889	.025
M001	MAUI I	-5466006.470	.012	-2404427.953	.012	2242187.475	.013
M002	RICHMOND	961319.050	.013	-5674090.966	.013	2740489.520	.014
M001	SANTIAGO	1769699.798	.029	-5044612.926	.030	-3468260.050	.029
M001	CERRO TO	1815517.158	.029	-5213464.875	.029	-3187999.408	.029
M001	ASKITES	4353444.929	.021	2082666.283	.021	4156506.648	.021
M001	KATAVIA	4573400.091	.022	2409322.200	.021	3723881.751	.022
M002	DIONYSOS	4595216.461	.020	2039435.345	.021	3912629.488	.021
M001	ROUMELLI	4728694.704	.018	2174373.372	.018	3674572.923	.018
M002	CAGLIARI	4893398.119	.019	772673.316	.019	4004140.939	.020
M002	BOLOGNE	4461399.729	.016	919566.848	.017	4449510.490	.018
S001	METSAHOV	2892595.614	.024	1311807.759	.024	5512610.719	.024
S001	ZIMMERWA	4331283.630	.017	567549.537	.017	4633139.941	.017
S002	WETTZELL	4075530.090	.013	931781.316	.013	4801618.172	.014
S001	CERGA GR	4581691.822	.015	556159.394	.015	4389359.383	.015
S001	SHANGHAI	-2831087.790	.019	4676203.485	.018	3275172.826	.021
S001	SIMOSATO	-3822388.354	.018	3699363.497	.017	3507573.095	.019
S002	LUSTBUEH	4194426.735	.017	1162693.874	.017	4647246.544	.017
S001	HERSTMON	4033463.849	.017	23662.354	.017	4924305.017	.017
S007	CANBERRA	-4446476.781	.018	2678127.080	.018	-3696251.774	.018
M002	FLAGSTAF	-1923976.627	.018	-4850871.713	.018	3658574.856	.018
M001	PASADENA	-2493211.862	.018	-4655229.531	.018	3565574.503	.018
S001	AREQUIPA	1942792.008	.018	-5804077.671	.018	-1796919.305	.018
S001	MATERA	4641965.108	.017	1393069.947	.017	4133262.218	.017
S003	CANBERRA	-4447548.616	.018	2677133.980	.018	-3694996.475	.019

Table 3. cont.

	STATION	X	m_x	Y [m]	m_y	Z	m_z
S002	FORT DAV	-1330781.240	.010	-5328755.585	.011	3235697.676	.011
S005	MAUI I	-5466006.954	.012	-2404428.165	.012	2242188.478	.013
S002	CERGA GR	4581692.400	.015	556195.867	.015	4389354.945	.016

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THE TRANSFORMATION FROM FK4 TO FK5 AND THE DEFINITION OF UT

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ABSTRACT. The different procedures of the transforation from FK4 to FK5 are reviewed. With these procedures some numerical examples for selected FK4 stars are given. Basing on the original work of Fricke (1985), we prefer to select the procedure developed by Standish (1982). One of the main reasons for the procedure of Aoki et al. (1983) is to consist with the new definition of UT. Since the corrections "FK5 - FK4" for many stars are significant, even if adopting this procedure, it is still impossible to avoid the effect of the right ascension discontinuities on UT.

1 Introduction

According to the resolution adopted by IAU in 1976, the FK4 should replaced by the FK5 from 1984 January 1 onward. The transformation from FK4 to FK5 includes the following processes: (1) eliminate the E terms of aberration. (2) correct the equinox error and its motion. (3) correct the proper motion for the new precession constant. (4) change the unit of the time from tropical to Julian centuries. (5) transform the positions and proper motions to J2000.0. (6) apply the systematic and individual corrections ("FK5 - FK4"). Because the "FK5 - FK4" were not available for the year 1984-1987, there was a transitional phase in which only the first five procedures have been applied.

For the transitional transformation several approaches have been developed by Standish (1982), Aoki et al. (1983), Lederle and Schwan (1984), Smith et al. (1989), Yallop et al. (1989), Murray (1989) and Soma et al. (1989). Among these there are two main differences: (1) The equinox correction is applied in the fixed frame (Standish) or in the rotating frame (Aoki et al.). (2) The transfer from FK4 to FK5 is performed at B1950.0 (Standish) or at 1984 January 1 (Aoki et al.).

In Sect.2 we suggest a criterion to review the differences. In Sect.3 some numerical examples are given for comparison. In Sect.4 we discuss the problem whether the transformation should or could consist with new definition of UT.

2 The Review of Different Procedures

In the transformation the equinox error of FK4 and the correction of precession constant, which are determined by Fricke, are adopted; therefore the procedure of the transformation should base on the original work of Fricke (1985).

2.1 THE DETERMINATION OF THE EQUINOX ERROR OF FK4

Fricke adopted the following formula for determining the equinox error of FK4

$$\Delta\alpha = E + f(\Delta\lambda, \Delta\varepsilon, \Delta h, \Delta k) \tag{1}$$

where $\Delta\alpha = (\alpha_\odot)_o - (\alpha_\odot)_c$. $(\alpha_\odot)_o$ is the observed right ascension of the sun, and $(\alpha_\odot)_c$ is the computed value provided by an ephemeris. $\Delta\lambda$, $\Delta\varepsilon$, Δh and Δk are corrections to the "solar orbit".

Combining a series of observational results from the sun and members of the planetary system, Fricke obtained the equinox error of FK4. Soma et al. (1989) show that the equinox correction E of Fricke is determined in the frame of date. On the contrary, we think that the equinox correction E so determined is in the fixed frame. In fact, deriving a numerical value of E needs the observational results of one year or even several years. They all must be reduced to the FK4 system. Moreover, the solar ephemeris is also founded in the fixed frame. It is convincing that Fricke oneself gave the transformation from FK4 to FK5.

2.2 THE DETERMINATION OF PRECESSION CORRECTION

Fricke made use of proper motion, μ_{FK4}, of FK4 stars to determine the precession correction. If the effect of precessional error and zero point error on proper motion is only considered, then the formula becomes

$$\Delta(\mu_\alpha cos\delta) = \Delta k \cdot cos\delta + \Delta n \cdot sin\alpha sin\delta \tag{2}$$

$$\Delta\mu_\delta = \Delta n \cdot cos\alpha \tag{3}$$

where $\Delta k = \Delta p_1 \cdot cos\varepsilon - \Delta\lambda - \Delta e$, $\Delta n = \Delta p_1 \cdot sin\varepsilon$. p_1 and λ are the lunisolar and planetary precessions respectively. Δe is "equinox motion" of FK4, and it corresponds to the derivative of E in Eq.(1).

From the above consideration it appears that Fricke took no account of the effect of equinox correction on μ_{FK4}. Accordingly the effect of equinox correction should not be involved when the proper motion relative to new precession constant is corrected.

As is given in the above discussion, it is obvious that the procedure adopted by Standish is in keeping with the original work of Fricke. We insist that Eq.(3) in the paper of Soma et al. be adopted.

3 Numerical Examples and Comparison

In order to compare numerical values obtained from different procedures of the transformation, we select four FK4 stars in the computation (from Table 1 of Smith et al.)

All authors have no objection to the removal of the elliptic terms in aberration. Using the formula given by Lederle and Schwan (1984), we make first the correction to the catalog position. Applying steps (2), (3), (4) and (5) in the Introduction, we compute the transformation matrices by Standish (St), Aoki et al. (Ao) and Soma et al. (So) (see Soma et al. (1989), Eqs.(1),(2),(10)). Numerical examples for J2000.0 are given in Table 1 for selected four FK4 stars.

Table 1. Numerical examples for J2000.0

No.	Proc.	α	δ	P.M.R.A.	P.M.DEC
10	St	$00^h20^m04^s.3088$	$-64°52'29''.333$	$26^s.8627$	$116''.284$
	Ao	.3096	.331	.8647	.286
	So	.3092	.332	.8635	.286
907	St	$02^h31^m49^s.8303$	$+89°15'50''.655$	$21^s.7784$	$-1''.581$
	Ao	.8113	.659	.7287	.571
	So	.8227	.658	.7634	.576
923	St	$21^h08^m46^s.0510$	$-88°57'23''.666$	$8^s.4080$	$0''.180$
	Ao	.0673	.669	.4481	.172
	So	.0601	.667	.4263	.177
1307	St	$11^h52^m58^s.7456$	$+37°43'07''.460$	$33^s.7149$	$-581''.214$
	Ao	.7460	.459	.7157	.215
	So	.7458	.459	.7152	.216

We note from Table 1 that

(1) For non-circumpolar stars the differences of J2000.0 positions and proper motions obtained by four approaches can be omitted.

(2) For circumpolar stars the four approaches differ systematically by as much as $0^s.02$ and $0^s.05$ per century in J2000.0 right ascensions and proper motions in right ascension respectively.

Another transformation on the removal of the E terms of aberration from the J2000.0 mean place was adopted in Merit Standard (1983) and Astronomical Alamanac (1984). We examine the effect of epoch change of stellar reference system on E terms of aberration. We find that for circumpolar stars the effect of 50-year epoch change of the reference system on E terms of aberration may reach to $0^s.4$ and $0''.07$ respectively. The effect is far larger than the differences from different procedures of the transformation in Table 1.

4 The Review of the New Definition of UT

Since the Eighteenth General Assembly of the IAU in 1982, there have been many reviews on the new definition of UT. Its main defects are as follows: (1) Lack of a clear physical cocept of UT. (2) Involving some determined constants. (3) Not applicable to the new techniques.

Aoki et al. (1983) emphasized that the transformation from FK4 to FK5 should consist with the new definition of UT, and demanded that the positions and proper motions of all FK4 stars referred to the new and old systems respectively are the same at 1984 January

1. This is logically unreasonable. In fact, the fundamental plane and origin of a stellar reference system can be defined, whereas the positions and proper motions of all stars cannot be specified. Their numerical values can only be determined by observations.

As is given in the Introduction, the transformation from FK4 to FK5 involves 6 steps. The systematic and individual correctons of FK4 stars should be applied to the result of transitional transformation. According to "Corrections FK5 - FK4" published by Wielen et al. (1987), for 1984 $\Delta\alpha$ of 69 stars are larger than $0^s.1$, among them $\Delta\alpha$ of 23 stars are larger than $0^s.2$. It is obvious that the effects of the systematic and individual corrections for many stars on the determination of UT cannot be omitted at the beginning of 1984.

From 1988 onwards the IERS system no longer includes the optical observations. So it seems that the new definition of UT is unnecessary in IERS Standard. We suggest that the Working Group of the Reference System should rediscuss the definition of UT.

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THE LINKAGE BETWEEN RADIO AND OPTICAL COORDINATE SYSTEMS: PROGRAM CONFOR

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ABSTRACT. The present status and prospects of program CONFOR-Connection of Frames in Optic and Radio, are described. It is supposed that positions of extragalactic objects and intermediate stars in 200 areas will be obtained in FK5 system.

Program CONFOR - Connection of Frames in Optic and Radio intends for the link of optical counterparts of compact extragalactic radio sources with fundamental coordinate system. It was described [1] at IAU Symp. No 141 'Inertial coordinate system on the sky ' at 1989 in Leningrad. It is planned that optical coordinates of about 200 radio sources will be determined in FK5. The radio sources are extracted from the list of Argue et al. [2], which considered now as a basis for construction of radio coordinate system.

One part of the program is a determination of reference stars coordinates in neighborhoods of radio sources by meridian observations. This part is considered in more detail in the paper by Tel'njuk-Adamchuk et al., presented to this Colloquium. Up to date meridian observations are carried out for 1600 stars, about two thirds specified in the program. The observatories of State Kiev University, State Odessa University, State Kazan University and also observatories in Belgrade, Bucharest and Bordo take part in this observations.

According to program CONFOR it is necessary to determine the coordinates of intermediate reference stars of 12 - 14 stellar magnitude for areas around radio sources, which is faint in optics . For a few dozen sources, fainter then 19 stellar magnitude the secondary systems of intermediate reference stars of 16 - 18 stellar magnitude are introduced. For this purpose the photographic observations are carrying out at observatories in the Abastumani (R. Ya. Inasaridze) and Kitab (L. I. Bashtova) with Zeiss - 400 astrographs. The photographic observations of the intermediate stars also are carrying out in Astronomical observatory of State Kiev University with 20 - cm astrograph (S. V. Pasechnick). Up to date it is obtained the plates for more than 170 areas. The measurements of images and data processing are executing.

Photographic observations of compact extragalactic radio sources are performing by means of 1-m RCC telescope of Institute of Astrophysics of Tajik AS (B. N. Irkaev) and Tautenburg Schmidt telescope of Central Institute of Astrophisyics (W. R. Dick). Astrometric investigations of these instruments, with mostly used for astrophysics observations have been fulfilled. It was shown [3] that accuracy of obtained positions is saticfactory for the task of the program namely for 1 - m RCC VMS is 0."04 and for Schmidt Telescope better then 0."15. For each area two or more plates are taken. At present it is obtained more than 270 plates for 100 objects.

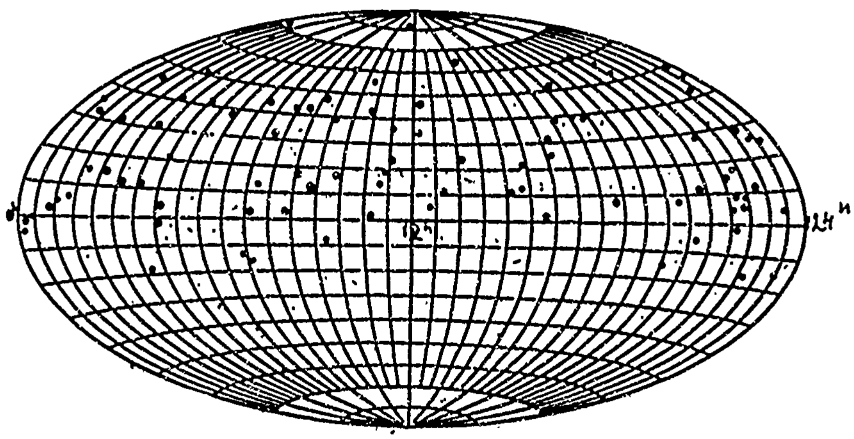

Fig. 1

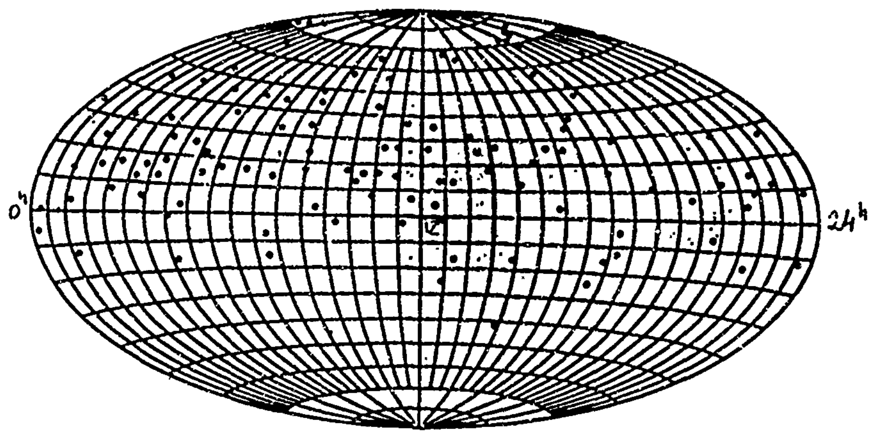

Fig. 2

Distribution of obtained plates with intermediate reference stars in celestial sphere is presented at Fig. 1,and with extragalactic objects at Fig. 2.

The program CONFOR also is caring out in Main Astronomical observatory of Ukrainian Academy of Sciences and in State Sternberg Astronomical Institute.

The main results of CONFOR is supposed to be the catalogues of positions of extragalactic objects in FK5 - system and coordinates of intermediate reference stars. It is supposed to determine the systematic differences between radio and optical coordinate systems and their relative orientation.We are intend to use the methods developed in previous works [4] specially for this program. These results of the program CONFOR are expected to the end of 1991 year.

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THE EXTRAGALACTIC RADIO/OPTICAL REFERENCE FRAME

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1.0 INTRODUCTION

The celestial positions of extragalactic radio sources may be determined to a precision of less than a milliarcsecond. Further, since these sources are believed to be at great distances from the galaxy, little or no proper motion is expected on scales of order a milliarcsecond. Therefore a reference frame based on the positions of carefully selected sources so that display compact radiation on scales less than a milliarcsecond will noticeably improve the precision of present celestial reference frames. If the radio objects making up the reference frame also emit radiation at optical wavelengths, and assuming the optical/radio radiation is coincident, the radio frame can update the optical frame to the accuracy of the individual optical positions.

The IAU Working Group on the Radio/Optical Reference Frame, formed by Commission 24 in 1978, has as its charter to draw up a list of suitable candidates. A candidate list of 234 sources was drawn up with their positions based on a weighted mean by error of 9 radio catalogs and had an average accuracy better than 0.01 arcsec (Argue et al. 1984).

A program to establish a radio/optical reference frame was undertaken in 1987 and is described in IAU Symposium 129 (Johnston et al. 1987). This program has as its goal the establishment of a reference frame based on the radio/optical positions determined for 400 sources uniformly distributed over the entire celestial sphere. These sources are to be compact in their radio/optical emission. They should be brighter than visual magnitude 19 and have a total flux density greater than 1 Jy at 5 GHz. The method of construction of the reference frame is to adopt the radio positions as the reference positions of the frame and upgrade the optical frame by adopting the radio positions. In particular this program will provide a major contribution to the absolute orientation of the anticipated HIPPARCOS stellar net currently under observation.

2.0 SOURCE SELECTION

The first step is the selection of appropriate sources that have compact emission at radio wavelengths. The spatial distribution of these sources is to be one every 100 square degrees.

At the initiation of this program in 1987 there was a much greater knowledge of the characteristics of compact radio sources in the northern hemisphere. This was simply due to the large number of VLBI facilities and organized VLBI networks such as the US Network and the European VLBI Network as well as extensive observing campaigns by NASA's Crustal Dynamics Program using numerous telescopes. There were approximately eighty sources north of the equator and forty sources south of the equator with positions of order a few milliarcseconds. However, no attempt was made in the programs which determined these positions to identify them and determine whether they had optical counterparts.

One of the primary tasks currently underway is the selection of the 400 sources with the appropriate radio/optical emission for the reference frame. We are currently sorting the sources into 3 lists -- primary, preliminary, and proposed sources -- which are described in detail at this meeting in Russell et al. The preliminary sources need further data and must be checked for source structure. The proposed sources have not been observed yet, but will be used as a list of candidates from which to fill in the sparse areas of the global distribution.

As of this meeting, the lists of primary, preliminary and proposed candidates contain 220, 139 and 160 sources, respectively. Many sources are also listed which will not be suitable for the reference frame because of problems with radio structure, optical magnitudes which are too faint, no identifiable optical counterparts, etc. The refinement of the lists will continue until the final catalog is completed, but we expect to have finished sorting sources into these lists by the time of the Twenty-First IAU General Assembly in 1991.

3.0 OBSERVING PROGRAM

The radio positions are determined using Very Long Baseline Interferometry (VLBI). All observations are made using the Mark III VLBI recording system and at two frequencies spanning S and X Bands. Stations in the northern hemisphere that have been employed in this program have been Green Bank and Maryland Point on the east coast of the U.S., Hat Creek on the West Coast of the U.S., Fairbanks, Alaska for a northern site and Hawaii for a far western site. Observations of the northern sources are made annually. In the southern hemisphere, the stations are Tidbinbilla, Australia, Hobart, Tasmania and Hartebeesthoek South Africa. Because of the limited resources available, the southern sources are not observed as often but whenever possible. The precision for the VLBI observations is 1 mas in the northern hemisphere and 2 to 10 mas in the southern hemisphere due to the limited observations and system calibration.

The optical program determines the position of the extragalactic radio source via a minimum of two long focus plates obtained with a 4 meter class prime focus telescope for the fainter sources of visual magnitude greater than 18, or various R.C. telescopes, as for example the Calar Alto 2.2 m for the brighter sources. The telescopes must have an astrometrically useable field of 30 arcminutes diameter to provide the necessary density of secondary reference stars, mainly in the magnitude range 12-15. A major problem is the modeling of the large optical distortion of most prime focus correctors. The system of secondary reference stars for each extragalactic source field is obtained from four wide field astrograph plates.

The prime focus plates allow the determination of the position relative to the "local" field stars located within one degree of the object. The astrograph plates with a field of approximately five degrees on a side make it possible to determine the relationship of the "local" field stars to FK5 system via the IRS reference stars. The average precision of the optical source positions is 10 to 50 mas.

The prime focus telescopes currently used are the Kitt Peak 4m, the AAT 3.9m and ESO 3.6m. The astrographs are the Hamburg Observatory astrograph, the USNO astrograph at Black Birch Astronomical Observatory and the Lick Observatory 20 inch astrograph. All astrographs are used in the yellow spectral range, as provided by 103aG+OG515 plate-filter combination (Hamburg and Lick) and 103aG+GG495 (Black Birch). Typically about 70-100 secondary reference stars inside a 1x1 degree field centered at the source position are measured. Based on 4 plates, the accuracy of a single reference star position is about 0.06 arcsec. In addition to the photographic plates deep CCD images are taken to evaluate possible optical structure, particularly for low redshift sources.

The source list is about 65% quasars, 10% BL Lac's and 10% compact galaxies; the remainder are unidentified or empty fields. The present distribution covers the whole sky, with slightly more sparse coverage in the south and with the largest gaps along the galactic plane. The unidentified sources are at present carried in the program until their identifications are made or, if not they will be removed from the program.

4.0 PRESENT STATUS

The source positions are adopted from the radio. The data are the Mark III group delays and phase delay rates. In the data reduction of the radio observations (Ma et al. 1990) the IAU sanctioned definitions of precession, sidereal time (J2000.0) and nutation (IAU 1980), and the PEP ephemeris for the solar system model are used. The troposphere is calibrated using the CfA 2.2 model or the Chao model. The differential pathlength due to the ionosphere is calibrated out by use of simultaneous dual frequency S/X observations. The cable delays in the systems are calibrated using a phase calibration signal at each site. Source positions are solved for globally, while site positions, nutation offsets in longitude and obliquity, clock offsets, zenith atmosphere delays and rates solved for. From the individual data bases, a reference date of 1980 October 17 is presently being used (Ma et al. 1990).

The first portion of the radio/optical data was published in Ma et al. (1990) and was based on the geodetic data available from the data base maintained at the Crustal

Dynamics Project (CDP). The zero point of right ascension was established from optical positions of 28 quasars determined on the system of the FK5, with a precision of about 20 mas. Additional publications in preparation contain additional sources. These publications contain radio positions for 325 sources with an average precision better than 1 mas. By mid-1990 we had obtained VLBI observations of 422 sources, including 69 whose positions are not yet reduced.

In general the optical photographic program has been more difficult to accomplish. Not only have we had to contend with the expected problems of cloudy weather and bad seeing, but the most difficult problem has been the availability of time on large telescopes needed to take the prime focus plates for these faint objects. These large instruments are needed to obtain plates with measurable positions of 19th magnitude objects relative to intermediate reference stars which can be tied to the fundamental system. It is imperative that more large telescope time be made available for this type of astrometric observations. Furthermore the data accumulation on the southern hemisphere has been even more difficult. The number of sources where sufficient source plates, normally 2-4, are available is about 150 on the northern hemisphere and less than 50 on the southern hemisphere at present. We expect substantial improvements from adding the ESO Schmidt telescope to the program on the southern hemisphere although the precise astrometric reduction of Schmidt-type plates is much more complicated.

5.0 ARCHIVAL DATA BASES

Now that a significant fraction of the data for the establishment of the radio/optical reference frame has been obtained, the long term archiving of this data is underway. The primary goal of the archive is to have available at one site and in an accessible format a data base of all of the original data obtained at both the optical and radio wavelengths. This can be easily accomplished with current computer technology, which should continue to improve.

Having centralized data bases for the optical and radio data would allow observations to be rereduced at will. It would avoid all of the problems associated with the formation of compilation catalogs and give future investigators complete flexibility with reference frame work.

The largest current collection of radio data is the Geodetic data base currently maintained by the Crustal Dynamics Project. (We will not include radio observations before about 1980 which were not made at dual frequencies since they cannot be reduced satisfactorily for ionospheric effects.) The U.S. Naval Observatory and the Naval Research Laboratory are beginning a new program to archive and maintain the radio data base, including the CDP collection and the additional astrometric VLBI data available to date. This is described at this meeting in McCarthy et al. With the archive the radio reference frame can be redefined as necessary. Finally the archive will provide a collection of research data for the astrometric community.

A similar data base of optical data is being created at Hamburg Observatory and will be duplicated at the USNO. It will include the original optical plate measurements and CCD frames. With the data centralized and available on-line, any change in the reference star catalog, including the Hipparcos net, can be immediately incorporated by rereduction of the original data.

6.0 FUTURE PLANS

The future plans for the project include finishing the original catalog of 400 sources by 1991. The establishment of the data bases of the original data will allow easy updating of the reference frames in a consistent manner and system when needed, about every 5 years, and allow for continued research in the field without the hazards of depending on compilation catalogs.

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CONSISTENCY OF NUTATION MODELING IN RADIO SOURCE POSITION COMPARISONS

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ABSTRACT. Comparisons between sets of radio source coordinates determined from independent VLBI measurements show that rotational offsets can be as large as several milliarcseconds (mas), and are considerably larger than the positional uncertainties (≤1 mas). The 1 to 2 mas discrepancies remaining after removal of the rotational offsets indicate the present true level of accuracy. Analyses of DSN and CDP astrometric data are performed to explore the effects of alternative schemes of nutation modeling on such catalog comparisons. It is concluded that rotations between different source catalogs are minimized or eliminated if the nutation models employed are consistent. Intrinsic source coordinate discrepancies, however, remain at the 1 mas level.

1. Introduction

During the past several years, frequent comparisons have been made between radio source positions determined from Deep Space Network (DSN), Crustal Dynamics Project (CDP), and International Radio Interferometric Surveying (IRIS) VLBI data. Such comparisons serve to establish the level of systematic errors. Comparisons for recent data are considered in Section 2. Typically, the three-dimensional rotational offsets that are required to bring these three source catalogs into best coincidence range up to several milliarcseconds (mas), and are considerably larger than the positional uncertainties (≤1 mas). Section 3 presents analyses of DSN and CDP astrometric data that were performed in an effort to understand orientational and intrinsic source coordinate discrepancies. All parameter estimation employs the JPL VLBI modeling and estimation software MODEST. Variants of nutation modeling include estimates of daily angles, estimates of precession and nutation amplitudes, and adoption of the recent Zhu *et al.* (1990) revision of 1980 IAU nutation.

2. Comparisons of Source Catalogs

The IERS Annual Report for 1989 (IERS, 1990) reports a radio reference frame based on combinations of four independently determined sets of radio source coordinates. These are based on the VLBI observing programs of the Goddard Space Flight Center (Crustal Dynamics Project (CDP), Ma *et al.*, 1990), the Jet Propulsion Laboratory Deep Space Network (DSN) (Sovers *et al.*, 1988), the National Geodetic Survey (International Radio

Interferometric Surveying (IRIS), Robertson *et al.*, 1986), and the U.S. Naval Observatory (USNO, Eubanks *et al.*, 1990). Although the modeling of VLBI observables in all four projects largely conforms to the IERS Standards (IERS, 1989), there is one major exception. It has been known for a number of years that VLBI data clearly demonstrate the inadequacy of the IAU models for the Earth's nutation and precession (Herring *et al.*, 1986). Consequently routine VLBI data analysis includes some means of correcting this deficiency. This usually takes the form of estimating the nutation angles in longitude and obliquity for each 24-hour observing session. In order to fix the relative orientations of the terrestrial and celestial coordinate systems, one session is adopted as the "reference day", for which the 1980 IAU nutation angles are adopted and not allowed to vary. In the past, there has been no agreement on a common reference day between the various observing programs, partly due to scarcity of overlapping experiments. This has contributed to some obscurity in interpreting comparison of results, and thus in assessing the true accuracies of the VLBI-measured coordinates of radio sources.

Table 1. Typical Source Position Comparisons

Pair	N_s	A_1	A_2	A_3	$\chi_\nu{}^2$	Δ arc
IERS90 - IERS89	169	0.1	0.3	0.1	2.1	2.2
IERS88	163	0.6	0.0	0.5	2.0	2.9
IERS90 - JPL90	175	-0.4	-0.6	0.0	1.6	1.3
GSFC90	71	-1.3	-2.5	0.3	0.7	0.6
NGS90	69	-1.4	-0.9	0.2	0.9	0.9
USNO90	74	-1.6	-2.4	-0.0	0.7	2.0
JPL90 - GSFC90	52	-1.9	-3.1	0.7	1.8	1.2
NGS90	51	-2.2	-1.4	0.5	2.1	1.6
USNO90	61	-1.9	-2.9	0.3	1.6	3.2

To give some examples of comparisons of various radio reference frames, Table 1 presents three groups of results. The first two pairs are comparisons of the three IERS combined catalogs for 1988, 1989, and 1990. Here, N_s is the number of common sources, A_{1-3} are the x, y, z rotation angles (applied to the second member of each pair) that bring the two catalogs into best coincidence, χ^2_ν is the chi-square per degree of freedom in coordinate differences after removal of the rotational offset, and Δarc is the RMS arc length difference for all pairs of sources. Since special pains are taken to ensure stablity of the IERS catalogs from year to year, it is not surprising that the rotational offsets between the current one and the 1988 and 1989 versions are only a few tenths of mas. The second group of comparisons in Table 1 are between the current IERS catalog and its four constituents. Here it is obvious that a number of rotational offsets can exceed the mean formal source coordinate uncertainties, and can be larger than 2 mas. The last group of results shows comparisons between the four constituents of the IERS90 catalog, with rotations ranging up to 3 mas. The accuracy level of present VLBI source coordinates (for 50 to 60 of the best-determined objects) appears to be 1-2 mas. Normalized chi-squares exceed unity, indicating that the formal uncertainties may be underestimates of the true accuracies.

3. Effects of Alternative Nutation Modeling

The four component catalogs of IERS90, whose mutual rotational offsets were shown in the last three rows of Table 1, all employ different nutation models. For GSFC90, NGS90, and USNO90, daily nutations in longitude and obliquity were estimated, with separate reference days, while the JPL fit estimated precession and amplitudes of selected nutation components. Exploratory fits were performed in order to determine whether the large rotational offsets and RMS arclength discrepancies are connected to the disparate nutation modeling. Subsets of the observations entering the determinations of JPL90 (9000 delay-delay rate pairs, 1978-89) and GSFC90 (25000 intercontinental D-DR pairs, 1979-89) were studied. All parameter estimation employed the JPL VLBI modeling and estimation software MODEST (Sovers and Fanselow, 1987). Fits were performed to obtain positional coordinates of radio sources, time rates of station motion, and the usual clock and troposphere parameters (one every 3 hours for the latter). Corrections to the IAU nutation model took various forms: separate as well as identical nutation reference days for separate fits, estimates of nutation amplitudes and precession, and a nutation model fixed at the results of Zhu *et al.* (1990).

Table 2. Source Position Comparisons
(Variants of Nutation Modeling)

Pair	N_s	A_1	A_2	A_3	χ_ν^2	Δ arc
JPL (B) - GSFC(A)	36	1.9	-1.1	-0.8	1.5	1.2
GSFC (C) - GSFC(A)	49	4.2	1.3	-0.7	(0)	(0)
JPL (C) - GSFC(C)	36	0.2	0.3	0.0	1.2	1.2
JPL (S) - GSFC(S)	36	-0.3	0.5	0.2	6.2	1.9
JPL (Z) - GSFC(Z)	36	-0.0	1.2	0.3	6.9	2.1
JPL (N) - GSFC(N)	36	0.0	0.0	0.0	3.1	1.4

A = 1980.9 reference day
B = 1983.4 reference day
C = 1983.9 reference day
S = Standard 1980 IAU nutation model
Z = Zhu *et al.* nutation model
N = Nutation amplitudes + precession estimated

Table 2 summarizes the results of source coordinate comparisons between selected pairs of catalogs yielded by the analyses. The first pair of catalogs was generated by fits estimating daily nutation angles, with separate reference days 2.5 years apart. To give an indication of the quality of the source coordinates, the RMS formal uncertainties for the 36 sources in common are 0.6 mas for RA and 0.7 mas for declination for JPL(B), and (0.5, 0.6) mas for GSFC(A). Average observation epochs are substantially different for some sources: the RMS epoch difference is 1.6 years. The rotations A_1 and A_2 are qualitatively consistent with the known differences in corrections to $\Delta\psi$ and $\Delta\epsilon$ between 1980.9 and 1983.4.

To give an even clearer demonstration of the role of the nutation reference day in catalog comparisons, the second pair in Table 2 is a comparison of two catalogs derived from identical data and identical modeling, with the sole exception of a different nutation reference day. A large rotational offset is seen. As the last of fits estimating nutation angles, the third pair in Table 2 compares JPL and GSFC source positions with the same nutation reference day. This day is 18 November 1983, when both JPL and GSFC had observing sessions between North America and Europe. The angles A_i are seen to be much reduced; they amount to only fractions of the catalog uncertainties.

The last three JPL-GSFC pairs were generated by fits that fixed the nutation model as either the 1980 IAU or the Zhu *et al.* series, or estimated precession and selected nutation amplitudes. With nutation fixed either at the 1980 IAU (S) or Zhu *et al.* (Z) series, the rotational offsets are small. The exception is the angle A_2, which is caused by the precession model defect in combination with source coordinate observation epoch differences. The normalized chi-square values with either the S or Z nutation models show that fits to the VLBI data are not as good as those in which daily angles are estimated. The last line in Table 2 shows a comparison between source positions derived from fits in which the precession constant and a number of in- and out-of-phase nutation amplitudes are estimated. Three nearly mutually orthogonal coordinates (RA and declination of OJ 287, declination of CTD 20) are constrained to the IERS90 values, and small rotations around the x and y axes are estimated, as in the procedure of Steppe *et al.*, 1989. Such fits ensure that any rotational offsets are eliminated internally, and the resulting source catalogs show zero relative rotation.

In conclusion, significant rotations between different source catalogs are minimized or eliminated if care is taken to ensure that the nutation models used in the analyses are consistent. Intrinsic source coordinate discrepancies are presently of the order of 1 mas.

4. Conclusions

Rotational offsets between catalogs of independently determined source positions may be eliminated either by adopting the same model of nutation, or by estimating precession and nutation amplitudes in the original data analyses. If daily nutations in longitude and obliquity are to be estimated instead, then source catalog rotational offsets are minimized if the same reference day is adopted. In the latter case, the rotation is not completely eliminated, but reduced to well below the level of formal catalog uncertainties. With either scheme, for the sample data considered here, the coordinate discrepancies after removal of the rotation are of the order of 1 mas (somewhat larger than the RMS catalog uncertainties.) Normalized chi-squares exceed unity, indicating that the formal σs are underestimates of the true accuracies. As additional experiments reduce formal uncertainties to the 0.1 mas level, improved modeling of source structure (Charlot, 1990) and dynamic troposphere variations (Treuhaft and Lanyi, 1987) will be imperative to bring accuracy to the same level.

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STABILITY OF THE EXTRAGALACTIC REFERENCE FRAME REALIZED BY VLBI

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ABSTRACT. The 318 compact extragalactic radio sources with positions derived from dual frequency Mark III VLBI data acquired by the geodetic and astrometric programs of NASA, NOAA, NRL and USNO form a celestial reference frame with stability in orientation and relative position at the 1 mas level. This paper examines the reference frame realized using 461,000 observations from 1021 observing sessions between 1979 August and 1990 August in the NASA Crustal Dynamics Project VLBI data base. Catalogs of positions estimated from subsets of data (annual, seasonal, network) show differences in orientation typically less than 1 mas provided precession and nutation are adjusted using a reference day. For 17 sources with >5 year time span and >200 one-day position estimates, the rates of change of right ascension and declination are generally less than 5 mas/century, giving upper limits on real motion.

1. Introduction

A kinematically fixed celestial reference frame can be realized from VLBI (Very Long Baseline Interferometry) observations of compact extragalactic radio sources. Since most of these radio sources have optical counterparts (albeit faint) that can be measured in the conventional FK5 frame, this radio reference frame can anchor the stellar frame. The level of stability of the radio reference frame is thus of considerable importance. This paper discusses two aspects of stability: the global orientation of the frame and individual source positions relative to the frame. The current distribution of the data and progress in analysis are briefly reviewed.

This paper is an extension of the work described in Robertson *et al.* (1986), Ma *et al.* (1986), Ma *et al.* (1990), Ma (1990) and Russell *et al.* (in press). A parallel and completely independent effort is described in Fanselow *et al.* (1984), Sovers *et al.* (1988), Sovers (1990), and Sovers (this volume). The comparison and convergence of these independent catalogs indicate that milliarcsecond (mas) accuracy has been achieved.

2. Data

The 461,000 dual frequency Mark III observations used for this work can be divided into four categories from three geodetic programs and the combined astrometric effort: 1) 206,000 observations from the Crustal Dynamics Project (CDP) of the National Aeronautics and Space Administration (NASA) acquired at irregular intervals beginning in 1979 and using many networks around the globe, 2) 210,000 observations from the IRIS program of the National Oceanic and Atmospheric Administration (NOAA) acquired at regular intervals beginning in 1980 using a small number of networks, 3) 29,000 observations from the

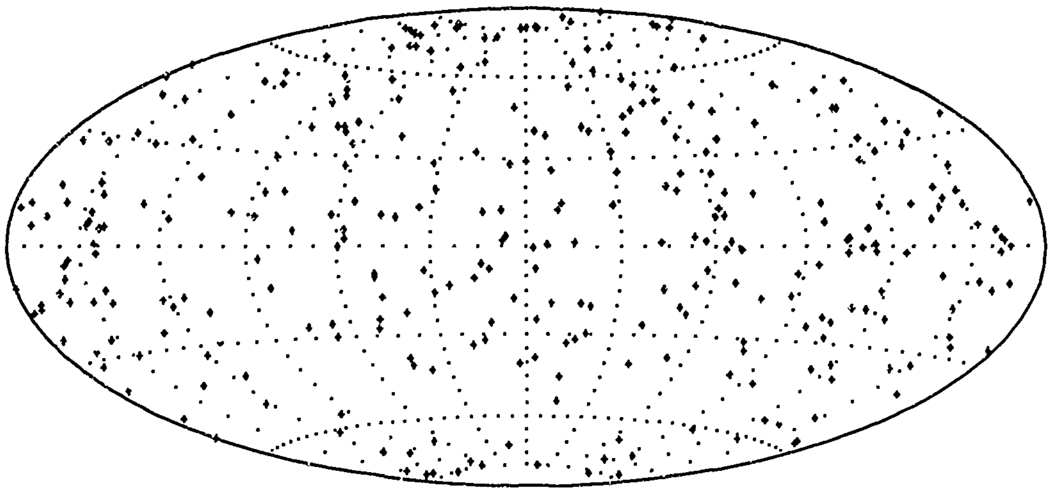

Figure 1. GLB677 sources on an Aitoff projection. 0 hr at right.

Navnet program of the US Naval Observatory (USNO) acquired at weekly intervals beginning in 1988 on a network normally including Hawaii, Alaska, West Virginia and Florida, and 4) 16,000 observations acquired by the CDP, Naval Research Laboratory (NRL), and NOAA astrometric programs from various northern and southern hemisphere networks in 48 sessions since 1980. Geodetic data from small (<2000 km) networks were not included. The distribution of the 318 sources with useful data in both observing frequencies is shown on an equal area projection in Figure 1. While the spatial distribution is quite uniform (see Ma, 1990), Table 1 shows that the distribution of the number of observations, the number of observing sessions and time spans is very uneven. Some sources used since the beginning of the geodetic programs have tens of thousands of observations from hundreds of sessions over many years while the majority of sources, which come from the astrometric programs, have very few observations from a small number of sessions. It is the intention of the NRL/USNO astrometric program. however, to observe each source on a continuing basis as scheduling permits. All these data are now contained in the CDP VLBI data base and can easily be re-analyzed.

Table 1. Data Distribution

observations	1-99	100-999	1000-9999	>10000
sources	231	39	34	14
sessions	1-2	3-9	10-99	>100
sources	130	96	53	39
span (days)	1-99	100-999	>1000	
sources	78	113	127	

3. Analysis

The data from the 1021 one-day sessions were analyzed as a whole and in subsets using the CALC/GLOBL software developed at Goddard. Details of the parametrization and estimation algorithm are given in Ma *et al.* (1990) and Ma *et al.* (in press). The *a priori* models generally follow the IERS (International Earth Rotation Service) standards although our theoretical VLBI delay model departs from the alternative IERS algorithm (McCarthy, 1989) at the few picosecond rms level. It should be noted that the development of the relativistic effects on VLBI observations is more complete than in the past. Adjusted parameters included source positions from the ensemble of data and station positions, celestial pole offsets in longitude and obliquity, and nuisance parameters (station clocks and residual atmospheres) for each session. The parametrization of the residual atmosphere (after calibration from real-time information) permitted better compensation for atmospheric fluctuations than previous astrometric work. The post-fit weighted rms residual from the GLB677 solution that generated the full catalog was 45 ps and the reduced χ^2 was 1.01. The standard errors of 261 sources are less than 1 mas in both right ascension and declination while another 39 sources have standard errors less than 3 mas.

Sovers (this volume) discusses different methods of adjusting the conventional precession/nutation (P/N) models. While the direct adjustment of the precession constant, nutation coefficients and free core nutation is possible from these data, this method requires the full time span to give a useful separation of the longer periods. In addition, it cannot handle variations which may not be at the modeled frequencies. Since this paper uses catalogs formed from short subsets of data, it is necessary to adopt the method of estimating daily pole offsets from the pole of a reference day. The position of the celestial ephemeris pole of the reference day, evaluated from the conventional P/N models, and an arbitrary choice of right ascension zero point (in this case the *a priori* right ascension of one source) define the orientation of the catalog axes with respect to the observed objects. The strength of the catalog, *i.e.*, the precision of the angles between sources, is improved as more days of data are added. All the sources observed on each day need not be the same but the full catalog must be constructed from days with overlapping source lists. A day with no sources in common with any other day cannot be included since the pole offset cannot be determined. It should be noted that the precision of source positions, as opposed to the precision of relative angles, depends on the precision with which the pole of the reference day can be determined from the data of the reference day. It is most desirable to use as the reference day one which has a large network and high intrinsic data quality. For this work the CDP observing session of 86 Nov 5 was arbitrarily chosen. The network included stations in Japan, Alaska, California, Massachusetts and Germany, and the pole determination had a standard error of 0.17 mas. There are, however, other CDP days with even better networks and data distribution.

4. Stability of Orientation

Tables 2 through 4 show the comparison of catalogs derived from subsets of data with the catalog constructed using all the data. It is important to understand that each subset also included the data from the reference day. The numbers of sources and observations refer to the subset catalog. The three angles A_1, A_2, and A_3 follow the conventions of Arias *et al.* (1988) and represent small, rigid rotations about the coordinate axes between catalogs. Sinδ is a systematic variation of declination with declination and $\Delta\delta$ is a declination offset. The large variation of source numbers in Table 2 and 3 arises from the irregular scheduling of astrometry sessions. A difference in observing strategy is reflected in the disparity of numbers for IRIS compared to Navnet. The slight drop in the number of observations for 1990 is caused by having data only through August.

It can be seen that the catalogs made from individual years, from different seasons, and from separate networks differ in orientation from the full catalog by less than 1 mas. The larger declination differences in the early years are probably caused by poor network distribution, predominantly single baseline sessions.

It is clear, however, that the observations of the reference day determine the orientation of a catalog even if the vast majority of the data are removed in time or use different networks. Similarly, incrementing a catalog with additional data will not significantly change its orientation. Errors in the P/N models which would cause spurious changes in position with time are entirely absorbed in the estimated pole offsets.

Table 2. Comparison with annual catalogs

Year	sources	obs	A_1 mas	A_2 mas	A_3 mas	$\sin\delta$ mas	$\Delta\delta$ mas
80	28	12000	.1	-.2	-.1	-1.7	1.8
81	52	10000	.1	-.2	-.2	-1.4	1.5
82	35	13000	.1	-.2	-.1	-1.5	1.6
83	45	15000	.1	-.4	-.4	-.2	.4
84	50	34000	.4	.2	.3	.1	.0
85	32	48000	.2	.4	.3	-.5	.4
86	43	52000	-.3	-.0	.2	-.1	-.1
87	166	68000	.3	.1	-.1	-.6	.6
88	213	72000	-.0	-.1	-.2	-.1	.1
89	185	73000	.2	.3	-.1	-.3	.0
90	146	66000	-.1	-.3	-.1	.1	-.1

Table 3. Comparison with seasonal catalogs

	sources	A_1 mas	A_2 mas	A_3 mas	$\sin\delta$ mas	$\Delta\delta$ mas
Winter	194	-.1	.1	.1	-.0	.1
Spring	243	-.0	-.1	.0	.2	-.2
Summer	230	-.0	-.1	-.1	-.3	.1
Autumn	101	.2	.2	.1	-.3	.3

Table 4. Comparison with network catalogs

	sources	A_1 mas	A_2 mas	A_3 mas	$\sin\delta$ mas	$\Delta\delta$ mas
IRIS	44	-.1	.0	-.0	-.1	.1
Navnet	80	-.0	-.3	.1	.3	-.4

If the conventional P/N model is not adjusted, both the orientation and the relative positions of the catalogs are compromised. Table 5 shows annual catalogs constructed using only the conventional P/N model compared to the full catalog with P/N adjusted. There are large variations in A_1 and A_2. The right ascension zero point constraint forces the much smaller variation in A_3. The column labeled fit is the post-fit weighted rms residual of the annual solution, to the left with only the conventional model and to the right with adjustment of P/N. Within the one-year interval the error in nutation causes considerable spurious variation in the source positions and a poor fit. The column labeled χ^2 gives the reduced χ^2 of the catalog comparison after the rotation has been applied. The larger value to the left from the comparison with annual solutions lacking P/N adjustment reflects distortions caused by P/N model errors and nonuniform temporal distribution of data for each source. The smaller value to the right shows that the difference between catalogs derived with adjusted P/N models is entirely a rotation.

Table 5. Comparison with annual catalogs lacking adjustment of precession/nutation

Year	A_1 mas	A_2 mas	A_3 mas	fit ps	χ^2
80	-3.1	-3.8	-.5	61/58	.95/.92
81	-1.6	-2.8	-.9	52/50	.98/.93
82	-1.0	-3.0	.0	55/53	1.01/.93
83	-1.1	-2.5	-.7	61/57	1.07/.97
84	- .9	-1.0	-.1	66/60	1.13/.97
85	- .8	-1.7	-.0	55/48	1.22/.94
86	- .2	-1.6	.3	51/45	1.18/.93
87	- .2	-1.0	.0	51/45	1.22/.94
88	-1.4	- .6	-.2	47/40	1.33/.97
89	-2.2	.3	.1	50/39	1.61/1.02
90	-4.4	- .3	-.2	54/44	1.63/1.09

5. Stability of Individual Source Positions

A small number of sources have been observed repeatedly in the geodetic programs. These include strong (> 1 Jy) radio sources with structure on the few mas scale that were used in the early years when the VLBI systems had less sensitivity. These sources have now been largely relegated to observations using small mobile antennas where sensitivity is still a problem but only on short baselines where structure is less important. Seventeen sources observed for more than 5 years in more than 200 sessions were used to study possible changes in position. Since some of these sources are now no longer used in long baseline networks, a catalog was constructed using both long baseline fixed antenna and short baseline mobile antenna data. The position of each test source was estimated relative to all the other sources for each day in which the test source was observed. Typical standard errors for these one-day estimates of position are < 1 mas from days with large networks. The time series of positions of two of these sources are shown

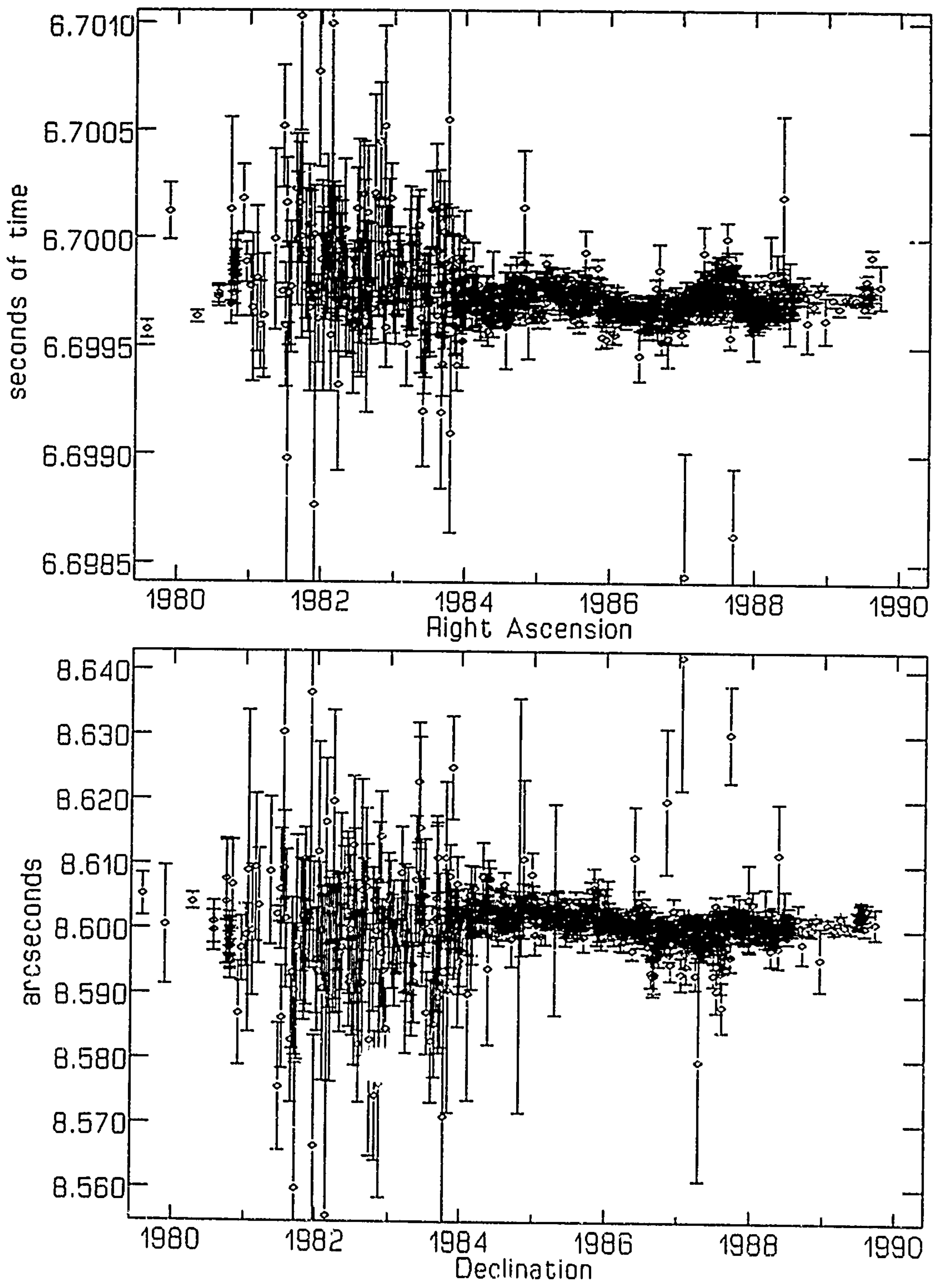

Figure 2. Estimated positions of 3C273B

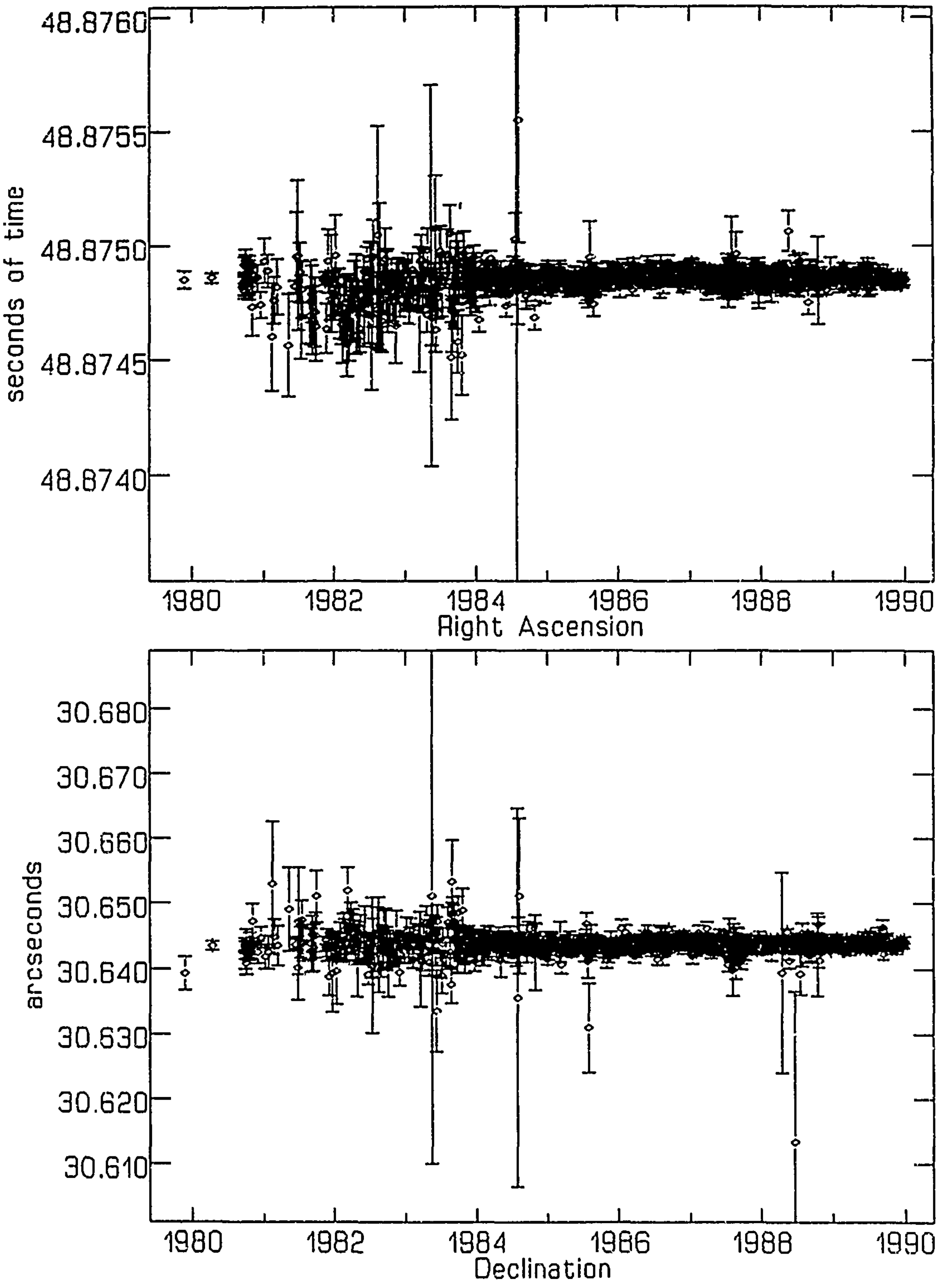

Figure 3. Estimated positions of OJ287

in Figures 2 and 3 with similar vertical scales. The source 3C273B is well known for variable structure while the source OJ287 is pointlike when mapped. Variations in position might arise from changes in structure, different networks from day to day, changes in observing schedules and geometry, *etc.* The actual changes are small. Tables 6 and 7 show the sources, the number of observing sessions, the total time span, the rate of change, the weighted rms scatter about the best-fit line, and the reduced χ^2 for right ascension and declination, respectively. The fit and χ^2 are generally worse for sources with much structure. While the rates are significantly different from zero in some cases, these should not be interpreted as actual proper motions of the real objects. Differential measurements of a pair of unrelated quasars separated by a small angular distance (Bartel *et al.*, 1986) place the upper limit on proper motion of the main components at 2 mas/century using essentially 2.5 years of data. More recent measurements reduce the upper limit by a factor of 4 (Bartel, 1990, personal communication). The rates given here should be viewed as upper limits of possible changes in this particular realization of the extragalactic frame and are illustrative of the stability of the positions of these sources, many of which have measurable, varying structure. Since the sources observed in the astrometric programs are generally fainter, it is expected that they are intrinsically smaller. Many have been selected because they have little or no structure visible on the long baselines.

Table 6. Rates of right ascension change

Source	sessions	span years	rate 0.1 ms/century	fit 0.1 ms	χ^2
0106+013	783	9	4.9 ± 0.8	.40	1.49
0212+735	678	8	-0.3 ± 1.7	.77	1.08
0229+131	539	5	-3.6 ± 0.7	.27	1.39
0234+285	276	9	-1.0 ± 0.8	.30	1.52
0420-014	233	5	5.4 ± 1.9	.32	2.48
0528+134	556	8	-2.7 ± 0.7	.28	1.37
0552+398	1068	10	-1.2 ± 0.4	.28	1.16
OJ287	811	10	0.7 ± 0.5	.26	1.52
4C39.25	771	10	3.2 ± 0.5	.28	1.40
3C273B	675	9	-4.6 ± 1.4	.56	3.13
OQ208	685	9	3.8 ± 0.6	.34	1.51
3C345	911	10	-9.1 ± 0.8	.46	2.12
1741-038	251	8	2.0 ± 2.0	.38	2.15
1803+784	629	6	5.8 ± 2.8	1.07	1.63
2134+004	504	10	2.3 ± 1.4	.50	1.86
VR422201	671	10	5.3 ± 0.6	.33	1.43
3C454.3	679	10	-5.7 ± 0.8	.35	2.39

Table 7. Rates of declination change

Source	sessions	span years	rate mas/century	fit mas	χ^2
0106+013	783	9	3.8 ± 2.7	1.13	1.09
0212+735	678	8	2.5 ± 0.7	.32	1.16
0229+131	539	5	1.2 ± 1.9	.70	1.21
0234+285	276	9	6.7 ± 1.6	.46	1.18
0420-014	233	5	9.1 ± 4.0	.68	1.26
0528+134	556	8	0.1 ± 2.3	.82	1.19
0552+398	1068	10	-3.4 ± 0.7	.45	1.16
OJ287	811	10	2.5 ± 1.1	.55	1.05
4C39.25	771	10	0.7 ± 0.9	.49	1.44
3C273B	675	9	-8.8 ± 4.0	1.33	2.33
OQ208	685	9	8.7 ± 1.6	.83	1.24
3C345	911	10	-6.2 ± 1.1	.60	1.31
1741-038	251	8	8.5 ± 5.5	.94	1.58
1803+784	629	6	3.4 ± 0.7	.28	1.17
2134+004	504	10	19.1 ± 5.2	1.76	1.65
VR422201	671	10	0.1 ± 1.1	.57	1.35
3C454.3	679	10	-2.7 ± 1.7	.63	1.20

6. Conclusions and Possibilities

The existing VLBI data can be used to realize an extragalactic radio reference frame with overall orientation stability at the mas level over periods of decades if not longer. Individual sources should be useful over similar intervals in terms of both position stability and continued visibility to VLBI. The current number of radio sources is limited by system sensitivity rather than by intrinsic number. Since the number of radio sources is proportional to (source flux density)$^{-1.5}$, the number can be considerably expanded with more sensitive instruments such as the VLBA or QUASAR. With 800 second observations on the VLBA, it could be possible to extend the radio reference frame to one source per square degree.

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DYNAMICAL REFERENCE FRAME AND SOME ASTRONOMICAL CONSTANTS FROM PLANETARY OBSERVATIONS 1769-1988 YY

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Modern planetary theory may be considred as a carrier of 4-dimensional dynamical reference frame which is to be confronted with stellar (or quasar) based fundamental frame. Of most interest is the problem of existance of secular trends between the above systems. We investigated the problem by discussing a vast set of planetary observations of different types. The set includes ranging observations of the inner planets (up to 1988 y), USNO meridian observations, transits, etc. The main resuls are the following:

1. We have confirmed our earlier findings on the corrections dT to the adopted system of the differences between the dynamical ("ephemeris") time and Universal Time :

   dT= -12.9±1.3 sec/cy (from transits)

   dT=-14.5±2.1 sec/cy (from USNO meridian obs.)

2. The time derivative $\dot{g}$ of the gravitational constant g (or a secular trend between the atomic and dynamical time) was estimated:

   $\dot{g}=0.32\pm0.45\ /10^{-11}$

3. By investigating of secular variations of the solar and planetary longitudes a correction to Newcomb's value of the constant of precession has been obtained (which is independet of errors in the proper motions of stellar catalogues) :

   dp=0.46±0.13 "/cy

4. For distance scaling factor AU of the dynamical frame the new estimate is

   AU=149597870.62±0.18 km

THE ORIENTATION OF THE DYNAMICAL REFERENCE FRAME

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ABSTRACT. We summarize the current status of the JPL ephemerides, focusing on the various data types utilized, especially the impact of the modern ranging data, and the resulting accuracies obtained. The dynamical equinox, as determined from the analysis of Lunar Laser Ranging data, is determined with an accuracy of 5 mas and the obliquity to a 2 mas level in ~1983, the weighted center of data. Knowledge of the lunar and planetary positions with respect to the dynamical equinox degrades to 10 mas at J2000. Twenty years of LLR data allow for the separation of the 18.6 yr nutation terms from the precession constant. The correction to IAU precession is found to be -2.7 ± 0.4 mas/yr, while the 18.6 yr nutation of the pole is 3.0 ± 1.5 mas larger in magnitude than the 1980 IAU series. The necessity of different reference systems and the accurate knowledge of the interconnections between frames is addressed.

1. Introduction

This paper highlights recent JPL activities in the area of lunar/planetary ephemerides and related investigations. Section 2 discusses current JPL ephemerides with emphasis on the data sets considered, the orientation of the ephemerides, and ephemeris uncertainties; for a fuller discussion, the reader is referred to *Williams and Standish* (1989). The corrections to the IAU nutation and precession constants are presented in Section 3; a more detailed account is given by *Williams et al.* (1990). Reference frame issues are addressed in Section 4 (see *Dickey*, 1989 for a complete discussion); concluding remarks and a summary are given in Section 5.

2. Ephemeris Considerations

2.1 OBSERVATIONAL DATA

The creation of modern ephemerides is quite a different process from a few decades ago. We can now safely assume that our equations of motion properly describe the physical laws of gravitation and that all of the significant forces which affect the motions of the planets and Moon are known. Furthermore, we are no longer overwhelmed with the process of integrating the equations of motion. Now modern computers do the integration numerically without concern about problems such as non-converging expansions, neglected terms, and truncated series. Numerical integration programs have been tested; they provide sufficient accuracy.

Full concentration is now given to the observational data: the accuracy, variety, coverage and the reduction processes — those are the most important ingredients in creating modern-day ephemerides.

Table 1 lists the observational data now being fit to generate ephemerides. The table shows many additions since the creation of JPL's DE200/LE200 in 1980 as described by *Standish* (1990). These further sets of data are in the form of additions to standard data types as well as completely new types of observations. The acquisition and utilization of observational data continues to be the most vital part of the ephemeris creation process.

As a result of the increasing strength of the observational data, JPL is presently engaged in the creation of its next major set of planetary and lunar ephemerides, expected to be available in late 1990. Significant improvements over DE200 are expected in the ephemerides of each planet as well as the Moon.

TABLE 1. THE SOURCES OF THE OBSERVATIONAL DATA

		Sun	Mer	Ven	Mar	Jup	Sat	Ura	Nep	Plu	Moon
Optical transits	1911–	S	M	V	M	J	S	U	N		
Photoelectric transits	1982–				M	J	S	U	N	P	
Astrolabe	1969–				M	J	S	U			
Radar ranging	1964–		M	V	M						
Mariner 9 Ranges	1971–72				M						
Mariner 10 Ranges	1974–75		M								
Viking Lander Ranges	1976–82				M						
Radio Astrometry	1983–					J	S	U	N		
Ring Occultations	1977–							U			
Disk Occultations	1968–								N		
Pioneer Tracking Data	1973–80					J	S				
Voyager Tracking Data	1979–89					J	S	U	N		
Pluto Astrometry	1914–									P	
Lunar Laser Ranging	1969–										M

2.2 ORIENTATION OF THE EPHEMERIDES

Two important features of the ephemerides deserve mention, especially since these are neither well-known nor intuitively apparent.

Inertial mean motions: The mean motions of the four inner planets and the Moon are determined from the ranging observations—for example, lunar laser, radar, and spacecraft—not by the optical observations; there are a number of explanations designed to show why (see, e.g., *Standish and Williams*, 1990). The most accurate planetary ranges are from Earth to Mars and the most accurate inertial mean motions are for these two planets (0$''$.01/cty as given in Table 2).

Dynamical equinox and obliquity: The lunar laser ranging observations are highly sensitive to the direction of the pole of the Earth's rotation and, therefore, to the celestial equator of date. Data over an extended period of time will provide determinations of the Earth's orientation. The observations are also highly sensitive to the solar perturbations on the lunar orbit, which effectively cause the lunar orbit to precess along the ecliptic. Thus, the location of the ecliptic of date is also determined from the lunar observations. As a result, for successful data reduction, the true equator must be represented accurately in the analysis program and the true ecliptic in the ephemerides, most importantly over the time span of the lunar laser ranging data. Finally, one may locate the mean ecliptic at an epoch by analyzing the true ecliptic, given by the ephemeris of the Earth-Moon barycenter orbit about the Sun. This determination has been done by *Standish* (1982) and by *Chapront-Touzé and Chapront* (1983) in order to extract the obliquity and dynamical equinox. The accuracy of the obliquity determination is better than 2 mas and, in the early 1980s, the Moon and planets are known with respect to the dynamical equinox to 5 mas. Recent obliquity determinations are close to the value in *Standish* (1982).

Thus, the ephemerides of the inner four planets and the Moon are created without reference to objects outside the solar system. The mean motions with respect to inertial space and the orientation to the mean equator and the dynamical equinox are determined strictly from ranging measurements coupled with dynamics.

2.3 EPHEMERIS UNCERTAINTIES

An analysis of the ephemeris parameter uncertainties by *Williams and Standish* (1989) has been made by considering the accuracies of the relevant observational data and by considering how sensitive such observations are to changes in each of the parameters. Those results are used here in producing Table 2, where the estimates are intended to be realistic uncertainties. We also include our estimates of the orbits of the outer planets, realizing that the uncertainty of extrapolating into the future is largest for the outermost planets.

TABLE 2. ESTIMATED EPHEMERIS ERRORS

	Moon	Merc & Ven	Mars	Jup...Nep	Pluto
longitude					
wrt earth in 1980	[0."001]	0."002	0."00002	0."05	0."5
wrt earth in 1990	[0."001]	0."02	0."001	0."05	0."5
wrt 1980 dyn eq	0."005	0."05	0."005	0."05	0."5
wrt 2000 dyn eq	0."01	0."01	0."01	0."05	0."5
latitude	0."002	0."02	0."0005	0."05	0."5
mean motion	0."04/cty 1."/cty^2	0."2/cty	0."01/cty	0."5/cty	2"/cty

3. Precession and Nutation

Twenty years (August 1969 to December 1989) of lunar laser ranges have been analyzed to extract corrections to the luni-solar precession constant and the 18.6 yr nutation coefficients. The two-decade span of data permits the precession and 18.6 yr nutations to be separated; the largest correlation between them being –0.64. The most recent ranges are an order of magnitude more accurate than the ranges from the early 1970s, and the data are weighted accordingly.

The solutions given in Table 3 are from *Williams et al.* (1990), where additional details may be found. The nominal precession and nutation expressions are given by the respective IAU working groups (*Lieske et al.*, 1977, *Seidelmann*, 1982). The table gives the corrections to these standard values.

Three solutions are presented. Case A is a solution for the precession, in-phase and out-of-phase 18.6 yr nutation coefficients, and the two in-phase annual components. Case B is based on the recent improvements to the nutation theory by *Kinoshita and Souchay* (1990). Their corrections to the 9.3 yr coefficients and the in-phase 18.6 yr obliquity coefficient were adopted; the out-of-phase 18.6 yr coefficient corrections were constrained to a fixed ratio, additional in-phase 18.6 yr coefficient corrections were constrained to a ratio which depends on the value of the precession correction; and the annual corrections were fixed at values similar to those found by VLBI. Case C is based on the recent improvements to the nutation theory by *Zhu et al.* (1990). Their corrections were adopted for the 18.6 yr (in-phase obliquity), 9.3 yr, annual (in-phase), semi-annual, and semi-monthly coefficients. Analogous to Case B, two constraints were set up for ratios of the 18.6 yr coefficients.

The three solutions of Table 3 show that the precession constant needs to be decreased and the magnitude of the 18.6 yr nutation needs to be increased. The out-of-phase corrections of the 18.6 yr terms are similar to the size expected from ocean tide influences (*Wahr and Sasao*, 1981, *Zhu et al.*, 1990), but the errors only permit us to say that it is suggestive. Case A indicates that the annual correction is detected. An additional solution with the two annual components constrained to be equal gives 1.8 ± 0.5 mas. If the recent theoretical improvements to nutations are an accurate representation of the motion of the Earth's axis of rotation, then the solutions of Cases B and C are expected to be improvements over the Case A solution. The uncertainties given in the table are intended to be realistic estimates; the formal errors are smaller.

The four LLR-derived corrections to the 18.6 yr coefficients are in good agreement with the recent VLBI results of *Herring et al.* (1990), and their precession correction of −3.2 ± 1.3 mas/yr is compatible with our value. This volume (*Charlot et al.*) has a combined LLR and VLBI solution which adds the strength of the LLR data for precession to the accuracy of the VLBI data for short periods.

The −2.7 mas/yr correction to the luni-solar precession constant gives a new value of 50.3851"/yr at J2000. It is the luni-solar precession rather than the general precession which is determined. The dynamical equinox is better determined during the past decade than at J2000. The uncertainty of the precession constant contributes to the 10 mas uncertainty at J2000 in Table 2.

TABLE 3. CORRECTIONS TO PRECESSION AND NUTATIONS

	Case A	Case B	Case C
Precession (mas/yr)	−2.8 ± 0.5	−2.7 ± 0.4	−2.7 ± 0.4
In-phase 18.6 yr terms (mas)			
$\Delta\varepsilon$	0.4 ± 1.9	3.1 ± 1.1	3.0 ± 1.2
$\sin\varepsilon\,\Delta\psi$	−3.4 ± 1.6	−3.4 ± 1.5	−3.1 ± 1.5
Out-of-phase 18.6 yr terms (mas)			
$\Delta\varepsilon$	1.2 ± 1.9	1.4 ± 1.6	1.3 ± 1.6
$\sin\varepsilon\,\Delta\psi$	1.5 ± 1.9	1.0 ± 1.2	1.0 ± 1.3
In-phase annual terms (mas)			
$\Delta\varepsilon$	1.6 ± 1.1	1.8*	1.79*
$\sin\varepsilon\,\Delta\psi$	2.0 ± 0.8	1.8*	1.89*

*Correction fixed at this value

4. Reference Frame Considerations

Each technique observing a particular class of objects can be expected to establish its own reference frame (see Table 4). Contemporary astronomy has led to the development of three types of celestial coordinate systems: the optical frame (e.g. FK4/FK5) based on the positions of galactic stars,

TABLE 4. FRAME DETERMINATIONS

FRAME	TECHNIQUE	TARGET
* Ephemeris	* Planetary/Spacecraft Ranging * Lunar Laser Ranging	* Planets * Moon
* Optical	* Optical Astrometry	* Stars, Sun and Planets
* Radio	* Very-Long-Baseline Interferometry	* Quasars, Radio Stars, Pulsars, and Inter-planetary Spacecraft
* Satellite	* Satellite Laser Ranging * Doppler * GPS	* Earth-Orbiting Satellite * Transmitting Satellites * GPS Satellites

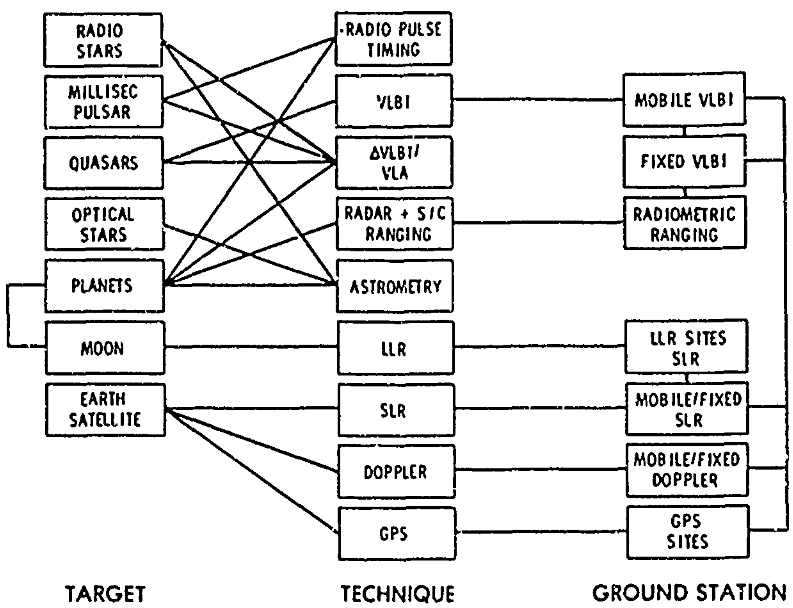

FIG. 1: Connections Between Reference Systems

planets, and the Sun; the planetary/lunar ephemeris frame based on the major celestial bodies of the solar system; and the radio frame constructed from observations of extragalactic sources/quasars. It should be noted that the radio, optical, and ephemeris frames generate complementary terrestrial frames as well. Other terrestrial frames are developed through the analysis of the data from Earth-orbiting satellites [e.g. GPS (Global Positioning System), Doppler, and laser-reflecting satellites such as LAGEOS]. The terrestrial frames must consider local deformations as well as tectonic motion; for example, most of the sites are moving at rates of several cm/year. The celestial and terrestrial coordinate systems from a single technique and class of target are related through adopted constants and definitions. Each frame is rotated with respect to the others, and this offset may be time variable (e.g. the radio vs the FK4 frame).

Measurements are inherently more accurate in their "natural" frame and hence should always be reported as such. However, to benefit from the complementarity of the various techniques, knowledge of the frame interconnections (both the rotation and the time-variable offset) is essential; these are summarized in Fig. 1 and Table 4 (after *Dickey*, 1989). Recent activity in this area is indicated by the number of boxes and lines in Fig. 1 (the accuracy cut-off here is 0.05 arcsec). The lunar/planetary system, integrated in a joint ephemeris, is by its nature unified by the dynamics. The radio frame is tied to the ephemeris frame in several ways: one is via differential VLBI measurements of planet-orbiting spacecraft and angularly nearby quasars; another is the determination of a pulsar's position in the ephemeris frame (via timing measurements) and the radio frame [via radio interferometry (VLA)]. VLA observations of the outer planets (Jupiter, Saturn, Uranus and Neptune) or their satellites provide an additional tie between these two frames. As for an optical-radio frame tie, a preliminary link has been established between the FK5 optical frame and the JPL radio reference frame via the differential VLBI measurement of optically bright radio stars and angularly nearby quasars coupled with comparisons of their optical positions, and also by the use of the optical positions of quasars. The optical and ephemeris frames are tied by optical observations of the planets. For a fuller discussion of these topics and referrals to references in the literature, the reader should see *Dickey* (1989).

5. Concluding Remarks

A summary of the current ephemeris developments at JPL was presented, stressing data types utilized and the accuracies obtained. Ranging observations are the dominant data for the inner four planets and the Moon; the most accurate ranges (and orbits) are for Earth, Mars, and the Moon. Optical data are significant for only the five outermost planets. The inclusion of Voyager tracking data and radio astrometry provide major improvements in the outer planet ephemerides.

Lunar laser ranging, being sensitive to the planes of the ecliptic, the lunar orbit, and the equator determines their mutual orientation to an accuracy of 2 mas and hence, the dynamical equinox (the intersection of the mean equator and the mean ecliptic) to an accuracy of 5 mas early in the 1980s. The uncertainty of the Moon and planets with respect to the dynamical equinox degrades to 10 mas at J2000. The dynamical equinox is used as our reference point for origin of the right ascension. The analysis of two decades of lunar laser ranging data has permitted the separation of the precession constant from the 18.6 year nutation terms. The correction to the IAU-adopted precession constant is -2.7 ± 0.4 milliarcseconds/yr, whereas the 18.6 yr nutation of the pole is found to be 3.0 ± 1.5 mas larger in magnitude than the 1980 IAU series. The nutation series corrections are consistent with theoretical expectations.

Reference frame issues were addressed briefly. It was stressed that each technique acquires measurements in its own reference frame, with results being most accurate in its own "natural" frame. To benefit from the complementarity of the various techniques, knowledge of the frame interconnections is essential.

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REPORT OF THE IAU WORKING GROUP ON THE THEORY OF NUTATION

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ABSTRACT. Modern observations of celestial pole offsets indicate that small but significant corrections to the current IAU theories of nutation and precession can be detected. These corrections are of the order of a few milliseconds of arc. To meet the needs of those who require millisecond of arc accuracy in reference frame applications, the International Earth Rotation Service (IERS) publishes in weekly and monthly bulletins the most recent observations, along with predictions of these corrections. These appear to meet the needs of users. In view of the fact that the geophysical community has not adopted a model which describes the nutation and precession theory and that the current procedure of the IERS meets the need of high-accuracy users, there does not appear to be a need to identify a new IAU nutation theory at this time.

1. Introduction

The Working Group on Reference Systems (WGRS) was instituted at the IAU General Assembly in Baltimore in 1988. The Subgroup on Nutation was organized then to examine the various approaches which can be, and have been, taken towards the characterization and specification of this motion of the Earth. From this examination were to come recommendations regarding procedures to be used in describing this phenomenon. A new, analytical closed theory of nutation was not necessarily the goal, although such an effort was not ruled out. The working group was charged with the responsibility of defining accessible, practical procedures regardless of the theoretical and observational foundations of the adopted approach to the problem in both its geophysical and astronomical aspects.

The International Earth Rotation Service (IERS) Standards (McCarthy 1989) recommends the use of the 1980 IAU Nutation Theory (Seidelmann 1982) based on the Wahr model (Wahr 1981). The IERS Central Bureau and the IERS Sub-bureau for Rapid Service and Predictions publish the observed corrections to the 1980 IAU nutation model and the predictions of these corrections. The predictions are based on a model utilizing derived corrections at nutation frequencies to extrapolate an improved nutation series.

The IAU 1980 Nutation Series is based on Kinoshita's (1977) rigid Earth theory, using Newcomb's Theory for the motion of the Earth, Brown's Theory for the motion of the Moon and the IAU 1976 System of Astronomical Constants. These theoretical coefficients are modified on the basis of Wahr's theory (1981) in the ratio of the amplitudes of each circular nutation relative to a realistic Earth model and to a rigid Earth model. This ratio is computed for an elliptical, rotating, elastic and oceanless

Earth with a fluid, hydrostatically pre-stressed core. While this nutation series has proven adequate for many astronomical reductions, the introduction of high-precision VLBI and LLR observing techniques has revealed some significant inadequacies (Herring *et al.* 1986, Herring 1987, Zhu *et al.* 1990, McCarthy and Luzum, 1990).

2. Considerations

Recently Kinoshita and Souchay (1990) have completed an improvement to Kinoshita's theory (1977) for a rigid Earth, using modern theories for the motion of the Earth and Moon, current values for the astronomical constants including the effect of the planetary perturbations. To enable the accurate reduction, on a common basis, of VLBI, LLR and SLR observations it would be desirable to arrive at an acceptable model of the Earth attempting to incorporate all known contributions at the one tenth milliarcsecond level. It should include the contribution of the non-hydrostatic core flattening as well as the oceanic and anelastic effects which have been shown to be significant and possible inner core effects. No geophysical model has become generally accepted although research in this area has been active (Molodenskiy and Kramer 1987, Mathews *et al.* 1989, Dehant 1990, Zhu *et al.* 1990,).

Until such a model is accepted it becomes necessary to provide standardized procedures to allow for systematic treatment of nutation for astronomy in general as well as the reduction of VLBI, LLR and SLR observations. It is the consensus of this Working Group that these corrections should be values based on geophysical models which produce corrections consistent with the observed results. This Working Group, then suggests the following recommendation.

3. Recommendation

The IAU Working Group on the Theory of Nutation

RECOGNIZING that a generally accepted non-rigid Earth theory of nutation including all known effects at the one tenth milliarc-second level is not yet available

RECOMMENDS

1. That those requiring accuracy of the nutation angles (ϵ or $\psi \sin\epsilon_0$) numerically greater than $\pm 0\rlap{.}''002$ should continue to use the 1980 IAU Nutation Theory (Seidelmann 1982).
2. That those requiring nutation angle descriptions more accurate than $\pm 0\rlap{.}''002$ should make use of the Bulletins of the IERS which publish observations and predictions of the celestial pole offsets which are accurate to about $\pm 0\rlap{.}''0006$ for a period of up to six months.
3. That the IUGG be encouraged to develop and adopt an appropriate Earth model to serve as the basis for a new IAU Theory of Nutation.

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THE ZMOA-1990 NUTATION SERIES

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ABSTRACT. We present a new nutation series for the Earth (ZMOA-1990) based on (1) the rigid Earth nutation series developed by *Zhu and Groten* [1989], (2) the normalized response for an elastic, elliptical Earth with fluid-outer and solid-inner cores developed by *Mathews et al.* [1990], and (3) corrections for the effects of ocean tides and anelasticity, computed to be consistent with the *Mathews et al.* [1990] normalized response function. In deriving this series, only two parameters of the geophysical model for the Earth have been modified from their values computed with PREM: the dynamic ellipticities of the whole Earth, e, and of the fluid outer core, e_f. The adopted values for these parameters, determined from the analysis of very long baseline interferometry (VLBI) data, are e=0.00328915 which is about 1% higher than the value obtained from PREM and 6×10^{-5} times larger than the IAU adopted value, and e_f=0.002665 which is 4.6% higher than the PREM value. The above values were obtained from an adjustment of -0.3 "/cent to the IAU-1976 luni-solar precession constant for e, and from the amplitude of the retrograde annual nutation for e_f. The ZMOA-1990 nutation series agrees with estimates of the in-phase and the out-of-phase nutation amplitudes obtained from VLBI data to within 0.5 mas for the terms with 18.6 year period, and to better than 0.1 mas for terms at all other periods except for the out-of-phase terms with annual period (differences 0.39 mas, retrograde, and 0.13 mas, prograde), and for the in-phase term with prograde 13.66 day period (difference -0.25 mas).

1. Introduction

It has been apparent for about 5 years now that the IAU-1980 nutation series [*Seidelmann*, 1982] is inadequate for defining the motion of the Earth in inertial space with sufficient precision for the analysis of modern spaced-based geodetic data [see e.g., *Herring et al.*, 1986]. There are a number of deficiencies in the IAU-1980 series arising both from the assumptions made in the geophysical models used in its derivation and from the nutation series of the rigid body that is convolved with the normalized response function of the Earth. We present here a new nutation series which overcomes, to a large degree, the deficiencies in the IAU-1980 series. This new series is based on the rigid Earth nutation series of *Zhu and Groten* [1989] with a slight modification discussed below, and the normalized response for an elastic, elliptical Earth with fluid-outer and solid-inner cores developed by *Mathews et al.* [1990*a* and *b*]. Effects of mantle anelasticity and ocean tides are explicitly included in this new series with the specific corrections used based on a re-computation of the models used by *Wahr and Sasao* [1981] for ocean tides and *Wahr and Bergen* [1986] for anelasticity. This new series, which we refer to as ZMOA-1990 (Zhu, Mathews, Oceans and Anelasticity), is discussed fully in *Herring et al.* [1990], and we restrict our discussions here to specific aspects of it. We also give the new series and two variants in their entirety.

The aim of the *Mathews et al.* [1990*a* and *b*] and the *Herring et al.* [1990] sequence of papers was to assess the geophysical signals which could be studied using nutations and to assess the agreement between an almost purely geophysical model for the nutations and the "observed" series obtained from the analysis of very long baseline interferometry (VLBI) data. Only two parameters of the Earth derived from the Preliminary Reference Earth Model (PREM) of *Dziewonski and Anderson* [1981] are modified in computing the normalized response function of the Earth for the ZMOA-1990 series: namely, the dynamical ellipticities of the whole Earth, e, and of the fluid core, e_f. It has been known for sometime that values of e computed assuming that the Earth is in hydrostatic equilibrium and using the radially symmetric density distribution from seismic-based Earth models are not consistent with the value required to explain the luni-solar precession constant. The difference is usually about 1%. We have further modified the standard astronomical value of e by a fractional change 6×10^{-5} to be consistent with a correction of -0.3 "/cent to the standard value of the precession constant that is now being obtained from the analysis of VLBI data (-0.32±0.13 "/cent) [*Herring et al.*, 1990] and lunar laser ranging data (-0.27±0.04 "/cent) [*Williams et al.*, 1990]. The need for a modification of the value of the dynamic ellipticity of the fluid core computed assuming that the Earth is in hydrostatic equilibrium is also well established from nutation studies [*Gwinn et al.*, 1986]; from seismology [*Hager et al.*, 1985; *Morelli and Dziewonski*, 1987], and from gravimetry [*Neuberg et al.*, 1987]. The 4.6% increase in the value of e_f used in the ZMOA-1990 series is consistent with values obtained in previous analyses. The models for the effects of ocean tides and mantle anelasticity have also been re-computed to be consistent with e_f and with the 0.01 mas truncation level used in the *Zhu and Groten* rigid Earth series and ZMOA-1990. The comparison of the VLBI derived series and the geophysical series did influence the choice of anelasticity models in that the QMU model of *Sailor and Dziewonski* [1978] is favored over the β model of *Sipkin and Jordan* [1980] because the former is more consistent with the VLBI derived series. The ZMOA-1990 series with ocean tide and anelasticity effects included in the series coefficients is given in Table 1. In addition to containing more terms than the IAU-1980 one, this series also contains out-of-phase terms which are necessitated by the incorporation of anelasticity and ocean tide contributions.

TABLE 1. The ZMOA-1990 nutation series.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
0	0	0	0	1	-6798.383	-17206.34	-17.43	9205.11	0.90	1.90	0.98
0	0	2	-2	2	182.621	-1317.18	-0.16	573.06	-0.31	-1.41	-0.46
0	0	2	0	2	13.661	-227.56	-0.02	97.79	-0.05	-0.02	0.01
0	0	0	0	2	-3399.192	207.50	0.02	-89.77	0.05	-0.05	-0.02
0	1	0	0	0	365.260	147.60	-0.36	7.30	-0.02	-0.12	0.09
1	0	0	0	0	27.555	71.11	0.01	-0.67	0.00	-0.03	0.03
0	1	2	-2	2	121.749	-51.69	0.12	22.44	-0.07	-0.06	-0.02
0	0	2	0	1	13.633	-38.71	-0.04	20.06	0.00	-0.01	0.00
1	0	2	0	2	9.133	-30.13	0.00	12.89	-0.01	0.03	0.01
0	-1	2	-2	2	365.225	21.61	-0.05	-9.60	0.03	0.02	0.01
1	0	0	-2	0	-31.812	-15.70	0.00	-0.13	0.00	-0.01	-0.01
0	0	2	-2	1	177.844	12.91	0.01	-6.98	0.00	0.02	0.00
-1	0	2	0	2	27.093	12.35	0.00	-5.33	0.00	0.01	0.00
1	0	0	0	1	27.667	6.32	0.01	-3.32	0.00	0.00	0.00
0	0	0	2	0	14.765	6.36	0.00	-0.12	0.00	0.00	0.00
1	0	2	2	2	9.557	-5.96	0.00	2.55	0.00	0.00	0.00
-1	0	0	0	1	-27.443	-5.81	-0.01	3.15	0.00	0.00	0.00
1	0	2	0	1	9.121	-5.15	0.00	2.64	0.00	0.00	0.00
2	0	0	-2	0	205.892	4.79	0.00	0.05	0.00	0.00	0.00
-2	0	2	0	1	1305.479	4.60	0.00	-2.43	0.00	0.00	0.00
0	0	2	2	2	7.096	-3.84	0.00	1.64	0.00	0.00	0.00
2	0	2	0	2	6.859	-3.09	0.00	1.32	0.00	0.00	0.00
2	0	0	0	0	13.777	2.93	0.00	-0.06	0.00	0.00	0.00
1	0	2	-2	2	23.942	2.86	0.00	-1.24	0.00	0.00	0.00
0	0	2	0	0	13.606	2.59	0.00	-0.05	0.00	0.00	0.00
0	0	2	-2	0	173.310	-2.19	0.00	-0.01	0.00	0.00	0.00
-1	0	2	0	1	26.985	2.05	0.00	-1.07	0.00	0.00	0.00
0	2	0	0	0	182.630	1.68	-0.01	0.02	0.00	0.00	0.00
0	2	2	-2	2	91.313	-1.58	0.01	0.69	0.00	0.00	0.00
-1	0	0	2	1	31.961	1.52	0.00	-0.80	0.00	0.00	0.00
0	1	0	0	1	385.998	-1.40	0.00	0.86	0.00	0.00	0.00
1	0	0	-2	1	-31.664	-1.29	0.00	0.70	0.00	0.00	0.00
0	-1	0	0	1	-346.636	-1.28	0.00	0.64	0.00	0.00	0.00
2	0	-2	0	0	-1095.175	1.10	0.00	0.01	0.00	0.00	0.00

TABLE 1 Continued. The ZMOA-1990 nutation series.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
-1	0	2	2	1	9.543	-1.02	0.00	0.52	0.00	0.00	0.00
1	0	2	2	2	5.643	-0.77	0.00	0.32	0.00	0.00	0.00
0	-1	2	0	2	14.192	-0.76	0.00	0.33	0.00	0.00	0.00
0	0	2	2	1	7.088	-0.66	0.00	0.34	0.00	0.00	0.00
1	1	0	-2	0	-34.847	-0.74	0.00	-0.01	0.00	0.00	0.00
0	1	2	0	2	13.168	0.76	0.00	-0.33	0.00	0.00	0.00
-2	0	0	2	1	-199.840	-0.58	0.00	0.31	0.00	0.00	0.00
0	0	0	2	1	14.797	-0.64	0.00	0.33	0.00	0.00	0.00
2	0	2	-2	2	12.811	0.65	0.00	-0.28	0.00	0.00	0.00
1	0	0	2	0	9.614	0.66	0.00	-0.02	0.00	0.00	0.00
1	0	2	-2	1	23.858	0.58	0.00	-0.30	0.00	0.00	0.00
0	0	0	-2	1	-14.733	-0.50	0.00	0.28	0.00	0.00	0.00
0	-1	2	-2	1	346.604	-0.48	0.00	0.28	0.00	0.00	0.00
2	0	2	0	1	6.852	-0.53	0.00	0.26	0.00	0.00	0.00
1	-1	0	0	0	29.803	0.47	0.00	-0.01	0.00	0.00	0.00
1	0	0	-1	0	411.784	-0.46	0.00	-0.07	0.00	0.00	0.00
0	0	0	1	0	29.531	-0.40	0.00	0.01	0.00	0.00	0.00
0	1	0	-2	0	-15.387	-0.44	0.00	-0.01	0.00	0.00	0.00
1	0	-2	0	0	-26.878	0.41	0.00	0.01	0.00	0.00	0.00
2	0	0	-2	1	212.323	0.41	0.00	-0.22	0.00	0.00	0.00
0	1	2	-2	1	119.607	0.36	0.00	-0.20	0.00	0.00	0.00
1	1	0	0	0	25.622	-0.34	0.00	0.01	0.00	0.00	0.00
1	-1	0	-1	0	-3232.862	-0.33	0.00	0.00	0.00	0.00	0.00
-1	-1	2	2	2	9.814	-0.29	0.00	0.12	0.00	0.00	0.00
0	-1	2	2	2	7.236	-0.26	0.00	0.11	0.00	0.00	0.00
1	-1	2	0	2	9.367	-0.29	0.00	0.12	0.00	0.00	0.00
3	0	2	0	2	5.492	-0.29	0.00	0.12	0.00	0.00	0.00
-2	0	2	0	2	1615.748	-0.31	0.00	0.14	0.00	0.00	0.00
1	0	2	0	0	9.108	0.34	0.00	-0.01	0.00	0.00	0.00
-1	0	2	4	2	5.802	-0.15	0.00	0.06	0.00	0.00	0.00
1	0	0	0	2	27.780	-0.20	0.00	0.08	0.00	0.00	0.00
-1	0	2	-2	1	-32.606	-0.20	0.00	0.11	0.00	0.00	0.00
0	-2	2	-2	1	6786.317	-0.15	0.00	0.08	0.00	0.00	0.00
-2	0	0	0	1	-13.749	-0.23	0.00	0.13	0.00	0.00	0.00
2	0	0	0	1	13.805	0.21	0.00	-0.11	0.00	0.00	0.00
3	0	0	0	0	9.185	0.16	0.00	-0.01	0.00	0.00	0.00
1	1	2	0	2	8.910	0.24	0.00	-0.10	0.00	0.00	0.00
0	0	2	1	2	9.340	0.16	0.00	-0.07	0.00	0.00	0.00
1	0	0	2	1	9.627	-0.10	0.00	0.05	0.00	0.00	0.00
1	0	2	2	1	5.638	-0.13	0.00	0.07	0.00	0.00	0.00
1	1	0	-2	1	-34.669	-0.06	0.00	0.03	0.00	0.00	0.00
0	1	0	2	0	14.192	-0.06	0.00	0.00	0.00	0.00	0.00
0	1	2	-2	0	117.539	-0.06	0.00	0.00	0.00	0.00	0.00
0	1	-2	2	0	-329.791	-0.09	0.00	0.00	0.00	0.00	0.00
1	0	-2	2	0	32.764	-0.06	0.00	0.00	0.00	0.00	0.00
1	0	-2	-2	0	-9.530	-0.06	0.00	0.00	0.00	0.00	0.00
1	0	2	-2	0	23.775	-0.07	0.00	0.00	0.00	0.00	0.00
1	0	0	-4	0	-10.085	-0.14	0.00	-0.01	0.00	0.00	0.00
2	0	0	-4	0	-15.906	-0.13	0.00	0.00	0.00	0.00	0.00
0	0	2	4	2	4.793	-0.07	0.00	0.03	0.00	0.00	0.00
0	0	2	-1	2	25.420	-0.07	0.00	0.03	0.00	0.00	0.00
-2	0	2	4	2	7.349	-0.12	0.00	0.05	0.00	0.00	0.00
2	0	2	2	2	4.684	-0.11	0.00	0.05	0.00	0.00	0.00
0	-1	2	0	1	14.162	-0.07	0.00	0.03	0.00	0.00	0.00
0	0	-2	0	1	-13.579	-0.06	0.00	0.03	0.00	0.00	0.00
0	0	4	-2	2	12.663	0.09	0.00	-0.04	0.00	0.00	0.00

TABLE 1 Continued. The ZMOA-1990 nutation series.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
0	1	0	0	2	409.234	0.07	0.00	-0.03	0.00	0.00	0.00
1	1	2	-2	2	22.469	0.13	0.00	-0.05	0.00	0.00	0.00
3	0	2	-2	2	8.745	0.09	0.00	-0.04	0.00	0.00	0.00
-2	0	2	2	2	14.632	0.13	0.00	-0.06	0.00	0.00	0.00
-1	0	0	0	2	-27.333	0.14	0.00	-0.06	0.00	0.00	0.00
0	0	-2	2	1	-169.002	0.09	0.00	-0.04	0.00	0.00	0.00
0	1	2	0	1	13.143	0.08	0.00	-0.04	0.00	0.00	0.00
-1	0	4	0	2	9.057	0.11	0.00	-0.05	0.00	0.00	0.00
2	1	0	-2	0	131.671	0.11	0.00	0.00	0.00	0.00	0.00
2	0	0	2	0	7.127	0.06	0.00	0.00	0.00	0.00	0.00
2	0	2	-2	1	12.787	0.10	0.00	-0.05	0.00	0.00	0.00
2	0	-2	0	1	-943.227	0.07	0.00	-0.04	0.00	0.00	0.00
1	-1	0	-2	0	-29.263	0.09	0.00	0.00	0.00	0.00	0.00
-1	0	0	1	1	-388.267	0.10	0.00	-0.04	0.00	0.00	0.00
-1	-1	0	2	1	35.026	0.07	0.00	-0.04	0.00	0.00	0.00
0	1	0	1	0	27.322	0.05	0.00	0.00	0.00	0.00	0.00
3	0	2	2	2	4.003	-0.01	0.00	0.01	0.00	0.00	0.00
1	0	2	4	2	4.083	-0.02	0.00	0.01	0.00	0.00	0.00
4	0	2	0	2	4.579	-0.03	0.00	0.01	0.00	0.00	0.00
2	0	2	2	1	4.680	-0.02	0.00	0.01	0.00	0.00	0.00
0	0	2	4	1	4.789	-0.01	0.00	0.01	0.00	0.00	0.00
3	0	2	0	1	5.488	-0.05	0.00	0.02	0.00	0.00	0.00
1	1	2	2	2	5.557	0.01	0.00	-0.01	0.00	0.00	0.00
1	-1	2	2	2	5.731	-0.06	0.00	0.02	0.00	0.00	0.00
-1	0	2	4	1	5.797	-0.03	0.00	0.01	0.00	0.00	0.00
-1	-1	2	4	2	5.895	-0.02	0.00	0.01	0.00	0.00	0.00
2	1	2	0	2	6.733	0.04	0.00	-0.02	0.00	0.00	0.00
0	0	4	0	2	6.817	0.02	0.00	-0.01	0.00	0.00	0.00
2	0	2	0	0	6.846	0.03	0.00	0.00	0.00	0.00	0.00
0	1	2	2	2	6.961	0.05	0.00	-0.02	0.00	0.00	0.00
1	0	2	1	2	6.976	0.03	0.00	-0.01	0.00	0.00	0.00
2	-1	2	0	2	6.991	-0.05	0.00	0.02	0.00	0.00	0.00
0	0	2	2	0	7.081	0.04	0.00	0.00	0.00	0.00	0.00
2	0	0	2	1	7.135	-0.01	0.00	0.01	0.00	0.00	0.00
0	-1	2	2	1	7.229	-0.04	0.00	0.02	0.00	0.00	0.00
-2	0	2	4	1	7.341	-0.02	0.00	0.01	0.00	0.00	0.00
0	-1	2	2	2	7.236	-0.01	0.00	0.01	0.00	0.00	0.00
0	0	0	4	0	7.383	0.05	0.00	0.00	0.00	0.00	0.00
0	0	0	4	1	7.391	-0.02	0.00	0.01	0.00	0.00	0.00
1	0	4	-2	2	8.676	0.02	0.00	-0.01	0.00	0.00	0.00
3	0	2	-2	1	8.734	0.02	0.00	-0.01	0.00	0.00	0.00
1	1	2	0	1	8.898	0.04	0.00	-0.02	0.00	0.00	0.00
-1	0	4	0	1	9.045	0.02	0.00	-0.01	0.00	0.00	0.00
0	1	2	1	2	9.107	-0.02	0.00	0.01	0.00	0.00	0.00
-3	0	0	0	1	-9.172	-0.01	0.00	0.01	0.00	0.00	0.00
-1	1	2	2	2	9.313	0.06	0.00	-0.02	0.00	0.00	0.00
0	0	2	1	1	9.327	0.03	0.00	-0.01	0.00	0.00	0.00
1	-1	2	0	1	9.354	-0.04	0.00	0.02	0.00	0.00	0.00
-1	0	0	-2	1	-9.600	-0.04	0.00	0.03	0.00	0.00	0.00
-1	-1	2	2	1	9.799	-0.05	0.00	0.02	0.00	0.00	0.00
1	-1	0	2	0	9.874	0.05	0.00	0.00	0.00	0.00	0.00
1	0	0	-4	1	-10.070	-0.01	0.00	0.01	0.00	0.00	0.00
-1	0	0	4	1	10.100	-0.02	0.00	0.01	0.00	0.00	0.00
-1	-1	0	4	0	10.371	0.01	0.00	0.00	0.00	0.00	0.00
2	1	2	-2	2	12.377	0.03	0.00	-0.01	0.00	0.00	0.00
0	0	4	-2	1	12.639	0.02	0.00	-0.01	0.00	0.00	0.00

TABLE 1 Continued. The ZMOA-1990 nutation series.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
1	0	2	-1	2	13.222	-0.03	0.00	0.01	0.00	0.00	0.00
2	1	0	0	0	13.276	-0.03	0.00	0.00	0.00	0.00	0.00
0	0	2	0	1	13.633	-0.01	0.00	0.00	0.00	0.00	0.00
0	0	2	0	3	13.688	0.02	0.00	0.00	0.00	0.00	0.00
0	1	0	2	1	14.221	0.02	0.00	-0.01	0.00	0.00	0.00
1	0	0	1	0	14.254	-0.03	0.00	0.00	0.00	0.00	0.00
2	-1	0	0	0	14.317	0.04	0.00	0.00	0.00	0.00	0.00
-2	0	2	2	1	14.600	0.02	0.00	-0.01	0.00	0.00	0.00
0	0	0	-2	2	-14.701	0.01	0.00	-0.01	0.00	0.00	0.00
0	0	0	2	2	14.830	-0.05	0.00	0.02	0.00	0.00	0.00
0	1	0	-2	1	-15.353	-0.03	0.00	0.02	0.00	0.00	0.00
0	-1	0	2	1	15.422	-0.02	0.00	0.01	0.00	0.00	0.00
2	0	0	-4	1	-15.869	-0.01	0.00	0.01	0.00	0.00	0.00
-2	0	0	4	1	15.943	0.01	0.00	-0.01	0.00	0.00	0.00
0	-2	0	2	0	16.064	0.02	0.00	0.00	0.00	0.00	0.00
0	0	2	-4	1	-16.102	-0.01	0.00	0.01	0.00	0.00	0.00
1	1	2	-2	1	22.395	0.03	0.00	-0.01	0.00	0.00	0.00
-1	1	2	0	2	25.222	0.04	0.00	-0.02	0.00	0.00	0.00
-1	-1	0	0	1	-25.525	0.02	0.00	-0.01	0.00	0.00	0.00
1	1	0	0	1	25.719	-0.03	0.00	0.02	0.00	0.00	0.00
1	0	-2	0	1	-26.772	0.03	0.00	-0.01	0.00	0.00	0.00
0	0	1	0	1	27.322	-0.02	0.00	0.00	0.00	0.00	0.00
-1	-1	2	0	2	29.263	-0.02	0.00	0.01	0.00	0.00	0.00
-1	1	0	2	1	29.390	-0.01	0.00	0.01	0.00	0.00	0.00
0	0	0	-1	1	-29.403	0.03	0.00	-0.02	0.00	0.00	0.00
0	0	0	1	1	29.659	-0.04	0.00	0.02	0.00	0.00	0.00
-1	1	0	0	1	-29.673	-0.02	0.00	0.02	0.00	0.00	0.00
1	-1	0	0	1	29.934	0.05	0.00	-0.03	0.00	0.00	0.00
1	0	0	-2	2	-31.517	0.03	0.00	-0.01	0.00	0.00	0.00
-1	0	0	2	2	32.112	-0.04	0.00	0.02	0.00	0.00	0.00
-1	0	2	-2	2	-32.451	0.03	0.00	-0.01	0.00	0.00	0.00
-1	1	2	-2	1	-35.803	-0.01	0.00	0.01	0.00	0.00	0.00
-1	-2	0	2	0	38.522	0.03	0.00	0.00	0.00	0.00	0.00
1	0	2	-4	1	-38.742	-0.04	0.00	0.02	0.00	0.00	0.00
0	3	2	-2	2	73.051	-0.05	0.00	0.02	0.00	0.00	0.00
0	3	0	0	0	121.753	0.03	0.00	0.00	0.00	0.00	0.00
-2	-1	0	2	1	-129.169	-0.02	0.00	0.01	0.00	0.00	0.00
0	0	2	-2	1	177.844	-0.09	0.00	0.07	0.00	0.00	0.00
0	-2	0	0	1	-177.852	-0.01	0.00	0.01	0.00	0.00	0.00
0	0	2	-2	3	187.662	0.13	0.00	-0.02	0.00	0.00	0.00
2	0	0	-2	2	219.167	-0.03	0.00	0.01	0.00	0.00	0.00
-2	1	2	0	1	285.406	-0.01	0.00	0.00	0.00	0.00	0.00
-2	1	2	0	2	297.913	-0.01	0.00	0.00	0.00	0.00	0.00
-1	0	2	-1	1	313.042	-0.04	0.00	0.01	0.00	0.00	0.00
0	-1	0	0	2	-329.819	0.04	0.00	-0.01	0.00	0.00	0.00
1	0	0	-1	1	438.335	0.03	0.00	-0.01	0.00	0.00	0.00
-2	-1	2	0	2	-471.950	0.02	0.00	-0.01	0.00	0.00	0.00
-2	-1	2	0	1	-507.157	0.03	0.00	0.00	0.00	0.00	0.00
-3	0	2	1	2	-552.625	0.02	0.00	-0.01	0.00	0.00	0.00
0	0	0	0	3	-2266.128	-0.02	0.00	0.00	0.00	0.00	0.00
-1	-1	2	-1	2	3230.131	0.13	0.00	-0.05	0.00	0.00	0.00
-1	0	1	0	1	3231.495	-0.15	0.00	0.03	0.00	0.00	0.00
-1	0	1	0	2	6159.135	0.03	0.00	-0.01	0.00	0.00	0.00
-1	1	0	1	1	6164.100	0.07	0.00	-0.04	0.00	0.00	0.00

The nutation series is evaluated using $\sin\theta j$ for the in-phase component of the nutation in longitude, $\Delta\psi$, and $\cos\theta j$ for the out-of-phase component. For the nutation in obliquity, $\Delta\varepsilon$, $\cos\theta j$ is used for the in-phase component and $\sin\theta j$ for the out-of-phase component, where θj is the argument of the nutation (see *Herring et al.* [1990] for details). The coefficients are referenced to J2000, and the rates are applied from this epoch.

Since the ZMOA-1990 series is largely a geophysical one, it does not match the VLBI derived coefficients exactly. In particular, there are four notable differences which are discussed in *Herring et al.* [1990]. In decreasing order of significance they are: (1) the out-of-phase correction to the retrograde annual nutation (0.39±0.04 mas); (2) the in-phase correction to the prograde 13.7 day nutation (-0.25±0.04 mas); (3) the amplitude of a nutation at the resonance frequency of the retrograde free core nutation (RFCN) mode (0.26±0.04 mas); and (4) the out-of-phase correction to the prograde annual nutation (0.13±0.04). Our aim here is to discuss the likely causes of these differences and the impact of these causes on the ZMOA-1990 nutation series. Items (3) and (4) are discussed in detail in *Herring et al.* [1990] and will not be discussed further here except to mention that item (3) indicates the detection of a signal, amplitude 0.26 mas, at the resonance frequency of the RFCN mode, and item (4) is disturbing in that there is no apparent explanation for its presence. We are now investigating the possibility that this latter term arises from the annual term associated with the same general relativistic effect which causes de Sitter geodetic precession.

An additional complication which has arisen in developing models for the motion of the Earth in inertial space is that the conventional nutations no longer appear to be the only "forced" motions of the Earth. Recent computations of the effects of tidally induced ocean currents on UT1 by *Brosche et al.* [1989] have shown that there should be diurnal and semidiurnal variations in the rotation rate of the Earth with amplitudes of order 0.3 mas (0.02 milli-time-seconds, ms). These signals have now been observed with VLBI [*Dong and Herring*, 1990], and are likely to be responsible for the second difference noted above. To further complicate matters, it also seems likely that the currently available solid-earth tide and ocean-tidal loading models are not adequate for modeling high precision geodetic data. Therefore not only must a new nutation series be obtained, but also models for prograde diurnal polar motion, prograde and retrograde semidiurnal polar motion, diurnal and semidiurnal UT1 variations, and tidal displacements in the diurnal and semidiurnal bands need to be adopted. Since these new tidal models are likely to arise from ocean effects, and thus contain multiple spherical harmonic coefficients for a given temporal frequency, a method needs to be adopted for separating deformations from rotational variations.

2. Discussion of differences between ZMOA-1990 and VLBI results

As discussed in *Herring et al.* [1990], the most likely cause for the difference at the retrograde annual frequency is a dissipative process acting at or very near the core-mantle boundary. It is unlikely that this process can be accommodated with the current anelasticity models for the mantle because these models affect the eigenfunction of RFCN mode more than the resonance frequency, and thus impact many terms in the nutation series and not just the retrograde annual nutation. Also, the anelasticity models affect the in-phase nutation amplitude more than the out-of-phase component and thus would change terms which currently show no disagreement between theory and observation. It is also unlikely that this difference arises from the ocean tide effects on UT1 discussed in the introduction because of the small size of the ψ_1 tide that drives this nutation.

The likely effects of dissipation on the core-mantle boundary have been studied by estimating selected coefficients of the normalized response function, $\eta(\sigma)$, directly from the nutation angle data. The functional form for $\eta(\sigma)$ used by *Mathews et al* is:

$$\eta(\sigma) = R + R'(1+\sigma) + \sum_{\alpha} R_{\alpha}/(\sigma - \sigma_{\alpha}) \tag{1}$$

where α sums over the four normal modes; CW, Chandler wobble; RFCN, retrograde free core nutation; PFCN, prograde free core nutation; and ICW, inner core wobble; σ_α and R_α denote the eigenfrequencies and "oscillator strengths", respectively; and σ is the frequency of the forced nutations seen from the rotating Earth. (See *Mathews et al.*, 1990*a* for detailed discussions of this form of expansion.) The frequencies are in cycles per sidereal day (cpsd). For studying core-mantle boundary dissipation the R_α and σ_α for the RFCN mode were estimated as complex values. The estimates of these four parameters are given in Table 2 for the entry ZMOA-1990-2 along with the values computed from the geophysical theory (simply ZMOA-1990) and the values obtained when the real part of R' is also estimated (ZMOA-1990-1). (For historical reasons this later solution which will be discussed below is entitled ZMOA-1990-1.) As expected, the estimation of these parameters reduced the difference between the observed value for the out-of-phase component of the retrograde annual nutation from -0.39 mas to 0.01 mas without affecting greatly the other terms in the nutation series. The most affected nutation amplitudes were the out-of-phase terms for the retrograde 18.6

year and prograde semiannual nutations. The nutation series terms which differ between ZMOA-1990 and ZMOA-1990-2 by more 0.01 mas are given in Table 3.

TABLE 2. Standard and Estimated values for selected terms in the Normalized Response Function Model (see text for discussion).

Series	R_{RFCN}		σ_{RFCN}		R'
	Real	*Imag.*	*Real* (cpsd)	*Imag* (cpsd)	*Real*
ZMOA-1990	-1.1978×10^{-4}	–	−1.0023203	–	-0.28034
ZMOA-1990-1	$-1.1978\times10^{-4} \pm 0.0012\times10^{-4}$	$0.0075\times10^{-4} \pm 0.0012\times10^{-4}$	−1.0023207 ± 0.0000020	0.0000168 ± 0.0000020	-0.2487 ± 0.0050
ZMOA-1990-2	$-1.1976\times10^{-4} \pm 0.0012\times10^{-4}$	$0.0075\times10^{-4} \pm 0.0012\times10^{-4}$	−1.0023206 ± 0.0000020	0.0000168 ± 0.0000020	-0.28034

TABLE 3. Coefficients of the ZMOA-1990-2 nutation series that differ from ZMOA-1990.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
0	0	0	0	1	-6798.383	-17206.70	-17.43	9205.25	0.90	3.75	1.68
0	0	2	-2	2	182.621	-1317.14	-0.16	573.05	-0.31	-1.50	-0.52
0	0	2	0	2	13.661	-227.56	-0.02	97.79	-0.05	-0.02	0.01
0	0	0	0	2	-3399.192	207.51	0.02	-89.77	0.05	-0.08	-0.03
0	1	0	0	0	365.260	147.64	-0.36	7.32	-0.02	0.82	-0.28
1	0	0	0	0	27.555	71.11	0.01	-0.67	0.00	-0.03	0.03
0	1	2	-2	2	121.749	-51.68	0.12	22.44	-0.07	-0.06	-0.02
0	0	2	0	1	13.633	-38.71	-0.04	20.06	0.00	-0.01	0.00
1	0	2	0	2	9.133	-30.12	0.00	12.89	-0.01	0.03	0.01

From the form of the normalized response function it is clear the difference between the in-phase part of the prograde 13.66 day nutation can be reduced by estimating a correction to the R' coefficient. The results of such an estimation are also given in Table 2 for the entry ZMOA-1990-1. (There was no significant difference to the out-of-phase term for this nutation amplitude and therefore the imaginary part of R' was not estimated.) The nutation series coefficients which differ from ZMOA-1990 are given in Table 4. However, as mentioned in the introduction, recent investigations have shown that the difference in the 13.66 day nutation is likely to arise from aliasing of the diurnal UT1 variations into the estimates of the nutations. Trial solutions, using about 250 of the approximately 1000 available VLBI experiments, have shown that estimating diurnal UT1 variations reduces the difference between the ZMOA-1990 value for the 13.66 day nutation amplitude and the estimated value to less than 0.05 mas: a difference consistent with uncertainty of the estimated value. For this reason, the ZMOA-1990-1 nutation series, while matching the existing VLBI nutation angles better than any other series, is not likely to be correct. The estimated values of the diurnal UT1 variations match those predicted by *Brosche et al.* [1989] to within about 10%; in the semidiurnal band, the VLBI estimates are about half the size of the *Brosche et al.* predictions.

For the solutions in which the coefficients of the normalized response function were estimated, the precession constant and linear rate of change of the obliquity of the ecliptic were more precisely estimated because the amplitudes of the 18.6 year nutations were effectively constrained by the normalized response function and rigid Earth nutation series. In Table 5, we give the estimates of the corrections to the luni-solar precession constant, Δp, and $\Delta d\varepsilon/dt$ for each of the ZMOA nutation series and for the solution given in *Herring et al.* [1990]. We also give the difference between the estimates of the out-of-phase component of the 18.6 year period nutation in longitude for each of these series and the ZMOA-1990 value. This particular nutation series coefficient is the one most highly correlated (-77%) with the correction to the precession constant. For both ZMOA-1990-1 and ZMOA-1990-2 the value of $\Delta d\varepsilon/dt$ is very close to zero as should be expected [see e.g., *Rochester*, 1976].

TABLE 4. Coefficients of the ZMOA-1990-1 nutation series that differ from ZMOA-1990.

Fundamental arguments					Period	$\Delta\psi_{in}$	$\Delta\dot{\psi}_{in}$	$\Delta\varepsilon_{in}$	$\Delta\dot{\varepsilon}_{in}$	$\Delta\psi_{out}$	$\Delta\varepsilon_{out}$
l	l'	F	D	Ω	(solar days)	(mas)	(mas/cy)	(mas)	(mas/cy)	(mas)	(mas)
0	0	0	0	1	-6798.383	-17206.53	-17.43	9205.18	0.90	3.71	1.66
0	0	2	-2	2	182.621	-1317.38	-0.16	573.14	-0.31	-1.50	-0.52
0	0	2	0	2	13.661	-228.11	-0.02	98.00	-0.05	-0.02	0.01
0	0	0	0	2	-3399.192	207.51	0.02	-89.77	0.05	-0.08	-0.03
0	1	0	0	0	365.260	147.65	-0.36	7.32	-0.02	0.82	-0.28
1	0	0	0	0	27.555	71.11	0.01	-0.70	0.00	-0.03	0.03
0	1	2	-2	2	121.749	-51.70	0.12	22.44	-0.07	-0.06	-0.02
0	0	2	0	1	13.633	-38.82	-0.04	20.10	0.00	-0.01	0.00
1	0	2	0	2	9.133	-30.23	0.00	12.93	-0.01	0.03	0.01
0	-1	2	-2	2	365.225	21.61	-0.05	-9.60	0.03	0.01	0.01
1	0	0	-2	0	-31.812	-15.70	0.00	-0.13	0.00	-0.01	-0.01
0	0	2	-2	1	177.844	12.92	0.01	-6.98	0.00	0.02	0.00
-1	0	2	0	2	27.093	12.36	0.00	-5.34	0.00	0.01	0.00
1	0	0	0	1	27.667	6.33	0.01	-3.33	0.00	0.00	0.00
0	0	0	2	0	14.765	6.36	0.00	-0.12	0.00	0.00	0.00
-1	0	2	2	2	9.557	-5.98	0.00	2.56	0.00	0.00	0.00
-1	0	0	0	1	-27.443	-5.80	-0.01	3.15	0.00	0.00	0.00
1	0	2	0	1	9.121	-5.17	0.00	2.65	0.00	0.00	0.00
2	0	0	-2	0	205.892	4.79	0.00	0.05	0.00	0.00	0.00
-2	0	2	0	1	1305.479	4.60	0.00	-2.43	0.00	0.00	0.00
0	0	2	2	2	7.096	-3.86	0.00	1.65	0.00	0.00	0.00
2	0	2	0	2	6.859	-3.11	0.00	1.32	0.00	0.00	0.00

TABLE 5. Estimated corrections to the IAU-1980 luni-solar precession constant, Δp; to the rate of change of the obliquity of the ecliptic, $d\varepsilon/dt$; and to the ZMOA-1990 value for the out-of-phase component of the 18.6 year period nutation in longitude; for the ZMOA-1990 nutation series and its two variants discussed in the text.

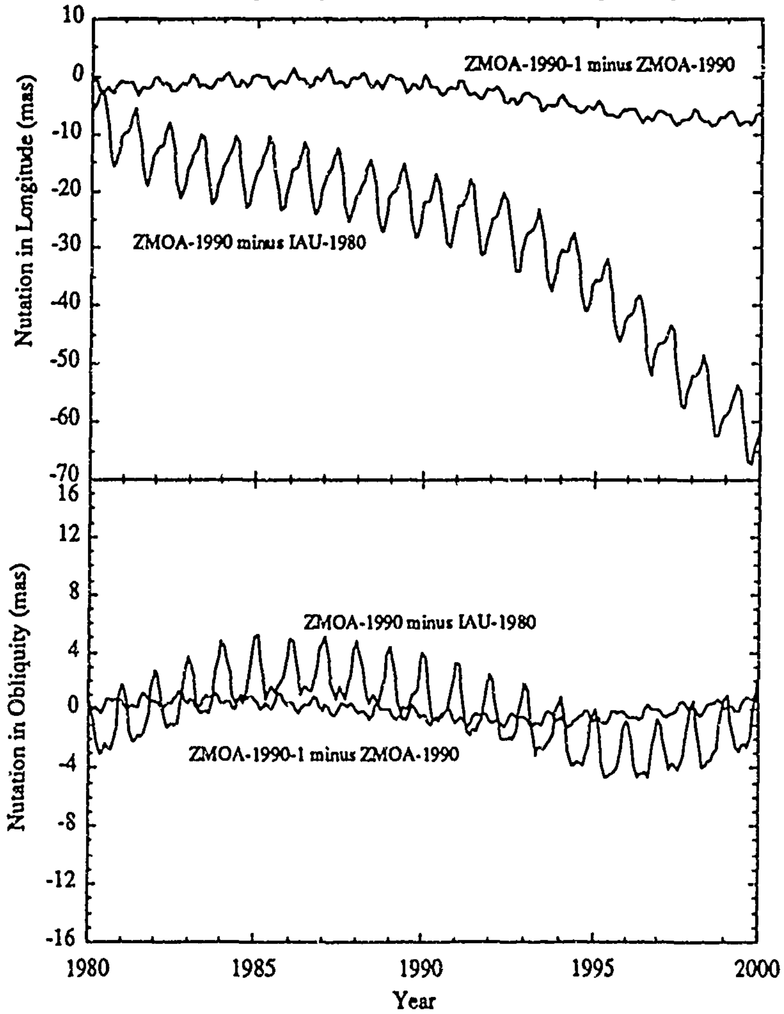
Series	Δp ("/cent)	$\Delta d\varepsilon/dt$ ("/cent)	$(\delta\Delta\psi_i)_{18.6\ \text{year}}$ (mas)
VLBI: coefficients estimated	-0.32 ± 0.13	-0.04 ± 0.05	2.3 ± 3.8
ZMOA-1990	-0.27 ± 0.02	-0.03 ± 0.00^6	—
ZMOA-1990-1	-0.30 ± 0.04	$0.01^5 \pm 0.01^4$	1.8
ZMOA-1990-2	-0.31 ± 0.04	$0.01^6 \pm 0.01^5$	1.9

The "VLBI: coefficient estimated" line refers to the solution given in *Herring et al.* [1990] obtained when complex components of 18 nutation amplitudes in the serie- are estimated; the ZMOA-1990 line gives the estimates when only Δp and $\Delta d\varepsilon/dt$ are estimated (in this case the series value for $(\Delta\psi_i)_{18.6\ \text{year}}$ is used and no estimate is given for the difference); and the ZMOA-1990-1 and ZMOA-1990-2 lines are for the series obtained when selected terms in the normalized response function are estimated. The latter two cases the values for $(\delta\psi_i)_{18.6\ \text{year}}$ are the differences between each of the series and ZMOA-1990.

3. Conclusions

Of the three nutation series presented here, ZMOA-1990-2 is likely to be the most accurate one, although this series should be used in conjunction with diurnal and semi-diurnal UT1 variations if the full motion of the Earth in inertial space is to be obtained. Associated with this series is a correction of -0.31 ± 0.04 "/cent to the luni-solar precession constant. This correction is consistent with the value obtained from LLR, -0.27 ± 0.04 "/cent. To understand the likely errors incurred in using the IAU-1980 nutation series and, to some extent, the uncertainty in ZMOA-1990-2 series, we show in Figure 1 the difference between the ZMOA-1990 and IAU-1980 series, and the difference between ZMOA-1990 and ZMOA-1990-1 over the twenty year interval between 1980 and 2000. (The ZMOA-1990-2 yields results almost identical to ZMOA-1990-1 except that the high frequency component arising from the 13.66 day nutati is not present. We do not show this difference for clarity.) Over this twenty year interval, the IAU-1980 nutation series and the IAU-1976 precession constant are likely to introduce errors in the the realization of an inertial frame of about 70 mas (approximately 28 mas in the celestial pole position). The differences between the ZMOA models is about 10% of this value, and thus while there is still uncertainty in the appropriate values of the parameters in the geophysical model of the nutations, these uncertainties are probably less that 10% of the errors in the current IAU nutation series.

FIGURE 1. Differences between ZMOA-1990 and the IAU-1980 nutation in longitude (top figure) and nutation in obliquity (lower figure). Also shown is the difference between ZMOA-1990-1 and ZMOA-1990. These latter values are indicative of the uncertainty in the parameters of the current geophysical models for nutations. The scales have been selected such that each figure represents the same amount of pole displacement.



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LONG-PERIOD PERTURBATIONS IN TERRESTRIAL REFERENCE FRAMES

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ABSTRACT. Oceanic and fluid core effects inherent in polar motion and l.o.d.-data were analyzed and related results are discussed in detail.

A new exact analytic solution to the hydrodynamic equations is obtained, which describes tidal motions at low frequencies in a homogeneous, incompressible, inviscid liquid core with arbitrary core-mantle topography. Some geophysical and astrometrical consequences of this solution are considered.

The numerical estimation of the deviations of the pole tide from the static one is obtained. A new hypothesis is proposed that the known interrelation between long-term amplitude and frequency variations in Chandler wobble may be attributed to the influence of turbulent friction of non-equilibrium pole tide.

1. INTRODUCTION

Astrometric space missions such as Hipparcos may substantially contribute to <u>relative</u> accuracy of stellar and other (fundamental) catalogues. On the other hand, the associated <u>absolute</u> systems of reference are still basically referred to the Earth. Consequently, any improvement in modeling the irregular rotation of the Earth with respect to the celestial frame or system or reference contributes also to the implementation of celestial reference systems. This holds for optical as well as for radio catalogues (de Vegt et al., 1988, Argue, 1989).

In this paper we focus on the long-period variations of Earth rotation which are related to the fluid parts of the Earth : (a) oceanic effects and (b) outer core perturbations. By "long-period" effects of periods longer than a few months up to about 14 months (Chandlerian period) is meant. The emphasis is here put, as far as the oceanic effects are concerned, on pole tide which is basically the variation of sea surface caused by the varying centrifugal force of polar motion. The latter reflects the changes between celestial and terrestrial (in the sense of CTS - Conventional Terrestrial System) systems and therefore represents the transformation parameters for the transition from celestial to terrestrial systems and vice versa. As pole tide is excited in a thin layer of liquid (in comparison with the Earth's radius) two-dimensional integrations of related differential equations along the Earth's surface are sufficient in most cases. The fundamental phenomena were investigated by J. Wahr, S. Dickman and others which led to the conclusion that basically pole tides are in equilibrium. Therefore, we focus here on the finer structure of such effects; this is necessary in view of the increased accuracy of modern observations and analysis methods. It affects mainly effects such as the dependence of polar motion frequency variations on its amplitude variations.

The influence of inner and outer core effects on polar motion and LOD-data is of particular importance because there is limited information on the Earth's core; there are relatively few phenomena such as Earth tides, free vibrations related to very big earthquakes, geomagnetism, seismology, to some extent, and a few others which can really give reliable information on the detailed structure of the Earth's core. Therefore, the analysis of Earth rotation data in terms of LOD- and polar motion data is of utmost importance. Mainly the detailed geometry and topography of the core-mantle-boundary (CMB) is important in that respect as well as the physics around it such as questions of hydrostatic equilibrium etc. which are closely related to its topography. In spite of impressive recent results and of rather general agreement, within certain limits, there is still a wide disagreement on details. This paper aims at contributing to a clearer understanding of related phenomena.

In the first part of this report the influence of the CMB, both on the Chandler wobble and on the length of day variation, is considered.

A new analytical solution to the hydrodynamic equations is obtained, which describes tidal motions at low frequencies in a homogeneous, incompressible, inviscid liquid core with arbitrary core-mantle topography. The result is applied to the estimation of the influence of the core-mantle boundary topography on the Chandler wobble and on the long-periodic tidal variations of the length of day.

It is found, that the influence of CMB topography is manifested not only in the changes of the parameters, describing these events (period and ellipticity of the Chandler wobble, amplitudes of the tidal variations of the length of day), but also leads to a new "cross-coupling" effect of (1) the excitation of the length of day variation with the Chandlerian period and (2) the excitation of polar motion with the periods of zonal tidal waves.

The orders of values of all these effects are strongly dependent on the values of gradients of the CMB topography. The numerical estimation for the reasonable models of CMB-topography shows, that the influence of CMB-topography on the period and ellipticity of the Chandler wobble as well as the excitation of the polar motion by the long-period tidal waves are, however, less than the errors of the modern VLBI-measurements. But at the same time, the excitation of the length of day variations by the Chandler wobble is significant. As a result, the analysis of the observed values of the length of day amplitudes at Chandler frequency makes it possible to obtain new information concer-

ning the CMB topography of the actual Earth. Some numerical estimations of this type are
obtained which are based on the analysis of modern VLBI data.

In the second part of this paper the influence of dynamical pole tide on the Chandler wobble is considered. The results of new numerical calculations of the planetary vorticity maps for the actual ocean are presented. Some qualitative and quantitative estimations of the possible dynamical effects are obtained.

An interesting consequence of the dynamical theory of the pole tide is the conclusion, that the system (Earth + ocean) is not linear. It is known, that the frequencies of free oscillations of such systems are dependent on the amplitudes of the oscillation. As a result, the frequency of the Chandler wobble must be dependent on its amplitude.

The qualitative theoretical analysis of this effect makes it possible to conclude, that the Chandler period is an increasing function of the amplitude. It is known, that a similar conclusion was made by Melchior (1957) (see also Munk & McDonald, 1960) based on the analysis of polar motion data since 1900,0. Thus we may conclude that it is possible to attribute this event to the influence of non-equilibrium pole tide.

2. THE SMALL LONG-PERIODIC OSCILLATIONS OF THE HOMOGENEOUS INCOMPRESSIBLE INVISCID LIQUID, CLOSED IN THE RIGID NONUNIFORMLY ROTATING CONTAINER WITH ARBITRARY GEOMETRY.

The small oscillations of the homogeneous incompressible inviscid liquid are described in the uniformly rotating system of Cartesian coordinates by the known system of governing equations and boundary conditions (Lamb, 1932):

$$\dot{\vec{v}} + 2\vec{\omega}\times\vec{v} + \vec{\omega}\times\vec{\omega}\times\vec{r} + \dot{\vec{\omega}}\times\vec{r} = - \nabla \left(\frac{p}{\rho} + V\right) ; \tag{1a}$$

$$\nabla \cdot \vec{v} = 0 \quad , \tag{1b}$$

$$(\vec{v},\vec{N})\big|_s = 0 \quad , \tag{2}$$

where $\vec{\omega}$ is the vector of the angular velocity of the system of coordinates, $\vec{r}$ is the radius-vector,

$\vec{v} = \dot{\vec{r}}$ is the velocity of the element of fluid with respect to this system, p = pressure, ρ = density, V is the gravitational potential, the dot above a symbol denotes the time derivative with respect to the non-uniformly rotating system of coordinates, $\vec{N}$ is the outer normal to the boundary surface s.

We adopt the system of Cartesian coordinates (x,y,z) which is rigidly connected with the container in such way, that $|\omega_x| \ll \omega_z$, $|\omega_y| \ll \omega_z$ (z is the direction of the uniform rotation). In this case, by taking into account only the linear term with respect to ω_x/ω_z, ω_y/ω_z, we write eq. (1a) in the form

$$\dot{\vec{v}} + 2\omega_z \vec{e}_z \times \vec{v} = -\nabla\psi + \vec{\chi} \quad , \tag{2}$$

$$\psi = \frac{p}{\rho} + V - \frac{\omega_z^2}{2}(x^2+y^2) + (\omega_x\omega_z + \dot{\omega}_y)\, xz + (\omega_y\omega_z - \dot{\omega}_x)\, yz \quad , \tag{3}$$

$$\vec{\chi} = \dot{\omega}_z\ (y\vec{e}_x - x\vec{e}_y) + 2\vec{e}_z\ (\dot{\omega}_y x - \dot{\omega}_x y), \tag{4}$$

$\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ are the unit vectors, which are oriented along the direction of the axes x,y and z, correspondingly.

To present eq. (1a) in a form which is suitable for the application of the method of perturbation, we calculate the curl of left and right sides of eq. (2). Taking into account conditions (1b) and (4), we get:

$$\mathrm{curl}(\vec{e}_z \times \vec{v}) = \vec{e}_z(\nabla\cdot\vec{v}) - (\vec{e}_z, \nabla)\,\vec{v} = -\frac{\partial\vec{v}}{\partial z} \; ,$$

$$\mathrm{curl}\ \vec{\chi} = -2\dot{\vec{\omega}} \; ,$$

and

$$\frac{\partial\vec{v}}{\partial z} = \frac{\dot{\vec{\omega}}}{\omega_z} + \frac{1}{2\omega_z}\,\mathrm{curl}\ \dot{\vec{v}} = \frac{i\sigma}{\omega_z}\left(\vec{\omega}_1 + \frac{1}{2}\,\mathrm{curl}\ \vec{v}\right) \quad , \tag{5}$$

where σ is the frequency of oscillations and $\vec{\omega}_1 = \vec{\omega} - \omega_z\,\vec{e}_z$ is the variable part of $\vec{\omega}$.

In the limiting case $\sigma \to 0$, the right side of (5) tends to zero, too, and eq. (5) is reduced to the well known Proudman-Taylor theorem, in accordance with which the stationary flows in rotating fluids ("geostrophic flows") satisfy the equation

$$\partial\vec{v}^{(0)}/\partial z = 0 \; ; \tag{6}$$

and as a result, the components $v_x^{(0)}$, $v_y^{(0)}$, $v_z^{(0)}$ are functions of x,y only.

It is known (Greenspan, 1969), that the geostrophic flows in the bounded volume are described by the conditions:

1) the lines of flow coincide with the isolines

$$\tilde{z} = z_2(x,y) - z_1(x,y) = \text{const} \quad , \tag{7}$$

where $z_2(x,y)$ and $z_1(x,y)$ are consequently the equations of the upper and lower boundary surfaces.

2) the velocities of geostrophic flows are described by the conditions:

$$v_x^{(0)} = v_x^{(0)}(x,y) = - \phi(\tilde{z}) \frac{\partial \tilde{z}}{\partial x} ,$$

$$v_y^{(0)} = v_y^{(0)}(x,y) = \phi(\tilde{z}) \frac{\partial \tilde{z}}{\partial y} , \tag{8}$$

$$v_z^{(0)} = v_z^{(0)}(x,y) = \gamma(x,y) \ \phi(\tilde{z}) \quad ,$$

where

$$\gamma(x,y) = \frac{\partial \tilde{z}}{\partial x} \frac{\partial \bar{z}}{\partial y} - \frac{\partial \tilde{z}}{\partial y} \frac{\partial \bar{z}}{\partial x} ,$$

$$\bar{z} = \bar{z}(x,y) = \frac{z_1(x,y)+z_2(x,y)}{2} \quad ,$$

and $\phi(\tilde{z})$ is an arbitrary function of $\tilde{z}$ which is determined by the initial conditions only. It is easy to see, that the components of $\vec{v}$ described by (8) satisfy the boundary condition (2), the condition of incompressibility (1b) and the dynamical equation (6).

Subsequently, we can use this solution as a zero approximation. Obviously, when we consider the case of forced oscillations instead of the case of stationary flows, then the function $\phi(\tilde{z})$ must be determined uniquely. Let us consider this condition:

To use the method of perturbations, we present the vector of velocity $\vec{v}$ as a sum of the zeroth-order term in the form (8) and as a first-order term:

$$\vec{v} = \vec{v}^{(0)} + \vec{v}^{(1)} \quad . \tag{9}$$

Substituting (9) into (5) and taking into account the first-order terms only, we get:

$$\frac{\partial \vec{v}_1}{\partial z} = \vec{K}(x,y) \quad , \tag{10}$$

where

$$\vec{K}(x,y) = \frac{i\sigma}{\omega_z} \left(\vec{\omega}_1 + \frac{1}{2} \operatorname{curl} \vec{v}^{(0)}(x,y)\right) . \tag{11}$$

After the integration of (8) with respect to z in the limits from $z_1(x,y)$ to $z_2(x,y)$ we get:

$$\vec{v}^{(1)}(x,y,z) = v_1(x,y) \,\Big|_{z=0} + \vec{K}z \ . \tag{12}$$

Let us consider now the condition of incompressibility and the boundary conditions for the vector $\vec{v}^{(1)}$. Substituting (9) into (1b) and (2) and taking into account, that, in accordance with (8), $\vec{v}^{(0)}$ satisfies the conditions (1b) and (2) automatically, we get:

$$\nabla \cdot \vec{v}^{(1)} = 0 \ , \tag{13a}$$

$$(\vec{v}^{(1)},\vec{N})\big|_s = 0 \ . \tag{13b}$$

Substituting (12) into (13a) and taking into account, that, in accordance with (11),

$$\nabla \cdot \vec{K} = 0 \ ,$$

we obtain:

$$\nabla \cdot \vec{v}_1 = \nabla \cdot \vec{v}_1(x,y)\big|_{z=0} + z\nabla \cdot \vec{K} + (\vec{K},\nabla z) = 0 \quad ,$$

$$\frac{\partial v_x{}^{(1)}(x,y)\big|_{z=0}}{\partial x} + \frac{\partial v_y{}^{(1)}(x,y)\big|_{z=0}}{\partial y} + K_z(x,y) = 0 \ . \tag{14}$$

Taking into account, that $\vec{N} = \vec{N}_1 = \left(\dfrac{\partial z_1}{\partial x}, \dfrac{\partial z_1}{\partial y}, -1\right)$

on the surface $z_1(x,y)$ and

$\vec{N} = \vec{N}_2 = \left(-\dfrac{\partial z_2}{\partial x}, -\dfrac{\partial z_2}{\partial y}, 1\right)$ on the surface $z_2(x,y)$, we can present the condition (13b) in the form:

$$(\tilde{v}_1(x,y)|_{z=0},\ \vec{N}_1) + z_1(\vec{K}\ ,\vec{N}_1) = 0, \tag{15a}$$

$$(\tilde{v}_1(x,y)|_{z=0},\ \vec{N}_2) + z_2(\vec{K},\vec{N}_2) = 0\ . \tag{15b}$$

To exclude from the equations (15) the component $v_z^{(1)}(x,y)|_{z=0}$, we sum up (15a) and (15b). We then get:

$$v_x^{(1)}(x,y)|_{z=0}\ \frac{\partial\tilde{z}}{\partial x} + v_y^{(1)}(x,y)|_{z=0}\ \frac{\partial\tilde{z}}{\partial y} + K_x\ \frac{\partial}{\partial x}\ (\tilde{z}\bar{z}) + K_y\ \frac{\partial}{\partial y}\ (\tilde{z}\bar{z}) - K_z\tilde{z} = 0. \tag{16}$$

The conditions (14) and (16) determine the unknown function $\phi(\tilde{z})$ uniquely. To prove this, let us reduce the system (14), (16) to a single integro-differential equation. It is easy to see, that, if eq. (14) is valid, then the components of $\vec{v}$ on the surface z=0 can be presented in the form:

$$v_x^{(1)}(x,y)|_{z=0} = \frac{\partial\xi(x,y)}{\partial y} - \int_0^x K_z(x',y)dx'\ , \tag{17}$$

$$v_y^{(1)}(x,y)|_{z=0} = -\ \frac{\partial\xi(x,y)}{\partial x}\ ,$$

where ξ is an arbitrary single-valued twice differentiable function of x,y. Substitution of (17) into (16) reduces the system (14), (16) to a single integro-differential equation:

$$\frac{\partial\tilde{z}}{\partial x}\frac{\partial\xi}{\partial y} - \frac{\partial\tilde{z}}{\partial y}\frac{\partial\xi}{\partial x} + F(x,y) = 0\ , \tag{18}$$

where

$$F(x,y) = K_x\ \frac{\partial}{\partial x}\ (\tilde{z},\bar{z}) + K_y\ \frac{\partial}{\partial y}\ (\tilde{z}\bar{z}) - K_z\tilde{z} - \frac{\partial\tilde{z}}{\partial x}\ \int_0^x\ K_z(x',y)dx'\ . \tag{19}$$

Now our analysis is very close to the analysis given in (Molodensky, 1989) for the two-dimensional case.

The form of eq. (18) makes it possible to find the increment of the function $\xi(x,y)$ along some contours in terms of F in a manner similar to Cauchy's method for the integration of first-order quasi-linear equations in partial derivatives (Kamke, 1966). Indeed, relation (18) may be regarded as an orthogonality condition for vectors with the Cartesian coordinates

$$\vec{e}_1 = \left(\frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y}, -1 \right) ,$$

and

$$\vec{e}_2 = \left(-\frac{\partial \tilde{z}}{\partial y}, \frac{\partial \tilde{z}}{\partial x}, -F \right) .$$

Since

$$d\xi = \frac{\partial \xi}{\partial x} dx + \frac{\partial \xi}{\partial y} dy,$$

the vector $\vec{e}_1$ is also orthogonal to the vector

$$\vec{e}_3 = (dx, dy, d\xi)$$

which is tangent to the surface $\xi(x,y)$. The vector $\vec{e}_2$ is thus perpendicular to the normal to the surface $\xi(x,y)$, hence it lies in a plane that is tangent to that surface. Consequently, the curves defined by the equations

$$\frac{dx}{\left(-\frac{\partial \tilde{z}}{\partial y} \right)} = \frac{dy}{\left(\frac{\partial \tilde{z}}{\partial x} \right)} = \frac{d\xi}{(-F)} \tag{20}$$

belong to the surface $\xi(x,y)$. The first part of this equation is equal to

$$\frac{\partial \tilde{z}}{\partial x} dx + \frac{\partial \tilde{z}}{\partial y} dy = 0 ,$$

i.e. the curves under consideration coincide with the geostrophic contours $\tilde{z} = \text{const}$. The second part of (20) determines the increment of ξ along these contours:

$$d\xi\big|_{\Gamma} = \frac{F\,dx}{\partial\tilde{z}/\partial y} = -\frac{F\,dy}{\partial\tilde{z}/\partial x} = -\frac{F\,d\ell}{\partial\tilde{z}/\partial n}\,. \tag{21}$$

where $d\ell = (dx^2+dy^2)^{1/2}$ is the element of length of this contour and $\partial/\partial n$ is the derivative along the outer normal to it.

Equation (21) is fully equivalent to the original integro-differential equation (18), in the sense that any integral curve of (21) belongs to the surface $\xi(x,y)$ defined by (18) and, conversely, any solution of (18) can be represented as a family of integral curves of (21). For this reason, the condition of existence for solutions of the partial equation is equivalent to that for solutions of the equation (21). It is easy to see, that this latter is reduced to the single requirement that

$$\oint_{\Gamma} d\xi = -\oint_{\Gamma} \frac{F\,d\ell}{\partial\tilde{z}/\partial n} = 0\,. \tag{22}$$

When (22) does not hold, the increment of ξ along a closed geostrophic contour Γ does not vanish, which is incompatible with the assumption of ξ being a single-valued function of the coordinates.

Let us show now that the condition of existence for the first-order terms in (22) determines uniquely the function $\phi(\tilde{z})$ which enters into the zero-order equation. To show this, one expresses the function F which enters into (22) in terms of ϕ and the known functions $\tilde{z}$, $\bar{z}$, γ.

Substitution of (8) into (11) yields

$$\vec{K}(x,y) = \frac{i\sigma}{\omega_z}\left[\vec{\omega}_1 + \frac{1}{2}\left[\vec{e}_x\left(\phi'\gamma\frac{\partial\tilde{z}}{\partial y} \quad \phi\frac{\partial\gamma}{\partial y}\right) - \vec{e}_y\left(\phi'\gamma\frac{\partial\tilde{z}}{\partial x} + \phi\frac{\partial\gamma}{\partial x}\right)\right] + \vec{e}_z\left(\phi'(\nabla\tilde{z})^2+\phi\,\Delta\tilde{z}\right)\right]\,,$$

where

$$\phi' = \frac{d\phi(\tilde{z})}{d\tilde{z}}\,.$$

Taking into account this expression and (19), one can write eq. (22) in the form:

$$\oint_\Gamma \frac{d\ell}{\partial\bar{z}/\partial n}\left[\left[\omega_x + \frac{1}{2}\left(\phi'\gamma\frac{\partial\tilde{z}}{\partial y} + \phi\frac{\partial\gamma}{\partial y}\right)\right]\frac{\partial(\tilde{z}\bar{z})}{\partial x} +\right.$$

$$+\left[\omega_y - \frac{1}{2}\left(\phi'\gamma\frac{\partial\tilde{z}}{\partial x} + \phi\frac{\partial\gamma}{\partial x}\right)\right]\frac{\partial(\tilde{z}\bar{z})}{\partial y} - (\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z})\,\tilde{z} - \tag{23}$$

$$\left. - \frac{\partial\tilde{z}}{\partial x}\int_0^x (\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z})\,dx'\right] = 0.$$

The last term in the left hand side of (23) can be transformed in the following way: By taking into account that on the contour Γ in accordance with (21)

$$\frac{d\ell}{\partial\bar{z}/\partial n}\frac{\partial\tilde{z}}{\partial x} = dy \quad ,$$

one can write

$$-\oint_\Gamma \frac{d\ell}{\partial\bar{z}/\partial n}\frac{\partial\tilde{z}}{\partial x}\int_0^x (\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z})\,dx' = -\iint_s(\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z})\,ds \quad ,$$

where $ds = dx\,dy$ and s is the area of the region, which lays in the plane (x,y) and is bounded by the contour Γ. Taking into account now that

$$\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z} = \mathrm{div}\,(\phi\nabla\tilde{z}) \quad ,$$

and using the well known Gaussian formula, one writes this term in the form:

$$-\iint_s(\phi'(\nabla\tilde{z})^2 + \phi\Delta\tilde{z})\,ds = -\oint_\Gamma \phi(\nabla\tilde{z},\bar{n})\,d\ell = -\oint_\Gamma \phi\frac{\partial\bar{z}}{\partial n}\,d\ell \;.$$

In accordance with (8), ϕ is a function of the single argument $\bar{z}$, and the values ϕ, ϕ' are constants on the contour Γ, where $\bar{z}$ = const. Therefore, the values ϕ,ϕ' can be shifted in front of the symbol of integration, and eq. (23) can be presented in the form of an ordinary differential equation with respect to the single unknown function $\phi(\tilde{z})$:

$$c_1(\tilde{z})\phi'(\tilde{z}) + c_2(\tilde{z})\phi(\tilde{z}) + c_3(\tilde{z}) = 0 \tag{24}$$

where

$$c_1(\tilde{z}) = \frac{1}{2}\oint_\Gamma \frac{d\ell}{\partial\tilde{z}/\partial n}\left[\gamma\left(\frac{\partial\tilde{z}}{\partial y}\frac{\partial(\tilde{z}\bar{z})}{\partial x} - \frac{\partial\tilde{z}}{\partial x}\frac{\partial(z\bar{z})}{\partial y}\right) - \tilde{z}\left(\frac{\partial\tilde{z}}{\partial n}\right)^2\right]$$

$$= -\frac{\tilde{z}}{2}\oint_\Gamma \frac{d\ell}{\partial\tilde{z}/\partial n}\left(\gamma^2 + \left(\frac{\partial\tilde{z}}{\partial n}\right)^2\right) ; \tag{25}$$

$$c_2(\tilde{z}) = \frac{1}{2}\oint_\Gamma \frac{d\ell}{\partial\tilde{z}/\partial n}\left[\left(\frac{\partial\gamma}{\partial y}\frac{\partial(\tilde{z}\bar{z})}{\partial x} - \frac{\partial\gamma}{\partial x}\frac{\partial(\tilde{z}\bar{z})}{\partial y}\right) - \tilde{z}\Delta\tilde{z} - \left(\frac{\partial\tilde{z}}{\partial n}\right)^2\right] ;$$

$$c_3(\tilde{z}) = -(\omega_1)_z\, s(\tilde{z}) + \oint_\Gamma \frac{d\ell}{\partial\tilde{z}/\partial n}\left(\omega_x\frac{\partial(\tilde{z}\bar{z})}{\partial x} + \omega_y\frac{\partial(\tilde{z}\bar{z})}{\partial y} - (\omega_z)_1\tilde{z}\right) .$$

Taking into account now that the element of the surface ds is equal to

$$ds = \frac{d\ell d\tilde{z}}{\partial\tilde{z}/\partial n} , \tag{26}$$

we present the expression for $c_2(\tilde{z})$ in the form:

$$c_2(\tilde{z}) = \frac{1}{2}\frac{d}{d\tilde{z}}\iint_s \left(\frac{\partial\gamma}{\partial y}\frac{\partial(\tilde{z}\bar{z})}{\partial x} - \frac{\partial\gamma}{\partial x}\frac{\partial(\tilde{z}\bar{z})}{\partial y} - \tilde{z}\Delta\tilde{z} - \left(\frac{\partial\tilde{z}}{\partial n}\right)^2\right) ds =$$

$$\frac{1}{2}\frac{d}{d\tilde{z}}\iint_s \operatorname{div}\bar{A}\, ds = \frac{1}{2}\frac{d}{d\tilde{z}}\oint_\Gamma (\bar{A},\bar{n})\, d\ell = \tag{27}$$

$$\frac{1}{2}\frac{d}{d\tilde{z}}\oint_{\Gamma}\frac{d\ell}{\partial\tilde{z}/\partial n}(\vec{A},\nabla\tilde{z}) ,$$

where

$$\vec{A} = \vec{e}_x\left(-\gamma\frac{\partial(\tilde{z}\bar{z})}{\partial y} - \tilde{z}\frac{\partial\bar{z}}{\partial x}\right) + \vec{e}_y\left(\gamma\frac{\partial(\tilde{z}\bar{z})}{\partial x} - \tilde{z}\frac{\partial\bar{z}}{\partial y}\right) . \tag{28}$$

Substituting (28) into (27), one realizes that

$$c_2(\tilde{z}) = \frac{dc_1(\tilde{z})}{d\tilde{z}} . \tag{29}$$

The expression for the coefficient $c_3(\tilde{z})$ is performed analogously. Using (26), we get:

$$c_3(\tilde{z}) = -(\omega_1)_z\left(s(\tilde{z}) + \tilde{z}\frac{ds(\tilde{z})}{d\tilde{z}}\right) + \frac{d}{d\tilde{z}}\left[\iint_s \operatorname{div}\vec{B}\,ds\right] =$$

$$= -(\omega_1)_z\frac{d}{d\tilde{z}}(\tilde{z}s(\tilde{z})) + \frac{d}{d\tilde{z}}\left[\oint_{\Gamma}\frac{d\ell}{\partial\tilde{z}/\partial n}\left(\frac{\partial\tilde{z}}{\partial x}B_x + \frac{\partial\tilde{z}}{\partial y}B_y\right)\right], \tag{30}$$

where

$$\vec{B} = \tilde{z}\bar{z}(\omega_x\vec{e}_x + \omega_y\vec{e}_y). \tag{31}$$

Substituting (31) into (30) and taking into account, that $\tilde{z}|_{\Gamma} =$ const., we get

$$c_3(\tilde{z}) = \frac{dD(\tilde{z})}{d\tilde{z}} , \tag{32}$$

where

$$D(\tilde{z}) = -\tilde{z}(\omega_1)_z s(\tilde{z}) + \tilde{z}\oint_{\Gamma}\frac{d\ell\bar{z}}{\partial\tilde{z}/\partial n}\left(\omega_x\frac{\partial\tilde{z}}{\partial x} + \omega_y\frac{\partial\tilde{z}}{\partial y}\right) . \tag{33}$$

The conditions (29), (32) make it possible to integrate the differential equation (24) analytically. Indeed, using (29), we get

$$c_1(\tilde{z})\phi'(\tilde{z}) + c_2(\tilde{z})\phi(z) = (c_1(\tilde{z})\phi(\tilde{z}))' ,$$

and equation (24) is reduced to

$$(c_1(\tilde{z})\phi(\tilde{z}) + D(\tilde{z}))' = 0 .$$

After the integration of this equation with respect to $\tilde{z}$ we get:

$$\phi(\tilde{z}) = -\frac{D(\tilde{z})}{c_1(\tilde{z})} + \frac{\text{const.}(\tilde{z})}{c_1(\tilde{z})} .$$

Using the expression for $c_1(\tilde{z})$ (25), one sees that the coefficient $c_1(\tilde{z})$ is equal to zero in the points of extrema of the function $\tilde{z}(x,y)$. Taking into account, that the velocities $\vec{v}^{(0)}$ are bounded in the vicinities of these points, we can counclude, that

$$\text{const.}\ (\tilde{z}) = 0 ,$$

and, as a final result,

$$\tilde{\phi(z)} = -\frac{D(\tilde{z})}{c_1(\tilde{z})} - 2\,\frac{-s(\tilde{z})(\omega_1)_z + \oint_\Gamma \frac{d\ell\bar{z}}{\partial\tilde{z}/\partial n}\left(\frac{\partial\tilde{z}}{\partial x}\omega_x + \frac{\partial\tilde{z}}{\partial y}\omega_y\right)}{\oint_\Gamma \frac{d\ell}{\partial\tilde{z}/\partial n}\left(\gamma^2 + \left(\frac{\partial\tilde{z}}{\partial n}\right)^2\right)} . \tag{34}$$

Equations (34) and (8) give the full solution of the problem under consideration.

2.1 THE QUALITATIVE ANALYSIS OF THE RESULTS

It is interesting to compare (34) with the well known Poincare's solution (Lamb, 1932). It is known that the last one describes the oscillations (generally, with finite amplitude and arbitrary frequency) of the homogeneous, incompressible, inviscid liquid, which is surrounded by a non-uniformly rotating rigid container with an ellipsoidal boundary.

Our solution (34) describes the more particular case in so far as we consider only the small oscilations for the limiting case of the very long periods ($\sigma/\omega \to 0$). At the same time, in some aspects, it is essentially more general because it describes the motions not only in the ellipsoidal cavity, but in the cavity of arbitrary geometry.

It is easy to show that this solution predicts some new effects, which are absent in the case of the Earth model with an elliptical core-mantle boundary (which is axially symmetrical with respect to the axis of rotation). They are as follows:

1. It is known, that the free Eulerian (Chandler) wobble of the Earth model with an elliptical core-mantle boundary excites the motions in the liquid core with an invariant z-component of angular momentum. Consequently, the Chandler wobble is not accompanied by the l.o.d. (length of day) variations at the same (Chandlerian) period.

2. Inversely, the tidal variations of the l.o.d. excite the currents inside the liquid core without the x- and y- components of angular momentum. As a result, the long periodic tidal waves don't excite the polar motion.

Using our expression (34), one can see, that these both properties don't take place in the general case of an arbitrary core-mantle boundary. Moreover, in the case of Chandler wobble the amplitude of x-component of the angular momentum in the liquid core does not generally coincide with the amplitude of the y-component. As a result, the trajectory of the Chandler wobble is not circular, but elliptical.

Taking these circumstances into account, it is possible to formulate the inverse problem of estimation of the possible core-mantle boundary heterogeneities based on modern astrometrical data. To make this, we shall consider first the dynamics of the liquid core for some very simple models of the core-mantle boundary.

Let us begin the qualitative analysis of equ. (34) from the consideration of some very simple cases.

2.1.1. If the container is symmetrical with respect to the plane $z = 0$, then $z_2(x,y) = -z_1(x,y)$, and $\bar{z} = \gamma = 0$. Substituting these values into (34), we get:

$$\phi(\tilde{z}) = -2(\omega_1)_z \frac{s(\tilde{z})}{\oint\limits_{\Gamma} \frac{\partial \tilde{z}}{\partial n} \, d\ell} . \tag{35}$$

It is interesting to note, that the geostrophic flow determined by (35) is not dependent on the components ω_x, ω_y. Probably, the physical sense of this conclusion can be interpreted as follows: it is known (Greenspan, 1969), that the geostrophic currents organize the system of Proudman-Taylor columns, which are similar to the rigid bodies in several aspects. For example, these columns have the tendency to conserve their form and sizes. It is, indeed, easy to see, that the stationary geostrophic flow described in section 1 is possible only in the case, where the sizes of the columns in the direction parallel to $\vec{\omega}$ is not dependent on time (in the opposite case the stationary flow along the geostrophic contours $\tilde{z}$=const. does not satisfy the condition (6)). If the boundary surfaces are mobile, with respect to the vector $\vec{\omega}$, then, in general, the flow is not stationary and the kinetic energy of the fluid is not constant.

The compression of the Proudman-Taylor columns in the z-direction is accompanied by a decreasing z-component of circulation $(\mathrm{curl}\ \vec{v})_z$ and of the total knetic energy; in the case of stretching the signs are opposite.

From the simple geometrical considerations it is easy to see, that for the case $z_2(x,y) = -z_1(x,y)$, the small tilt of the vector $\vec{\omega}$ is not accompanied by any compression or stretching of the Proudman-Taylor's columns, and the geostrophic flows are not excited. This is why $\phi(\tilde{z})$ is not dependent on the components ω_x, ω_y.

Using the general relation (34), one can see that the ratio of the velocities of geostrophic flow to the velocities of the column's compression or stretching are in the general case of the order of ω/σ. In the limiting case $\sigma \to 0$ this ratio tends to infinity. It means, that even very small long-periodic polar motion (such as the Chandler wobble) results in significant geostrophic motions. In the case of Chandler wobble of the real Earth the boundary of the liquid core is close to the sphere, and the geostrophic contours are close to the circles with centers on the axis of the Earth's rotation. Such motion has an angular momentum mainly in the direction of the z axis and must result in variations of the length of day with Chandler period. We shall consider the numerical estimation of this effect in section 2.3.

2.1.2 For the most simple case when the container is symmetrical both with respect to the plane $z = 0$ and to the axis $x = y = 0$, the values $\partial\tilde{z}/\partial n$ are constant on the contour Γ, and relation (35) is reduced to

$$\phi(\tilde{z}) = -\frac{2s(\tilde{z})(\omega_1)_z}{\ell(\tilde{z})\ \partial\tilde{z}/\partial n},$$

where $\ell(\tilde{z})$ is the length of the contour line $\tilde{z} =$ const. Taking into account, that this contour coincides with the circle, we get:

$s(\tilde{z}) = \pi r^2$, $\ell(\tilde{z}) = 2\pi r$, $\phi(\tilde{z}) = -r/\partial z/\partial n$, and $\vec{v}^{(0)}$

$$= -(\omega_1)_z\ \vec{e}_z \times \vec{n},$$

(where $r = (x^2+y^2)^{1/2}$ is the radius of circle, and $\vec{r}$, as before, is the radius-vector). This result has a trivial physical meaning: Obviously, the non-uniform rotation of the container with respect to its axis of symmetry does not excite any differential motion in the liquid, and the liquid conserves its uniform rotation around the z axis in space. In the non-uniformly rotating system of coordinates (x,y,z) this motion is described similarly to the non-uniform rotation of a rigid body.

Using the relation (35), one can see, that the dynamic coupling between the liquid core and mantle is determined not by the value of the deviation of the core-mantle boundary with respect to the axially symmetric geometry but only by the ratios of the bounded surfaces $s(\tilde{z})$ inside the contours Γ to the lengths of the contours

$$\ell = \oint_{\Gamma} d\ell \; .$$

The dynamic coupling is significant when the ratios s/ℓ are small enough. This situation takes place, for example, when the contours $\tilde{z}$ = const. present the system of closed contours with a relatively small scale of lengths.

2.1.3. Now we can consider the more realistic case where the core-mantle boundary is close enough to the sphere. If we propose, in addition, that the partial derivatives of the core-mantle boundary (with respect to x,y) are close enough to the same derivatives of the unperturbed (spherical) boundary, then we can write

$$\tilde{z} \approx 2b\cos\theta, \; \frac{\partial\tilde{z}}{\partial n} \approx -2 \operatorname{tg} \theta \quad ,$$

and

$$\gamma^2 \ll \left(\frac{\partial\tilde{z}}{\partial n}\right)^2 ,$$

where θ is the co-latitude and a is the mean radius of the core-mantle boundary.

The geostrophic contours Γ are close to the circles, $d\ell = a\sin\theta \, d\lambda$ and one can estimate the contribution to the integral in (34) as follows:

$$\oint_{\Gamma} \frac{d\ell}{\partial\tilde{z}/\partial n} \left(\gamma^2 + \left(\frac{\partial z}{\partial n}\right)^2\right) \approx \oint_{\Gamma} \frac{\partial\tilde{z}}{\partial n} \, d\ell \approx -4\pi a \, \frac{\sin^2\theta}{\cos\theta} \; .$$

Taking into account that

$$\frac{\partial\tilde{z}/\partial x}{\partial\tilde{z}/\partial n} = \cos\lambda \; , \quad \frac{\partial\tilde{z}/\partial x}{\partial\tilde{z}/\partial n} = \sin\lambda \quad ,$$

where λ is longitude, equ. (34) is presented in the form:

$$\phi(\tilde{z}) = -\frac{1}{4\pi} \operatorname{ctg} \theta \int_0^{2\pi} \overline{z}(\cos\lambda\delta\omega_x + \sin\lambda\delta\omega_y) \, d\lambda \; . \qquad (36)$$

Using the presentation of function $\overline{z}(x,y)$ in the form of a Fourier series

$$\bar{z} = \sum_{n=0}^{\infty} \bar{z}_n^c (R) \cos n\lambda + \bar{z}_n^s (R) \sin n\lambda \quad ,$$

$$\left[\text{where } R = \sqrt{x^2+y^2} \right] \quad ,$$

one realizes that the integral (36) does not vanish only for the harmonics of degree $n = 1$. After integration of (36) with respect to λ, we get:

$$\phi(\bar{z}) = - \frac{1}{4\pi} \operatorname{ctg} \theta \, (\bar{z}_1^c \, \delta\omega_x + \bar{z}_1^s \, \delta\omega_y) \quad . \tag{37}$$

The substitution of this expression into (8) yields all three components $v_x^{(0)}$, $v_y^{(0)}$, $v_z^{(0)}$ and the angular momentum cf the liquid core uniquely. Using the definition of the z-component of the angular momentum, we estimate:

$$M_z = \int_\tau (xv_y^{(0)} - yv_x^{(0)}) \, \rho \, d\tau \sim \frac{c_1 \bar{z}_1}{a} \delta\omega \quad , \tag{38}$$

where c_1 is the moment of inertia of the liquid core, $\bar{z}_1 = (\bar{z}_1^{c\,2} + \bar{z}_1^{s\,2})^{1/2}$,

a is the mean radius of the liquid core and $\delta\omega = (\delta\omega_x^2 + \delta\omega_y^2)^{1/2}$.

Using the numerical value $\delta\omega \sim 10^{-6} \omega_z$ (which corresponds to an amplitude of Chandler wobble of ~ 0,2 arc sec), and $c_1 \sim 0.1$ c (where c is the moment of inertia of the mantle) we find

$$\frac{\delta M_z}{M} = - \frac{(\delta\omega_z)_{mantle}}{\omega_z} \sim 10^{-7} \frac{\bar{z}_1}{a} \quad , \tag{39}$$

where $M = c\omega_z$ is the angular momentum of the mantle and $(\delta\omega_z)_{mantle}$ is the amplitude of the periodic variation of $(\omega_z)_{mantle}$ at Chandler frequency.

To make this relation more obvious let us consider the case where $\bar{z}_1$ is a linear function of R, i.e. $\bar{z}_1 = K R$. It is easy to see that the value $d = K/e$ (where e is the geometric flattening of the liquid core) is equal to the tilt of the main axis of the liquid core's ellipsoid of inertia with respect to the axis of rotation $\bar{\omega}$. Taking this relation into account and introducing the values $\delta\lambda = \int(\delta\omega_z)_{mantle} \, dt$, which is equal to the angular displacement of the mantle in the direction of longitude, and $(\delta\lambda_0)$ which is the amplitude of $\delta\lambda$, then we may write (39) in the form

$$\delta\lambda_0 = 6 \text{ m arc sec.} \cdot \alpha \, . \tag{40}$$

If we assume that the accuracy of modern VLBI-measurements of periodical processes is of the order of 0.15 $\tau\rho$ 0.18 m arc sec. (Gwinn, and Shapiro, 1986), then the measurements of $\delta\lambda_0$ make it possible to determine α with the accuracy of the order of 0.025 - 0.030 rad or 1.5 - 2 degrees.

We may thus conclude that the measurements of l.o.d.-variations at the Chandler period can be considered as a very sensitive method for investigating the core-mantle boundary. Some numerical estimations and examples are given in Section 3.

Inversely, using (34), it is easy to show, that the influence of the tidal l.o.d. variations on polar motion is extremely weak. Under no circumstances do they exceed a value of the order of 10^{-3} m arc s, which is two orders of magnitude smaller than the current accuracy inherent in the harmonic analysis of VLBI measurements.

2.1.4. The influence of the core-mantle topography on the Chandler period and on the ellipticity of the Chandler wobble can be estimated as follows. Using Poincare's presentation for the velocities within the ellipsoidal liquid core in the form

$$\tilde{v}_x \sim - \frac{z\sigma}{\omega_z} \delta\omega_y \, ,$$

$$\tilde{v}_y \sim \frac{z\sigma}{\omega_z} \delta\omega_x \, ,$$

$$\tilde{v}_z \sim \frac{\sigma}{\omega_z} (x\delta\omega_y - y\,\delta\omega_x) \, ,$$

and comparing these expressions with our solutions (8) and (37), we realize that

$$v_x^{(0)}/\tilde{v}_x \sim v_y^{(0)}/\tilde{v}_y \sim \frac{\bar{z}_1}{z} \frac{\omega_z}{\sigma} \, ;$$

$$v_z^{(0)}/\tilde{v}_z \sim \frac{\bar{z}_1 \, \partial\bar{z}_1/\partial\lambda}{2R^2} \frac{\omega_z}{\sigma} \, .$$

In the case of the Chandler wobble ($\omega_z/\sigma \sim 400$) the first ratio is equal to unity if $\bar{z}_1/z \sim 1/400$, i.e. whenever the deviations of core-mantle boundary with respect to the ellipsoid are of the order of only 10 km. Consequently, the topography of the core-mantle boundary for the real Earth model can exert an extremely strong influence on the distribution of the currents in the liquid core. Nevertheless, the influence of these currents on the Chandler period T_{Ch} and on the ellipticity of the Chandler wobble is comparatively weak.

To show this it is enough to remember that the hydrodynamic motions under consideration have mainly z-component of the angular momentum and, consequently, they influence mainly the l.o.d. variations.

2.2 THE COMPARISON WITH THE MEASUREMENTS

The results of the Maximum Entropy Spectrum Analysis of the modern V.L.B.I.-l.o.d. data are presented in Fig. 1. It is necessary to note, that the amplitudes obtained by MESA-technique are well known to be problematic in general. One can see, that some peak with very small amplitude in the vicinity of Chandler frequency probably exist, but its ratio to the level of noise is too small to identify it with the necessary reliability.

3. LINEAR AND NONLINEAR MODELS OF THE DYNAMICAL POLE TIDES

The asymtotic behaviour of solutions to Laplace's tidal equations (L.t.e.) at low (for example, Chandlerian) frequencies was considered in recent years in may papers (see, for example, Dickman 1985, 1986; O'Connor, 1986; Carton, Wahr, 1986; Molodensky, 1989; Groten, Lenhardt, Molodensky, 1990). It was shown in the last two papers, that for the limiting case $\sigma/\omega \to 0$ (where σ is the tidal frequency and ω is the angular velocity of the Earth's diurnal rotation)) these solutions are unstable in that the functions involved in the zero-order approximation are not uniquely determined by the zero-order equations, but depend on first-order terms (terms of the order of σ/ω) as well. As a result, the solutions of L.t.e. significantly depend on the very small terms entering the L.t.e. In the most general case the equations describing the pole tides in the thin (in comparison with the Earth's radius r) layer of the liquid are described by the known system of governing equations (see, for example, Kagan, Monin, 1978) read:

$$\dot{\vec{v}} + (\vec{v},\nabla)\vec{v} + 2\omega\cos\theta\ [\vec{e}_r\vec{v}] = -g\nabla(\zeta-\bar{\zeta}) + \vec{F}\ ,$$

$$\dot{\zeta} = -\operatorname{div}_2(\vec{v}h)\quad , \tag{41}$$

where $\vec{v}=(v_\theta,\ v_\lambda)$ is the two-dimensional vector of tidal velocity, ζ is the level of the ocean, $\bar{\zeta}=\bar{\zeta}(\theta,\lambda)$ is the associated equipotential surface, ω is again the angular velocity of the Earth's diurnal rotation, $\vec{e}_r$ is the radius-vector, g is the acceleration due to gravity at the Earth's surface, $h = h(\theta,\lambda)$ is the ocean depth, $\mathrm{div}_2(\vec{v}h)$ is the divergence of two-dimensional vector $\vec{v}h = h\cdot(v_\theta, v_\lambda)$, $\vec{F} = (F_\theta, F_\lambda)$ are the components of the force of friction, which act on the element of the liquid. In the most general case the vector $\vec{F}$ is presented in the form

$$\vec{F} = -k_0\vec{v} + k_h\Delta\vec{v},$$

where k_0 is the coefficient of the bottom friction and k_h is the coefficient of turbulent horizontal friction. In case of turbulent motion the coefficient k_0 is proportional to $|\vec{v}| = (v_\theta^2 + v_\lambda^2)^{½}$ being a function of the depth distribution $h(\theta,\lambda)$; in case of laminar motion k_0 is independent of $\vec{v}$ and is a function of the distribution $h(\theta,\lambda)$ only.

3.1 LINEAR MODEL

The asymtotic behaviour of solutions to eq. (41) for the linear approximation is described by the conditions:

1) the lines of flow and isolines $\tilde{\zeta} = \zeta - \bar{\zeta}$ coincide with the geostrophic contours Γ as determined above which, for the case of a thin layer of liquid, are described by

$$\alpha = \frac{g}{2\omega a^2}\,\frac{h(\theta,\lambda)}{\cos\theta} = \mathrm{const.}$$

2) the dependence of $\zeta(\alpha)$ is determined by the ordinary differential equation (Molodensky, 1989):

$$[c_1(\alpha)\,\zeta'(\alpha)]' + c_3(\alpha)\zeta(\alpha) = b(\alpha)\ , \tag{42}$$

where the prime denotes differentiation with respect to α and c_1, c_3, b are the known functions of the depth distribution, which are described by the relations:

$$c_1(\alpha) = \alpha \oint_\Gamma \frac{(\kappa+i\sigma)\partial\alpha/\partial n}{\cos\theta}\,d\ell\ ,$$

$$c_3(\alpha) = -\frac{2i\sigma\omega}{a^2}\oint_\Gamma \frac{d\ell}{\partial\alpha/\partial n}\ , \tag{43}$$

$$b(\alpha) = \frac{2i\sigma\omega}{a^2} \oint_\Gamma \frac{\bar{\zeta} d\ell}{\partial\alpha/\partial n},$$

where a is the mean radius of the earth, $d\ell$ is the element of length of the geostrophic contour Γ (here for simplicity we put $k_h=0$).

The solutions of this equation depend mainly on the dimensionless value

$$H = \frac{g}{4\omega^2 \cos^2\theta} \frac{h}{r_0{}^2} = \frac{73}{\cos^2\theta} \frac{ha}{r_0{}^2},$$

r_0 is the horizontal scale of length of the closed contours α = const. Taking into consideration the dimensionless value γ, which is equal to the ratio of the mean values of ζ to the mean value of $\bar{\zeta}$ in the same region, one obtains a simple estimate of the relation $\gamma(H)$ for the dissipationless case as follows (Molodensky, 1989):

H = 0,1	0,2	0,5	1,0	2,0	5,0
γ = 0,520	0,656	0,812	0,893	0,944	0,976

From this table one realizes that, when depth increases (or in, other words, equivalently: when the horizontal dimensions of the closed geostrophic contours α = const. decrease), the dynamic tide approaches the static one. When $\cos^2\theta=0.5$ and h=4 km, H=0.1 corresponds to $r_0=6\cdot10^3$ km and H=5 to $r_0=8\cdot10^2$ km. One sees from the aforementioned table that, for the first case, the deviation of the dynamic pole tide from the statical one is significant, whereas for the second case it is very small.

The isolines α=const., for the real ocean model, are given in Levitus (1982) and were recalculated by us on the ground of the spherical harmonics expansion of the depths distribution for degrees $\ell\leq180$. The results are shown on Fig. 20, 20b. These pictures reveal that in most regions of the real ocean the characteristic scale of the length r_0 is in almost any case less than $(2\text{-}3)\cdot10^3$ km. Moreover, the regions, where isolines α=const. are closed, cover a comparatively small part of the oceanic surface. Using the simple estimates based on the aforementioned relation $\gamma(H)$, one sees that, in linear approximation, everywhere in the ocean we have

$$\zeta < 0.1\,\bar{\zeta},$$

and the influence of the dynamic pole tide on the Chandler period is less than the errors inherent in modern measurements: $\delta T_{Ch} < 1$ day.

3.2 NON-LINEAR MODELS

As was mentioned above, the instability of the solutions of the equations (41) results in the strong dependence of the solutions on small perturbing terms. To estimate the non-linear effects in the dynamic theory of pole tide, it is necessary to compare two groups of small perturbations.

1) the linear terms $\dot{\vec{v}}$, $(\vec{F})_{laminar}$ and

2) the non-linear terms $(\vec{F})_{turbulent}$, $(\vec{v},\nabla)\,\vec{v}$.

To estimate the non-linear terms, it is necessary to take into account not only the tidal currents, but also the nontidal stationary currents in the real ocean, i.e. to present the velocity vector $\vec{v}$ as a sum:

$$\vec{v} = \vec{v}_0 + \vec{v}_1$$

where $\vec{v}_0$ is the vector of velocity of the nontidal stationary ocean currents and $\vec{v}_1$ is relatively small vector tidal flow, in comparison with $\vec{v}_0$. By taking into account that, for the real ocean, $\vec{v}_0$ is of the order of a few cm s^{-1} one realizes that the non-linear group of perturbing terms is greater than, or of the same order as, the linear group, and they must consequently be included in our considerations. The influence of non-linear terms is manifested in the following new properties of the governing equations:

1) The geostrophic contours (lines of flow) are determined not only by the depth distribution $h(\theta,\lambda)$, but also by the distribution of the world ocean currents $\vec{v}_0$. Consequently, instead of contours $h/\cos\theta$ = const., it is necessary to consider the isolines of "potential vorticity"

$$P = \frac{2\omega\cos\theta + (\mathrm{rot}\,\vec{v}_0)_r}{h} = \mathrm{const.};$$

in this case, the role of the parameter r_0 plays the role of horizontal scale of length of the closed contours P=const.

2) As the values k_0 are functions of $|\vec{v}|$, entering into (43) the coefficients c_1, c_3 are functions of $|\vec{v}|$ too. This means that the governing equations (42) are significantly non-linear.

The distribution of the surface currents $\vec{v}_0(\theta,\lambda)$ in the actual global ocean (Fahrbach et al., 1985) is presented in Fig. 3. Comparing Fig. 2 and 3, one realizes, that the difference between the isolines α=const. and P=const. is significant and that it is manifested in the bigger scale length of the regions bounded by isolines P=const. in comparison with the regions, bounded by α=const. As a result, the in fluence of the dynamic pole tide on the Chandler period is essentially greater for the non-linear model than for the linear one.

From Fig. 3 is seen that, for the non-linear model, the typical values r_0 are of the order of $(3\div5)\cdot10^3$ km. Taking into account the relation $\gamma(H)$ given in the aforementioned table we conclude, that for the non-linear model without dissipation (i.e. for the model which takes into account the non-linear term $(v,\nabla)\, v$ only) the possible values of γ are of the order of $0.6\div0.8$. Consequently, the influence of the dynamical pole tides on the period of Chandler wobble may be of the order of $6\div12$ days.

The more exact numerical estimation of the dynamical pole tides for the non-linear approximation is complicated mainly by the following circumstances:

1. The absence of the rigorous mathematical models of oceanic bottom and horizontal turbulent friction. As a result, the models of the dependence of the coefficients k_1, k_2 on $|\vec{v}|$, h are based on some inexact empirical and semi-empirical laws (Schwiderski, 1980);

2. The absence of the detailed models of the world-ocean currents distribution with the depth.

As a result, the exact estimation of the bottom friction is impossible even in the case when the law of bottom friction is known.

Nevertheless the sign of the effects of the ocean friction is determined uniquely. As a matter of fact, when $\sigma/k \to 0$, then the deviation of the dynamic pole tide from the statical one is negative and tends to zero (Molodenski, 1989). Conseqeuntly, the period of Chandler wobble is an increasing function of the coefficients of friction k_0 and k_h. Moreover, it is known that, for turbulent motions, the values of these coefficient present an increasing (usually linear) function of the velocities (Kagan, Monin, 1978). As a result, we may claim, that the period of Chandler wobble must present an increasing function of its amplitude, and the range of the variation of the period is of the order of $6\div12$ days. It is interesting to note, that exactly the same conclusion was obtained by Melchior (1957) based on the analysis of empirical data.

It is interesting to compare the Melchior's results with the results of the latest analysis. The maximum entropy spectral analysis of the intervals 1900-1920; 1920-1940; 1940-1960; 1960-1978 and 1967-1984 was performed by Lenhardt, Groten (1985). The results are presented on Fig. 4 (circles). The results of Melchior are presented on the same picture as points. One can realize, that there is a very high probability, that hte correlation between the amplitudes and periods of Chandler wobble exist indeed.

Thus we may conclude that the interrelation between long-tern amplitude and frequency variations in polar motion may be attributed with the high probability to the influence of turbulent friction for the non-equilibrium pole tide.

CONCLUSIONS

The efforts discussed in this paper basically refer to the axis of rotation and the associated modulus of the earth rotation vector which might be expressed in terms of LOD. Insofar the title of this paper might be questioned if we assume that, by definition, the quantities considered here are related to the celestial system of reference instead of the terrestrial frame. As polar motion defines two of the Eulerian angles relating celestial to terrestial system it is more or less a matter of personal judgment whether we discuss those perturbations with respect to terrestrial or celestial frames. The sources of these perturbations are so closely related to the earth itself that our choice of title appears appropriate in order to avoid misunderstanding.

Consequently, we did not refer to a particular type of a CTRS (Boucher, 1990) such as IERS-TRF and rather treated the topic in general terms.

Two aspects have to be stressed : (1) It still appears possible to clear up existing open problems related to the fluid parts of the earth-fluid outer core and ocean - by precise analysis of polar motion and LOD-data. Consequently, astrometry has not yet been fully exploited in giving information on geophysics in a domain of frequencies where little alternative information is available and (2) by still improving the accuracy in measuring polar motion and LOD we may still get substantially better information on the physics of the earth which, together with improved atmospheric (AAM etc.) data, could lead to the possibility to model and predict polar motion and LOD better than it is carried out now. This paper fills a gap in that respect.

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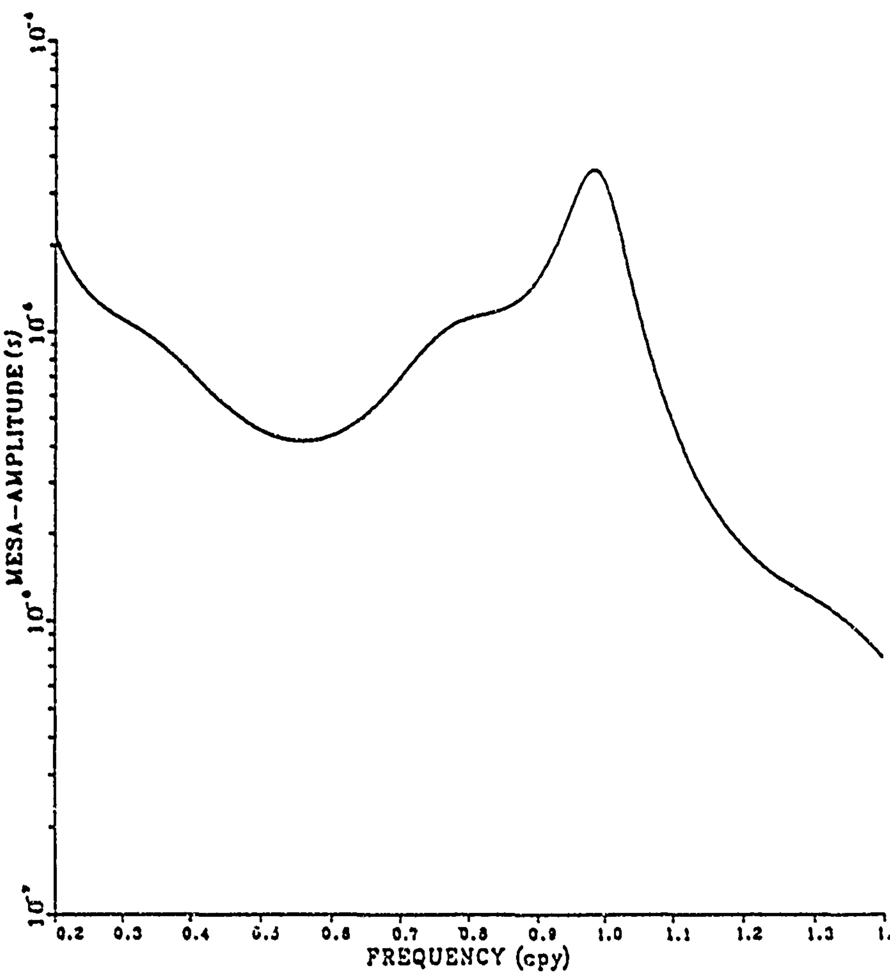

FIG.1 SPECTRUM OF L.O.D

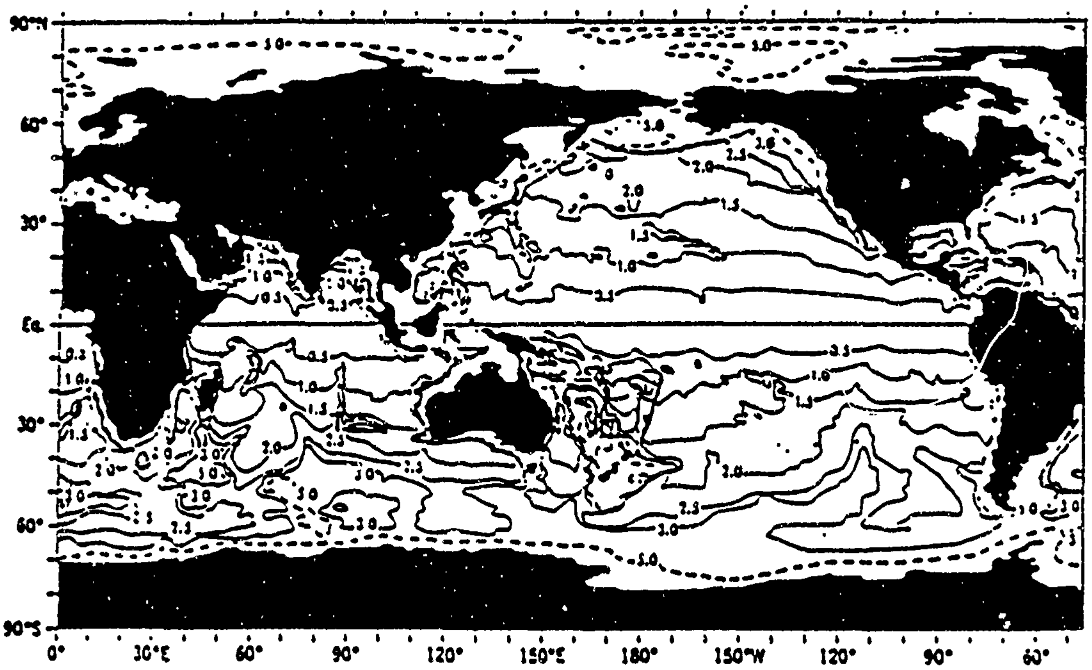

Fig.2a - Planetary vorticity ($10^{-8}m^{-1}sec^{-1}$)
(from Levitus, 1982)

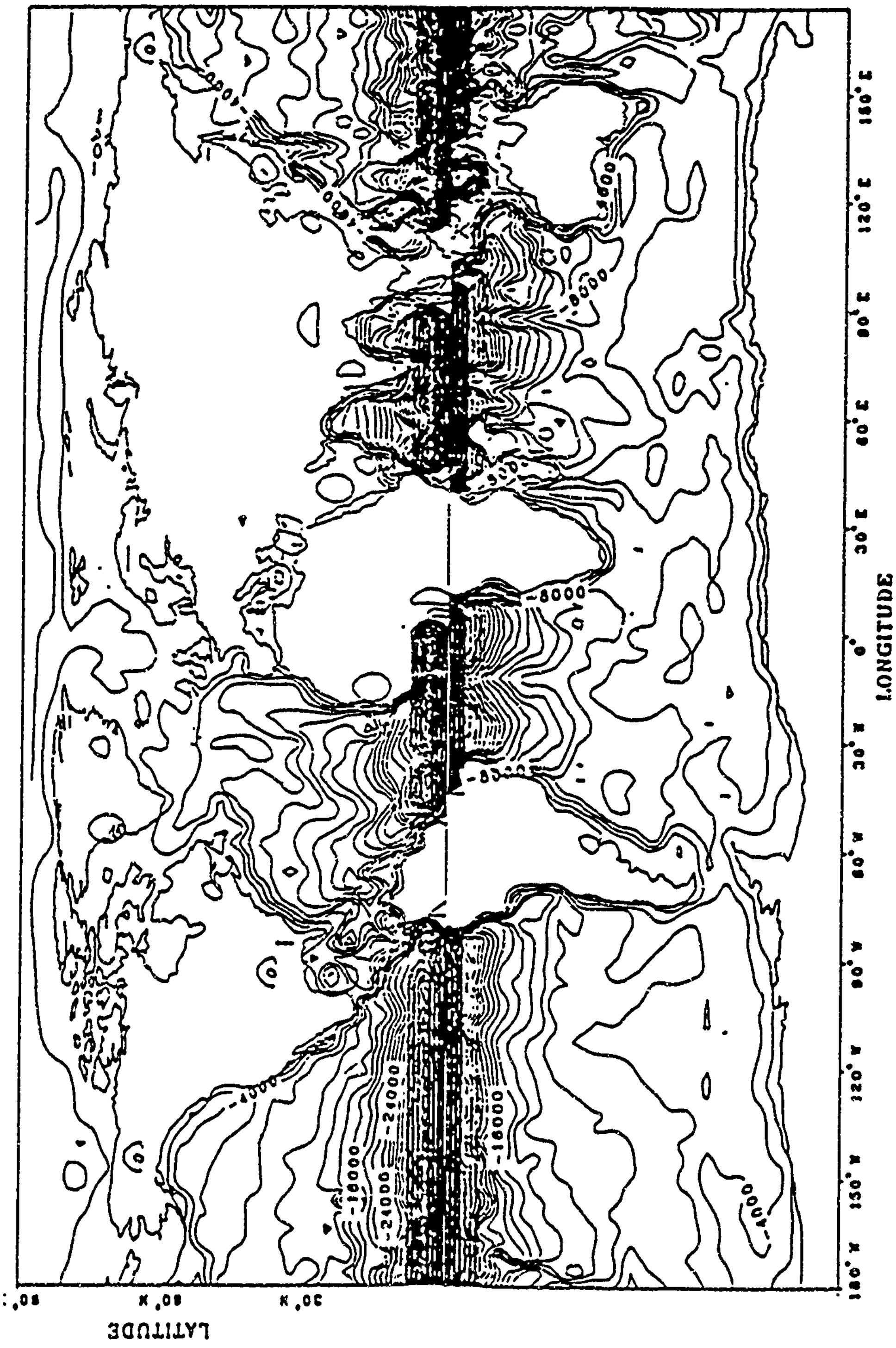

Fig. 2b. Isolines $h/\cos\theta = \text{const}$ (m).

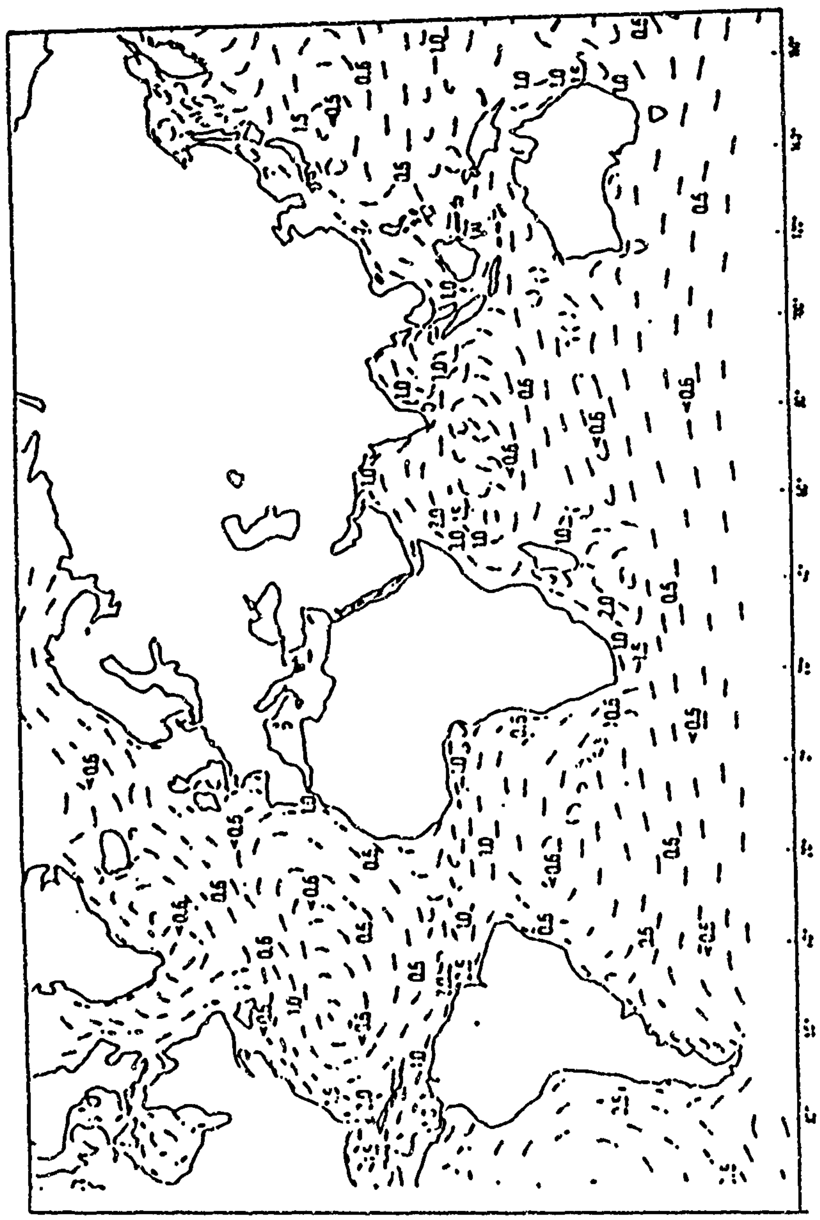

Fig. 3. World ocean surface currents, in knots, compiled from ship drift estimates for northern summer. From USA Navy (1974, 1976–1979).

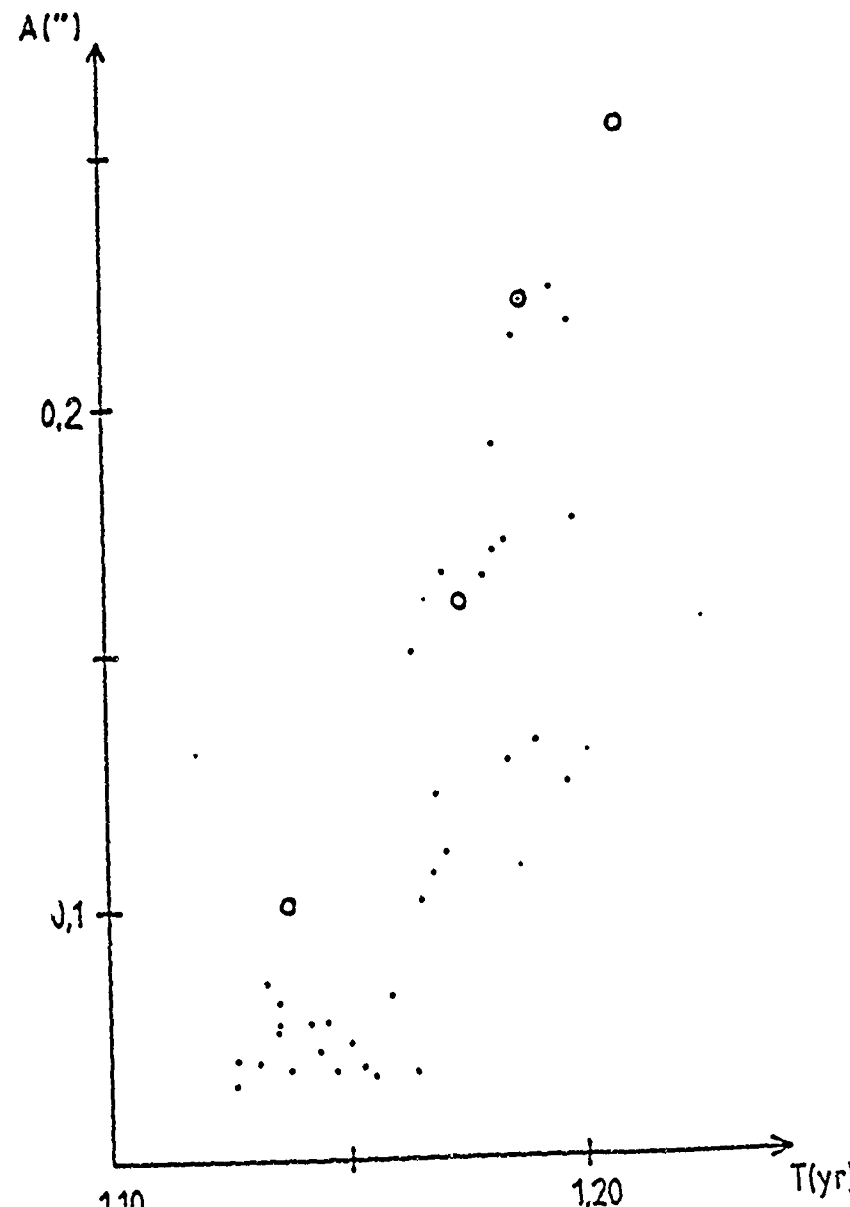

Fig. 4

The correlation between the amplitudes and periods of the Chandler wobble (the small second peak in the spectrum for the period 1940-1960 (Lenhardt, Groten, 1985) is excluded)

A LUNI-SOLAR PRECESSION AND NUTATION ANALYSIS FROM RADIO ASTROMETRIC OBSERVATIONS

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The Cambridge 5-km Radio Interferometer astrometric results from 1972 to 1987 were analysed, leading to corrections to the main terms of luni-solar precession and nutation. The method used follows Elsmore's 1976 suggestion (MNRAS **177**, 291) which takes pairs of sources nearly 12-h apart in right ascension.

Using sources whose positions would maximize the effects, 773 observations, all at 5GHz, of 21 extragalactic radio sources were considered. No selection criterion was used other than that of a minimum standard of quality of the observations. The results were grouped per source, per season, forming a total of 110 groups.

The main result shows the following corrections to the 1964 IAU System. Constant of luni-solar precession, centennial: $\Delta p_1 = 1''.11 \pm 0''.15$, 18.6y term of nutation in longitude $\Delta N = 0''.05 \pm 0''.01$. Details are in preparation for publication in Monthly Notices of the Royal Astronomical Society.

THE CELESTIAL SYSTEM AND FRAME OF IERS

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M. Feissel (Observatoire de Paris, Central Bureau of IERS)

ABSTRACT. The celestial system maintained by the International Earth Rotation Service is described in terms of physical properties of the fiducial objects, internal consistency of the frame, and agreement with the FK5 and dynamical systems.

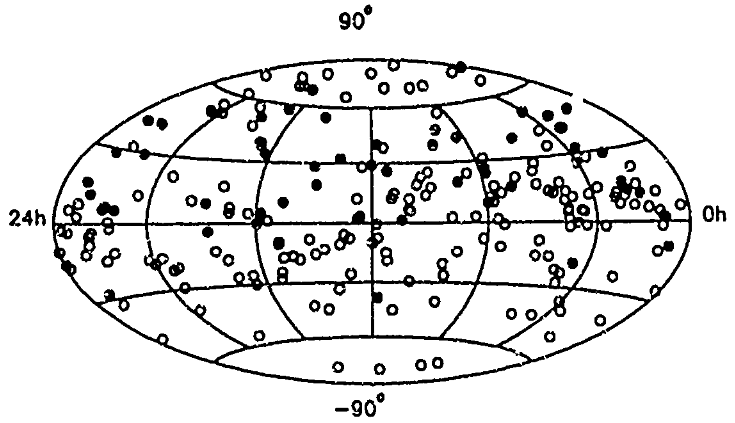
1. Introduction

The celestial reference frame of IERS is based on compact extragalactic objects observed by VLBI. It is maintained on the basis of several independent Earth rotation programs analysed by various analysis centres. We present hereafter its latest realization, which is a combination of individual frames obtained by four groups: GSFC (Ma *et al.*, 1990), JPL (Steppe *et al.*, 1990), NGS (Carter and Robertson, 1990), and USNO (Eubanks *et al.*, 1990). The combination is based on a three rotation angle model applied to a selection of radio sources common to the individual frames. The initial definition of the system and the maintenance process is described by Arias and Feissel (1990); the connection to the conventional terrestrial system of IERS at the level of 0.001" is studied by Feissel (1990). In this paper, we discuss some aspects of interest to the WGRS: the physical properties of the objects, the precision and internal consistency of the frame, and the definition and maintenance of the system axes.

2. Physical properties of the fiducial objects

The distribution on the sky of the latest realization (IERS, 1990) of the IERS celestial reference system is shown on figure 1; the declination interval covered is from -80° to +85°; filled circles represent the 51 primary sources.

Figure 1. Distribution of the 228 radio sources of RSC(IERS)90 C 01.



The physical properties of the sources have been investigated on the basis of the VLBI survey published by Preston *et al.* (1985). This survey covers the full sky; it contains 1398 objects with the following information: redshift (850), total flux (740), spectral index (1200), optical identification (1200), optical magnitude (1000). In addition, the list of IERS sources has been compared to the list of quasars selected for the HIPPARCOS/HST link. Statistics on the type of objects are given in Table 1. The redshifts span the interval 0.1-2.5 quite evenly. The total flux is in general over 1 Jansky, while the survey has a larger proportion of weaker sources. The spectral indices are between -0.8 and +1.4; this represents a biasing of the sampling with respect to the complete survey, which has a majority of spectral indices in the interval -1.0 to 0. The distribution in optical magnitudes is similar to the one of the survey, with a peak around the 18th magnitude. A part of the IERS sources has been mapped at S and X bands (e.g. Charlot 1990). The primary sources mapped show no significant structure at the angular scale of 0.001".

Table 1. Numbers of sources according to their characteristics

Characteristics	IERS sources		Detected in
	Primary (51)	Other (177)	Survey (917)
Optical identification			
Quasar	40	115	544
BL Lacertae	5	15	56
Galaxy	2	8	102
Other/unident	4	39	215
HIPPARCOS/quasars (95)	14	47	

3. Internal consistency of the frame

It is well known that the use of the conventional IAU 1976 Precession and IAU 1980 Theory of Nutation in the analysis of VLBI observations would give rise to systematic errors in the source position, and to the misorientation of the axes of the frames, both at the level of a few milliarcseconds. Therefore the common practice in catalogue work is to estimate additional parameters which describe the motion of the celestial pole relative to its conventional position, either by estimating celestial pole offsets for each session, or by estimating a precession correction and the amplitudes of some of the nutation terms, as allowed by the length and time density of the series of observations analysed. Sovers (1990b) studies in detail the effects of these procedures on the resulting source positions. In the combination performed by the Central Bureau of IERS, only individual frames obtained by one of these procedures are used. The slight offsets between the poles of the catalogues, that are due to inconsistent fixing of the celestial pole offsets at some reference day, are accounted for by the adjustment of rotation angles; they have no influence on the consistency of the individual frames with the combined one.

The noticeably different network geometries of the various observing programs are expected to cause regional deformations in the derived celestial frames. From detailed comparisions of large JPL and GSFC catalogues, Sovers (1990a) concludes that the only systematic difference between the two catalogues is an increase of right ascension differences for lower declinations, amounting to about 0.002" over the entire range of declinations. This estimation sets an upper limit to the regional errors to be expected in the combined frame.

The realization of the celestial reference system published in the Annual Report of IERS for 1989 contains 228 sources with different status: primary, secondary and complementary. The 51 *primary sources* were chosen on the basis of consistency of their estimated coordinates in the four individual frames, after removing the relative rotations: only sources which showed position differences under 0.0015" in all comparisons two by two were retained as primary. Their position uncertainties in the IERS frame, derived from this consistency, are smaller than 0.0007". The other sources common to at least two frames but with larger position discrepancies, are considered *secondary*; there are 40 of them in the realization described here. Finally, 137 *complementary* sources in the IERS frame were available from only one individual catalogue. Altogether 113 sources have a position uncertainty smaller than 0.001", 104 between 0.001" and 0.003", and 11 over 0.003".

4. Definition and maintenance of the system

The IERS celestial reference system is barycentric through the appropriate modelling of observations by the analysis centres which contribute individual catalogues. The condition that the sources have no proper motion is also applied by the analysis centres; however, checks are regularly performed to insure the validity of this constraint (Ma, 1990) to avoid spurious (more likely than real) motions of some sources.

The Ox axis was implicetely defined in the initial realization (Arias *et al.*, 1988) by the adoption of the right ascensions of 23 radio sources in catalogues obtained by the GSFC, the JPL, and the NGS. As these catalogues had been compiled by fixing the right ascension of 3C273B to the usual conventional FK5 value (12h 29m 6.6997s at J2000.0), the IERS Ox axis is in agreement with the FK5 origin of right ascensions (see Feissel, 1990). In addition, according to Dickey (1989), it is in agreement with the equinox of the JPL planetary frame DE200/LE200 within 0.02".

The Oz axis points in the direction of the mean pole at J2000.0 as defined by the IAU conventional models for precession and nutation. As a result of the inaccuracy of the conventional models (Herring, 1990), the Oz axis of the IERS celestial system is shifted from the expected position of the mean pole at J2000.0 by about 0.01" in longitude $\cdot \sin \varepsilon$ and 0.001" in obliquity.

The new astrometric techniques and the availability of large computers used to analyze data raise the question of how one should understand the conventional character of the celestial frame. In practice, it can be understood in two ways:

- the source coordinates themselves are considered conventional, *i.e.*, their numerical values are fixed for some time (years); this is the FKn philosophy,
- the axes of the system are considered conventionally as fixed to their initial directions, but improved or additional source coordinates available at the time of analysis are provided to the users; this is required for space navigation, Earth orientation programs, geodetic applications, linkage of celestial frames, etc. The system maintenance method applied by the Central Bureau of IERS is along these lines. It should be mentionned that such a procedure is made possible by the high model standardization in the operation of IERS, and by the fact that the implementation of an extragalactic celestial frame implies much less geophysical and astronomical modelling than does the FK series of catalogues.

New realizations of the IERS celestial reference system are produced whenever justified by the progress in the observations or in the modelling. The successive realizations produced up to now have maintained the initial definition of the axes within 0.0001"; the coordinates of the 51 primary sources have changed by less than 0.0007" (rms).

5. Conclusion: contribution of IERS to astronomy

The celestial system of IERS was defined and is maintained for the study of the Earth's orientation at a level better than 0.001". It contains presently over 100 objects with position uncertainties at this same level.

The contribution of Earth rotation observations to the extragalactic celestial frame includes the monitoring of the sources. In the near future, as a consequence of the extension of the networks in the southern hemisphere for a better coverage of the planet, the sky coverage in the southern hemisphere will be improved, and the systematic errors due to uneven latitude distribution of stations will be diminished. The networks are operated in parallel by groups which tend to extend their lists of observed objects both with common and non common sources, in order to perpetuate their celestial references on a sound basis; this results in a progressive improvement in precision and distribution of the IERS celestial frame.

The physical properties of the fiducial objects will be further investigated in view of the merging into a celestial frame with a larger scope. In its present state, the IERS celestial reference frame has a majority of quasars in the primary sources, an even sampling of redshifts, more than half of the sources having optical counterparts up to the 18th magnitude, and two-thirds of the HIPPARCOS/HST quasars belonging to it. It can be considered as a valuable contributor to general astronomical references.

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LINK BETWEEN HIPPARCOS AND VLBI CELESTIAL REFERENCE FRAMES

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ABSTRACT. The celestial frame of HIPPARCOS astrometric satellite will be materialized by the positions and proper motions of about 120000 stars relative to arbitrary origins. As the HIPPARCOS reference frame is not naturally related to fixed directions in space, it has to be linked to an inertial frame of similar quality. The technique of VLBI determines the coordinates of extragalactic radio sources precise at the milliarcsecond level in an equatorial frame. The precision expected for HIPPARCOS normal mission is 0.002" for the positions, yearly proper motions and parallaxes.

1. Introduction

The link between the VLBI and HIPPARCOS reference frames will a) "stop" the rotation of reference frame produced by the satellite, b) densify the extragalactic reference frame in optical frequencies, c) unify the radio and optical coordinate systems and allow direct comparison of the images of celestial objects obtained with radio and optical techniques with the same angular resolution.

Radio stars provide a direct link since they can be observed with both techniques without intermediate objects. To assure good observational conditions by the satellite, they must be brighter than m = 11. Their flux densities must be at least of some mJy to be adapted to the VLBI sensitivity. Besides that, their optical and radio images must be coincident at 0.002". Following these criteria Lestrade *et al.* (1982) have established a list of 22 radio stars candidate to the link. Most of the objects in this list are close binaries RSCVn with angular separations < 0.004" and flux densities < 50 mJy.

2. The method

We have studied the link between HIPPARCOS and VLBI frames on the basis of observations of radio stars.

In absence of regional deformations, the link between two frames with the same origin can be mathematically expressed by a simple rotation. If the relative orientation of the frames evolves linearly with time, the link is given by a global rotation at an arbitary epoch, represented by a matrix $[R]$ and by a matrix $[\dot{R}]$ which represents the angular velocity of rotation of one frame relative to the other.

For each radio star i, the technique of VLBI provides a vector $\vec{\sigma}_{Vi}$ at an epoch t_{Vi} of observation of the star, and an associated proper motion $\dot{\vec{\sigma}}_{Vi}$. On the other hand, the observation of the satellite will provide vector $\vec{\sigma}_{Hi}$ at an epoch t_{Hi} of observation and an associated proper motion $\dot{\vec{\sigma}}_{Hi}$.

At an arbitrary instant t, the link equations between both frames given by one commonly observed radio star are:

$$\vec{\sigma}_{Vi}(t) = [R(t)] . \vec{\sigma}_{Hi}(t) \qquad (1)$$

$$\dot{\vec{\sigma}}_{Vi} = [R(t)] . (\dot{\vec{\sigma}}_{Hi} + [\dot{R}] . \vec{\sigma}_{Hi}(t)) \qquad (2)$$

The relative orientation between the two frames at an initial epoch t_0 is represented by three rotations A_1, A_2, and A_3 around the axes of one frame, and the angular velocity of rotation is represented by the time - derivatives $\dot{A}_1$, $\dot{A}_2$ and $\dot{A}_3$. Expressions (1) and (2) provide four independent equations in the coordinates. Two radio stars are enough to evaluate the unknowns; however we will consider a greater number of objects to perform a least squares adjustment.

We have adopted in this analysis the equatorial coordinate system because it is the natural system of the extragalactic VLBI frame. The FAST consortium has adopted the ecliptic coordinate system to reduce HIPPARCOS data. Equations (1) and (2) represent the link between the ecliptic coordinate system of HIPPARCOS and the VLBI equatorial coordinate system. It will be more expeditive to transform the ecliptic coordinates of HIPPARCOS observation into equatorial coordinates system before realising the link. In these conditions, the angles A_1, A_2, and A_3 will be small quantities at the level of 10^{-7} rad and the matrices, to the first order in the angles and in the velocity components, can be written:

$$[R(t_0)] = \begin{bmatrix} 1 & A_3 & -A_2 \\ -A_3 & 1 & A_1 \\ A_2 & -A_1 & 1 \end{bmatrix} \qquad [\dot{R}] = \begin{bmatrix} 0 & \dot{A}_3 & -\dot{A}_2 \\ -\dot{A}_3 & 0 & \dot{A}_1 \\ \dot{A}_2 & -\dot{A}_1 & 0 \end{bmatrix}$$

3. The radio stars selected for the link

14 radio stars with mv < 11 have been used for the link. Their distribution in the sky is shown in figure 1. Their VLBI coordinates at J2000.0, their proper motions and the corresponding epochs of observation are in Table 1. Data of the eight stars indicated with (1) had been obtained with the MarkIII data acquisition system and analysed in the MASTERFIT by Lestrade *et al.* (1988). The four objects at negative declinations indicated with (2) have been observed by White *et al.* (1990). The positions of the two stars indicated with (3) in Table 1 were also provided by Lestrade and have been determined with the VLA. All the positions are in J2000.0 by means of the standard precession-nutation models IAU 1976/1980.

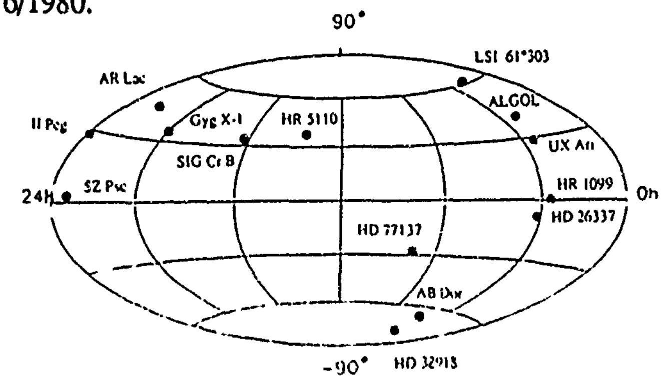

Figure 1.
Radio stars considered.

Table 1: VLBI positions and proper motions of the radio stars

Star	Right Asc. J 2000.0 h m s	Declination J 2000.0 ° ' "	σ_α 10^{-4}s	σ_δ 10^{-3}"	Proper Motion μ_α (s/y)	μ_δ ("/y)	$\sigma_{\mu\alpha}$ 10^{-4} s/y	$\sigma_{\mu\delta}$ 10^{-3} "/y	Ep. -1900
(1) LSI 61°303	02 40 31.686	61 13 45.56	0.3	0.5	0.0000	0.000	0.2	0.3	90.5
(1) ALGOL	03 08 10.1308	40 57 20.359	0.7	1.0	+0.0003	+0.002	0.5	0.7	88.5
(1) UX ARI	03 26 35.3375	28 42 56.026	0.3	0.5	+0.0016	- 0.106	0.2	0.3	88.5
(1) HR 1099	03 36 47.3251	00 35 18.60	0.3	5.0	- 0.002	- 0.161	0.2	0.3	90.5
(2) HD 26337	04 09 40.864	-07 53 35.64	3.3	5.0	+0.001	+0.130	2.7	4.0	92.5
(2) HD 32918	04 55 11.044	-74 56 16.02	3.3	5.0	+0.0072	+0.058	2.7	4.0	92.5
(2) AB Dor	05 28 44.53515	-65 27 02.1914	3.3	5.0	+0.0084	+0.133	2.7	4.0	92.5
(2) HD 77137	08 59 42.765	-27 48 58.11	3.3	5.0	- 0.0036	- 0.049	2.7	4.0	92.5
(1) HR 5110	13 34 47.6893	37 10 56.859	0.3	0.5	+0.0069	- 0.013	0.2	0.3	89.5
(1) SIG CrB	16 14 41.2011	33 51 32.47	0.3	0.5	- 0.0221	- 0.080	0.2	0.3	87.5
(1) Cyg X-1	19 58 21.6804	35 12 05.887	0.3	0.5	- 0.0016	- 0.018	0.2	0.3	91.5
(3) AR Lac	22 08 40.871	45 44 31.51	0.3	0.5	- 0.0028	+0.035	0.2	0.3	89.5
(1) SZ Psc	23 13 23.7645	02 40 31.31	0.3	5.0	- 0.0002	+0.041	0.2	0.3	92.5
(3) II Peg	23 52 29.0891	28 21 17.74	0.3	0.5	+0.0437	+0.038	0.2	0.3	92.5

4. The simulation of HIPPARCOS observations

We have simulated the positions and proper motions $\vec{\sigma}_{Hi}(t_{Hi})$, $\dot{\vec{\sigma}}_{Hi}$ for each radio star "observed" with HIPPARCOS from the known VLBI vectors $\sigma_{Vi}(t_{Vi})$, σ_{Vi} given by Lestrade *et al.* (1988). The relative orientation between the two frames at the arbitrary epoch t_0 = 1991.0 has been fixed by adopting the values

A_1 = +0.030", A_2 = +0.025", A_3 = -0.045".

The time evolution of the frames from the epoch t_0 is given by

$\dot{A}_1$ = +0.002"/year, $\dot{A}_2$ = -0.003"/year, $\dot{A}_3$ = +0.001"/year.

The equations (1) and (2) for t = t_{Hi} have been used to simulate HIPPARCOS observations.

To make the simulation realistic, we have added gaussian noise to the VLBI and HIPPARCOS positions and proper motions. The uncertainties are also realistic; for HIPPARCOS data they are those expected for the mission, and for the VLBI observations, they have been assigned according to the caracteristics of the networks.

5. Results

We have analysed the quality of the link in two cases, considering either all the stars or only objects in the Northern hemisphere.

First, we have simulated HIPPARCOS as described in the previous section. Then, we have reevaluated A_k, $\dot{A}_k$ (k = 1, 3) from the vectors $\vec{\sigma}_{Vi}$, $\dot{\vec{\sigma}}_{Vi}$ and the simulated $\vec{\sigma}_{Hi}$, $\dot{\vec{\sigma}}_{Hi}$. The results of the two cases of link analysed are shown in Table 2.

The relative orientation at the initial epoch (A_k) is determined with a precision at the level of 0.001". The precision of the components $\dot{A}_k$ of the angular velocity is in the range 0.0007" - 0.0009".

Concerning the distribution of objects, the inclusion of radio stars at negative declinations, with uncertainties several times greater than those in the North, does not improve the link and introduces a bias in A_1, A_2 and A_3.

Table 2: Results of the link between HIPPARCOS and VLBI reference frames via radio stars. Units of A_k = 0.001", units of $\dot{A}_k$: 0.001"/year. σ are the post-fit residuals, in units of 0.001", R is the goodness-of-fit parameter.

N	14 All stars	10 Northern Stars
A1	+31.09 ± 0.91	+31.45 ± 0.87
A2	+23.23 ± 0.86	+23.27 ± 0.79
A3	-44.93 ± 0.73	-44.65 ± 0.71
$\dot{A}1$	+1.49 ± 0.82	+1.44 ± 0.79
$\dot{A}2$	-2.78 ± 0.77	-2.98 ± 0.72
$\dot{A}3$	+1.08 ± 0.67	+1.55 ± 0.66
σ	1.92	1.55
R	1.55	1.25

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OBSERVATIONS OF THE PLANETARY SATELLITES EUROPA AND TITAN BY HIPPARCOS

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ABSTRACT. Observations of the satellites Europa and Titan will be obtained from the European Astrometric Satellite HIPPARCOS. These observations will be used to obtain "observed" positions of the planets Jupiter and Saturn; these positions have the advantages, first to be given in the reference frame defined by the Hipparcos system, and second, the accuracy on these positions is better than the accuracy of the ground-based observations of these planets. To obtain the positions of the planets from those of their satellites, we have to take the calculated positions of these satellites by means of their ephemerides. The accuracy of the computed positions of Europa and Titan relatively to their primary is of the order of 0".1, better than the one of the direct observation of Jupiter and Saturn. An efficient comparison of the different ephemerides of these planets will be possible that way.

1. INTRODUCTION

The European Astrometric Satellite HIPPARCOS has observed the satellites Europa (of Jupiter) and Titan (of Saturn). In spite of the motion of these objects, the observation has been made as for the stars, using the ephemerides of these two satellites relatively to their primary, the used ephemerides of which being BDL82. The sky is scanned by rotating the telescope slowly around an axis which is nominally perpendicular to the two directions of observation (the acute angle between these directions being the basic angle). At the same time, this axis of rotation rotates at a constant angle close to 43° around the direction of the Sun (fig. 1). The transformation of the data obtained by Hipparcos on the modulating grid (fig. 2) are transformed by the two "consortia of data reduction". The final data will be obtained only after the end of the mission and only provisional results will be available during the next months.

2. COMPARISON WITH EPHEMERIDES

The Hipparcos data are referred to reference great circles (RGC) which define intermediate reference frames (fig.3). One RGC is identified by its pole and a corresponding epoch; it is used for objects near this reference great circle (a few degrees). The duration of validity of

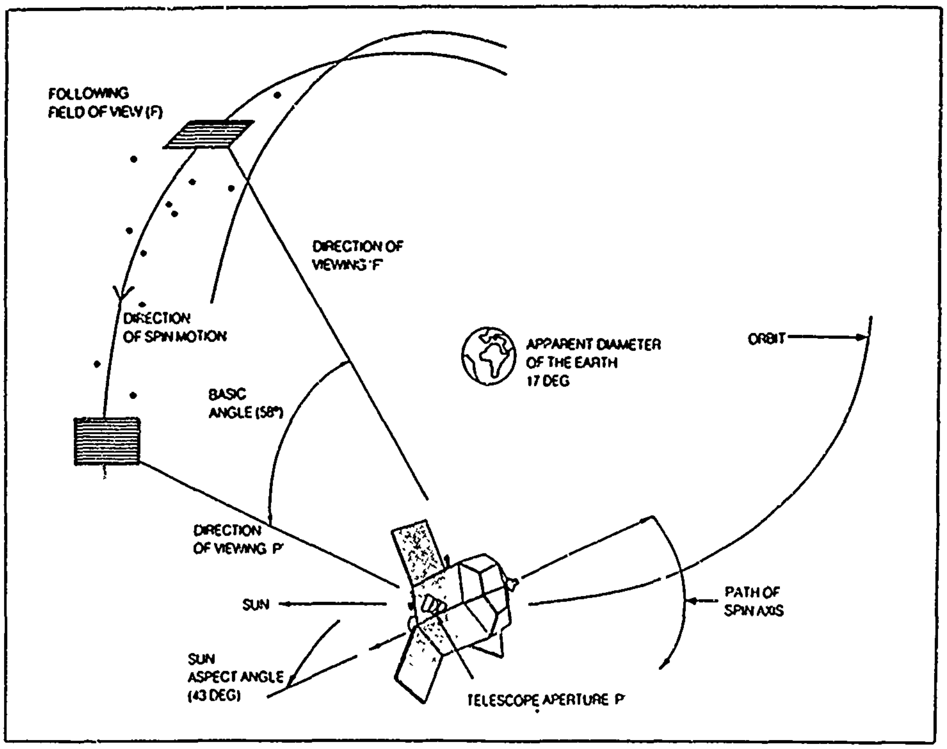

Figure 1.

such a circle is about tc ·; hours. Grid coordinates give informations mainly on the spherical coordinate along the reference great circle, named the abscissa.

Therefore, the spherical coordinates of the planetary satellites, Europa or Titan on a reference great circle have to be calculated. This is carried out as for minor planets (Bec-Borsenberger, 1990). For an observation of a planetary satellite by Hipparcos, the reference great circle number and the "observed" abscissa are given. From this RGC number, the julian date (t1) and the modified julian date (t2) of the epoch of the observation may be obtained as well as the pole coordinates of this RGC. Then from t1, the ecliptic astrometric position of the planetary satellite is computed, and from t2, the position and the velocity of the Hipparcos satellite may be obtained. Next, the parallax, light deflection, relativistic observation corrections are to be applied to obtain precise apparent ecliptic position. Finally, by rotating the ecliptic frame to the reference great circle frame, the calculated abscissa on the reference great circle is obtained.

3. OBSERVED POSITION OF THE PLANET JUPITER

The first observations for Europa, have been made on March 13, 1990. Absolute positions of Europa have been obtained in the reference frame defined by the Hipparcos system. These positions need theoretical positions of the planet Jupiter in order to obtain (O-C), but using theoretical positions of Europa relative to its primary, one will obtain an observed position of Jupiter itself. Since the ephemerides of Europa relative to Jupiter have a better

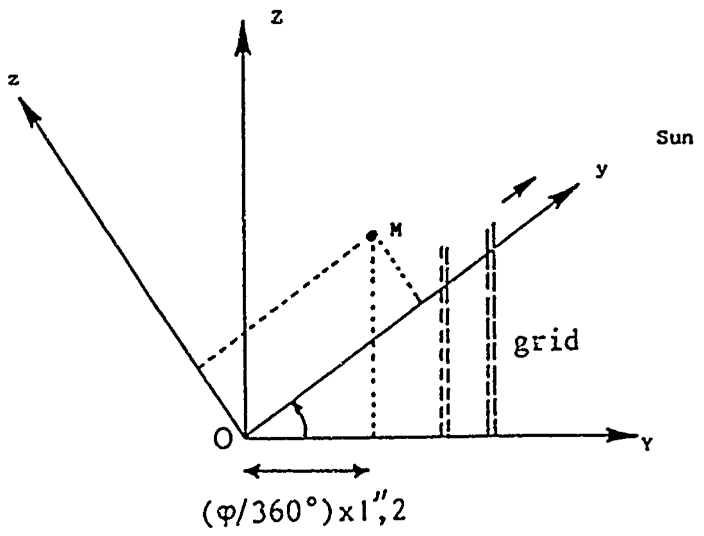

Figure 2. Illustration of the focal plane of Hipparcos. M is a point of the image of a planetary satellite; Oy is the direction of the Sun.

accuracy (near 0".1) than the ephemerides of Jupiter (near 0".4) in the J2000 reference frame, the observed position of Jupiter that we will obtain is valuable for further development. The ephemerides used are built using the following theoretical works: for Europa, theory of Sampson-Lieske (Lieske, 1977), adjusted with photographic observations (Arlot, 1982); for Titan, theoretical works recently adjusted with observations (Dourneau, 1987, 1990).

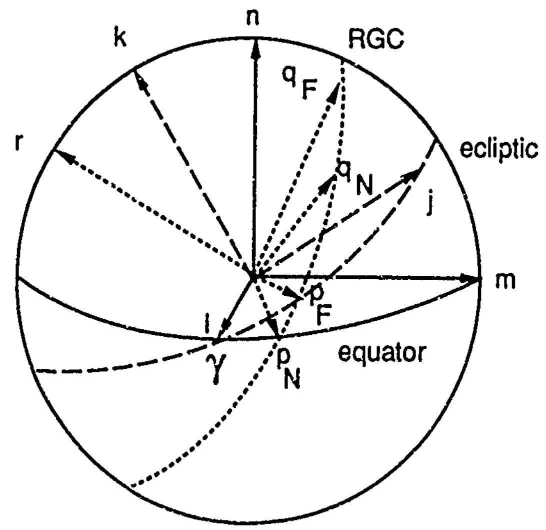

Figure 3. Reference frames for Hipparcos objects.

4. CONCLUSION

The observation of Europa and Titan by the astrometric satellite Hipparcos will induce observational data leading to the position of the planets Jupiter and Saturn because of the accuracy of the theoretical positions of Europa and Titan (0".1) relatively to their primary. These positions of Jupiter and Saturn will be of great interest in order to make a comparison between the different ephemerides of these planets and to appreciate their precision.

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COMPARISON BETWEEN ASTRONOMICAL AND GEODETIC COORDINATES

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A requirement for solving Earth rotation problems is the definition of a terrestrial reference system (or systems) and the determination of a terrestrial coordinate frame (or frames). In this context it is important to determine the positions of the observing sites and their temporal variations and establish the dependence of the observations on local conditions and observing methods.

In situ concurrent observations are therefore important. In this paper we report the results of a comparison between astronomical and geodetic coordinates derived from different observations at the Cagliari Observatory and the Carloforte latitude station.

A TERRESTRIAL REFERENCE SYSTEM CONSISTENT WITH WGRS

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ABSTRACT. The IAU and IUGG has jointly established in 1988 an International Earth Rotation Service (IERS) which is in charge of the realization of conventional celestial and terrestrial reference systems, together with the determination of earth orientation parameters which connect them.
The theoretical definition of the terrestrial reference system which is realized by IERS through a conventional terrestrial reference frame formed by SLR, LLR, VLBI and GPS stations is presented. In particular its origin, scale, orientation and evolution with time are reviewed, taking into account relativistic and deformation effects.

1. IERS Requirements for a Conventional Terrestrial Reference System (CTRS)

a) to permit realizations at millimeter level at the Earth's surface or 10^{-10},

b) consistent with IAU WGRS,

c) to take into account geodynamical effects at mm level,

d) to take into account relativistic effects at mm level.

2. Coordinate Systems

Four types of Coordinate Systems may be defined :

- <u>Barycentric celestial CS</u> (b) :

$$X^{\alpha} \qquad X^{0} = cT$$

- <u>Quasi inertial geocentric CS</u> (e) :

$$\tilde{x}^{\alpha} \qquad \tilde{x}^{0} = c\tilde{t}$$

where $\tilde{t}$ = TCG , Geocentric Coordinate Time

- Terrestrial geocentric CS (t) :

$$x^{\alpha} \qquad x^0 = ct$$

where $\quad t = TT = TAI + 32.184s$

- Terrestrial CS (T)

$$\bar{X} = \bar{x} + \bar{T}_G(t)$$
$$= (1 - \ell)\, R(t)\, \tilde{x} + \bar{T}_G(t)$$

where R is a rigid spatial rotation, T_G a translation and ℓ a scale factor.

The transformation between these Coordinate Systems can be done as follows :

- from (e) to (t) :

$$x^i = (1 - \ell) R^i_j(\tilde{t}) \tilde{x}^j$$

$$t = (1 - \ell)(\tilde{t} - t_T) + t_T$$

with

$$\begin{cases} \ell = 6.97 \times 10^{-10} \sim \dfrac{W_0}{c^2} \\ t_T = 1977 \text{ January } 1 \end{cases}$$

- from (t) to (T)

$$X = x + T_G(t)$$

Applying Tisserand condition on the Earth's crust, the CTRS should have

- no global translation with regards to the Crust :

$$T_G(1988.0) = 0 \quad , \qquad \int_{\text{Crust}} \rho\, \dot{X}^k\, d^3X = 0$$

- no global rotation with regards to the Crust :

$$R(1988.0) = R_0 \quad , \quad \varepsilon_{ijk} \int_{\text{Crust}} \rho\, X^j\, \dot{X}^k\, d^3X = 0$$

where R_0 is a conventional orientation and ρ is the density of the Crust.

3. Realizations of CTRS

3.1. PURPOSE

Within IERS, analysis centers and the Terrestrial Reference Frame Section produce realizations of the CTRS or close systems, by set of station coordinates, or Terrestrial Reference Frames:

- the TRF Section combines data to obtain a realization of the CTRS, named IERS Terrestrial Reference Frame (ITRF), see (Boucher, Altamimi, 1989, 1990a and 1990b)

- individual analysis centers
  . either compute directly a realization of the CTRS,
  . or compute a specific frame, which is compared to ITRF and then converted as a realization of the CTRS.

3.2. SPECIFICATIONS FOR THE REALIZATIONS OF THE CTRS

+ List of coordinates of points at epochs in TT, with velocity, or time series of coordinates :

$$X(t)\ ,\ V(t) \qquad t \text{ in TT}$$

or

$$X(t_k) \qquad t_1, t_2 \ldots t_k \text{ in TT}$$

+ Corrected from periodic tidal deformation :

$$X(t)_{\text{instantaneous}} = X(t) + \Delta X_{\text{tid,periodic}}(t)$$

or in time average :

$$< X(t)_{\text{inst}} >_{\text{time}} = < X(t) >_{\text{time}}$$

+ Tisserand condition on residual velocities / CCVF (see below)

+ orientation from BIH/IERS

3.3. CONVENTIONAL CRUSTAL VLEOCITY FIELD MODEL (CCVF)

Tisserand condition (given in § 2) should be applied to a CCVF ensuring no net rotation/translation of the CTRS with regards to the Crust. CCVF will be established from plate (NUVEL 1 ?) and vertical (post glacial rebound ?) motion models. This leads to a velocity field $\bar{V}(P_k,t)$, for a network of stations P_k ($k = 1,n$) and epoch t.

3.4. USE FOR A DISCRETE NETWORK

In order to tie reference frames, one has to :

- define a residual velocity :

$$\bar{v}_k = \dot{\bar{X}}_{P_k}(t) - \bar{V}(P_k, t)$$

- apply a discrete Tisserand condition to the residual velocities :

$$\begin{cases} \sum_k \bar{v}_k = 0 \\ \sum_k \bar{X}_k \wedge \bar{v}_k = 0 \end{cases}$$

Note : For the ITRF89 results presented in this poster during the colloquium, see the IERS Technical Note n° 6 (Boucher, Altamimi, 1990b).

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QUASAR LINK CONDITIONS FOR HIPPARCOS[1]

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ABSTRACT. Fictitious proper motioms of quasars and of their surrounding stars have been determined based on plates with an epoch difference of up to 90 years. From the fact that the true proper motions of the quasar are vanishingly small, we obtain conditions for the extragalactic calibration of the preliminary Hipparcos system. We present results for the fields of 3C 273, OQ 208, 3C 371 and 3C 390.3. With the data it is possible to achieve the link with random errors smaller than 0"15 per century.

[1]Accepted for publication in Astron.Astrophys., 1990.

PHOTOGRAPHIC CATALOGUE OF 200000 SOUTHERN STARS - FOCAT-S

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ABSTRACT. The catalogue of about 200000 stars in the zone from equator to southern pole was compiled from photographic observations carried out during 1982 - 1989 years in USSR and Bolivia by Pulkovo astrograph (D=23 sm, F=230 sm, working field $4^{\circ} \times 4^{\circ}$). The plates were measured by Zeiss Ascorecord measuring mashines in Pulkovo, Tarija, and others observatories. The process of measurement and compilation of catalogue were carried out in Pulkovo by ES-1033 computer. The observations were made with a four-fold overlap of the sky. Reference catalogue is SRS catalogue (FK5 system). The differencial refraction, differencial aberration and objective distortion were taken into account. The mean accuracy of catalogue FOCAT-S positions is ±0.123″.

1. INTRODUCTION.

The project of the reference photographic catalogue of the southern sky was proposed by D.Polojentsev and H.Potter in 1977 [1]. Its aim was to extend the fundamental system for 8-11 magnitude stars as uniformly as possible over the sky. The planned density of the catalogue was 10 stars per 1 square degree.

Up to now the accurate stars position determination is completed; its brief description is given below. The proper motion determination will be continued in the near future. The FOCAT-S catalogue is now being used by astronomers: its preliminary version is the main part of PPM-S compiled catalogue which was created in Heidelberg [2].

2. THE SOURCE STARS LIST.

The FOCAT-S source list is to contain about 210000 stars in declination zone from 0° to -90° according to the planned density of 10 stars per 1 square degree. SAO catalogue was used as the basis for the source list. It numbers about 130000 stars. The rest 80000 stars were taken from Bonner Durchmusterung ($+2^{\circ}$, -18° declination zone) and Cape Durchmusterung (-18°, -90° declination zone).

SAO catalogue stars were included into source list as a whole. Durchmusterung stars were choosen under the following conditions:

- magnitudes range is 8^{m} - 13^{m};

- the stars number in every $1^{\circ} \times 1^{\circ}$ area is no less then 10;
- the neighouring stars distance is no less then 3 arcminutes.

The total number of FOCAT-S source list is 214475 stars. Their magnitude and spectrum distributions are shown in Table 1.

Table 1. Source list stars distribution

spectr	num.of stars	mag	num.of stars
B	19572	<7	4646
A	18307	7- 8	12274
F	29183	8- 9	52347
G	32648	9-10	120477
K	20589	10-11	12547

3. OBSERVATIONS.

Observations were started in Marth, 1982 at Pulkovo Ordubad station ($\lambda = -3^{h}02^{m}25^{s}$, $\phi = +39^{\circ}06'20''$, H = 2000 m) by Pulkovo astrograph (D = 23 sm, F = 230 sm, working field $4^{\circ} \times 4^{\circ}$). The observations in Ordubad were carried out in declination zone from 0° to -30° and lasted till August, 1982. Then astrograph was transported to Tarija, Bolivia ($\lambda = 4^{h}18^{m}32^{s}$, $\phi = -21^{\circ}35'08''$, H = 2100 m). The observations there began in April, 1983 and continued till the mid of 1989.

The observations were made with a four-fold overlap for obtaining the star image in different parts of plate. The neighouring plates intersection is 2°: that is a half of the working field of astrograph. Such technique permits to reduce systematic errors due to objective field such as coma, astigmatizm, etc.

All observations were carried out near meridian on photographic plates ZU-1 and ZU-21. The exposure-times are 12 min and 4 min accordingly. The total number of FOCAT-S programm plates is 6440.

4. MEASUREMENTS AND PRELIMINARY PROCESS.

Plates were measured by Ascorecord measuring mashines. The measurement procedure was divided into two steps. Approximate coordinates of two bright stars on plate edges were measured during the first step. Then those data were used for the determination of the plate orientation in Ascorecord and for calculation of measurement ephemerides of FOCAT-S stars. Using these ephemerides the search of FOCAT-S stars and their measurements was made during the second step. SRS reference stars were measured twice: before and after FOCAT-S stars for stability control of plate position in Ascorecord.

Data base on magnetic disks was obtained during preliminary measurement, with possible instrumental and personal errors being controlled. The plates wich failed to meet any controle condition were re-observed.

5. REDUCTION OF OBSERVATIONS.

The measured stars coordinates were corrected by differential refraction and differential aberration. Then equatorial coordinates of stars were determined by the 8 constant method, which connects the measured (X, Y) with ideal (x', y') coordinates by the following expression

$$
\begin{aligned}
x' &= a_1X + b_1Y + c_1 + dx'X + ex'Y \\
y' &= a_2X + b_2Y + c_2 + dy'X + ey'Y
\end{aligned}
\qquad (1)
$$

Unknown parameters (a_1, b_1, c_1, a_2, b_2, c_2, d, e) were determined by least square method as applied to the joint solution of equations (1) for reference stars. The rough data were not taken into account.

The subsequent analysis of residual vectors of reference stars in different parts of plates for all plates showed us that equation (1) wasn't satisfactory for the whole working field of the plate (Figure 1). There are the systematical errors in the form of the positive distortion in residuals which can be up to 0.1" at the edges of plates and can be aproximated by

$$v = -0.051" + 12.75" \times r^3,$$

where r is the distance of the star from the plate centre. The above errors were excluded from all the measured coordinates as well.

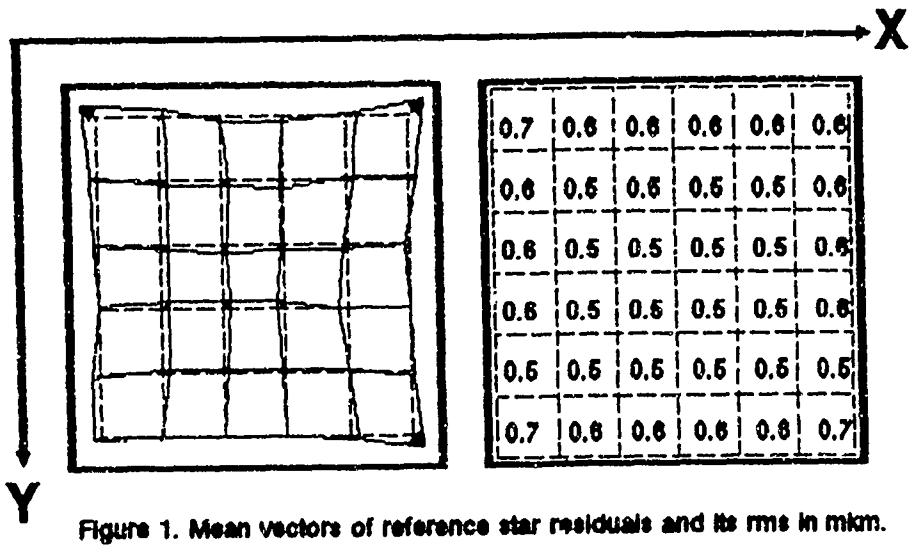

Figure 1. Mean vectors of reference star residuals and its rms in mkm.

Right part of Figure 1 (rms of residuals) shows us the good quality of the astrograph objective randomly. We can see that the random error of the corner points of the plates are no more than twice as large as the ones in the centre. At the same time for the greater part of plate the measurement error is practically the same.

Rms of unit weight in the plate as obtained by least square method charactirizes the quality of our data in a different way. Thus the analysis of the distribution of these errors for different plates shows that the measurement accuracy doesn't depend on the observational season and temperature, it is the same for different observers and measuring mashine operators and at the same time it differs for different declination zones. The latter evidently reflects the properties of the refernce catalogue. The mean rms of unit weight for all FOCAT-S catalogue is ±0.27".

6. FINAL PROCEDURE.

The coordinates of every star were determined indepently 4 time due to four-fold overlap during observations. Final catalogue positions of stars were obtained as mean-weighted of all determinations by the expression

$$\bar{X} = \sum P_l X_l \ / \sum P_l,$$

where $\bar{X}$ - a star position (right ascencion or declination),

X_l, P_l - a star coordinate on l-th plate and its weight.

The mean epoch and star position rms were computed by the same formula. The weight of every separate solution was computed as the function of two parameters: rms of unit weight on l-th plate S_l and the star distance from the plate centre in millimetres r_l:

$$P_l = (0.27'' \ / \ S_l)^2 \times (1 - (r_l/182)^2).$$

The first multiplier characterizes the plate property in general. The second one is connected with the star disposition on the plate and was obtained empirically using Figure 1 data.

The two-steps procedure was used for sorting out rough determinations. Firstly, the Dicson method for small samples was applied. Rough data were tested by 3σ criterion, and external evaluation being taken as σ (mean for all declination zone). In case both criteria give the same result, data were omitted. About 1% data obtained were discarded from the catalogue.

7. CATALOGUE INVESTIGATION.

The study of the magnitude equation and color equation provided by reference stars shows that the mentioned dependences are practically absent within the magnitudes and spectral classes of SRS catalogue.

External analisys, that is the comparison of FOCAT-S with Carlsberg meridian catalogue CMC [3] by U.Bastian and S.Roeser (Figure 2) confirms this result and at the same time shows that there are the possibility of the magnitude equation in declination for the most faint stars.

Figure 2 shows the declination dependent and right ascention dependent differences (FOKAT-S - CMC) too.

Mean rms of star position is $\pm 0.123''$ for the whole FOCAT-S catalogue. Figures 3,4 show its dependence on the declination and its value distribution.

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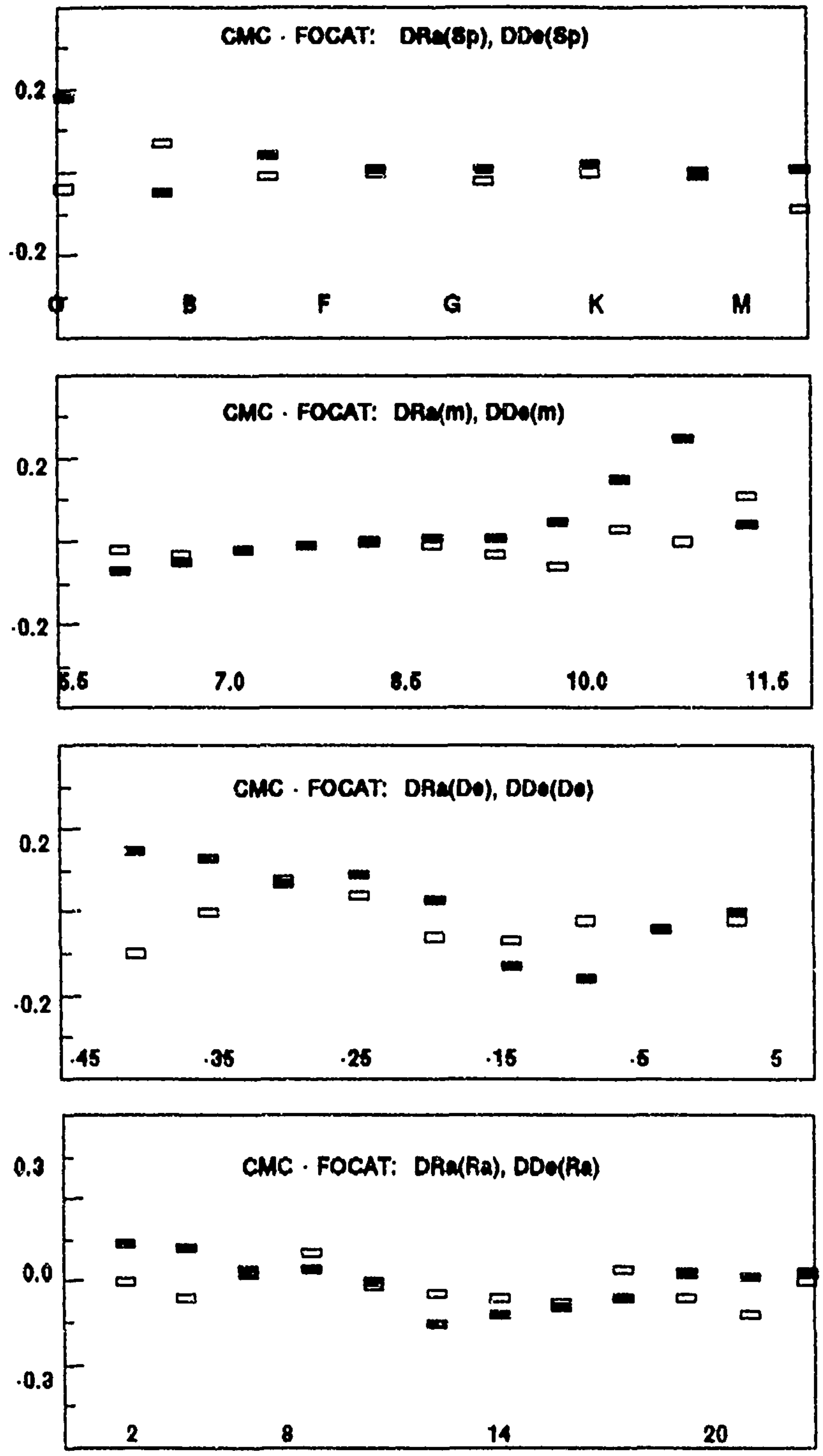

Figure 2. The systematic differences CMC - FOCAT.

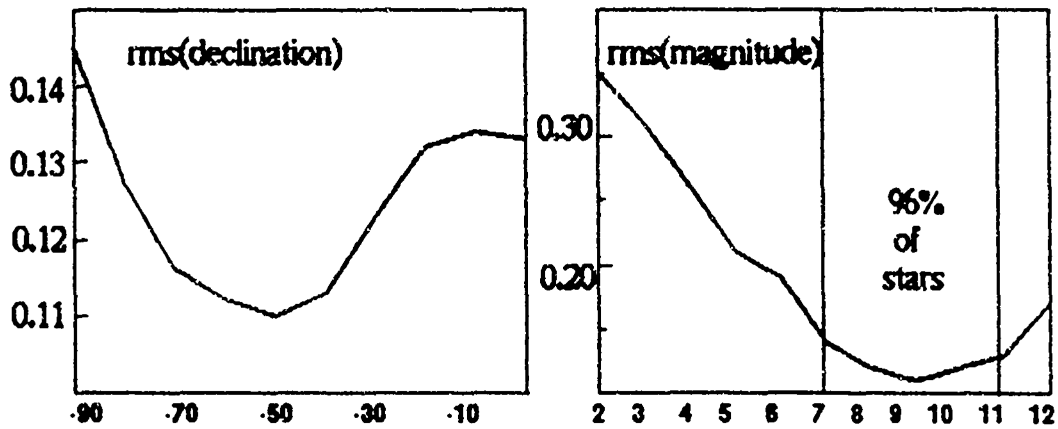

Figure 3. FOCAT-S rms-declination and rms-magnitude dependence (in arcsec).

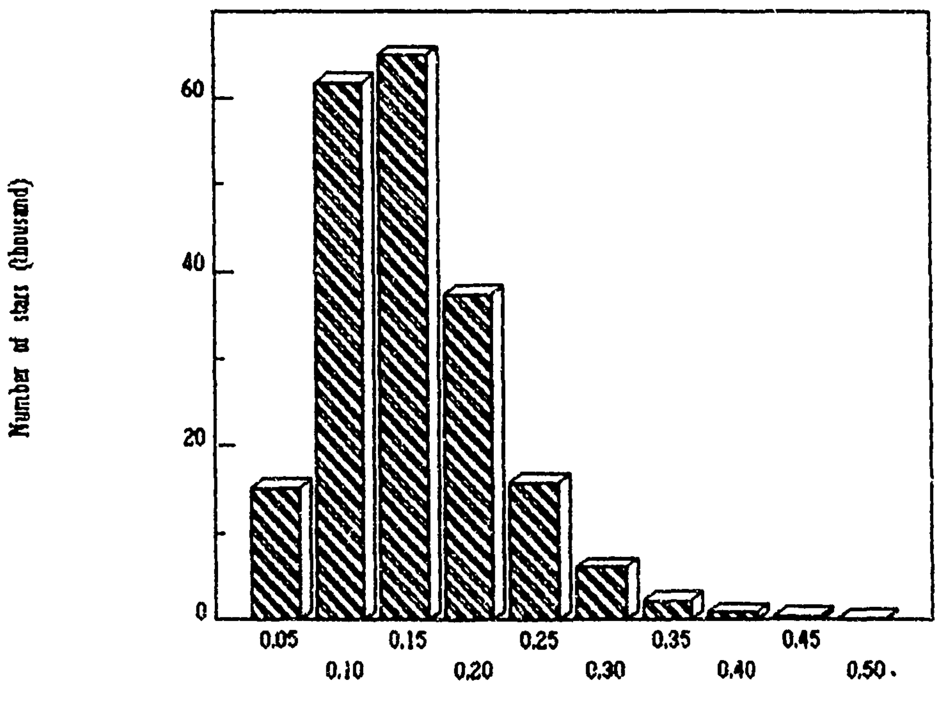

Figure 4. FOCAT-S positions rms distribution.

DEFICIENCIES IN THE MODEL FOR THE CELESTIAL MOTION OF THE CEP AS DERIVED FROM A GODDARD/VLBI SERIES OF POLE OFFSETS FROM 1979 TO 1989

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ABSTRACT. A VLBI series of celestial pole offsets (designated as ERP(GSFC)90 R 01 in the IERS series of Earth Rotation Parameters) spanning 10 years has been used for deriving the corrections to the IAU constant of precession and to the main terms of the 1980 IAU series of nutation using the same method as Herring (1988) for constraining the corrections to the 9.3yr nutation and to the 18.6yr nutation in longitude. The estimated corrections have been found to be - 0.279"/c ± 0.02 "/c (formal error) for the IAU constant of precession and +3.13±0.15mas for the 18.6 yr nutation coefficient in obliquity. A circular term with a radius of the order of 0.25mas at a period of 430 days has moreover been revealed in the residuals as obtained after applying the estimated corrections to the series. If such a circular term in the celestial motion of the pole is due to the free core nutation, its estimated period would correspond to a core flattening of 2.687×10^{-3}, which is in good agreement with the one of 2.674×10^{-3} corresponding to the estimated amplitude of the retrograde annual nutation.

1. INTRODUCTION

The celestial pole offsets as derived, with a milliarsecond accuracy, from VLBI observations of radio-sources, include the deficiencies in the conventional models for precession and nutation at the date of the observations and are therefore well adapted to the estimation of the corrections to the IAU model for precession and nutation.

A VLBI series of celestial pole offsets (designated as ERP(GSFC)90 R 01 in the IERS series of Earth Rotation Parameters) from 1979.6 to 1990.0 has been analysed in this purpose. As this series spans only over 10 years, large correlations appear between the 9.3yr and 18.6yr terms as well as between the precession in longitude and the 18.6 yr nutation in longitude. Calculated corrections have therefore been applied both to the 9.3 yr nutation coefficients and to the coefficient in longitude of the 18.6 yr nutation, using the same method as Herring (1988), before estimating the corrections to the other terms.

This short paper summarizes the results obtained in such a study, which will be presented in further details in a more complete paper (Capitaine and Caze 1990).

2. ESTIMATED CORRECTION TO THE IAU PRECESSION AND NUTATION MODEL

Using the theoretical corrections for the nutations of a rigid Earth as given by Kinoshita and Souchay (1989), as well as the correction to the amplitude of the main nutations corresponding to a first estimated value of -0.87 mas/yr for the secular correction in longitudexsinε_0, allows us to compute the total theoretical correction to be applied, due to the rigid Earth model : $+ 0.194$ mas sinΩ + 0.462 mas sin 2Ω to dψ sinε_o ,

and: $- 0.550$ mas cosΩ - 0.224 mas cos 2Ω to dε,

and then to compute, from the estimated correction to the principal term in obliquity ("in-phase" and "out of phase" components), the "geophysical" part of the correction in dε from which the "geophysical" part of the correction in dψ sinε_0 can be derived (Herring 1988); this allows to constraint the total correction to the principal term in longitudexsinε_o to:

-2.276 mas for the "in-phase" component, and +1.602 mas for the "out of phase" component.

Using such corrections to the coefficients in longitude of the 18.6 yr nutation, as well as the corrections, as previoulsy computed, to the 9.3 yr nutation, allowed us to estimate the corrections to the precession and nutation model. The final correction to the constant of precession has been found to be: $\delta f = - 0.279''/c \pm 0.02''/c$ and the final estimated corrections to the 1980 IAU nutation series are given, with their formal errors, in Table 1.

Argument	Period	$\delta\ \psi \sin \varepsilon_o$	$\delta\ \varepsilon$
2☾	13.66 d		
in-phase		- 0.40 ± 0.02	0.34 ± 0.02
out of phase		- 0.03 ± 0.03	- 0.14 ± 0.02
☾ - p	27.55 d		
in-phase		- 0.10 ± 0.02	- 0.06 ± 0.02
out of phase		- 0.03 ± 0.02	- 0.01 ± 0.02
2⊙	182.63 d		
in-phase		0.66 ± 0.02	- 0.49 ± 0.02
out of phase		- 0.46 ± 0.03	- 0.45 ± 0.02
⊙- p	365.25 d		
in-phase		1.99 ± 0.03	1.86 ± 0.02
out of phase		0.49 ± 0.02	- 0.27 ± 0.02
Ω	18.61 yr		
in-phase		constrainted	3.13 ± 0.15
out of phase		correction	1.21 ± 0.17

Table 1: Estimated corrections to the main IAU 1980 nutation coefficients from the ERP(GSFC)90 R 01/VLBI series of celestial pole offsets from 1979.6 to 1990.0

The residuals as obtained after applying the previous estimated corrections to the series have been analysed for by a least squares adjustment of periodic terms which has clearly revealed a circular term, with a radius of the order of 0.25 mas at a period of 430 days. If such a circular term in the celestial motion of the pole is due to the free core nutation, its estimated period would correspond to a core flattening of 2.687×10^{-3}, which is in good agreement with the one of 2.674×10^{-3}, which can be derived from the amplitude of the retrograde annual nutation (-32.98 mas) as estimated from the present analysis.

3. REFERENCES

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THE USE OF THE NONROTATING ORIGIN IN THE COMPUTATION OF APPARENT PLACES OF STARS FOR ESTIMATING EARTH ROTATION PARAMETERS

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ABSTRACT. A new procedure has been devised for computing apparent places of stars in the intermediate frame (Capitaine 1990) linked to the nonrotating origin (Guinot 1979) for estimating the Earth rotation parameters (ERP) from astrometric observations. The latitude and time parameters as derived by this procedure have been compared to the parameters as derived from the classical procedure used for the reduction of Paris astrolabe observations (Chollet 1984). The consistency between the two procedures has been found to be of the order of a few 10^{-4}", which is under the order of precision of the computations in the classical procedure. The new procedure, which is more directly related to the Earth rotation, is proposed to be used for the derivation of the ERP in the Hipparcos frame from existing astrometric observations, which is planned for the near future (IAU WG on "Earth Rotation in the Hipparcos Reference Frame").

1. INTRODUCTION

The latitude and time parameters are clasically derived from astrolabe observations of one group of stars by comparing the computed apparent places of the stars in an intermediate frame linked to the Celestial Ephemeris Pole (CEP) and true equinox with their corresponding observed places.

The use of the nonrotating origin (NRO) as proposed by Guinot (1979) instead of the equinox, for reckoning the Earth's angle of rotation, associated with the use of the celestial coordinates of the Celestial Ephemeris Pole to account for the effects of precession and nutation, instead of the classical precession and nutation parameters, provides an intermediate frame which is more directly related to the observation of Earth rotation.

A new procedure has thus been devised for computing the instantaneous latitude and time parameters from Paris astrolabe observations referred to such an intermediate frame. This procedure is based on the matrix transformation of vector components (denoted by [ ]) between the geocentric celestial and terrestrial reference systems (denoted by CRS and TRS, respectively) which can be written (Capitaine 1990) as:

$$[TRS] = W(t).\, R(t).\, NP(t)\, [CRS], \qquad (1)$$

W(t) being for the terrestrial displacement of the CEP (i.e. "polar motion"), R(t) for the celestial Earth's angle of rotation (i.e. "stellar angle") and NP(t) for the celestial displacement of the CEP (i.e. precession/nutation) from the epoch t_0 to the date t.

2. PROCEDURE FOR COMPUTING APPARENT POSITIONS OF STARS REFERRED TO THE NRO

The new procedure, compiled from the classical procedure for computing apparent positions of stars for astrolabe reductions (Chollet 1984), uses the following steps:

- expression, in rectangular coordinates, of the unit vector determined by the standard mean place of the star in the FK5 (epoch J2000.0),

- correction for proper motion and normalization of the resulting vector,

- rotation (ε_o) of the vector from the mean equator of epoch to the ecliptic of epoch,

- correction for annual parallax and aberration using the position and velocity of the Earth, with respect to the solar system barycenter, referred to the ecliptic and mean equinox of J2000.0 and normalization of the resulting vector,

- rotation ($-\varepsilon_o$) of the vector from the ecliptic of epoch to the mean equator of epoch,

- rotation of the vector from the mean equatorial frame of epoch to the celestial intermediate frame of date, t, linked to the CEP and the NRO, using the matrix transformation (Capitaine 1990) (see Figure 1): $NP(t) = R_3(s).R_3(-E).R_2(d).R_3(E)$. (2)

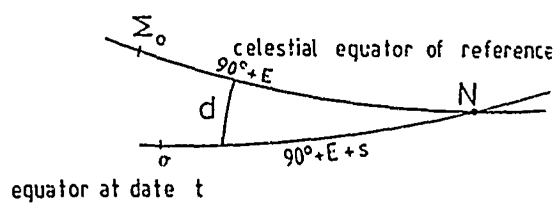

Figure 1: *The angular parameters for the transformation in the new procedure*

3. PROCEDURE FOR DERIVING LATITUDE AND TIME PARAMETERS REFERRED TO THE NRO

The new procedure uses the following steps:

- rotation of the vector from the intermediate celestial frame of date to the local terrestrial frame linked to the CEP and to the local meridian, using the rotation matrix $R_3(\theta+\lambda)$, where $\theta = \theta_0 + k\,(UT1 - UT1_0)$ is the stellar angle (Guinot 1979, Capitaine *et al.* 1986) and λ is the longitude of the station,

- computation of the spherical coordinates of the resulting apparent direction to be compared with the observed spherical coordinates of the star,

- least squares adjustment of latitude and time parameters: φ and UT1 -UTC for each group of stars.

4. NUMERICAL CHECKS

Numerical checks of this procedure were performed for the derivation of the latitude and time parameters from Paris astrolabe observations spanning several months (see example).

(i) Classical procedure

N° 226
FK 5
Groupe 2
Lam
19901009

PARIS

Latitude 48 50 8.5000
Longitude 0 9 21.0453

$\boxed{\gamma}$

Date	Gr	Obs.	UT0 UTC	Pds.UT	UT Moy.	UT0 TAI	UT Ref.	dφ	Pds.φ	Rayon	Nb.*	Pds.Gr
19901009	2	19	$\boxed{0.19564}$	4.3	24.600	25.19560	24.00	$\boxed{0.9145}$	3.1	15.9701	26	3.6
Sigmas correspondants				0.0044				0.051		0.033		0 168

Rang	N°FK	Résidu	CLI	UT Calculé	Val.Obs	A(x)	Cte.	dh	SinZ	CosZ	Refr
1	851	0.135	0.000	85809.4014	9.8670	0.0	1	1.7118	0.35771	0.93383	0.0203
2	165	0.324	0.000	86113.6947	13.8713	0.0	0	1.5661	0.88061	0.47384	0.0234
3	2336	0.105	0.000	86232.0380	12 1353	0.0	0	0.6522	0.65505	0.75558	0 0246
4	3794	0.287	0.003	86430.5889	31.0333	0.0	0	2.0212	0.46137	0 88721	0.0267
5	1600	0.099	0.000	86698.0323	58.2107	0.0	0	1.7073	0.98855	0.15090	0.0294
6	89	0.139	0.000	86929.1566	49.4731	0.0	1	1.6364	0.53286	0.84620	0.0318
7	844	0.169	0.000	87093.3840	33.6487	0.0	0	2.3408	0.90775	0.41952	0 0335
8	1613	0 295	0.000	87361.4125	1.5494	0.0	0	1.2193	0.93468	0.35549	0.0363
9	178	0.066	0.000	87481.8031	1.8533	0.0	0	0.3281	0.59760	0.80179	0.0376
10	192	0.026	0.000	87726.3025	6.4036	0.0	0	0.7906	0 74976	0.66171	0.0401
11	869	0.175	0.000	88105.0381	25.2301	0.0	0	1.8451	0.99800	0.06329	0.0440
12	3891	0.054	0.000	88235.3128	35.4936	0.0	0	1.6481	0.95204	0.30598	0.0454
13	158	0.247	0.000	88402.4695	22.6548	0.0	0	1.8294	0.96620	0.25779	0 0471
14	863	0.053	0.000	88738.3189	18.6467	0.0	1	1.9797	0.60246	0.79815	0 0508
15	2248	0 119	0.000	88866.3719	6.5556	0.0	0	1.8692	0.99507	0.09917	0 0519
16	1032	0.258	0.000	89046.8787	6.9544	0.0	1	0.3460	0.45130	0.89237	0.0538
17	202	0 111	0.000	89381.3768	41.4890	0.0	1	0 7439	0.68586	0.72773	0 0572
18	27	0.032	0.000	89487.5151	27.6195	0.0	1	0.7049	0.67892	0.73422	0.0583
19	193	0.035	0 000	89640.5470	0.7171	0.0	0	1.7136	0 97889	0.20439	0 0599
20	139	0.147	0.000	89867.0846	47.4008	0.0	1	2.0951	0.67145	0.74105	0.0623
21	1094	0.044	0.000	90017.5692	17.9442	0.0	1	1 7221	0.46359	0.88605	0 0638
22	875	0.031	0 000	90393.5972	33.8732	0.0	0	2.1734	0.82107	0.57083	0 0677
23	1619	0.030	0 000	90726.9163	7.1331	0.0	0	2.0462	0 99005	0 14073	0 0712
24	19	0.000	0.000	90878.7614	38.8306	0 0	1	0.5929	0.86866	0.49542	0.0728
25	173	0.165	0.000	91005.0263	44.9364	0.0	1	0 2492	0.25771	0.96622	0.0741
26	1126	0.080	0.000	91318.9675	59.1965	0.0	0	2 0178	0 85028	0.52653	0 0774
27	66	0 168	0 000	91455.1823	15.1552	0.0	1	0 1334	0 42878	0 90341	0 0788

UTC H 0.0000 TAI UTC 25. m H/UTC 0.0000 m UT/UTC 0.0015
CHR H 0.0000 DTC 0.0 R approché 16.000

N° 226
FK 5
Groupe 2
Lam
PARIS
19901009

(ii) New procedure

N° 226
FK 5
Groupe 2
Lam
19901009

PARIS

Latitude 48 50 8.5000
Longitude 0 9 21.0453

$\boxed{\text{NRO}}$

Date	Gr	Obs.	UT0 UTC	Pds.UT	UT Moy.	UT0 TAI	UT Ref	dφ	Pds.φ	Rayon	Nb *	Pds.Gr
19901009	2	19	$\boxed{0.19567}$	4.3	24.600	25 19563	24.00	$\boxed{0.9140}$	3.1	15.9703	26	3.6
Sigmas correspondants				0.0044				0.051		0 033		0 168

Rang	N°FK	Residu	CLI	UT Calculé	Val.Obs	A(x)	Cte	dh	SinZ	CosZ	Refr
1	851	0 135	0.000	85809.4016	9.8670	0.0	1	1 7112	0.35771	0.93383	0 0202
2	165	0.324	0.000	86113.6947	13.8713	0 0	0	1.5666	0.88061	0.47384	0 0233
3	2336	0.105	0.000	86232.0379	12.1353	0.0	0	0.6529	0.65505	0.75558	0.0245
4	3794	0.286	0.000	86430.5890	31.0333	0.0	0	2.0206	0.46137	0.88721	0.0266
5	1600	0.099	0.000	86698.0329	58.2107	0.0	0	1.7073	0.98855	0.15090	0.0294
6	89	0.139	0.000	86929.1566	49.4731	0.0	1	1.6363	0.53286	0.84620	0.0318
7	844	0.169	0.000	87093.3841	33.6487	0.0	0	2 3406	0 90775	0.41952	0.0335
8	1613	0.295	0.000	87361 4125	1.5494	0.0	0	1.2195	0.93468	0.35549	0.0362
9	178	0.066	0 000	87481.8029	1.8533	0.0	0	0 3288	0.59760	0.80179	0 0375
10	102	0.026	0.000	87726.3024	6.4036	0.0	0	0.7913	0.74976	0.68171	0.0400
11	869	0 175	0.000	88105.0382	25.2301	0.0	0	1.8451	0.99800	0.06329	0 0439
12	3891	0.054	0.000	88235.3128	35.4936	0.0	0	1.6483	0.93204	0.30598	0.0453
13	158	0 247	0 000	88602 4694	22.6548	0.0	0	1.8297	0 96620	0.25779	0 0470
14	863	0.053	0.000	88738.3190	18.6467	0.0	1	1.9793	0 60246	0 79815	0.0507
15	2148	0.118	0.000	88866.3719	6.5556	0.0	0	1.8696	0.99507	0.09917	0 0518
16	1032	0.258	0.000	89046.8787	6.9544	0.0	1	0.3463	0.45130	0.89237	0.0537
17	203	0.111	0.000	89381.3767	41.4890	0.0	1	0 7446	0.68586	0.72773	0.0572
18	27	0 032	0.000	89487 5160	27.6195	0 0	1	0 7052	0.67892	0.73422	0.0583
19	193	0.035	0.000	89640.5470	0.7171	0.0	0	1.7142	0.97889	0.20439	0 0598
20	139	0.147	0.000	89867.0846	47.4008	0.0	1	2.0952	0.67145	0.74105	0 0622
21	1094	0 044	0 000	90017 5692	17.9442	0.0	1	1 7221	0 46359	0 88605	0 0638
22	875	0 032	0.000	90393 5972	33.8732	0.0	0	2.1732	0.82107	0.57083	0 0677
23	1619	0.029	0.000	90726.9163	7.1331	0.0	0	2.0463	0 99005	0 14073	0.0711
24	19	0.000	0.000	90878.7614	38.8306	0 0	1	0.5924	0.86866	0.49542	0 0727
25	173	0 165	0.000	91005.0260	44.9354	0.0	1	0 2485	0 25771	0 96622	0 0740
26	1126	0.080	0 000	91318.9675	59.1965	0.0	0	2.0181	0.85028	0 52633	0.0773
27	66	0.167	0 000	91455.1822	15.1552	0.0	1	0 1331	0.42878	0.90341	0 0787

UTC H 0.0000 TAI UTC 25. m H/UTC 0.0000 m UT/UTC 0 0015
CHR H 0 0000 DTC 0 0 R approché 16 000

N° 226
FK 5
Groupe 2
Lam
PARIS

Table 1: Example of the estimated parameters φ and UT0-UTC using the two procedures

These checks show our procedure referred to the NRO to give results consistent to a few 10^{-4} arcseconds with the estimations of the same parameters from the classical procedure referred to the true equinox and using the Sidereal Time computed with the complete equation of the equinoxes (Aoki and Kinoshita 1983). Such a consistency is under the order of precision of the computations in the classical procedure.

The practical advantages of this new procedure for deriving variations of latitude and time from astrometric data are that the calculations are simpler and that the derived quantities are more directly linked to the kinematic parameters representing Earth Rotation.

5. CONCLUSION

The present study shows that the computation of the apparent places of stars referred to the nonrotating origin provides latitude and time parameters which are consistent with those derived from the classical method with an accuracy of a few 10^{-4} arcseconds. Such a procedure, which is more directly related to the Earth rotation, can be used with advantage for the estimation of the Earth Rotation Parameters (ERP) from astrometric observations. It is proposed to be used for the computation of the ERP in the Hipparcos frame from existing astrometric observations, which is planned for the near future (IAU WG on Earth Rotation in the Hipparcos Reference Frame).

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A GLOBAL VLBI/LLR ANALYSIS FOR THE DETERMINATION OF PRECESSION AND NUTATION CONSTANTS

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ABSTRACT. Two decades of LLR data and one decade of VLBI data are combined in a global analysis to yield improved estimates of the Earth's precession and nutation. In this analysis, LLR provides a strong determination of precession, while VLBI is stronger in fixing nutation terms with short periods. In all, 24 nutation amplitudes are estimated. The largest correlation coefficient, between precession and 18.6 yr out-of-phase nutation in longitude, is 0.88. With the exception of some 9 yr and 18.6 yr terms, formal uncertainties are 0.1 to 0.2 milliarcseconds.

1. Introduction

In the past five years, it has become obvious that the current precession and nutation theory recommended by the IAU is not adequate for reduction of modern astrometric observations, such as those acquired by the VLBI and LLR techniques. Discrepancies at the level of a few milliarcseconds, especially for the precession constant and 18.6 yr, 1 yr, 0.5 yr and 14 day nutation terms, have been revealed by VLBI and LLR observations (*e.g.* Herring *et al.*, 1986; Williams *et al.*, 1990). It is of great importance to establish more accurate precession and nutation constants in order to improve VLBI and LLR analyses for astrometry and geodesy. However, based on theoretical considerations, any improvement of the quality of these constants is presently difficult because geophysical properties of the Earth's interior (*e.g.* core flattening, mantle inelasticity) are not known well enough. The only way to obtain more accurate precession and nutation constants is to estimate them from the VLBI and/or LLR observations themselves. Comparing precession and nutation constants estimated from LLR and Deep Space Network VLBI data (see Section 4) shows that estimates of the short-period nutation terms (1 yr, 0.5 yr, 14 day) are more accurate when derived from VLBI data, while estimates of the precession constant and the long-period nutation terms (18.6 yr, 9 yr) in longitude are more accurate when derived from LLR data. Therefore, combining VLBI and LLR data is the best way to improve the estimation of precession and nutation constants with the present data, because it takes advantage of the unique and complementary strengths of the two techniques.

For the purpose of estimating precession and nutation constants, we have developed a global analysis of VLBI and LLR observations. This global analysis (described in Section 2) is performed by combining the VLBI and LLR information matrices derived from the data equations with the VLBI and LLR software of the Jet Propulsion Laboratory (JPL). We emphasize that, in this analysis, the full VLBI and LLR information matrices are retained to produce combined VLBI/LLR estimates of the precession constant and 24 nutation amplitudes. Section 3 describes the data sets and modeling used in our analysis, while Section 4 presents and discusses our results, including a comparison between the VLBI, LLR, and VLBI/LLR estimates and correlation coefficients.

2. The Algorithm Used to Combine VLBI and LLR Data

In the past decade, VLBI software (MODEST) (Sovers and Fanselow, 1987) and LLR software (LPRED) has been developed at JPL. Both of these programs use the Square Root Information Filter (SRIF) algorithm, based on repeated Householder transformations, to triangularize the data equation matrix. This algorithm is extensively described in Bierman (1977). One advantage of the SRIF algorithm is that it allows one to add new data equations to a previously analyzed set of data equations and produce "updated" parameter estimates without reanalyzing all the data. In practice, this saves much computer time when large amounts of data are to be processed and updated. In our analysis, this capability has been used to combine observations of different types (VLBI and LLR).

The main steps of our joint VLBI/LLR analysis are shown in Figure 1. First, theoretical values of LLR ranges and VLBI delays and delay rates are calculated and differenced with observations to produce O-C's. This step is performed separately for VLBI and LLR measurements by using MODEST and LPRED. Then, the SRIF algorithm is applied to LLR O-C's and the R-matrix is retained before the calculation of the LLR parameter estimates. This matrix is an upper triangular matrix obtained by applying Householder transformations to the data equations (see Bierman, 1977). It is further used as an *a priori* information matrix in a way that mimics a VLBI R-matrix, when the SRIF algorithm is applied to VLBI O-C's. This latter step requires identification of parameters common to VLBI and LLR and matching their names and units in order to produce consistent joint VLBI/LLR parameter estimates. It is to be noted that for our analysis, only precession and nutation constants have been considered as common VLBI/LLR parameters.

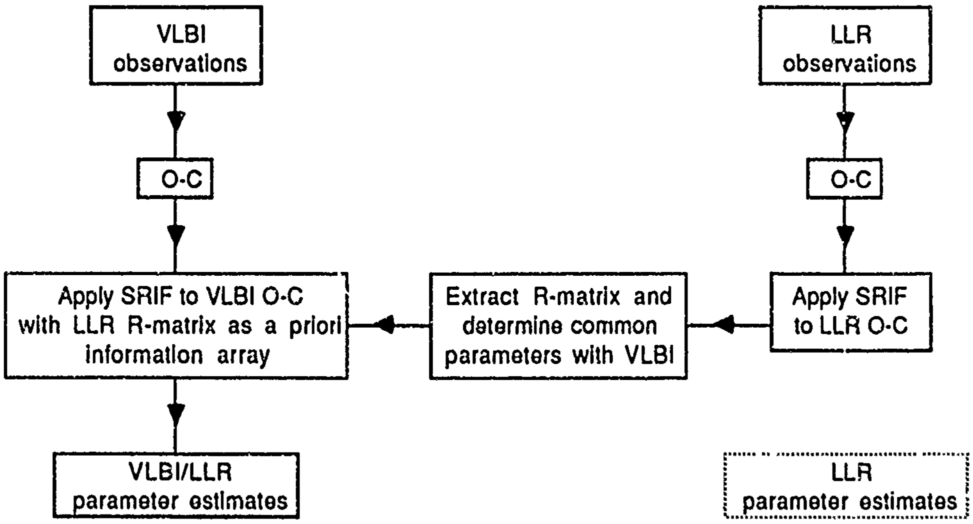

Figure 1: Main steps of the joint VLBI/LLR analysis.

3. Observations and Modeling

Our VLBI data set consists of 68 dual-frequency (2.3 and 8.4 GHz) VLBI observing sessions carried out by the Deep Space Network (DSN) on two intercontinental baselines (Goldstone-Madrid and Goldstone-Tidbinbilla) between October 1978 and October 1989. It is basically the same data set described by Sovers (1990), and includes 8578 dual-frequency Mark II delay and delay rate pairs and 423 Mark III pairs. The LLR data (described in more details by Williams *et al.*, 1990) consist of 6293 ranges (distance measurements) from telescopes on the Earth to reflectors on the Moon acquired between August 1969 and December 1989. VLBI and LLR modeling was performed in Solar System Barycentric coordinates in terms of the mean equator and equinox of J2000.0, and largely adhered to the IERS standards (1989) for astronomical constants. The LLR analysis is based on the DE303/LE303 planetary and lunar ephemeris. Earth orientation parameters were adopted from a combination-of-techniques series obtained by Kalman filtering (Gross 1990, private communication). More information about VLBI and LLR modeling is found in Sovers (1990) and Williams *et al.* (1990), respectively.

In our analysis, the precession constant and both the in-phase and out-of-phase nutation terms of periods 18.6 yr, 9 yr, 1 yr, 0.5 yr, 14 days, and 433.2 days (retrograde free core nutation with reference epoch J2000) have been estimated. We emphasize that this is the first unconstrained analysis with the precession constant and the 18.6 yr and 9 yr nutation terms estimated together in the same fit. Such unconstrained analysis is possible because of the time span of our data, especially the LLR data which cover 20 years.

4. Results

Table 1 shows our combined VLBI/LLR estimates of 25 precession and nutation constants together with the individual VLBI and LLR estimates of these constants. The values indicated in this table are the corrections to the IAU nutation coefficients; all errors quoted in this paper are formal 1σ uncertainties. Units are milliarcseconds (mas) for nutation, and mas/yr for precession. The nutation terms are labeled with subscripts corresponding to term numbers in the 1980 IAU series, while the superscripts indicate the phase (*s*ine or *c*osine).

Comparing the formal uncertainties of the individual VLBI and LLR estimates shows that LLR data provide more accurate estimates of the precession constant and the 18.6 yr and 9 yr nutation terms in longitude, while VLBI data provide more accurate estimates of the other nutation terms, especially the short-period nutation terms (1 yr, 0.5 yr, 14-day). Notably, the formal uncertainty of the precession constant is smaller by an order of magnitude when derived from LLR data (0.17 mas/yr) than when derived from VLBI data (1.87 mas/yr). Estimates of the precession constant and the 18.6 yr and 9 yr nutation terms in longitude based on VLBI data have large uncertainties caused by extremely high correlation coefficients between these parameters as shown in Table 2. In this table, only correlations exceeding 0.5 are printed; within each triplet, the order (top to bottom) is LLR, VLBI, and VLBI/LLR. For example, the correlation coefficient between the precession constant and the 18.6 yr out-of-phase nutation in longitude ($\Delta\psi_1^c$) is 0.997 in the VLBI analysis, while it is only 0.64 in the LLR analysis. Such a huge correlation coefficient is explained by the shape of $\Delta\psi_1^c$, which is almost linear over the period of our VLBI observations (see Figure 2). Besides this correlation coefficient, 14 other correlation coefficients are larger than 0.50 in the VLBI analysis, 4 of them larger than 0.90. By contrast, only 3 correlation coefficients are larger than 0.50 (maximum value of 0.64) in the LLR analysis.

Table 1. Comparison of the VLBI, LLR, and Combined VLBI/LLR Estimates of 25 Precession and Nutation Constants

	LLR	VLBI	VLBI/LLR
Precession	-2.66 ± 0.17	-5.17 ± 1.87	-2.83 ± 0.15
18.6 yr $\Delta\varepsilon_1^c$	0.47 ± 0.59	1.57 ± 0.26	1.44 ± 0.20
$\Delta\psi_1^s$	-7.60 ± 1.49	-10.21 ± 3.62	-6.07 ± 0.85
$\Delta\varepsilon_1^s$	2.46 ± 0.74	2.80 ± 0.54	2.28 ± 0.37
$\Delta\psi_1^c$	3.12 ± 1.52	12.52 ± 8.29	3.05 ± 0.96
9 yr $\Delta\varepsilon_2^c$	-0.43 ± 0.49	0.18 ± 0.23	-0.12 ± 0.18
$\Delta\psi_2^s$	-0.22 ± 1.04	-0.88 ± 1.76	1.17 ± 0.62
$\Delta\varepsilon_2^s$	-0.29 ± 0.50	-0.17 ± 0.34	0.04 ± 0.24
$\Delta\psi_2^c$	1.09 ± 1.02	-1.24 ± 1.56	0.08 ± 0.41
1 yr $\Delta\varepsilon_{10}^c$	1.59 ± 0.34	1.60 ± 0.13	1.69 ± 0.11
$\Delta\psi_{10}^s$	5.03 ± 0.66	4.26 ± 0.34	4.25 ± 0.27
$\Delta\varepsilon_{10}^s$	-1.87 ± 0.27	-0.25 ± 0.12	-0.54 ± 0.10
$\Delta\psi_{10}^c$	-0.39 ± 0.47	0.85 ± 0.33	0.81 ± 0.25
0.5 yr $\Delta\varepsilon_9^c$	-0.65 ± 0.27	-0.58 ± 0.11	-0.50 ± 0.10
$\Delta\psi_9^s$	1.09 ± 0.50	1.70 ± 0.33	1.37 ± 0.25
$\Delta\varepsilon_9^s$	1.02 ± 0.29	-0.61 ± 0.12	-0.44 ± 0.10
$\Delta\psi_9^c$	-0.02 ± 0.52	-0.28 ± 0.30	-0.27 ± 0.24
14 day $\Delta\varepsilon_{31}^c$	0.80 ± 0.47	0.18 ± 0.11	0.07 ± 0.10
$\Delta\psi_{31}^s$	-2.97 ± 0.93	-0.49 ± 0.30	-0.74 ± 0.23
$\Delta\varepsilon_{31}^s$	0.91 ± 0.42	0.05 ± 0.12	-0.12 ± 0.10
$\Delta\psi_{31}^c$	0.75 ± 0.82	0.68 ± 0.31	0.56 ± 0.25
FCN $\Delta\varepsilon_F^c$	-0.12 ± 0.26	-0.29 ± 0.13	-0.34 ± 0.11
$\Delta\psi_F^s$	-0.71 ± 0.42	1.21 ± 0.42	0.04 ± 0.28
$\Delta\varepsilon_F^s$	-1.03 ± 0.25	0.10 ± 0.15	-0.13 ± 0.13
$\Delta\psi_F^c$	0.19 ± 0.50	-0.93 ± 0.35	-0.61 ± 0.26

LLR data cover a period long enough to properly separate the precession constant and the 18.6 yr and 9 yr nutation terms.

In the combined VLBI/LLR analysis, correlation coefficients are significantly reduced relative to those of the VLBI analysis alone: only 8 correlation coefficients are larger than 0.50 (instead of 15 in the VLBI analysis), and the maximum value is 0.88 (instead of 0.997 in the VLBI analysis). It is also to be noted that formal uncertainties of parameters are

Table 2. Comparison of Correlation Coefficients $|\rho| > 0.5$ for the Precession Constant and the 18.6 yr and 9 yr Nutation Terms.

	$\Delta\varepsilon_1^c$	$\Delta\psi_1^s$	$\Delta\varepsilon_1^s$	$\Delta\psi_1^c$	$\Delta\varepsilon_2^c$	$\Delta\psi_2^s$	$\Delta\varepsilon_2^s$	$\Delta\psi_2^c$
Prec.	... ...	... 0.91 ...	... ...	0.64 0.997 0.88	... ...	... 0.86 0.52	... ...	... 0.91 ...
$\Delta\varepsilon_1^c$		... ...	... 0.77 0.66	... ...	... ...	... ...	... 0.74 0.72	... ...
$\Delta\psi_1^s$			... ...	... 0.89 ...	... ...	0.57 0.60 0.72	... ...	... 0.97 ...
$\Delta\varepsilon_1^s$				... ...	... 0.82 0.74	... ...	0.58 0.89 0.82	... ...
$\Delta\psi_1^c$					... ...	... 0.88 0.69	... ...	... 0.90 ...
$\Delta\varepsilon_2^c$						... ...	... 0.64 ...	... ...
$\Delta\psi_2^s$							... ...	... 0.65 ...

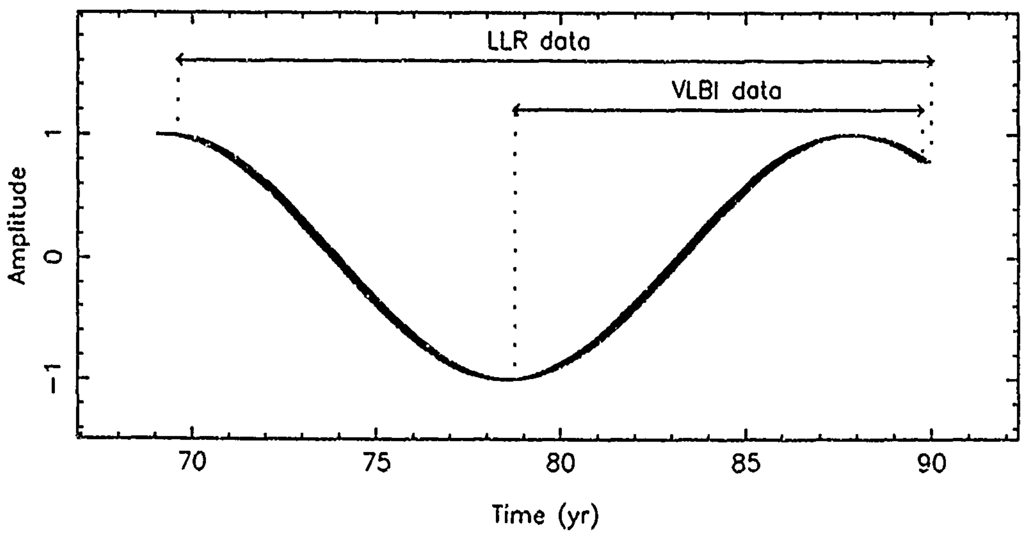

Figure 2: Shape of $\Delta\psi_1^c$ and coverage of VLBI and LLR data between 1969 and 1990.

smaller in the combined analysis than in the individual VLBI and LLR analyses, with improvements of up to a factor of 2.5 (case of $\Delta\psi_2^c$). Based on the decrease of the correlation coefficients and the improvement of formal uncertainties, we argue that the combined VLBI/LLR analysis is stronger than either analysis alone. Comparing our VLBI/LLR estimates with the independent VLBI results of Herring (1988) and Zhu *et al.* (1990) indicates good overall agreement, with most of the differences at the $1-2\ \sigma$ level. Because of the addition of the LLR data, our estimates of the precession constant and the 18.6 yr nutation terms are stronger than those of Herring (1988) and Zhu *et al.* (1990). It is also worth noting that for the 9 yr nutation terms, not previously estimated from VLBI data, our estimates of the corrections (-0.12 ± 0.18, 1.17 ± 0.62, 0.04 ± 0.24, 0.08 ± 0.41 mas) are in good agreement with those from the deformable-Earth series of Zhu *et al.* (1990) (-0.25, 1.23, 0.0, 0.0 mas).

5. Conclusion

25 precession and nutation parameters are estimated in a combined analysis of 20 years of LLR data and 11 years of VLBI data. This global VLBI/LLR analysis combines the information matrices derived separately from the VLBI and LLR data equations. Comparisons of formal uncertainties and correlation coefficients between various precession and nutation parameters show that the combined VLBI/LLR analysis is stronger than either the VLBI or the LLR analysis alone. This combined analysis provides estimates of the precession constant and the 18.6 yr nutation terms that are stronger than any previous results. It also provides, for the first time, estimates of the 9 yr nutation terms. We emphasize that our results do not depend on any geophysical hypothesis, that they are unconstrained, and that they have been obtained with the full VLBI and LLR information matrices retained. The overall agreement between our estimates and the independent results of Herring (1988) and Zhu *et al.* (1990) is good.

Acknowledgments

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INTERNAL PRECISION IN CCD ASTROMETRY OF QSO OPTICAL COUNTERPARTS

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ABSTRACT. The main purpose of this contribution is twofold. First, we report on a preliminary assessment of the astrometric properties of the CCD system we are presently using to extend the magnitude limit of the Torino program on optical positions of QSO's. This assessment is based on the internal consistency of the positions derived from different CCD frames of the same objects. Then, we give a first evaluation of the precision of CCD derived QSO positions after extending a plate based reference frame (secondary frame) onto the CCD images (tertiary frame).

Special emphasis is given to comparisons with plate positions of the same faint objects. It is shown that use of the CCD camera promises a factor of five (or larger) improvement in the precision of QSO positions.

1. Introduction

The extention to fainter magnitudes ($V > 16.5$) of the Torino program on optical positions of extragalactic radio-sources (Chiumiento et al. 1987, Chiumiento et al. 1989) brought us to experiment with the 320 x 512 pixels RCA CCD (1 square pixel = 30 μm on a side) attached to the 1.5 m Ritchey-Chretien of the University of Bologna. The CCD system operated at a temperature of about 120 °K and the telescope scale at the CCD chip is 0.51"/pixel.

Several targets exist in our observing list such that 3 or more stars (besides the QSO) are measurable on our photographic plates within the CCD field (2'.7 x 4'.4). The plates are calibrated via CAMC stars (La Palma 1989); thus the CCD frames can be tied to the CAMC (FK5) system by using the stars surrounding the QSO as the link stars.

In the next section we will briefly describe the available CCD observations. Then, we report on the internal precision of our CCD astrometry. Finally, a preliminary evaluation of the precision of CCD based QSO positions is given in section 5. There, we also compare the quality of our CCD astrometry to photographic errors we had previously obtained for the same targets.

2. Observations

During four nights (Jan 24, Feb 5-7, 1990) partially devoted to this program, we obtained 34 CCD frames, B and V colors, of 10 radiosources of our main program. The exposure times ranged from 5 to 30 minutes. The February nights were during full Moon; thus about one third of the available exposures have an abnormally high sky background. The number of usable link stars in the frames varies from 3 to 10.

3. Reduction of the CCD frames

As usual, the CCD frames are first corrected for dark current, bias, and pixel-to-pixel sensitivity variations (flat fielding). Then, the astrometric image processing is done using the PC-based software ROBIN developed at Torino Observatory (Lanteri 1990). ROBIN is a package specifically designed for astrometric reductions of both CCD and PDS images.

ROBIN fits a bidimensional gaussian-like function to the star images, plus a linear (both in x and y coordinates) polynomial to take into account the sky background. The fits are performed on *windows* extracted from the frames, whose sizes vary with object magnitudes.

ROBIN outputs the estimated x and y coodinates (in pixel units) of the objects successfully centered (i.e. the center of the bidimensional gaussian model) and their dispersions (σ 's). Instrumental magnitudes of the same objects are also derived.

Finally, these quantities are processed through our standard astrometric software.

4. Internal precision of CCD Astrometry

For this preliminary study of the internal precision of CCD based positions, we used frames of comparable exposure times on the two QSO's 1148-001 and 1611+343 (Argue *et al.*, 1984). Both targets have a B magnitude close to $17^m.5$. Ten and seven star-like images (besides the QSO's) were successfully centroided (see sec. III) in the fields of 1148-001 and 1611+343 respectively. The signal-to-noise ratios of the different images varied from $S/N \simeq 25$ (for objects of $B \simeq 16^m$) to $S/N \simeq 3$ (for $B \simeq 20^m$).

Also, through the estimated σ's of the stars, we computed the FWHM of the PSF of each frame ($FWHM \simeq 2.4 \times \sigma$). A typical σ during our observing nights was 1.6 pixel, which gives a typical seeing of $\simeq 2''$.

The internal astrometric error was determined by comparison of the different exposures after bringing each of them into the same instrumental reference frame via a 6-constant least squares adjustment.

The average error in one coordinate ranges from 0.02 pixel ($\simeq 0''.01$) for the "bright" objects ($< B > \simeq 17^m$) to about 0.15 pixel ($\simeq 0''.07$) for those close to the frame limit ($< B > \simeq 20^m$).

Estimates of the photometric internal precision were calculated as well. The average errors vary from $0^m.01$ for the bright objects to $0^m.15$ for the faint ones.

5. CCD based QSO positions

In order to evaluate the internal consistency of QSO positions, we selected test objects from our main list for which we have both plate and CCD series. Unfortunately, the CCD run covered a range in right ascension poorly covered by our reduced photographic material. Anyway, we had available 2 plates of 1611+343. Both plates were taken at the same telescope used for the CCD observations. One of the two plates covers a square field of about 65' on a side and contains 13 CAMC stars. On this plate we measured both the secondary (15 stars) and the tertiary (7 stars) reference frames. The secondary link stars were used to reduce the other plate which is only 9x12 cm. The magnitudes of the CAMC stars range from V=10 to V=13. Secondary link stars have magnitudes between 14^m and 16^m and the Tertiary stars cover the interval from 16.5^m to 17.5^m.

Given the photographic tertiary frame, the three CCD frames available for 1611+343 were independently reduced. Then, the three positions were intercompared to evaluate the internal consistency. The resulting root-mean-square errors are ±0".038 in RA and ±0".047 in DEC. This result, although very preliminary, points to an improvement of about a factor of 10 over the traditional photographic technique at $B \simeq 17^m.5$, as shown in Figure 1.

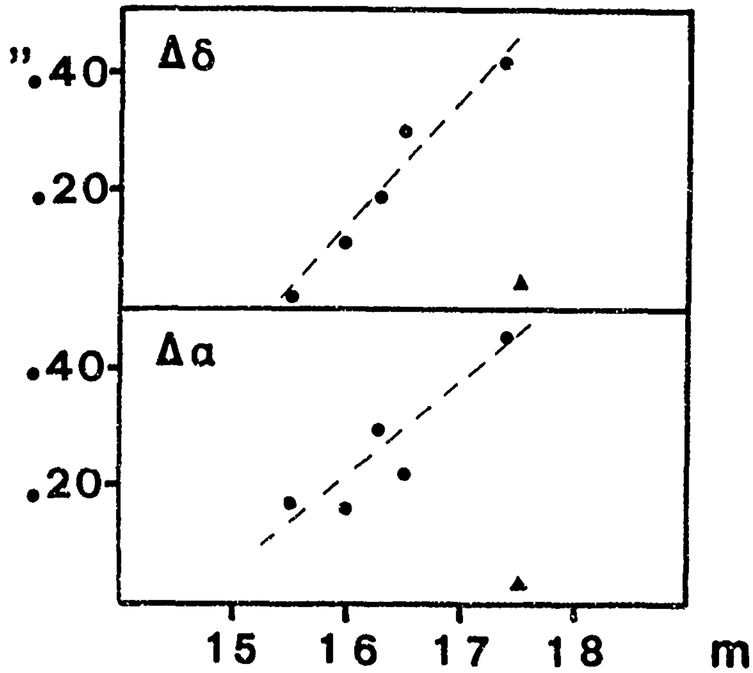

Figure 1. Photographic vs CCD astrometric precision

• = Photographic; ▲ = CCD

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PRESENT STATUS OF WORK ON THE FK5 EXTENSION

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ABSTRACT. The FK5 Extension, consisting of 3117 new fundamental stars selected from the FK4 Sup catalogue and the IRS list, will extend the FK5 system to about 9.5th visual magnitude. The construction of the FK5 Extension is briefly described and the main characteristics are given.

1. INTRODUCTION

The Fifth Fundamental Catalogue (FK5) will consist of two parts, namely the "Basic FK5" (Fricke et al., 1988) and the "FK5 Extension". The Basic FK5 contains the classical 1535 fundamental stars already given in the FK4. Systematic and individual corrections to the mean positions and proper motions of the FK4 have been derived and the IAU(1976) System of Astronomical Constants has been introduced. The Basic FK5 defines the system of the new fundamental catalogue; it has been constructed with the aim to represent an inertial system as far as possible. Details are given in the introduction to the Basic FK5 as well as by Schwan (1987) and in the literature quoted there.

One important shortcoming of the FK4 is the predominance of bright stars. Only about 100 stars are fainter than magnitude 6.5 and thus the FK4 is not well defined at magnitudes fainter than this. The extension of the optical system to fainter magnitudes was therefore an indispensable task in constructing the FK5.

2. SELECTION OF THE NEW FUNDAMENTAL STARS

It was realized by Fricke (1973) that there are essentially two star lists from which the new fundamental stars could be selected: the International Reference Stars (IRS) for extending the system to about magnitude 9.5 and the FK4 Sup stars which had to fill a remaining gap in the magnitude distribution from about 5th to 7th magnitude.

Mean positions and proper motions were determined for all IRS at the U.S. Naval Observatory (Corbin and Urban, 1990) and for all FK4 Sup stars at the Astronomisches Rechen-Institut (Schwan, 1987). On the basis of the mean errors of the positions and proper motions and the distribution over the sky coupled with the distribution in magnitude we have selected 992 stars from the FK4 Sup and 2125 stars from the IRS, altogether 3117 new fundamental stars. These new fundamental stars represent the FK5 Extension and they are to define the FK5 system for fainter stars up to about 9.5th mag.

It seems to be worth mentioning that the FK5 Extension includes 12 FK4 Sup stars not yet in the tape version of the bright stars in the FK5 Extension which has been distributed since 1988.

3. DERIVATION OF MEAN POSITIONS AND PROPER MOTIONS

The FK5 Extension was primarily derived in the system of the FK4. All observations which could be used for the derivation of mean positions and proper motions had therefore to be referred to that system. This transformation was comparatively easy in the case of the FK4 Sup stars since most of them do not exceed seventh magnitude. On the basis of the FK4 stars in an observational catalogue the systematic relations Cat-FK4 were determined and the observed positions were directly transformed to the FK4 system.

In the case of the IRS, in particular of the southern IRS, this simple procedure was not possible, since most of these stars are outside the limit of FK4 magnitudes. The systematic relations Cat-FK4, determined on the basis of the FK4 stars alone, could not be directly applied to the fainter stars. It was necessary to construct first an intermediate system which represents the FK4 system for fainter magnitudes. The observations which could be used for that purpose had either to be free of magnitude dependent errors or their systematic errors at faint magnitudes had to be determined. North of -30 degrees the catalogues observed with screens (which eliminate magnitude equations) could be used to derive such an intermediate system. There was, however, an insufficient number of appropriate catalogues for deriving a corresponding system south of -30 degrees. In that region an extrapolation of the magnitude equation from northern declinations to the southern region was necessary. This extrapolation could be performed by making use of southern catalogues observed with a moving-wire micrometer. Such catalogues have been found to have magnitude equations that are not declination dependent.

The construction of an intermediate system extending the FK4 system to about magnitude 9.5 was the essential step in deriving the astrometric data for the IRS stars. This extended FK4 system could be used to reduce many other catalogues with observed faint stars to the FK4 (see also Corbin and Urban, (1990)).

After having transformed all relevant observations to the FK4 system we have performed weighted least squares solutions for deriving the mean positions and proper motions from the various observed catalogue positions.

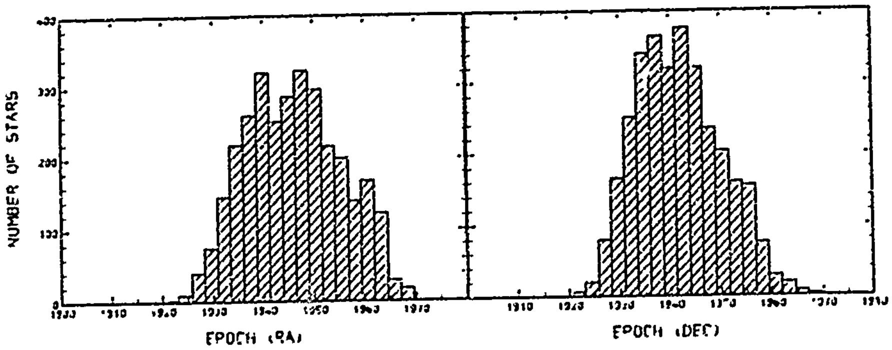

Fig. 1: FK5 Extension: Distribution of mean epochs.

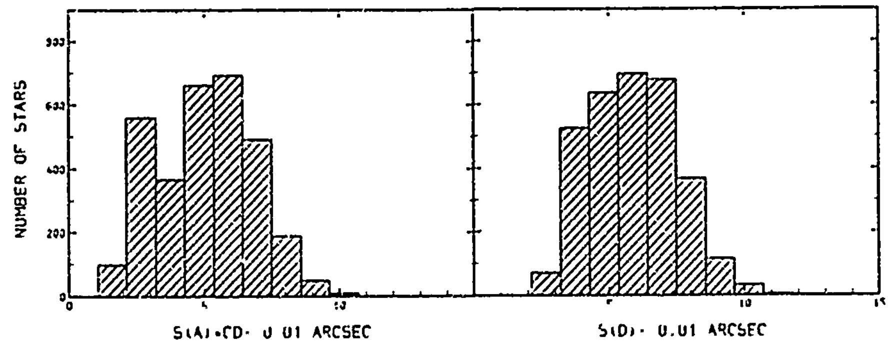

Fig. 2. FK5 Extension: Distribution of mean errors of mean positions in RA (left) and DEC (right), respectively; units: 0.01 arcsec.

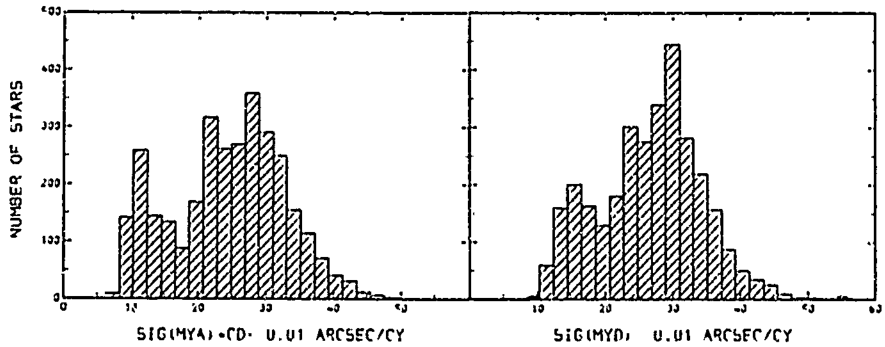

Fig. 3. FK5 Extension: Distribution of mean errors of the proper motions in RA (left) and DEC (right), respectively; units: 0.01 arcsec/cy.

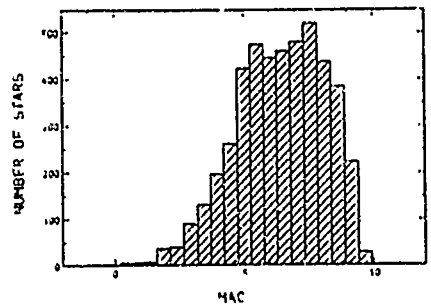

Fig.4. Distribution of apparent magnitudes of the whole FK5 (Basic FK5 plus Extension)

4. MAIN CHARACTERISTICS OF THE FK5 EXTENSION

The main characteristics of the FK5 Extension are presented in the four figures. In Fig. 1 we show the distribution of mean epochs in right ascension (left) and declination (right), respectively. There is a small dip in both distributions indicating that the FK4 Sup stars in the FK5-Extension have, on the average, more recent mean epochs than the IRS. This is a consequence of the fact that the Sup stars were preferentially observed after 1955 when they had been proposed as candidates for a future extension of the fundamental system. The two subgroups can more or less also be identified in Fig. 2 and Fig. 3. The average mean epoch of the FK5 Extension is 1944.

In Fig. 2 are given the distributions of the mean errors of mean positions in right ascension (left, multiplied with cos(delta)) and declination (right), respectively, and in Fig. 3 one finds the corresponding distribution of the proper motion errors. The FK4 Sup stars are a little more precise than the IRS. The overall precisions are 0.055 arcsec for the mean positions and 0.255 arcsec/cy for the proper motions.

In Fig. 4 we present the distribution of apparent visual magnitudes of the whole FK5 (Basic plus Extension). Preliminary magnitudes were used in the star selection and also in Fig. 4. It is, however, unlikely that the final magnitudes will alter this distribution significantly.

5. PRESENTATION OF THE FK5 EXTENSION

The FK5 Extension will be given, as far as possible, in the same format as the Basic FK5. We plan to publish the following data for each star: FK5 number, apparent visual magnitude, spectral type, position and proper motion for the epoch and equinox J2000 in accordance with the IAU (1976) System of Astronomical Constants, the corresponding values transformed to epoch and equinox B1950, mean epochs of observation, mean errors of position and proper motion at the mean epochs, and identifications with some other important star lists. Paralaxes and radial velocities will be given in the catalogue for all stars with significant foreshortening terms.

We hope that a tape version of the FK5 Extension can be made available around the beginning of the next year.

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β PERSEI, A FUNDAMENTAL STAR AMONG THE RADIOSTARS

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ABSTRACT. Optical fluctuations of the radiostar β Perseï are seen from 13 campaigns performed with the astrolabe located at the Paris Observatory.

1. INTRODUCTION

Among the radiostars, βPerseï (Algol) - a fundamental star - was chosen by radioastronomers as a zero reference for right ascensions in radioastrometry. Since 1975 this fundamental star has been included in the observing programme performed by the "Astrolabe et systèmes de référence" group in charge of the instrument at the Paris Observatory. The eight first campaigns published have been presented at the IAU Colloquium n° 100 (Belgrade 1987). The average of the mean square errors given were 0.004s in right ascension and 0.13" in declination, according to the FK4 and the constants in use at that time.

2. DETERMINATIONS AND ERRORS

There are now thirteen campaigns available from 1975/76 to 1987/88 and they have been reduced in the FK5 system with the new fundamental constants according to the formulas established by Chollet (1984). Due to the fact that the group and the internal smoothing corrections (according to Débarbat et Guinot, 1970) are not yet available in the case of the FK5, the reduction have been performed for both FK4 and FK5. As an example of residuals, for the zenith distance, to which accuracy this quantity is obtainable when 12 transits (at east and at west) are observed, Table I gives the values for the 1983/1984 campaign (J 2000, FK4 and FK5).

Table I

East residuals	FK4	- 0.114" ± 0.085"	12 transits
J 2000	FK5	- 0.123" ± 0.096"	
West residuals	FK4	+ 0.077" ± 0.078"	12 transits
J 2000	FK5	+ 0.093" ± 0.081"	

The residuals are not significantly different when the FK4 and the FK5 quantities are used. Also their mean square errors have the same order of magnitude.

For each of the 13 campaigns (1975/1976-1987/1988), $\Delta\alpha$ and $\Delta\delta$ have been derived, the calculation being made in two cases J 2000, FK4 and J 2000 FK5. The probable errors calculated for each campaign correspond to an average which is the same in the case of the FK4 and in the case of the FK5 : ± 0.0058s (right ascension), ± 0.116" (declination).

3. FLUCTUATIONS IN RIGHT ASCENSION AND IN DECLINATION

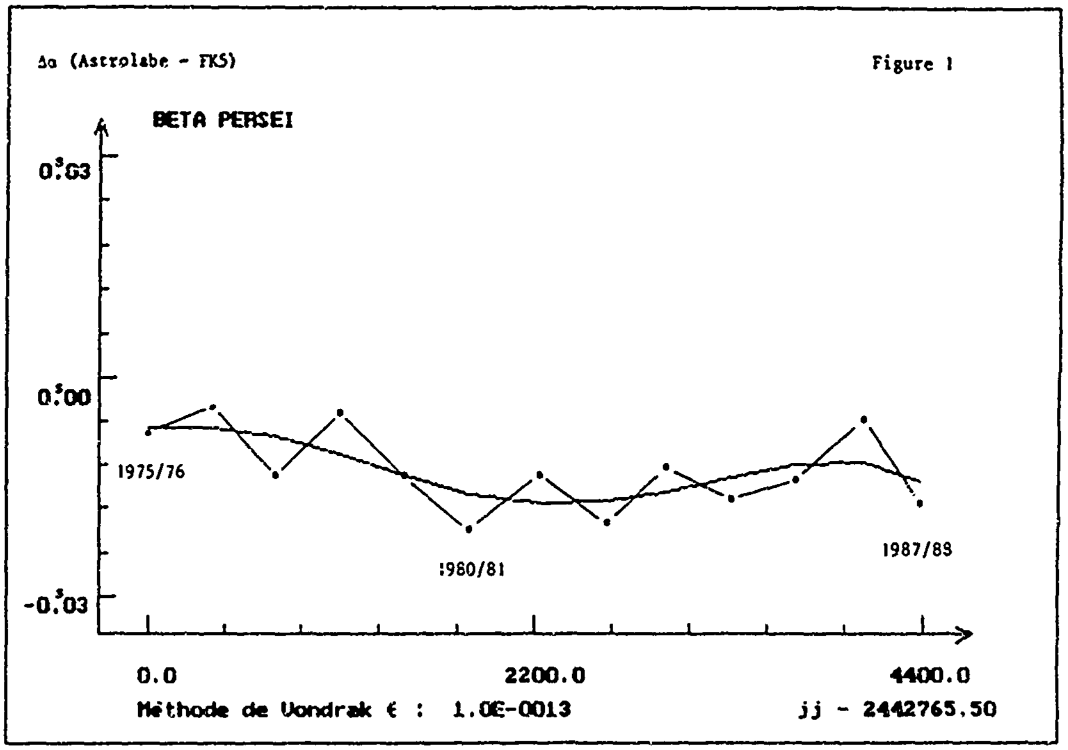
As fluctuations appeared in both coordinates (fig.1 and fig.2), smoothing curves (according to Vondrak 1969) have been determined with the same smoothing factor ; they are reported on figures 1 and 2. The corresponding mean square errors, together with the errors with which the curves are mathematically given, are in Table II.

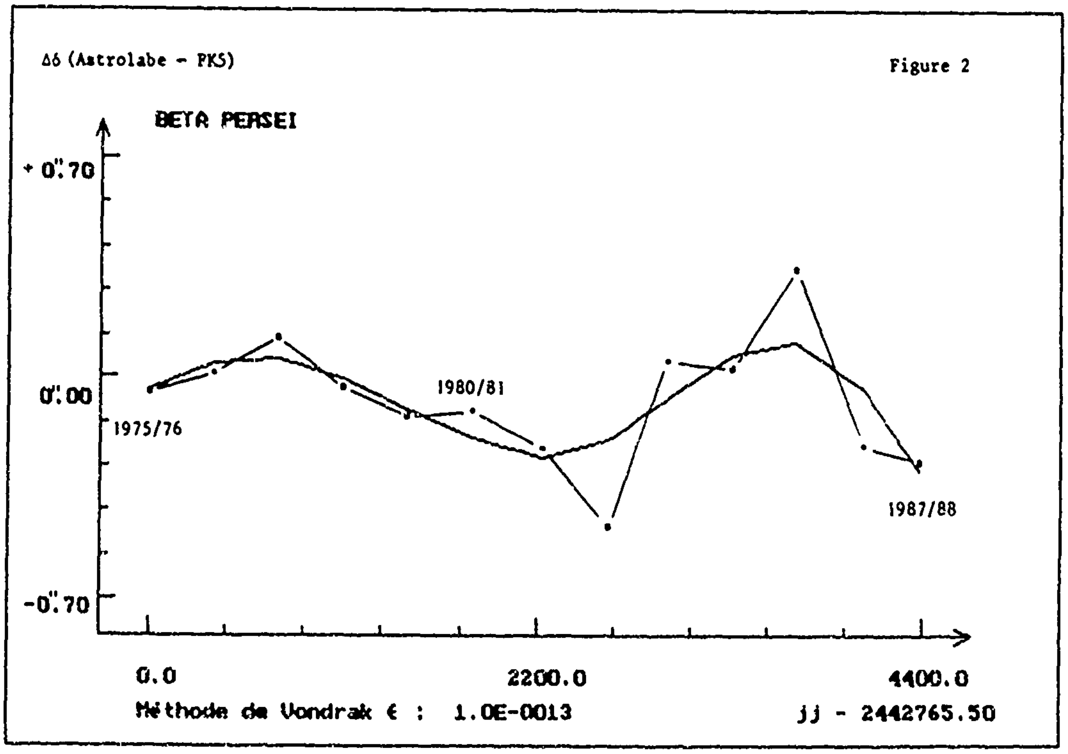
Table II

	Mean square error	Error of the curve
Right ascension	± 0.0042s	± 0.0012s
Declination	± 0.140"	± 0.039"

The amplitudes for the fluctuations (0.010s in right ascension, 0.25" in declination) and the associated errors (0.0012s and 0.039") show that the optical variations appears to be real.

The optical positions of this radiostar (which is also a multiple star), no longer used as a zero reference in right ascension, but still an object of interest for radioastronomers and double star specialists, must be compared with VLA and/or VLBI determinations for the same period.


Δα (Astrolabe - FK5)
Figure 1
BETA PERSEI
0ˢ.03
0ˢ.00
-0ˢ.03
1975/76
1980/81
1987/88
0.0
2200.0
4400.0
Méthode de Vondrak ∈ : 1.0E-0013
jj - 2442765.50


Δδ (Astrolabe - FK5)
Figure 2
BETA PERSEI
+ 0".70
0".00
-0".70
1975/76
1980/81
1987/88
0.0
2200.0
4400.0
Méthode de Vondrak ∈ : 1.0E-0013
jj - 2442765.50

4. CONCLUSION

β Perseï, as a fundamental star among the radiostars will be used for the linkage of the "optical" and the "radio" system of reference. β Perseï represents an example of the problems to which the link will have to face due to the fluctuations this star is showing after 13 years of optical observations.

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Improving the Reference Frame by Radio-and Optical Astrometry of Radio Stars

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ABSTRACT. A longterm program of precise radio - and optical astrometry of selected radio stars has been conducted in the last decade by our observatories using the VLA and astrographs on both hemispheres. Positions of 54 stars north of -26 deg. declination including 6 MASER stars have been obtained. The program status and some results from the southern hemisphere are reported.

1. Introduction

The VLA has been used since the early '80 to determine precise astrometric parameters of selected radio stars in the primary extragalactic VLBI reference frame. For a recent detailed description of the observing program and previous results we refer to /1/ and further references therein.
The main goal of the program is to provide a net of about 100 radio stars globally to link the present groundbased optical reference frame to the VLBI based extragalactic reference frame and to provide a similar link for the space based HIPPARCOS stellar net /2/.
As most stars display radio emission at cm wavelengths only on the level of a few mJ, presently only the VLA can provide the necessary sensitivity although the accesible sky coverage is limited to > -26 deg. decl.

2. Status of Radio Work

At present precise positions (±0.01-0.02 arcsec) have been obtained for 54 stars, including 6 MASER stars. Radio proper motions (±0.004 mas/yr) have been determined now for the stars HR1099 and UX Ari and second epoch observations have begun for additional stars.

The quoted accuracies may be improved finally to the 1 mas level by incorporating additional calibrator sources in the close vicinity of the stars. In addition work on radio parallaxes has been started for UX Ari. The diagram displays the distribution of the present radio stars. The lack of observations on the southern hemisphere is obvious, however, the coverage extends already far enough to the south to allow for a rotation solution in the comparision of the reference frames.

3. Status of Optical Observations

Almost all radio stars of the present sample are optically brighter than visual magnitude 12 . Therefore they can be tied to the optical fundamental reference frame as given by the IRS-catalog easily by use of high quality wide field astrographs on both hemispheres.
Due to the favourable field size of these instruments which is at least 25 sq.deg. and corresponding number of IRS-reference stars, the positions of the radio star can be determined with high systematic accuray (<0.05 arcsec) in the FK5 system. Table 1 summarizes the main instrumental parameters.

Table 1. Main Instrumental Parameters of Astrographs

Northern Hemisphere	Southern Hemisphere
Site: Hamburg +53.5 Lat.	New Zealand -41.8 Lat. *)
Aperture: 23 cm	20 cm
Scale: 100"/mm	100"/mmm
Field: 6x6 deg.	5x5 deg.
Spectral range: 5200 - 5800 Å	5000 - 5800 Å
Gratings: 4 mag / 6 mag	4 mag / 6 mag
Emulsion + Filter: 103aG+OG515	103aG+GG495
Plate size: 240x240x6.4 mm III	203x254x6.4 mm III III=(micro-flat)
Reference star Catalog: AGK3RN	SRS **)

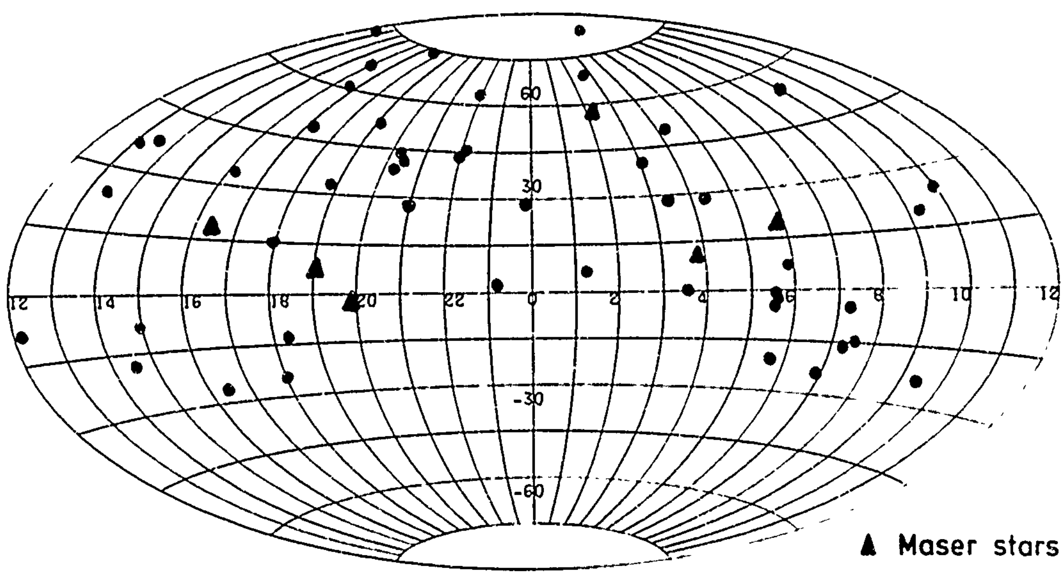
*) USNO Southern Hem. Station at Black Birch Astron. Observatory

**) SRS=final Version USNO /3/, both cat. FK4/B1950 and FK5/J2000 version.

Normally 4 plates (single exposure) or 2 plates (double exp.) have been used for each object and 2 (1) plates have been combined in pairs, exposed symmetrical to the meridian and in opposite orientation of the telescope to the sky to minimize possible residual color dependent effects of the optics and atmosphere /4/. Depending on the magnitude of the radio star, a 6 or 4 mag. grating has been used. Most of the plates have been measured on the recently completely automated and CCD-camera based MANN-Comparator of Hamburg Observatory /5/. Typical mean errors of 0.05 arcsec for the optical positions have been achieved. Both the AGK3RN and SRS catalogs have been transfered to the FK5-system using the analytical expressions, as developed at ARI-Heidelberg /6/. As the systematic differences FK5-FK4 reach their largest amplitudes on the southern hemisphere, radio stars may provide an excellent test, at least locally. Table 2 shows some examples. The m.e. of a single "O-R" difference is 0.05 arcsec wich demonstrates the systematic quality of the SRS catalog obviously. Similar results have been obtained recently for a small sample of extragalactic radio sources /7/.

Table 2. Examples of Southern Hemisphere System Comparisons "Optical - "Radio" (SRS cat.FK5/J2000)

Star	DA*cos(dec)	DD [arcsec]
TW Lep	+0.05	+0.04
RV Lib	+0.11	-0.17
HU Vir	-0.06	-0.07



Distribution of stars from VLA-program

4. Acknowledgements

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INFLUENCE OF THE SOLID INNER CORE AND COMPRESSIBILITY OF THE FLUID CORE ON THE EARTH NUTATION

Sergei DIAKONOV

1 INTRODUCTION

While calculating low frequency oscillations of the Earth liquid core spherical harmonic representation of the deformation field is usually used [1-3]:

$$\mathbf{u} = \sum \sum [S_l^m + T_l^m] \ . \tag{1}$$

Substitution of (1) into the equations of motion gives an infinite system of differential equations for scalar functions S_l^m and T_l^m. Approximate solutions of such a system are obtained by truncating of the system. But results of [4] show that sometimes such method divergences.

2 NUTATION OF THE EARTH WITH COMPRESSIBLE CORE

2.1 MODEL. Let us calculate a forced nutation amplitude for the Earth model consisting of the rigid mantle and compressible liquid core with simple liquid density distribution:

$$\rho(r_\circ) = \rho_0(1-\delta r_\circ^2), \tag{2}$$

where $r_\circ$ is dimensionless radius.

Let us investigate behavior of such a system affected by tide-generating potential. The mantle angular velocity may be written in the form:

$$\Omega = \{ \eta(\mathbf{i}-i\mathbf{j})\mathrm{ext}(i\sigma t) + \mathbf{k} \}\omega \quad , \tag{3}$$

and nutation amplitude η must be found.

2.2 EQUATIONS OF LIQUID OSCILLATIONS. Small oscillations of the fluid core are described by the following equations:

$$\Delta\psi - \frac{4\omega^2}{\sigma^2}\frac{\partial^2\psi}{\partial z^2} = \tag{4}$$

$$= -(\sigma^2-4\omega^2)\frac{\psi-[V_1+V_t+\eta\omega^2(xz-iyz)\exp(i\sigma t)]}{\alpha^2} - \frac{\sigma^2-4\omega^2}{i\sigma}\mathbf{v}\frac{\nabla\rho}{\rho} ,$$

$$\Delta V_1 = 4\pi G\rho(1/\alpha^2)(\psi-V_1-V_t-\eta\omega^2(xz-iyz)\exp(i\sigma t)) \tag{5}$$

$$\psi = P_1/\rho+V_1+V_t+ \eta\omega^2(xz-iyz)\exp(i\sigma t) \ , \tag{6}$$

Here $\mathbf{v}$, P_1, V_1 are perturbations of liquid velocity, pressure and gravitansional potential correspondingly, α is speed of sound and V_t is tide-generating potential.

2.3 METHOD OF SOLUTION. Let us represent the solution in the form:

$$\psi = \psi_p + \delta\psi \quad , \tag{7}$$

where ψ_p is the Poincare solution:

$$\psi_p = -\eta\sigma\omega \frac{1-1/k-\tau^2/(1-\varepsilon_c^2)}{1-1/k+\tau^2/(1-\varepsilon_c^2)} (xz-iyz), \quad k=\sigma/2\omega, \quad \tau^2=1-1/k^2 . \tag{8}$$

Let represent $\delta\psi$ by expansion on characteristic functions of the Poincare operator:

$$\delta\psi = \sum_m \sum_l \sum_k \left\{ a^m_{lk} \Psi^m_{lk} \right\} \exp(i\sigma t) \quad , \tag{9}$$

and Ψ^m_{lk} functions satisfy to the equation

$$(\Delta - \frac{4\omega^2}{\sigma^2} \frac{\partial^2}{\partial z^2}) \Psi^m_{lk} = \lambda^m_{lk} \Psi^m_{lk} \quad . \tag{10}$$

Substitution of (7) and (9) into the system (4)-(5) gives a system of equations for a^m_{lk}. Nondiagonal elements of the matrix of this system, as numerical results show, are small compared to the diagonal ones. This allows the truncation the expansion (9) in order to obtain an approximate solution.

2.3 MAIN RESULTS AND DISCUSSION. Nutation amplitudes for different maximum numbers of characteristic functions in (9) are given in table 1. For comparison, nutation amplitudes for the rigid and Poincare models are also shoun.

Table 3. Nutation amplitudes η in angular milliseconds (δ=0.2, ε_c=0.0715) for an Earth with rigid mantle.

$\frac{\omega}{\sigma+\omega}$	Solid model	Poincare model	Compressible core model		
			M=2	M=4	M=6
-6800	8051.05	7999.60	8000.851	8000.848	8000.848
-365.3	24.94	-38.633	-25.536	-25.552	-25.554
-182.6	22.60	28.197	28.329	28.329	28.329

The results obtained show that the compressibility of the fluid Earth core may significantly affect theoretical nutation amplitudes and must be taken into account while calculating nutation amplitudes. Comparison of theoretical nutation amplitude with radiointerferometer data may give some information about the Earth core.

3 NUTATION OF EARTH WITH SOLID INNER AND FLUID OUTER CORES

3.1 FORMULATION OF THE PROBLEM. Let us investigate a sympli-fied earth model consisting of: (i) rigid mantle with ellipsoidal cavity; (ii) ideal homogeneous incompressible liquid, filling the cavity; (iii) solid inner core, which is under the influence of the gravitation field in the center of the cavity.

Let the surface of the cavity and the surface of the inner core be discribed by the equations:

$$x^2 + y^2 + z^2/(1-\varepsilon_1^2) = R_1^2 \quad , \tag{11}$$

$$x^2 + y^2 + z^2/(1-\varepsilon_2^2) = R_2^2 \quad . \tag{12}$$

Here R_1 and R_2, ε_1 and ε_2 are equatorial radiuses and eccentricities of cavity and solid core correspondingly.

Let us examine the system behavior under the action of the tide-generating potential. The angular velocity of mantle rotation is of the form:

$$\Omega = \{\ \eta(\mathrm{i}-\mathrm{ij})\mathrm{ext}(\mathrm{i}\sigma t) + \mathbf{k}\ \}\omega, \tag{13}$$

Motion of the inner core with respect to the moving coordinate system may be described by instantaneous angular velocity:

$$\delta\omega = \eta_1\omega\ (\mathrm{i} - \mathrm{ij})\exp(\mathrm{i}\sigma t) \tag{14}$$

Unknown amplitudes of nutation η and η_1 must be determined from the solution of the problem.

The small oscillations of the fluid core are described by the following equation:

$$\Delta\psi - \frac{4\omega^2}{\sigma^2}\frac{\partial^2\psi}{\partial z^2} = 0 \quad , \tag{15}$$

with boundary conditions of the form:

$$\hat{\mathbb{B}}_0\psi = \frac{\mathrm{i}k\sigma\eta}{2(1-k^2)R_1}\left(1 - \frac{1}{k} - \frac{\tau^2}{1-\varepsilon_1^2}\right)(xz - \mathrm{i}yz) \ , \qquad \mathbf{r} \in \mathrm{S}_1$$

$$\hat{\mathbb{B}}_0\psi = \left[\frac{\mathrm{i}k\sigma\eta}{2(1-k^2)R_2}\left(1 - \frac{1}{k} - \frac{\tau^2}{1-\varepsilon_2^2}\right) + \frac{\mathrm{i}\eta_1\omega}{R_2}\frac{\varepsilon_2^2}{1-\varepsilon_2^2}\right](xz-\mathrm{i}yz), \ \mathbf{r} \in \mathrm{S}_2$$

$$\hat{\mathbb{B}}_0\psi = \frac{-\mathrm{i}k}{2\omega(1-k^2)R}\left\{\left(x - \frac{y}{\mathrm{i}k}\right)\frac{\partial\psi}{\partial x} + \left(y + \frac{x}{\mathrm{i}k}\right)\frac{\partial\psi}{\partial y} + \frac{\tau^2 z}{1-\varepsilon^2}\frac{\partial\psi}{\partial z}\right\} \tag{16}$$

$$k = \sigma/2\omega \ , \qquad \tau = 1 - 1/k^2 \ .$$

Here S_1 and S_2 are the surfaces of the cavity and the solid core correspondingly.

3.2 RESULTS. Approximate solution of the boundary value problem (7), (9) may be written in the form:

$$\psi \approx (a_1 + a_2)(xz - iyz). \tag{17}$$

$$a_1 = -\eta\sigma\omega \frac{1-1/k-\tau^2/(1-\varepsilon_1^2)}{1-1/k+\tau^2/(1-\varepsilon_1^2)}$$

$$a_2 = -\frac{\eta\sigma\omega}{h_2^*}\left\{\left[1-\frac{1}{k}-\frac{\tau^2}{1-\varepsilon_2^2}\right]-\left[1-\frac{1}{k}+\frac{\tau^2}{1-\varepsilon_2^2}\right]\frac{1-1/k-\tau^2/(1-\varepsilon_1^2)}{1-1/k+\tau^2/(1-\varepsilon_1^2)}\right\} - $$

$$-\frac{2\eta_1\omega^2(1-k^2)}{kh_2^*}\frac{\varepsilon_2^2}{1-\varepsilon_2^2}$$

$$-h_2^* R_2^5 = \left\{\left[1-\frac{1}{k}+\frac{\tau^2}{1-\varepsilon_1^2}\right]R_1^5 - \left[1-\frac{1}{k}+\frac{\tau^2}{1-\varepsilon_2^2}\right]R_2^5\right\}$$

In table 2 there are represented the values of nutation amplitudes for: (i) the solid Earth model, (ii) the Poincare model, (iii) the examined model. Numbers in Table 2 shows that correction to forced nutation amplitude due to the existance of the inner core, may be greater than the accuracy of observations.

Table 1. Nutation amplitude in angular miliseconds (r_2=0.357r_1, ε_1=0.0715, ε_2=0).

$\frac{\omega}{\sigma+\omega}$	Rigid model	Poincare model	Solid inner core model
-6800	8051.05	7999.92	8000.23
-365.3	24.91	-38.63	-31.24
-182.6	22.60	28.16	28.20
182.6	530.80	571.71	571.63

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COMMENT ON THE DEFINITION OF THE NONROTATING ORIGIN

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ABSTRACT. The paper gives a rigorous and purely formal derivation for the relationship between the "nonrotating" origin and the x-axis of the Q_t system, i.e., the true equator system. Neglecting nutation, a nonrotating origin could also be achieved by putting m=0 in the formula for the time derivative of right ascension.

The condition defining the nonrotating origin is stated by Capitaine, Guinot and Souchay (1986) as "σ is kinematically defined in such away that, as P moves in the CRS, [$Oxyz$] has no component of instantaneous rotation with respect to the CRS around Oz." This obviously means that the axis of instantaneous rotation must lie in the y-z plane of [$Oxyz$].

We denote, for brevity, the CRS by K and the system [$Oxyz$] by k. It is therefore clear, that the matrix M(k,K), which transforms a given vector from k to K is

$$\mathrm{M(k,K)} = \mathrm{R}_3(-E\text{-}90°)\mathrm{R}_1(-d)\mathrm{R}_3(S+90°) =$$

$$\begin{pmatrix} \sin E\sin S+\cos E\cos S\cos d & -\sin E\cos S+\cos E\sin S\cos d & \cos E\sin d \\ -\cos E\sin S+\sin E\cos S\cos d & \cos E\cos S+\sin E\sin S\cos d & \sin E\sin d \\ -\cos S\sin d & -\sin S\sin d & \cos d \end{pmatrix} = \begin{pmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{pmatrix}$$

In this expression, E and d are longitude and colatitude, respectively, of the z^k-axis with respect to K, and S, which replaces $E + s$ of Capitaine, Guinot and Souchay, is the angle between the x^k-axis and the direction of the vector $(001)^T \times \hat{x}(E,d)^T$ with respect to K, i.e., that along the direction in which the x-y planes of K and k, respectively, intersect.

Since the matrix M(k, K) is orthogonal, we have

$$\mathrm{M(K,k)} = \mathrm{M^T(k,K)}.$$

We therefore have

$$x^k = \mathrm{M^T(k, K)}x^K \quad \text{and} \quad x^K = \mathrm{M(k, K)}x^k.$$

Since we assume x^K not to vary with time, we have

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$$\dot{x}^k = \Big[\Big(\frac{\partial}{\partial E}M(K,k)\Big)\dot{E} + \Big(\frac{\partial}{\partial S}M(K,k)\Big)\dot{S} + \Big(\frac{\partial}{\partial d}M(K,k)\Big)\dot{d}\Big]\,M(k,K)x^k,$$

which expresses the components of $\dot{x}^k$ in terms of x^k itself, as well as of E, S, d, $\dot{E}$, $\dot{S}$ and $\dot{d}$.

Routine calculations show that

$$\Big(\frac{\partial}{\partial E}M(K,k)\Big)M(k,K) = \begin{pmatrix} 0 & a_{33} & -a_{32} \\ -a_{33} & 0 & a_{31} \\ a_{32} & -a_{31} & 0 \end{pmatrix},$$

$$\Big(\frac{\partial}{\partial S}M(K,k)\Big)M(k,K) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$\Big(\frac{\partial}{\partial d}M(K,k)\Big)M(k,K) = \begin{pmatrix} 0 & 0 & -\cos S \\ 0 & 0 & -\sin S \\ \cos S & \sin S & 0 \end{pmatrix}$$

One obtains the angular velocity vector ϖ (whose direction is in the axis of rotation) by taking the cross product of the vector with its velocity. Thus we get

$$\varpi^k = \begin{pmatrix} -(\dot{E}\sin S\sin d - \dot{d}\cos S)xy + (\dot{E}\cos S\sin d + \dot{d}\sin S)(y^2+z^2) + (\dot{E}\cos d - \dot{S})xz \\ (\dot{E}\sin S\sin d - \dot{d}\cos S)(x^2+z^2) - (\dot{E}\cos S\sin d + \dot{d}\sin S)xy + (\dot{E}\cos d - \dot{S})yz \\ -(\dot{E}\sin S\sin d - \dot{d}\cos S)yz - (\dot{E}\cos S\sin d + \dot{d}\sin S)xz - (\dot{E}\cos d - \dot{S})(x^2+y^2) \end{pmatrix}$$

This shows that ϖ depends on the vector; the requirement stated by Capitaine, Guinot and Souchay could therefore be changed to read:

"σ is kinematically defined in such a way that, as P (i.e., the z-axis of k) moves with respect to K, the equatorial plane of k has no component of instantaneous rotation with respect to the z-axis of k." Only for $z = 0$ will $\dot{E}\cos d = \dot{S}$ satisfy this requirement.

(Note that what I have done is to regard the motion of a vector (supposedly fixed in K) with respect to k, this mirrors the motion of the system with respect to the vector and is practically the same thing.)

There is a certain analogy of the whole situation with the precessional motion of the Q_m system with respect to the Q_0 system. The derivative of α with respect to time is given by $\dot{\alpha} = m + n \sin\alpha \tan\delta$. Even if we had a nonmoving origin for the right ascensions, which would be accomplished by setting the origin such that $m = 0$, we see that in general, $\dot{\alpha} = 0$ only on the instant equator, quite analogous to the situation we have described above.

Acknowledgement

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THE RADIO REFERENCE FRAME OF THE U.S. NAVAL OBSERVATORY RADIO INTERFEROMETRY PROGRAM

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ABSTRACT. The U.S. Naval Observatory Navnet program monitors changes in the rotation of the Earth on a regular basis using radio interferometric observations acquired with telescopes in Alaska, Hawaii, Florida, West Virginia and, in the past, Maryland; other radio telescopes have also participated occasionally. These observations have been used to derive a radio interferometric celestial reference system, Navy 1990-5, using two years of dual frequency measurements from 24-hour-duration observing sessions. A total of 84 extragalactic radio sources, mostly quasars, have been observed by the Navnet program to date, of which 70 currently have source position formal errors of one milli second of arc or less. The root mean square of the difference between source position estimates from the Navnet data and an independently derived catalog using completely different data is less than one milli second of arc in both right ascension and declination after the adjustment of an arbitrary rotational offset between the two celestial reference frames.

I. Introduction

As part of its participation in the National Earth Orientation Service (NEOS) the U.S. Naval Observatory (USNO) operates the Navnet program in Very Long Baseline Interferometry (VLBI) to monitor changes in the orientation of the Earth on a regular basis. NEOS is a joint cooperative effort of the USNO and the National Geodetic Survey (NGS), and this VLBI program is designed to complement the observations coordinated by the NGS as part of the International Radio Interferometric Surveying (IRIS) subcommission. The Naval Research Laboratory (NRL) and the Crustal Dynamics Project VLBI group at the National Aeronautics and Space Administration Goddard Space Flight Center (GSFC) have also assisted in the development of the program. The USNO program measures the orientation of the Earth in space from interferometric observations acquired with telescopes at Gilmore Creek, Alaska, Kokee Park, Hawaii, Richmond, Florida, Green Bank, West Virginia and, in the past, Maryland Point, Maryland. Radio telescopes at Mojave, California, Westford, Massachusetts, Algonquin Park, Canada, and Medicina, Noto, and Matera, Italy, have also participated in these experiments on an occasional basis. Current Navnet operations consist of one 24-hour-duration observing session per week together with three-hour duration observing sessions on two other days of the week. The Navnet data are used to estimate UT1, polar motion and nutation for inclusion in the International Earth Rotation Service (IERS) combined solutions.

One 24-hour-duration experiment was observed in September, 1988; monthly 24-hour Navnet experiments commenced in April, 1989, and all of the weekly Navnets have been 24 hours in duration since June 27, 1989. At present, one 24-hour Navnet experiment is scheduled every week using at least three of the Florida, Alaska, Hawaii and West Virginia antennas. These data are reduced and released to NEOS and the IERS on a regular basis; typical processing times are now on the order of 8 to 14 days from the acquisition of data to release of the final Earth orientation results.

The Navnet VLBI data are acquired using the Mark III VLBI data acquisition system as described by Clark *et al.* (1985). The Navnet VLBI data have all been correlated at the Washington Correlator and reduced and processed using the GSFC Calc and Solve computer program package. After correlation, fringe fitting, and the removal of any remaining bandwidth synthesis delay ambiguities, the data are used in a series of weighted least-squares solutions to define a Navnet VLBI reference frame and to estimate the Earth orientation within that reference frame. The data are processed with IERS standard models to the maximum extent possible (see IERS Technical note number 3). Unmodeled variations in the tropospheric propagation delays and the relative time offset between the station clocks are a significant source of error in geodetic VLBI. The surface pressure, temperature and relative humidity are recorded at each station and used to estimate the variations in the hydrostatic zenith tropospheric propagation delay. Further variations in these quantities are treated by the estimation of piecewise linear models directly in the least squares solutions. A new piecewise linear function is introduced every 60 minutes for the zenith tropospheric propagation delay and every 90 minutes for the station clocks.

II. The Navnet Radio Reference Frame

An attempt is made to align the Navnet reference frame as closely as possible with both

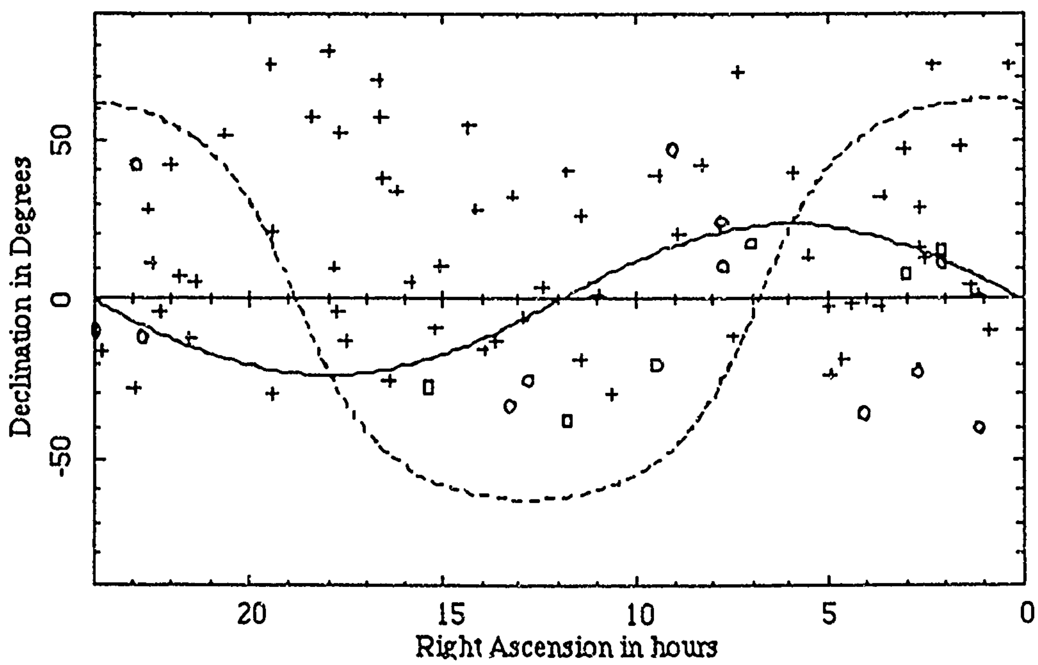

Figure 1 : The location in the sky of the sources currently used in Navnet operations. The primary sources used in the comparison with the GSFC GLB 353 catalog are indicated with crosses, other sources with open circles. The ecliptic and the galactic equator are denoted by the solid and dashed lines, respectively.

the terrestrial and celestial reference frames maintained by the IERS, and to minimize bias offsets between the Navnet Earth orientation parameter estimates and those of the IERS combined solutions. Due to software limitations, the Navnet reference frame is obtained from the USNO VLBI data in a two solution process. In the first solution, the IERS Rapid Service (Bulletin A) estimates of the UT1 and polar motion are treated as a priori measurements and the nutation in longitude and obliquity is fixed at the IERS Rapid Service values, while the Right Ascension (α) of the source 2216-038 is fixed at the value given in the IERS combined celestial reference frame for 1989 (R(IERS) 89 C 01). All other source coordinates, and all of the station coordinates except for the Richmond station, were adjusted in this solution. The purpose of this solution is to align as closely as possible the Navnet celestial reference frame with the celestial pole implied by the IERS nutation series. The coordinates of two sources, 0202+149 (in both α and Declination, δ) and 0742+103 (in δ only) are then fixed in a second solution to the values obtained in the first solution. This second solution globally adjusts the coordinates of all of the other sources and all of the station coordinates except for the Richmond station, together with a separate adjustment for UT1, Polar Motion, both components of nutation and piecewise linear clock and troposphere models for each experiment. The IERS Rapid Service estimates for UT1 and polar motion are used as a priori measurements with weights as given by the IERS formal errors, but the two components of nutation are estimated freely for each observing session. The position of the Richmond station at the epoch 1988.0 is fixed at the value given by the ITRF-88 for that epoch, and all stations are required to move at the AM0-2 rate. The USNO reference frame 1990-5 (IERS designation RSC (USNO) 90 R 05) was obtained from such a solution using all of the Navnet VLBI data from 67 24-hour-duration observing sessions from September 10, 1987, through September 20, 1990; this solution used a total of 29216 observation pairs

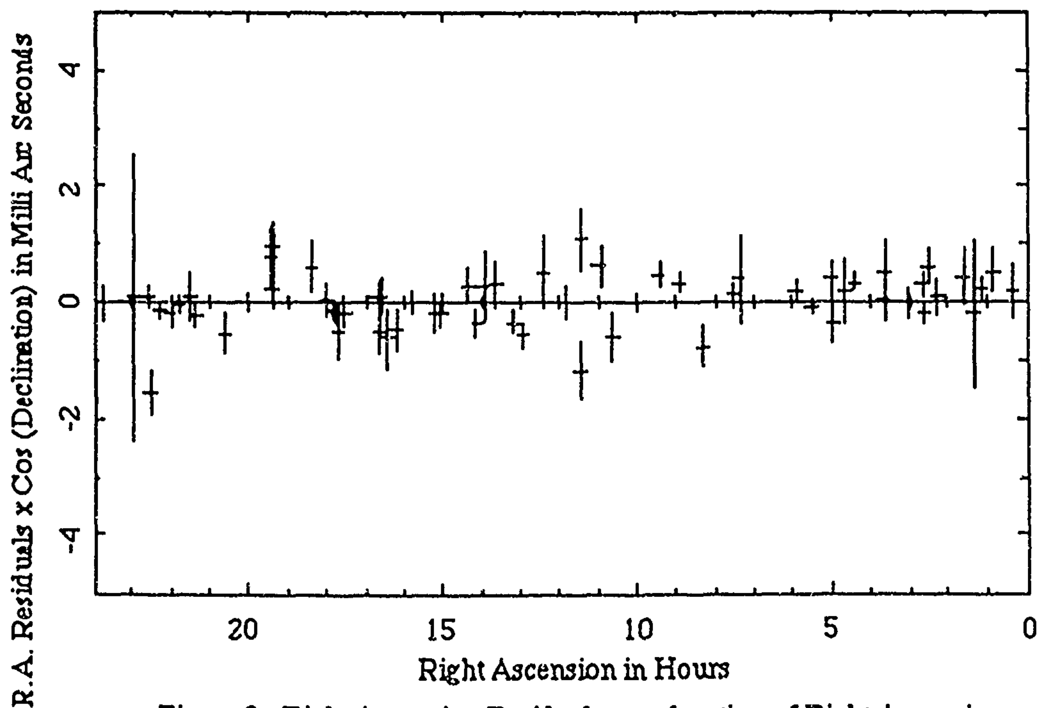

Figure 2 : Right Ascension Residuals as a function of Right Ascension

(delay and delay rate) with a weighted rms residual scatter of ±49.7 picoseconds for the delay data and ± 104.2 femtoseconds per second for the delay rate data. A total of 84 sources have been observed by the Navnet program; four radio sources (3C84, 3C273B, 3C345 and 3C454.3) were judged to have too much source structure to be usable for geodetic work and are no longer routinely observed; these sources were treated as "arc" parameters in the solution, with a separate position being estimated for each experiment in which they were observed. The Navy 1990-5 celestial reference frame thus consists of the positions of 80 radio sources, including the two reference sources, 68 sources with formal uncertainties of less than one mas in both components of position ("primary" sources), and 10 others with worse position estimates. The distribution of these sources in the sky is shown by Figure 1.

III. Comparison of Independent Radio Reference Frames

Several groups have derived Radio Interferometric reference frames using VLBI observations from transcontinental and intercontinental baselines. (see, e.g., Robertson *et al.*, 1986, Sovers *et al.*, 1988, and Ma *et al.*, 1990). Reference frames derived from modern VLBI observations usually claim positional uncertainties on the order of 0.1 to 1 milli arc seconds (mas). Robertson *et al.* (1986), using a variety of internal repeatability checks, showed that the precision (but not necessarily the accuracy) of the Mark III derived NGS catalog is in the order of 0.5 mas or less. Sovers *et al.* (1988) and Ma *et al.* (1990) compared catalogs derived from Mark III observations with a catalog from the less sensitive JPL geodetic Mark II system and found root mean square (rms) agreement at the 2 mas level. Sub milli second of arc agreement has been found between various GSFC and NGS catalogs (see, e.g., the IERS Annual Reports for 1988 and 1989), but these catalogs, although independently processed, share a considerable amount of common data (all of the NGS IRIS data are used in the GSFC solutions), and thus these

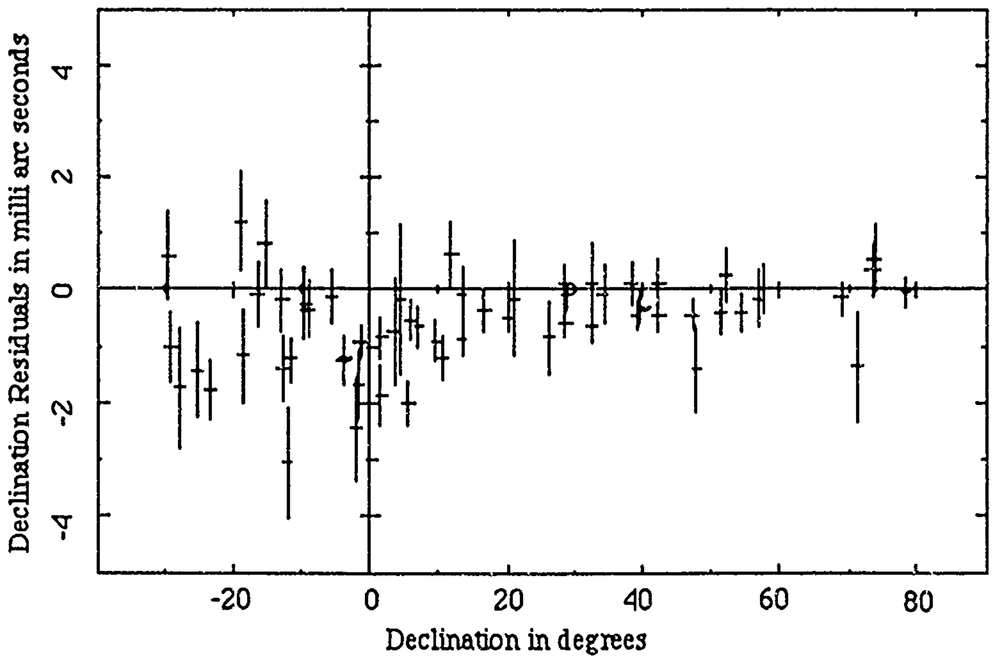

Figure 3 : Declination Residuals as a function of Declination

comparisons are not a true measure of the accuracy of these catalogs (Arias *et al.* 1988).

Recently, Ma *et al.* (1990) published a catalog based on previous GSFC and NGS geodetic observations plus the first results of a GSFC/NRL/USNO program to determine a dense celestial reference frame using radio sources with optical counterparts. This catalog, denoted GLB 353, contains estimates of the positions of 182 sources including 62 of the 68 primary Navnet sources. The GLB 353 positions were subtracted from the Navy 1990-5 positions and an arbitrary rotation offset removed by estimating the A1, A2 and A3 parameters described in Arias *et al.* (1988). The agreement in α, shown in Figure 2, was excellent, with an rms scatter of only 0.46 mas after scaling the α residuals by the cosine of the declination, and very little evidence for systematic (non-random) patterns in the residuals. The scatter in the δ residuals is larger, with an rms value of 0.97 mas. The scatter (Figure 3) clearly increases towards lower declinations, probably a result of the smaller coverage in hour angle available for those sources from a network in the Northern hemisphere. The correlation, - 0.25, between the α and δ residuals is small and not significant.

The scatter between the Navy 1990-5 and the GSFC GLB 353 source positions, although small, is significantly larger in both components than would be expected on the basis of the formal errors provided by the two solutions. This effect was modeled by estimation of a constant additive variance; separate constants were estimated for the α residuals and δ residuals. The square root of these constants (the "additive noise") is 0.1 mas for the α residuals (again scaled by the cosine of the declination) and 0.55 mas for the δ residuals. The larger additive noise required for the δ residuals is associated with the increased residual scatter at declinations less than 10^{o}, and may be related to failures in the adopted troposphere propagation delay model at low elevation angles, since the sources in the extreme South must be observed at low elevation angles from networks in the Northern hemisphere.

IV. Conclusions

The Navnet celestial reference frame agrees with independent radio source position estimates at the level of one milli second of arc in declination and one half milli second of arc in right ascension. There is evidence that the models and procedures currently used may not be adequate at low declinations. The Navnet program continues to acquire weekly measurements and thus the Navnet observations will play an increasingly important role in the determination of the celestial radio reference frame.

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ON THE DETERMINATION OF AN ASTROMETRIC CENTER OF A QUASAR[1]

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ABSTRACT. The influence of optical structure of compact extragalactic radio sources on the definition of their centroids is discussed. Numerical calculations are performed using 4C 31.63 as an example. The results show that the presence of the underlying luminous galaxy does not lead to an unusual decrease of the accuracy in the optical astrometric measurements.

[1]Will appear in the Proceedings of the Colloquium "Systemes de Reference Spatio-temporels". Eds.: N.Capitaine, S.Debarbat. Paris, 1990.

CONNECTION BETWEEN NON-ROTATING LOCAL REFERENCE FRAMES BY MEANS OF THE WORLD FUNCTION

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ABSTRACT. By means of the world function an approximate transformation showing the Riemann tensor between the Fermi coordinates associated to two non-rotating local reference frames is derived in a General Relativistic space-time. One of the observer's world lines is resticted to be a time-like geodesic of the space-time, and the other is a time-like curve of a general character. The space-time where the transformation is evaluated is supposed to be of small curvature, and the calculations are carried out in a first order of approximation with respect to the Riemann tensor.

1. Introduction

The technique based on the world function can be found in Synge (1960). The transformation is derived in three steps: first, under the restrictions of this technique a general transformation is obtained. Next, the small-curvature hypothesis on the space-time is applied; and finally, the hypothesis of quasi-parallelism between the base lines of the reference frames is applied to derive the result.

2. General Transformation

Denoting by Π a time-like curve and by Γ a time-like geodesic, both in a region $\mathcal{N}$ where the cited technique can be used; parametrizing these base lines with the afin parameters s and s', so that A and D represent two generic points in Π and Γ respectively; denoting by n^{A}_{1} and n^{D}_{1} the unit tangent vectors of Π and Γ at A and D respectively; and finally, denoting by $\Sigma(A)$ and $\Sigma(D)$ the hypersurfaces of $\mathcal{N}$ defined by

$$\Sigma(A):= \{P \in \mathcal{N} \,/\, \underset{1}{n}{}_{A}\, \Omega^{A}(AP) = 0\} \;, \quad \Sigma(D):= \{Q \in \mathcal{N} \,/\, \underset{1}{n}{}_{D}\, \Omega^{D}(DQ) = 0\}$$

then the Fermi coordinates of a point $B \in \Sigma(A) \cap \Sigma(D)$ can be written in terms of the world function, Ω, in the following way : First, with respect to the base line Π and to the tetrad $\lambda^{i}_{(a)}$ Fermi transported along Π, which we shall denote by $x^{(\alpha)}(B)=x_{(\alpha)}(B)$, $x^{(4)}(B)=-x_{(4)}(B)=s$, as

$$X^{(\alpha)}(B)|_A = X_{(\alpha)}(B)|_A = -\,\Omega_{i_A}(AB)\,\lambda^{i_A}_{(\alpha)}\ ,\quad X^{(4)}(B)|_A = -\,X_{(4)}(B)|_A = s$$

and second, with respect to the geodesic line Γ and the parallel transported tetrad $\lambda'^{i}_{(a)}$ along Γ ,which will be denoted by $x'^{(\alpha)}(B)= x'_{(\alpha)}(B)$, $x'^{(4)}(B)= -\,x'_{(4)}(B) = s'$, as

$$X^{(\alpha)}(B)|_D = X_{(\alpha)}(B)|_D = -\,\Omega_{i_D}(DB)\,\lambda^{i_D}_{(\alpha)},\qquad X^{(4)}(B)|_D = -\,X_{(4)}(B)|_D = s'$$

where Ω_{i_A} and Ω_{i_D} are the covariant derivatives of Ω with respect to A and D respectively.

In order to obtain the relationship between the Fermi coordinates $[x_{(\alpha)}(B),x_{(4)}(B)]$ and $[x'_{(\alpha)}(B),x'_{(4)}(B)]$ we consider the point C, intersection of $\Sigma(A)$ and Γ, and it is supposed that $X_{(4)}(C)|_D= -\Delta s'$. Then, by using the two solutions for the geodesic triangles ADB and ACD, and the parallel propagator , $g_{i_A j_D}$, applied to the tetrad $\lambda^{i}_{(a)}$, we have

$$\boxed{\begin{aligned}
&X_{(\alpha)}(B)|_A - X_{(\alpha)}(C)|_A - X_{(\beta)}(B)|_D\, L^{(\beta)}_{(\alpha)} = \Omega_{i_D}(CD)\, L^{(4)}_{(\alpha)}\, \lambda'^{i_D}_{(4)}\\
&-[\chi_{i_A}(ABD) - \chi_{i_A}(ACD)]\,\lambda^{i_A}_{(\alpha)} + [\chi_{i_D}(ABD) - \chi_{i_D}(ACD)]\, L^{(b)}_{(\alpha)}\, \lambda'^{i_D}_{(b)}\\
&+\frac{1}{2}[\phi_{i_A}(ABD) - \phi_{i_A}(ACD)]\,\lambda^{i_A}_{(\alpha)} - \frac{1}{2}[\phi_{i_D}(ABD) - \phi_{i_D}(ACD)]\, L^{(b)}_{(\alpha)}\, \lambda'^{i_D}_{(b)}
\end{aligned}}$$

where $\chi(ABD)$, $\chi(ACD)$, $\phi(ABD)$ and $\phi(ACD)$ are the 3-point invariants given by

$$\chi(ABD) = \Omega_{i_B}(AB)\,\Omega^{i_B}(BD)\ ,\qquad \chi(ACD) = \Omega_{i_C}(AC)\,\Omega^{i_C}(CD)\ ,$$

$$\phi(ABD) = \frac{1}{3}\int_0^1 (1-\zeta)^3 \frac{d^4\Omega(\zeta)}{d\zeta^4}\, d\zeta\ ,\qquad \phi(ACD) = \frac{1}{3}\int_0^1 (1-\theta)^3 \frac{d^4\Omega(\theta)}{d\theta^4}\, d\theta$$

and $\mu^{i_D}_{(a)} = g^{i_D}_{j_C}\, g^{j_C}_{k_A}\, \lambda^{k_A}_{(a)} = L^{(b)}_{(a)}\, \lambda'^{i_D}_{(b)}$, provided that Γ_{AB} and Γ_{BD} are parametrized so that ζ takes the values 0 at B, and 1 both at A and D, and that Γ_{CA} and Γ_{CD} are parametrized so that $\theta = 0$ at C, and $\theta = 1$ at A and D.

3. Approximation by small curvature

By introducing the hypothesis that at any point P of the hypersurfaces $\Sigma(A)$ and $\Sigma(D)$ the Riemann tensor, R_{ijkl}, and its covariant derivatives are small, of the first order, or O_1, and denoting by O_2 second-order smalles with respect to the Riemann tensor, we obtain

$$\boxed{\begin{aligned}
&X_{(\alpha)}(B)|_A - X_{(\alpha)}(C)|_A - X_{(\beta)}(B)|_D\, L^{(\beta)}_{(\alpha)} = L^{(4)}_{(\alpha)}\, \Delta s' \\
&+ \frac{1}{2}\, \underset{1}{K}_{(\alpha b\gamma\delta)}\, [X^{(\gamma)}(C)|_A]\, [X^{(\delta)}(C)|_A]\, [L^{(b)}_{(4)}\, \Delta s' + L^{(b)}_{(\mu)}\, X^{(\mu)}(B)|_D\,] \\
&+ \frac{1}{2}\, \underset{2}{K}_{(ab44)}\, L^{(b)}_{(\alpha)}\, L^{-1(a)}_{(\beta)}\, (\Delta s')^2\, [\, X^{(\beta)}(C)|_A - X^{(\beta)}(B)|_A\,] \\
&+ \frac{1}{4}\, [\phi_{i_A}(ABD) - \phi_{i_A}(ACD)]\, \lambda^{i_A}_{(\alpha)} - \frac{1}{4}\, [\phi_{k_D}(ABD) - \phi_{k_D}(ACD)]\, \mu^{k_D}_{(\alpha)} \\
&+ \frac{1}{4}\, [\phi^{i_A}_{k_D}(ABD) - \phi^{i_A}_{k_D}(ACD)]\, [-\Omega^{k_D}(BD)\lambda_{(\alpha)i_A} + \Omega_{i_A}(BA)\, \mu^{k_D}_{(\alpha)}\,] + O_2
\end{aligned}}$$

where

$$\underset{1}{K}_{(ab\gamma\delta)} = -\frac{3}{2}\int_0^1 (1-\sigma_1)\sigma_1\, S_{(ab\gamma\delta)}\, d\sigma_1, \quad \underset{2}{K}_{(ab44)} = -\frac{3}{2}\int_0^1 (1-\sigma_2)\sigma_2\, S_{(ab44)}\, d\sigma_2$$

the first integral being taken along the geodesic Γ_{AC}, the second along the geodesic Γ_{CD} ($S_{(abcd)}$ being the symmetrized Riemann tensor) and

$$\phi(ABD) = \phi_0(ABD) + \phi_1(ABD) + \phi_2(ABD) + O_2$$

$$\phi_0(ABD) = 3k^3\int_0^1 (1-\zeta)^3\, d\zeta \int_{u_1}^{u_2} [(u_2-u)^2 + (u-u_1)^2]\{1122\}du$$

$$\begin{aligned}
\phi_1(ABD) = 2k^3\int_0^1 \zeta(1-\zeta)^3\, d\zeta \int_{u_1}^{u_2} [\, & 2(u_2-u)^3\{11221\} + 3\,(u_2-u)^2(u-u_1)\{11222\} \\
& + 3(u_2-u)(u-u_1)^2\{22111\} + 2(u-u_1)^3\{22112\}]du
\end{aligned}$$

$$\phi_2(ABD) = \tfrac{1}{2}k^3\int_0^1 \zeta^2(1-\zeta)^3\, d\zeta \int_{u_1}^{u_2}[(u_2-u)^4\{112211\}+4(u_2-u)^3(u-u_1)\{112212\}$$
$$+3(u_2-u)^2(u-u_1)^2(\{112222\}+\{221111\})$$
$$+4(u_2-u)(u-u_1)^3\{221121\}+(u-u_1)^4\{221122\}]du \quad , \qquad k^{-1}= u_2-u_1$$

with a similar expression for $\phi(ACD)$, where

$$\{1122...\}= S_{(abcd)}\ [X^{(a)}(C)|_A]\ [X^{(b)}(C)|_A]\ [X^{(c)}(D)|_A]\ [X^{(d)}(D)|_A]\ [...]$$

4. Approximation by quasiparallelism

Denoting by U^{1_C} the tangent vector to the geodesic Γ at C, by U^{1_A} the tangent vector to Π at A, and introducing the hypothesis of quasi-parallelism, i.e.,

$$g^{1_A}{}_{1_C}\, U^{1_C}\, \lambda_{(a)1_A} = U^{1_A}\, \lambda_{(a)1_A} + O_1$$

then the final transformation reads

$$\boxed{\begin{aligned} & x_{(\alpha)}(B) - x_{(\alpha)}(C) - x'_{(\beta)}(B)\, L^{(\beta)}_{(\alpha)} = [1^{(4)}_{(\alpha)} + \bar{\underset{1}{K}}_{(\alpha 4)}]\, \Delta s' \\ & + \bar{\underset{1}{K}}_{(\alpha\beta)}\, L^{(\beta)}_{(\mu)}\, x'^{(\mu)}(B) + \frac{1}{4}\, [\phi_{1_A}(ABD)\, \lambda^{1_A}_{(\alpha)} - \phi_{k_D}(ABD)\, L^{(\beta)}_{(\alpha)}\, \lambda'^{k_D}_{(\beta)}] \\ & + \frac{1}{4}\, [\phi^{1_A}_{k_D}(ABD)]\, [x'_{(\beta)}(B)\, \lambda'^{k_D(\beta)}\, \lambda_{(\alpha)1_A} - x_{(\gamma)}(B)\, \lambda^{(\gamma)}_{1_A}\, L^{(\beta)}_{(\alpha)}\, \lambda'^{k_D}_{(\beta)}] \\ & \qquad + O_2 \end{aligned}}$$

where

$$\bar{\underset{1}{K}}_{(\alpha 4)} = \frac{1}{2}\, \underset{1}{K}_{(\alpha 4\gamma\delta)} x^{(\gamma)}(C) x^{(\delta)}(C)\ ,\quad \bar{\underset{1}{K}}_{(\alpha\beta)} = \frac{1}{2}\, \underset{1}{K}_{(\alpha\beta\gamma\delta)} x^{(\gamma)}(C) x^{(\delta)}(C)$$

and $L = 1 + O_2$.

This transformation is ready to input the value of the Riemann tensor evaluated on the accelerated observer and the behaviour of the two reference frames as characteristics derived from their particular selection and from the dynamical model to be chosen.

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LATERAL IMAGE PZT. DESCRIPTION AND OPERATING METHOD

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ABSTRACT. We faced the problem of designing and building a Photographic Zenith Tube with mercury basin and lateral plate-holder system providing enough accuracy to allow the determination of astronomical coordinates. This instrument is intended to work as a prototype of a bigger and more accurate one to be designed and developed in the future (Lopez , A. *et al.*: 1983).

From the beginning, we thought to make use of the scheme and the operating method of a classical PZT (Markowitz, W.: 1969), although we are aware of other new designs (Kühne, C.: 1978). The design of a classical PZT is easy, from a theoretical point of view, but any modification of that design must keep the optical principle on which it is based. Holding up this principle was our first problem because the PZT lens is designed with an outer nodal point on which the photographic plate is situated. This does not allow the use of normal objectives, which generally have the two nodal points inside. Our design is based on the idea of avoiding this inconvenience.We use a plain mirror placed near the objective that deflects the light of a zenith-star in such a way that the image of the second nodal point is located in the lateral of the system, where we place the photographic plate. Moreover, the design has the advantages of easy focusing, adjustment and levelling, and we can use it visually as well as photographically.

As the main function of this instrument is intended to be similar to classical PZT, we have preserved the same operating method of it.

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DIGITAL IMAGE PROCESSING IN THE ANALYSIS OF ASTROMETRIC PLATES.

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ABSTRACT. In this paper, we display an improvement to our process of semi-automatic measuring of astrometric plates, in which the photometric sensor is substituted by a CCD system of image grabing and digitalization. The advantages of this method are analyzed taking into account the possibilities of the image analysis in the space and frequency domain.

1. Introduction

In the Astronomical Observatory of Valencia we have a program of selected minor planets observations since 1984. To this aim we have developed both the photographic equipment and the mechanical and electronic devices for plate measuring as well as the software support to whole process. The optical system includes an ocular and photoelectric photometer heading.

Once two stars (E1 and E2) have been selected, the several expositions observed and the sequence of the objects to be measured defined, the process of measuring in each position of the plate-holder is automatic. The process is repeated after a turn of 180° of the plate-holder.

An automated position measure is applied for each star and for the minor planet. It includes reading local plate background transparency, an approximate and fine centering and the automatic measuring process of each image.

As the separation of the two or three expositions of the objects is about 0.3 to 1 mm, all the images cannot be seen in the ocular field (1 mm wide) at the same time, making tedious star finding and centering, and easy to get in troubles in the full process.

Details of the measure process are shown in (Lopez , A. *et al.*: 1988), (Lopez , A. *et al.*: 1990) and (Lopez , A. *et al.*: 1989a).

In this paper we present the new process of image detection and identification, giving better results and being easier, quicker and more robust than the previous one with the photometer.

2.- CCD Image detection

An alternate centering process reading all the pixels around the image with a two dimensional sensor (CCD camera plus digitizer board) and getting the center of transmitted light has given better results, as shown in (Lopez , A. *et al.*: 1989b).

From that, several changes have been applied in order to improve the plate measuring process.

The binocular reading head has been substituted by a small CCD camera and a digitizer board on the computer, so that the field containing the several exposures can be seen on the screen, the pixels in a window around the images read and the image positions detected.

The plate measuring process begins with the centering of the two initial stars on the screen (3x4 mm across) (Fig. 1).

The scale and tilt of the CCD camera versus the microscope X,Y screws are obtained by making movements of the first image of E1 and determining its position over four cross small windows.

The expositions of E1 and E2 are measured in a small central window and the relative position of the images is determined for every object to be measured.

The sequence of stars and asteroid is defined and the measure process begins.

3.- Digital image analysis

For each object (star and asteroid) the detection process is global, including all the expositions at the same time.

Our purpose is to get positions of all images relative to the camera field center in a way as fast and sure as possible working in a real time process. So, applied steps are simple and efficient.

A central window is defined including all the images of the object (Fig. 2). As we measure the plates in two opposite positions, the window is elongated in the horizontal direction of screen, as the several images of the object appear.

The window is divided into 10x6 squares with ten by ten pixels each, and the images are identified by the signal detected in these squares. As the size of the squares is similar to the images diameters, each image will give positive signal at least in one or two adjacent squares, side by side or in diagonal. Other spurious marks of the plate will active squares isolated or in irregular chains (Fig. 3).

The process of identifying and measuring the stellar images is the following:

1.- The maximum and minimum pixel signal inside the window described above are determined.

2.- In each square the pixels with signal below a threshold give, when added, the square signal value. The greatest value is needed to select the active squares.

3.-The connectivity of active squares is determined, classifying them into isolated and chained.

4.- For the chained squares, an analysis is made and the members of each group are determined.

5.- All the active groups are measured again, determining the X,Y coordinates of its light center.

6.- A small square window is defined around this center and the final X,Y position determined. Some other parameters can be obtained if necessary (brightness, shape, orientation, etc) (Fig. 4).

7.- The identification of active groups with object images is the final step of this process. For that, we compare the X and Y distances of the several expositions with the theoretical values calculated from E1 and E2 (Fig. 5).

4.- Future improvements

We will also apply other routines to special problems involved in our minor planets work, specially when the asteroid image is very weak. Masks and Fourier analysis techniques will be investigated and applied in this context in the next future.

The use of masks in the previous analysis of images is extended and digitizer boards hardware and software have many possibilities. Nevertheless the use of masks is dangerous as it modifies the real structure of signals.

It will be possible to identify the object images structure in the frequency domain comparing its principal components with the Fourier coefficients of the theoretical positions, size and shape of the object images, although the measure process time will increase substantially (Gonzalez, R.C.; Wintz, P., 1987).

5.- Conclusions

This method of identifying and measuring image positions in the context of astrometric plates, improves substantially the results of our previous photometric measuring process.

It can by extended to other similar problems such as the measure of PZT plates and astronomical plates in general.

Major astronomical image processing systems (MIDAS, IRAF, INVENTORY, COSMOS, etc) include many commands that can perform the analysis described here (Murtagh, F.: 1989).

Nevertheless, we think it is better in our work to develop a special software, that will be improved in the next future, in order to extend the confidence of the full process of measurement of astrometric plates.

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Fig. 1.- Visible plate field in the CCD camera

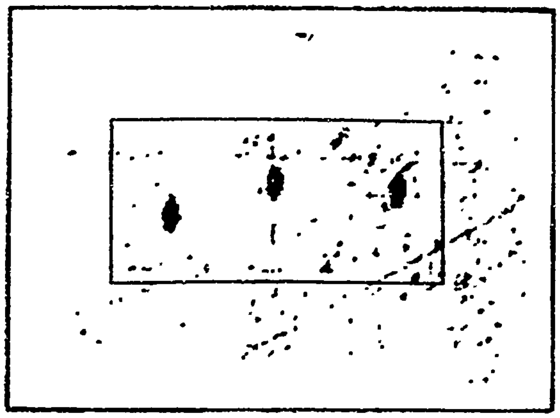

Fig. 2.- Central window where the analysis is performed.

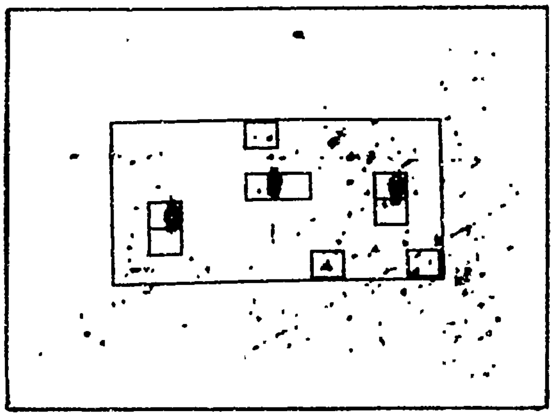

Fig. 3.- Squares with active transmitted light signal.

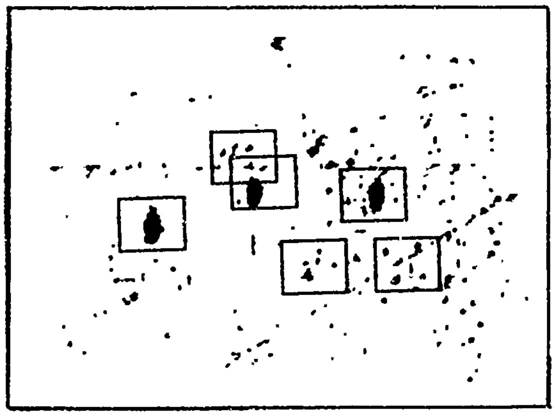

Fig. 4.- Result of the active groups re-centering process.

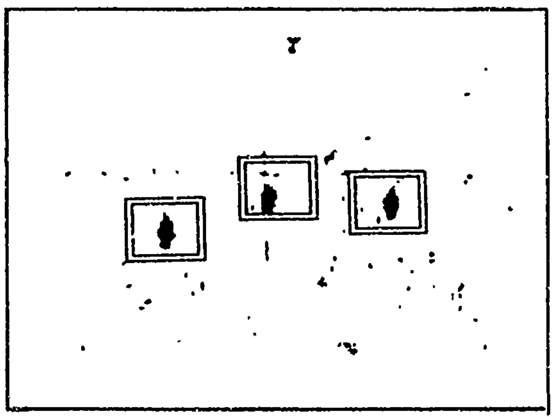

Fig. 5.- Object images finally identified.

CORRECTION OF FUNDAMENTAL CATALOGUE CONSTANTS.

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ABSTRACT

One of the main problems in positional astronomy is determining the equator and vernal equinox of the reference system.
In this paper we display a method of amendment of minor planets elements taking into account the perturbations in the coefficients of the equations of condition, also including the corrections of the vernal equinox and obliquity in the fitting.

TECTONICALLY-INDUCED DIVERGENCES OF EARTH ROTATION SERIES

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ABSTRACT. The stability of the Terrestrial Reference Frame (TRF) is studied by analyzing divergences of independently determined Earth rotation series. These series, which have been (nominally) determined within the same TRF, are found to differ by up to 4.7 mas in bias and up to 0.80 mas/yr in rate. These bias-rate differences could reflect errors in the various models used during the data reduction process.

1. Introduction

A Terrestrial Reference Frame (TRF) can be defined as a right-handed Cartesian reference frame that has been tied to the solid Earth in some prescribed manner (e.g., *Kovalevsky et al.*, 1989). It is realized in practice by specifying the coordinates of a number of globally distributed observing stations. On a tectonically active body such as the Earth, it is not sufficient to just specify the static location of each station, but the vector velocity of each site must be specified as well. As the tectonic plates move about, the positions of the stations located upon them will change. This change in station position must be taken into account when defining the TRF in order for it to be stable and not drift with the stations.

The locations of the observing stations are principally determined by the modern space-geodetic techniques of Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), and Lunar Laser Ranging (LLR). In addition (e.g., *Lambeck*, 1988), these techniques can determine the orientation of the Earth in space, as well as the locations of the laser reflectors (in the case of SLR and LLR), or the locations of the radio sources (in the case of VLBI). Each such solution for the time-dependent station locations and/or Earth rotation parameters defines its own Terrestrial Reference Frame. These solution-specific TRFs can be offset from each other (exhibit differences in bias) and can drift away from each other (exhibit differences in rate) even though each analysis center has followed the procedures and used the constants recommended by the International Earth Rotation Service (IERS; *McCarthy*, 1989).

These differences in bias and rate between different realizations of the Terrestrial Reference Frame are probably largely caused by errors in the models used when reducing the data. For example, the rate differences are likely to be largely caused by errors in modeling the plate tectonic-induced motions of the observing stations. The different observing techniques (VLBI SLR, or LLR) use different sets of observing stations that are located on different subsets of the tectonic plates. If the model describing the motions of the tectonic plates is not accurate, then errors in modeling the plate (and hence station) motions are likely to be different for each set of stations, giving rise to rate differences between the solutions.

Differences in bias and rate between different realizations of the Terrestrial Reference Frame can give rise to differences in bias and rate between different solutions for the Earth rotation parameters that have been determined within the different realizations of the TRF. In this paper, the stability of the Terrestrial Reference Frame is studied by analyzing the divergences of independently determined Earth rotation series.

TABLE 1. DATA SETS STUDIED

DATA SET NAME	DATA TYPE	ANALYSIS CENTER	DATA SPAN	NUMBER POINTS	PLATE MODEL
LLR (L1707; ΔΦ, UT0)					
McDonald 2.7m	LLR	JPL	4/15/70 – 6/29/85	810	AM0-2
McDonald LRS	LLR	JPL	3/2/85 – 1/27/88	35	AM0-2
McDonald LRS (New Site)	LLR	JPL	3/28/88 – 12/9/89	20	AM0-2
CERGA	LLR	JPL	4/7/84 – 12/20/89	271	AM0-2
Haleakala	LLR	JPL	11/14/84 – 12/17/89	105	AM0-2
TEMPO (90 R 01; T, V)					
CA (12) – Spain (63)	VLBI	JPL	7/18/82 – 8/25/84	57	AM0-2
CA (14) – Spain (61)	VLBI	JPL	9/19/82 – 8/1/87	42	AM0-2
CA (14) – Spain (63)	VLBI	JPL	11/26/79 – 12/10/89	142	AM0-2
CA (15) – Spain (63)	VLBI	JPL	10/4/87 – 7/8/90	42	AM0-2
CA (12) – Australia (43)	VLBI	JPL	7/2/82 – 7/22/84	58	AM0-2
CA (14) – Australia (42)	VLBI	JPL	2/15/83 – 9/20/87	34	AM0-2
CA (14) – Australia (43)	VLBI	JPL	10/28/78 – 3/3/90	170	AM0-2
CA (15) – Australia (43)	VLBI	JPL	11/8/87 – 7/9/90	36	AM0-2
CDP (EOP.629)					
Multi-Baseline	VLBI	GSFC	8/4/79 – 12/29/89	700	AM0-2
Westford – Ft. Davis	VLBI	GSFC	6/25/81 – 1/1/84	103	AM0-2
NAVNET (NAVY 1990-2)					
Multi-Baseline	VLBI	USNO	9/11/88 – 2/21/90	38	AM0-2
CSR (89 L 02; PMX, PMY)	SLR	U. Texas	5/15/76 – 1/3/89	970	AM1-2

2. Approach

Some particulars about the independently determined Earth rotation data sets chosen for this study are given in Table 1. Each series listed was chosen because it has been obtained, at least nominally in rate, within the same Terrestrial Reference Frame, namely that one determined by applying (without adjustment) the plate tectonic motion model AM0-2 of *Minster and Jordan* (1978) as a model for the motions of the observing stations. Note that the SLR results chosen were obtained by applying (without adjustment) the plate tectonic motion model AM1-2 of *Minster and Jordan* (1978). However, these SLR results can be (and have been) analytically corrected to the AM0-2 frame by applying a rate adjustment of –0.52 milliarcseconds per year (mas/yr) to the x-component of polar motion (PMX), and of –0.24 mas/yr to the y-component (PMY; *IERS Annual Report for 1989*, pp. II-26). Also note that the SLR UT1 results were not used in this study due to problems associated with separating this component from the effects of unmodelled forces acting on the satellite causing the node of its orbit to change.

These chosen Earth rotation data sets have been intercompared in order to determine bias and rate corrections needed to be applied to each component of each data set in order for it to agree (in bias and rate) with a combination of all the other data sets. At the same time, scale factors have been determined that need to be applied to the stated uncertainties of each component of each series so that its residual (when differenced with a combination of all other series) has a reduced chi-square of one. This intercomparison has been done in an iterative, round-robin fashion in which each series is compared, in turn, to a combination of all others. The incremental bias-rate corrections and uncertainty scale factors determined

TABLE 2. ADJUSTMENTS TO DATA SETS

DATA SET NAME	BIAS (mas)			RATE (mas/yr)			UNCERTAINTY (σ) SCALE FACTOR		
LLR (L1707; ΔΦ, UT0)	ΔΦ	UT0		ΔΦ	UT0		ΔΦ	UT0	
McDonald 2.7m	−1.954	3.994		----	----		1.086	1.199	
McDonald LRS	0.170	0.634		----	----		0.917	1.687	
McDonald LRS (New Site)	−0.260	3.822		----	----		0.716	1.001	
CERGA	0.479	2.998		0.328	0.650		1.204	1.104	
Haleakala	−1.308	3.262		0.801	0.269		1.204	1.192	
TEMPO (90 R 01; T, V)	T	V		T	V		T	V	
CA (12) – Spain (63)	−1.877	4.293		----	----		1.240	1.181	
CA (14) – Spain (61)	−0.123	2.827		----	----		1.196	0.991	
CA (14) – Spain (63)	−0.344	2.109		0.258	−0.524		1.295	1.144	
CA (15) – Spain (63)	−0.879	1.193		----	----		1.021	1.130	
CA (12) – Australia (43)	−2.495	−4.004		----	----		1.142	1.160	
CA (14) – Australia (42)	−1.842	−3.651		----	----		1.168	1.143	
CA (14) – Australia (43)	−1.767	−2.529		0.103	−0.014		1.456	1.114	
CA (15) – Australia (43)	−1.085	−4.028		----	----		1.139	0.852	
CDP (EOP.629)	T	V		T	V		T	V	
Westford – Ft. Davis	−0.947	−4.687		----	----		1.649	1.021	
CDP (EOP.629)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Multi-Baseline	1.402	−1.465	−1.616	0.009	0.775	0.559	1.567	1.540	2.181
NAVNET (NAVY 1990-2)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Multi-Baseline	0.971	−0.569	−1.475	----	----	----	1.278	1.436	1.274
CSR (89 L 02; PMX, PMY)	PMX	PMY	UT1	PMX	PMY	UT1	PMX	PMY	UT1
Satellite Laser Ranging	−2.176	3.512	----	−0.025	0.412	----	0.838	0.835	----

REFERENCE TIME FOR RATE ADJUSTMENT IS 1988.0

during each iteration are applied to the series and the process repeated until convergence is achieved, which is indicated by the incremental bias-rate corrections for each component of each series converging to zero, and the uncertainty scale factors converging to one. Note that rate corrections were obtained only for those series spanning a great enough length of time (and having enough overlap with the other series) that the determination could be reliably obtained.

The combination and intercomparison is done using a Kalman filter approach developed at the Jet Propulsion Laboratory (JPL) for just such a purpose (*Eubanks*, 1988; *Morabito et al.*, 1988). This approach facilitates comparing and combining data sets of disparate quality, sampling rate, and data type. The comparison is done (and the results reported) in the "natural" reference frame for each data type. For single station LLR results this is the variation of latitude (ΔΦ), UT0 frame. For single baseline VLBI results this is the transverse (T), vertical (V) frame (*Eubanks and Steppe*, 1988). For multi-baseline VLBI and SLR results this is the usual UTPM (PMX, PMY, UT1) frame.

3. Results, Discussion, and Conclusions

The results obtained by this approach are shown in Table 2 which gives the total bias-rate corrections and uncertainty scale factors that must be applied to each component of each raw data set in order for all the data sets to agree with each other. The adjustments to the bias range (in absolute value) up to 4.7 mas, with an average (absolute) value of 2.0 mas and a median (absolute) value of 1.8 mas. The adjustments to the rate range (in absolute value) up to 0.80 mas/yr, with an average (absolute) value of 0.36 mas/yr and a median (absolute) value of 0.33 mas/yr.

For each baseline of the single baseline VLBI results, the bias adjustment to the vertical component of Earth orientation is larger (in absolute value) than the adjustment to the transverse component. This could be reflecting the property that the vertical component of Earth orientation is more sensitive to errors in the models affecting the local vertical position of the station, such as tropospheric path delay effects, atmospheric and oceanic loading effects, etc. Also note that for each LLR station, the bias adjustment to the UT0 component is greater (in absolute value) than the adjustment to the $\Delta\Phi$ component. These results suggest that modeling errors (of different kinds) are affecting the solution for the Earth rotation parameters in both bias and rate.

Thus, in summary, Earth rotation series that have been determined within the same Terrestrial Reference Frame have been found to differ from each other on average by 2.0 mas in bias and 0.36 mas/yr in rate. All of the series studied here have been determined by modeling station velocities using a global plate tectonic motion model. This model indicates that the tectonic plates (and hence the stations located upon them) move with respect to each other by up to about 4 mas/yr (*Minster and Jordan*, 1978). The divergences of the Earth rotation series found in this study are likely to be caused (at least in part) by errors in modeling the motions of the stations. Thus these divergences could perhaps be reduced by applying an improved plate tectonic motion model (such as the NUVEL-1 model of *DeMets et al.*, 1990) or by directly solving for the station velocities (e.g., *Ma and Clark*, 1990).

ACKNOWLEDGMENTS. The work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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ECLIPTICAL COORDINATE SYSTEM AS THE BASIS OF REGATTA-ASTRO SPACE ASTROMETRY PROGRAM

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One of the most important problems of modern astrometry is constructing of an ideal inertial coordinate system. A bulk of up-to-date star catalogues is referenced to an equatorial frame, which is far from being really inertial. Some attempts were made to remove this defect by excluding both nutation and precession in right ascention [Guinot 1979, Murray, 1990]. This proposal seems to be a compromise between the existing tradition and the new requirements met by the modern astrometry.

Perhaps, the use of non-rotating ecliptical coordinates would be more practical. There are two reasons at least supporting this idea. First, this frame is physically defined much better than the equatorial one due to a pure knowledge and variability of the great number of parameters involved in the Earth's rotation. Secondly, many basic problems of astronomy and astronautics deal with the ecliptical coordinates rather than the equatorial one. Take for instance the theories of the orbital motion of planets and minor selestial bodies. In addition, resent space-born astrometrical instruments provide the data which can be more conveniently referenced to the ecliptical frame. Furthemore, the equatorial plane cannot be defined in principle from the extraterrestial astrometric measurements, while the ecliptical plane is "peculiar" because of its connection with the heliocentric orbital motion of the observatory. Note that historically the very first star catalogues by Ptolemeus and Hipparchus did use the ecliptical coordinates. Thus, what we propose is a return to origin.

The central idea of the approach considered herein is the observability of the ecliptical plane and thus the straightforward availability of ecliptical coordinates of stars. Practically, one can do it by simultaneous observations of stars and the Sun. A location of a spacecraft carrying the astrometrical instrumentation and orbiting around the Earth may be determined very accurately by means of radio tracking. Continuous on-board obserations of the Sun provide the heliocentric orbital plane of the spacecraft (generally, it is quite near to the ecliptical plane) Necessary reductions may be computed from the radio tracking data. Following this way, the true ecliptic plane will be defined. There is no problem to reduce it to the mean ecliptical plane. Unlike the interrelation between the true and mean equators, it may be easily calculated.

An approach described will be implementen in REGATTA-ASTRO project of Space Research Institute, Moscow [Avanesov et al, 1990]. An astrometric module will be mounted onboard the Small Space Laboratory a lightweight spacecraft rigged with the solar seil providing the attitude mode with one spacecraft axis permanently pointing the Sun. The spacecraft is rotating slowly (about 1 revolution per day) around this direction. The instrument consist of four rigidly connected star telescopes and a solar telescope (Fig.1). The astrometric module is involved into the complicated angular motion. Following it, the field of view of the star telescopes scan the celestial sphere and cover it totally during half of year (Fig.2). The Sun is staying permanently inside the FOV of the solar telescope. An attitude mode is highly benefit for the constanty of temperatures onboard. The spacecraft is designed in a way minimizing the perturbing torgues, thus providing an extremely high degree of smoothness of its rotation. The choice of an operating orbit is meeting the same requirement (it is so-called quasy-satellite orbit). These means make it possible to process long measure arcs and achieve a significant updating of the star positions. The a priori appraisals forecast the output accuracy of the ecliptical star positions, proper motions and parallaxes to be about 10 mas for approximately 100000 stars.

In addition to the astrometric program, an experiment on the solar physics is planned for the REGATTA-ASTRO mission. It includes the measurements of the apparent diameter of the solar disc, it oblatness, it brightness and their temporal variations with the resolution of some 10 seconds. The level of accuracy will overcome the ground-based measurements by order of magnitude as minimum. The resul anticipated may spread light on problems of helioseismology.

It seems to be evident that the technical implementation of the project is not more sophisticated than that of other spaceborn experiments, while its benefits may be very significant. For example, the information of the solar telescope simplifies greatly the data processing and catalogue compilation. Besides, the whole procedure of catalogue orientation may be excluded.

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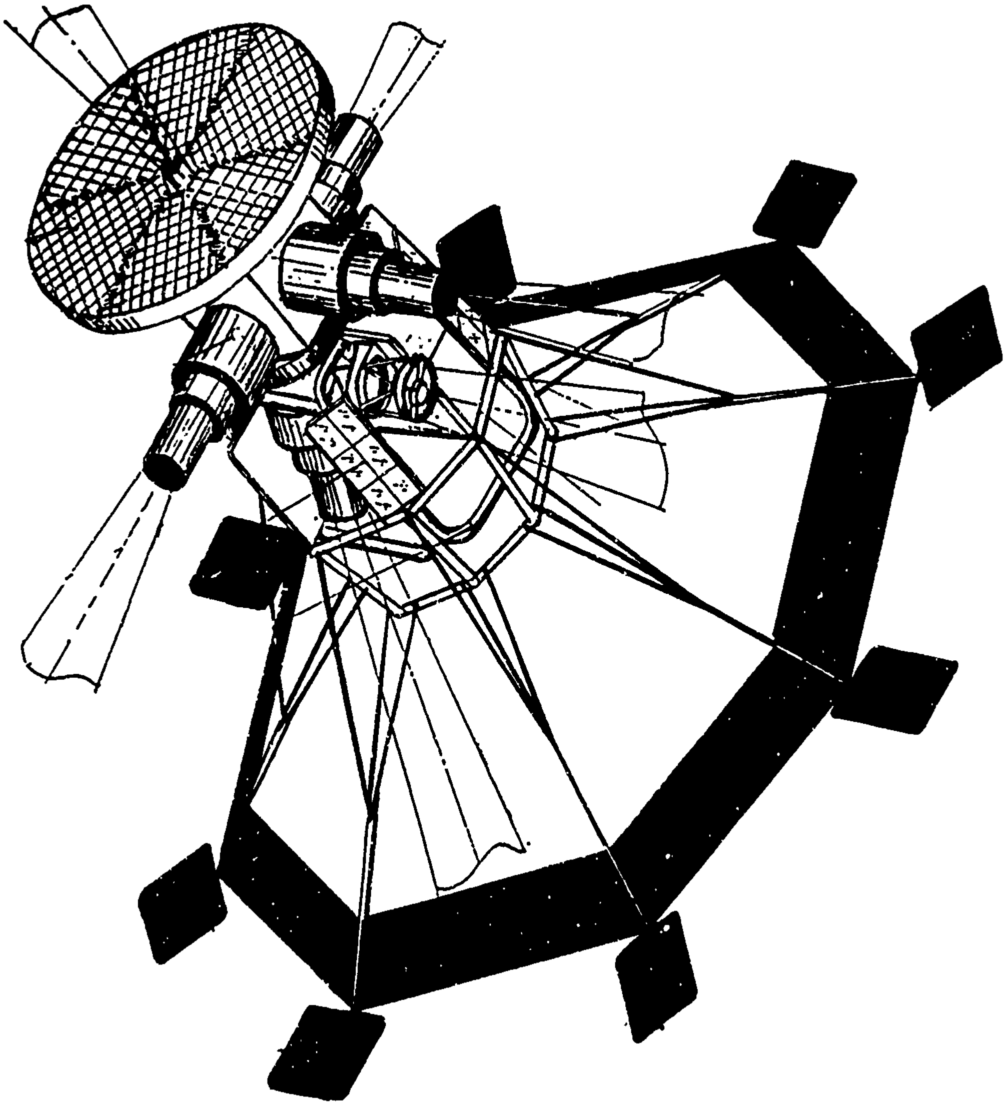

Fig. 1

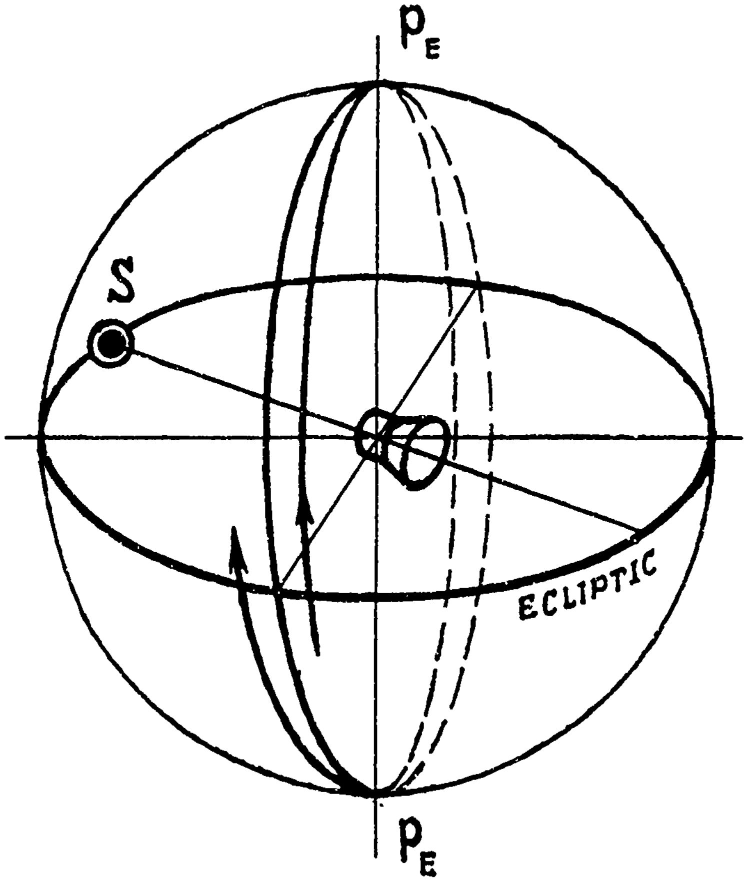

Fig. 2

INFLUENCE OF SYSTEMATIC DIFFERENCES OF FK4 ON DETERMINING EARTH ORIENTATION PARAMETERS

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ABSTRACT. In this paper the corrections of positions and proper motions (J2000) of FK4 were obtained. For different kinds of observational methods the formulae of calculating the influence of systematic differences of FK4 on time and latitude determinations were derived. The influence of systematic differences on the universal time and polar coordinates (1980-1988) of Chinese Joint System (CJS) and the global optical solution of earth orientation parameters (1987-1988) were calculated and their external precisions referred to BIH or IERS were also re-estimated.

1. Introduction

Just because till now only about 100 radio sources have the positions with the precision of 1 mas a project of determining the positions of 400 or more radio sources is being carried out over a five year period starting from July 1987 (Johnston,R.J. et al. 1987). It is known that the connection between the radio and optical reference frame is being studied so the optical reference frame FK5 is still used in many research fields.

The stellar positions and proper motions of FK4 were used during 1962-1988 for time and latitude determinations. According to the observational status, the influence of systematic differences of FK4 on the universal time and polar coordinates (1980-1988) of Chinese Joint System (CJS) and the global optical solution of earth orientation parameters (1987-1988) should be considered.

2. Determinations of Systematic Differences Between FK5 and FK4

As it is known the positions and proper motions of FK4 are referred to the equinox and equator of B1950.0. By means of the formula (McCarthy, D.D 1989) those are changed to referred equinox and equator of J2000.

The analytical method (Bien, R. et al. 1978) was adopted for calculating the systematic differences of positions and proper motions, corresponding epoch being J2000, between FK5 and FK4. The positions and proper motions of 1535 and 1987 stars, corresponding to fundamental and supplement stars of FK4, were used to calculate the systematic differences separately.

3. Influence of Systematic Differences of Catalogue On Time and Latitude Determinations

There are several methods for time and latitude determinations. The influence of catalogue systematic differences for different methods are as follows:

$$\begin{array}{ll} \text{transit} & \Delta u = \Delta\alpha \\ \text{zenith distance measured} & \Delta\phi = \Delta\delta \end{array} \qquad (1)$$

For equal altitude method if the observing condition is assumed that the stars are homogeneous distribution along the equal altitude and are observed each 10^{o} except $\pm 15^{o}$ around meridian $\Delta u = \Delta\alpha$, $\Delta\phi = \cos z\ \Delta\delta$, where $|\phi| <= 40^{\circ}$.

4. Influence of Systematic differences of FK4 On Determining Earth Orientation Parameters

There are several independent joint universal time system such as BIH (before 1988), Standard Time of USSR and Chinese Joint System (CJS).

The influence of systematic differences of FK4 on CJS was calculated during 1980-1988.Because the observing range of declination is different for transit, astrolabe etc. the systematic differences are interpolated by using the interpolating factors of declinations such as $\phi-10^{\circ}$, $\phi-5^{\circ}$, ϕ , $\phi+5^{\circ}$, $\phi+10^{\circ}$. The interpolating factor in right ascension will be $0^{h}.5$, $1^{h}.5$... etc. according to the observing interval of the group. The systematic difference of catalogue for the observing group is obtained from all systematic differences, which are reduced the proper motions, in observing range of declination and right ascension.

By using the pratical weight, which equals $\sqrt{N}$ P, the influence of systematic differences of FK4 on the universal time of CJS during 1980-1988 were obtained and drawn in Fig.1. After reduction of the systematic differences of FK4, the precisions referred to BIH and IERS were re-estimated and are listed in Table 1.

It can be seen that the precision of universal time of CJS in 1988 is not improved because of the less observations made with seven instruments.

By using the Orlov method the instantaneous coordinates are obtained with the observations of single station. The instantaneous polar coordinates referred to the mean pole of the epoch were used in CJS. As we know the relation between stationary polar coordinates and the pole coordinates of epoch is $X = Xo + X1$, $Y = Yo + Y1$,where Xo and Yo are the mean pole coordinates of epoch in 1968.0 (Polar Motion Collaboration Group 1976). For Orlov method X1 and Y1 are the sum of the annual and Chandler components, i.e. $X1 = Xa + Xc$ and $Y1 = Ya + Yc$, in which Xa and Ya are calculated with the stationary formulae. Therefore, the influence of systematic differences of FK4 on $\Delta X1$ and $\Delta Y1$ are the same as that on ΔXc and ΔYc.

$$\Delta\phi_c = \Delta X_c \cos\lambda + \Delta Y_c \sin\lambda \qquad (2)$$

where $\Delta\phi_c$ is the influence of systematic differences of FK4 on the latitude determinations. After solving the observing weighted equation (2) ΔXc and ΔYc, i.e. $\Delta X1$ and $\Delta Y1$, are obtained each month.

In 1976 there were 41 instruments adopted to determine the mean polar coordinates of epoch 1968.0. Since 1983 there were only 26 instruments, such as Ottawa (PZT),Quito (AST),Tianjin (ZTL-180) etc. to continue observations. It is assumed that the observations are symmetric to meridian of the station and the clear night are homogeneous for whole year and month. The influence of systematic differences of FK4 on Xo and Yo were calculated with the following formula.

$$\Delta\phi_{\Delta\delta} - \Delta\phi_c = \Delta X_0 \cos\lambda + \Delta Y_0 \sin\lambda \qquad (3)$$

where $\Delta\phi_{\Delta\delta}$ are the influence of systematic differences on latitude determinations measured with 26 instruments and $\Delta\phi_c$ are calcalated by formula (2). The values of ΔX and ΔY are shown in Fig.1.

Since 1988 Shanghai Observatory was assigned as an analysis center of optical technique by IAU commission 19(jin Wenjing and Liao Dechun, 1989). The global solution of earth orientation parameters,in which the influence of systematic differences of FK4 was taken into account, was calculated. The influence of those on earth orientation parameters are shown in Fig. 2.

5. Conclusion

The influence of systematic difference of FK4 on the previous results of time and latitude determinations should be considered.

After reduction of systematic difference of FK4 the values of earth orientation parameters, whether the universal time of CJS or the global solution, are close to those of BIH and IERS. The improvement of precisions in the universal time of CJS is slightly better than that in the global solution of earth orientation parameters because of the reduction of influence of catalogue on time and latitude determinations measured by the optical instruments, which are located at the narrow longitudinal region in China.

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Table 1 Precisions of Universal time of CJS Unit: $0\overset{s}{.}001$

	original precision		precision after considering the systematic difference	
	x	ε	x	ε
1983	-4.6	±2.1	-3.4	±2.0
1984	-4.9	2.8	-3.7	2.4
1985	-4.4	3.0	-3.2	2.5
1986	-2.4	2.4	-1.3	1.9
1987	-2.2	2.3	-1.1	1.7
1988	-3.5	2.0	-2.4	2.7

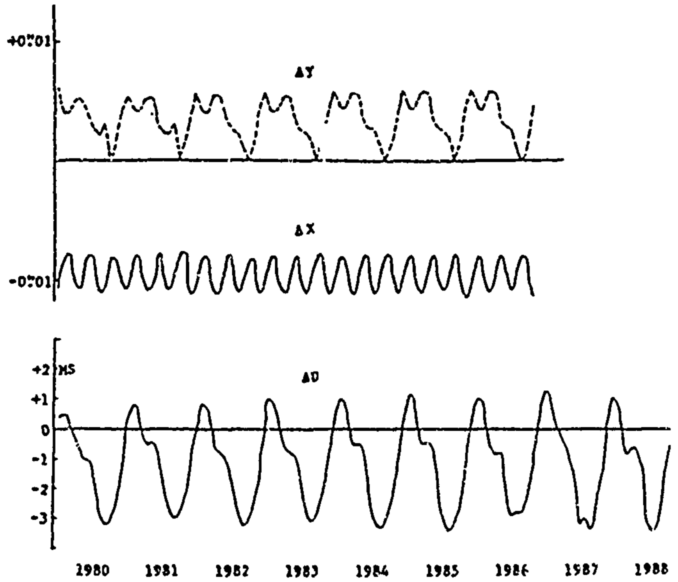

Fig. 1 The influence of FK4's Systematic Differences on Earth Orientation Parameters of CJS during 1980 -- 1988

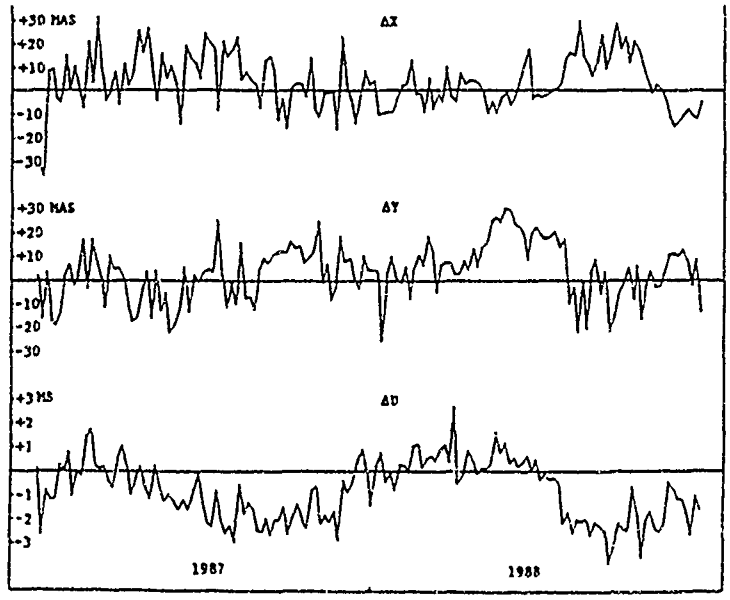

Fig. 2 Influence of Systematic Difference of FK4 on Global Solution of EOP

OPTICAL ASTROMETRY AND THE GLOBAL POSITIONING SYSTEM

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ABSTRACT. Astrometric accuracies of a few tens of milliarcseconds are expected to be attainable within five years by calibrating astrograph plates with optical observations of Global Positioning System (GPS) satellites against a stellar background. The line of sight from an observer on the Earth's surface to a GPS satellite may be calculated with high accuracy. Motion on each day of the line of sight to the satellite and changes from day to day in the apparent path of the satellite are sufficiently slow to make it possible to reduce atmospheric errors by averaging. Advanced ground-based optical sensors, probably using charge coupled device technology, will be required for GPS optical astrometry.

1. Global Positioning System

The Global Positioning System (GPS) is designed to provide three-dimensional positions and velocities to navigational users worldwide. Although primarily a military system, many of its products are available to the civilian community. Its "clear access" (C/A) signals give to users who know their height above sea level (e.g. mariners) two-dimensional positions to 100 meters (two sigma). More interesting to scientific users are the precise ephemerides, which are declassified after 48 hours. At present, they are computed by the Defense Mapping Agency (DMA) and distributed by the National Geodetic Survey (NGS). US civilian centers now active in improving GPS ephemeris accuracy include the Center for Space Research of the University of Texas at Austin (Schutz et al., 1989) and the Jet Propulsion Laboratory (Lichten and Border, 1987).

Each GPS satellite is in a near circular orbit of which the period of revolution is 12 sidereal hours. Orbital inclinations are high -- 63 to 64 degrees for Block I GPS satellites and 54 to 55 degrees for Block II GPS satellites. When satellite longitudes and latitudes are plotted in cartesian coordinates, the plotted positions nearly form a square wave with amplitude equal to the orbital inclination (Figure 1). Since the distance of a GPS satellite from the Earth's center is approximately four Earth radii, the line of sight from an observer on the Earth's surface to the satellite is within 20 degrees of that point of the celestial sphere with declination equal to satellite latitude and hour angle equal to the difference of observer and satellite longitudes. A GPS satellite is visible when the Sun-satellite-Earth angle is less than 90 degrees and brightest just before eclipse, when that angle is about 13 degrees.

A GPS satellite is maneuvered so that its ground track is nearly fixed in terrestrial latitude and longitude, while its orbital plane moves slowly relative to the stars. Thus, for each point on the Earth, observed values of the satellite's declination and hour angle repeat daily, given values occurring about four minutes earlier each day. Figure 2 shows altitudes at Washington, D.C. of GPS satellites 16, 6, 17, 10, 19, 14, and 21 for late September 1990.

The GPS satellites have several advantages over other satellites for astrometric control. They are approximately magnitude 9, and satellite motion relative to the stars is of the order of 45 arc minutes per minute of time. The sky coverage is good -- twenty-one operational satellites are planned, plus three spares, with full global coverage. The path of a GPS satellite scans slowly through a star field over a period of months -- the satellite's ground track is nearly fixed, while the orbital plane moves westward at a rate of 0.03 to 0.04 degree per day relative to the stars. Orbital eccentricity is small, and the apogee advances at a rate of less than 0.02 degree per day.

The position of the optical center of a GPS satellite relative to an observer on the Earth's surface can currently be calculated to 3- to 5-meter accuracy, relative to an Earth-centered coordinate system oriented by VLBI observations, corresponding to an angular accuracy of 30 to 50 milliarcseconds in the line of sight direction. GPS satellite precise ephemerides now give accuracies of 2 to 4 meters for the center of mass. Errors in the calculated displacement from satellite center of mass to center of light are believed to be at present approximately 1 meter. Over the next five years, significant improvements in orbit determination and in modelling of reflectance are expected for GPS satellites. These should lead to the ability to calculate the line of sight to 5 milliarcseconds.

Stationkeeping maneuvers are occasionally performed. The mean frequency of such maneuvers is one per 1.19 years per satellite. Such maneuvers limit the predictability of satellite motion, but are fully reflected after the fact in the precise ephemerides.

2. Optical Astrometry Relative to Global Positioning System Lines of Sight

In order to determine star positions with high accuracy relative to the line of sight to a GPS satellite, one may accumulate into pointlike charge images the photoelectrons arising in a charge-coupled device (CCD) both from photons in an optical image of the satellite and photons in optical images of stars. Pointlike charge images of stars may be formed (Figure 3) by placing the CCD in a telescope which follows the stars. Aligning the columns of the CCD with the direction of motion of the image of the satellite and moving charge in half of the CCD from row to row at an appropriate rate causes (Figure 4) the photoelectrons coming from the image of the satellite to be accumulated into a pointlike charge image. Such a method of image motion compensation has been used previously (Monet, 1988) to measure star positions with a fixed telescope. Motions of the telescope are reflected identically in both star and satellite charge images.

An instrument for GPS astrometry might consist of a 0.5-meter aperture telescope of 4-meter focal length with a 2000 by 2000 pixel CCD, having 20 micron pixels, in its focal plane (Figure 3). The time required for the image of a GPS satellite to cross the CCD, which corresponds to a 30 by 30 arc minute region of sky, is approximately 40 seconds. A narrow bandpass filter is shown, which reduces the number of photoelectrons to 8000 per pixel for an image of 3 pixel radius, exposed 40 seconds, of a magnitude 10 star (filter transmission is 50% and CCD efficiency is 40%). Such a narrow bandpass makes refraction differences with color insignificant.

The standard deviation of observation errors with the described instrument is expected to be approximately 50 milliarcseconds. This includes a one-fortieth pixel error due to causes within the CCD (Monet, 1988), an error of 25 milliarcseconds due to atmospheric turbulence (Han, 1989), and an error of 25 milliarcseconds (2.5 meters) due to inaccuracies in the satellite ephemeris.

Since the path of a GPS satellite scans slowly among the stars, covering approximately one hour of right ascension per year, tens of observed GPS positions can be placed in the region covered by a single astrograph plate. These observations can be used to calibrate the plate to significantly better than the 50-milliarcsecond standard deviation of a single observation.

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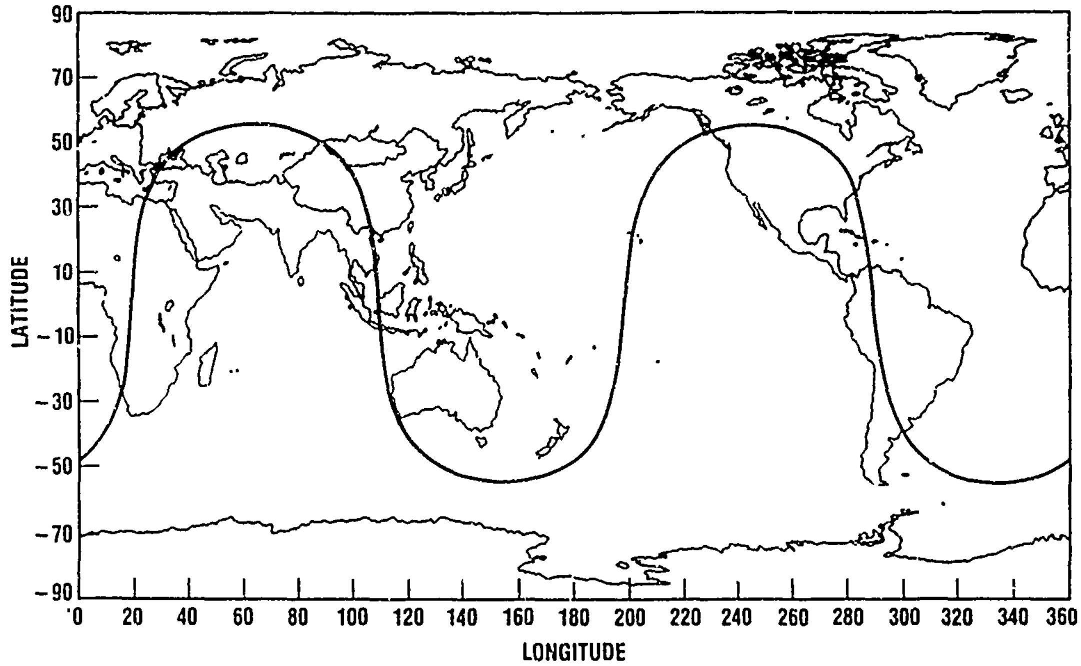

Figure 1. Ground Track of GPS Satellite Number 21 in Late September, 1990

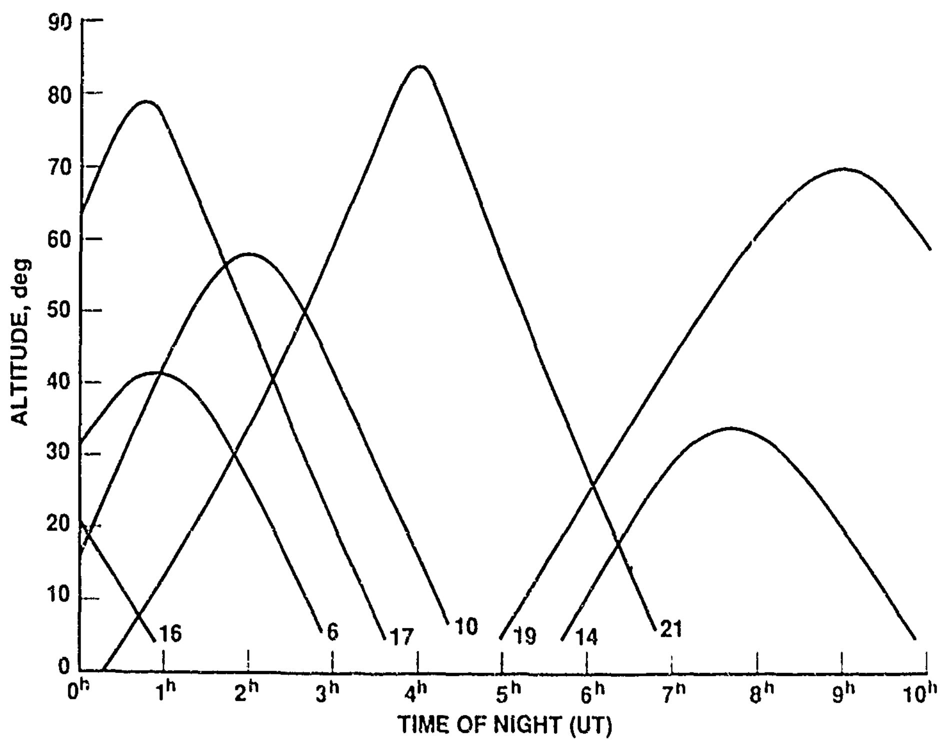

Figure 2. GPS Satellite Altitudes for Washington, D.C. in Late September, 1990

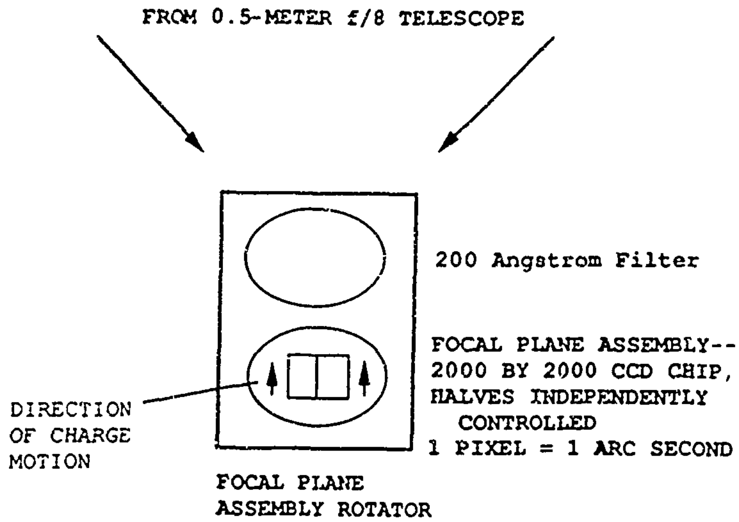

Figure 3. Focal Plane Assembly of GPS Astrometry Telescope

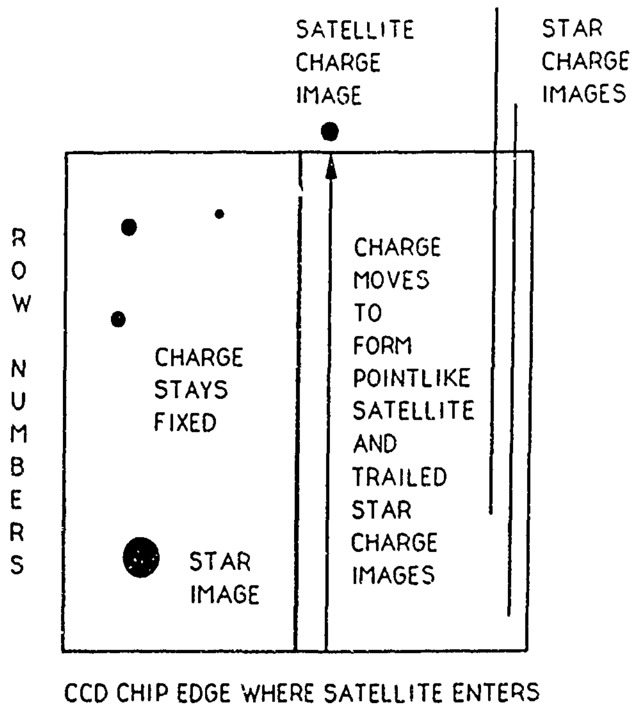

Figure 4. Operation of Focal Plane CCD Chip

STATUS OF THE HIPPARCOS DATA REDUCTION

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1. PRESENT SITUATION OF THE SATELLITE

HIPPARCOS is now well settled on its $10^{h}40^{m}$ eccentric orbit and the data is received by three telemetry stations that are also transmitting commands: Odenwald (F.R.G.), Perth (Australia) and Goldstone (California). Although during about 80 to 90% of the time, data are recovered, in practice only the observations made from higher than the outer Van Allen belt are usable. Sometimes even at that height, on-board attitude determination does not converge due to the noise that covers the star mapper signal. Finally, about between 55 and 65% of the mission time is usable for reduction. The actual routine data recovery started on November 27, 1989.

2. WORK OF THE REDUCTION TEAMS

Both data reduction consortia (FAST and NDAC) have received and reduced about 1,5 months of data sampled thoughout the first eight months of mission. The main objectives were :

- To modify the software in order to accomodate the new situation arising from the unexpected orbit and the increased noise when the satellite is still close to the radiation belts or when it is in lengthy occultation situation. More generally, the treatment of the real data often showed features that were not correctly dealt with by software built using some *a priori* model.

- To calibrate the instrument: intensity and modulation transfer functions, grid to field and field to grid transformation, basic angle, etc...

- To test the correctness of the software by comparison between various intermediary results obtained by the two consortia.

This initial phase is essentially complete, and it expected that routine data reduction might start sometime in November 1990. It should last at least 4 years, possibly more if the duration of the mission is extended.

3. PRECISION OF THE RAW DATA

Raw data consists of photo-electron counts produced by the photomultipliers. The reduction using a calibrated model gives the times of crossings of the star mapper grids and the modulation phases that are used to determine the position of a star image on the main grid. Both can be readily transformed into milliseconds of arc on the sky (fig. 1 and 2).

In addition, the amplitude of the modulation curve on the main grid gives the intensity of the light received from the star and, hence, its magnitude in the HIPPARCOS photometric system. Using as calibrators a certain number of *standard stars* that have been accurately observed from the ground, one can derive the accuracy of magnitude determinations in addition to their precision (see fig. 3). Actually, there are two independent methods to determine the magnitude using different terms in the modulation function. They give the same results for single stars, but there are differences in case of non-single stars and this is one of the ways to identify double or multiple stars.

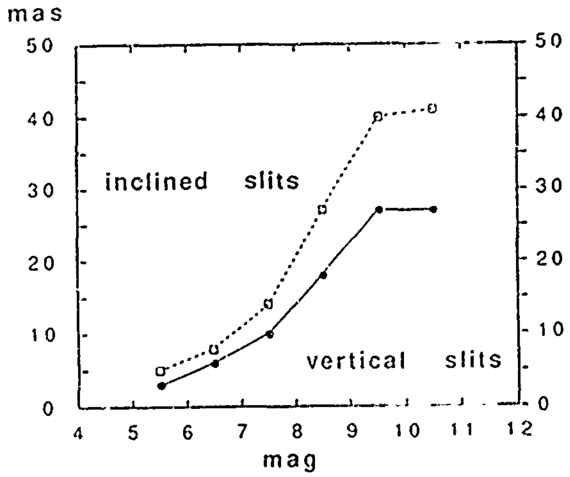

Figure 1 - Typical average r.m.s. errors of the determination of the times of transit of a star through vertical and inclined slits of the star mapper in function of magnitudes.

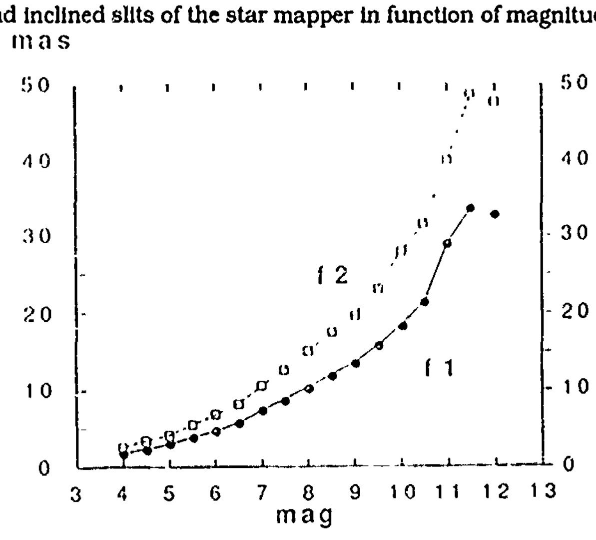

Figure 2 - Typical average r.m.s. errors on the determination of the phases of the first and second harmonic of the modulated signal from the main grid in function of magnitudes. The corresponding observation times range from 0.2s for bright stars to 1 second or more for the fainter ones.

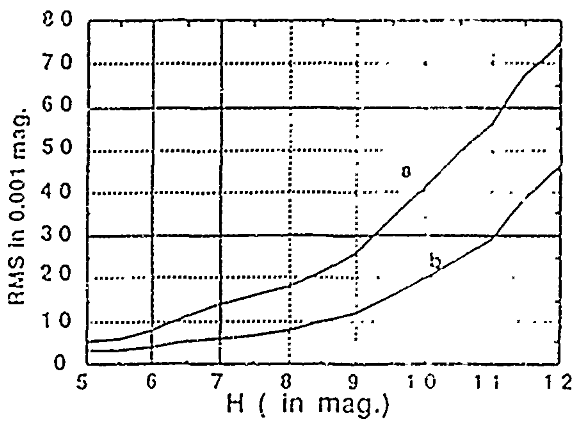

Figure 3 - Standard deviation per transit as a function of the magnitude for the two methods (a). (b) of determining it.

Up to a couple of dozen new double stars are discovered by this methods from the analysis of 5 to 8 hours of data. The separation power depends of course on the magnitude difference between the components. In the most favourable cases it could be as low as 0".05.

4. REDUCTION ON A GREAT CIRCLE

The reduction procedure calls for a synthetic treatment of the results of the raw data analysis presented in the preceding section, as accumulated during a data set consisting of the useful part of the satellite revolution. Two steps are performed.

a) Attitude determination from the star mapper

About 3000 to 6000 transit times through each group of slits are obtained. The attitude is determined (this is a first approximation) using the star positions provided by the Input Catalogue. The following r.m.s. are typically obtained for the three orientation angles :

ψ : 60 to 100 mas (along track)
θ : 100 to 150 mas (around oy axis)
ϕ : 200 to 400 mas (around ox axis)

However, these precisions are primarily affected by the uncertainties of the Input Catalogue. The attitude so obtained may be used to improve the along scan positions deduced from the Input Catalogue. The results show that indeed the catalogue induced errors are of the order of the attitude error obtained. It also shows the quality of the Input Catalogue which is of the order of 0".2 to 0".3.

b) Abscissae on the reference great circle

This last global step in the data set reduction is the determination of the mean abscissa of each observed star on the reference great circle using the attitude and the grid coordinates provided earlier. Simultaneously, the ψ component of the attitude is redetermined as well as some instrumental parameters. A typical r.m.s. of the order of 10 to 16 mas characterizes the precision of the abscissae on the great circle. The same precision is obtained for the along scan attitude ψ.

5. FINAL PRECISION EXPECTATIONS

The determination of positions, proper-motions and parallaxes requires a new step in which the results obtained for a large number of reference great circles are combined. This could not be done since too few data have been reduced so far. However, it is possible to extrapolate the precisions presented above using our experience with simulated data. In particular we can use the fact that these positions are better than the nominal ones, and this partially compensates the loss of data acquisition. Table 1 gives the expected precisions as a function of the duration of the mission.

The behaviour of the satellite proves to be such that a 4-year duration is not to be excluded. In addition, one estimates that various improvements of the reduction method would improve the results by a factor of 0.7. From this we conclude that reaching the nominal precisions (and seemingly also accuracies) is a reasonable expectation if the satellite survives another couple of years. If, as it is quite possible, finances permitting, HIPPARCOS remains absolutive another couple of years, a final precision of the order of one millisecond of arc is not impossible.

TABLE 1

Expected precisions in m.a.s. for a 9 magnitude star

Duration of the mission	18	24	30	40	50	Nominal mission (30 months)
Position	6	4.5	3	2	1.5	2
Annual proper motions	10	4	2.5	1.7	1.0	2
Parallaxes	6	4.5	3	2	1.5	2

MAIN LITTERATURE

The HIPPARCOS Mission, Prelaunch status.

Vol. 1: "The HIPPARCOS satellite", M.A.C. Perryman, H. Hassan and 22 other authors, 353 pages.

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Vol 3 : "The Data Reduction", M.A.C. Perryman, L. Lindegren, C.A. Murray, E. Hoeg, J. Kovalevsky and 49 other authors, 515 pages.
ESA Publication SP1111, July 1989.

VLBI ASTROMETRY OF THE HIPPARCOS LINK RADIO STARS

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The VLBI extrgalactic reference frame contains 280 radio sources distributed evenly in both hemispheres. The average internal astrometric accuracy of this frame has reached 1 milliarcsecond. The link between this stable VLBI reference frame and the rotating HIPPARCOS frame is important to unify the radio and optical coordinates systems. We are determining the tie between the two frames by conducting VLBI observations of optically bright radio emitting stars which are common objects to both frames. We are presently monitoring 12 such radio stars with a high-sensitivity and high-accuracy VLBI technique for differential astrometry. We present several tests as an assessment of this astrometric accuracy.

NOTE ON THE DEFINITION OF THE INTERNATIONAL ATOMIC TIME TAI

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ABSTRACT. Relations among the three concepts (TAI,coordinate time,and proper time) are discussed and comments of the definition of the TAI are given.

1. Introduction

Since the establishment of the atomic time scale in 1956 at the BIH and the presence of its notation TAI in 1971 (Guinot,Seidelmann,1988),two possible definitions of the TAI(Guinot,1990) are being concerned from time to time: a coordinate time in a non-rotating geocentric reference system; or the proper time at a specified location on the Earth. Although TAI was defined as a coordinate time in 1980 by the CCDS, discussions between different opinions are still carrying on frequently. It is quite possible that a clearer understanding of the TAI is still needed in nowadays. The purpose of this paper is to make a further explanation on the relations among the three concepts: TAI,coordinate time,and proper time,then some comments on the definition of the TAI are given.

2. Relation between the proper time τ' of a Earth fixed clock and the coordinate time T in a non-rotating reference system

The relation between the proper time τ'_i of a clock i,which is fixed on the Earth, and the coordinate time T of the non-rotating geocentric reference system is:

$$d\tau'_i = \left(1 - \frac{2\Phi_i}{c^2} \right)^{\frac{1}{2}} dT, \qquad (1)$$

Φ_i is the sum of gravitational and centrifugal potential of the i clock. If the same clock is moved to the geoid, the proper time τ_i of the corresponding fictitious clock can be related to the coordinate time T as:

$$d\tau_i = \left(1 - \frac{2\Phi_o}{c^2} \right)^{\frac{1}{2}} dT, \qquad (2)$$

Φ_o represents the corresponding potential of the fictitious clock.

So, we have:

$$d\tau_i \,/\, d\tau'_i \doteq \left(1 - \frac{\Phi_o - \Phi_i}{c^2} \right) \qquad (3)$$

From (3), the time scale established by the i clock can be changed to the fictitious clock's.

The existence of equation (2) demonstrates that τ_i is not the coordinate time T itself,but it is one of the members of the coordinate time family and relates to T with a constant factor k. Let us use t to express it and the following relation can thus be written:

$$dt = kdT = d\tau_i \qquad (4)$$

It can be concluded here,the proper time τ'_i of a Earth fixed clock, after being reduced to the fictitious clock's on the geoid, has its duality: it is the proper time τ_i of the fictitious clock on the geoid, but it is also the coordinate time t of the non-rotating geocentric reference system at the same time.

3. Relations among TAI,proper time τ,and the coordinate time t

As a result from the proper times τ_i (i=1,2,...,n) of a certain number of fictitious clocks on the geoid,what is the nature of the TAI now?

In practice, we can not establish the TAI without using the concept of coordinate synchronization. So, it is easy to believe that TAI is only a coordinate time now, and not a proper time anymore(Huang et al.,1989). Is it the only answer that we can have?

We first discuss the case in which only real clocks are concerned. Let us suppose that clocks are fixed on the geoid and located at the same place. Due to the existence of the equation (4),it is clear that the obtained TAI will be no difference whether the concept of coordinate synchronization or the concept of so-called standard synchronization (Huang et al.,1989) is used in comparing clocks. Then, clocks are moved to different locations but still fixed on the geoid. The equivalence of the two kinds of synchronization will still exist. So, if the TAI is obtained from the readings of the fixed real clocks on the geoid,it can be regarded as coordinate time and also proper time as well at the same time even we have only used the concept of coordinate synchronization in comparing clocks.

Now we come to the case that we are actually confronting in establishing TAI. The answer depends on the understanding of the "proper time of fictitious clocks". If it can be regarded as a proper time, why we will then be not able to regard the TAI as a proper time? It should be noticed that the approximation of regarding TAI as a proper time is the same as that of the actual expression which we have used to define the coordinate time.

4. Discussion

(1) In considering the problem of the definition of TAI, it is possible to understand it in a wider sense in practice. It is not the unique solution that we have to restrict ourself to a unchangeable definition.

(2) It is correct to define the TAI as a coordinate time in the non-rotating geocentric reference system, but it is not necessary to exclude the TAI absolutely from the concept of proper time at the same time. It depends on the accuracy of the demanding in theory. It is possible to define TAI as a proper time at a specified location on the Earth in practice. Its actual meaning is the mean, or weighted mean, of the readings of fictitious clocks which are fixed on the geoid.

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THIRD GENERAL CATALOGUE OF STARS OBSERVED WITH THE PHOTOELECTRIC ASTROLABES

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Abstract

On the basis of the data observed with the Photoelectric Astrolabes in China observed during the period from 1982 to 1990, the general catalogue is compiled. With magnitudes ranging from $0^m.1$ to $7^m.3$, the general catalogue consists of 2577 stars spreading from $\delta=$ -3° to $\delta=$ 69°, in which 955 are FK5 stars and 1622 are GC stars.The mean precisions of 2577 $\Delta\alpha$'s and 1892 $\Delta\delta$'s are $\pm$4.0 ms and $\pm$0".065, respectively. The mean epoch of the catalogue is 1987.4. Finally systematic corrections of (CAT−FK5) are given.

I. Introduction

The equal altitude method brings good results not only for the determination of the Earth's rotation parameters, but also in the improvement of catalogues. The potentiality of this method in the field of position astronomy has been shown by several general catalogue [1,2] of stars observed with the different marks of chinese photoelectric astrolabes since 1973. The general catalogue reported here is compiled on the basis of the preliminary catalogue of the photoelectric astrolabe mark II of Beijing, Yunnan and Shanghai Astronomical Observatories and mark I photoelectric astrolabe of Shaanxi Astronomical Observatory since 1982.

The chinese photoelectric astrolabes mark II have been modified to automatic ones and photo-counters are used for recording the time of almucanter transit of a star with the limiting magnitude of at least $9^m.5$. The photoelectric astrolabe mark III are adjusting and will be installed at Beijing, Yunnan Astronomical Observatories respectively. These instruments will be used for observations of faint stars and some minor planets.

II. Reduction of the data

Adopting the positions at J2000.0 of the Fifth Fundamental Catalogue (FK5) and new astronomical constants (IAU, 1976), the data observed with the Photoelectric Astrolabes are reduced to the FK5 system.

All preliminary catalogue are compiled using same method. The equation defined the position corrections are

$$\Delta\alpha = \frac{V_e - V_w}{30\cos\varphi_0|\sin A|} + \Delta A$$

$$\Delta\delta = -\frac{V_e + V_w - 2K}{2\cos q} + D\cos\delta$$

where

V_e, V_w – the residuals reduced to the mean instrumental system at both eastern and western passages;

φ_0 – the adopted value of latitude at the site of the instrument;

A – the azimuth of a star observed, measured eastwards from north ;

q – the parallactic angle of a star as it transits the almucantar of the astrolabe.

ΔA and D$\cos\delta$ are constans to be determined. 2K can be calculated using the stars of $|\cos q| < 0.2$.

After correcting for the magnitude equations, spectral type equations, difference in the constant of right ascensions, the value of 2K's and the systematic error of delclinations of the form D$\cos\delta$, the analytic method developed by R.Bien et al[3] have been used to obtain the systematic differences respectively of α, δ and magnitude M in the sence of (CAT−FK5) for each preliminary catalogues. The systematic difference may be given by

$$f_i(\delta, M, \alpha) = R_{pnml} L_n(X < \delta >) J_p(Y < M >) F_{ml}(\alpha),$$

where

R_{pnml} – normalizing factor;

$L_n(X < \delta >)$ – Lgendre polynomial;

$J_p(Y < M >)$ – Jacobi polynomial;

$F_{ml}(\alpha)$ – Fourier term.

Meanwhile, a computerized numerical method has been used as a comparison. It is considered that these results ara agree with each other quite well.

The composite system of the general catalogue is formed by weighting each FK5 star of different preliminary catalogue according to their systematic differences

$$f_i(X < \delta >, Y < M >, \alpha).$$

And all GC stars have been obtained with the same process.

III. The Results

The general catalogue consits of 2577 stars in which there are 955 FK5 stars and 1622 GC stars. The magnitudes are from $0^m.1$ to $7^m.3$. The declinations are from $\delta = -3°$ to $\delta = 69°$. The mean precisions of 2577 $\Delta\alpha$'s and 1892 $\Delta\delta$'s are ± 4.0 ms and $\pm 0''.065$, respectively. In the catalogue, among these stars appearing in more than one preliminary catalogue, there are 885 common in $\Delta\alpha$ and 369 in $\Delta\delta$. The mean accordances for the common stars in $\Delta\alpha$ and $\Delta\delta$ are $\pm$4.5ms and $\pm 0''.061$, respectively. The mean epoch of the catalogue with 188 thousants observations is 1987.4.

By the method [1] and [3] and with the $\Delta\alpha$ and $\Delta\delta$ of FK5 stars, the systematic corrections of the catalogue of stars (CAT–FK5) are analyzed. The systematic corrections on the right ascension, declination, and magnitude $\Delta\alpha_\alpha$, $\Delta\delta_\alpha$, $\Delta\alpha_\delta$, $\Delta\delta_\delta$, $\Delta\alpha_m$,and $\Delta\delta_m$, are given in Tables 1, 2, and 3, respectively.

Table 1. The Systematic Errors (CAT–FK5) $\Delta\alpha_\alpha$ and $\Delta\delta_\alpha$

α^h	0	1	2	3	4	5	6	7	8	9	10	11
$\Delta\alpha_\alpha(0^s.0001)$	0	7	12	16	17	16	13	9	5	2	0	0
$\Delta\delta_\alpha(0''.001)$	17	28	15	-5	-12	1	7	16	-5	-29	-32	-11
α^h	12	13	14	15	16	17	18	19	20	21	22	23
$\Delta\alpha_\alpha(0^s.0001)$	0	0	0	-2	-5	-9	-13	-16	-17	-16	-12	-7
$\Delta\delta_\alpha(0''.001)$	17	28	15	-5	-12	1	7	16	-5	-29	-32	-11

Table 2. The Systematic Errors (CAT–FK5) $\Delta\alpha_\delta$ and $\Delta\delta_\delta$

δ°	0.0	5.0	10.0	15.0	20.0	25.0	30.0
$\Delta\alpha_\delta(0^s.0001)$	-3	6	-29	-14	22	29	12
$\Delta\delta_\delta(0''.001)$	47	45	19	-3	-11	-9	-4
δ°	35.0	40.0	45.0	50.0	55.0	60.0	65.0
$\Delta\alpha_\delta(0^s.0001)$	3	8	11	3	-7	-16	-50
$\Delta\delta_\delta(0''.001)$	-3	-10	-29	-33	-38	-31	-9

Table 3. The Systematic Errors (CAT–FK5) $\Delta\alpha_m$ and $\Delta\delta_m$

M	0.33	2.14	3.11	4.02	5.04	5.94	6.76
$\Delta\alpha_m(0^s.0001)$	-9	1	-3	0	0	-2	-8
M	0.06	2.16	3.08	4.01	5.04	5.91	6.73
$\Delta\delta_m(0''.001)$	6	-13	3	3	0	1	1

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THE APPROACH OF IMPROVEMENT TO STELLAR COORDINATE

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ABSTRACT

This article presents a new method for observing minor planets.The observation is operated with a photo–electronic imaging devise CCD and a lower latitude meridian circle.

The CCD is mainly used except it is during the favourable opposition of the minor planet when the meridian circle is mainly used in this method.The method can improve precision of observation of planetary position and enlarge scope of observation of planetary orbit.Therefore,the measured precision of zero point of stellar coordinate could be increased.The key of succeeding is that more precise result is got using the CCD.

The experiment in this article indicates that this method is a good way:While a minor planet and calibration stars locate in the same image of the CCD,the measured precisions of the minor plant are

$$\sigma_\alpha = \pm\ 0''.03, \qquad \sigma_\delta = \pm\ 0''.035;$$

While the minor planet and calibration stars are located in different images of the CCD,the precisions are

$$\sigma_\alpha = \pm\ 0''.06, \qquad \sigma_\delta = \pm\ 0''.08.$$

1.INTRODUCTION

At present,the precision of the coordinate based on the stellar coordinate is affected because precision of observations on the celestial bodies in the solar system is not able to reach precision of observations on the stars.So raising precision of observations on the celestial bodies in the solar system is one of the important way to improve the stellar coordinate.Since 1986,we have been making a series of orientation observations on minor planets using CCD receiving system of 1M telescope in Yunnan Astronomical Observatory.The observations indicate[1]: When the tested minor planet and the calibration stars locate in the same image of CCD,the measured precision of the minor planet is: $\sigma_\alpha = \pm 0''.03$, $\sigma_\delta = \pm 0''.035$.On the other hand,when the measured minor planet is connected with calibration stars by means of overlap measurement,the precision becomes: $\sigma_\alpha = \pm 0''.06$, $\sigma_\delta = \pm 0''.08$.The increasing of observational frequency can no doubt improve the orientation precision of the minor planets.When we want to determine the orbits of those minor planets,we can adopt CCD.It has not only higher precision comparing with photograph,but also higher efficiency.On this basis,the author presents an idea to determine zero point of stellar coordinate,that is observing minor planets by using conjugation of CCD and the meridian circle[1].This paper discusses it in the concrete.

2.THE ADVANTAGES AND DISADVANTAGES OF DETERMING ZERO POINT OF THE STAR CATALOGUE BY MEANS OF OBSERVING MINOR PLANETS.

In history,zero point of the star catalogue was determined by means of observing the sun and the planets.However,considering the observational conditions now,it is difficult to improve the observational precision of zero point of the star catalogue.The minor planets are near to point sources,and also they are observed at night.Theoretically,the position of the minor planets can be arrived with the stellar measured precision.So minor planets are the ideal celestial bodies to determine zero point of star catalogue.In the 20′s of the 20th century,Dyson had presented a proposal to determine zero point of the coordinate through observing minor planets[2].For dozens of years,the precision of zero point of the star catalogue determined by minor planets is not high.The reason might be divided into three sides[3–4]:

(1).The adopted stars in the fundamental coordinate are mainly those stars absolutely and relatively measured through meridian circle.The minor planets are very dim,so not many of them can be observed by using meridian circle.

(2).In practice,photograph method is usually used.But the precision of the star catalogue adopted by this method is too low,the measured precision of the minor planets reduces.

(3).The minor planets are never observed systematically by using photograph or meridian circle,and the theory of movement of the minor planets are not perfect.

The conjugation of CCD and meridian circle to observe minor planets can improve the measured precision,and also enlarge the observed range.If the full use of the observational data of high precision of minor planets is made and the accurate orbits of the minor planets are determined,then the main disadvantages of the measurement of observing minor planets in the past can be overcome and the precision of zero point of the star catalogue determined by minor planets can be increased,and also our aim of improving stellar coordinate will be realized.

3.CONJUGATION OF CCD AND MERIDIAN CIRCLE TO OBSERVE MINOR PLANETS

In this paper,the observational means is CCD receiving system of 1M telescope and low latitude meridian circle in Yunnan Astronomical Observatory.The meridian circle can observe celestial bodies of about 13^m[5].

During the opposition of minor planets,our main observational instrument is meridian circle,CCD helps it.They are used to observe for a period of time simultaneously.The aim is normalization calculation.When the minor planet's location is not good for meridian circle,CCD can be full used.In order to assure coincidence of the system and improve CCD orientating precision of minor planets,calibration stars in CCD observation should mainly be those stars observed by using low latitude meridian circle.CCD observation is an important assurance to get more observational points on the whole orbits of the minor planets.

The experiment indicates[1]:Observing minor planets by using CCD have high precision and high efficiency.It can assure the orientating precision, enlarge the length of observational arcs for minor planets and reach high precision of zero point of the star catalogue.

4. SOME CONSIDERATIONS ON THE TREATMENT OF THE DATA

Conjugation of CCD and meridian circle realized the improvement on the observational means and methods.It is the important assurance of raising the measuring precision of the minor planets.Similarly,the improvement of the treating methods of the observational data can also reach the aim of raising precision.

(1).The measurement and correction of the systematic error

CCD receiving system and meridian circle are two different observational means.Their normalization calculation is very important.So it must be solved through experiments.According to the fact already known,number 1—4 minor planets can be selected as the observed celestial bodies.Even though,in the practice of observations on the minor planets,CCD and meridian circle must be simultaneously used for a period of time.Thus the high precision of measuring minor planets by using meridian circle can be assured.

(2).The improvement of the measuring precision of minor planets.

The experiments indicate:In order to raise the measuring precision of the minor planets' locations on the selected moment,it is necessary to increase CCD observational frequency around that moment as many as possible.Multinomial fitting is used for treating the data,and (α, δ) changing as time varies could be got from following function:

$$\alpha = \alpha_{\circ} + at + bt^2 + ct^3 + \ldots\ldots$$
$$\delta = \delta_{\circ} + a't + b't^2 + c't^3 + \ldots\ldots$$

where a,b,c,d,......,a′,b′,c′,d′,......are coefficients to be determined.Such observational methods and data processing can assure the high precision of minor planets′ locations on the selected moment[1].

(3).The improvement of average orbital elements

Since the use of normalization calculation increases the precision of minor planets observed by meridian circle,and enlarges the length of observed arcs of minor planets,more observed values can be got on the enlarged arcs of minor planets′ orbits.Making use of the observed values of high precision,the average orbital elements of minor planets will be improved,and the determination of accurate orbit of minor planets will be realized.Since the theory of minor planets′ movement is not perfect,methods of the experiment are:make full use of the observed values that have high precision on the orbit of minor planets,take an envelope of osculating ellipses for approximations to minor planets′ real orbits,realize the improvement of the minor planets′ average orbital elements.

5. CONCLUDING REMARKS

As the transition from fundamental coordinate to dark stars,as the improvement of observational precision of zero point on the fundamental coordinate,it will have important significance to determine zero point of star catalogue by observing minor planets and have much more affection in practice.The method presented in this paper is a new way to determine zero point with minor planets.

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OBSERVATIONS OF LUNI-SOLAR AND FREE CORE NUTATION

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ABSTRACT. An analysis of the differences between observed nutation angles and the 1980 IAU Nutation Theory shows the existence of currently unmodeled effects. An empirical set of corrections to the 1980 IAU Nutation Theory is presented and compared with current geophysical models. A prograde periodic variation (period $\approx$ 420 days), which may be related to the theoretical free core nutation, is apparently seen.

1. Introduction

The nutational motion of the Earth's axis of figure is modeled by theories based initially on descriptions of the motion of a rigid Earth caused by luni-solar torques (Woolard 1953, Kinoshita 1977, Zhu and Groten 1989, Souchay and Kinoshita 1990). Corrections computed from geophysical theories (Wahr 1981, Molodenskiy and Kramer 1987, Mathews *et al.* 1989, Dehant 1990, Zhu *et al.* 1990) are applied to the rigid Earth models in order to produce non-rigid Earth nutation models (*e. g.* ZMOA-1990 in Herring 1990) describing the actual motion of the Earth more closely.

The International Earth Rotation Service (IERS) Standards (McCarthy 1989) recommends the use of the 1980 IAU Nutation Theory (Seidelmann 1982) based on the Wahr model (Wahr 1981). However, astronomical observations made by using Very Long Basline Interferometry (VLBI) and Lunar Laser Ranging (LLR) of the nutation angles ψ (longitude) and ϵ (obliquity) show discrepancies ($d\psi$ and $d\epsilon$) with the model.

Complicating the analysis of the observations is the possible existence of the free core nutation (FCN), which has sometimes been referred to as the nearly diurnal free wobble. This motion, due to the rotating, elliptical, fluid core, should appear, according to Sasao and Wahr (1981) as a retrograde periodic variation in nutation with a period of about 460 sidereal days.

2. Nutation Observations

The observations of $d\psi$ and $d\epsilon$ from the CALC 7.1 solution of International Radio Interferometric Surveying (IRIS) five-day VLBI data set, as provided by the National Geodetic Survey (NGS) (*NEOS Annual Report for 1989*, p. 3), were used. Nutation observations from the CALC 7.1 solution of the Crustal Dynamics Project (CDP) VLBI data set, as provided by the National Aeronautics and Space Administration (NASA) at Goddard Space Flight Center, were also utilized (*IERS Annual Report for 1989*). NASA made two

VLBI solutions: one with the plate motion fixed and one with the plate motion treated as a "solve for" variable.

Nutation coefficients can also be obtained from LLR observations (Williams *et al.* 1990). These coefficients are not produced for every observation, as is the case with the VLBI data, but are instead produced when a global solution is made. Due to the sparse nature of the LLR data, it is not possible to determine a full set of nutation coefficients. However, the long span of data makes it useful in the attempt at separating the precession term from the main 18.6-year nutation term.

3. Computation of Nutation Coefficients

In step 1, the separate VLBI solutions were used in simultaneous least-squares solutions to determine a bias, rate, and corrections to the primary 18.6-year periodic term in nutation for both $d\psi$ and $d\epsilon$ for each series. In step 2, the observations are adjusted by the corrections found in step 1 and a solution is made for the high-frequency (periods less than nine years) terms only. Final estimates were then obtained by iterating steps 1 and 2 for the long- and short-period terms, including the 420-day term, until convergence was obtained.

When the final nutation coefficients were determined for each of the VLBI series, they were combined with the coefficients provided by LLR (Williams *et al.* 1990). In this combination, the results from only one NASA series (the fixed) were included to avoid overweighting the NASA CDP solution. As can be seen in Tables II, III, and IV, there is no significant difference between the two NASA CDP series. A weighted mean was computed for each of the coefficients, where the weight is the inverse square of the formal error. The results are shown in Tables I, II, and III. The error listed is a combination of the internal errors of the contributors to the mean and the standard error of the weighted mean. There appear to be significant unexplained differences between VLBI and LLR coefficients.

Table I. Correction in bias and slope to the $d\psi$ and $d\epsilon$ found from the analysis of the VLBI and the LLR nutation series. The units of the coefficients are msec. of arc for bias and msec. of arc/century for slope.

$d\psi$	IRIS	NASA (Fixed)	NASA (Free)	LLR	MEAN
Bias	-37.2 ± 2.5	-38.6 ± 3.1	-37.7 ± 3.1		-37.8 ± 2.1
Slope	-274.6 ± 17.6	-295.0 ± 21.3	-288.8 ± 21.2	-260 ± 50	-282.7 ± 14.8
$d\epsilon$					
Bias	-5.1 ± 0.9	-4.6 ± 1.4	-5.2 ± 1.4		-5.0 ± 0.8
Slope	-11.1 ± 6.5	-6.8 ± 9.9	-11.0 ± 9.8		-9.8 ± 5.8

Table II. Free Core Nutation model derived from IRIS and NASA VLBI observations. The coefficients are in the sense of a $\sin\theta$ + b $\cos\theta$ where $\theta = 2\pi(\mathrm{MJD}-51544.5)/420.0$. The units of the coefficient are msec. of arc.

	IRIS	NASA (Fixed)	NASA(Free)	MEAN
$d\psi$				
a	0.859 ± 0.085	1.012 ± 0.084	1.013 ± 0.084	0.936 ± 0.097
b	-0.122 ± 0.086	-0.214 ± 0.085	-0.223 ± 0.085	-0.169 ± 0.076
$d\epsilon$				
a	0.160 ± 0.033	0.201 ± 0.031	0.200 ± 0.031	0.182 ± 0.030
b	0.281 ± 0.034	0.227 ± 0.031	0.227 ± 0.031	0.252 ± 0.035

Table III. Corrections to the nutation terms $d\psi$ and $d\epsilon$ found from the analysis of the VLBI and the LLR solutions. The units of the coefficients are msec. of arc.

CORRECTIONS IN LONGITUDE

MULTIPLE OF							IRIS	NASA (Fixed)	NASA (Free)	LLR	MEAN
L	L'	F	D	Ω	Coeff.	PERIOD (DAYS)	(0.001")	(0.001")	(0.001")	(0.001")	(0.001")
0	0	0	0	1	sin	6798.4	-4.41 ± 0.55	-4.67 ± 0.58	-4.43 ± 0.58	-8.5 ± 4.0	-4.57 ± 0.49
					cos		4.53 ± 0.47	5.88 ± 0.60	5.74 ± 0.60	3.8 ± 4.8	5.04 ± 0.60
0	0	2	-2	2	sin	182.6	1.500 ± 0.084	1.514 ± 0.079	1.510 ± 0.079		1.507 ± 0.058
					cos		-1.241 ± 0.085	-1.240 ± 0.082	-1.241 ± 0.082		-1.240 ± 0.059
0	0	2	0	2	sin	13.7	-0.298 ± 0.145	-0.626 ± 0.153	-0.626 ± 0.154		-0.453 ± 0.195
					cos		-0.211 ± 0.145	-0.122 ± 0.154	-0.101 ± 0.154		-0.169 ± 0.115
0	0	0	0	2	sin	3399.2	0.059	0.062	0.047		0.060
					cos		-0.068	-0.085	-0.054		-0.076
0	1	0	0	0	sin	365.3	5.079 ± 0.087	5.310 ± 0.086	5.306 ± 0.086	5.0 ± 2.0	5.196 ± 0.102
					cos		0.987 ± 0.085	1.334 ± 0.084	1.321 ± 0.084		1.163 ± 0.183
1	0	0	0	0	sin	27.6	-0.135 ± 0.084	-0.119 ± 0.079	-0.125 ± 0.079		-0.127 ± 0.058
					cos		-0.105 ± 0.085	-0.143 ± 0.082	-0.140 ± 0.082		-0.125 ± 0.062
0	1	2	-2	2	sin	121.7	0.100 ± 0.084	-0.122 ± 0.080	-0.115 ± 0.081		-0.016 ± 0.125
					cos		-0.036 ± 0.085	0.046 ± 0.080	0.042 ± 0.080		0.007 ± 0.071
0	0	2	0	1	sin	13.6	-0.534 ± 0.143	-0.359 ± 0.156	-0.360 ± 0.156		-0.454 ± 0.137
					cos		-0.144 ± 0.146	0.021 ± 0.152	-0.005 ± 0.152		-0.065 ± 0.134
1	0	2	0	2	sin	9.1	-0.317 ± 0.084	-0.287 ± 0.080	-0.290 ± 0.080		-0.301 ± 0.060
					cos		-0.217 ± 0.084	0.014 ± 0.080	-0.003 ± 0.080		-0.096 ± 0.129

CORRECTIONS IN OBLIQUITY

MULTIPLE OF							IRIS	NASA (Fixed)	NASA (Free)	LLR	MEAN
L	L'	F	D	Ω	Coeff.	PERIOD (DAYS)	(0.001")	(0.001")	(0.001")	(0.001")	(0.001")
0	0	0	0	1	sin	6798.4	1.97 ± 0.20	2.23 ± 0.27	2.12 ± 0.27	1.2 ± 1.9	2.06 ± 0.19
					cos		2.75 ± 0.17	2.75 ± 0.28	2.83 ± 0.28	0.4 ± 1.9	2.74 ± 0.19
0	0	2	-2	2	sin	182.6	-0.419 ± 0.033	-0.446 ± 0.029	-0.443 ± 0.029		-0.434 ± 0.026
					cos		-0.497 ± 0.033	-0.513 ± 0.030	-0.511 ± 0.030		-0.506 ± 0.024
0	0	2	0	2	sin	13.7	-0.153 ± 0.057	-0.187 ± 0.056	-0.181 ± 0.056		-0.170 ± 0.043
					cos		0.207 ± 0.057	0.240 ± 0.056	0.234 ± 0.056		0.224 ± 0.043
0	0	0	0	2	sin	3399.2	-0.012	-0.015	-0.014		-0.013
					cos		-0.036	-0.036	-0.037		-0.036
0	1	0	0	0	sin	365.3	-0.263 ± 0.034	-0.252 ± 0.031	-0.253 ± 0.032		-0.257 ± 0.024
					cos		1.988 ± 0.033	1.993 ± 0.031	1.994 ± 0.031	1.6 ± 1.1	1.990 ± 0.023
1	0	0	0	0	sin	27.6	0.039 ± 0.033	0.021 ± 0.029	0.022 ± 0.029		0.029 ± 0.024
					cos		-0.034 ± 0.033	-0.068 ± 0.030	-0.069 ± 0.030		-0.053 ± 0.028
0	1	2	-2	2	sin	121.7	-0.102 ± 0.033	-0.023 ± 0.030	-0.025 ± 0.030		-0.059 ± 0.045
					cos		0.081 ± 0.033	0.073 ± 0.029	0.075 ± 0.029		0.076 ± 0.022
0	0	2	0	1	sin	13.6	-0.036 ± 0.056	0.058 ± 0.057	0.054 ± 0.057		0.010 ± 0.062
					cos		0.121 ± 0.057	0.147 ± 0.056	0.149 ± 0.056		0.134 ± 0.042
1	0	2	0	2	sin	9.1	-0.035 ± 0.033	0.005 ± 0.029	0.006 ± 0.029		-0.012 ± 0.029
					cos		0.067 ± 0.033	0.029 ± 0.029	0.032 ± 0.029		0.046 ± 0.029

4. Comparisons

Table IV compares the observed corrected constants with theoretical rigid and non-rigid Earth values of various authors. Recall that the nutation angles can be represented by

$$\psi = \psi_r \sin\theta + \psi_i \cos\theta,$$
$$\epsilon = \epsilon_r \cos\theta + \epsilon_i \sin\theta,$$

where ψ_r and ϵ_r correspond to the estimates shown in Table IV. Here, θ represents any of the principal angular nutation arguments.

Table IV. Comparison of theoretical models with the observed values. Note that the models of Kinoshita and Kinoshita and Souchay are for a rigid Earth. The column labelled IAU lists the 1980 IAU Nutation Theory coefficients. The units are sec. of arc.

ψ_r

Period (days)	Kinoshita	Kinoshita & Souchay	IAU	This Paper	ZMOA 1990	Molodenskiy & Kramer	Dehant	Zhu & Groten
9.1	-0.0296	-0.0296	-0.0301	-0.0304	-0.0301	-0.0302	-	-0.0302
13.6	-0.0378	-0.0379	-0.0386	-0.0391	-0.0387	-	-	-0.0389
13.7	-0.2215	-0.2216	-0.2274	-0.2279	-0.2276	-0.2275	-0.2260	-0.2282
27.6	0.0678	0.0678	0.0712	0.0711	0.0711	0.0711	-	0.0712
121.7	-0.0500	-0.0498	-0.0517	-0.0517	-0.0517	-0.0517	-	-0.0518
182.6	-1.2775	-1.2732	-1.3187	-1.3172	-1.3172	-1.3178	-1.3138	-1.3172
365.3	0.1255	0.1255	0.1426	0.1478	0.1476	0.1472	0.1464	0.1474
3399.2	0.2079	0.2090	0.2062	0.2063	0.2075	0.2062	-	0.2074
6798.4	-17.2815	-17.2807	-17.1996	-17.2042	-17.2053	-17.2058	-17.2097	-17.2062

ϵ_r

Period (days)	Kinoshita	Kinoshita & Souchay	IAU	This Paper	ZMOA 1990	Molodenskiy & Kramer	Dehant	Zhu & Groten
9.1	0.0126	0.0126	0.0129	0.0129	0.0129	0.0129	-	0.0129
13.6	0.0194	0.0194	0.0200	0.0201	0.0201	-	-	0.0201
13.7	0.0949	0.0950	0.0977	0.0979	0.0978	0.0978	0.0973	0.0981
27.6	-0.0010	-0.0010	-0.0007	-0.0008	-0.0007	-0.0007	-	-0.0007
121.7	0.0216	0.0216	0.0224	0.0225	0.0224	0.0224	-	0.0225
182.6	0.5534	0.5534	0.5736	0.5731	0.5731	0.5734	0.5716	0.5732
365.3	-0.0001	-0.0001	0.0054	0.0074	0.0073	0.0072	0.0069	0.0072
3399.2	-0.0902	-0.0903	-0.0895	-0.0895	-0.0898	-0.0895	-	-0.0898
6798.4	9.2276	9.2286	9.2025	9.2052	9.2051	9.2045	9.2066	9.2053

5. Discussion

The large unexplained differences between the coefficients derived from VLBI and those derived from LLR point to the possibility of systematic errors in both techniques. Ignoring possible systematic errors, then, it would appear that VLBI observations of the major components of the variation of $\psi \sin\epsilon_0$ and ϵ are precise to better than ± 0.1 msec. of arc. Combining VLBI and LLR results shows that the accuracy of the derived nutation coefficients is better than ± 1.0 msec. of arc.

The 420-day term is included because it is statistically significant in the analysis of the residuals. One might suspect that this is evidence for the FCN, which would be expected to contribute a variation in the observations with a period of about 460 sidereal days. The observed motion is mainly prograde, contrary to the expected motion (Sasao and Wahr 1981) with the prograde amplitude being 0.33($\pm$0.03) msec. of arc and the retrograde being 0.08($\pm$0.03) msec. of arc. Herring (1987) mentions the FCN term but finds that its amplitude is too small to be included in his solution. Zhu, *et al.* (1990) finds a 433.2-day term which they call free core nutation. Additional study is necessary to explain the difference in observed direction of rotation in the FCN term between theory and observation.

6. Conclusion

The solution for luni-solar nutation presented here combines VLBI and LLR observations. The rms of the fit with respect to the observations in $\psi\sin\epsilon_0$ and ϵ are at the level of about ±0.6 milliseconds of arc. Accuracy of the derived constants, as well as the agreement between the fit to the observations and theory, seems to be at the level of about ±1.0 millisecond of arc in $\psi\sin\epsilon_0$ and ϵ.

Using only data from MJD 45700 (January 1984), a prograde "free core nutation" term is found with a period of 420 days and amplitude of 0.33 msec. of arc.

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THE EXTRAGALACTIC REFERENCE FRAME

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ABSTRACT. A cooperative program has been established to produce a reference frame of at least 400 suitable extragalactic sources. This is an extension of collaborative efforts (Johnston *et al.*, Russell *et al.*, Ma *et al.*, and Reynolds *et al.*) reported at this meeting. In Phase One, a catalog of radio source positions will be constructed using original VLBI observations already obtained by NASA, NGS, NRL, JPL and USNO. Observations will be re-reduced in a consistent system to avoid some of the more serious problems associated with the formation of compilation catalogs. In Phase Two, the USNO will maintain the system, including the monitoring of source structure and/or variation. During both phases cooperation among the various agencies will continue. A list of sources will be made available to optical observers so that the frame will have benchmarks in both the optical and radio regimes.

1. Introduction

The IAU Working Group on the Radio/Optical Reference Frame was established in recognition of the importance of an extragalactic radio reference frame to the improvement of the optical one. In Argue *et al.* (1984), the working group published a list of 233 sources with positions compiled from several published radio catalogs as a starting point. The importance of the completion and maintenance of the Extragalactic Reference System Catalog was emphasized at a recent workshop on the Celestial Reference System sponsored by the National Earth Orientation Service and held at the U. S. Naval Observatory (USNO). While all agreed on the importance of an on-going reference frame program and all had made observations which were useful for it, none of the agencies represented had long term plans for maintaining the catalog for astronomy.

The current Navy program (Johnston, *et al.*, 1988), is the only active program whose goal is to produce an extragalactic radio frame suitable for ties to the other astronomical reference frames, although others have announced plans toward the same goal. The Navy program incorporates some data which had been obtained previously for other purposes (*e.g.* Crustal Dynamics Program (CDP), IRIS) and new data obtained solely for the reference frame project, all with the cooperation of individuals from several agencies (NRL, USNO, NASA/CDP, NASA/JPL, NOAA/NOS/NGS) and countries (USA, Australia, Germany, Japan, South Africa). Optical positions of the sources having optical counterparts are also being measured as part of the current program. In accordance with the goal of providing a radio/optical tie-in, the RA zero point for the extragalactic radio reference frame is fixed using positions of optical counterparts of 28 quasars in the system of the FK5.

The aim of the Navy program is to establish a reference system of at least 400 extragalactic sources (about one per 100 square degrees), which are compact, have a flat spectrum in the radio, display optical emission, and are evenly distributed about the sky. It began in 1987 with a 5-yr plan. As of September, 1989, 347 sources have been observed in VLBI geodetic or astrometric experiments. The maintenance of the large data bases for this work have been done as a cooperative effort with the NASA Crustal Dynamics Project, scheduled to end in December 1991.

The radio positions published so far are in Ma, *et al.* (1990, 182 sources) and Russell, *et al.* (1991a, 55 more; 1991b, 39 more for a total of 276 published sources). At a recent workshop on the Celestial Reference System sponsored by the National Earth Orientation Service and held at the Naval Observatory participants concluded that a standard VLBI reference system should be created, maintained and related to other systems. While other catalogs of radio source positions exist, none are comprehensive and most were created for other purposes, solving for various other parameters in their derivations. The proposed catalog would fill this need for the entire astronomical community.

2. Objectives

The object of this program is to assume responsibility for the production and maintenance of a standard celestial reference frame defined by VLBI observations of extragalactic radio sources. This project will be a joint USNO/NRL project with extensive cooperation among other agencies (NASA/GSFC, NGS, NASA/JPL, CSIRO, and Hamberger Sternwarte). At the beginning of the project much of the work will be performed at NRL but transition to operation at USNO will begin immediately.

Complete transition to an operational USNO program will occur at the beginning of 1993. The project can be considered to be divided into two separate phases.

Phase One is the initialization of the radio program at USNO. This will consist of establishing a data base with all available dual frequency Mark III VLBI data at USNO. This will be the Crustal Dynamics Data Base with small modifications for precise astrometric analysis or something very similar to the CDP data base. The software necessary to reduce this data currently used in the Navy program will be reviewed with the aim of establishing a uniform set of software for the program. There are two objectives in Phase One. First, an initial catalog of source candidates and source positions from the present data base will be chosen. This work will supercede the Argue *et al.* list and give a preliminary reference frame. Second, a first catalog of radio source positions of all 400 sources and alternates will be produced.

This First Radio Reference Frame Catalog will be reduced in a consistent system by NRL/USNO personnel using all the collected data in the data base to avoid some of the problems associated with previous compilation catalogs. In Phase One of the program, the universal radio data base will be established at USNO and the software algorithms necessary to produce excellent astrometric results from this data base will be verified. Observations will be ongoing. Phase One is expected to end in 1992.

In Phase Two, following the initial catalog, the USNO will make VLBI observations to densify and maintain the system. The monitoring of source structure and/or variation will be carried out by NRL/USNO. Most of these observations would be made in conjunction with routine observations made for the determination of Earth orientation. Use of the VLBA by NRL/USNO would be anticipated to investigate possible source structure problems. Observations required to improve celestial reference frame ties would continue to be made by USNO/NRL. These might include, for example, observations of radio stars in radio and optical wavelengths, optical observations of quasars, radio observations of solar system objects, millisecond pulsar observations, etc. During both phases, cooperation among the various agencies would continue.

3. Specifications

The VLBI reference frame will consist of the positions of approximately 400 sources distributed evenly in the sky (Figure 1). The accuracy of these positions exceed the level of ±1 millisecond of arc. The frame will be updated annually with announcements of problems which may be caused by varying source structure or brightness. Complete revisions of the system will occur approximately every five years. The zero point of the right ascension coordinate will be specified by the IAU working group on origins. The pole is to be defined by the IERS pole or an alternate proposed by the IAU Working Group on Reference Systems. IERS standards will be used in the solutions unless specified otherwise. If other values for critical constants or procedures are used other than the IERS values, they will be described in full detail.

4. Conclusions

The proposed catalog could serve as the radio source catalog for use as the extragalactic reference frame. Phase one, to be completed in 1992, will produce a catalog of 400 sources all reduced in the same system. Phase Two will continue this work with periodic updates.

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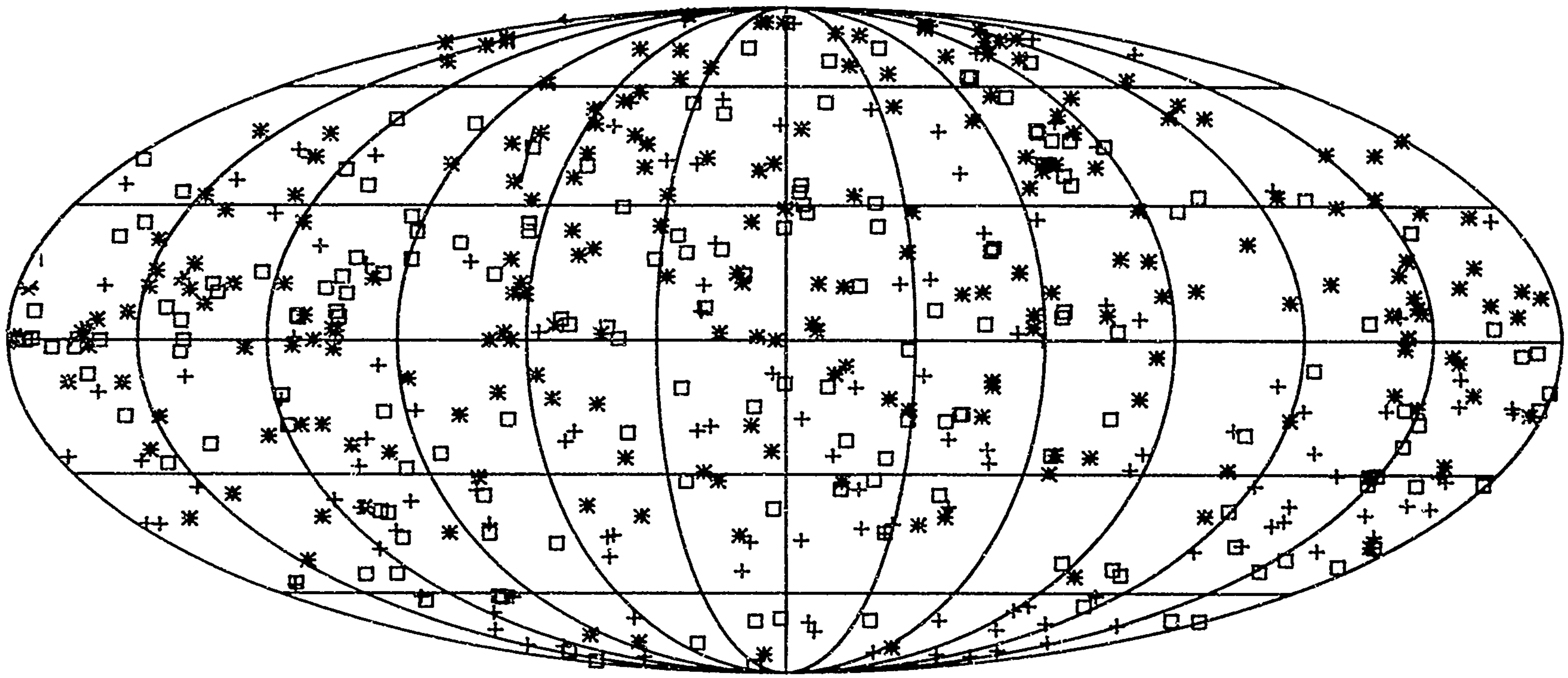

Figure 1. Distribution of sources which are candidates for the radio reference frame. The symbols represent the number of Mark III VLBI observations, small squares having the most, plus signs fewer and the asterisks having the least.

A Source of Systematic Error, $\Delta\delta_\alpha$, in Absolute Catalogs Compiled from Meridian Circle Observations

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Abstract

Automated photoelectric meridian circles are able nowadays to have the full set of graduation errors of the declination circle determined within a few days. Thus, the modern meridian circles can be monitored, and the annual and secular changes of the graduations can be easily detected to provide the graduation corrections at any date. In order to remove systematic declination errors in the form $\Delta\delta_\alpha$ from absolute catalogs, these continuous changes should be taken into account.

Background

The **meridian circle** has been one of the main instruments on the ground for realizing the **absolute stellar reference frame** tieing it to the dynamical reference frame defined by the planetary ephemerides. In meridian circle work, the attitude of the telescope (the so-called instrumental constants such as nadir, azimuth, collimation, flexure, etc. of the telescope) is calibrated frequently through the night during an observational tour, in a consistent fashion. Therefore, the attitude of the telescope is considered as known at any observational instant.

However, the calibration of the **graduations** of the **declination circle**, with which the declination measurement is made, has been handled quite differently. Since a full determination of the graduation errors required lengthy effort for the classical meridian circle, the determination has usually been carried out only once in several years in each observing program or at best, once a year. Thus, if the **diurnal, annual,** and **secular changes** of the graduations went undetected during a long-period observing program, then, these changes would have caused systematic errors in the declination system of the compiled absolute catalog.

In the fully automated **photoelectric meridian circle**, the entire set of graduations can be calibrated within a few days (Einicke *et al.* 1971, and Miyamoto and Suzuki 1985). In the case of the **Tokyo Photoelectric Meridian Circle** (hereafter referred to as **Tokyo PMC**) with a declination circle of 3600 divisions, Miyamoto *et al.* (1986) have shown that the amplitude of the annual change in the "**circle error**" amounts to about 0.″05. (Fig.1). Since this amount of change would cause, in compiling a modern **absolute catalog** of stellar positions, a **systematic declination error** $\Delta\delta_\alpha$ depending on the season, that is, on right ascension, the annual change of all the graduations is approximated by the use of sinusoidal curves and is corrected the annually published observational catalogs of the Tokyo PMC.

Since 1985, all of the 3600 divisions of the Tokyo PMC have been calibrated with a frequency of one or two times a month during our regular observing program. Recently, we have found that the annual change of the graduations is clearly different in its amplitude and phase from year to year, that is, the set of graduations is **changing secularly** as well(Fig.2). Since the meridian observation of stars and members of the solar system in an observing program is usually carried out over several years (the Washington series, for example), not only the annual change, but also the secular change should be taken into account in compiling an absolute catalog of stellar positions.

In compiling an absolute catalog, we need daytime observations of the sun, planets, and bright stars. Usually, the day- and night- observations have presupposed the constancy of the graduations within the 24hour day, whatever the temperature difference may be. But, it is not unusual to find temperature differences larger than 10°C within a 24hour observing period. Thus, the above finding implies further the possibility of a **diurnal change** of the graduation errors as well.

Based on the fact that the dominant contributions of the **diameter errors** in the Tokyo PMC are always limited to the Fourier components with harmonics lower than 10^{th}, we have applied a simplified method of graduation measurement to detect the diurnal change of only these dominant components. The measurement, whose cycle-time is 15 minutes, was carried out during a 24hour period in Feb. 1986(Fig.3). We have proved that the diurnal change of the graduations is fortunately negligible (Miyamoto *et al.* 1986).

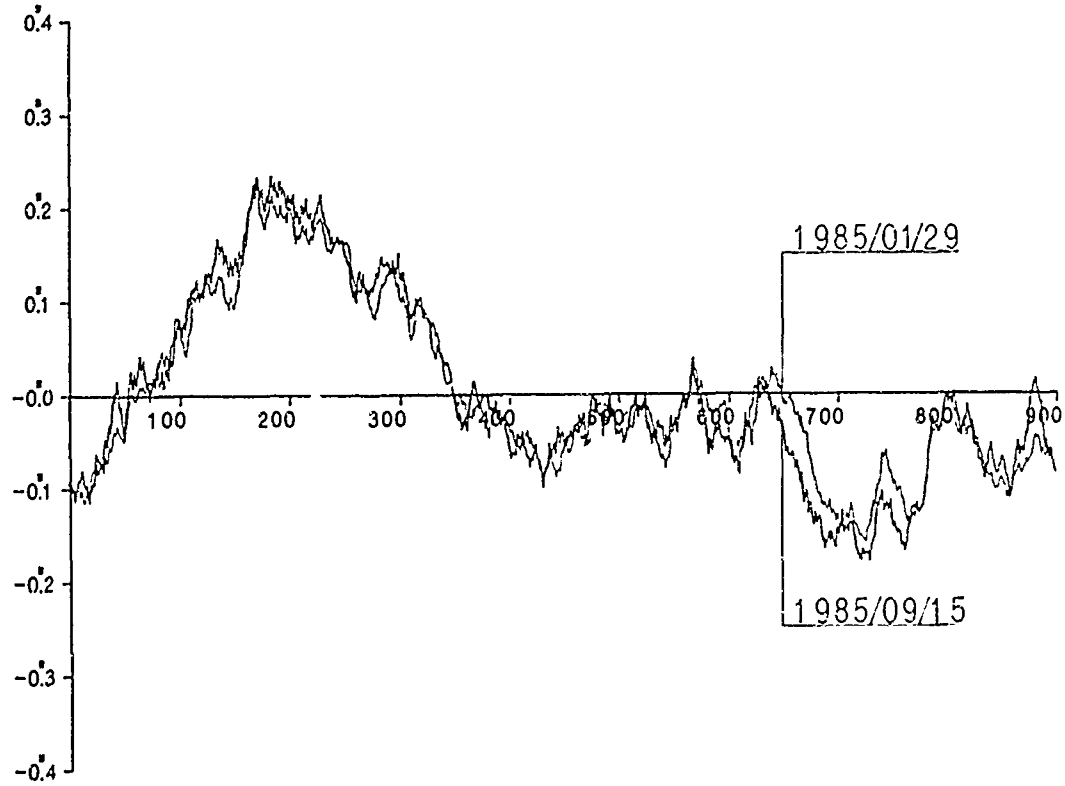

Figure 1. A Seasonal Change of the Circle Errors

Two sets of circle corrections (with opposite sign to the circle errors) derived from the diameter corrections in Fig.1 are illustrated. These corrections to be added to 900 circle readings respectively are given as a function of the numbering ($n = 1, 2, 3, \cdots, 900$) of the circle readings. The figure shows the circle errors dependent on the season.

Note : Averaging each set of two readings of 3600 divisions separated by 180°, we have 1800 diameter readings. Averaging again each set of four readings of 3600 divisions separated by 90°, we have 900 circle readings. In routine observations, only these 900 circle readings are used,which are distinguished by the numbering $n = 1, 2, 3, \cdots, 900$. Thus, we need 900 circle corrections at any observational instant (J.D.).

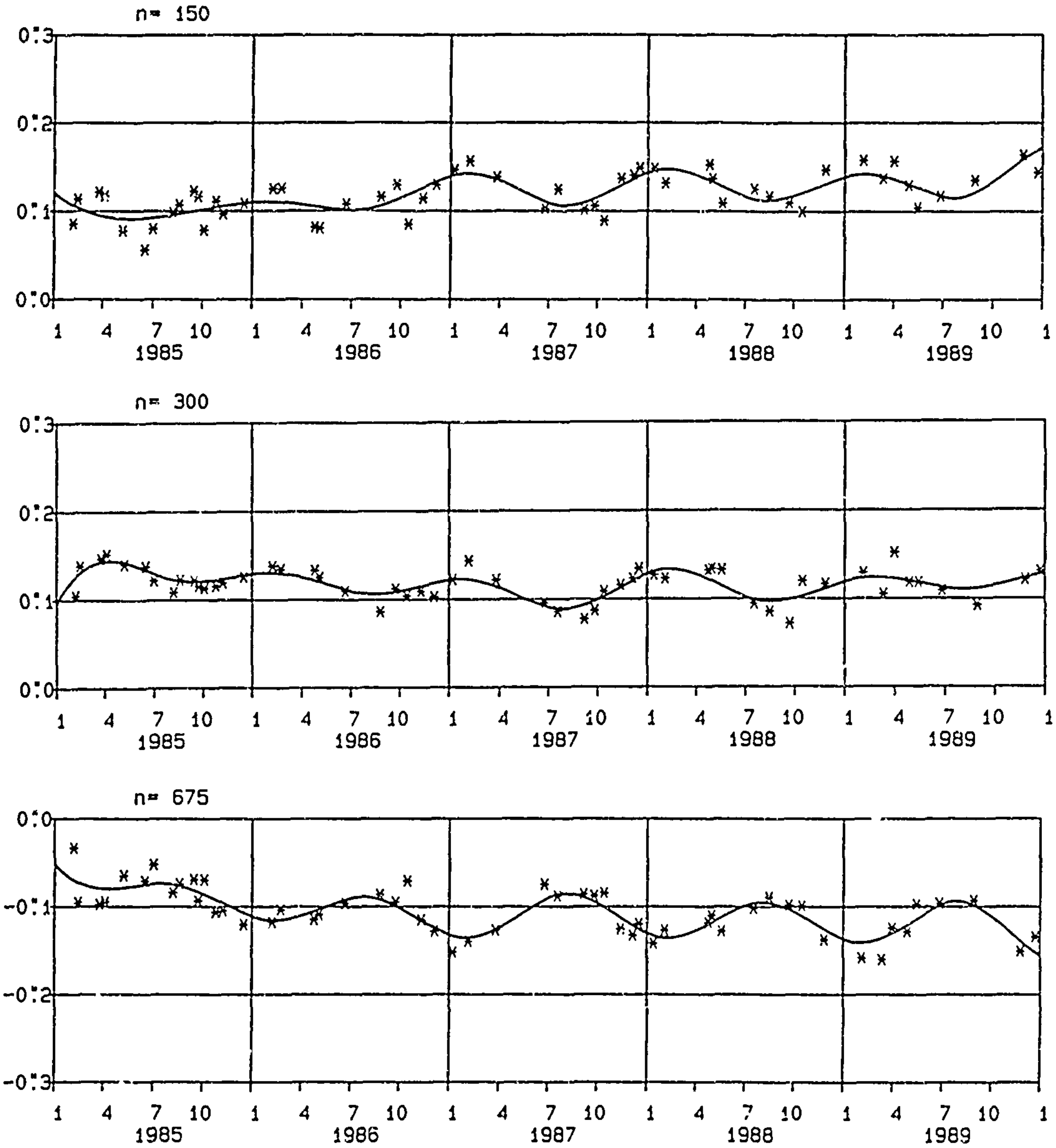

Figure 2. A Secular Change of the Circle Errors

As is shown in Fig.1, all the circle errors (in total 900) change annually in a characteristic sinusoidal fashion. But, the amplitude and phase of the sinusoidal curve is different from year to year. That is, the circle errors (and therefore, the graduations) are changing secularly. The figure shows the secular change superposed on the annual one of the circle errors for the n^{th} circle readings indicated. In this figure, months and years are indicated along the abscissa. The symbol $*$ indicates measurements, and a semiannually overlapped smoothing is applied. These continuous changes of the circle errors will be corrected in the Tokyo PMC Catalogs.

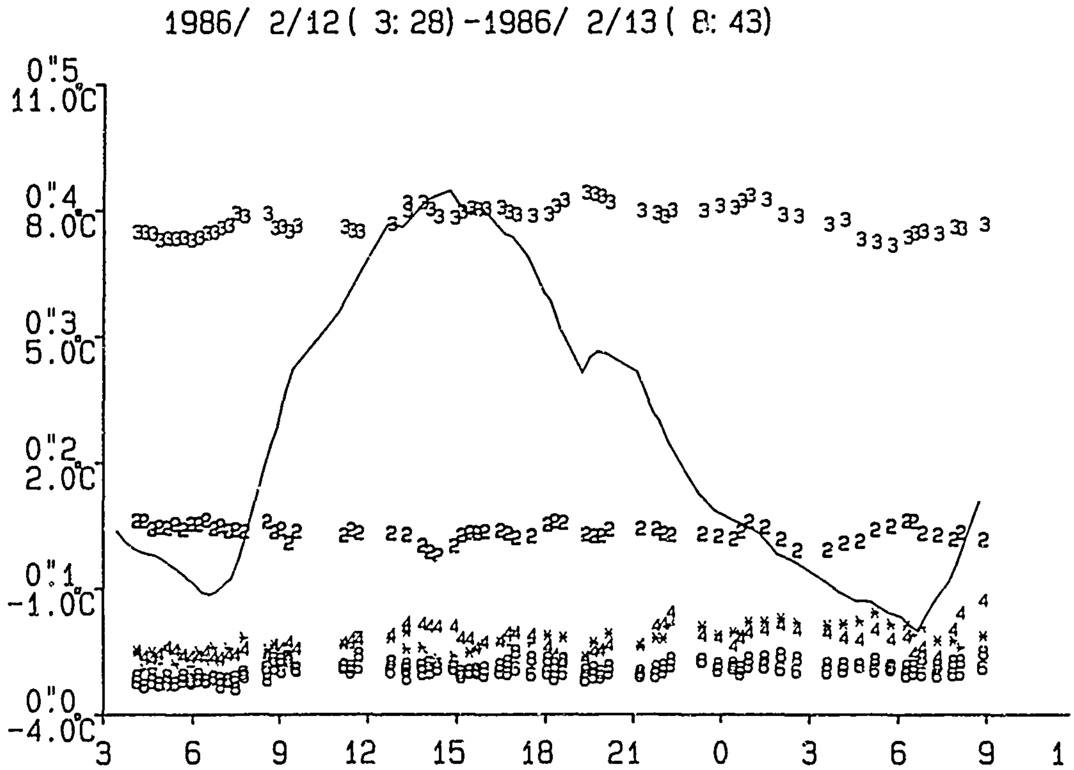

Figure 3. A diurnal change of the diameter errors obtained on 12–13 February 1986
The ordinate denotes the amplitude (in arcsec) of each harmonic component of the diameter errors and the temperature (°C), and the abscissa the hours in Japan Standard Time. The components of respective harmonics are plotted by the corresponding harmonic numbers every 15 minutes (* indicates the 10^{th} harmonic component). This experiment was carried out, with the dome-slit opened on a clear day. One finds no obvious correlation, within the measuring error, of the diameter errors with the temperature.
Note : Only even harmonic Fourier components of the diameter errors contribute to the circle errors.

Nowadays, we can determine the graduation errors within an accuracy of ±0."01 on any date. The annual and secular changes of the graduations will be taken into account in the Tokyo PMC Absolute Catalog.

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EVIDENCE OF SYSTEMATIC ERRORS IN FK5

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The primary purpose of the FK5 is to provide an absolute optical reference frame against which to measure the positions and motions of other objects. One of the main tasks of meridian circles is to check and improve that reference frame by repeated observation of the fundamental stars. The meridian circle at Bordeaux, France and the Carlsberg Automatic Meridian Circle (CAMC) at La Palma are carrying out programmes of observation of FK5 stars partly with this aim in view.

The positions from which the systematic differences are derived are the means of at least 10, but normally about 30 (Bordeaux) and 50 (CAMC), independent observations of each FK5 star made in the period 1984–87. The systematic differences are displayed in pairs of 3-dimensional plots, Bordeaux–FK5 and CAMC–FK5. The similarities of the corresponding pairs of plots is striking, particularly in declination. This implies that systematic errors exist in the FK5 which reach about 0.″1 in places at the epoch 1986.0.

A discussion of the results of other contemporary meridian circles and astrolabes at different latitudes is required in order to arrive at definitive corrections to the FK5. Perhaps this should be done before fitting the zero points of the Hipparcos catalogue to the FK5 system.

CLOSE APPROACH ASTROMETRY
A Method for Improving Reference Systems

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ABSTRACT

For improved establishment of the dynamical reference frame and tying it to the stellar reference frame, a method to observe close approach (CA) events between satellite and satellite and/or satellite and star is proposed here. The accuracy of measurements of angular distance is 0".02 - 0".03 for a single data point. We apply this technique fairly successfully to the CA events of the Galilean satellites and a preliminary orbital longitude correction to the E2X3 constants of J4 is obtained from eight CA observations ranging from 1987 to 1989.

1. Introduction

For high-precision observations of planetary positions, two technical improvements were recently proposed. The first is to observe the positions of the satellites, rather than planet itself (e.g., Pascu and Schmidt, 1990). This is because diameters of satellites are small and thus their phase effects are negligible. The second is to use intersatellite observations; (e.g., Taylor and Shen, 1988); this has the advantage that possible systematic errors of the adopted catalogue for plate solution can be eliminated.

If this line of improvement is pursued for the purpose of establishing the dynamical reference frame and tying it to the stellar reference frame, the following two step procedures will apply: high-precision intersatellite observations and satellite-to-star observations. Once a high-precision ephemeris of the satellite is available, determination of the right ascension and declination of stars based on the dynamical reference frame is fairly easy through measurements of CA events between the stars and the satellite. Therefore we restrict ourselves to high-precision measurements of satellites positions in this paper.

It is well known that, due to the strong correlation of turbulent motions of adjacent air masses, an apparent angular distance of two close celestial bodies can be determined much more accurately than that between telescope's cross threads and a single object. Lindegren (1980) predicts that 0".03-0".04 accuracy is achieved for a relative distance of 30"-40" with a time integration of 20 sec. Our experience in double star observations gives a little better accuracy than Lindegren's theory. Han (1989) also reports a few times better accuracy than Lindegren's formula.

In this paper this advantage of short distance measurements was applied to the observations of the CA events of the Galilean satellites and the results were compared with the Sampson-Lieske theory. From the observations of the events including J4, a preliminary orbital longitude correction was determined. As far as we know, there seems to be no such applications of this technique yet to practical observations of satellites, though the similar ideas have been proposed (e.g., Kammeyer et al., 1990).

2 Prediction of close approach events

For observations of any CA events, prediction calculations are essential. The CA events of the Galilean satellites were calculated by M. Soma using GALSAP program provided by J.H. Lieske and it is found that the CAs of less than 30" take place as many as 60-100 times every year at an observing site. This is due to the ecliptic plane being close to the orbital planes of the Galilean satellites.

3 Observations and data reduction

A telescope of long focal length is essential to achieve high accuracy measurements. Observations were made at the Cassegrain focus (f=16.5m) of a 36-inch reflector (Dodaira station, Japan) with a CCD-type TV camera (field of view: 110"x83") and the images were recorded in a SONY U-matic (2/3 inch) video recorder, with time signals synchronized to UTC up to about 10 microsec. The play-backed image signals were time-integrated on a frame memory as digital data of pixel size of 14 micron square which corresponds to 0".16 square on the sky. In order to determine scale value, we observed the diurnal motion of Polaris by stopping the sidereal drive of the telescope and compared the measured lengths of Polaris' arc with the daily positions of the star given in the Japanese Ephemeris. We also sometimes observed a double star Sigma CAM 1694 to check Affin geometry of pixel spacing of the CCD chip used. A Gaussian profile with a quadratic- polynomial background was fitted to the marginal brightness distribution of star images. After ordinary flat- fielding, centroiding accuracy of 1/10 pixels is easily realized. Typical integration time was 30 sec (900 frames) for the satellites and 20 sec for Polaris.

Twenty two CAs of less than about 60" were observed from October 1987 through March 1989. Observation time of an event ranges from 1 hr to 3.5 hr depending on the circumstance of each CA, nearly centered at the closest approach time in most cases. From these data several frames of 30 sec integration were obtained at the interval of 10-15 min.

The relative distance d of two close satellites 1 and 2 can be written, with enough accuracy, as

$$d = (q^2 + p^2 \cos^2 \delta_1 - p^2 q \sin \delta_1 \cos \delta_1)^{1/2}, \tag{1}$$

where $p = \alpha_2 - \alpha_1$, and $q = \delta_2 - \delta_1$. Then the difference between observed and calculated distance Δd is expressed using the errors in α and δ as follows:

$$\Delta d = p/d \cos^2 \delta_1 \Delta p + q/d \Delta q, \tag{2}$$

where $\Delta p = \Delta\alpha_2 - \Delta\alpha_1$, and $\Delta q = \Delta\delta_2 - \Delta\delta_1$. Since $\Delta\alpha$ and $\Delta\delta$ in equation (2) can be connected through GALSAP program to the corrections to the Lieske's orbital parameters ϵ_i and β_i (Lieske, 1980), we adopt equation (2) as an observation equation for orbital improvement.

If we wish to compare observations with theory at the error level of say 0".005, various small corrections to the observed distance are necessary. Among them the effects due to differential refraction are most important. Since refraction is larger for shorter wave length, the observations without filter or with a broad-band filter cause stellar images to elongate in the altitude direction, especially for large zenith distances; this is called the atmospheric prism effect. This effect is troublesome because the surface brightness distribution in the elongated images depends on both the wave-length sensitivity of the detector system used and color of the object observed. However, our model calculations showed that, for zenith distance of less than 75° and relative distance of less than a few arcmin in the altitude direction, correction of monochromatic differential refraction is sufficient as far as the error level of 0".005 is allowed.

4 Preliminary longitude correction

An example of the time variation of $\Delta d (= d_{obs} - d_{cal})$ is given in Fig.1. In actual solutions of equation (2), the correction to scale value (f) had to be incorporated as a form of $d_{cal} \Delta f / f$. This is probably because Jupiter was in the southern sky whereas Polaris in the northern sky and thus the atmospheric effect on them was different.

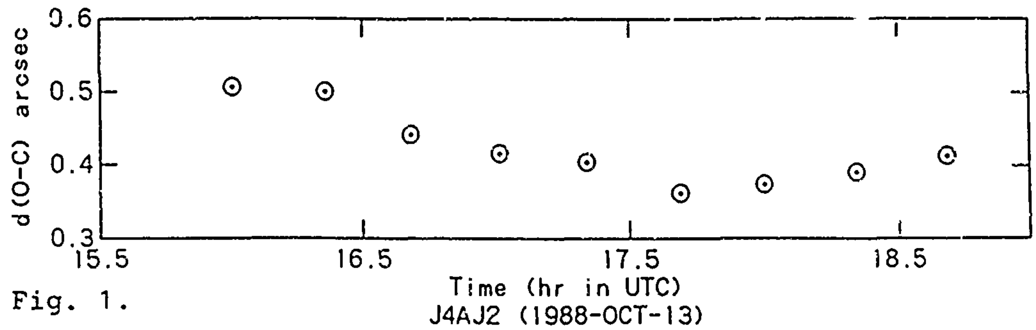

Fig. 1.

For a preliminary orbital analysis, we assume that only the mean orbital longitudes (β_1, β_2 and β_4) of the satellites need improvement. $\partial d / \partial \beta$ is largest at the inferior conjunction of the satellites with Jupiter and zero near elongation. In case of the CA events involving J4, the relevant inner satellite tends to be near elongation. This means that unless the longitude errors of the satellites other than J4 are unusually large (this is actually the case for the Galilean satellites), $\partial d / \partial \beta_4 \Delta \beta_4$ is much larger than other terms in equation (2). Actually in most of our observations involving J4, $\partial d / \partial \beta_4$ is proved to be 10 to a few 10 times larger than other derivatives. Moreover, in the mutual events, the phenomena involving J4 are least observed ones (e.g., Aksnes and Franklin, 1976). Therefore we have tried to obtain a preliminary correction to the E2X3 constants of J4 using eight J4 CA events whose partner is J1 or J2. We have neglected $\Delta \beta_1$ or $\Delta \beta_2$ because of the reason mentioned above. Since $\Delta \beta_4$ and $\Delta f / f$ were not correlated each other during our observation period because of the Earth being far from the orbital planes of the satellites, we c[illegible]d solve $\Delta \beta_4$ from each single event. Then we have $-0''.027 \pm 0''.021$ for the averaged $\Delta \beta_4$. An overall solution is now being undertaken by using all the observed events and by taking account of other errors of orbital elements.

Finally we emphasize that CA events will have wide applications in Saturnian and Uranian satellites, asteroid-asteroid, asteroid-star, satellite-star, and so on.

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PRECESSION THEORY USING THE INVARIABLE PLANE OF THE SOLAR SYSTEM

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ABSTRACT. Standard precession theory builds up the precession matrix $\mathbf{P}$, which rotates coordinates from the mean equator and equinox of epoch to the mean equator and equinox of date, by a sequence of three elementary rotations by the accumulated Euler angles ζ_A, θ_A, and z_A: $\mathbf{P} = \mathrm{R}_3(-z_A)\,\mathrm{R}_2(\theta_A)\,\mathrm{R}_3(-\zeta_A)$. This scheme works well provided both the epoch and the date are within a few centuries of J2000. For long-term applications, the alternative formulation using the accumulated luni-solar and planetary precession, $\mathbf{P} = \mathrm{R}_3(\chi_A)\,\mathrm{R}_1(-\omega_A)\,\mathrm{R}_3(-\psi_A)\,\mathrm{R}_1(\varepsilon)$, is more stable.

Yet another formulation for $\mathbf{P}$ is possible, using the invariable plane of the Solar System as an intermediate plane: $\mathbf{P} = \mathrm{R}_3(-L)\,\mathrm{R}_1(-I)\,\mathrm{R}_3(-\Delta)\,\mathrm{R}_1(I_0)\,\mathrm{R}_3(L_0)$. The angles I_0 and L_0 are the inclination and ascending node of the invariable plane at epoch; I and L are the same quantities at the date. Only the angle Δ is a function of both times. This scheme works for both short-term and long-term applications.

For the short term, polynomial coefficients for I, L, and Δ are derived from the currently-accepted coefficients of the angles ζ_A, θ_A, and z_A. For the long term, these angles are expressed as sums of Chebyshev polynomials obtained from analysis of a million-year numerical integration.

If the intersection of the mean equator and the invariable plane were adopted as the origin of right ascensions, the theory would be simplified further: since L_0 and L would no longer be required, $\mathbf{P}$ would again consist of the minimum three rotations.

1. Introduction

This paper is a brief report of my doctoral research (Owen 1990) into the consequences of using the invariable plane of the Solar System as an intermediate plane in the formulation of precession theory. Space limitations prevent the inclusion of any details.

The work described here comprises three major topics: the determination of the orientation of the invariable plane, the "short-term theory" based on the precession angles of Lieske *et al.* (1977) (hereinafter denoted "L77"), and the "long-term theory" based on numerical integration of Kinoshita's (1977) model for the speed of luni-solar precession and Laskar's (1990) formulation for the ecliptic. A comparison of the long-term results near J2000 with the short-term theory reveals possible improvements to the currently-adopted precession theory, particularly in the motion of the ecliptic and in the rate of change of Newcomb's Precessional Constant.

2. The Orientation of the Invariable Plane

The invariable plane of the Solar System is rigorously defined as the plane which passes through the Solar System barycenter and is normal to the total angular momentum. The rotational angular momenta are poorly known (especially for the Sun), and the orbital angular momenta of the satellites are subject to precession; these were therefore ignored, and only the orbital angular momenta of the planets and Sun were kept. The planets were thus assumed to be point masses (including the masses of their satellites) located at their respective planet-satellite barycenters.

Positions and velocities for the nine planetary barycenters, the Sun, and five asteroids were interpolated from the M04786 planetary ephemeris (Jacobson *et al.* 1990), the most recent one produced at JPL and the only one so far to use the Voyager 2 determination of Neptune's mass. The total angular momentum vector (after Burkhardt 1982), rotated into J2000 coordinates, is directed toward

$$\alpha_0 = 273°51'09''.262 \pm 0''.038, \tag{1}$$

$$\delta_0 = 66°59'28''.003 \pm 0''.013. \tag{2}$$

The right ascension L_0 of the ascending node of the invariable plane on the mean equator of J2000 and the inclination I_0 of the invariable plane to the mean equator of J2000 are consequently

$$L_0 = 3°51'09''.262 \pm 0''.038, \tag{3}$$

$$I_0 = 23°00'31''.997 \pm 0''.013. \tag{4}$$

It is worth noting that the standard errors above have decreased nearly a hundredfold due solely to the improvements in the planetary masses provided by Voyager 2.

3. Short-term Precession Theory

The precession matrix P to transform from the mean equator and equinox of J2000 to that of date is given by L77 as

$$\mathsf{P} = \mathsf{R}_3(-\zeta_A)\,\mathsf{R}_2(\theta_A)\,\mathsf{R}_3(z_A), \tag{5}$$

where $\mathsf{R}_i(\alpha)$ is the 3×3 orthogonal matrix which rotates the coordinate axes by the angle α about axis i. The angles in equation (5) are approximated by

$$\zeta_A = \zeta_1 T + \zeta_1' T^2 + \zeta_1'' T^3, \tag{6}$$

$$\theta_A = \theta_1 T + \theta_1' T^2 + \theta_1'' T^3, \tag{7}$$

$$z_A = z_1 T + z_1' T^2 + z_1'' T^3, \tag{8}$$

with T measured in Julian centuries from J2000 (JED 2451545.0).

From Figure 1, P can also be represented by the sequence of rotations

$$\mathsf{P} = \mathsf{R}_3(-L)\,\mathsf{R}_1(-I)\,\mathsf{R}_3(-\Delta)\,\mathsf{R}_1(I_0)\,\mathsf{R}_3(L_0), \tag{9}$$

where L is the right ascension of the ascending node of the invariable plane on the mean equator of date, I is the inclination of the invariable plane to the mean equator of date, and Δ is the angle in the invariable plane from the mean equator of J2000 to that of date.

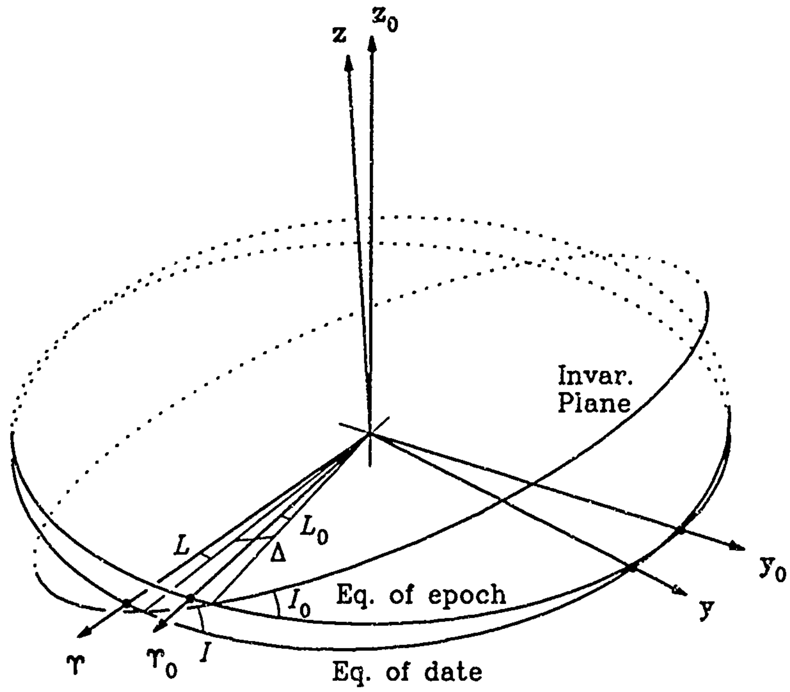

Figure 1. Precession Angles Using the Invariable Plane

Equating the right-hand sides of equations (5) and (9) and expanding the matrix products yields exact expressions for L, I, and Δ:

$$L = \text{plg}[\cos\theta_A \sin(L_0 + \zeta_A)\sin I_0 - \sin\theta_A \cos I_0,\ \cos(L_0 + \zeta_A)\sin I_0] + z_A, \tag{10}$$
$$I = \cos^{-1}[\cos\theta_A \cos i_0 + \sin\theta_A \sin(L_0 + \zeta_A)\sin I_0], \tag{11}$$
$$\Delta = \text{plg}[\sin\theta_A \cos(L_0 + \zeta_A),\ \cos\theta_A \sin I_0 - \sin\theta_A \sin(L_0 + \zeta_A)\cos I_0]. \tag{12}$$

In equations (10) and (12), plg(y, x) is Eichhorn's (1987/88) notation for the four quadrant arctangent, expressed in Fortran as ATAN2(Y,X).

Equations (10) through (12) can be expanded in powers of T to yield approximation polynomials for the angles L, I, and Δ. The L77 coefficients of the precession angles imply

$$L = 3^\circ 51' 09''.262 - 96''.7230T - 1''.94824T^2 + 0''.006539T^3, \tag{13}$$
$$I = 23^\circ 00' 31''.997 - 134''.6685T + 0''.49754T^2 + 0''.006173T^3, \tag{14}$$
$$\Delta = 0''.000 + 5116''.1809T + 2''.92466T^2 - 0''.005636T^3. \tag{15}$$

4. Long-Term Precession Theory

Since the L77 approximations for ζ_A, θ_A, and z_A begin to break down after a few centuries, numerical integration was used to obtain the precession angles over longer time spans. Kinoshita's (1977) model supplied the speed of luni-solar precession, and the orientation of the ecliptic came from Laskar (1990). The integration covered one million years centered at

J2000. The obliquity ε and the precession angles ψ_A, χ_A, ω_A, L, I, and Δ were obtained every century, and Chebyshev polynomials were fit to these results. Computer-readable tables of the Chebyshev coefficients may be obtained from the author.

Two substantial differences are apparent when the long-term results are compared with the short-term ones. First, Laskar's motion of the ecliptic near J2000 differs from that in L77. This changes the speed of planetary precession and therefore ζ_1, θ_1, and z_1: ζ_1 and z_1 decrease from $2306''.2181$/cy to $2306''.2174$/cy while θ_1 increases from $2004''.3109$/cy to $2004''.3141$/cy. The rate of change of the obliquity changes by a greater amount, from $-46''.8150$/cy to $-46''.8065$/cy. Second, Kinoshita's terms containing M_1 and M_3 are absent in the L77 work; their presence causes P_1, the derivative of P at J2000, to change from $-0''.00369$/cy in L77 to $-0''.00393$/cy.

5. Conclusions

One notes several desirable properties in equation (9) for **P**. Foremost among these is that the initial and final times are isolated rather cleanly; when one is precessing between two arbitrary times, **P** takes the form

$$\mathbf{P} = \mathbf{R}_3[-L(T_2)]\,\mathbf{R}_1[-I(T_2)]\,\mathbf{R}_3\{-[\Delta(T_2) - \Delta(T_1)]\}\,\mathbf{R}_1[I(T_1)]\,\mathbf{R}_3[L(T_1)]. \tag{16}$$

The angles L and I are each functions of only one time, and only $[\Delta(T_2) - \Delta(T_1)]$ would be evaluated using both initial and final times.

Finally, it is obvious that if right ascensions were measured from the ascending node of the invariable plane on the mean equator (instead of from the traditional vernal equinox), the first and last $\mathbf{R}_3$ rotations in equation (9) would vanish, leaving once again a sequence of three rotations. Now, however, the three rotation angles would require only two formulas for their evaluation, and only one of those two would require two arguments. Such a scheme is simpler computationally than that of the L77 paper.

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A NOTE ON THE ASTROMETRIC PRECISION OF MINOR PLANET OBSERVATIONS

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ABSTRACT. For years, interest in precise positions of minor planets has centered on tying the dynamical reference frame with the stellar frame and determining catalog zone errors. Photographic methods are generally used in obtaining observed spherical equatorial coordinates (R.A., Dec.) or crossing-point observations. Estimates of the external precision of the equatorial coordinates are overly pessimistic, while those for crossing-point observations, too optimistic.

It is estimated that equatorial positions for the brighter ($m < 11$) minor planets can be determined with an external precision not worse than +/-0.2 arcsec (m.e.), and perhaps as low as +/-0.1 arcsec (m.e.), depending on the reference catalog zonal errors.

Intersatellite observations are a type of crossing-point observation in which the images of two different objects appear in the same exposure. This is the most precise type of crossing-point observation and gives an estimate for the lower limit to the external precision of this observation type. Recent studies of satellite observations indicate that this lower limit is in the +/-0.05 to +/-0.08 arcsec (m.e.) range.

1. Introduction

The history of the use of minor planets for the improvement of the celestial coordinate system has been well documented by Orelskaya (1974), Pierce (1978), Hemenway (1980) and Whipple et al. (1988). Following IAU resolutions presented at Brighton (Trans. IAU XIVB, 1971) several programs of photographic observation were begun/continued for a selected list of minor planets. Estimates of the external precision (accuracy) of these observations were often made by redetermining positions for catalog reference stars and comparing these positions to the catalog values. This method simply returned the (accidental) catalog error for the epoch and did not adequately allow for the benefit gained from using many reference stars. Large external errors were often attributed to catalog zone errors. To eliminate zone errors, Hemenway (1980) and Whipple et al. (1988) have advocated the use of crossing-point (x-pt) observations. The external errors involved in these two techniques are discussed below.

2. Spherical Equatorial Coordinates

A better estimate of the external error can be found from the expression (see Pascu and Schmidt 1990):

$$e^2_t = \frac{e^2_{*c} + S^2 e^2_{*o}}{(n-m)} + (S\, e_{po})^2$$

where e_t is the total, single-image external error in position of an object p near the center of a photographic plate; e_{*c} is the mean catalog error of the reference stars (arcsec); e_{*o} is the mean, total observational error of the reference stars; e_{po} is the total observational error of object p (in mm); S is the plate scale (in arcsec/mm); n is the number of reference stars used in the plate solution; and m is the number of degrees of freedom--the number of constants determined in the plate solution. In this expression, the first term represents the accidental error due to the reference frame (catalog and observational), while the second term is due to the plate and night errors for object p. Note that this formulation does not account for zone errors in the reference catalog, nor for some unmodelled observational effects.

Using AGK3 parameters and current values for the other quantities, the expected external precision is between +/-0.1 and +/-0.3 arcsec (m.e.), depending on instrumental focal scale. A telescope with a focal scale of about 30 to 40 arcsec/mm will provide the optimum precision with these parameters. To this value must be added the systematic portion due to the zone errors of the catalog. These numbers are supported by photographic observations of Saturn (Pascu and Schmidt 1990) and agree with the assessment of E. Hog (quoted by Hemenway 1980), and indicate that AGK3 zone errors may not be greater than about 0.1 arcsec at epoch 1980 along the northern ecliptic.

With the introduction of the new high density, high precision photographic catalogs (see de Vegt 1988), the error interval on e_t will decrease further to (+/-0.06 to +/-0.2 arcsec m.e.), the advantage then lying with the long-focus instruments. It is clear that the systematic catalog errors will be the limiting factor in the use of these catalogs for this application.

3. Crossing--Point (X-Pt) Observations

Whipple et al. (1988) define a x-pt observation as "the observed angular separation between two minor planets as measured against the same stellar background". This observation type has the advantage that systematic catalog errors are eliminated. Usually, the observation of each minor planet is made on different plates and on different nights. However, should both objects appear on the same plate, such observations would be the most precise because the observational errors due to the reference stars would also be eliminated. Intersatellite observations are this type of x-pt observation if reference stars are used to reduce the plates. Thus, discussions of this observation type can yield a value for the lower limit to the external error for x-pt observations.

Taylor and Shen (1988) recently analyzed a very large dataset of the intersatellite observations of the eight brightest moons of Saturn. Their Table 3 shows no individual dataset with an external mean error smaller than +/-0.10 arcsec for all satellite/Titan pair observations taken together. Taylor (1985,1986) also computed external mean errors for individual satellite/Titan pairs. The smallest external mean error was +/-0.08 arcsec for the Rhea/Titan observations made with the 26-inch refractor at the Naval Observatory. There was also an indication in Taylor's computations that image density or magnitude difference contributed to this error and that a lower limit, say +/-0.05 arcsec (m.e.), was reasonable.

4. Conclusions

Recent studies of the external precision of photographically obtained equatorial positions for Saturn (Pascu and Schmidt 1990) and photographic intersatellite observations of the Saturnian moons (Taylor and Shen 1988, Taylor 1985, 1986) yield im⌃ortant information on the expected external precision of astrometric obse⌃ations of minor planets. For bright (m < 11 mag), slow-moving minor planets, photographic spherical equatorial coordinates can be obtained with an external precision of +/-0.25 arcsec (m.e.) along the northern ecliptic using the AGK3 catalog and a long-focus refractor. With the introduction of the modern high-density reference star catalogs (Douglass and Harrington 1990, de Vegt 1988) this precision can be increased to +/-0.1 arcsec (m.e.). Meanwhile, x-pt observations can be obtained with an external precision not much better than +/-0.08 arcsec (m.e.). Considering the value of the spherical equatorial coordinates in fixing the minor planet orbit in space (Whipple et al. 1988), x-pt observations should only be made when systematic catalog errors larger than 0.1 arcsec are suspected.

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OPTICAL POSITIONS OF RADIOSTARS BY THE DANJON ASTROLABE AT CAGLIARI AND FURTHER PROJECTS

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ABSTRACT . Precise positions of optical counterparts of radiosources are needed in connecting optical and radio reference frames. The Cagliari Astronomical Observatory has planned systematic observations with the Danjon astrolabe of objects from the high priority Hipparcos radio-stars list. A preliminary result of the position of radiosource HR 5110 is presented.

1. Introduction

Precise positions and proper motions of optical counterparts of radiosources are needed in order to determine a direct link between the radio reference frame and the ground-based optical reference frame and will be of great importance when the Hipparcos catalogue becomes available, making it possible to link together the Hipparcos and radio reference frames (Froeschlé and Kovalevsky, 1982). During the past decade, within the framework of international cooperation, a great number of observations have been carried out to provide high quality astrometric data on radiostars.

An international programme of systematic observations of radiostars included in the list prepared by the IAU Commission 24 working group "On Identification of Radio/Optical Astrometric Sources" with prismatic Danjon astrolabes was proposed at the beginning of this decade (Debarbat 1980,1981) and several observation campaigns have been conducted (Clauzet et al.,1985,1986; Noel, 1986) which have confirmed the opportunity of astrolabe programmes devoted to the connection of the optical and radio reference frames.

2. Preliminary Results

Some radiostars on the list of Commission 24 Working Group have been observed with the Danjon astrolabe in Cagliari these objects being included in the fundamental groups of the astrolabe program for time and latitude observations (Poma and Gusai , 1987). Unfortunately , because of the contraints required for a uniform distribution in azimuth of the stars in each group, the observations of these radiostars have been made at one of two transits solely. In this case, as is known (Debarbat and Guinot, 1970), it is possible to determine the right ascension only for those stars with a sufficiently small parallactic angle . In our case this was true only for radiosource HR 5110 (FK5 502, HD 118216 in Table 1) which was observed 21 times at west transit. Thus, the only preliminary result we can report here is the value of its right ascension . We obtain, by using standard reduction procedures (Debarbat and Guinot, 1970)

$$\Delta\alpha = 0^{s}.004 \pm 0^{s}.0036$$

for the mean epoch 1984.1 and in the sense Astrolabe - FK5.

3. Subsequent Projects

The Danjon astrolabe of the Cagliari Astronomical Observatory has recently been taken out of routine time and latitude observations. It was decided to use the instrument, after suitable improvements, mainly for astrometric purposes (Poma and Zanzu, 1990). In this context we have set up a programme to determine accurate optical positions of radiostars included in the " High priority Hipparcos radiostars " list proposed for the Hipparcos input catalogue. The objects that we have selected are listed in Table 1.

TABLE 1

Radiostars currently observed by the Cagliari astrolabe at the zenith distance of 30°

Radiostar	M_v	α (1950.0)	δ (1950.0)	
		h m s	° ′ ″	
HD 004502	4.1	00 44 40.968	23 59 43.95	FK5 27
HD 008634	6.1	01 22 51.423	23 15 07.42	
HD 019356	2.1	03 04 54.356	40 45 52.46	FK5 111
HD 026961	4.6	04 14 28.441	50 10 28.87	b Per
HD 039587	4.4	05 51 25.196	20 16 07.39	
HD 062044	4.3	07 40 11.386	29 00 22.58	sig Gem
HD 091480	5.2	10 31 57.350	57 20 27.14	FK5 398
HD 118216	5.0	13 32 33.917	37 26 16.62	FK5 502
HD 127739	5.9	14 30 16.090	22 28 45.49	26 Boo
HD 146361	5.2	16 12 48.251	33 59 02.63	TZ Crb
HD 174638	3.4	18 48 13.936	33 18 12.51	FK5 705
HD 179094	5.8	19 07 15.438	52 20 42.81	
HD 206860	6.0	21 42 06.546	14 32 35.62	HN Peg
HD 208816	5.0	21 [illegible] 1.45	63 23 13.5	VV Cep
HD 210334	6.1	22 06 39.425	45 29 45.82	AR Lac
HD 216489	5.6	22 50 34.454	16 34 31.28	IM Peg
HD 217476	5.0	22 57 58.164	56 40 36.60	
HD 222107	3.8	23 35 06.520	46 11 13.83	FK5 890
HD 223460	5.9	23 47 09.719	36 08 52.56	

The list takes into account the latitude (39°.1) and the capabilities of the instrument at the present time (zenith distance = 30° $M_v < 6.2, 11° < \delta < 67°$). The number of objects which could be observed will be increased when in the near future the astrolabe is equipped with a 135° reflecting prism, at present being built by Soptel of Paris, allowing observations at 45° for the zenith distance to cover a larger declination zone. The list of the radiostars that will be added to those reported in Table 1 is given in Table 2.

It is important to note that the possibility of observations at the two different zenith distances of 30° and 45° will also allow us to obtain absolute determinations of declinations (Kreinin,1968) with good accuracy because of the favourable geographic position of the Cagliari astrolabe on the ILS parallel (Kreinin,1986). A study in this direction is in progress.

TABLE 2

Radiostars to be observed by the Cagliari astrolabe at the zenith distance of 45°

Radiostar	M_v	α (1950.0)	δ (1950.0)	
		h m s	° ' "	
HD 001061	5.8	00 12 24.123	08 32 36.10	UU Psc
HD 007672	5.4	01 14 03.829	-02 45 46.66	AY Cet
HD 017138	6.1	02 44 22.747	69 25 32.89	RZ Cas
HD 018884	2.5	02 59 39.744	03 53 41.15	FK5 107
HD 022468	5.7	03 44 12.88	00 25 32.6	
HD 036486	2.2	05 29 27.017	-00 20 04.41	FK5 206
HD 037128	1.7	05 33 40.476	-01 13 56.30	FK5 210
HD 039801	0.5	05 52 27.809	07 23 57.92	FK5 224
HD 109387	3.9	12 31 21.550	70 03 48.96	FK5 472
HD 111456	5.8	12 46 29.276	60 35 32.12	
HD 115383	5.2	13 14 17.524	09 41 05.57	59 Vir
HD 124224	5.0	14 09 43.762	02 38 38.22	CU Vir
HD 169985	5.2	18 24 38.88	00 09 53.8	d Ser

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COMPARISON OF THE FK5 PROPER-MOTION SYSTEM WITH A KINEMATIC DISTRIBUTION FUNCTION

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ABSTRACT. A model for the kinematic distribution function of our Galaxy can be used as an independent confirmation that a reference system is free of Earth motions and retains the true kinematics of the stars. Maximum likelihood can simultaneously estimate the parameters required to calibrate distances to the stars, represent the kinematic distribution function, and check on residual Earth rotations in the proper-motion system. The global maximum-likelihood analysis uses all available information: photometry, trigonometric parallax, proper motion, and line-of-sight velocity for a well-defined catalog of stars. Awaiting observations from HIPPARCOS, preliminary testing of the algorithm on available ground-based observations is discussed.

1. Introduction

Ideally, proper motions should be independent of Earth motions and retain the true kinematics of the stars. The direct way to link the reference system realized by FK5 or HIPPARCOS to an inertial coordinate system is to measure extragalactic sources; however, these sources have significantly fainter apparent magnitudes than most of the reference stars in these fundamental catalogs.

An independent confirmation that the resulting reference system is inertial can be obtained using a Galaxy model for the kinematic distribution. Line-of-sight velocities for the reference stars are independent of the slow motions of the Earth and sample the same stellar kinematics in our Galaxy as the proper motions.

The currently-adopted 1976 IAU precession constant is based on a Galaxy model that is simple to compute. In most analyses the velocity distribution function was assumed isotropic. However, Eichhorn (1974) warns of the error that the observed anisotropy could introduce into the estimated precession constants.

With currently available computing resources, it is possible to estimate simultaneously both the parameters required to represent the kinematic distribution function as well as to look for systematic errors in the reference system and other observables. The global maximum-likelihood analysis uses all available information; photometry, trigonometric parallax, proper motion, and line-of-sight velocity for a well-defined catalog of stars.

We are currently developing the required Numerical Algorithms for Maximum Likelihood Estimation (hereafter referenced by the acronym NAMaLiE) for this analysis. Hopefully, observations from HIPPARCOS will generate a well-defined catalog with sufficient accuracy and uniformity appropriate for detailed statistical analysis. The algorithms are being tested on available catalogs of ground-based observations.

1.1. ADVANTAGES OF THE NAMaLiE APPROACH

The classical approach for the analysis is to transform observed quantities to a theoretical reference frame, such as to evaluate a distance from an observed trigonometric parallax, or to compute space velocities using proper motions, line-of-sight velocities, and parallax estimates. The only advantage is that the analysis is simple to compute, but the procedure requires the combination of many different types of observations and errors selected with various criteria. This makes rigorous statistical analysis and investigation of systematic errors impossible.

The likelihood function used in NAMaLiE is defined using the directly observed quantities, which allows for proper handling of errors of observation, sample selection, and censorship. We overcome sample incompleteness by using conditional probabilities, a procedure that discards the 'difficult-to-use' information (Casertano, Ratnatunga & Bahcall 1990).

NAMaLiE also allows the analysis of samples for which trigonometric parallax, proper motion and line-of-sight velocity are not all available for every star. In magnitude-limited reference catalogs, photometry and proper motions are typically available for most stars, while trigonometric parallaxes and line-of-sight velocities are available only for a subset.

Numerical simulations are used to show that the NAMaLiE gives unbiased estimates for parameters and their associated errors. The procedure is a simple compute intensive approach that does not require any clever analytical solutions. Most analytical solutions require simplifying assumptions and approximations, do not solve the real problem, and may give biased estimates of the parameters.

2. The Galaxy Model

The distribution function is represented by a sum of stellar populations, the kinematics of each being represented by a trivariate-shifted Gaussian, as used in the IAS-Galaxy model (Ratnatunga, Bahcall & Casertano 1989). The kinematic distribution for each population is then represented by six free parameters, consisting of three velocity dispersions and three streaming motions.

The model must also include the parameters required to calibrate stellar distances from their observed photometry and available trigonometric parallaxes, as described in Ratnatunga & Casertano (1991).

For simplicity, consider a selected sample of stars from a single luminosity class and a limited range of temperature (color) over which the color/absolute-magnitude calibration can be assumed as linear. Assume that the cosmic scatter in absolute magnitude is Gaussian and constant over that color range. The absolute-magnitude calibration can then be represented by three free parameters.

Additional parameters are required to define the density distribution and relative normalizations. For stars in the solar neighborhood, the kinematics and distribution of stars can be assumed as uniform, and a single free parameter is sufficient to define the relative normalization for each extra component.

The residual systematic error in proper motion can be represented as rotations about the three axes. A systematic error in trigonometric parallax may be represented, for example, as errors in the zero point and in the size of the published error estimates.

Hopefully, the catalog has been generated free of systematic errors in photometry and line-of-sight velocity, which are relatively simpler measurements derived from single observations, rather than the difference of two time-separated observations. The analysis uses the coordinates of the stars only to define the line of sight and, as such, is insensitive to typical errors in coordinates.

For example, a spectroscopically selected sample of K and M dwarfs in the solar neighborhood contains stars forming the thin- and old- disk populations (halo stars being only a small 0.2% contamination) and we would need a total of 24 free parameters. Some of the parameters associated with the mean streaming motion may be held zero by assuming that the density components don't drift relative to the local standard of rest (LSR) radially or vertical to the galactic plane. Then a value for the peculiar motion of the Sun with respect to the LSR could be derived or adopted if poorly constrained by a particular sample.

3. Characteristics of the Catalog of Observations

NAMaLiE can make use of incomplete samples of stars and observations as long as the sample selection characteristics are well defined. However, complete samples are desirable to derive reliable information about relative normalizations of stellar components. Although the best available observations should be analyzed, NAMaLiE can make use of data of any accuracy.

The following are required: good coordinates and reliable photoelectric photometry (apparent magnitude and color) for all stars in the sample, all available proper motions transformed to a single uniform reference system, heliocentric line-of-sight velocities, and trigonometric parallaxes for the stars.

To avoid the need for extra free parameters in the Galaxy model associated with any variation of the density and kinematic distribution as a function of location in the Galaxy, a solar neighborhood sample within, say, 100 pc from the Sun is useful. Available trigonometric parallax estimates are also relatively more accurate, and effects of galactic extinction on the photometry are less important. For young, early-type stars, the distribution (example: Gould belt) or kinematics (example: moving groups) may be nonuniform. In an older, late-type stellar population, inhomogeneities in rate and location of star formation have probably time-averaged to a uniform distribution.

However, precise proper motions for a more distant population of stars give a tighter constraint on the residual rotations of the proper-motion system being estimated to link to an inertial reference system. Another advantage of a more distant population is the space-averaging of the distribution sampled; i.e., independent of any peculiarities in local kinematics. The sample selected for analysis must optimize these diverse characteristics with the parameters that need to be estimated and the available accuracy of the observations.

3.1. INADEQUACY OF AVAILABLE GROUND-BASED SAMPLES

The analysis requires a sample with good trigonometric parallaxes as well as proper motions in the FK5 system.

The FK5 and Extension, which was the catalog that we hoped to analyze for this paper, consists primarily of intrinsically bright stars with a sampling distance to about 250 pc from the Sun. Line-of-sight velocities and trigonometric parallaxes are available for a subset of the sample. However, the accuracy of the currently published parallaxes and the intrinsic width of the giant branch prevented deriving any meaningful estimates for the residual systematic rotations of the proper-motion system.

The spectroscopically selected Vyssotsky K and M dwarf sample has sufficient trigonometric parallaxes and a much smaller intrinsic width of the main sequence to define the absolute calibration. However, the available proper motions have not as yet been transformed to the FK5 system.

4. Progress To Date

Most of the required numerical algorithms have been coded and the solutions converge. The procedure has been applied to the Vyssotsky sample of K and M dwarfs. The samples need to be represented by two stellar populations with velocity dispersions and asymmetric drift corresponding to the thin and old disk populations. The two populations appear to have statistically different absolute-magnitude calibrations for main-sequence dwarfs with the old disk stars intrinsically fainter and in the Kron $(R - I)$ color range $[0.35, 1.3]$. Detailed results will be published in Ratnatunga & Upgren (1991). The parallax errors seem to be underestimated by $20\% \pm 7\%$. The 'mean' zero point of the published parallaxes could be 3.4 ± 1.4 mas.

Using this particular catalog of about 750 nearby disk dwarfs with available proper motions, we find no systematic error in the reference system with an rms accuracy of 6, 7 and 9 mas per year for rotations about the U,V,W axes, respectively. Although this is not a particularly useful constraint, it reflects the total uncertainty, including those from kinematics and the distance calibration. It is derived entirely from information available for this small sample of stars. Observations from HIPPARCOS of a much larger sample of more distant K and M giants should compare favorably with the 0.4 mas per year accuracy achieved for the precession constant using lunar laser ranging (Williams *et al.* 1990).

5. Conclusion

A numerical algorithm for maximum-likelihood estimation has been developed to simultaneously estimate the parameters that define the rotation of the reference system and those required to model the disk kinematics and the absolute-magnitude calibration as a function of color. Two stellar components with anisotropic velocity dispersions are required.

Application of this procedure to a sample of K and M dwarfs shows no significant deviations of the FK5 reference system with an rms accuracy of about 7 mas per year. The procedure will be of more practical importance for this problem after observations from HIPPARCOS generate a larger, well-defined catalog of parallaxes and proper motions with sufficient accuracy and uniformity appropriate for detailed statistical analysis.

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SOUTHERN HEMISPHERE VLBI ASTROMETRY

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Until relatively recently the number of radio source positions in the Southern sky measured to an accuracy significantly better than 1 arcsec was very small, with virtually no measurements south of -45 Declination. A program of VLBI astrometric observations is currently underway in the Southern Hemisphere with the aim of establishing an all-sky precision reference frame of extragalactic sources at the milliarcsecond level. Substantial progress has been made in the last few years towards raising the number and precision of Southern VLBI measurements towards the level obtained in Northern programs. Most of the measurements have been made on the 10,000 km baseline between Canberra, Australia and Hartebeesthoek, South Africa using dual-frequency wideband (Mark-III S/X) recording systems. A second important VLBI program to link the radio and optical frames in the South through radio stars will commence in 1991.

THE ESTABLISHMENT OF THE RADIO/OPTICAL REFERENCE FRAME

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This 5-yr program which began in 1987 is to establish a link between the radio and optical reference frames, using about 400 extragalactic sources which are compact and flat spectrum in the radio and which are observable in the optical. An overview of the program is presented at this meeting by Johnston *et al.* Various specific aspects of it are also discussed in Ma *et al.*, McCarthy *et al.* and Reynolds *et al.*

The observations have been continuing in both hemispheres and in both the radio and the optical. We have observed 348 sources with VLBI. There is at least one astrograph plate available for 369 sources and at least one prime focus plate available for 138 sources. Altogether there are 450 sources with some data.

In the selection of the final 400 radio/optical link sources, which will replace the list from Argue *et al.* (1984), we have been categorizing the sources into three lists. The first is the primary sources, those with sufficient data of good quality. The second includes the preliminary sources, those with some good data, but needing more. The third is those sources chosen based on survey data which have not been observed yet. Implicitly, there is a fourth list of reject sources.

At this meeting we have shown the first drafts of lists of sources in each classification, including a total of 564 sources -- 220 primary, 139 preliminary, 160 proposed and 45 rejects. The final list of sources will be presented at the upcoming 1991 IAU General Assembly.

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CQSSP: A NEW TECHNIQUE FOR ESTABLISHING THE TIE BETWEEN THE STELLAR AND QUASAR CELESTIAL REFERENCE FRAMES

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ABSTRACT. Using artificial satellites as transfer objects the project "Coupled Quasar-Satellite-Star Positioning" represents an independent method for linking quasar and stellar reference frames. Optical observations of close approaches between reference stars and satellites yield satellite positions in the stellar reference frame. On the other hand high precision satellite orbits in the International Earth Rotation Service (IERS) terrestrial reference frame are obtained from laser or radiometric observations. Using IERS earth rotation parameters and adopted transformation models the satellite and eventually the star positions can be expressed in the IERS quasar celestial reference frame. In this paper we describe the CQSSP project and assess its capability for providing an accurate tie between tho two metioned celestial reference frames.

1. Introduction

In recent years techniques have been proposed to tie the conventional inertial coordinate reference frames defined by the very long baseline (radio) interferometry (VLBI) positions of selected extragalactic radio sources, primarily quasars, to the reference frames established through optical observations of stars. The need to tie these two types of frames stems from the high accuracy but limited accessibility of the quasar frames on the one hand and the less accurate but more readily accessible stellar frames on the other. If the relationship between the radio source and stellar frames can be accurately established optical positions could be readily expressed in the radio source frame. The procedure of relating the two types of frames should reveal the errors in position of individual stars and zonal errors in the optical catalogues as well as giving the global orientation of one frame with respect to the other.

The classical techniques used to tie the radio to the optical frames all make use of radio-emitting object which are observable by VLBI as well as in the visible part of the electromagnetic spectrum. Several ongoing projects observe either extragalrctic objects (EGOs) or radio stars in the optical using astrograph cameras or CCD sensors on large telescopes. The priciple of these methods is outlined in Figure 1. Unfortunately there are some limitations of this technique. The first is that there are relatively few compact EGOs which are strong enough to be accurately positioned by VLBI and sufficiently bright in the visible to provide good optical images. The second is that for most EGOs there will not be a nearby FK5 star in the image. Hence the positions of secondary and tertiary reference stars must be established with attendant positional uncertainty. Third one must assume that the radio centres and optical photocentres of the objects coincide. Many radio-emitting EGOs show an elaborate structure on the scale of a few mas or more that is both time and frequency dependent. For example, Charlot et al. [1988] found for 3C273B that at 1985.37 the 90 % of peak brightness contour spanned over 28 mas at 2.3 GHz but only about 5 m as at 8.4 GHz.

2. The CQSSP Technique

The key idea of the "Coupled Quasar-Satellite-Star Positioning" project is to use artificial earth satellites as transfer objects (see Figure 2). The positions of these satellites can be observed very accurately in the International Earth Rotation Service (IERS) Terrestrial Reference Frame (ITRF) by Satellite Laser Ranging (SLR) or radiometric observations (e.g GPS). On the other hand VLBI observations of quasars define the International Celestial Reference Frame (ICRF) and a series of earth orientation parameters as determined by the International Earth Rotation Service (IERS). Therefore we can assume the transformation from the IERS Terrestrial Reference Frame (ITRF) to the quasar frame to be given by VLBI at any epoch. This enables us to express the coordinates of the transfer objects (the satellites) in the quasar frame although they are measured by Satellite Laser Ranging (SLR) or GPS in the ITRF. The optical observations of the satellites with respect to the reference stars will finally result in coordinates of these stars in the quasar reference frame (ICRF).

2.1 OPTICAL OBSERVATIONS OF STARS AND SATELLITES

Close encounters between satellites and stars are recorded on

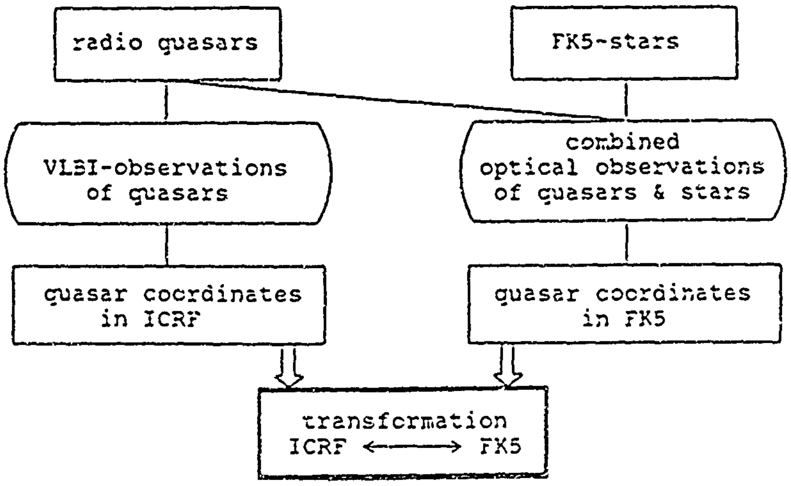

Figure 1: Classical Method

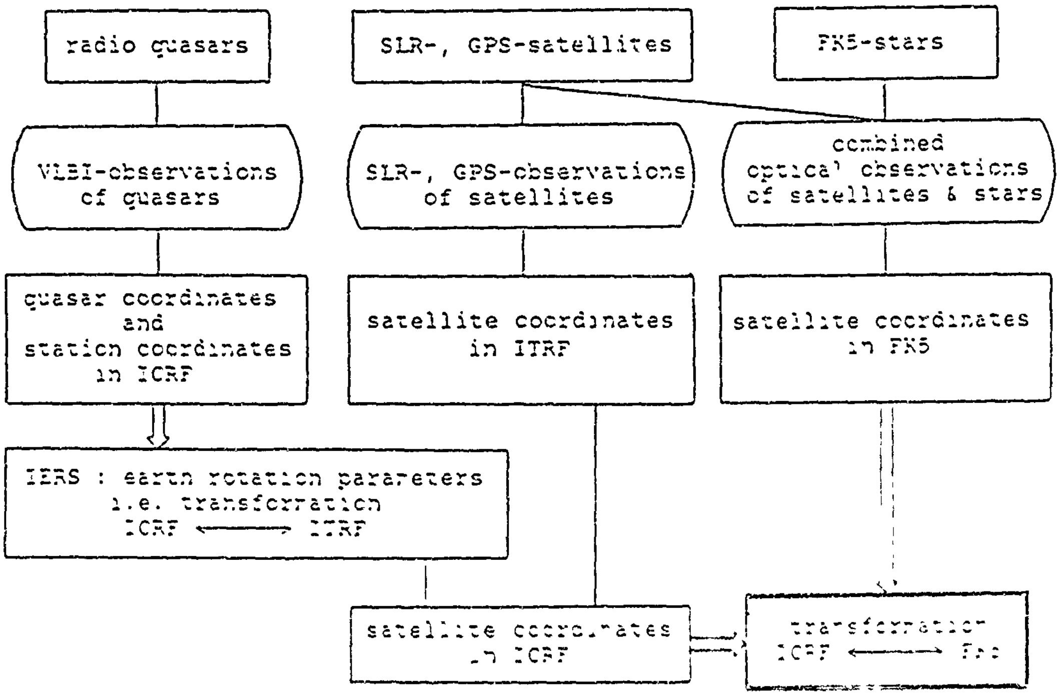

Figure 2: CQSSP

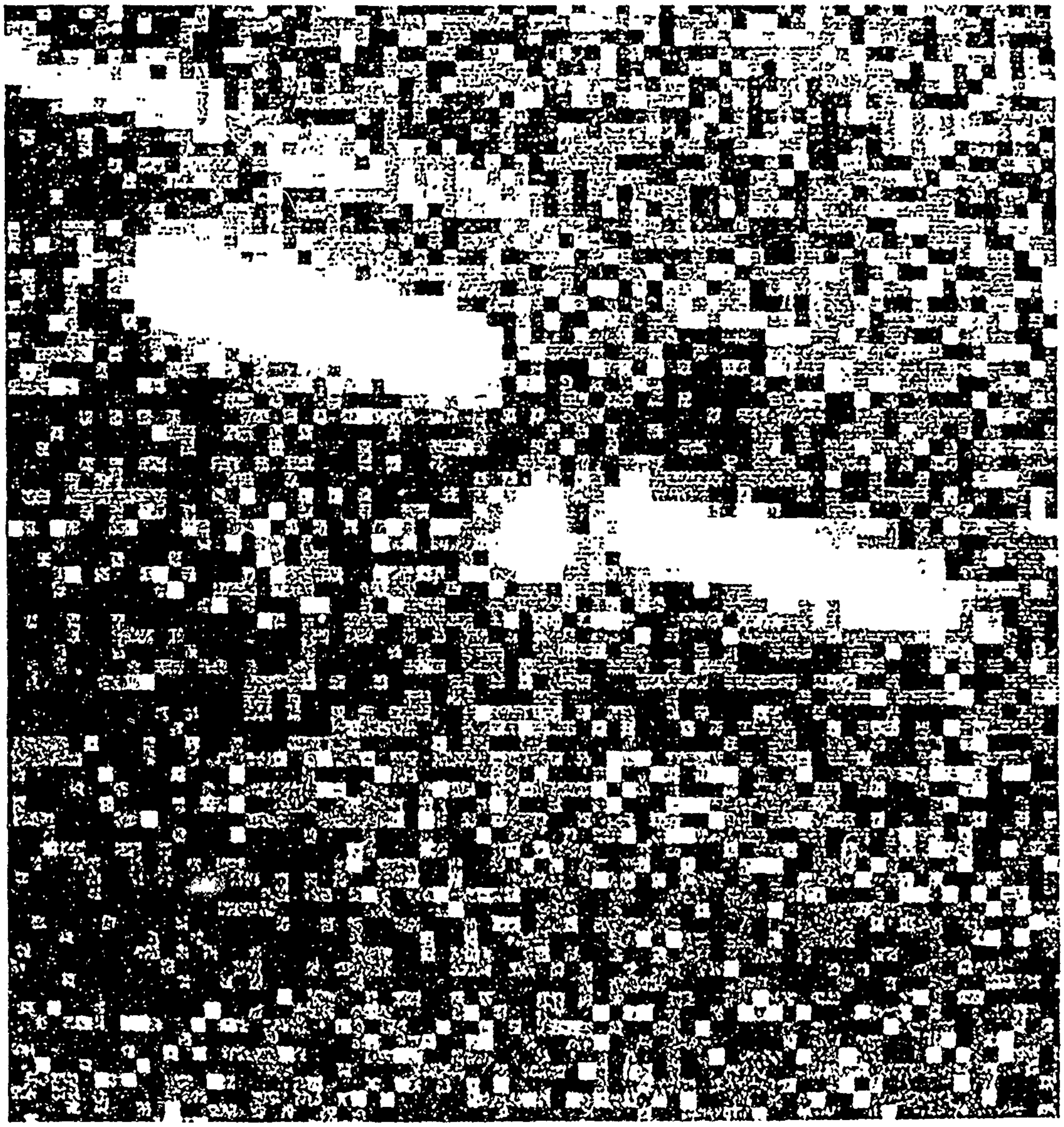

Figure 3. 0.5 second exposure of the geodynamics satellite LAGEOS and several star trails at the 0.5 m laser ranging telescope in Zimmerwald. The telescope was tracking the 13.5 mag. satellite moving with about 200"/sec relative to the stars. The image represents approximately a 7' x 7' area.

a CCD detector. The idea is to select such events with very small angular distances between the two objects thus minimizing the influence of differential refraction. Pushing the approach to the limit would mean that we allow for occultations only which however would restrict the number of events to much. Assuming a small field of view of 10 arcminutes and 200'000 reference stars homogeniously distributed over the entire celestial sphere (HIPPARCOS catalogue) we end up with about 50 observable encounters per satellite pass (from 30 deg elevation over zenith to 30 deg elevation). (See Bauersima, 1984).

First observations have been acquired with the 0.5 m f2 Cassegrain telescope at the Zimmerwald Laser Ranging observatory where the useful field of view is about 10 arcminutes. The detector is an 512 x 512 pixel front side illuminated CCD with 20 x 20 f pixel size resulting in a linear scale of 4.1 arcseconds per pixel.

Depending on the height of the used satellites (1000 to 6000 km for LASER, 20000 km for GPS satellites) an encounter may be observed for 10 to 50 seconds at maximum (determined by the fact that both objects have to be simultaneously in the small field of view of 10 arcminutes). Due to the movement of the satellites with respect to the stars we will obtain trailed images at least for one object (depending on the tracking mode).

Figure 3 shows the situation where the telescope was tracking a satellite and thus the stars moved with respect to the telescope. Thinking in terms of signal to noise ratios of the single images we prefer to concentrate the light of the fainter object on a few pixels of the detector which in most cases means that we have to track the satellite. The importance of this point is evident considering an optical brightness of 12 to 14 mag for the satellites in contrast to the brightness of the reference stars (< 10 mag). We should also point out, that a 0.5 m telescope equipped with a CCD will collect only about 4500 photons per second from an 14 mag object.

With exposure times < 1 sec we end up with a series of 10 to 50 single images of the type shown in Figure 3 per close encounter. Each image results in a pair of coordinates for the star and the satellite in a CCD coordinate frame. These coordinates can be transformed to give satellite positions in the reference star system (FK5) or vice versa to give in star positions in the satellite reference frame (ITRF).

3. Advantages of the Method / Error Estimation

There are several distinct advantages of a method using satellites as transfer objects over methods using some sort of triangulation techniques (e.g. secondary and tertiary refe-

rence stars):
- The large angular distances between the quasars and the reference stars are bridged by high precision satellite orbits.

- Satellites allow for a selection of a homogenious set of reference stars.

- Only differential quantities are observed optically which reduces the influence of refraction to a negligible amount.

Errors of the radiointerferometric observations of the quasars and the determination of the satellite orbits are not concerned because they are well below 0.01 arcs and thus of minor importance compared with the errors associated with the optical observations.

For the error budget of the optical part we considered the following contributions:
- Determination of the observation epoch (the satellites move with up to 250 arcs/s).
- Determination of the centroids.
- Estimation of the parameters of the transformation CCD coordinates <--> celestial reference frame (FK5).

Assuming a 1 m, f4 to f6 telescope with high quality (astrograph) optics we end up for a single close encounter between a reference star and a 14 mag satellite with an accuracy for the position of the satellite in the stellar reference frame of 0.06 arcs.

Using the Zimmerwald Laser Ranging telescope (see section 2) as a test instrument first results already indicate a accuracy of about 0.4 arcs. We have to point out in this context that the mentioned telescope gives a scale of 4.1 arcs/pixel and produces images of very poor quality! Nevertheless an accuracy of 0.4 arcs is in the range of what is achievable with small Schmidt cameras. Since the exposure times are up to 2000 times shorter with the CCD system this opens a variety of new applications.

However, we should always remember that the CQSSP goal is the determination of 3 Eulerian angles (and their first derivatives) of the transformation ICRF <--> stellar reference frame (FK5) (and not positions of individual sources!) using hundreds of single "close encounter" observations.

4. Conclusions

CQSSP gives a link between the quasar and stellar reference frames which is independent from the classical link using radio sources.

The technique has the advantage to depend on a comparatively homogeneous distribution of reference stars.

We expect an accuracy which is comparable with the classical approaches using astrographic techniques.

In view of the importance of the topic, the specific advantages and the expected accuracy of the technique, we expect CQSSP to make a valuable contribution to the establishment of transformation parameters between optical and quasar reference system.

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TESTING THE ASTROMETRIC ACCURACY OF THE 2m TELESCOPE OF ROZHEN OBSERVATORY AFTER CHANGING THE PRIME MIRROR

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ABSTRACT. The astrometric accuracy of the recent configuration of the (D=2m, f=16.5m) RC telescope of the Rozhen obesrvatory (Bulgaria) was tested. We compared x and y positions from a plate of the RC telescpe with positions obtained from a plate of the (D=0.7m,f=3m) CERGA Schmidt telescope at CERGA (France).The positions coincided on a level of 0.09 arcsec, which is caused mainly by the error of the measurements of the CERGA plate.

INTRODUCTION

This investigation has a long history. It began in 1981.Then the first of the authors was in agreement with prof .J.Kovalevsky that the astrometric characteristics of the field of the 2m RC telescope at Bulgarian National Observatory could be determined using plates,taken with the CERGA Schmidt telescope. As it is well known a astrometric test of a telescope can be done by comparison of measurements from plates of that telescope with corresponding ones from a telescope which is known to be of good astrometric quality. The observations were carried out by the French colleagues but the plates were not jet received in Bulgaria, owing to the "competence" of the "special authorities" of the Bulgarian Embassy in Paris. And the history started again. In 1987, Nov. 17, only one plate was taken on the CERGA Schmidt telescope by V. Shkodrov and C. Pollas. The coordinates of the 1909 plate center are RA = $3^h59^m40.0^s$, D = $+41^\circ47'39.99''$.The area culminate near to the zeniths of the two telescopes at CERGA and in Rozhen. The plate had III a-J emulsion with hypersensitization 310. The exposure time was 51^m. At the same time the changing of the mirror of 2m Rozhen telescope again delayed the realization of the test problem.It continued from the beginning of 1988 till the summer of 1989,so we succeeded to take plates of the same area at the end of 1989. Among the Rozhen plates we used only plate 1577, taken on October 27,1989. Kodak III a-O emultion with a GG385 filter was used. The exposur time was 60^m. The coordinates of the plate center are RA $= 04^h 01^m 41^s$ D $= +42^{\circ}16'59''$.

COMPARISON OF THE PLATES

On the Rozhen plate 121 stars equally distributed over the plate and nearly of the same magnitude have been marked for the comparison. They were also found on the CERGA plate. The CERGA plate was measured on the ZEISS Ascorecord of Rozhen observatory by Shkodrov and Ivanova,For each star four settings were done. The mean error of the mean was $\pm 2.0\mu$. In addition, on the CERGA plate also 50 reference stars from the AGK3 equally distributed over the plate have been measured. They were used for the transformation of the rectangular coordinates into spherical coordinates α and δ. For this transformation we had to use a reduction model with terms up to the third order of the rectangular coordinates x and y. The deviations of the measuremants from the catalogue for different reduction models are given in Table 1.

Table 1. Mean deviations of measurements from the AGK3 positions for diferent reduction models.

Reduction model with		$\Delta X(\mu)$	$\Delta Y(\mu)$
linear	terms	16	16
+quadratic	terms	13	13
+cubic	terms	6	7

The internal error of the catalogue from two independent measurements of the CERGA plate was of the order of $\pm$ 0.08 arcsec.

The Rozhen plate was independently measured by both of us at the ASCORECORD of Hoher List Observatory (Geffert, 1986). We measured the plate in four orientations.The measuremants of 0° (respectively 90°) were transformed into the corresponding ones in a 180° (270°) orientation and the mean was taken. After that a mean of the two measuremants ($0^{\circ}/180^{\circ}$ and $90^{\circ}/270^{\circ}$) was determined for each of us.The mean differarces of the various comparisons are given in Table 2.

Table 2. Comparisons of measurements of the Rozhen plate

Measurement 1	Measurement 2	$\Delta X(\mu)$	$\Delta Y(\mu)$
Gef 0°	Gef 180°	4	6
Gef 90°	Gef 270°	4	6
Gef $0^{\circ}/180^{\circ}$	Gef $90^{\circ}/270^{\circ}$	4	6
Shk 0°	Shk 180°	6	10
Shk 90°	Shk 270°	9	9
Shk $0^{\circ}/180^{\circ}$	Shk $90^{\circ}/270^{\circ}$	6	5
Mean Shk	Mean Gef	3	2

Table 2 indicates that the systematic errors can be reduced by measuring the plate in all four orientations. Since the errors in this table were caused by both measurements, the mean accuracy of our independent mean measurements was better than 2μ which is an excellent value for the visual centering of large star images.

Comparison of our measurements with the catalogue obtained from the CERGA plate led to large (>100μ) deviations for reduction models with linear and quadratic terms of the rectangular coordinates. Only the reduction with third order polynomials resulted in errors of 6μ (in α) and 7μ (in δ) which correspond to 0.07 and 0.08 arcsec for a focal length of 16.5m.

DISCUSSION

If we compare these deviations of 0.07/0.08 arcsec with the 0.08 arcsec internal error of the positions obtained from the CERGA plate, the Rozhen telescope can be used for further precise astrometric work.

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CONSISTENT RELATIVISTIC VLBI THEORY WITH PICOSECOND ACCURACY†

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1. Introduction

The accuracy of Very Long Baseline Interferometry (VLBI), representing one of the most important space techniques of modern geodesy, especially for the determination of the Earth's rotation parameters and baselines, is steadily increasing. Presently, delay residuals are of the order of 30 - 50 ps, corresponding to an uncertainty in length of about 1 centimeter e.g. in the determination of baselines or the position of the rotation pole. As has already been stressed by many authors, at this level of accuracy a relativistic formulation of the VLBI measuring process is indispensable (e.g. the gravitational time delay for rays getting close to the limb of the Sun amounts to 170 ns!). Starting with the work by Finkelstein et al. (1983) a series of papers has meanwhile been published on a relativistic VLBI theory (Soffel et al., 1986; Hellings, 1986; Zeller et al., 1986; Herring, 1989). However, possibly apart from Brumberg's treatment in his new monograph (Brumberg, 1990) all of these theories have one fatal drawback: they are not based upon some consistent theory of reference frames, which relates the global, barycentric coordinates, in which the measuring process is primarily formulated and in which positions and velocities of the bodies of the solar system are computed, with the local, geocentric coordinates, comoving with

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the Earth, in which the geodetically meaningful baselines are defined. Furthermore, none of these theories (including Brumberg's (1990) treatment) have the accuracy of one picosec which seems desirable with respect to the achieved residual values.

This article presents a relativistic VLBI theory with an accuracy of better than 1 ps. It is based upon a consistent theory of reference frames in the solar system which first has been introduced by Damour, Soffel and Xu (1990a-c) and complete at the first post-Newtonian level. For an accuracy of 1 ps our result differs from all results that have been published earlier.

2. Post-Newtonian VLBI Theory For The Group Delay

We consider some radio signal being emitted from some remote source at barycentric coordinate time t_0 and position $\boldsymbol{x}_0$. We consider two "light-rays", contained in the signal, which arrive at two VLBI antennas (called 1 and 2) at coordinate time t_1 and t_2. This barycentric coordinate time t is also called TCB. Let us denote the Euclidean unit vector from the source to the barycenter by $\boldsymbol{k}$ ($k^i k^i = 1$). Then the barycentric coordinate arrival time difference $t_2 - t_1$ to first post-Newtonian order is given by

$$\Delta t \equiv t_2 - t_1 = -\frac{1}{c}\left[\boldsymbol{x}_1(t_1) - \boldsymbol{x}_2(t_2)\right] \cdot \boldsymbol{k} + (\Delta t)_{\text{grav}}, \tag{1}$$

where $\boldsymbol{x}_i(t_i)$ denotes the barycentric coordinate position of antenna i at coordinate time t_i. $(\Delta t)_{\text{grav}}$ is the gravitational time delay, resulting from solving the equation for null geodesics (light rays) in some background metric describing the gravitational influence of the Sun and the planets. To sufficient accuracy $(\Delta t)_{\text{grav}}$ can be written as a sum over the contributions of the various massive bodies in the solar system.

Now, for picosecond accuracy it is sufficient to consider the spherical part of the gravitational potential in $(\Delta t)_{\text{grav}}$ only. Taking earlier results from Richter and Matzner (1983) we estimate the contribution from the quadrupole moment of the Sun to the time delay to be much less than a picosecond ($\sim 10^{-18}$ s). The effect from the angular momentum of the Sun (a gravitomagnetic effect of 1.5 post-Newtonian order) is of the same order, while the dominant post-post Newtonian terms are expected to be less than about 0.5 picoseconds.

For the solar contribution we can neglect the motion of the Sun about the barycenter and the usual "light-time equation" for the spherical field can be written in the form (e.g. Soffel, 1989)

$$(\Delta t)_{\text{grav}}^{\odot} \simeq \frac{2m_\odot}{c} \ln\left(\frac{|\boldsymbol{x}_1| - \boldsymbol{x}_1 \cdot \boldsymbol{k}}{|\boldsymbol{x}_2| - \boldsymbol{x}_2 \cdot \boldsymbol{k}}\right), \tag{2}$$

where $\boldsymbol{x}_i$ refers to t_i and $m_\odot \equiv GM_\odot/c^2 = 1.48\,\text{km}$. The time difference Δt can be neglected in the ln-term and writing

$$\boldsymbol{x}_i = \boldsymbol{x}_\oplus + \Delta\boldsymbol{r}_i$$

we obtain (Finkelstein et al., 1983; Zeller et al., 1986):

$$(\Delta t)^{\odot}_{\mathrm{grav}} \simeq \frac{2m_{\odot}}{c}\ln\left[\frac{r_{\oplus}(1-e_{\oplus}\cdot k)+\Delta r_1\cdot(e_{\oplus}-k)+(\Delta r_1)^2/2r_{\oplus}-(e_{\oplus}\cdot\Delta r_1)^2/2r_{\oplus}}{r_{\oplus}(1-e_{\oplus}\cdot k)+\Delta r_2\cdot(e_{\oplus}-k)+(\Delta r_2)^2/2r_{\oplus}-(e_{\oplus}\cdot\Delta r_2)^2/2r_{\oplus}}\right] \quad (3)$$

with

$$e_{\oplus}\equiv x_{\oplus}/r_{\oplus};\quad r_{\oplus}=|x_{\oplus}|=(x^i_{\oplus}x^i_{\oplus})^{1/2}.$$

For baselines of $\sim 6000\,\mathrm{km}$, the $(\Delta r_i)^2$-terms are of order 3×10^{-14} sec and can be neglected for picosec-accuracy.

For the gravitational time delay due to the Earth one finds ($m_{\oplus}=GM_{\oplus}/c^2=0.44\,\mathrm{cm}$)

$$(\Delta t)^{\oplus}_{\mathrm{grav}} \simeq \frac{2m_{\oplus}}{c}\ln\left[\frac{|\Delta r_1|-\Delta r_1\cdot k}{|\Delta r_2|-\Delta r_2\cdot k}\right], \quad (4)$$

if the motion of the Earth during signal propagation is neglected. Similarly, for any other planet A, if its motion is neglected, one obtains

$$(\Delta t)^{A}_{\mathrm{grav}} \simeq \frac{2m_A}{c}\ln\left[\frac{|x_{A1}|-x_{A1}\cdot k}{|x_{A2}|-x_{A2}\cdot k}\right], \quad (5)$$

where $x_{Ai}=x_i-x_A$. Note that the maximal gravitational time delays due to Jupiter, Saturn, Uranus and Neptune are of order 1.6(Jup), .6(Sat), .2(U) and .2(N) nanosec resprectively, but these values decrease rapidly with increasing angular distance from the limb of the planet. E.g. 10 arcmin from the center of the planet the gravitational time delay amounts only to about 60 picosec for Jupiter, 9 picosec for Saturn and about one picosec for Uranus. To consider the barycentric motion of the planet during signal propagation the position of the planet might be taken at the time of closest approach (e.g. Hellings, 1986); it is, however, unclear how good this correction for the planet's velocity really is.

Let us define baselines at signal arriving time t_1 at antenna 1. Let the barycentric baseline b be defined as

$$b(t_1)\equiv x_1(t_1)-x_2(t_1), \quad (6)$$

then a Taylor expansion of $x_2(t_2)$ about t_1 yields ($O(n)\equiv O(c^{-n})$)

$$\begin{aligned}\Delta t = &-\frac{1}{c}(b\cdot k)\left[1+\frac{1}{c}(\dot{x}_2\cdot k)+\frac{1}{c^2}(\dot{x}_2\cdot k)^2-\frac{1}{2c^2}(b\cdot k)(\ddot{x}_2\cdot k)\right]\\ &+(\Delta t)_{\mathrm{grav}}+O(4),\end{aligned} \quad (7)$$

all quantities now referring to barycentric coordinate time t_1. We call this relation the "VLBI-delay equation", describing the barycentric coordinate time delay Δt entirely by quantities defined in the global system.

We will now relate the various barycentric quantities with corresponding geocentric ones apart from the propagation vector k. This will remind us that the process of signal propagation from the source to the antennas cannot be formulated in the local, accelerated, geocentric system. We now write the time transformation in the form (Damour et al., 1990a-c)

$$
\begin{aligned}
ct &= z^0_{\oplus}(T) + e^0_a(T)X^a + O(3) \\
&= \int_{T_0}^{T} c e^0_0 \, dT' + e^0_a(T)X^a + O(3) \\
&= c(T - T_0) + \frac{1}{c}\int_{T_0}^{T} \left(\overline{U}(z_{\oplus}) + \frac{1}{2}v^2_{\oplus} \right) dT' + \frac{1}{c} R^i_a(T) v^i_{\oplus}(T) X^a + O(3).
\end{aligned}
$$

Here, T = TCG is the geocentric coordinate time and X^a the geocentric spatial coordinate. $\overline{U}$ is the *external* gravitational potential which does *not* include the contribution from the Earth. Replacing T' by t' in the integral and considering that R^i_a is a slowly time dependent matrix we can relate $\Delta t = t_2 - t_1$ with the corresponding local time interval $\Delta T = \Delta\mathrm{TCG} = T_2 - T_1$:

$$
\begin{aligned}
\Delta t = \Delta T &+ \frac{1}{c^2}\int_{t_1}^{t_2} \left(\overline{U}(z_{\oplus}) + \frac{1}{2}v^2_{\oplus} \right) dt' \\
&+ \frac{1}{c^2} R^i_a v^i_{\oplus}(T_2) X^a_2(T_2) - \frac{1}{c^2} R^i_a v^i_{\oplus}(T_1) X^a_1(T_1) + O(4).
\end{aligned}
$$

With

$$
v^i_{\oplus}(T_2)X^a_2(T_2) \simeq v^i_{\oplus}X^a_2 - v^i_{\oplus}V^a_2 \left(\frac{b \cdot k}{c} \right) - a^i_{\oplus}X^a_2 \left(\frac{b \cdot k}{c} \right) + O(2),
$$

where quantities on the right hand side now refer to T_1, we formally get the relation

$$
\begin{aligned}
\Delta t = \Delta T &+ \frac{1}{c^2}\left[-\mathbf{v}_{\oplus} \cdot \mathbf{B} + \int_{t_1}^{t_2} \left(U(z_{\oplus}) + \frac{1}{2}v^2_{\oplus} \right) dt' \right] \\
&- \frac{1}{c^3} [(\mathbf{v}_{\oplus} \cdot \mathbf{v}_2)(\mathbf{b} \cdot \mathbf{k}) + (\mathbf{a}_{\oplus} \cdot \Delta \mathbf{r}_2)(\mathbf{b} \cdot \mathbf{k})] + O(4), \qquad (8)
\end{aligned}
$$

where

$$
\mathbf{B}(T_1) = \mathbf{X}_1(T_1) - \mathbf{X}_2(T_1)
$$

and

$$B^i \equiv R^i_a B^a$$

$$v^i_2 \equiv R^i_a V^a_2$$

$$\Delta r^i_2 \equiv R^i_a X^a_2.$$

v_2 is the geocentric velocity of antenna 2. Next, we will relate the barycentric baseline vector $b(t_1)$ appearing in the VLBI-delay equation (7), with the corresponding geocentric one $B(T_1)$. Using the notation of Damour et al. (1990a-c) we find

$$x^i_1(t_1) - x^i_2(t_2) = z^i_\oplus(t_1) - z^i_\oplus(t_2) + R^i_a\left(X^a_1(T_1) - X^a_2(T_2)\right) - \frac{1}{c^2}\left(\frac{1}{2}v^i_\oplus v^k_\oplus + \overline{U}(z_\oplus)\delta_{ik}\right)B^k + \xi^i(T_1, X^a_1) - \xi^i(T_2, X^a_2) \quad (9)$$

with

$$\xi^i = \frac{1}{c^2}e^i_a(T)\left[\frac{1}{2}A^a_\oplus X^2 - X^a(A_\oplus \cdot X)\right] + O(4) \quad (10)$$

and

$$A^a_\oplus(T) \equiv e^i_a(T)\frac{d^2 z^i_\oplus}{dT^2}.$$

With

$$z^i_\oplus(t_1) - z^i_\oplus(t_2) \simeq -v^i_\oplus \Delta t - \frac{1}{2}a^i_\oplus(\Delta t)^2,$$

$$x^i_1(t_1) - x^i_2(t_2) \simeq b^i(t_1) - (\Delta t)\dot{x}^i_2(t_1) - \frac{1}{2}(\Delta t)^2\ddot{x}^i_2(t_1)$$

and

$$X^a_1(T_1) - X^a_2(T_2) \simeq B^a(T_1) - \left(\frac{d}{dT}X^a_2\right)\Delta T - \frac{1}{2}\left(\frac{d^2}{dT^2}X^a_2\right)(\Delta T)^2,$$

where

$$\Delta T = \Delta t + \frac{1}{c^2}v_\oplus \cdot B + O(3),$$

(the integral in (8) is practically of order c^{-3}) the desired relation between barycentric and geocentric baselines reads

$$b^i = B^i - \frac{1}{c^2}(v_\oplus \cdot B)v^i_2 - \frac{1}{c^2}\left(\frac{1}{2}v^i_\oplus v^k_\oplus + \overline{U}(z_\oplus)\delta_{ik}\right)B^k + \Delta\xi^i + O(3) \quad (11)$$

with

$$\Delta\xi^i \equiv \xi^i(T_1, X^a_1) - \xi^i(T_2, X^a_2).$$

Using equation (8), the VLBI-delay equation (7) for (Δt) and the relation (11) for the baselines we obtain the formal expression ($B \cdot k = B^i k^i$ etc.):

$$
\begin{aligned}
\Delta T = \\
& -\frac{1}{c}(B \cdot k) - \frac{1}{c^2}(B \cdot k)k \cdot (v_\oplus + v_2) + \frac{1}{c^2}(v_\oplus \cdot B) \\
& + \frac{1}{c^3}(v_\oplus \cdot v_2)(B \cdot k) - \frac{1}{c^3}(B \cdot k)[k \cdot (v_\oplus + v_2)]^2 + \frac{1}{c^3}(B \cdot k)\overline{U}(z_\oplus) \\
& + \frac{1}{2c^3}(v_\oplus \cdot k)(v_\oplus \cdot B) + \frac{1}{c^3}(v_\oplus \cdot B)(v_2 \cdot k) - \frac{1}{c^2}\int_{t_1}^{t_2} \left(\overline{U}(z_\oplus) + \frac{1}{2}v_\oplus^2\right) dt' \\
& - \frac{1}{c}k^i \Delta\xi^i + \frac{1}{2c^3}(B \cdot k)^2 k(a_\oplus + \dot{v}_2) + \frac{1}{c^3}(B \cdot k)(a_\oplus \cdot \Delta r_2) \\
& + (\Delta t)_{\text{grav}} + O(4).
\end{aligned}
\tag{12}
$$

Keeping only terms with amplitudes greater than 1 picosec for baselines of the order of 6000 km, we approximately find:

$$
\begin{aligned}
\Delta T = \\
& -\frac{1}{c}(B \cdot k) - \frac{1}{c^2}(B \cdot k)k \cdot (v_\oplus + v_2) + \frac{1}{c^2}(v_\oplus \cdot B) \\
& + \frac{1}{c^3}(v_\oplus \cdot v_2)(B \cdot k) - \frac{1}{c^3}(B \cdot k)[k \cdot (v_\oplus + v_2)]^2 + \frac{1}{c^3}(B \cdot k)\left(2\overline{U}(z_\oplus) + \frac{1}{2}v_\oplus^2\right) \\
& + \frac{1}{2c^3}(v_\oplus \cdot k)(v_\oplus \cdot B) + \frac{1}{c^3}(v_\oplus \cdot B)(v_2 \cdot k) + (\Delta t)_{\text{grav}}.
\end{aligned}
\tag{13}
$$

Finally, the geocentric coordinate time T can be related with proper time τ as indicated by some (atomic) clock located at some VLBI station. Neglecting all tidal effects on local clock rates for clocks at rest at the Earth's surface we find

$$
\frac{d\tau}{dT} \simeq 1 - \frac{1}{c^2}\left(U_\oplus(X) + \frac{1}{2}(\Omega \times X)^2\right) = 1 - \frac{1}{c^2}U_{\text{geo}}(X), \tag{14}
$$

where $U_\oplus$ is the gravitational potential of the Earth and Ω is the angular velocity of the Earth's rotation. This can be written in the form

$$
d\tau \simeq \left[1 - \frac{1}{c^2}U_{\text{geo}}^0 + \frac{1}{c^2}g(\psi)h\right] dT, \tag{15}
$$

where U_{geo}^0 is the geopotential at the geoid, $g(\psi) = (9.78027 + 0.05192\sin^2\psi) \times 10^2\ \text{cm/s}^2$ is the latitude dependent gravity acceleration and h is the height above the geoid. Instead of using this formula for the $T \leftrightarrow \tau$ relation, we split it into two parts, defining $\tau^* = \mathrm{TT}$ as proper time on the geoid:

$$d\tau^* = d(\mathrm{TT}) \equiv \left(1 - \frac{1}{c^2} U^0_{\mathrm{geo}}\right) dT \equiv \kappa_0 \, dT = \kappa_0 \, d(\mathrm{TCG}) \tag{16a}$$

$$d\tau = \left(1 + \frac{1}{c^2} g(\psi) h\right) d\tau^*. \tag{16b}$$

The constant κ_0 relating τ^* with the geocentric coordinate time T has the numerical value

$$\kappa_0 \simeq 1 - 6.9 \times 10^{-10}.$$

Finally we would like to adress the question of the orientation of spatial coordinates of the local geocentric system. This orientation is determined by the matrix R^i_a (remember that in eqs.(12) and (13) $B \cdot k = B^i k^i$ with $B^i = R^i_a B^a$). There are two preferred choices for R^i_a leading to geocentric coordinates which are either

- fixed star oriented (kinematically non-rotating)
- or locally inertial (dynamically non-rotating).

In the first case of kinematically non-rotating coordinates we can take $R^i_a = \delta^i_a$. Then the geodesic precession will be in the precession-nutation matrices as well as in the dynamical equations (e.g. for satellies orbiting the Earth); it will not appear in the group delay equations (12) or (13). On the other hand if dynamically non-rotating geocentric coordinates are chosen then the geodesic precession (secular and annual term) has to be included in the R^i_a matrix. In this case the precession-nutation matrices (and dynamical equations) do not contain the geodesic precession.

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SYSTEMATIC CORRECTIONS TO THE FUNDAMENTAL CATALOGUE DUE TO THE PRECESSION ERROR AND THE EQUINOX CORRECTION

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ABSTRACT. Fundamental catalogues of stars have systematic errors due to the precession error and the equinox correction. The formula for these errors to be applied to the FK4 is presented.

1. Introduction

Fundamental catalogues of stars have been compiled mainly from meridian observations. The adopted precession constant is obtained from analyses of stars' proper motions, and the adopted position at any epoch of the zero-point in right ascension (equinox) is obtained from analyses of positional observations of the Sun, Moon and planets in addition to a study of stars' proper motions. When a new fundamental catalogue is constructed, a decision is made whether the precession constant and/or the position of the equinox should be changed or not. In the case of the Fifth Fundamental Catalogue (FK5) (Fricke *et al.* 1988), the correction to Newcomb's precession constant obtained by Fricke (1977) and the correction to the position of the FK4 equinox derived by Fricke (1982) are adopted.

In the course of investigating the transformation from FK4 system to FK5 system, Sôma and Aoki (1990) have obtained the formula for the systematic correction to the FK4 due to the errors in the precession constant and in the location of the equinox. In deriving the formula they assumed the equinox correction (error in the location of the equinox) to be expressed by a linear function of time, but the linearity cannot be assumed *a priori*.

In this paper we derive the formula for the systematic correction to the FK4 without assuming the linearity of the equinox correction and show that Sôma and Aoki's assumption is valid up to the 10^{-10}/century^2.

2. Derivation of the Formula

We will deal with the systematic corrections to the FK4 due to the errors in the precession and in the location of the equinox. For this purpose we will ignore the systematic corrections to the FK4 at the epoch of B1950.0. In this paper it is assumed that the E terms of aberration (the elliptic part of aberration due to the eccentricity of the Earth's orbit) are already removed from the positions and proper motions.

The position vector r and velocity v in this paper are related to the right ascension α, declination δ, proper motions in right ascension μ_α (in radians/tropical century) and in declination μ_δ (in radians/tropical century), radial velocity V (in km/sec), and parallax π (in radians) by the following formulae:

$$r = \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \quad \text{and} \quad v = \mu_\alpha \begin{pmatrix} -\cos\delta\sin\alpha \\ \cos\delta\cos\alpha \\ 0 \end{pmatrix} + \mu_\delta \begin{pmatrix} -\sin\delta\cos\alpha \\ -\sin\delta\sin\alpha \\ \cos\delta \end{pmatrix} + 21.094502\pi V r.$$

The 3×3 unit matrix is denoted by I.

The position vector r and velocity v of a star in the FK5 system are related to the position vector r_1 and velocity v_1 of the star in the FK4 system by the following equations:

$$\begin{aligned} &P_{\mathrm{IAU76}}(\mathrm{B1950.0},t)(r+vt)|_{t=0} = R_3(-E(t))P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(r_1+v_1t)|_{t=0}, \\ &\frac{d}{dt}[P_{\mathrm{IAU76}}(\mathrm{B1950.0},t)(r+vt)]|_{t=0} = \frac{d}{dt}[R_3(-E(t))P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(r_1+v_1t)]|_{t=0}, \end{aligned} \tag{1}$$

where the vectors r, v, r_1, and v_1 are evaluated at the epoch of B1950.0, $P(T_1, T_2) = R_3(-z_A)R_2(\theta_A)R_3(-\zeta_A)$ is the 3×3 precession matrix for the equatorial coordinates from the epoch T_1 to the epoch T_2 based on either Newcomb's precession (subscript NEWC) or the IAU (1976) precession (subscript IAU76), R_i is the standard 3×3 rotation matrix about the i_{th} axis, $E(t)$ is the equinox correction, and t is the time reckoned from B1950.0 in tropical centuries. The variables ζ_A, z_A and θ_A denote the equatorial precession parameters (see Lieske *et al.* 1977). The above equations were given by Aoki *et al.* (1983) and their correctness has been confirmed by Sôma and Aoki (1990).

Since

$$P(\mathrm{B1950.0},t)|_{t=0} = I \quad \text{and} \quad \frac{d}{dt}P(\mathrm{B1950.0},t)|_{t=0} = \begin{pmatrix} 0 & -m & -n \\ m & 0 & 0 \\ n & 0 & 0 \end{pmatrix}, \tag{2}$$

we obtain from solving Eq. (1) the following:

$$r = Mr_1 \quad \text{and} \quad v = Nr_1 + Mv_1, \tag{3}$$

where

$$\begin{aligned} M &= \begin{pmatrix} \cos E_0 & -\sin E_0 & 0 \\ \sin E_0 & \cos E_0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ N &= \begin{pmatrix} (m^N - m^O - \dot{E})\sin E_0 & (m^N - m^O - \dot{E})\cos E_0 & n^N - n^O\cos E_0 \\ -(m^N - m^O - \dot{E})\cos E_0 & (m^N - m^O - \dot{E})\sin E_0 & -n^O\sin E_0 \\ -n^N\cos E_0 + n^O & n^N\sin E_0 & 0 \end{pmatrix}. \end{aligned} \tag{4}$$

In the above equations E_0 and $\dot{E}$ are the values of the equinox correction and its first derivative at B1950.0 derived by Fricke (1982), and m and n are the rates of general precession in right ascension and declination, respectively, based on the Newcomb precession (superscript O) and the IAU (1976) precession (superscript N). The quantities m and n are obtained from

$$m = \frac{\partial}{\partial t}(\zeta_A(T,t) + z_A(T,t))|_{t=0} \quad \text{and} \quad n = \frac{\partial}{\partial t}(\theta_A(T,t))|_{t=0}, \tag{5}$$

where T is evaluated at the epoch of B1950.0.

The position vector $s_{\rm FK4}(t)$ in the FK4 system with respect to the mean equinox of date is given by

$$s_{\rm FK4}(t) = P_{\rm NEWC}({\rm B1950.0}, t)(r_1 + v_1 t). \tag{6}$$

The position vector $s_{\rm FK5}(t)$ in the FK5 system with respect to the mean equinox of date is given by

$$\begin{aligned} s_{\rm FK5}(t) &= P_{\rm IAU76}({\rm B1950.0}, t)(r + vt) \\ &= P_{\rm IAU76}({\rm B1950.0}, t)[(M + Nt)r_1 + Mtv_1]. \end{aligned} \tag{7}$$

The difference between (6) and (7) is mainly due to the equinox correction at the epoch t.

The systematic correction other than the equinox correction expressed as $E + \dot{E}t$ is obtained by

$$\begin{aligned} & s_{\rm FK5}(t) - R_3(E_0 + \dot{E}t)s_{\rm FK4}(t) \\ = & \left[\begin{pmatrix} +0.0003 & +0.3170 & -0.1235 \\ -0.3170 & +0.0001 & +5.9231 \\ +0.1234 & -5.9233 & +0.0002 \end{pmatrix} \times 10^{-8} t^2 \right. \\ & + \begin{pmatrix} +0.0059 & +0.0991 & -0.0657 \\ -0.0415 & +0.0071 & +0.0009 \\ -0.0668 & -0.0005 & -0.0013 \end{pmatrix} \times 10^{-8} t^3 \\ & \left. + \begin{pmatrix} +0.0016 & -0.0001 & +0.0000 \\ +0.0001 & +0.0016 & +0.0000 \\ +0.0000 & +0.0007 & +0.0001 \end{pmatrix} \times 10^{-8} t^4 \right] r_1 \\ + & \left[\begin{pmatrix} +0.0000 & +1.1539 & -2.1113 \\ -1.1539 & +0.0000 & +0.0247 \\ +2.1113 & -0.0247 & +0.0000 \end{pmatrix} \times 10^{-6} t^2 \right. \\ & + \begin{pmatrix} +0.0053 & +0.0034 & -0.0018 \\ -0.0032 & +0.0258 & +0.0120 \\ +0.0012 & -0.0480 & -0.0205 \end{pmatrix} \times 10^{-6} t^3 \\ & \left. + \begin{pmatrix} +0.0001 & +0.0006 & +0.0000 \\ -0.0004 & +0.0001 & +0.0000 \\ -0.0006 & +0.0000 & +0.0000 \end{pmatrix} \times 10^{-6} t^4 \right] v_1, \end{aligned} \tag{8}$$

where the difference is expanded to the polynomial of t. The r_1-term is the regional systematic correction, and v_1-term is the correction depending on the star's velocity which contributes to the individual correction.

3. Comparison with the Previous Result

Eq. (8) shows the systematic corrections to the FK4 due to the errors in precession and in the location of the equinox. The corresponding difference in accord with the discussion by

Sôma and Aoki (1990) is obtained from their Eqs. (12a) and (13):

$$
\begin{aligned}
&P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)[(X_2 - X_1X_0^{-1}X_1)X_0^{-1}t^2 + (X_3 - X_2X_0^{-1}X_1)X_0^{-1}t^3]r_1 \\
&\quad + P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(X_1X_0^{-1}t^2 + X_2X_0^{-1}t^3)v_1 \\
&= \left[\begin{pmatrix} +0.0002 & +0.3148 & -0.1354 \\ -0.3148 & +0.0001 & +5.9231 \\ +0.1354 & -5.9232 & +0.0002 \end{pmatrix} \times 10^{-8}t^2 \right. \\
&\quad + \begin{pmatrix} +0.0058 & +0.1006 & -0.0579 \\ -0.0430 & +0.0070 & +0.0007 \\ -0.0746 & -0.0007 & -0.0014 \end{pmatrix} \times 10^{-8}t^3 \\
&\quad \left. + \begin{pmatrix} +0.0017 & -0.0001 & -0.0001 \\ +0.0001 & +0.0016 & +0.0002 \\ +0.0000 & +0.0007 & +0.0001 \end{pmatrix} \times 10^{-8}t^4 \right] r_1 \\
&+ \left[\begin{pmatrix} +0.0000 & +1.1539 & -2.1112 \\ -1.1539 & +0.0000 & +0.0247 \\ +2.1112 & -0.0247 & +0.0000 \end{pmatrix} \times 10^{-6}t^2 \right. \\
&\quad + \begin{pmatrix} +0.0053 & +0.0034 & -0.0019 \\ -0.0031 & +0.0258 & +0.0120 \\ +0.0014 & -0.0480 & -0.0205 \end{pmatrix} \times 10^{-6}t^3 \\
&\quad \left. + \begin{pmatrix} +0.0001 & +0.0002 & -0.0007 \\ +0.0001 & +0.0001 & -0.0001 \\ +0.0000 & +0.0000 & +0.0000 \end{pmatrix} \times 10^{-6}t^4 \right] v_1. \qquad (9)
\end{aligned}
$$

Note that Eq. (12a) of Sôma and Aoki is with respect to the reference frame of B1950.0 and therefore the difference is multiplied by P_{NEWC} to express the difference with respect to the frame of date.

The difference between (8) and (9) is less than 10^{-10}/century^2 which is equal to 2×10^{-5}arcsec/century^2. Therefore the linearity assumption of the equinox correction is valid within this accuracy.

The calculations were carried out on the FACOM M780/10S of the National Astronomical Observatory of Japan.

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INTERMEDIATE STAR REFERENCE SYSTEMS IN THE VICINITY OF RADIO SOURCES

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ABSTRACT. In the framework of CONFOR program the formation of star lists of two intermediate reference star systems is being carried out. The first list, RRS2, contains meridian stars in the fields centered at extragalactic radio/optical sources. The second one is formed on the base of 12-14 magnitude stars. The observations are in progress now. The main purpose of this program is to form a base for investigation of mutual orientation of fundamental reference system and new ones.

In the frame of program CONFOR described earlier by Gubanov et al.(1989) the work on establishment of intermediate reference systems for photographic determinations of extragalactic radiosource positions is being carried out. This program has as an aim the investigation of connection wetveen radiointerferometric and optical coordinate frames. The work is

Fig.1. Distribution of RRS2 stars
by visual magnitude

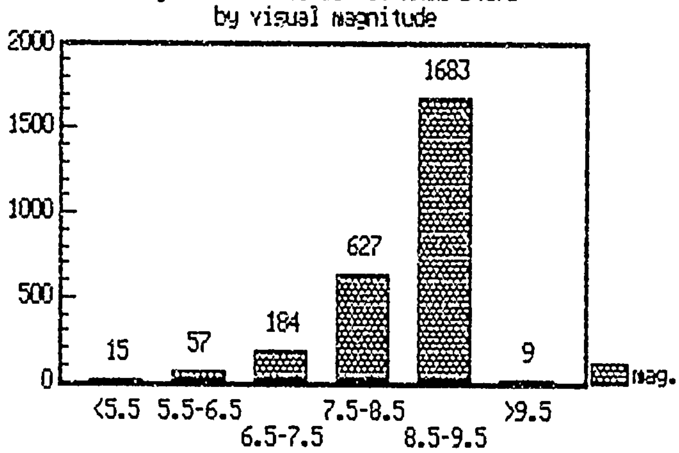


Fig.2. Displacement of radiosources
from the center of reference star
systems

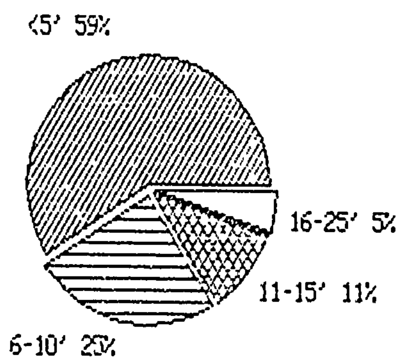


Fig.3. Density of RRS2 stars versus declination (number of stars per square degree)

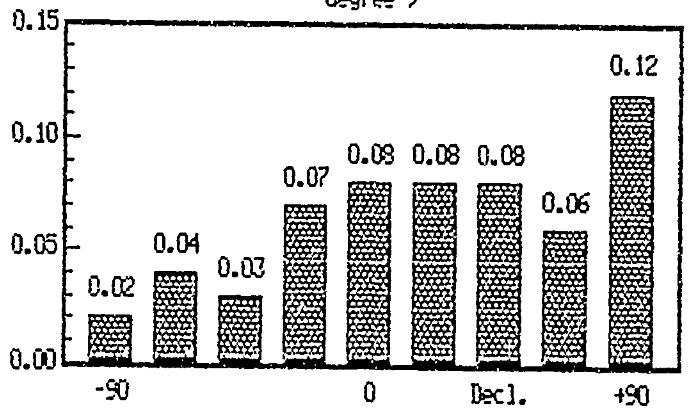


Fig.4. Distribution of RRS2 stars by spectral classes

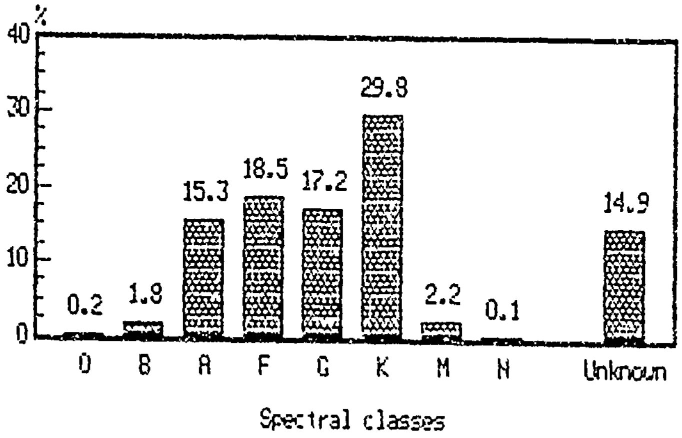
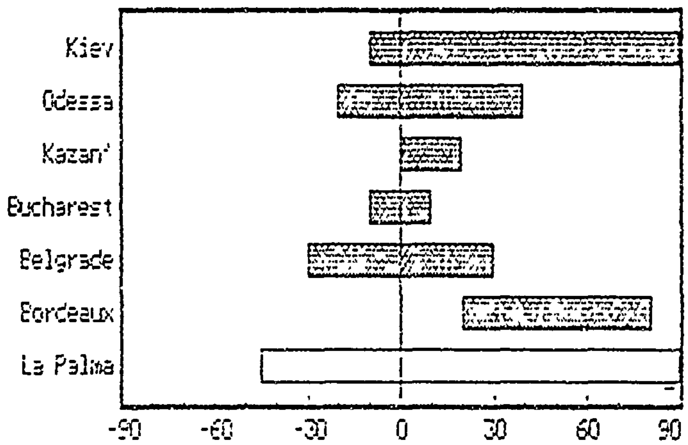


Fig.5. Status of RRS2 observations



in progress now concerning formation of observational lists and position determinations of stars belonging to two systems of intermediate reference stars in the vicinity of 238 extragalactic sources by Argue et al. (1984).

The first system (RRS2 list) contains from 9 to 13 stars per field. They are attainable for visual TCs, see Tel'nyuk-Adamchuk and Molotay (1989). The prepared list RRS2 contains 2575 stars throughout the whole sky. The mean visual magnitude of RRS2 stars is 8.5. There are near 90 per cent stars brighter than 9.1 visual magnitude in the RRS2 list (Fig.1). The angular distances between sources and RRS2 stars do not exceed 40 arcmin as a rule. Displacement of reference stars center from a source in the field is less than 5 arcmin as a rule (Fig.2). The distribution of RRS2 stars in declination zones and in spectral classes are represented in Fig.3 and Fig.4.

The corresponding observations started in 1989 using several TCs (Kiev, Kazan', Odessa, Bucharest, Belgrade). Several other observatories (e.g. Bordeaux) are planning to join this campaign. There is more than a half of RRS2 stars in the program of Carlsberg meridian circle. The zones of observations are shown in Fig.5. Meanwhile it is very important to use modern instruments,especially southern ones. All the data concerning the stars of RRS2 list can be sent by Kiev Observatory to any obobservatory which is interested in.

On the base of the material available now and that obtained as a result of the given program it is planned to compile the combined catalogue of RRS2 stars for the purposes of astrometry of extragalactical sources.

In 1988 Kiev Univ.Observatory started the astrophotographic observations (astrograph 20/430 cm) of these fields to choose and determine the positions of stars of 12-14 mag.in the fields of 20-30 arcmin centered at each radiosource.There are 10-20 stars of this kind in each field. The coordinates of the stars mentioned will be obtained on the base of RRS2 stars as the reference ones. For the sake of positional accuracy we plane some spectrophotometrical investigations of 12-14 mag. stars.

It is reasonable to specify clearly the star list of these two systems and to observe them intentionally collecting the positional data. This will enable to spread the optical frame onto the intermediate stars as well as onto the sources themselves. As a result a good base for investigation of the mutual orientation of the fundamental frame and new ones will be given. The astrometrical data for the sets of stars in two magnitude ranges in many fields of the sky will be useful for purposes of stellar astronomy too.

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satellites and quasars, gives us the possibility of determining astronomical coordinates with greater precision than the classical optical techniques. If we are aiming at precision of 1 mas (0".001), we have to be extremely careful about our definitions of models, constants, and reference systems.

2. Observed values of the nutations

The values adopted for the main term of the nutation in obliquity, the so-called constant of nutation , during the last hundred years, were based on the observed values while the following terms (semiannual and fortnightly) were based on theoretical values. This approach is, of course, inconsistent from a logical point of view but it was justified for two main reasons:

1. The main term, due to its importance, was derived from observations because in any transformation of coordinates we should have the best possible value available in order to minimize the errors; the observed value, corresponding to the real structure of the Earth, satisfied that requirement because the theoretical value corresponds to an imperfect theory.
2. The semiannual and fortnightly nutations were derived from theory because their magnitudes were so small that it was very difficult to determine reliable values from the classical optical observations.

Let us describe briefly the history of this subject to see not only the limitations imposed by the theory of forced precession and nutation, but also to avoid the mistakes made in the past. Some of the observed values of the main term of the nutation in obliquity (N), determined from classical optical observations, are shown in Table I.

TABLE I

AUTHOR	INTERVAL OF THE OBSERVATIONS	N
Newcomb(1895)	old observations of greater confidence	9".210±0".008
Spencer Jones (1939)	1911-1936	9".2066±0".0055
Morgan(1943)	1903-1925	9".206±0".007
Fedorov(1958)	1900-1934	9".1980±0".0018

The great difficulty in determining this nutation from the observations stems from the fact that it has a long period of about 19 years, and, therefore, the observations should cover several periods in agreement with the best principles of statistical analysis. Unfortunately, the great majority of observations, so far done, do not satisfy this fundamental requirement.

Let us comment briefly on the values of N listed in Table I. The value determined by Newcomb was based on observations made during several periods employing different observational techniques, but it has the inconvenience common to all observations made more than a century ago, that is, the low precision of the techniques employed at that time. The values determined by Morgan (1943) and Spencer Jones (1939) correspond to 22 and 25 years of observations and, therefore, are not as reliable. The value obtained by Fedorov (1958) covers a period of 34 years, which is therefore better than any other values but, again, not reliable enough from a proper statistical point of view.

The short period nutations, semiannual and fortnightly, have been difficult to determine from the classical optical observations because of their smaller amplitudes. All these values suffered from the fact that the instruments were localized at only one observatory or that they were determined from the International Latitude Service (ILS) chain of instruments situated at the same latitude. Nevertheless, the results were remarkable and we must remember that, in those days, we had scanty knowledge about geodynamics, namely, the behavior of the core and plate tectonics.

The advent of the modern technique of Very Long Baseline Interferometry (VLBI) has opened the possibility to determine the values of nutation from this type of observation. Here we have to distinguish the case of the short period nutations from the main nutation. So far, the VLBI observations (Herring et al. 1986) suffer from the same difficulties we have already pointed out for the classical observations, that is, very few instruments and the localizations of the observatories are not the best from the point of view of global tectonics and the stability of the sites.

We know very well the need for regular and systematic observations and, therefore, the more reliable VLBI observations are the ones corresponding to the International Radio Interferometric Surveying (IRIS) network which started around 1984. This short interval of observations conditions immediately the determination of the values of the two types of nutation.

1. It rules out the possibility of determining the main nutation, with a period of about 19 years, or of any other long period nutation. This is in agreement with the proper use of statistical techniques. A number of research papers have been published, dealing with the determination of the main nutation term, but the results cannot be trusted for the above mentioned reasons.
2. The case of the short period nutations, namely, the semiannual and fortnightly, is slightly better because the observations of the IRIS network already cover several periods. The difficulty with these nutations is that their amplitudes are very small and, therefore, the need for a well-distributed network of observatories is very important. This last condition is not yet satisfied by the IRIS network and, therefore, the results so far obtained should be considered with great care.

3. Comparison of theoretical and observed values of the nutations

Let us consider briefly the history of the comparison between theoretical and observed values, and the relationships between luni-solar precession and nutation. The research of Hill (1893) gave for the theoretical expressions of the constants of luni-solar precession P and nutation N the following :

$$P = \frac{C - A}{C}\left(\alpha_1 + \alpha_2 \frac{\mu}{1+\mu}\right), \qquad N = \frac{C - A}{C}\,\alpha_3 \frac{\mu}{1+\mu} \qquad (1)$$

where A and C are the principal moments of inertia of the Earth, μ is the ratio of the mass of the Moon to that of the Earth, and α_1, α_2, and α_3 are accurately known constants for a certain epoch. These expressions show the close relationships between precession and nutation which is logical because they correspond to complementary parts of the theory of the Earth's rotation.

The rigid body theory developed by Woolard (1953) employed Newcomb's value of the constant of nutation (9".2100) for the determination of the amount of the other nutations, for instance, the semiannual (0".5522) and fortnightly (0".0884) in obliquity.

The several comparisons of the observed and theoretical values of the nutations, done, for instance, between 1930 and 1950, showed that the theoretical values were always different from the observed ones. One of the first scientists to point out this discrepancy was Jackson (1930). The explanation of this disagreement was one of the outstanding difficulties of the system of astronomical constants. The approach of Herring (1988) to derive observational corrections to the principal nutations from VLBI observations is not valid for all the reasons already mentioned. Incidentally, We should notice that most VLBI results are derived employing the same computer program, for instance "CALC", and, therefore, the final values are similar.

Another comparison has been obtained by McCarthy and Luzum (1991) employing VLBI data and Lunar Laser Ranging (LLR) data. In spite of the fact that the VLBI series is not yet of sufficient duration, the combination with the longer LLR series improves the results. The authors use the correct procedure of employing the adopted IAU theory of nutation (Seidelmann 1982) and determining corrections for the principal nutation terms. In this way we can improve the knowledge of the nutation coefficients from the observations which represent the behavior of the real earth.

Recent research on the comparison of catalogues of extragalactic radio sources employed in VLBI observations (Walter 1991), point out that the nutation models applied to the reduction for the original measurements have not been used in a consistent way. This is obviously an additional cause of concern.

4. Influence of the Earth's structure on precession and nutation

The theory of the Earth's rotation up to the time of Woolard (1953) was based on models of the Earth considered as a solid and rigid body. One of the reasons for admitting this hypothesis was the lack of detailed knowledge about the internal structure of our planet. The advances made in internal geophysics, especially based on seismological research, permitted the determination of the main parameters for all the layers which were considered significant for defining the internal structure (Bullen 1963).

One of the main features of the structure of the Earth is the existence of a core divided in two regions - outer and inner core. The outer core is considered as fluid while the inner core is considered as solid; the expressions fluid and solid for the core have seismological meaning, referring to the behavior of the core when seismic waves propagate through it. We do not yet have the possibility of reproducing in our laboratories the conditions of pressure and temperature which exist inside the core. The importance of a fluid core on studies dealing with precession and nutation of rotating bodies have been pointed out in some classical papers (Hough 1895, Poincaré 1910). Unfortunately, all these studies, when applied to the case of the Earth, only considered the case of a rigid Earth, and, therefore, they were not very adequate models because they ignored the existence of elasticity. The research of Jeffreys' (1949, 1950) emphasized the importance of the liquid core.

The theory of nutation that first considered the existence of a liquid core and the elasticity of the mantle was developed by Jeffreys and Vicente (1957). Another important feature of this theory refers to the relevance of the ellipticity of the core for the values of certain types of nutation. This theory also pointed out that the external forces (due to the Sun and Moon) which cause the forced nutation also give rise to tidal attractions, deforming the Earth, and corresponding to the tides of the solid Earth. For instance, the forced nutations correspond to diurnal tides. This theory shows that, because of the ellipticity of the core, there is a great difference between the displacements that alter the direction of the principal axis and other displacements that do not affect its position. All diurnal tides tend to alter the position of the axis of rotation and, therefore, apply to the forced nutation.

We must remember that in the dynamics of rotating bodies we only have nutation, that is, oscillations of the axis of the body around a main axis considered as fixed. The fact that some of the nutations present special features led to their classification as precession. In the case of the Earth, the luni-solar precession corresponds to a nutation with a period of about 26,000 years and that implies certain features for this motion.

The solution of the equations of motion corresponding to the theory of the Earth's rotation, adopting a Lagrangian formulation, that is, in terms of displacements, reveals that the roots are grouped in pairs near certain values. This is an important feature in this theory, producing a phenomenon which was called "double resonance" by Poincaré (1910). We have to consider two cases.

1. If there is only resonance, that is, only one root very near to the period of one of the free oscillations of the system, the system behaves nearly as a solid body and the fluid core has no influence on the period of the oscillation.
2. If there is double resonance, that is, a pair of roots very near to the period of one of the free oscillations of the system, the influence of the fluid core becomes important and the amplitude of the nutation will be different in comparison with a solid body.

Case 1 applies to the luni-solar precession and, therefore, the existence of the fluid core does not affect the values obtained for the precession because the Earth behaves as a solid body. Case 2 applies to the forced nutations, namely, having periods of 6798.4, 182.6 and 13.7 days. The existence of elasticity does not alter the conclusions referring to double resonance. The fundamental difference between Case 1 (resonance - valid for the luni-solar precession) and Case 2 (double resonance - valid for the luni-solar nutation) shows that it is far more difficult to consider suitable models of the Earth's structure for the theory of nutation than for the theory of precession (Vicente, 1964).

The more recent theory of nutation was developed by Wahr (1981) and adopted by the International Earth Rotation Service (IERS) Standards. The

Earth's model adopted is not the best one available at the moment but the differences will not probably be significant if another model were adopted. For the question of consistency, it would be convenient to consider the Preliminary Reference Earth Model (PREM) which has been widely adopted by the international community (Dziewonski and Anderson, 1981). The adoption of the PREM model will not introduce very significant differences in agreement with Déhant (1990, Table 4) and the same happens considering the mantle inelasticity (Déhant, 1990, Table 3) when we consider precision of 1 mas (0".001).

There are a number of studies (Wahr and Bergen, 1986; Déhant 1988) dealing with different aspects of the behavior of the core and the mantle; they are interesting studies in mathematical physics but do not offer much improvement for the theory of nutation. On the other hand, a number of geophysical studies are based on Wahr's computer programs and, therefore, the results have to be similar.

We must remember that the equations of fluid dynamics and elasticity are partial differential equations and we do not know exact solutions. For this reason, all the solutions presented for different Earth models are approximations and, sometimes, it is very difficult to compare their results, especially when we are aiming at precisions of 1 mas. One very important point is the treatment of the boundary conditions for elliptical, rotating Earth models which has not been emphasized in most studies, especially in view of the critical importance of the ellipticity of the core.

There are attempts to combine new nutation series derived from rigid body dynamics with geophysical data. One of them (Zhu et al., 1990) tries to incorporate the very short series of VLBI observations and, therefore, is not reliable for the reasons already stated (Zhu et al., 1990, Fig 5 and 6). The comparisons of nutation corrections, considering different geophysical hypotheses, as was done by Zhu et al. (1990, Table XIV) justifies the criticism already mentioned - the different behavior of the principal nutation terms in longitude and obliquity, and the unreliable values of the compared results for precisions of 1 mas.

5. Theoretical and observed values of the luni-solar precession

We have shown that the Earth's structure does not have much influence on this type of precession and, therefore, the theory of precession does not need to consider such complex models as the theory of nutation. There is general agreement about the theory, and the expressions for precession have been published for several systems of astronomical constants (Lieske et al., 1976). Due to the fact that this precession has a very long period, it is necessary to accumulate observational data covering an interval of time as long as possible. This is one of the great difficulties in its determination, and the subject has been studied during the last decades (Fricke 1967, 1971). The relationship between the luni-solar precession and the general precession in longitude shows the importance of this precession and, therefore, for the determinations of the equinox and equator for any given epoch (Fricke 1982).

In any case, the values derived from the observations correspond to a better representation of the phenomenon than the theory where a number of approximations had to be made in order to be amenable to a numerical solution of the system of equations. A recent estimation of the principal precession and nutation terms has been done by Charlot et al. (1991) based on two decades of LLR and one decade of VLBI data. The improvement in the values obtained for the precession and the longer period nutations is noticeable, especially in the combined solution due to the longer series of LLR data. Nevertheless, we have to remember the past difficulties of determining precession from a short series of observations and so we have to be careful about these results. We must remember that the Deep Space Network (DSN) and the LLR network have very few stations, and we know already the implications of this fact for determining the short period nutation especially.

6. Conclusions

We have seen that the theory of luni-solar precession does not depend so much on the existence of the fluid core and its ellipticity and, therefore, it is easier to obtain agreement between theory and observation. The reasons given before show the need for a suitable model of the Earth's structure in order to define an adequate theory of the nutation. Wahr's theory is adequate for the time being and it is not convenient, for the purposes of a system of astronomical constants, to change fundamental values very often.

In any case, the theories of precession and nutation can only give us approximations to the real behavior of the Earth, because the models adopted do not include all the variables needed to describe such phenomena. For this reason, it is necessary that the values of the principal terms in precession and nutation be derived from observations. The observational series have to be done in a most careful and well planned way, covering adequate intervals of time and applying proper statistical analysis. The modern techniques of LLR and VLBI do not yet have series of observations that satisfy all these important requirements, including a well distributed network of observing stations, especially satisfying the stringent requirements of geophysical stability needed when we try to attain a precision of 1 mas.

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ANALYSIS AND SYNTHESIS OF CATALOGUES OF EXTRAGALACTIC RADIO SOURCES

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ABSTRACT. Recent observation catalogues of extragalactic radio sources obtained by Very Long Baseline Interferometry agree in the mean to a few milliarcseconds (mas). Within this range the position differences show constant, linear and periodic offsets. To cast light on these offsets the differences between some of the representative observation catalogues are plotted. Especially, the periodic variations of declination differences of observation catalogues having significantly different epochs are tentatively explained by an uncertainty of the general precession in declination. In terms of the luni-solar precession this uncertainty is estimated to be of the order of -1 to -2 mas per year.

1. Introduction

Observation catalogues of extragalactic radio sources set up by VLBI reflect the properties of the respective instrumental systems leading to catalogue differences of 1 to 2 milliarcseconds (mas). The more recent VLBI observation catalogues are assessed in pairs by simply plotting right ascension (RA) and declination (Dec) differences vs RA and Dec, respectively.

The difference patterns evidence an uncertainty of the luni-solar precession and, for some catalogues, a treatment of nutation not strictly complying with the IAU recommendations.

An independent approach to catalogue comparison aims at a compilation catalogue derived from the individual observations by weighted least squares adjustment of right ascensions, declinations, systematic catalogue differences and, optionally, precession terms. Results are presented below.

2. Analysis

Table 1 lists the observation catalogues taken into consideration. Each catalogue pair gives rise to four plots. Only a few plots of $\Delta\delta$ vs RA are displayed here for discussing some marked features.

TABLE 1. Selection of radio interferometric catalogues

Designation	Reference	Number of objects *)	Mean standard deviation (mas)		Mean epoch (1900+)	
			RA	Dec	RA	Dec
Short baseline interferometry:						
Q07	Wade and Johnston, 1977	30	32.3	30.3	75.4	75.4
Q15	Kaplan et al., 1982	14	16.7	13.9	79.9	79.9
Very long baseline interferometry:						
Q17	Fanselow et al., 1984	98	3.6	3.8	79.2	79.1
Q19B	Ma et al., 1986	75	1.2	2.8	81.9	81.9
Q22A	Sovers et al., 1988	106	1.3	1.6	83.4	83.3
Q23	Ma, 1988	164	0.8	0.9	85.2	85.4
Q27	Sovers, 1989	150	1.5	2.0	84.7	84.5
Q28	Ma et al., 1990	182	0.6	0.6	85.4	85.5

*) outliers omitted

2.1 CATALOGUES Q17 AND Q27

Apart from an offset of -2 mas in Fig. 1 there is evidence of a periodic variation given by

$$\Delta\delta = \delta(Q17) - \delta(Q27) = A \cos \alpha$$

which may be attributed to the variation of Dec as a function of the general precession in declination (n). Bearing in mind the different epochs t_1 and t_2 of Q17 and Q27, the amplitude A seems to be caused by the uncertainty Δn of the general precession in declination.

Enforcing agreement of the declinations by disposing of Δn yields

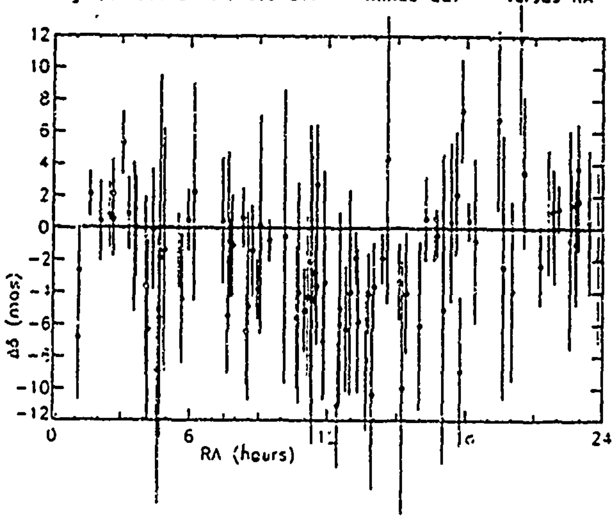
$$A \cos \alpha + (t_2 - t_1) \; \Delta n \cos\alpha = 0.$$

Taking $t_2 - t_1 = 5.4$ years and $A \simeq 4$ mas from Fig. 1, one gets by this geometrical interpretation the rough estimate of

$$\Delta n = - 0.74 \text{ mas/yr}$$

which is equivalent to the correction $\Delta\psi = - 1.9$ mas/yr of the luni-solar precession ψ.

Fig. 1. Dec differences Q17 minus Q27 versus RA



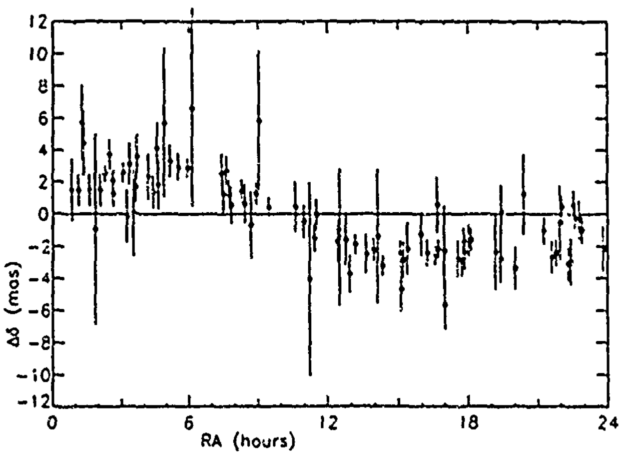
For comparison, a weighted least squares fit of precession in declination to the coordinate differences Q17 minus Q27 resulted in corrections of the luni-solar precession of

$$\Delta\psi_\alpha = -0.98 \pm 0.44 \text{ mas/yr} \quad \text{and} \quad \Delta\psi_\delta = -1.17 \pm 0.20 \text{ mas/yr}$$

for RA and Dec, respectively.

2.2. CATALOGUES Q27 AND Q28

Fig. 2. Dec differences QUA27 minus QUA28 versus RA



Only a small epoch difference exists between catalogues Q27 and Q28. In these circumstances the sinusoidal variation in declination of Fig. 2 could be associated with a modification of the nutation in obliquity ($\Delta\varepsilon$) in the course of data reduction for the one or the other of the two catalogues. To first order, the declination difference can be associated with

$$\Delta\delta = \Delta(\Delta\varepsilon) \sin\alpha.$$

The relative change of $\Delta\varepsilon$ in the models of nutation would amount to 3-4 mas indicating a rotation of that order about the x-axis of the conventional coordinate system.

3. Synthesis

A compilation catalogue has been derived from catalogues of Table 1 by simultaneously solving for source positions and systematic catalogue differences in a least squares adjustment process.

By having recourse to the compilation catalogue an uncertainty of the luni-solar precession could be determined by solving for a precession correction in a post-fit analysis (Walter, 1990). Moreover, a global solution of the compilation catalogue with the precession as additional unknown could be accomplished. Table 2 presents some typical results of these two approaches. For testing the effect of lengthening the time baseline, two catalogues built from measurements of connected interferometry, Q07 and Q15, have been included.

TABLE 2. Corrections $\Delta\psi_\alpha$ and $\Delta\psi_\delta$ to the luni-solar precession derived independently from RA and Dec measurements

Observation catalogues	Number of objects	$\Delta\psi_\alpha$ (mas/yr)	$\Delta\psi_\delta$ (mas/yr)	Number of obs. in RA and Dec
<u>Post-fit analysis</u>				
Q17	40	-1.69 ± 0.46	-1.60 ± 0.28	40
Q17, Q22A	40	-1.60 ± 0.52	-1.20 ± 0.35	80
Q17, Q22A, Q27	40	-1.28 ± 0.46	-0.89 ± 0.35	120
Q17, Q19B, Q22A, Q23, Q27, Q28	40	-0.71 ± 0.21	-0.74 ± 0.21	240
<u>Simultaneous solution</u>				
Q17, Q22A, Q27	40	-0.86 ± 0.36	-0.82 ± 0.22	120
Q17, Q19B, Q22A, Q23, Q27, Q28	40	-0.96 ± 0.18	-0.55 ± 0.15	240
Q07, Q15, Q17, Q19, Q22A, Q23, Q27, Q28	8	-3.13 ± 0.27	-2.42 ± 0.15	64

4. Conclusions

Observation catalogues have reached a level of internal accuracy allowing the identification of uncertainties in the physical model of precession provided that the catalogue epochs cover a baseline of at least 5 years. This uncertainty has been confirmed by three different methods: (1), direct comparison of observation catalogues, (2), post-fit analysis of the luni-solar precession by referring to a compilation catalogue, and, (3), global solution for source position, systematic differences and precession.

The results have in common the negative sign for the correction of the luni-solar precession while its magnitude ranges from -1 to -3 mas/yr depending on the data selected for the various case studies.

From comparing catalogues of nearly equal epochs periodic variations of milliarcsecond amplitude are detected. They may be ascribed to specific nutation models applied to the reduction of the original measurements, confirming the occurence of relative rotations between coordinate systems (Ma et al., 1990).

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LONG-TERM NUTATION AND THE LENGTH DAY VARIATION FROM VLBI OBSERVATIONS

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ABSTRACT. The shift of the celestial pole with respect to its 1980 position in longitude ($\Delta\psi$) and obliquity ($\Delta\varepsilon$) from the available VLBI measurement of 1984 year within IRIS project is estimated. Formal uncertainties of the angles $\Delta\psi$ and $\Delta\varepsilon$ are 5.9 mas and 2.7 mas respectively. The day length variation was obtained from the same data. Then attempt was made to obtain annual and semiannual nutation amplitudes. The values are in reasonable agreement with the determinations of other workers.

The aim of the work is to test a software developed in the IAA (Krasinsky et al.,1989) by processing the VLBI observations which were available for us within IRIS project during 1984. In IRIS project 19 sources were observed by 3-5 stations every 5-days (24-hour session).The analysis of about 28000 delay and delay rate pairs provides of different parameters.The reduced observations of every session were processed and 64 parameters were estimated.

They included source coordinates, site locations. time behavior parameters, tropospheric correctoons, day length variation. The root-mean-square residuals are 0.2 ns for delay and 0.1 ps/s for delay rate. The main purpose of the present paper is to analyze these results and to determine shift of the celestial pole with respect to its 1980 IAU nutation in longitude and obliquity and the day length variation. It's known that VLBI measurements provide good determinations of some amplitudes in the current model the Earth nutation (see, for instance, William E.Carter, 1987). We had the total of 1798 residuals of α and δ of the below source coordinates for the determination of celestial pole shift in longitude and obliquty during 1984 year. The residuals for every 5-days intervals numbered 64 points. Formal uncertainties range from 0.001" for regularly observed sources up to 0.01" for sources which were observed only 1-2 times.

Table 1. The list of source names in 1984 IRIS project and their errors of our determination.

Source name	σ_α (0.001")	σ_δ (0.001")
2134+00		±2.9
1637+574	±4.0	2.1
OQ208	16.2	1.8
3C345	1.7	1.4
3C454.3	1.2	2.4
VR422201	1.8	2.0
0106+013	1.0	3.2
0212+735	5.8	2.1
0234+285	2.2	2.6
NRAO150	4.2	2.4
0552+398	1.7	2.1
OJ287	1.1	2.2
4C39.25	1.6	1.7
3C273B		2.8
1803+784	7.6	1.5
0528+134	1.9	3.0
2216-038	3.8	11.9
0229+131	1.3	3.2
3C279		

For obtaining $\Delta\psi$ and $\Delta\varepsilon$ we used the standard formulae

$$\alpha-\alpha_0 = \Delta\psi(\cos\varepsilon+\sin\varepsilon\sin\alpha_0 tg\delta_0)-\Delta\varepsilon\cos\alpha_0 tg\delta_0$$
$$\delta-\delta_0=\Delta\psi\sin\varepsilon\cos\delta_0+\Delta\varepsilon\sin\alpha_0$$

By weighted least square method 167 parameters such as $\Delta\psi$ and $\Delta\varepsilon$ for every 24-hour session, corrections to 18 coordinates of sources (18 for $\Delta\alpha$ and 18 for $\Delta\delta$) were determinated. The estimated values display a marked systematic variation and are shown in Figure 1 and 2 for $\Delta\psi$ and $\Delta\varepsilon$ respectivly. The RMS value of correction to $\Delta\psi$ is 5.9 mas and to $\Delta\varepsilon$ is 2.7 mas as mentioned before.

The attempt was made to determine annual and semi-annual values of nutation amplutudes. In Table 2 the annual values are shown as compared to other workers' determinations. It may be notices that the agreement is reasonably good (O.J.Sovers et C.D.Edvards). Figure 3 shows the day length variations which is also of good agreement with other determinations (G.O.Dicky, 1988). The accuracy of determinations at every points is no worse then 0.1 msec.

We plan to continue our work in the same direction using a great number of observations and we hope to improve our technique.

Table 2. Corrections to 1980 IAU annual nutation amplitude. (0.001 arcsec)

Term	IRIS 1980-87	CDP 1979-87	CDN 1978-86	IRIS (IAA) 1984,IAA	Herring Observ.	Adopt.
$\Delta\psi$	4.30	4.26	5.16	4.01	5.22	5.23
	±0.07	±0.07	±1.08	±0.72	±0.25	
$\Delta\varepsilon$	2.01	1.84	1.93	2.18	2.08	2.08
	±0.02	±0.02	±0.43	±0.90	±0.10	

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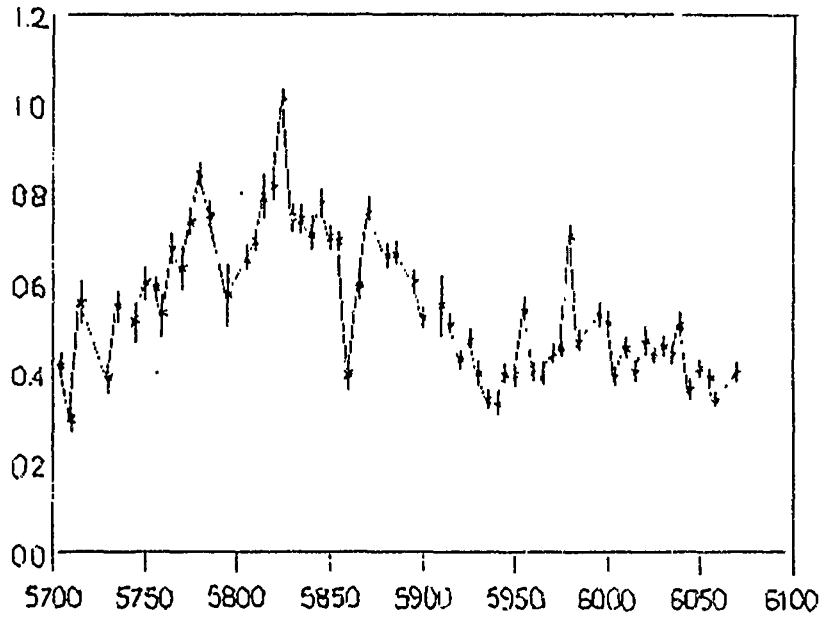
Length of Day

(MSec)



Date (1984 year) Fig. 3

Figure 1. Nutation in longitude

(in 0.001 arcsec)

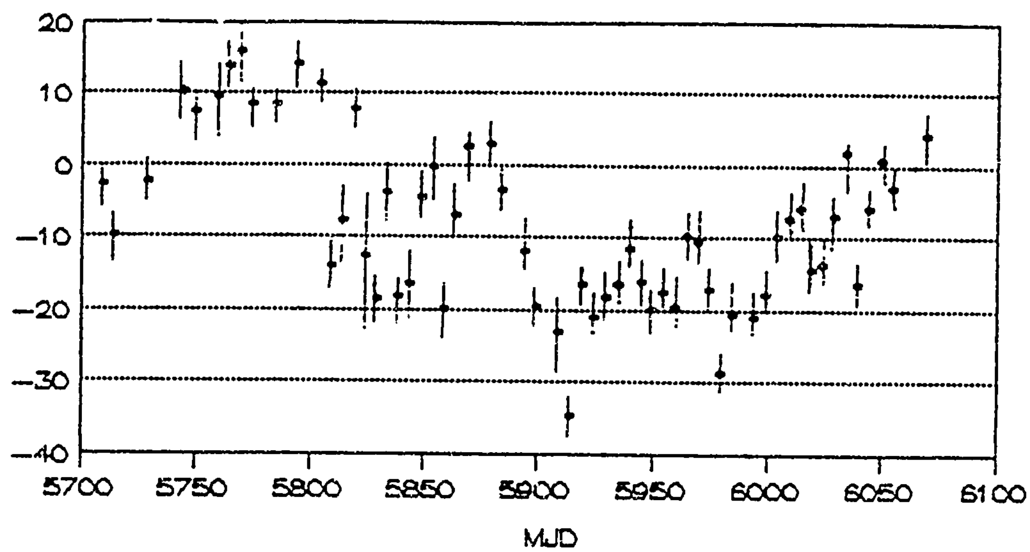
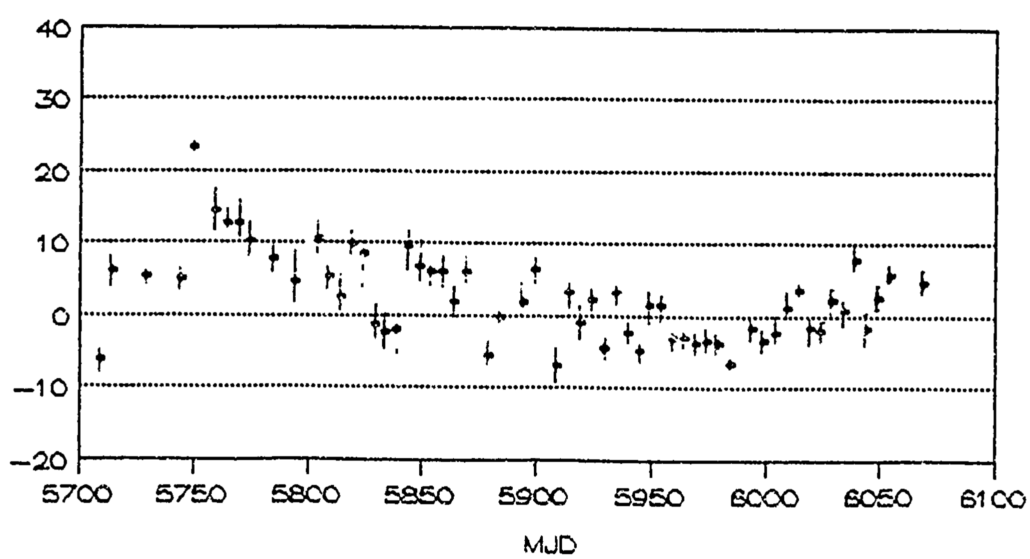


Figure 2. Nutation in inclination

(in 0.001 arcsec)



NOTE ON THE DETERMINATION OF THE NEARLY DIURNAL FREE NUTATION (CORE NUTATION) AND THE PRINCIPAL TERM OF NUTATION BASED ON THE ASTROMETRIC DATA

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1. INTRODUCTION

Due to the highly accurate determination of the corrections to the nutation terms (the IAU-1980 nutation theory) by VLBI observations, the proposal for adoption of the new nutation theory or standard model of nutation is being discussed. At this point there are two unsolved problems:

(a) VLBI observations can't provide reliable estimates of the principal nutation term. The reason is that the correlation between this term and the precession constant is high due to the insufficient time span of these observations (about 10 years).
(b) period of free core nutation (FCN) which depends on the ellipticity of the mantle-core boundary has not been, so far, reliably determined from direct observations.

In the present paper, we would like to draw attention to the results of the analysis of astrometric latitude and time observations which could be useful for resolving these problems.

2. SEARCH FOR THE NEARLY DIURNAL FREE NUTATION

From the papers by Yatskiv et al., 1975; Yatskiv, 1979; Kovbasyuk, 1984, 1985, 1988; and by some other authors, it is clearly seen that the spectra of the nearly diurnal variations of latitudes and longitudes of the observatories have a complicated and time-stable structure. The frequency and amplitude characteristics of the spectra don't depend on the observatory location and method of observation. The values of the amplitudes of main oscillations vary from 0.003 to 0.010 arcseconds. (These values exceed by one order of magnitude the amplitude of the FCN derived from the VLBI observations.) Moreover, the phase characteristics seem to indicate the existence of both the retrograde (predicted by the theory) and prograde (suggested by Yatskiv et al.,1975) components of nearly diurnal polar motion.

Thus, for the most general model of the nearly diurnal variations of the astronomical coordinates we take

$$\sum_j A_{ji} \cos[\pm\sigma_j(S_i - S_{0i}) + \beta_{ji} \pm \sigma_j \lambda_i], \tag{1}$$

where A_{ji} is the amplitude of the j-th harmonic derived from observations of the i-th observatory; $\sigma_j = 2\pi/\tau_j$ is the frequency of the j-th harmonic, where τ_j is the period of the nearly diurnal variation expressed in mean or sidereal days; the "+" sign corresponds to the clockwise retrograde wobble; S_i is the local sidereal or mean time reckoned from the origin of this time, S_{0i}; β_{ji} is the initial phase of the harmonic (west longitude of the nearly diurnal free wobble, reckoned from Greenwich meridian $\lambda_G = 0$ at a moment S_{0G}); λ_i is the west longitude of the i-th observatory.

Depending on the type of observations, one can determine either separate harmonics of (1) or their combination. We would note that there are three different modes of observations involved in the search for FCN:

(a) the day-and-night observations of the bright zenith stars and pairs of stars (program of station at Gorky, USSR);
(b) the observations of selected bright zenith stars at constant moment of sidereal time (program of observatory at Poltava, USSR);
(c) the observations of stars and groups of stars at constant (or nearly constant) moments of mean time (the ILS program).

All these methods have their own limitations. In case (a), it is impossible to determine separately the prograde and retrograde wobble components from the observations of a single station. In the other two cases, one can determine only the transformed oscillations with frequencies $(\sigma_0 - \sigma_j)$, where σ_0 is the sampling frequency which is equal to 1 cycle per sidereal day or 1 cycle per mean day for cases (b) and (c), respectively. The amplitudes and phases of transformed oscillations are the combinations of the amplitudes and phases of the nearly diurnal variations of astronomical coordinates given in eq. (1).

Comparing the results of analysis of different observational data allowed us to make the above-mentioned conclusions on the spectra of the nearly diurnal latitude variations.

Let us consider some examples.

2.1. EXAMPLE 1

Giver the spectrum of the nearly diurnal variations of latitude at the Gorky station, calculate a spectrum convolved about the frequency $\sigma_0 = 1$ cycle per sidereal day and compare it with the observed spectrum of latitude variation at Poltava (Popov and Yatskiv, 1979).

Based on the latitude observations at Gorky from 1961 to 1976, the spectrum of nearly diurnal variations was derived in the frequency domain $\sigma_j = 2\pi(1-0.000058K)$, where K is equal to ±1, ±2, ±3, etc. The initial phases of harmonics were converted to the initial epoch of the Poltava observations, i.e., $S_{op} = 0^h$ of sidereal time on March 1, 1950.

The symmetric harmonics in eq. (1) (for the case of the theoretically predicted retrograde component) were summed up:

$$A_j' \cos[(\sigma_o + \Delta\sigma_j)(S - S_o) + \beta_j'] + A_j'' \cos[(\sigma_o - \Delta\sigma_j)(S - S_o) + \beta_j''], \qquad (2)$$

where $\Delta\sigma_j = (\sigma_o - \sigma_j)$ is the frequency of the transformed oscillation. After that, the amplitudes and phases of the resulting oscillations $\tilde{A}_j$ and $\tilde{\beta}_j$ were derived.

When summing up the symmetric harmonics in the case of the prograde nearly diurnal wobble, the difference between the longitudes of the Gorky and Poltava stations has to be taken into account.

In Table 1, the values of the amplitudes and phases of the transformed harmonics which are in common for the Gorky and Poltava stations are given. To each value of aliasing period T_j, corresponds two values of the nearly diurnal periods τ_j. The agreement of results, except for the harmonic with the period T=454 sidereal days, is satisfactory, taking into account the standard errors of amplitude and phase which are ±3 marcsec and ±30 degrees, respectively. As for the harmonic T=454 sidereal days, the phase difference could be explained partly by the presence of the prograde nearly diurnal wobble.

Table 1. Amplitudes (in marcsec) and phases (in degrees) of the nearly diurnal latitude variations of the Gorky and Poltava stations.

Period (sidereal days)		Poltava		Gorky	
T_j	τ_j	$\tilde{A}_j$	$\tilde{\beta}_j$	$\tilde{A}_j$	$\tilde{\beta}_j$
321	0.996894 1.003125	12	113	9	160
391	0.997449 1.002564	19	290	7	350
454	0.997802 1.002207	10	190	7	308
961	0.998960 1.001042	8	177	6	200

2.2. EXAMPLE 2

Given the spectrum of the nearly diurnal latitude variations of the Gorky station, calculate a spectrum convolved about the frequency σ_o=1 cycle per mean day and compare it with the observed spectrum of variations of the ILS stations (Yatskiv et al.,1975).

For this purpose, we have adopted the values of aliasing periods as well as the direction of polar motion given in Yatskiv et al.,1975. The results are summarized in Table 2.

Table 2. Amplitudes (in marcsec) and phases (in degrees) of the nearly diurnal latitude variations of the Gorky and ILS stations.

Period (mean days)		ILS		Gorky	
T_j	τ_j	$\tilde{A}_j$	$\tilde{\beta}_j$	$\tilde{A}_j$	$\tilde{\beta}_j$
retrograde component					
169.1	0.994121 1.005948	3	33	3	5
192.1	0.994822 1.005232	4	95	5	69
209.6	0.995253 1.004793	6	266	9	235
235.6	0.995773 1.004263	5	348	12	282
prograde component					
186.3	0.994660	7	208+2λ	8	255+2λ
204.4	0.995169	7	204+2λ	7	216+2λ
246.9	0.995966	10	298+2λ	5	253+2λ

The agreement of results is remarkable. Similar results based on the data of other observatories were given in Kovbasyuk, 1980.

2.3. DISCUSSION

From the results presented above, we can conclude that the nearly diurnal variations of the latitudes really exist. They can be explained by:

(a) variations of meteorological conditions of observatory sites;
(b) variations of vertical lines of observatories;
(c) motion of the pole (within the Earth body and in space) which is due to the existence of the Earth's liquid core.

The first reason seems to be unreal because we have used the data of different observatories (with different conditions and methods of observation). As for the second reason, it has some physical meaning, but it has to be excluded, otherwise the corresponding changes of gravity would be observed. Finally, the third reason at the present time can't be confirmed because of the lack of theoretical modelling of the wave motions in the Earth's core (Dehant, 1988; Melchior et al.,1988). If we suppose that the oscillations with periods of 0.994822 and 0.995253 mean days (see Table 2) result from amplitude variations (damping) of theoretically predicted FCN, we find the average value of the period of this nutation is 446 sidereal days and the damping factor Q~2500 (the value of the average period, 0.995038 mean days, corresponds to a period of 23h 56m 46.5s of sidereal time). These estimates of period and Q-factor are in agreement with estimates based on recent observations.

Now the question arises, why do FCN based on the VLBI observations show amplitudes one order of magnitude smaller than the amplitudes derived from astronomical observations? To answer this question, it is necessary to consider the problem of the VLBI observables and search for FCN. First of all, the prograde nearly diurnal wobble will result in the time delay observable as the semidiurnal variation. Such variation was never searched for before. In the case of retrograde, nearly diurnal wobble (and corresponding long-period FCN), the effect of correlation of this wobble with other solved-for parameters has to be investigated for different observational programs.

3. DETERMINATION OF CORRECTIONS TO THE COEFFICIENTS OF THE PRINCIPAL TERM OF NUTATION

In Table 3, the most accurate (from our point of view) astrometric determinations of corrections to the amplitudes of the prograde and retrograde circular nutations with a period of 18.6 years are given.

Taking the average values of these amplitudes, we find the values of the coefficients of the principal term of nutation to be:

in longitude $-\Delta\psi \sin\theta = 6.8444 \pm 0.0006$ arcsec,
in obliquity $\Delta\theta = 9.2052 \pm 0.0006$ arcsec.

These values agree, within the errors, with the modern determinations of correction to nutation by VLBI observations and Lunar Laser Ranging (LLR) (Charlot et al.,1990).

So, the necessity of adopting corrections to coefficients of the principal term of nutation is evident.

Table 3. Amplitudes (in 0.0001 arcsec) of the retrograde and prograde circular nutations with a period of 18.6 years at J 2000.0

Author and reference	Mean epoch of observations	Retrograde	Prograde
Yatskiv (1980, 1989) The mean values from the astrometric determinations carried out before 1980	1950	80250±10	11790±10
Capitaine et al., (1988) Z-term based on the BIH data from 1962 to 1982	1972	80245±09	11813±09
Glebova et al,(1990) Observations of the latitude at Pulkovo from 1948 to 1989	1968	80236±29	11792±29
Lapaeva (1988) Observations of the latitude at the Engelhardt observatory from 1957 to 1976	1967	80238±22	11766±22
Mean values	1965	80248±06	11804±06
IAU-1980 theoretical values		80220	11804

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ASTROMETRIC AND GEODETIC GOALS FOR THE CHINESE VLBI NETWORK PROJECT

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ABSTRACT. This paper briefly introduces the current status of the Chinese VLBI Network (CVN) Project and its astrometric and geodetic goals.

1. Current Status of CVN Project

At present, the CVN project consists of three dedicated VLBI stations, two part-time VLBI stations, and a data analysis center. More information about the progress of the CVN project is as follows:

1.1 VLBI Stations

(1) Sites

Code	Station	Antenna (m)	Lat. N (°,′)	Long. E (°,′)
SH	Sheshan Radio Astronomy Station	25	31 06	121 12
UR	Urumqi Radio Astronomy Station	25	43 30	87 13
KM	Kunming Radio Astronomy Station	32	25 01	102 47
MY	Miyun M-wave Synthesis Radio Astronomy Station	47 (equiv.)	40 40	117 58
QH	Qinghai Mm-wave Radio Astronomy Station	13.7	37 21	97 36

(2) Equipment						
Items	Station	Dedicated			Part-Time	
		SH	UR	KM	MY	QH
Antenna		†	+	–	†	†
Receivers	330-MHz	†	+	–	+	
	610-MHz	+	+	–		
	1.4-GHz	†	+	–		
	1.6-GHz	†	+	–		
	2.3-GHz	†	+	–		
	4.9-GHz	†	+	–		
	8.4-GHz	†	+	–		
	10.7-GHz	†	+	–		
	12.2-GHz	†	+	–		
	22.2-GHz	–	–	–		†
Data Acquisition Terminals	Mk II	†	†	†	+	+
	Mk III or VLBA	†	–	–		–
Frequency Standards	H maser	†	+	+		–
	Rb	†	†	†	–	
Timing Receivers	Loran-C	†	†	†		
	GPS	†	+	–	–	–
Operation Start		87	92	94	92	92

Codes: † Available; + Fabrication started or ordered; – Planned.

1.2 VLBI Data Analysis Center

The data analysis center of CVN is located at Shanghai and operated by Shanghai Observatory, Chinese Academy of Sciences. The main facilities in the VLBI data analysis center are as follows:

Facilities		Available	Remark
VLBI Correlator:	S–2	1988	Compatible with Mk II
	S–3	1993/94	Compatible with Mk III and VLBA
Computer:	HP-1000 F series	1985	For the data analysis of astrometric and geodetic VLBI Mk III experiments.
	MicroVAX II	1988	For the postprocessing of the data from S–2 correlator and the data analysis of continuum and line VLBI experiments.
	VAX 3800	1991	VLBI data analysis
	Sun 4/11	1991	VLBI data analysis

2. Astrometric and Geodetic Goals for the CVN Project

- To measure the positions of the extragalactic compact radio sources (one source/$5° \times 5°$; declination: $-30°$ to $+90°$) for the establishment of the radio reference system;
- To monitor the structure variations and the proper motions of the radio sources for the maintenance of the radio reference system;
- To measure the positions and proper motions of radio stars and cosmic masers associated with late-type stellars for the linkage between the radio and optical reference systems;
- To measure the positions and proper motions of the pulsars combining the pulsar-timing data for the determination of the equinox;
- To measure the Earth rotation parameters;
- To measure the positions of the CVN stations with mm accuracy combining the Chinese SLR and GPS networks for the establishment of the Chinese Crustal Deformation Monitoring Network (CCDMN) and the global terrestrial reference system;
- To measure the crustal motions between the eastern, north-western, and south-western regions of China;
- To measure the relative motions between the Chinese continent and the surrounding plates, e.g., Pacific, North American, Australian, Indian, and Philippine plates and to monitor the stability of the Eurasian plate;
- To control the orientation and scale of the Chinese National Geodetic Control Network.

PROGRESS IN FIRST TOKYO PMC PROGRAM, 1985-1991

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ABSTRACT. Presented is a brief introduction to advances in the first Tokyo PMC program which began in 1985 and is to end in 1991.

1. Star List of First Tokyo PMC Program

The Tokyo Photoelectric Meridian Circle (Tokyo PMC) has been observing its first program since 1985. The *First Tokyo PMC Program* comprises about 33,000 program stars, as well as the observations of the solar system objects, i.e., the Sun, six major and nine minor planets, to locate the dynamical reference frame. The program stars are composed of several subprograms of different priorities; the FK5 stars (FK5 basics and FK5 bright extension), NPZT stars, OB stars, zodiacal stars, NIRS (AGK3RN), and several minor subprogram stars (H_2O masers, faint stars around extragalactic radio sources, and so on). The nine minor planets are: Ceres, Pallas, Juno, Vesta, Hebe, Iris, Flora, Metis, and Eunomia.

Tables 1 and 2 show the details of the star list together with some informations on advances of our first program: Table 1 is for normal stars and table 2 for the solar system objects. Shown in figure 1 are the distributions in equatorial coordinates of all program stars in First Tokyo PMC Program.

Table 1. First Tokyo PMC Program from Dec. 1985 to Mar. 1992†

	FK5 basic stars	FK5 bright extension	NPZT stars	OB stars	Zodiacal stars	Northern IRS	minor programs‡
Source Cat.	FK5	FK4 Suppl.	Tokyo PZT	Rubin, Yale,AGK3	SAO	AGK3RN	AGK3, Mira var.
Obs. period	'86-'91	'86-'91	'87-'91	'87-'91	'87-'91	'87-'91	'87-'91
Star number	1234	1523	1717	3204	3284	20195	1800
Stars observed more than n times§; n	1218 16	1365 6	831 2	2620 2	1806 4	10807 2	1234 2

†The expected total number of the effective observations is 200,000 up to March 15, 1992.
‡H_2O masers, faint stars around QSOs, and others.
§Up to Sep. 1, 1990.

Table 2. Observations of the solar system objects¶

Sun and major planets

Sun	Venus	Mars	Jupiter	Saturn	Uranus	Neptune
334	91	51	140	87	52	62

Minor planets

Cerres	Pallas	Juno	Vesta	Hebe	Iris	Flora	Metis	Eunomia
73	61	59	58	53	64	61	46	57

¶From Jan. 1987 to Dec. 1989.

2. Annual Catalog Series of First Tokyo PMC Program

The first annual catalog (*Tokyo PMC 85*) was published in 1987 (Yoshizawa et al. 1987); it contains the positions of 1007 stars observed in 1985 and referred to the FK4 system. The second annual catalog (*Tokyo PMC 86*) was published in 1989 (Yoshizawa and Suzuki 1989). The Tokyo PMC 86 catalog is composed of the positions of 3974 stars based on the FK5 system. A possible local systematic error of the order of 0.1 arcsec of the FK5 catalog is evident in Tokyo PMC 86, suggesting an erroneous proper motion of the FK5 system in some parts of the sky. It is expected that the modern photoelectric meridian circles can reveal local systematic errors of fundamental catalogs (e.g. FK5), if they are greater than a few hundredth of arcsec. even from the observations of just one year duration. Reproduced in figure 2 are the systematic differences $\Delta\alpha_\delta \cos\delta$ (upper panel) and $\Delta\delta_\delta$ (lower panel) observed for the FK5 basic stars in 1986 with three modern photoelectric meridian circles (the same as figure 2 of the Tokyo PMC 86 catalog).

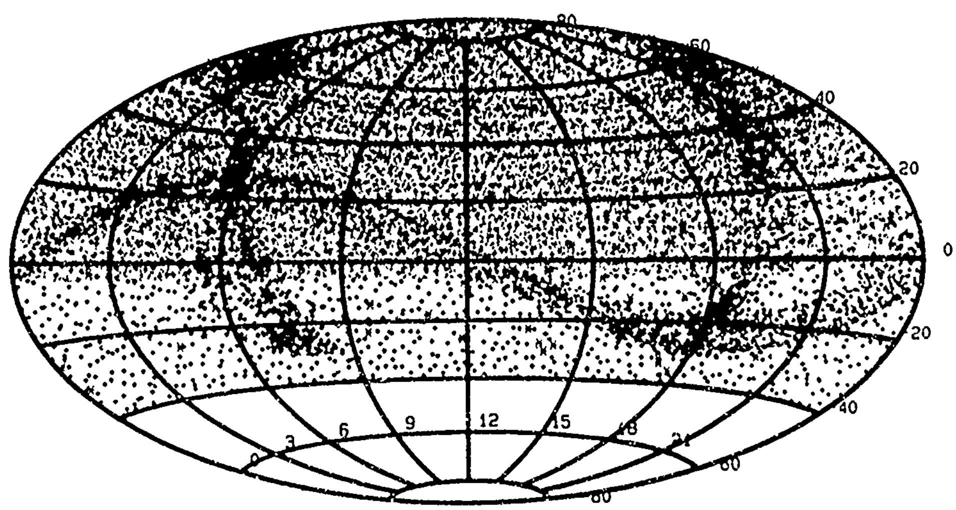

Fig. 1. The distributions of the First Tokyo PMC Program stars on the celestial sphere given in equatorial coordinates.

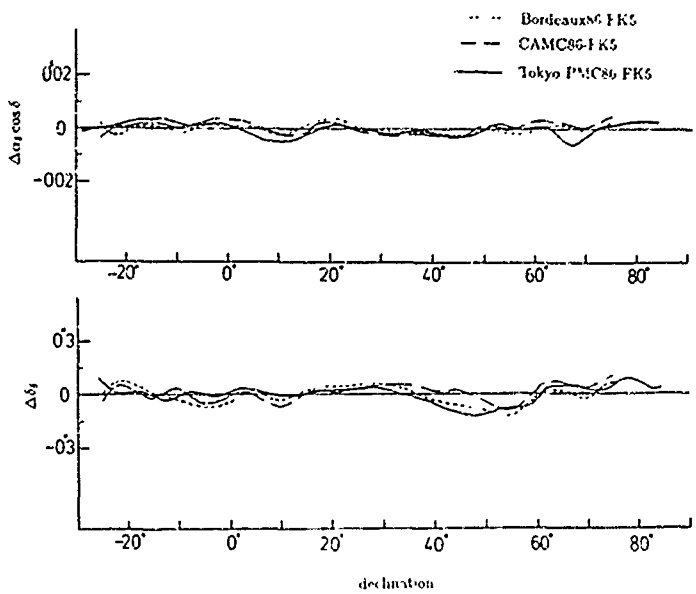

Fig 2 The systematic differences $\Delta\alpha_\delta \cos\delta$ (upper panel) and $\Delta\delta_\delta$ (lower panel) for the FK5 basic stars observed in 1986

The adjustments of the observed star positions in an annual catalog are made globally based on the FK5 system, providing that unknown orientation parameters of the telescope vary regularly with time over the period during which the each catalog was observed. We approximate the variation of the orientation parameters over one year by cubic spline functions with (s) nodes at $t = t_1, t_2, \ldots, t_s$. Then the observation equations relating the $(O-C)$ of a star to true catalog error are given by the followings:

$$\Delta\alpha\cos\delta = -\sum_{p=1}^{s}\{\Delta T_p\cos\delta + f_{culm}\Delta k_p \sin Z\}g_p(t) - f_{culm}(\Delta c_1 \sin 2Z + \Delta c_2 \cos 2Z) +$$

$$+\varepsilon_\alpha \cos\delta \text{ , and}$$

$$\Delta\delta = -f_{culm}\sum_{p=1}^{s}\{\Delta\varphi_p + \Delta R_p \tan Z\}g_p(t) - f_{culm}\Delta f_H \sin Z + \varepsilon_\delta$$

(See Yoshizawa and Suzuki 1989 for details.)
$\Delta T_p, \Delta k_p, \Delta\varphi_p$, and ΔR_p in the above equations denote, respectively, the values of ΔT (clock correction), Δk_{az} (azimuth of the artificial azimuth marks), $\Delta\varphi$ (correction to an assumed value of latitude), and ΔR (correction to refraction constant) at $t = t_p (p = 1, 2, \ldots, s)$. By using least squares methods we adjust all the observations made for FK5 basic stars in one year to get the most plausible values of $\Delta T_p, \Delta k_{az}, \Delta\varphi_p$, and $\Delta R_p (p = 1, 2, \cdots, s)$ together with three constants Δf_H(flexure correction), and Δc_1 and Δc_2(coefficients of the 2nd harmonics of collimation errors) of the year. The present annual catalog series are (and going to be) constructed with $s = 10$. Thus the total number of the fitted parameters in one annual catalog is 22 in R.A. system, and 21 in Decl. system.

3. Compilation of First Tokyo PMC Absolute Catalog

The annual catalogs for the observations made in 1987 and 1988 will soon appear in publications. They will contain the positions of about 5,000 stars based on FK5. In figure 3

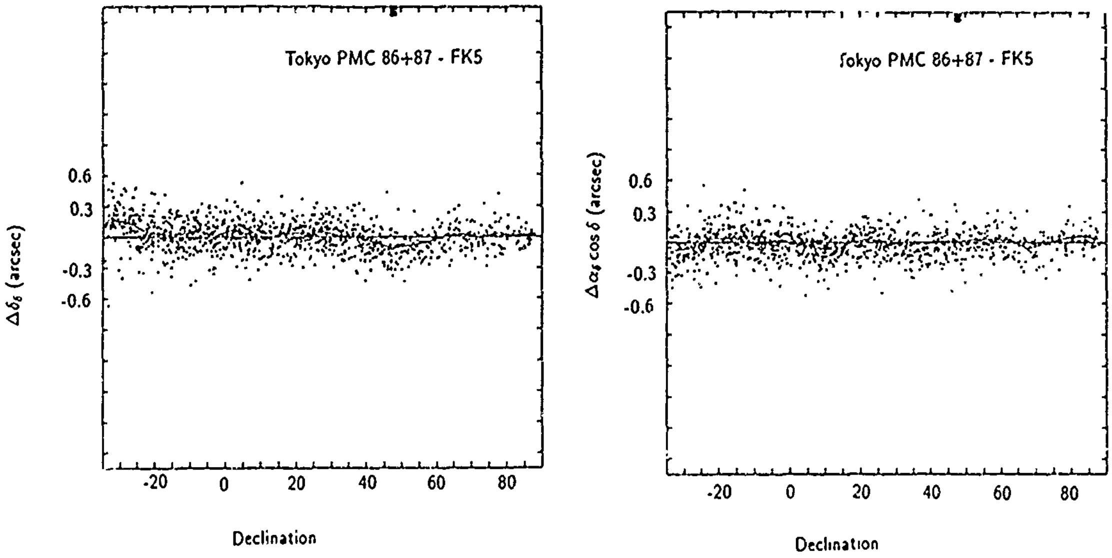

Fig. 3 Plotted are the observed (O-C)s of 1194 FK5 basic stars obtained through the global adjustment of all observations made in 1986 and 1987.

plotted are the observed $(O-C)$s of 1194 FK5 basic stars found from the observations made in 1986 and 1987, adjusting all the observations globally (cf. section 2.2).

The final goal of First Tokyo PMC Program is to provide absolute positions of the program stars that are consistent with the dynamical theory of the planetary system. The observations of the first program are to be finished by March, 1992. The compilation of the absolute Tokyo PMC catalog will begin in 1992.

4. A New CCD Micrometer of Tokyo PMC under Development

The development of a new CCD micrometer for Tokyo PMC is started. The so-called drift scanning method is the basic electrical architecture for detecting and accumulating the incoming photons from celestial objects. This architecture, if realized with a CCD chip of, say, 400x1000 pixels and Q.E. higher than 30%, enables us to achieve direct astrometric observations of faint objects up to 15th mag, e.g., some bright QSOs (or extragalactic compact objects), faint galactic stars, and faint minor planets. These faint objects are essential for connecting the optical and radio reference frames, and for the studies of the dynamical reference frame.

Now we have an experiment model of the drift scanning CCD micrometer, and have been testing it. The experiment model consists of a single-field CCD image sensor TH7883 (Thomson-CSF) cooled by liquid nitrogen down to around 200K, a clock-drive board, 16 bit ADC (Analog-to-Digital Converter), and an engineering work station to control the system The performance of the experiment model of the CCD micrometer is going to be tested in this winter through the observations of real stars by using *Gotier meridian circle* at Mitaka, Tokyo. Second Tokyo PMC program (1993-) will be observed with a CCD micrometer.

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IMPROVEMENT OF THE OPTICAL REFERENCE SYSTEM BY PHOTOGRAPHIC ASTROMETRY: FIRST RESULTS OF SIMULATIONS WITH GLOBAL BLOCK ADJUSTMENT METHODS

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ABSTRACT. Block adjustment (BA) simulations of an entire hemisphere show major improvements in individual as well as systematic accuracy of star positions obtained by photographic astrometry, independent of systematic errors of the reference star catalog used. Results for the accuracy estimates obtained from a patch-like plate pattern in the sky are not valid for closed plate pattern. The importance of BA methods for the realisation of a reference system is stressed.

1 Introduction

A review of the principles and appli ations of the simultaneous adjustment of overlapping plates, known as block adjustment (BA), has been published recently (Eichhorn 1988). The algorithm adopted here is based on that of Googe et.al. (1970); further details can be found elsewhere (Zacharias 1987).

This paper deals only with simulations, but the data structure of the CPC2 project (Nicholson et.al. 1984, Zacharias et.al. 1991) is widely used. Contrary to most other investigations of the accuracy obtainable with a BA, we investigate here a global pattern, i. e. a closed zone and an entire hemisphere, following Ebner (1970). Furthermore, typical systematic errors of the current FK5 based system (Morrison et.al. 1990), depending on the coordinates in the sky, are introduced to show their influence on the positions obtained from a BA solution.

2 Cape Zone Simulation

For a first investigation, the data structure of the Cape Zone of the CPC2 was used, with details listed in Table 1. The positions of that catalog are defined as error free (true) and then projected onto the corresponding plates. Random errors with normal distribution $N(\sigma, 0)$ are added to the x, y-coordinates as well as to the reference star positions according to Table 1.

Furthermore, a constant offset is added to the declinations of all reference stars within $-50° \leq \delta \leq -48°$, thus

$$\delta_{ref.cat} := \delta_{true} + N(\sigma_{ref}, 0) + 100mas$$

to simulate a systematic error in the reference star catalog.

Results of a BA with appropriate weights but no additional constraints on the 8 parameter plate model (full linear + plate tilt terms) are summarized in Table 2. Fig. 1 shows the mean of the declination differences (BA-true), taken over all stars along right ascension. The amplitude of the systematic error is reduced by about a factor of 3. Similar calculations with the Cape Zone, adding long periodic systematic waves along right ascension, show that these could not be removed by BA methods. Details can be found elsewhere (Zacharias 1987).

3 Global I

This first test covering an entire hemisphere with a 4-fold overlap pattern uses a large plate size to minimize CPU time. In addition to the plate model already mentioned, an orthogonal plate model was used as well. Details are given in Table 1 and 2. The following systematic error in declination was added to the reference star positions for those stars within the RA range of $6^h \leq \alpha \leq 12^h$:

$$\Delta\delta_{sys\ err} = 150mas \ \cos\,(4\ \delta)$$

The resulting declination differences (ref.cat.-true) and (BA-true) for the 8 and 4 parameter plate model are displyed in Fig. 2,3 and 4 respectively. Note the amplitude of the systematic error is reduced by a factor of about 3 and 6 respectively in the two BA solutions !

4 Global II

The full CPC2 data structure with 5695 plates is used in the Global II simulation. Similar to Global I, a systematic error was added to the reference star positions, now for those stars within the RA range of $6^h \leq \alpha \leq 9^h$:

$$\Delta\delta_{sys\ err} = 150mas \ \cos\,(8\ \delta)$$

For all reference stars within this RA range, the resulting differences (BA-true) and (ref cat.-true) are combined in Fig.5. The corresponding differences (BA-true) for all stars of this RA range are shown in Fig. 6. The amplitude of the systematic error is reduced by at least a factor of 10 in the BA solution, despite the small plate size and large number of plates used here. Note also the very low value for $\sigma(BA - true)$ which is even below $\sigma_{xy}/\sqrt{n}$ due to the powerful constraint set by the global closed pattern of plates (Ebner 1970).

5 Conclusions

A BA of a global closed 4-fold overlap pattern of plates gives a further major improvement in random and systematic accuracy of star positions as compared to BA results obtained from smaller unclosed patches of overlapping plates. Catalog accuracies below $\sigma_{xy}/\sqrt{n}$ are possible even with a large number of plates (many thousands).

We like to stress the potential of current photographic astrometry for the homogenisation of the present reference *system* on scales of about 20°, independent of transit circle observations. A complete and homogeneous catalog for all stars down to about 13^m could be established by these means. More details can be found elsewhere (de Vegt 1988).

Furthermore the BA results seem to be largely *independent* of the *plate size*. Thus a future dedicated astrometric telescope (de Vegt 1989) with only a 2° field but which will reach magnitude 18^m, could provide a direct link to the extragalactic reference frame, independent of present reference star catalogs (Clube 1968) which will be used only for the conventional single plate adjustment to provide starting values for the final BA.

Of course all results presented here are valid only when there are no systematic errors in the photographic data. Using modern techniques and a calibration with quasi error–free positions from a future HIPPARCOS catalog, it will be possible to meet these requirements to at least one micron level of precision.

6 Acknowledgements

The author thanks Prof. Dr. K. Hasselmann, director of the DKRZ at Hamburg, for allocation of computer time on the Cray-2S. The friendly assistance of the advisory team at the DKRZ is gratfully acknowledged. Comments and corrections on the draft paper by A.Murray and L.Morrison have been very helpful.

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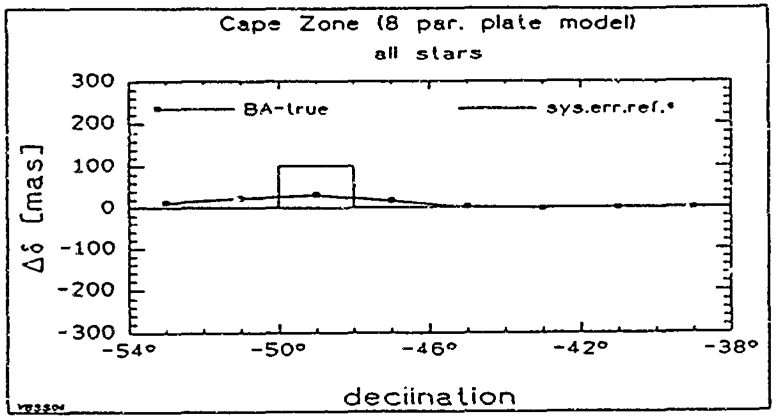

Fig.1: Simulation of a systematic error in the reference star catalog in the CPC2 Cape Zone and the corresponding BA result.

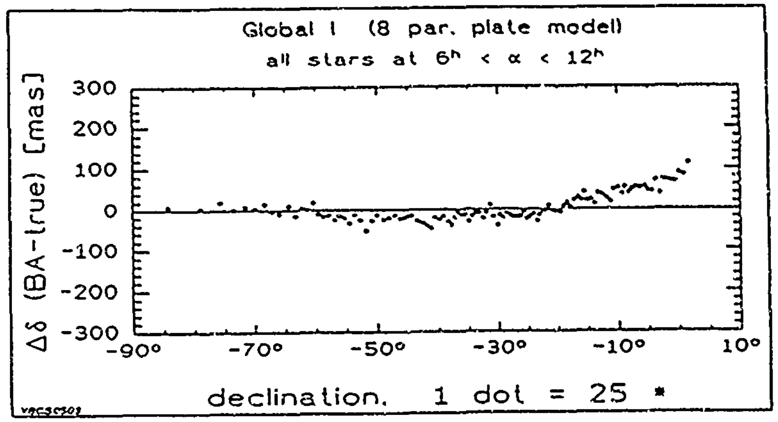

Fig.3: Results of the BA simulation Global I with an 8 parameter plate model for the same region in the sky as for Fig.2.

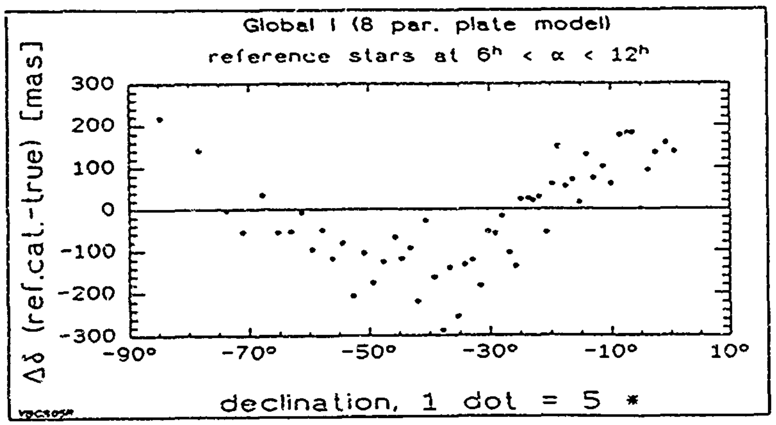

Fig.2: Declination error of reference stars as function of declination used in the simulation Global I.

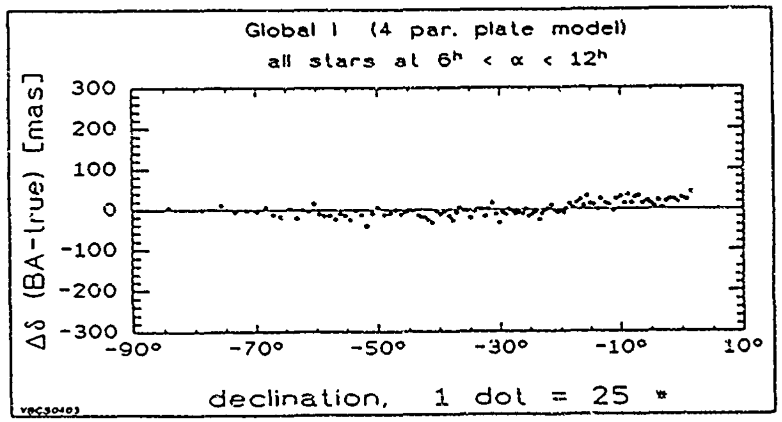

Fig.4: Similar to Fig.3 but with an orthogonal 4 parameter plate model.

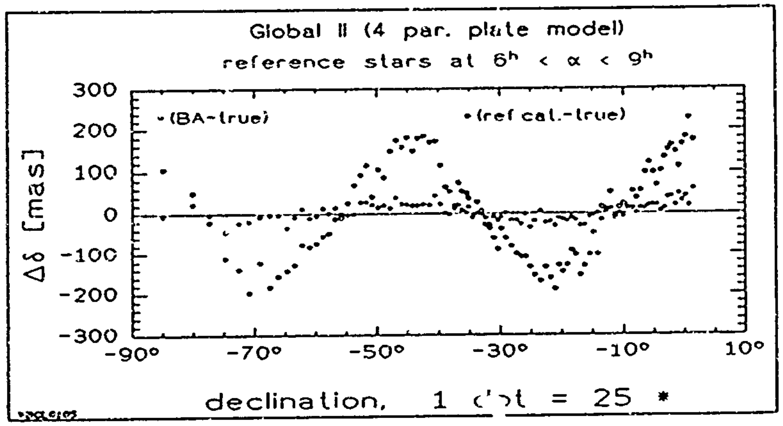

Fig.5: Declination error of reference stars as function of declination used in the simulation Global II. Superposed is the BA result of the same stars.

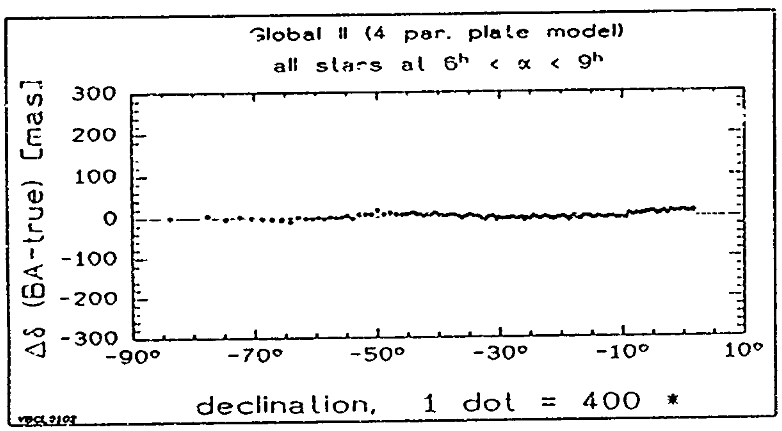

Fig.6: Results of the BA simulation Global II with a 4 parameter plate model for all stars in the same region in the sky as for Fig.5.

Table 1. Data structure of simulations

	Cape Zone	Global I	Global II
overlap pattern	reg. 4-fold	reg. 4-fold	reg. 4-fold
plate center	$-52° \leq \delta \leq -40°$	$-90° \leq \delta \leq 0°$	$-90° \leq \delta \leq 0°$
plate size	4°	20°	4°
n of plates	1008	265	5695
n of stars	51000	9209	276259
n of ref. stars	3300	920	18816
stars / plate	200	140	200
ref.stars / plate	13	14	13
σ_{ref}	136 mas	150 mas	150 mas
σ_{xy}	110 mas	120 mas	120 mas

Table 2: Results of BA simulations (units are mas)

	Cape Zone	Global I	Global I	Global II
n of plate const.	8	8	4	4
n of unknowns	8064	2120	1060	22780
differences (BA-true) for all stars:				
$\Delta\alpha\cos\delta$	−18 *	−3.9	−1.9	−0.1
$\Delta\delta$	11	3.8	3.2	−0.1
$\sigma(\Delta\alpha\cos\delta)$	51	62.2	59.5	55.5
$\sigma(\Delta\delta)$	53	66.1	61.0	55.4

*) NOTE: there are also system. errors added in right ascension, not dealt with here.

APPENDIX

IAU

Working Group on Reference Systems

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19 April 1991

Dr. Derek McNally
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Dear Dr. McNally,

As instructed by the Commission Resolution which founded the Working Group on Reference Systems (WGRS) at the XX General Assembly (GA) in Baltimore in 1988, I herewith forward to you the Recommendations of the WGRS. The recommendations, as presented, were drafted during IAU Colloquium No. 127, *Reference Systems*, which took place in October 1990 at Virginia Beach, USA.

After final editing by the Chairman of the WGRS, these recommendations were submitted to a vote by the members of the WGRS. A majority of the members voted in favor of submitting the recommendations to the IAU. Although in their present form the recommendations do represent a consensus, it should not be assumed that they are universally accepted. Legitimate and thoughtful objections have been put forward by members of the WGRS and others, regarding some of the recommendations. It is for this reason that the Joint Discussion (JD) on Reference Systems, to be held at the XXI GA in Buenos Aires in July 1991, is so important. From this JD will come the Resolutions to be presented to the GA for its consideration. The enclosed recommendations will serve as the starting point for the discussion. It is my personal hope that the resolutions, based in large part on the recommendations, will be sufficiently discerning and insightful so that their provisions may be incorporated into the work of those astronomers whose interests are addressed by the resolutions. This is, after all, the *raison d'être* of the WGRS.

Sincerely,

James A. Hughes

Encl: WGRS Recommendations

RECOMMENDATIONS OF THE

WORKING GROUP ON REFERENCE SYSTEMS

Drafted at IAU Colloquium 127, Virginia Beach, October 1990

RECOMMENDATION I

The Working Group on Reference Systems,

considering,

that it is appropriate to define several systems of space-time coordinates within the framework of the General Theory of Relativity,

recommends,

that the four space-time coordinates ($x^0 = ct$, x^1, x^2, x^3) be selected in such a way that in each coordinate system centered at the barycenter of any ensemble of masses, the squared interval ds^2 be expressed with the minimum degree of approximation in the form:

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{2U}{c^2}\right)(dx^0)^2 + \left(1 + \frac{2U}{c^2}\right)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right],$$

where c is the velocity of light, τ is proper time, and U is the sum of the gravitational potentials of the above mentioned ensemble of masses, and of a tidal potential generated by bodies external to the ensemble, the latter potential vanishing at the barycenter.

Notes for Recommendation I

1. This recommendation explicitly introduces The General Theory of Relativity as the theoretical background for the definition of the celestial space-time reference frame.

2. This recommendation recognizes that space-time cannot be described by a single coordinate system because a good choice of coordinate system may significantly facilitate the treatment of the problem at hand, and elucidate the meaning of the relevant physical events. Far from the space origin, the potential of the ensemble of masses to which the coordinate system pertains becomes negligible, while the potential of external bodies manifests itself only by tidal terms which vanish at the space origin.

3. The ds^2 as proposed gives only those terms required at the present level of observational accuracy. Higher order terms may be added as deemed necessary by users. If the IAU should find it generally necessary, more terms will be added. Such terms may be added without changing the rest of the recommendation.

4. The algebraic sign of the potential in the formula giving ds^2 is to be taken as positive.

5. At the level of approximation given in this recommendation, the tidal potential consists of all terms at least quadratic in the local space coordinates in the expansion of the Newtonian potential generated by external bodies.

RECOMMENDATION II

The Working Group on Reference Systems,

considering,

a) the need to define a barycentric coordinate system with spatial origin at the center of mass of the solar system and a geocentric coordinate system with spatial origin at the center of mass of the Earth, and the desirability of defining analogous coordinate systems for other planets and for the Moon,

b) that the coordinate systems should be related to the best realization of reference systems in space and time, and,

c) that the same physical units should be used in all coordinate systems,

recommends that,

1. the space coordinate grids with origins at the solar system barycenter and at the center of mass of the Earth show no global rotation with respect to a set of distant extragalactic objects,

2. the time coordinates be derived from a time scale realized by atomic clocks operating on the Earth,

3. the basic physical units of space-time in all coordinate systems be the second of the International System of Units (SI) for proper time, and the SI meter for proper length, connected to the SI second by the value of the velocity of light $c = 299792458\ ms^{-1}$.

Notes for Recommendation II

1. This recommendation gives the actual physical structures and quantities that will be used to establish the reference frame and time scales based upon the ideal definition of the system given by Recommendation I.

2. The kinematic constraint for the rate of rotation of both the geocentric and barycentric reference systems cannot be perfectly realized. It is assumed that the average rotation of a large number of extragalactic objects can be considered to represent the rotation of the universe which is assumed to be zero.

3. If the barycentric reference system as defined by this recommendation is used for studies of dynamics within the solar system, the kinematic effects of the galactic geodesic precession may have to be taken into account.

4. In addition, the kinematic constraint for the state of rotation of the geocentric reference system as defined by this recommendation implies that when the system is used for dynamics (e.g., motions of the Moon and earth satellites), the time dependent geodesic precession of the geocentric frame relative to the barycentric frame must be taken into account by introducing corresponding inertial terms into the equations of motion.

5. Astronomical constants and quantities are expressed in SI units without conversion factors depending upon the coordinate systems in which they are measured.

RECOMMENDATION III

The Working Group on Reference Systems,

considering,

the desirability of the standardization of the units and origins of coordinate times used in astronomy,

recommends that,

1. the ... of measurement of the coordinate times of all coordinate systems centered at the barycenters of ensembles of masses be chosen so that they are consistent with the proper unit of time, the SI second,

2. the reading of these coordinate times be 1977 January 1, $0^h\ 0^m\ 32\overset{s}{.}184$ exactly, on January 1, $0^h\ 0^m\ 0^s$ TAI exactly (JD = 2443144.5, TAI), at the geocenter,

3. coordinate times in coordinate systems having their spatial origins respectively at the center of mass of the Earth and at the solar system barycenter, and established in conformity with the above sections (1) and (2), be designated as Geocentric Coordinate Time (TCG) and Barycentric Coordinate Time (TCB).

Notes for Recommendation III

1. In the domain common to any two coordinate systems, the tensor transformation law applied to the metric tensor is valid without re-scaling the unit of time. Therefore, the various coordinate times under consideration exhibit secular variations. Recommendation 5 (1976) of IAU Commissions 4, 8 and 31, completed by Recommendation 5 (1979) of IAU Commissions 4, 19 and 31, stated that Terrestrial Dynamical Time (TDT) and Barycentric Dynamical Time (TDB) should differ only by periodic variations. Therefore, TDB and TCB differ in rate. The relationship between these time scales in seconds is given by:

$$TCB - TDB = L_B \times (JD - 2443144.5) \times 86400$$

The present estimate of the value of L_B is 1.550505×10^{-8} ($\pm 1 \times 10^{-14}$) (Fukushima et al., Celestial Mechanics, 38, 215, 1986).

2. The relation TCB - TCG involves a full 4-dimensional transformation:

$$TCB - TCG = c^{-2} \left[\int_{t_o}^{t} (v_e^2/2 + U_{ext}(x_e))dt + v_e \cdot (x - x_e)\right]$$

x_e and v_e denoting the barycentric position and velocity of the Earth's center of mass and **x** the barycentric position of the observer. The external potential U_{ext} is the Newtonian potential of all solar system bodies apart from the Earth. The external potential must be evaluated at the geocenter. In the integral, t = TCB and t_o is chosen to agree with the epoch of Note 3. As an approximation to TCB - TCG in seconds one might use:

$$TCB - TCG = L_C \times (JD - 2443144.5) \times 86400 + c^{-2}v_e \cdot (x - x_e) + P$$

Notes for Recommendation III (continued)

The present estimate of the value of L_C is 1.480813×10^{-8} ($\pm 1 \times 10^{-14}$) (Fukushima et al., Celestial Mechanics, 38, 215, 1986). It may be written as $[3GM/2c^2a] + \epsilon$ where G is the gravitational constant, M is the mass of the Sun, a is the mean heliocentric distance of the Earth, and ϵ is a very small term (of order 2×10^{-12}) arising from the average potential of the planets at the Earth. The quantity P represents the periodic terms which can be evaluated using the analytical formula by Hirayama *et al.*, ("Analytical Expression of TDB-TDT_0", in *Proceedings of the IAG Symposia*, IUGG XIX General Assembly, Vancouver, August 10-22, 1987). For observers on the surface of the Earth, the terms depending upon their terrestrial coordinates are diurnal, with a maximum amplitude of 2.1 μs.

3. The origins of coordinate times have been arbitrarily set so that these times all coincide with the Terrestrial Time (TT) of Recommendation IV at the geocenter on 1977 January 1, $0^h\ 0^m\ 0^s$ TAI. (See Note 3 of Recommendation IV.)

4. When realizations of TCB and TCG are needed, it is suggested that these realizations be designated by expressions such as TCB(xxx), where xxx indicates the source of the realized time scale (e.g., TAI) and the theory used for the transformation into TCB or TCG.

RECOMMENDATION IV

The Working Group on Reference Systems,

considering,

a) that the time scales used for dating events observed from the surface of the Earth and for terrestrial metrology should have as the unit of measurement the SI second, as realized by terrestrial time standards,

b) the definition of the International Atomic Time, TAI, approved by the 14th Conférence Générale des Poids et Mesures (1971) and completed by a declaration of the 9th session of the Comité Consultatif pour la Définition de la Seconde (1980),

recommends that,

1) the time reference for apparent geocentric ephemerides be Terrestrial Time, TT,

2) TT be a time scale differing from TCG of Recommendation III by a constant rate, the unit of measurement of TT being chosen so that it agrees with the SI second on the geoid,

3) at instant 1977 January 1, $0^h\ 0^m\ 0^s$ TAI exactly, TT have the reading 1977 January 1, $0^h\ 0^m\ 32^s\!.184$ exactly.

Notes for Recommendation IV

1. The basis of the measurement of time on the Earth is International Atomic Time (TAI) which is made available by the dissemination of corrections to be added to the readings of national time scales and clocks. The time scale TAI was defined by the 59th session of the Comité International des Poids et Mesures (1970) and approved by the 14th Conférence Générale des Poids et Mesures (1971) as a realized time scale. As the errors in the realization of TAI are not always negligible, it has been found necessary to define an ideal form of TAI, apart from the $32^s\!.184$ offset, now designated Terrestrial Time, TT.

2. In order to define TT it is necessary to define the coordinate system precisely, by the metric form, to which it belongs. To be consistent with the uncertainties of the frequency of the best standards, it is at present (1991) sufficient to use the relativistic metric given in Recommendation I.

3. For ensuring an approximate continuity with the previous time arguments of ephemerides, Ephemeris Time, ET, a time offset is introduced so that TT - TAI = $32^s\!.184$ exactly at 1977 January 1, 0^h TAI. This date corresponds to the implementation of a steering process of the TAI frequency, introduced so that the TAI unit of measurement remains in close agreement with the best realizations of the SI second on the geoid. TT can be considered as equivalent to TDT as defined by IAU Recommendation 5 (1976) of Commissions 4, 8 and 31, and Recommendation 5 (1979) of Commissions 4,19 and 31.

4. The divergence between TAI and TT is a consequence of the physical defects of atomic time standards. In the interval 1977-1990, in addition to the constant offset of $32^s\!.184$, the deviation probably remained within the approximate limits of $\pm\ 10\mu s$. It is expected to increase more slowly in the future as a consequence of improvements in time standards. In many cases, especially for the publication of ephemerides, this deviation is negligible. In such cases, it can be stated that the argument of the ephemerides is TAI + $32^s\!.184$.

Notes for Recommendation IV (continued)

5. Terrestrial Time differs from TCG of Recommendation III by a scaling factor, in seconds:

$$TCG - TT = L_G \times (JD - 2443144.5) \times 86400$$

The present estimate of the value of L_G is 6.969291×10^{-10} ($\pm 3 \times 10^{-16}$). The numerical value is derived from the latest estimate of gravitational potential on the geoid, $W = 62636860$ ($\pm$ 30) m^2/s^2 (Chovitz, Bulletin Geodesique, 62, 359, 1988). The two time scales are distinguished by different names to avoid scaling errors. The relationship between L_B and L_C of Recommendation III, notes 1 and 2, and L_G is, $L_B = L_C + L_G$.

6. The unit of measurement of TT is the SI second on the geoid. The usual multiples, such as the TT day of 86400 SI seconds on the geoid and the TT Julian century of 36525 TT days, can be used provided that the reference to TT be clearly indicated whenever ambiguity may arise. Corresponding time intervals of TAI are in agreement with the TT intervals within the uncertainties of the primary atomic standards (e.g., within $\pm 2 \times 10^{-14}$ in relative value during 1990).

7. Markers of the TT scale can follow any date system based upon the second, e.g., the usual calendar date or the Julian Date, provided that the reference to TT be clearly indicated whenever ambiguity may arise.

8. It is suggested that realizations of TT be designated by TT(xxx) where xxx is an identifier. In most cases a convenient approximation is:

$$TT(TAI) = TAI + 32^{s}.184$$

However, in some applications it may be advantageous to use other realizations. The BIPM, for example, has issued time scales such as TT(BIPM90).

RECOMMENDATION V

The Working Group on Reference Systems,

considering,

that important work has already been performed using Barycentric Dynamical Time (TDB), defined by IAU Recommendation 5 (1976) of IAU Commissions 4, 8 and 31, and Recommendation 5 (1979) of IAU Commissions 4, 19 and 31,

recognizes,

that where discontinuity with previous work is deemed to be undesirable, TDB may still be used.

Note to Recommendation V

Some astronomical constants and quantities have different numerical values depending upon the use of TDB or TCG. When giving these values, the time scale used must be specified.

RECOMMENDATION VI

The Working Group on Reference Systems,

considering,

the desirability of implementing a conventional celestial barycentric reference system based upon the observed positions of extragalactic objects, and,

noting,

the existence of tentative reference frames constructed by various institutions and combined by the International Earth Rotation Service (IERS) into a frame used for Earth rotation series,

recommends,

1. that intercomparisonss of these frames be extensively made in order to assess their systematic differences and accuracy,

2. that an IAU Working Group consisting of members of Commissions 4, 8, 19, 24 and 31, the IERS, and other pertinent experts, in consultation with all the institutions producing catalogs of extragalactic radio sources, establish a list of candidates for primary sources defining the new conventional reference frame, together with a list of secondary sources that may later be added to or replace some of the primary sources, and,

requests,

1. that such a list be presented to the XXII General Assembly (1994) as a part of the definition of a new conventional reference system,

2. that the objects in this list be systematically observed by all VLBI and other appropriate astrometric programs.

Note for Recommendation VI

This recommendation essentially describes the first part of the work that must be done to prepare the realization of the reference system defined by Recommendations I and II. The choice of objects must be made in the first place by considering their observability by VLBI, but special care should be taken to include a large proportion of extragalactic radio sources with well identified optical counterparts.

RECOMMENDATION VIII

The Working Group on Reference Systems,

recognizing,

a) the importance to astronomy of adopting conventional values of astronomical and physical constants,

b) that values of these constants should be unchanged unless they differ significantly from their latest estimates,

c) that estimates of these constants should be improved frequently to represent the current status of knowledge,

d) the necessity of providing standard procedures using these numerical values, and,

noting,

a) that the MERIT Standards and IERS Standards have contributed significantly to the progress of astronomy and geodesy,

b) that numerical values in these standards have served as a system of constants in analyzing observations of high quality, and

considering,

that procedures in these standards do not cover the whole of fundamental astronomy,

recommends,

that a permanent working group be organized by Commissions 4, 8, 19, 24 and 31, in consultation with the IAG and the IERS, in order to update and improve the system of astronomical units and constants, the list of estimates of fundamental astronomical quantities and standard procedures; this group shall:

1. prepare a draft report on the system of astronomical units and constants at least six months before the XXII General Assembly (1994),

2. prepare a draft list of best estimates of astronomical quantities at least six months before each following General Assembly,

3. prepare, at least six months before each following General Assembly, a draft report on standard procedures needed in fundamental astronomy, which,

    a) should have a maximum degree of compatibility with the IERS Standards,

    b) should include the implementations of procedures in the form of tested software and/or test cases,

    c) should be available not only in written form, but also in machine-readable form,

4. prepare a draft report on possible electronic access to these units, constants, quantities and procedures at least six months before the XXII General Assembly (1994).

Notes for Recommendation VII (continued)

3. The dynamical equinox in this recommendation is defined as the intersection of the mean equator and the ecliptic. The latter is defined as the uniformly rotating plane of the orbit of the Earth-Moon barycenter averaged over the entire period for which the ephemerides are valid. Since it is ephemeris dependent, the choice of the equinoctial point will be made using the most accurate and generally available ephemerides of the solar system at the time.

4. The definition given to the reference system by Recommendations I and II implies the stability in time of the system of coordinates realized by the celestial reference frame. The directions of the coordinate axes should not be changed even if at some later date the realization of the dynamical equinox or CEP are improved. Similarly, modifications to the set of extragalactic objects realizing the reference system should be made in such a way that the directions of the axes are not changed. This means that once the coordinate axes have been specified, in the way described in the last part of the recommendation, the connection between the definition of the conventional reference system and the peculiarities of the Earth's kinematics will have been severed.

5. As long as the relationship between the optical and the extragalactic radio frames is not sufficiently accurately determined, the FK5 catalog shall be considered as a provisional realization of the celestial reference system in optical wavelengths.

RECOMMENDATION IX

The Working Group on Reference Systems,

recognizing,

that a generally accepted non-rigid Earth theory of nutation, including all known effects at the one tenth milliarcsecond level, is not yet available,

recommends,

1. that those satisfied with accuracy of the nutation angles (ϵ or ψ sinϵ_0) numerically greater than ± 0."002 (one sigma rms) may continue to use the 1980 IAU Nutation Theory (P.K. Seidelmann, Celestial Mechanics, 27, 79, 1982),

2. that those requiring values of the nutation angles more accurate than ± 0."002 (one sigma rms) should make use of the Bulletins of the IERS which publish observations and predictions of the celestial pole offsets accurate to about ± 0."0006 (one sigma rms) for a period of up to six months in advance,

3. that the IUGG be encouraged to develop and adopt an appropriate Earth model to be used as the basis for a new IAU Theory of Nutation.